INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2019 Examinations

Subject SP6 - Financial Derivatives Principles

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision

Mike Hammer
Chair of the Board of Examiners
July 2019

A. General comments on the *aims of this subject and how it is marked*

1. The aim of Financial Derivatives Principles (SP6) is to develop a student's ability to understand different types of financial derivatives and their uses, the markets in which they are traded, methods of valuation of financial derivatives, and the assessment and management of risks associated with a portfolio of derivatives. It builds on material covered in earlier subjects, particularly Loss Reserving and Financial Engineering (CM2).

2. Candidates are reminded to ensure that their answers are sufficiently detailed to demonstrate understanding, as there were instances where inadequate explanations led to candidates scoring less well on questions than they might have done. The model solutions are intended to reflect the level of detail that a high scoring candidate might be able to produce. For many questions there are more marks available than the question requires to achieve full marks. This reflects that the examiners will give credit for valid alternative solutions, particularly in questions focussed on higher level skills.

3. Candidates who give well-reasoned points, not in the marking schedule, are awarded marks for doing so.

B. Comments on *student performance in this diet of the examination*

This paper was reasonably well answered by better prepared candidates.

Candidates in general demonstrated a good grasp of Core Reading and were able to apply this knowledge in familiar situations. The better attempted questions were questions 3, 4, 6 and 7.

Candidates overall scored less well on questions 1, 2 and 5, in particular where more detailed application skills were being assessed or in applying theoretical principles to an unfamiliar scenario. In an attempt to help future candidates we note below areas where candidates could have improved their marks:

Question 1: Few candidates managed to clearly demonstrate the differences between the two hedges in part (iii) and (iv). Marks could have been improved by stating obvious points, such as what would happen to the option value, delta and gamma in the scenario described.

Question 2: Only the strongest candidates were able to provide sufficient salient points to address part (ii) and (iii). Marks could have been improved linking the differences between risk-neutral and real-world to the central bank.

Question 5: While most candidates showed a good understanding of binomial tree principles, in general marks could have been improved by correctly incorporating the dividend impact. Candidates who were able to correctly follow through with their calculations were able to score highly with their attempts, even if the exact answer in section (iii) was not always found, many marks were available for each step.

Pass Mark

The Pass Mark for this exam was 60

Solutions for Subject SP6- April 2019

Q1

(i) The delta of the stock is 1 … [0.5]
…and the gamma is 0. [0.5]

Let N be the number of strike price 40 call options in the portfolio and M be the number of strike price 50 call options in the portfolio. In order for the new portfolio to be delta and gamma neutral the following equations are satisfied:

$$100\times 1+N\times 0.8018+M\times 0.4276=0,$$ [1]
$$100\times 0+N\times 0.0278+M\times 0.0392=0.$$ [1]

These can be solved simultaneously to give to the nearest integer:

$N=-201,$

$M=142.$

This means that the investor must sell 201 of the call options with strike price 40 and purchase 142 of the call options with strike price 50. [1]

(Markers: Alternative working, for example using put-call parity, should be given credit to.)

[Max 3]

(ii) Let Q be the number of strike price 40 put options in the portfolio and R be the number of strike price 50 put options in the portfolio. In order for the new portfolio to be delta and gamma neutral the following equations are satisfied:

$$100\times 1-Q\times 0.1981-R\times 0.5724=0,$$ [1]
$$100\times 0+Q\times 0.0278+R\times 0.0392=0.$$ [1]

These can be solved simultaneously to give to the nearest integer:

$Q=-481,$

$R=341.$

This means that the investor must sell 481 of the put options with strike price 40 and purchase 341 of the put options with strike price 50. [1]

(Markers: Alternative working, for example using put-call parity, should be given credit to.)

[Max 3]

(iii) You can test the effectiveness of the hedge by considering how much the portfolio loses when there are large changes in the share price. [0.5]

The call portfolio in part (i) has limited downside protection for large falls in the share price, as both sets of call options will have only a small time value. The portfolio value will essentially be the value of shares, as the expenses of setting-up the hedge are to be ignored. [1]

The put option portfolio requires significantly more put options to be written at 40 than bought at 50. As a result when there is a large fall in the share price the put options are heavily in-the-money and have a net value close to their intrinsic value. [1]

For very large moves the call options become ineffective as a hedge, whereas the put options even become risk-additive. [1]

[Max 2]

(iv) The value of the puts are approximately:

$(40 - S(t)) \times (-481) + (50 - S(t)) \times 341 = -2{,}190 + 140S(t).$ [0.5]

Here $S(t)$ is the share price. The portfolio value is approximately:

$-2{,}190 + 140S(t) + 100S(t) = -2{,}190 + 240S(t).$ [0.5]

Comparing the value of the call portfolio and the put portfolio results in the put portfolio being less effective than the call portfolio when:

$100S(t) > -2{,}190 + 240S(t),$ as the call options are approximately worthless. [0.5]

Therefore when $S(t) < 15.6$ (rounded to one decimal place),…

… the put portfolio is less effective than the call portfolio. [0.5]

[Max 2]

[Total 10]

Candidates overall scored less well on this question. Few candidates managed to clearly demonstrate the differences between the two hedges in part (iii) and (iv). Marks could have been improved by stating obvious points, such as what would happen to the option value, delta and gamma in the scenario described.

