INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2020 Examinations

Subject SP6 - Financial Derivatives Principles

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Mike Hammer
Chair of the Board of Examiners
December 2020

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A. General comments on the *aims of this subject and how it is marked*

The aim of Financial Derivatives Principles (SP6) is to develop a candidate's ability to understand different types of financial derivatives and their uses, the markets in which they are traded, methods of valuation of financial derivatives, and the assessment and management of risks associated with a portfolio of derivatives. It builds on material covered in earlier subjects, particularly Loss Reserving and Financial Engineering (CM2).

Candidates are reminded to ensure that their answers are sufficiently detailed to demonstrate understanding, as there were instances where inadequate explanations led to candidates scoring less well on questions than they might have done. The model solutions are intended to reflect the level of detail that a high scoring candidate might be able to produce. For many questions there are more marks available than the question requires to achieve full marks. This reflects that the examiners will give credit for valid alternative solutions, particularly in questions focussed on higher level skills.

Candidates who give well-reasoned points, not in the marking schedule, are awarded marks for doing so.

B. Comments on *candidate performance in this diet of the examination*

The majority of candidates made a good attempt at the exam and were able to score at least some marks in each of the 7 questions. However, only well-prepared candidates managed to score above the required 60 pass mark.

This was the second SP6 exam which was undertaken in the online / open book format. No particular issues were identified by the examining team and it was noted that candidates seem to have adapted to this format.

In general, candidates demonstrated good knowledge of the core reading materials and its application to familiar situations. This was evident in questions 2, 3 and 5 where most candidates managed to pick up significant marks.

Questions 4, 6 and 7 proved to be more challenging, although well-prepared candidates were still able to pick up the majority of the available marks.

Question 4 (v) – most candidates were able to explain why the assumptions underlying the Black-Scholes methodology were questionable in this situation. However, most candidates missed marks by not providing a sufficiently broad assessment.
Question 4 (vi) – most candidates struggled to generate ideas and describe how they could be used as a hedge.
Question 6 (ii) – on average, candidates only provided enough description of the method to pick up half the available marks.
Question 6 (v) – this was the lowest scoring question on the paper with many candidates failing to attempt it.
Question 7 (ii) – although the numerical calculations were straightforward, the use of multiple contracts led to some errors.

Pass Mark

The pass mark for this exam was 60.
68 candidates presented themselves and 23 passed.

Solutions

Q1

(i)

In a payer swaption, the buyer has the right to enter into a swap where they would pay the fixed rate.

The payoff from a payer swaption is $\frac{L}{m}max(R - R_X, 0)$ [1]

In a receiver swaption, the buyer has the right to enter into a swap where they would receive the fixed rate

The payoff from a receiver swaption is $\frac{L}{m}max(R_X - R, 0)$ [1]

Where L = contract notional value
m = number of interest payments made each year
R_X = strike rate
R = prevailing n-year swap rate

[Total marks available 2]

(ii)

Assuming
v= n-year, forward swap to pay fixed rate of X% and receive floating rate, starting at time=T
p= t,n - payer swaption with strike rate of X%
r = t,n – receiver swaption with strike rate of X% [1]

Consider two portfolios;

- Portfolio 1: p
- Portfolio 2: r+v [½]

At the time of expiry, assume the prevailing interest rate is Y% >X%.
Therefore, the payer swaption would be exercised but the receiver swaption would not be.[½]

- Portfolio 1 = swap paying X%, receive floating rate
- Portfolio 2 =swap paying X%, receive floating rate [1]

At the time of expiry, assume the prevailing interest rate is Y%< X%.
Therefore, the payer is not exercised but the receiver would be. [½]

- Portfolio 1 = 0
- Portfolio 2 = swap receiving X% and paying floating and swap paying X%, receive floating rate
  =0 [1]

Therefore, as the payoffs at time T are equal, the starting values of the portfolios must be equal under the no-arbitrage principal. [1]

Therefore, p=r+v [½]

[Total marks available 6, maximum 4]

(iii)
A liquidity need will arise where the value of the derivative moves against the insurer and requires extra collateral to be posted. [1]

Initial margin requirements will create demand for liquid assets, particularly if the insurer has multiple exposures. [½]

