INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

17 April 2019 (pm)

Subject SP6 – Financial Derivatives Specialist Principles

Time allowed: Three hours and fifteen minutes

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all questions, begin your answer to each question on a new page.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 An investor holds 100 shares in a company. The current share price is 46.25 per share. There are two traded option series which expire in nine months. The deltas and gammas of the option series are given below:

	Strike = 40		***Strike = 50***	
	Call	*Put*	*Call*	*Put*
Delta	0.8018	–0.1982	0.4276	–0.5724
Gamma	0.0278	0.0278	0.0392	0.0392

Each option is written on the same share as the investor holds. Throughout this question you can assume that any expenses and transaction costs can be ignored.

(i) Determine how the investor can make the portfolio delta and gamma neutral by calculating the number of call options to hold, to the nearest integers. [3]

(ii) Determine how the investor can make the portfolio delta and gamma neutral by calculating the number of put options to hold, to the nearest integers. [3]

(iii) Contrast the effectiveness of the delta and gamma neutral hedges in parts (i) and (ii) when there is a large fall in the share price. [2]

(iv) Show that the put portfolio is less effective than the call portfolio when the share price is less than 15.6 (rounded to one decimal place), assuming no rebalancing to the portfolios. [2]
Hint: you may ignore very small time values of these options.

[Total 10]

2 (i) Describe the similarities and differences between the volatility and the implied volatility of an asset price. [3]

The central bank of a country monitors the price of financial instruments and uses the information when setting policy.

(ii) Explain how and why the central bank might use the derivatives markets to create a risk-neutral distribution of future asset prices. [3]

The central bank generates an implied volatility surface for a given asset on a given day. The central bank uses this implied volatility surface to generate a risk-neutral distribution and a real-world distribution for the future price of the asset.

(iii) Assess the relevance to the central bank of a real-world and a risk-neutral distribution for the future price of the asset. [3]

[Total 9]

3 The risk department of a large financial institution is developing a Value at Risk ("VaR") model using the Historical simulation method, focusing on the 99% one-day VaR. The financial institution holds a wide range of complex derivatives including equity options, interest rate swaps and swaptions.

(i) State the key steps in the Historical simulation method to calculate the 99% one-day VaR. [3]

(ii) Explain the weaknesses of the Historical simulation approach for calculating VaR. [3]

(iii) List additional analyses, other than VaR metrics, that could help the risk department understand the market risks in the derivative portfolio. [2]

The institution's derivative traders are concerned that the first draft of VaR figures overstate the true market risk. In a one-day period they would expect to undertake "management actions" to mitigate falls in asset prices. These management actions include closing out positions, exercising embedded options in the derivatives and executing new hedges.

(iv) Propose, with reasons, the items the risk department should consider before allowing for the benefit of management actions in the one-day 99% VaR figure. [4]

[Total 12]

4 As part of a new solvency regime, a regulator is determining the risk-free yield curve which will be used by financial institutions to value their liabilities. Historically, institutions have used the continuously compounded LIBOR spot curve, but the regulator is now proposing to use the continuously compounded LIBOR spot curve less a "Credit Risk Adjustment" (CRA) which aims to reflect the component of LIBOR rates that is due to credit risk. The CRA will be expressed in basis points (bps) as a constant deduction at all terms and will be calibrated by the regulator each year in advance.

(i) Describe why credit risk is a component of LIBOR rates. [2]

(ii) Suggest one method which the regulator could use to calibrate the size of the CRA. [1]

(iii) Show that introducing the CRA reduces all LIBOR continuously compounded forward rates used by financial institutions by an amount equal to the CRA. [2]

(iv) Justify why the result in part (iii) is desirable from a regulatory perspective. [2]

The regulator has calculated the CRA to be 20bps. A Financial Institution (FI) seeks to understand how the surplus and matching position will change following the introduction of the CRA. The FI invests in a portfolio of swaps and cash. Swaps will always be valued using the LIBOR curve. The FI defines delta as the impact of a 1bp fall in interest rates. The table below shows values of the liabilities and assets under different interest rate curves:

	All figures in £m		
Curve	*Swaps*	*Cash*	*Liabilities*
LIBOR	100	900	1,000
LIBOR – 0.01%	110	900	1,010
LIBOR – 0.10%	220	900	1,120
LIBOR – 0.20%	380	900	1,280
LIBOR – 0.21%	398	900	1,298

(v) Demonstrate that the Financial Institution is both delta and gamma hedged prior to the CRA being introduced. [2]

(vi) Calculate the deficit that will emerge following the introduction of the CRA. [1]

(vii) Calculate the difference in delta between the assets and liabilities that will emerge once the CRA is introduced. [2]

The FI has a risk management policy of matching the value and sensitivity of the regulatory value of its liabilities.