Q2

(i) Volatility and implied volatility are both measures of uncertainty of the asset returns over time. [1]

Neither volatility nor implied volatility can be directly observed in the financial markets. [0.5]

Both have to be estimated from information contained in the financial markets. [0.5]

Volatility is usually estimated based on historical asset prices, whereas … [0.5]

… implied volatility is estimated from option prices (on the underlying asset) observed in the market. [0.5]

This implies that there needs to be (liquid) options traded with the asset as the underlying in order to be able to estimate an implied volatility. [0.5]

Further, implied volatilities can only be estimated for a subset of assets due to the limited number of assets which have options traded on them. [0.5]

An option pricing model is required for estimating implied volatility, and this is not required for estimating volatility. [0.5]

Volatility is a historical measure while implied volatility is the market price of future volatility when using a particular option model. [0.5]

Implied volatility can produce a volatility surface of values (where volatility varies with time and underlying asset price)… [0.5]

… but the volatility estimated form historical data will often produce a single (or a small set) of values. [0.5]

Volatility (both implied and historic) has been criticised as a risk measure as it has proven to be non-predictive of the range of future underlying price movements ahead of periods of market turmoil. [1]

[Max 3]

(ii) Derivative markets, and option markets in particular, can provide information to enable implied volatilities to be estimated. [0.5]

This enables the central bank to have a better understanding of possible changes to future asset prices. [0.5]

That is, the implied volatility, along with current asset prices, to create a model, or distribution, of future asset prices. [0.5]

By creating models, or distributions, for various asset prices it can assess the amount of uncertainty around future asset prices and hence the economic outlook or market sentiment. [1]

The models can be updated each trading day to provide 'real-time' information on economic outlook. [0.5]

[Max 3]

(iii) The risk-neutral asset distribution does not provide a true indication of the asset distribution in the real world. [0.5]

It only provides an indication of the future asset price if future investors in the asset market are risk-neutral… [0.5]

… and not investors uncertainty around the asset's future pay-offs when valuing the asset. [0.5]

In the real-world, investors will have a view on the risk of the asset's future value. [0.5]

This results in a different aggregate view of the future asset distribution for the investors. [0.5]

The bank could use the risk-neutral distribution to assess the price of options on the underlying asset. [0.5]

If the bank is able to determine a real-world asset distribution then possible advantages over the risk-neutral asset distribution include:

- It can provide the central bank with an insight into the investors' actual view on future asset prices, … [0.5]
  … which may help in understanding the impact on various economic/financial variables in setting policy. [0.5]

- A comparison between the two distributions may provide information about how the investors' risk preferences are affecting derivative prices (and hence those markets). [1]
  For example, a term structure of the equity risk premium or credit spread could be derived. [0.5]
  This comparison could be used over a period of time to track changes in risk preference. [0.5]

- The real-world asset distribution can be used in looking at a real-world projection of an asset value, [0.5]
  … which can be directly compared to other information the central bank may have (e.g. inflation or GDP). [1]
  [Max 3]

[Total 9]

Candidates overall scored less well on this question. Only the strongest candidates were able to provide sufficient salient points to address part (ii) and (iii). Marks could have been improved linking the differences between risk-neutral and real-world to the central bank.

Q3

(i) Historical time series data of daily market values is compiled over an observation period of at least one year… [0.5]
…using either exact prices… [0.5]
… or "proxy" values for those instruments that cannot be valued precisely [0.5]
The dataset is chosen to enable the entire portfolio to be recalculated on each of the days in the observation period. [0.5]
Then the profit/loss is obtained for each scenario as a change from the previous day across the entire dataset. [0.5]
Alternatively, profit/loss each day can be calculated by Taylor expansions of risk sensitivities (delta and gamma). [0.5]
The resulting profit/loss "vectors" are ordered from greatest loss to greatest profit. [0.5]
Select the VaR from the cell which contains the 99th percentile [0.5]
The profit/loss "vectors" can be calculated for individual assets and asset classes as well as the whole portfolio. [0.5]

[Unit 16, 1.1.2]
[Max 3]

(ii) The premise of the Historical simulation is that history will repeat itself in a similar pattern of losses… [0.5]
… but there may have been substantial changes which mean history is a poor guide to the future. [1]
The VaR figures are heavily influenced by the length and type of period chosen… [0.5]
… and if the history includes a significant stress event… [0.5]
… or if it includes changes in financial regimes or legislation. [0.5]
The use of a constant lookback window can lead to spikes in the VaR figure as stress events drop out of or enter the lookback period. [1]

Choosing the right length of observation period can be subjective and it is generally subject to regulatory scrutiny. This will require additional work in justifying this assumption. [0.5]

There can be substantial challenges in re-calculating the historical values of certain instruments…. [0.5]
… such as those recently purchased or issued. [0.5]
… and complex derivatives. [0.5]

There can be challenges in getting good data [0.5]
... such as availability, cost, time consuming, subjective [0.5]

Historical simulation cannot accurately account for any event that might occur in future but has not happened in the sample period. [0.5]

(Markers: please award markets for other reasonable answers)
[Unit 16, 1.1.2]
[Max 3]

(iii) The risk function should analyse market risk sensitivities to the various main market parameters such as … [0.5]

- Delta
- Gamma
- Vega
- Theta
- Rho
- Basis Risk [0.5 each]