A significant collateral call is also likely to come during a period of market stress which may inhibit the insurer's ability to source additional liquidity needs (for example via repo transactions). [½]

Reduced liquidity may also make it difficult/costly to transact derivatives e.g. if the insurer was adjusting its derivative portfolio. [½]

This is more likely to be the case for more complex derivative structures. [½]

[Total marks available 3, maximum 2]

(iv)
Liquidity risk arising from derivatives should be monitored as part of the insurer's liquidity risk management framework ie liquidity risk should be assessed holistically [½]

To monitor liquidity risk, the insurer must first set its liquidity risk appetite – this is the amount of liquidity risk it is willing to accept. [½]

To measure the liquidity risk it is likely that the insurer will use liquidity scenario/stress testing. [1]

Assessing the hypothetical liquidity needs of its derivative portfolio (alongside the wider liquidity needs) under a series of stressed market conditions will enable the insurer to develop a robust liquidity risk management plan. [1]

Appropriate systems and processes should be put in place to ensure that the derivative portfolio is being monitored. [½]

Any changes to the derivative portfolio, collateral requirement or discounting approach may impact the derivative portfolio's liquidity needs. [½]

As liquidity is a significant threat to the functioning of the insurance company, it would be expected that the board or executive function would have to review and sign-off the liquidity risk management strategy. [½]

The insurer may also use a series of quantitative indicators to assess how the liquidity needs of the derivative portfolio are developing. [½]

For example, the level of swap yields would be a key determinant in the collateral (and liquidity) requirements of the swap portfolio. [½]

Transaction costs (bid-offer spreads) should be monitored to assess the potential increased trading costs. [½]

Metrics should be developed to ensure that the insurer can assess the depth/liquidity of each derivative market in which it operates (eg. number/size of transactions). [½]

[Total marks available 6½, maximum 3]

[Total 11]

Most candidates were familiar with parts (i) and (ii) and scored well. Parts (iii) and (iv) proved to be challenging for most candidates with few candidates achieving full marks.

Q2

(i)

In T_1 years the purchaser of the option has the choice between a call and a put option, the chooser date. A rational investor will make a choice based on $\max\{c, p\}$. [1]

Here, c is the price of the call option as defined in the question and p is the price of the put option which has strike price K, $T_2 - T_1$ years to maturity and is written on the underlying asset with price S_t. [1]

From page 47 of the Formula and Tables book the put-call parity formula is:

$$c + Ke^{-r(T_2-T_1)} = p + S_{T_1}.$$ [1]

Substituting the put-call parity formula into $\max\{c, p\}$: [½]

$$\max\left\{c, c + Ke^{-r(T_2-T_1)} - S_{T_1}\right\}.$$ [½]

Extracting c gives the solution: $c + \max\left\{0, Ke^{-r(T_2-T_1)} - S_{T_1}\right\}$ [½]

[Total marks available 4½, maximum 4]

(ii)

The equation in (i) can be viewed as a portfolio at time T_1 consisting of a European call option with strike price K, time to maturity $T_2 - T_1$ and underlying asset with price S_t. [1]

Plus the **payoff** from (the second term of) a European put option written on the underlying asset with price S_t and [½]

strike price $Ke^{-r(T_2-T_1)}$, [½]

At the current time the equation (and interpretation) in (i) can be viewed as a European call option with strike price K, time to maturity T_2 and underlying asset with price S_t. [½]

Plus a European put option written on the underlying asset with price S_t, strike price $Ke^{-r(T_2-T_1)}$, and [½]

and time to maturity T_1. [½]

These two options can then be valued using standard models (for example Black Scholes) [½]

[Total marks available 4]

(iii)
There is an uncertain event which may impact upon the stock price of a company. [½]

To hedge this risk, the shareholder may want to buy a put and a call option eg. a straddle. However, it will be slightly cheaper to purchase a chooser. [½]

In order to hedge this risk the shareholder can use a chooser option as follows. [½]

The result of the company's bid is unlikely to be known with certainty. [½]

The strike price of the chooser could be chosen as the current price of the stock. [½]