(viii) Assess the extent to which the current risk management policy will mitigate the impact of the introduction of the CRA based on your answers to (v), (vi) and (vii). [3]

[Total 15]

5 A six-month American put option has been written on a stock with a known dividend amount and ex-dividend date.

(i) Describe how the binomial model can be used as a numerical method for valuing this stock option, by comparing it to valuing the same stock option without a dividend payment. [4]

In order to value this option, a three-step binomial tree is used with a time interval of two months. The ex-dividend date is in three months with a dividend of 0.55. The current stock price is 100 with a strike price of 95. The risk-free interest rate is 3% p.a. continuously compounded. The implied volatility that will be used is 29% p.a.

(ii) Construct a recombining stock price tree which can be used for valuing the option. You are required to calculate the node values, and state any assumptions made. [5]

(iii) Show that the price of the option at time 0 is 5.89. [5]

(iv) Set out how the method used in part (i) might change if the time to expiry of this option is changed to five years. [2]

[Total 16]

6 A futures contract exchange is introducing a temperature future for the first time. The payoff of one contract at expiry in \$m in one year will be defined as:

$$Payoff = T - K$$

where K is the delivery price and T is the temperature in Celsius on maturity of the contract. Risk-free rates are assumed to be 0% p.a. at all terms.

(i) Describe the further items that the exchange would need to specify in the temperature futures contract. [3]

(ii) Define the cost of carry for an investment asset. [1]

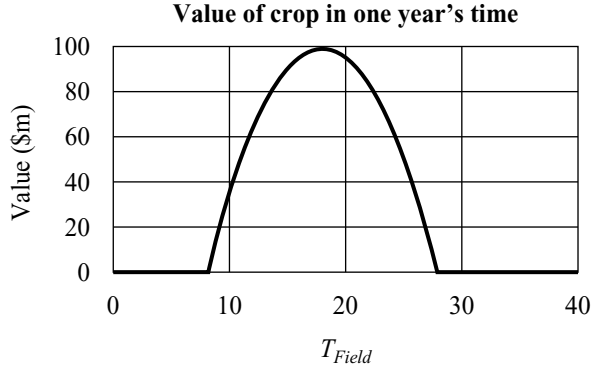
(iii) Define the convenience yield for a consumption asset. [1]

(iv) Comment on the challenges of using a cost of carry and convenience yield to derive arbitrage-free pricing for the temperature future. [3]

A farmer wishes to use the temperature future to dynamically delta hedge the impact of temperature on the value of his crop which will be harvested in one year's time. The farmer models the value of his crop as:

$$Value(\$m) = Max\left[0, 100 - (T_{Field} - 18)^2\right]$$

where T_{Field} is the temperature of the field in Celsius in which the crop is being grown on the day the crop is harvested. The value of the farmer's crop can therefore be shown graphically as follows:



(v) Explain the sources of the basis risk between the temperature future and the modelled value of the farmer's crop. [2]

(vi) Determine the delta of the modelled value of the farmer's crop with respect to T_{Field} for all T_{Field}. [3]

(vii) Explain the challenges in dynamically delta hedging the value of the farmer's crop using the temperature future. You should assume the delta of the temperature futures contract is \$1m with respect to T_{Field}. [4]

(viii) Suggest ways the exchange could restructure the payoff of the contract to make it more attractive to the farmer and other institutions. [2]

[Total 19]

7 A central counterparty (CCP) is setting the initial margin requirements for interest rate derivatives by modelling the behaviour of interest rates in the future. It wishes to carry out scenario analysis. The CCP proposes to use the Vasicek model for interest rates.

(i) Describe why the CCP should carry out both real-world and risk-neutral projections of interest rates. [2]

(ii) Write down the stochastic differential equation (SDE) of the Vasicek model in the real world, defining all terms used. [2]

(iii) Describe how the CCP could calibrate the parameters of the real world SDE in part (ii). [2]

(iv) Define the market price of risk for an interest rate derivative f, including equations and defining all terms where appropriate. [2]

(v) Show how the risk-neutral SDE for the Vasicek model can be derived from the real-world SDE for the Vasicek model and the market price of risk. [2]

(vi) Explain why the Hull-White two-factor model may be more appropriate than the Vasicek model for the CCP's purpose. [4]

The CCP has derived the following levels of initial margin for both payer and receiver swaps for all counterparties. All initial margin will be cash only.

Term outstanding on swap	*Level of initial margin required for payer and receiver swaps*
Less than 5 years	1% of notional
More than 5 years	2% of notional

(vii) Assess how effective these initial margin requirements will be for mitigating counterparty risk to the CCP from a counterparty with a range of payer and receiver swaps. [5]

[Total 19]

END OF PAPER