Scenario tests [0.5]
Stress tests [0.5]
Conditional VaR, Expected Shortfall and Tail Var [max 0.5]
Volatility [0.5]
Variance [0.5]

(Markers: please award marks for other reasonable answers)
[Max 2]

(iv) The risk department should consider whether the management actions can be demonstrably executable in stressed market conditions… [1]
… in a period of 1 day. [0.5]

The risk department may also want to test how the impact of the management actions has been quantified. [0.5]

The risk department might want independent advice [0.5]
... or test the management actions for robustness [0.5]

Particular consideration should be given to…
… amount of liquidity that will be available during a 1in100 stress… [0.5]
… especially given the recent history of the financial crisis. [0.5]
…whether the infrastructure to execute the actions is in place… [0.5]
… for example ISDA/CSAs with new counterparties. [0.5]
... having the required systems [0.5]
... and operational processes [0.5]
… the time of day that the management action needs to be completed by… [0.5]
… as during the 1 day period markets may move unfavourably… [0.5]
… the costs of implementing each management action… [0.5]
… any knock-on impacts / risks need to be fully allowed for… [0.5]
… for example any increased counterparty risk… [0.5]
… whether the "management actions" are consistent with past practice… [0.5]
… and consistent with each other… [0.5]
… and compliant with any internal policies and regulation. [0.5]

(Markers: please award credit for other appropriate suggestions)
[Max 4]

[Total 12]

This was one of the better attempted questions. While part (i) to (iii) were generally well answered as bookwork questions, few candidates were able to provide a sufficient range of relevant arguments from risk management perspective.

Q4

(i) LIBOR reflects the rate at which banks borrow from each other… [1]
… on an unsecured basis (i.e. no exchange of collateral). [0.5]
As banks are not risk free, a component of LIBOR will therefore relate to credit risk. [1]
This component will be commensurate with the maturity of the LIBOR rate. [0.5]
[Unit 5, 2.2, 2.3]
[Max 2]

(ii) The regulator could assess the long term / instantaneous difference between LIBOR rates and an alternative risk free measure (e.g. government bond yield curves or OIS [1]
The regulator could analyse the credit ratings /historical level of defaults of banks that quote LIBOR. [1]
(Please award credit for any appropriate suggestion)
[Max 1]

(iii) The formula for a continuously compounded forward interest rate RF between times T1 and T2 is:

$$R_F = \frac{R_2T_2 - R_1T_1}{T_2 - T_1}$$ [0.5]

Now, consider the continuously compounded forward rate post introduction of the CRA, RF_CRA:

$$R_{F\_CRA} = \frac{(R_2 - CRA)T_2 - (R_1 - CRA)T_1}{T_2 - T_1}$$ [0.5]

$$R_{F\_CRA} = \frac{R_2T_2 - R_1T_1}{T_2 - T_1} - CRA\frac{T_2 - T_1}{T_2 - T_1}$$ [0.5]

$$R_{F\_CRA} = R_F - CRA$$ as required [0.5]

[Sub-total 2]

(iv) This means that the introduction of the CRA will not distort the shape of the forward rates of the underlying LIBOR curve … [1]

… and so is unlikely to significantly affect institutions' behaviour… [0.5]
…such as the way they carry out interest rate risk management (e.g. hedging)... [0.5]
… except that the overall value of the liabilities will be affected by the parallel shift in interest rates. [0.5]

A parallel shift in forward rates may also be straightforward for institutions to implement. [0.5]
[Max 2]

(v) Delta(Assets) = £110m-£100m = £10m = £1,010m-£1,000m = £10m = Delta(Liabilities) [1]
The position will be gamma hedged if it is delta hedged and the movement in surplus following a larger shock is zero. [0.5]
At LIBOR-10bps, we can see that the impact on surplus is (£220m-£100m) - (£1,120m - £1,000m) = £0m [1]
[Note to markers: please award equivalent credit for candidates showing that gamma hedging is satisfied at LIBOR-20bps, or that it is satisfied by showing that delta = zero at two points on the curve (LIBOR and LIBOR - 20bps)]
[Max 2]

(vi) Prior to the CRA, the surplus is £900m+£100m-£1,000m = £0m [0.5]
Therefore the deficit that emerges is £1,280m - £1,000m = £280m [0.5]
... as the asset value does not change following the introduction of the CRA [0.5]
[Max 1]

(vii) Define Delta as the impact of a 1bps change in interest rates. [0.5]
Delta (Assets) = £10m (as before) [0.5]
Delta (Liabilities) = £1,298m-£1,280m = £18m [0.5]
Hence Delta mismatch is £18m - £10m = £8m. [1]
[Max 2]

(viii) The risk management policy only provides a minor mitigation against the impact of the CRA… [1]
... given the deficit and delta differences between the assets and liabilities the CRA has created. [0.5]

This is because the CRA is a "liability side" impact determined by the regulator rather than a market based impact on both the assets and liabilities… [1]
… so the swap hedges do not change in value as they are valued using the LIBOR curve. [1]

However, the existing swaps that ensured a delta and gamma match of the liabilities under the base LIBOR curve should provide a degree of protection should LIBOR rates move post introduction to the CRA. [1]
[Max 3]

[Total 15]

This was one of the better attempted questions. Most candidates were able to score well in this bookwork question, while part (viii) is the least well answered as few assessed the implications in a holistic manner beyond referencing to earlier parts.