Therefore a shareholder could buy a chooser option on the stock of the company which has a chooser date (T_1) greater than 2 months. [½]

The exact expiry date may be 3 or 4 months, in this way the stock market should have reacted to the results of the contract and the impact on the company. [½]

When T_1 arrives the shareholder could then decide to exercise the chooser option as a call option if the stock price of the company rises… [½]

as the market has reacted positively to the results and the stock price is greater than the strike price. [½]

Or a put option if the stock price of the company falls, [½]

as the market has reacted negatively to the results and the stock price is lower than the strike price. [½]

As the shareholder is already exposed to the price movement, the put option would mitigate the losses incurred. The call option would enhance the positive return. [½]
There is still a risk in the period between the chooser date and the date of expiry that other market movements move the call or put option out of the money. [1]

[Total marks available 7, maximum 4]

[Total 12]

This question was well answered by most candidates.

Q3

(i)
A property swap is a type of total return swap which aims to replicate the financial consequences of a physical property transaction. [½]

The buyer of a property swap receives the total return on the property or index, i.e. all the benefits (e.g. rent, capital repayments) associated with owning the property or properties underlying the swap, just like a physical buyer would. [½]

For this set of cash flows, the swap buyer pays a (usually) fixed coupon to cover the cost of the above benefits, such as financing (e.g. mortgage) and other outgoings (e.g. stamp duty, legal fees, agency fees). [½]

Property swaps typically reference a property index. [½]

[Total marks available 2]

(ii)
A property swap replicates the total cashflow returns from a physical property, so if the pension fund enters into a property swap it can remove (most of) the financial exposure to the property. [1]

The pension fund would be the seller of the property swap and would receive fixed or floating cashflows in return for the returns linked to properties/ property index. [½]

The pension fund should enter a swap which references a property index that is the most representative of its portfolio. [½]

This would enable the pension fund to minimise its market exposure while still retaining ownership of the property fund. [½]

[Total marks available 2½, maximum 2]

(iii)
From a risk perspective, the swap effectively provides liquidity to the pension fund to allow it to remove its exposure to the pension fund without having to wait for the disinvestment period to expire. [1]

The pension fund will still be exposed to the fund manager's alpha ie the difference in performance between the property fund and the designated index. [½]
Property swaps also introduce some risk to the pension fund:

- Basis risk: as noted above the fund would pay the return on a reference index
- Liquidity: the liquidity of property derivatives market is very limited
- Transparency of pricing: the pricing of contracts may differ across providers
- Credit risk: there may be collateral requirements which need to be met to set up the swap contract [½ each]

Under a property swap, the trustee can usually elect to receive fixed or floating payments. The trustee should make an assessment of which payment pattern fits its other investments/obligations. [½]

[Total marks available 4, maximum 2]

(iv)
The coupon and redemption payments of inflation-linked bonds are calculated based on the increases in a designated inflation index such as the Retail Price Index. [½]

The market for inflation linked bonds is usually smaller than the market in fixed rate bonds. [½]

This means that the liquidity is usually lower, and the choice is more limited which can lead to concentrations, particularly if inflation linked corporate bonds are being used. [½]

Inflation bonds provide hedging for real interest rate (i.e. they hedge both inflation and interest rate risk). [1]

The pension fund would need to buy inflation-linked bonds to hedge its inflation risk [½]

Inflation swaps are OTC contracts. They typically are constructed to swap a stream of fixed cashflows for inflation linked cashflows. [½]

Inflation swaps only hedge inflation and so the pension fund would also need to either 1) hold or 2) enter into appropriate fixed interest rate bonds/swaps to manage the inflation risk. [1]

Inflation swaps are more flexible than inflation bonds and as such may offer a improved hedging accuracy. [½]

Limited Price Indexation (LPI) swaps hedge inflation risk based on the RPI index, capped at 5% and floored at 0%. [1]

This is a common form of pension increase in the UK, hence why there are swaps available which can be used to hedge this risk. [½]

The trustee should only consider using LPI-swaps if it had LPI-linked liabilities. [½]

LPI swaps are less liquid than inflation swaps and generally more costly than inflation swaps [½]