Q5

(i) A basic attempt at constructing a tree for the dividend paying stock is as follows:

The parameters u (the proportionate increase in a stock value at each node), d (the proportionate decrease in a stock value at each node), and p (the probability of an up movement in a risk-neutral world) can be calculated as though no dividends are expected. [0.5]

The initial stock price nodes, up to the ex-dividend date will be the same as for a stock option without dividends. [0.5]

From this point onwards the tree becomes non-recombining, resulting in many more nodes than the non-dividend paying stock price tree. [0.5]

This is known as node proliferation, resulting in a complex tree with an increased number of calculations compared to a recombining tree. [0.5]

At the first node on or after the ex-dividend date the stock price nodes at this time correspond to the nodes at the same time on the non-dividend paying stock price tree less the known dividend amount. [0.5]

The resulting non-recombining tree is not valued but a combining tree is created using the method below. [0.5]

It is assumed that the stock price has two components: a part that is uncertain and a part that is the present value of the dividend. [0.5]

The new tree is drawn as follows:

- Draw an initial tree of the uncertain component, which has the first node equal to the initial stock price less the present value of the dividend; and then [0.5]
- create the final tree with the certain component, by adding the present value of the future dividend at each node. [0.5]

In order to construct this the volatility of the uncertain component is required and it is assumed constant. [0.5]

This leads to new values of u, d and p which can be used to calculate the tree. [0.5]

Standard binomial techniques for valuing the American put option can then be applied to this tree. [0.5]

(Note to markers: marks should be awarded for students using Black-Scholes as a basis and then with a binomial model to approximate the impact of dividends if explained appropriately as per core reading / Hull)

[Max 4]

(ii) At time 0 the present value of the dividend is $0.55 \times e^{-0.03 \times 3/12} = 0.55$. [0.5]

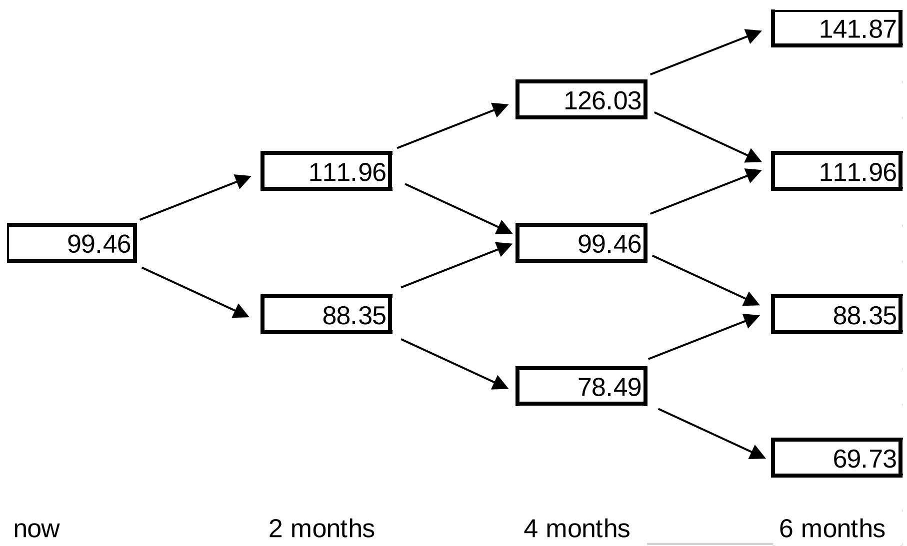
Here it is assumed that each month has length $1/12$ of a year, resulting in steps of $\Delta t = 2/12$ of a year. [0.5]

The initial value of the uncertain part of the stock price (denoted by S^*) is:
$S^* = 100 - 0.54589 = 99.45$. [0.5]

The implied volatility (σ^*) given in the question is 29%.

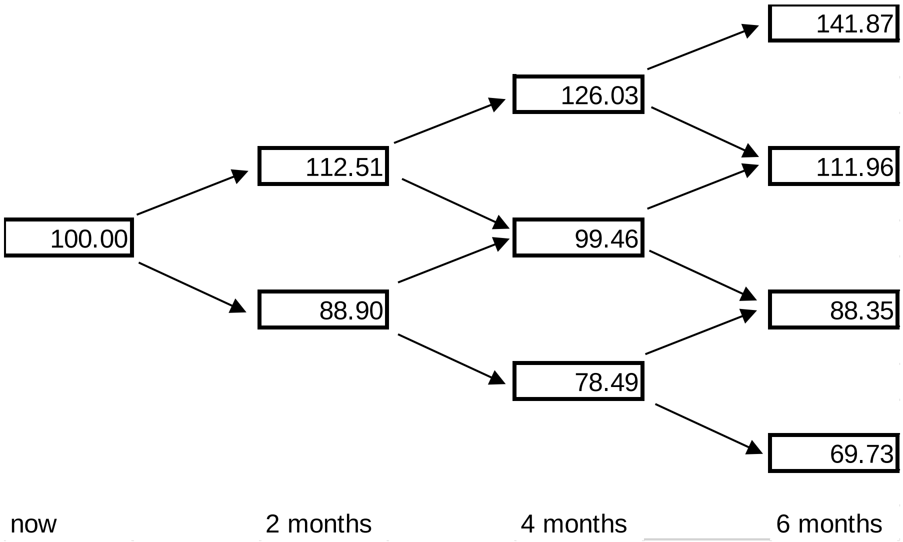
Using the formulas: $u = e^{\sigma^* \sqrt{\Delta t}}$, $d = e^{-\sigma^* \sqrt{\Delta t}}$, $a = e^{r \Delta t}$ and $p = \dfrac{a-d}{u-d}$ gives:
$u = 1.1256$, $d = 0.8883$, $a = 1.0050$ and $p = 0.4916$. [1]

This leads to the initial tree (as described in part (i)) of:



[1.5]

The final tree is completed by adding in the present value of the future dividend at each node (based on the method in part (i)):



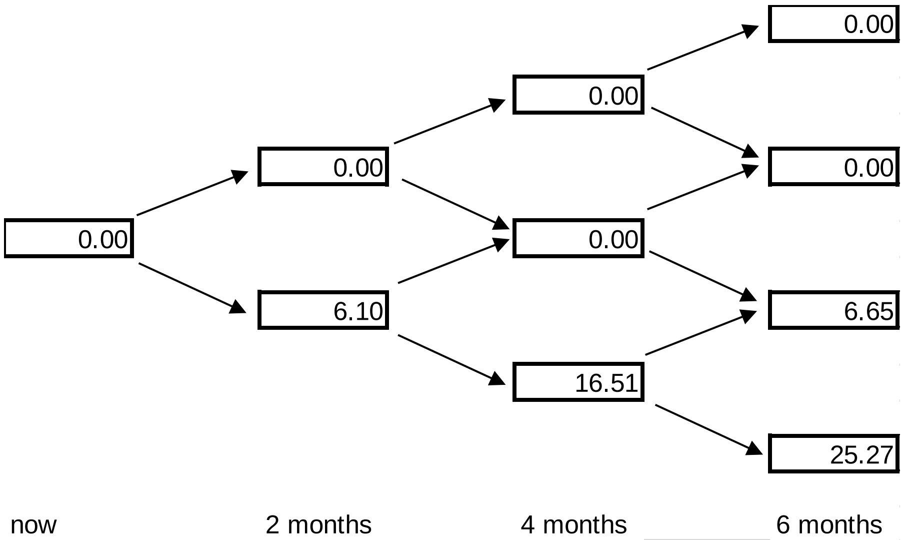
[1.5]
[Max 5]

(iii) The option price at the final nodes (time = 6 months) are calculated as $\max\left\{0, K - S^*\right\}$. [0.5]

The nodes at time = 4 months are firstly calculated assuming no exercise is made at the node. The option price is calculated as the present value of the expected option price at time = 6 months. A comparison is then made against the value of exercising at time = 4 months and the larger value is chosen. [1]

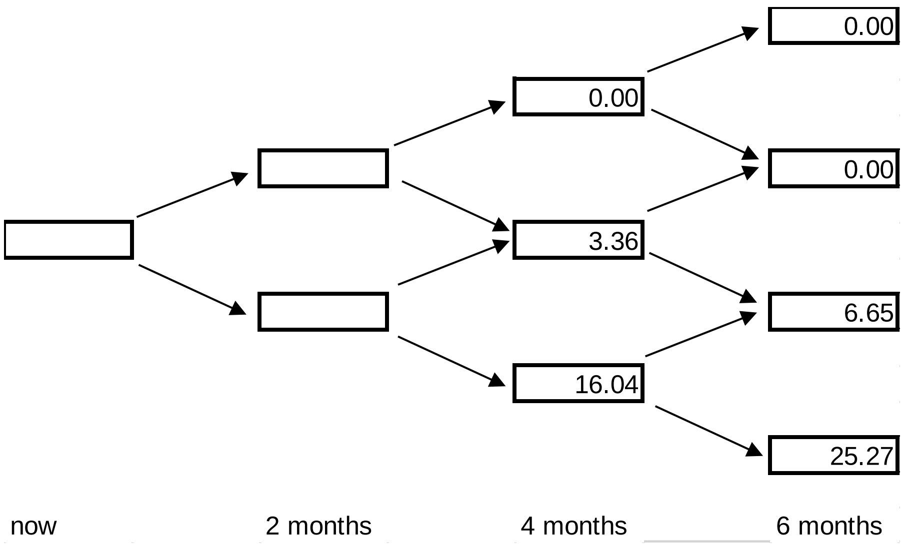
This method is repeated at time = 2 months and at time = now, with the value of the option given at time = now. [0.5]

The following tree shows the value at each node if the option is exercised at that node:



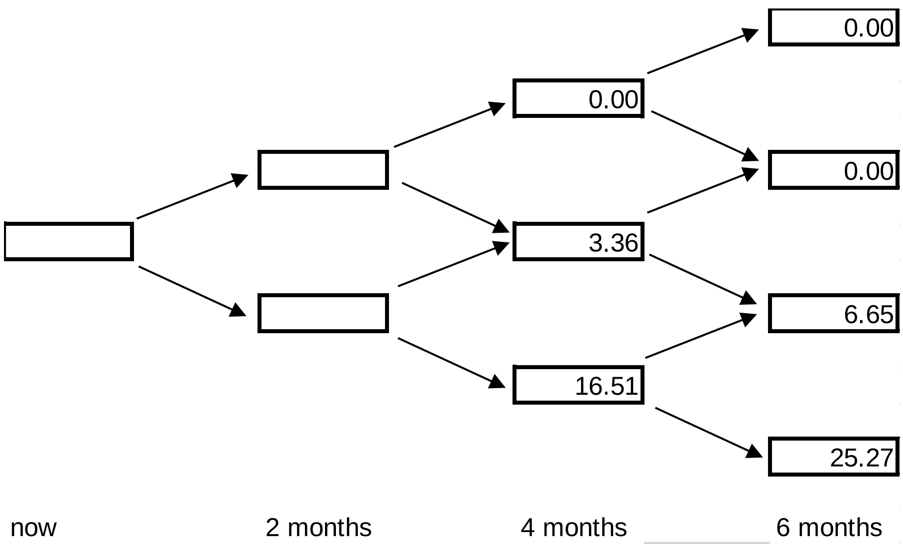
[1]

The following tree shows the nodes at time = 4 months, with the option price calculated as the present value of the expected option price at time = 6 months.



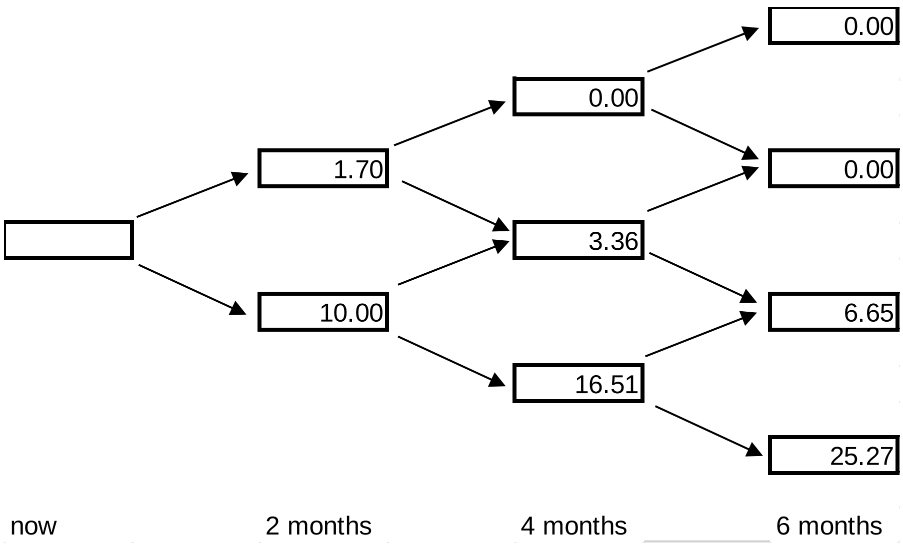
[1]

Comparing the two trees above leads to 3.36 being chosen for the middle node and 16.51 being chosen for the bottom node at time = 4 months:



[0.5]

The following tree shows the nodes at time = 2 months, with the option price calculated as the present value of the expected option price at time = 4 months.



[0.5]

By comparing with the values if the option is exercised at time = 2 months shows that not exercising is the better approach. [0.5]

The option is not exercised at time = now as it has no value so the value of the option at time = now is: $e^{-0.03*2/12} \times (0.4916 \times 1.70 + (1-0.4916) \times 10.00) = 5.89$. [0.5]

[Max 5]

(iv) With a longer term to expiry the amounts and timing of dates of future dividends will be known with less certainty. [0.5]

This is due to the dividends depending on the financial results of the company in the future. [0.5]

These can be projected but unless the company has made firm commitments on future dividends then there will be uncertainty. [0.5]

A more appropriate method may use a dividend yield rather than assuming discrete dividends. [0.5]

This can be based on historic data and it may provide a more accurate estimate of the option price. [0.5]

Further, a binomial model can be used to value an option using the dividend yield method rather than discrete known dividends. The resulting tree is also recombinant and it easily allows for dividend yields changing over the lifetime of the option. [1]

The tree becomes very complex [0.5]
and therefore we might use computational methods such as Monte-Carlo [0.5]
or use a larger time step for the model [0.5]

[Max 2]

[Total 16]

Candidates overall scored less well on this question. While most candidates showed a good understanding of binomial tree principles, in general marks could have been improved by correctly incorporating the dividend impact. Candidates who were able to correctly follow through with their calculations were able to score highly with their attempts, even if the exact answer in section (iii) was not always found, many marks were available for each step.

Q6

(i) With regards to T, the futures exchange will need to specify…
… when it is measured (e.g. midday/average during day)… [0.5]
… where it is measured (e.g. capital city?) [0.5]
… who will measure it (e.g. external agency?) [0.5]
… how it will be measured (e.g. what equipment?) [0.5]

The exchange would also need to specify…
… the precise period when final settlement must be made… [0.5]
… any daily price limits… [0.5]
… position limits… [0.5]
… how the price is quoted… (e.g. price increments / number of decimal places) [0.5]
... margin requirements [0.5]
[Hull pp24-26]
[Max 3]

(ii) The cost of carry for an investment asset is the storage costs plus the interest that is paid to finance the asset… [0.5]
…less the income earned on the asset. [0.5]
[Hull p120]
[Max 1]

(iii) The convenience yield measures the benefits of holding the physical consumption asset …. [0.5]
… that are not obtained by the holder of a long futures contract on the asset. [0.5]
[Hull p119]
[Max 1]