It is normal practice to use inflation swaps and interest rate swaps to delta hedge the LPI liabilities rather than use LPI-swaps. [1]

The pension fund would need to buy inflation/LPI swaps (ie. receive inflation-linked payments) to hedge its inflation risks [½]

[Total marks available 9, maximum 5]

(v)
The use of overseas inflation-linked bonds introduces currency risk. [½]

Given the trustee's desire to minimise risks, this currency exposure would also need to be hedged. [½]

The inflation index underlying the overseas inflation-linked bond is unlikely to be a direct match for the pension fund's liabilities. This is likely to result in considerable basis risk for the pension fund. [1]

Over the long-term, the trustee may be comfortable with the basis risk given the drivers of inflation are often of a global nature (eg. oil prices). [½]

However, as the trustee expects to transfer the portfolio to an insurer within 3 years, this risk is unlikely to be acceptable. [1]

Any changes in the overseas interest rates relative to changes in the domestic interest rates will also introduce a new risk to the pension fund. [1]

This rate risk can be hedged through the use of interest rate swaps which remove the overseas interest rate exposure and reintroduce domestic interest rate exposure. [1]

There is clearly a higher degree of complexity in the use of overseas inflation linked bonds for this purpose. [½]

However, overseas markets may provide opportunities and liquidity which are not available domestically. [½]

[Total marks available 6½, maximum 4]

[Total 15]

This question was well answered by most candidates.

Q4

(i)

The only cashflow is a lump sum payment from the insurance company to the customer. [1]

[Total marks available 1]

(ii)

There is a lump sum payment from the house sale to the insurer consisting of the original lump sum borrowed and [½]

the interest [½]

of $(e^{rt}-1)\times \text{lump sum}$, where t is the time of death. [½]

If there is insufficient money from the house sale to pay the loan in full then the outstanding amount will be paid by the customer's estate to the insurance company. [½]

[Total marks available 2]

(iii)

At time T the customer is required to pay back the original loan amount L(0) with interest:

$L(T)=L(0)\times e^{rT}$. [½]

The guarantee means that if $H(T)\le L(T)$ then the customer will pay is $H(T)$. [½]

If $H(T)>L(T)$ then the guarantee means the customer will pay $L(T)$. [½]

In each case the minimum of, $L(T)$ and $H(T)$ is paid by the customer to the insurance company, hence the result. [½]

[Total marks available 2]

(iv)

From part (iii):

$min\{H(T),L(T)\}=L(T)-max\{L(T)-H(T),0\}$. [½]

The guarantee at time T has the payoff of a put option. [½]

The underlying asset is the market value of the house price. [½]

The strike price is the value of the loan rolled-up with interest. [½]

The time to expiry of the option is T. [½]

[Total marks available 2½, maximum 2]

(v)
For this answer the word *option* relates to the guarantee described in this whole question.

Exercise Date
The option has an unknown exercise date when the customer buys the product. [½]

This is due to the date of death for the customer being unknown. [½]

The option is not European as the exercise date is not fixed, [½]

neither is it purely American in nature as the date of exercise is not a choice of either the customer or the insurance company. [½]

This will make the evaluation using Black-Scholes more complex. [½]

Assumptions will be needed for longevity. [½]

Market Value of the Property

The underlying asset of the option is the market value of the customer's property. Under Black-Scholes this market value is assumed to be geometric Brownian motion. [½]

From the basic investment characteristics of property as an asset class, it is likely to be real in nature. Given that stock prices often use geometric Brownian motion then using this for property may be suitable. [1]

Another characteristic of property, especially for a customer's house, is that it is generally unique and subject to many idiosyncratic factors. [½]

This could make estimating parameters to fit the geometric Brownian motion difficult. [½]

There is likely to be lots of aggregate data for property prices but very little market data for an individual property (if any). [½]

The distribution of geometric Brownian motion may also be unsuitable as although assets like equity can become 0 or close to 0 (for example if a company goes bust), property generally does not. [1]

Even if a house is destroyed there is insurance and underlying value in the land it is built on. [½]

Local, national and federal governments have a keen interest in the housing markets and policies are also likely to impact to prevent excessively low housing prices. [½]

Together this suggests that geometric Brownian motion may be an unreasonable assumption for the lower tail. [½]

Underlying assumptions

Some of the fundamental assumptions underlying the Black-Scholes option pricing formula could be questioned. [½]

- It is not possible to buy and sell any amount of the property. This is due to the property being unique and lived in by the customer. [1]
- There is likely to be transaction cost and fees when the property is sold. [½]
- On a wider point there is no market for these options, there are no derivatives for individual property. [½]
- There are also no speculators or arbitrageurs. [½]
- Overall, these combine to make it not possible to perfectly dynamically hedge the option. [½]

The Black Scholes model is a familiar and standard model used to value options… [½]

However, overall there does appear to be many factors which suggest that the Black-Scholes model may not be the most suitable for valuing this option. [1]

(Note to markers: other appropriate points should be given credit.)

[Total marks available 13½, maximum 7]

(vi)
Although the underlying asset for the guarantee (option) is a customer's property which (as discussed in (iii)) is unique and rarely traded hedging should be possible for a portfolio of these guarantees (options) [1]

As a starting point any hedging portfolio should have similar characteristics to the portfolio of guarantees. [½]

This should include having a similar delta, and [½]

gamma. [½]

There are particular assets which could be suitable in the hedging portfolio:

- residential property index funds;
- stock of residential property building companies;
- commercial property index funds;
- stock of commercial property building companies; and
- real estate trust funds.

[½ marks each, up to a maximum of 1½ marks]
(Note to markers: other suitable assets should be given credit)

In addition, derivatives using the above assets as the underlying could also be suitable in the hedging portfolio. [½]

All of the above assets are traded and the returns and behaviour will be similar to the overall residential property market. [½]

This hedge is unlikely to be perfect due to the following factors:

- There will be transaction costs in order to rebalance the portfolio. [½]
- These costs will increase if there is dynamic hedging. [½]
- The portfolio is subject to significant longevity risk which may be difficult to hedge against. [½]
- There will be basis risk as the individual properties are not actively traded. [½]

[Total marks available 8, maximum 4]

[Total 18]

Most candidates scored well in earlier parts of the question.

For part (v), candidates who scored highly provided a broad range of points covering the different aspects detailed above.

Q5

(i)

The Black model is widely used to value European-style interest rate options.
European futures options are equivalent to European spot options when the option contract matures at the same time as the futures contract. [1]

A spot option can then be valued using a futures contract which matures at the same time as the option. [1]

The futures price contains the market's estimate of the future income which would be derived from the underlying asset during the option period. [1]

[Total marks available 3, maximum 2]

(ii)

$$p = e^{-rT}[K\Phi(-d_2) - F_0\Phi(-d_1)]$$

[1]

Where

$$d_1 = \frac{ln\left(\frac{F_0}{K}\right)+\sigma^2 T/2}{\sigma\sqrt{T}}$$ [½]

$$d_2 = d_1 - \sigma\sqrt{T}$$ [½]

r = risk-free rate
T = term of option

K = strike price
σ = forward volatility
F_0 = forward price of bond [½]

[Total marks available 2½, maximum 2]

(iii)
Market value of bond = $3x[e^{-0.25*0.035} + e^{-0.75*0.035} + \ldots + e^{-3.25*0.035}] + 100e^{-3.25*0.035} = 109.0$ [2]

Forward bond price = $F_B = e^{1.5*0.035}[109.0 - 8.7] = 105.6$ [2]

(full 4 marks awarded for calculating in alternative manner)

K = 110
r = 3.5%
T = 1.5 years
σ = 0.12

$d_1 = -0.2012$ [1]
$d_2 = -0.3481$ [½]

$\Phi(-d_2) = 0.636$ [½]
$\Phi(-d_1) = 0.580$ [½]

$p = 0.949 * [110 * 0.636 - 105.6 * 0.580] = 8.28$ [1]

[Total marks available 7½, maximum 6]

(iv)
(a)Lower… [½]
because a lower strike price means the put is less likely to be exercised, hence a lower value [½]

(b)Lower… [½]
because the forward value of the bond will be higher and therefore the put is less likely to be exercised [½]

(c)Higher… [½]
because the forward value of the bond will be lower and therefore the put is more likely to be exercised [½]

[Total marks available 3]

[Total 13]

This question was well answered by many candidates.