(iv) The cost of carry and convenience yield approaches are based on the possibility of being able to storing or hold the underlying asset… [1]
… but the underlying "temperature" of the temperature future is not storable in any meaningful way… [1]
… hence it is difficult to see how one could determine a storage cost for "temperature" or the benefit of holding "temperature". [0.5]
An alternative model would therefore be required… [0.5]
… such as a statistical model to generate the possible range of outcomes of the temperature. [0.5]

[Max 3]

(v) For example, the two temperatures may be determined in different locations
(please award max one mark for any plausible example)
[1]

There may be additional basis risk if the farmer does not know exactly when the crop will be harvested… [1]
… or if the futures contract is required to be closed out before its delivery month. [1]
[Hull pp52-53]
[Max 2]

(vi) Firstly, determine the two points at which the value of the crop is zero:

$$T_{Field} = \sqrt{100} + 18$$

$$T_{Field} = 8, 28$$

[0.5]

As the main function is a quadratic (i.e. has max one turning point) and by inspection of the chart… [0.5]
Delta = 0 for TField >28 and TField <8 [1]

Now consider the body of the function between 8 and 28 degrees.

$$Delta = \frac{\partial}{\partial T_{Field}}\left[100 - (T_{Field} - 18)^2\right]$$

[0.5]

$$Delta = -2(T_{Field} - 18) = 36 - 2T_{Field} \text{ for } 8 < T_{Field} < 28$$

[1]

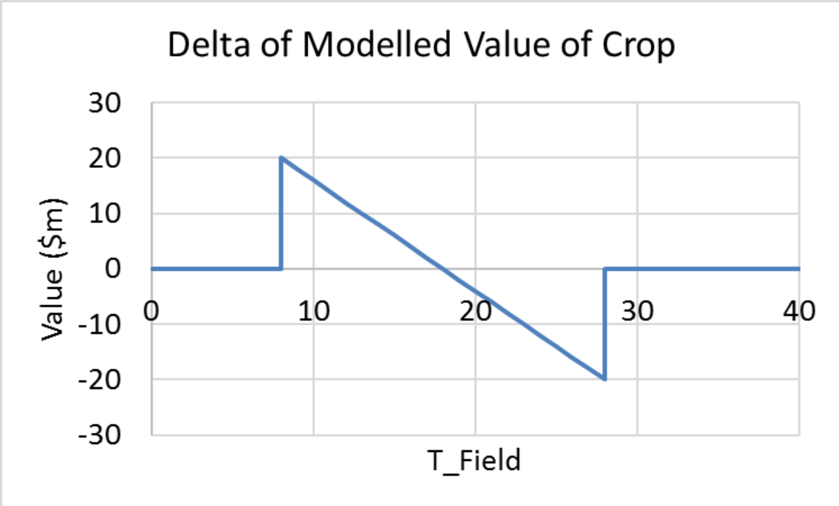
There is a discontinuity in Delta at T = 8, and T = 28. [0.5]
[Max 3]

(vii) The Delta of the modelled crop is extremely variable with respect to temperature.[0.5]
In contrast, the future has constant delta of $1m… [0.5]
… which means the farmer will need to rapidly adjust the number of contracts purchased as TField varies… [1]
… in particular at points TField =8 and TField =28 where the Delta "jumps" from zero to the minimum/maximum level [0.5]
There is a difference in Gamma between the futures value and the modelled value of the crop… [0.5]
…and so Delta mismatches can therefore easily emerge before the hedge can be rebalanced. [1]
Dynamically delta hedging will also …
… be complex and time-consuming to maintain… [0.5]
… incur transaction costs on rebalancing … [0.5]
… require readily available margin to be posted. [0.5]

There may be insufficient liquidity in the temperature future when the hedge needs to be adjusted in the future. [0.5]

The exchange may become concerned at the frequency of rebalancing and could impose higher margin requirements. [0.5]

Correctly drawing the delta pay-off diagram shown below. [1]



[Max 4]

(viii) The farmer's ideal contract would be one that replicates the opposite Delta of the modelled value of the crop. [0.5]

The exchange would be unlikely to offer this though, as this future may not directly appeal to other institutions affected by temperature. [0.5]

As many institutions are affected by deviations of temperature from a central estimate (e.g. heating costs increase if temperature reduces but air conditioning costs increase if temperature increases)… [0.5]

… the contract could be restructured to have a payoff that depends on the deviation of the temperature from a central estimate… [1]
… such as 18 degrees in the farmer's case. [0.5]

The exchange could consider adding caps and floors in the payoff of the future…[0.5]
… to reduce the delta of the contract beyond certain temperatures. [0.5]
(Markers: please award credit for sensible suggestions)

[Max 2]

[Total 19]

This was one of the better attempted questions. This was a question to apply typical derivative concept on an atypical situation. While bookwork questions were generally well scored, only a small group of candidates could fully apply the knowledge into the application questions in the last few parts.