Q6

(i)

Finite difference methods value a derivative by solving the differential equation that the derivative satisfies. [½]

The differential equation is converted into a set of difference equations, and the difference equations are solved iteratively. [½]

[Total marks available 1]

(ii)

A discrete rectangular grid of price changes ΔS and time steps ΔT spanning all possible outcomes of the stock evolution over the time 0 to T is set-up, where S is the underlying asset price and T is the life of the option. [1]

These steps must be small enough to make the approximation accurate, but not so small that the number of steps is computationally intense (leading to rounding errors). [1]

Let the steps in time go from $i = 1$ to N, and in stock price from $j = 1$ to M where $T = N\Delta t$, $S_{\max} - S_{\min} = M\Delta S$, ($S_{\max}$ / $S_{\min}$ are the highest / lowest prices in the grid). [1]

The *implicit* approach approximates a difference by taking values at the nearest previous time step. [½]

For an interior point (i,j) on the grid, $\frac{\partial f}{\partial S}$ (where f is the value of the derivative) can be approximated as: [½]

$\frac{\partial f}{\partial S} = \frac{f_{i,j+1} - f_{i,j}}{\Delta S}$ (forward difference approximation), or as [½]

$\frac{\partial f}{\partial S} = \frac{f_{i,j} - f_{i,j-1}}{\Delta S}$ (backward difference approximation). [½]

A more symmetrical approximation is used by averaging:

$\frac{\partial f}{\partial S} = \frac{f_{i,j+1} - f_{i,j-1}}{2\Delta S}$. [½]

For $\frac{\partial f}{\partial t}$, a forward difference approximation is used so that the value at time $i\Delta t$ is related to the value at time $(i+1)\Delta t$: $\frac{\partial f}{\partial t} = \frac{f_{i+1,j} - f_{i,j}}{\Delta t}$. [½]

A finite difference approximation for $\frac{\partial^2 f}{\partial S^2}$ uses the backward difference approximations of $\frac{\partial f}{\partial S}$ at (i,j) and (i,j+1): [1]

$$\frac{\partial^2 f}{\partial S^2} = \left(\frac{f_{i,j+1} - f_{i,j}}{\Delta S} - \frac{f_{i,j} - f_{i,j-1}}{\Delta S} \right) \Big/ \Delta S \text{ , or}$$ [½]

$$\frac{\partial^2 f}{\partial S^2} = \frac{f_{i,j+1} + f_{i,j-1} - 2 f_{i,j}}{\Delta S^2}.$$ [½]

These values can then be substituted into the partial differential equation to give a set of difference equations. [½]

Using suitable boundary conditions a set of simultaneous equations can be derived and solved. [½]

The computations work back from the end of the life of the derivative to the beginning. [½]

The implicit finite difference method is complicated but has the advantage that no special precautions are needed to ensure convergence. [½]

(*Note to markers: if a candidate described the explicit finite difference method then award a maximum of 3.5 marks.)*

[Total marks available 10, maximum 7]

(iii)

The price of the call option at time T is $\max\{S_T - K, 0\}$. [½]

The current price of the call option is the expected value of the payoff at time T discounted back to the current date. [½]

Under the risk-neutral measure the expectation is taken with respect to risk-neutral probabilities and the discounting is at the risk-free rate. [½]

$$C = E_Q\left[e^{-rT} \max\{S_T - K, 0\}\right].$$ [½]

Here Q is the risk-neutral measure and E is the expectation.