Q7

(i) Real World projections of interest rates will allow the CCP to understand how interest rates could likely evolve in different scenarios. [1]

For example, the CCP may wish to understand whether any initial margin will cover extreme interest rate stresses. [0.5]

Risk neutral projections would be helpful for the CCP to understand the complete term structure of interest rates a given point during the real world projection. [1]

For example, the CCP may seek to calculate arbitrage-free prices of the securities after 10 years of the real-world projection. [0.5]

[Hull p688]

[Max 2]

(ii) $dr(t) = \left[a(b - r(t))\right]dt + \sigma dz(t)$ [1]

Where…
r(t) is the short interest rate under the real world measure [0.5]
a, b and σ are constants [0.5]
z(t) is Brownian motion [0.5]

[Hull p688]

[Max 2]

(iii) The CCP needs to calibrate the parameters based on historical movements in the short rate. [1]

Data can be collected using a daily, monthly or annual frequency. [0.5]
Parameters can then be determined by either a maximum-likelihood approach… [0.5]
… or using linear regression. [0.5]
The CCP will determine values for μ, a and σ. [0.5]

[Hull p688, top]

[Max 2]

(iv) Suppose that the price of the security has a SDE of $df = f\left[\mu dt + s dz\right]$ [0.5]

The market price of risk is defined as:

$$\lambda = \frac{\mu - r}{s}$$ [1]

Where:
μ and s are the expected growth rate and volatility of f respectively [0.5]
r is the risk free rate. [0.5]

[Max 2]

(v) By applying Girsanov's theorem… [0.5]
… moving to the risk neutral world sets the instantaneous return to

$r = \mu - \sigma\lambda$ [0.5]

… but the volatility of r will remain the same. [0.5]
Substituting gives:

$dr = \left[a(b - r) - \sigma\lambda)\right]dt + \sigma dz$ [0.5]

Hence:

$$dr = \left[a(b^* - r)\right]dt + \sigma dz$$

Where

$b^* = b - \lambda\sigma / a$ [0.5]

[Hull p689]

[Max 2]

(vi) The Vasicek model only contains one source of randomness (one factor) [0.5]

The Hull and White two factor model (HW2F) contains two sources of randomness… [0.5]

… which provides a richer pattern of both term structure movements and volatilities than one factor models. [1]

The HW2F should therefore provide the CCP with a greater range of interest rate scenarios to model. [0.5]

For example, in the Vasicek model, all forward rates along the curve are perfectly correlated, which could mask exposures to counterparties that have different interest rate positions at different parts of the curve. [1]

The Vasicek model struggles to fit some short rate term structures... [0.5]

... as the parameters do not vary with time. [0.5]

The HW2F uses a deterministic function for the mean reversion level set to be consistent with the current yield curve and bond prices… [0.5]

… so the CCP will be able to replicate current prices of instruments. [0.5]

However, replicating current prices may be of secondary importance to the CCP if we are projecting interest rates many years into the future in a real world scenario. [0.5]

[Max 4]

(vii) The CCP has set the initial margin levels with respect to the notional of the contract which is sensible … [0.5]

… given the market value will likely be zero on execution. [0.5]

The level of initial margin requirements could equate to a c20bps movement in interest rates given that (taking a 5 year swap) 1%/5 years = 20bps. [0.5]

After 5 years, the basis point equivalent stress will materially depend on the duration of the instrument. [0.5]

If we take a 10 year instrument as an example, 2%/10 years = 20bps. [0.5]

The key function of the initial margin is to provide a buffer for movements over a one day period before the margin account is marked to market at the end of the day. [0.5]

Daily interest rate moves are typicaly below 20bps so this should add a substantial amount of protection for typical day … [0.5]

… but it could prove insufficient in the extreme stresses which pose the most material counterparty risk. [0.5]

Specifying different margin requirements by term is sensible as longer dated swaps will be more sensitive to a given parallel movement in interest rates. [0.5]

However, the breakdown by term is limited to just two "buckets"… [0.5]

… which may not sufficiently reflect the differnet underlying risk by term. [0.5]

Counterparties can also "game" the margin requirements (e.g. by executing a 4.9 year swap with a larger notional rather than a 6 year swap). [0.5]

Determining the initial margin using the outstanding term will mean there will be a sharp reduction in the margin accounts as soon the term on longer dated instruments contracts to below 5 years, which may not be ideal. [1]

Requiring cash to be posted should reduce any second-order counterparty risk from the collateral defaulting at the same time as the counterparty. [1]

However, counterparty risk could still be present from the cash is this includes short term money market instruments. [0.5]

The margin requirements are the same for all counterparties which may not incentivise risk management by the counterparties. [1]
For example, the CCP could require more margin from riskier counterparties with lower credit ratings. [0.5]

The margin requirements make no allowance for offsetting between payer and receiver swaps… [0.5]
… which will mean large amounts of margin is collected where there is limited counterparty exposure. [0.5]

When setting the initial margin levels, it may be more effective to allow for offsetting between different instruments… [0.5]
… and possibly different terms to ensure greater initial margin is collected for the largest exposures. [0.5]

The margin requirements are easy to understand for counterparties, so there should be limited risk of insufficient or incorrect margin being posted. [0.5]

(*Markers: Please award credit for other appropriate points.)*
[Max 5]

[Total 19]

This was one of the better attempted questions. Generally a bookwork question except for the last part. Candidates that do not appear to have insufficient time for this question generally scored well. Few candidates were able to score full points for the application question in part (vii) when most attempts focus on generic points with little reference to the question's differentiation between <5 and >5 years.

[Paper Total 100]

END OF EXAMINERS' REPORT