Writing this as an integral:

$$C = \int_{-\infty}^{\infty} e^{-rT} \max\{S_T - K, 0\} f(S_T) dS_T .$$ [½]

Simplifying the limits and removing the max function gives the result:

$$C = \int_{K}^{\infty} e^{-rT} (S_T - K) f(S_T) dS_T .$$ [½]

[Total marks available 3]

(iv)

One reason is that increasing the strike price narrows the range of underlying asset prices for which the call option has a positive payoff. [1]

Another reason is that by increasing K reduces the payoff by dK for every option which is in the money at maturity. [1]

[Total marks available 2]

(v)

$$\frac{\partial C}{\partial K} = \frac{\partial}{\partial K}\left(\int_K^\infty e^{-rT}(S_T - K)f(S_T)dS_T\right).$$ [½]

Differentiating the integral (using page 3 of the Formula and Tables book) gives:

$$\frac{\partial C}{\partial K} = e^{-rT}\left(-(K-K)f(K) + \int_K^\infty -f(S_T)dS_T\right).$$ [1]

From the definition of $F(x) = \int_{-\infty}^{x} f(z)dz$: [½]

$$\frac{\partial C}{\partial K} = e^{-rT}\left(\int_K^\infty -f(S_T)dS_T\right) = -e^{-rT}\left(1 - F(K)\right).$$ [½]

Rearranging gives the required expression:

$$F(K) = e^{rT}\frac{\partial C}{\partial K} + 1.$$ [½]

[Total marks available 3]

[Total 16]

Candidates generally displayed good knowledge of the finite difference method but many missed marks by not providing a sufficient level of detail to their answers in part (ii).

Part (v) proved to be the most challenging question with very few candidates picking up marks.

Q7

(i)

A forward contract is an agreement to buy or sell an asset at a certain future time for a certain amount.

Forwards are traded in the OTC market

Forwards can be non-standardised so can be adapted to meet the needs of individual market participants

Forwards are typically settled at the end of each contract

[½ each, max 1½]

A future contract is an agreement between two parties to buy or sell an asset at a certain time in the future for a certain price

They are standardised contracts

Futures are normally traded on an exchange

Contracts are settled daily via a margin account

Contracts are closed out at maturity of the contract

[½ each, max 1½]

Central clearing of forward contracts through a CCP reduces the differences between futures and forward contracts [½]

[Total marks available 5, maximum 3]

(ii)
Buy the following heating oil contracts: [1]
Mar = 20,000,000/42,000 = 476
Jun = 762 [½]

Cost in Euro
Mar = (20 000 000*1.67)/1.11 = 30 090 090
Jun = (32 000 000*1.58)/1.13 = 44 743 363 [2]

Therefore, need to sell the following Euro futures contracts [1]
Mar = 30 090 090/125 000 = 241
Jun = 358 [½]

[Total marks available 5]

(iii)
The airline could adopt a stack and roll strategy. This involves using shorter-dated futures contracts to hedge a long-dated exposure and then to roll these positions forward as the shorter-dated contracts approach expiry. [1]

This avoids the use of long-dated contracts where liquidity is generally lower. By the time the positions are rolled forward, the level of liquidity will have increased in the longer-dated contracts. [1]

For example, the airline could therefore purchase 2,762 (=476+762+857+667) March heating oil contracts and then prior to expiry of the March contract, it would simultaneously sell 2,762 March contracts and purchase 2,286 (=476+762+857) June contracts. [1]

[Total marks available 3]

(iv)
Hedging the cost of its fuel enables the airline to provide certainty about one of its main expenses. This enables the airline to get on the with its core activities and to be (relatively) sure that the price of oil should not impact on its P&L. [1]

However, there is a school of thought which says that companies should not hedge the risk of financial variables. This is because shareholders, can hedge this risk themselves if they were so inclined. [1]

The competitor may be of the view that oil prices will reduce over the next 12 months. If this occurred, a hedging programme would be detrimental to the company's P&L. [1]

Historical hedging policies may have been to the detriment of the company and so it may have taken a decision to not hedge this risk. [½]

The competitor airline may be lacking the specialist knowledge or experience to implement a hedging programme. [½]

There may be accounting reasons why the company chooses not to hedge this risk. [½]

The competitor airline's pricing structure may allow it to pass on the increased costs of oil to its passengers, for example if it operated monopolistic routes. [½]

It may have sought to offset some of the risk through strategic partnerships or purchases eg. purchase of an oil refinery. [½]

(Note to markers: other appropriate points should be given credit.)

[Total marks available 5½, maximum 4]

[Total 15]

Where candidates attempted parts (ii) and (iii), they generally scored well.

Most candidates also made good attempts at part (iv).

END OF EXAMINERS' REPORT