INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2019 Examinations

Subject SP6 - Financial Derivatives Principles

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Mike Hammer
Chair of the Board of Examiners
December 2019

A. General comments on the *aims of this subject and how it is marked*

1. The aim of Financial Derivatives Principles (SP6) is to develop a student's ability to understand different types of financial derivatives and their uses, the markets in which they are traded, methods of valuation of financial derivatives, and the assessment and management of risks associated with a portfolio of derivatives. It builds on material covered in earlier subjects, particularly Loss Reserving and Financial Engineering (CM2).

2. Candidates are reminded to ensure that their answers are sufficiently detailed to demonstrate understanding, as there were instances where inadequate explanations led to candidates scoring less well on questions than they might have done. The model solutions are intended to reflect the level of detail that a high scoring candidate might be able to produce. For many questions there are more marks available than the question requires to achieve full marks. This reflects that the examiners will give credit for valid alternative solutions, particularly in questions focussed on higher level skills.

3. Candidates who give well-reasoned points, not in the marking schedule, are awarded marks for doing so.

B. Comments on *student performance in this diet of the examination*

Overall the paper was well attempted by most candidates. Suitably prepared candidates were able to score highly across all questions demonstrating the ability to apply their ST6 knowledge and techniques to unfamiliar situations.

The better attempted questions were 1, 2, 3 and 4, where candidates on average scored well over half of the available marks. Well prepared candidates were also able to score highly in the other questions.

Pass Mark

The Pass Mark for this exam was 60.

Solutions for Subject SP6- September 2019

Q1

(i) $\lambda_1 = \dfrac{\mu_1 - r}{\sigma_1}$ [0.5]

where r is the risk-free rate [0.5]

[Total 1]

(ii) $dg = rgdt$ [1]

[Total 1]

(iii) Over the next instant of time, the change in P is:

$dP = (\sigma_2 f_2)df_1 + (-\sigma_1 f_1)df_2$ [1]

$dP = (\sigma_2 f_2)(\mu_1 f_1 dt + \sigma_1 f_1 dz) + (-\sigma_1 f_1)(\mu_2 f_2 dt + \sigma_2 f_2 dz)$ [0.5]

$dP = (f_1 f_2)(\mu_1\sigma_2 - \mu_2\sigma_1)dt$ [the dz terms cancel] [A] [0.5]

This portfolio is instantaneously riskless, and therefore must earn the risk-free rate. [1]

[Total 3]

(iv) From (ii) therefore:

$dP = rPdt = r(\sigma_2 f_2 f_1 - \sigma_1 f_1 f_2)dt = rf_1 f_2(\sigma_2 - \sigma_1)dt$ [B] [0.5]

Equating A & B: $(f_1 f_2)(\mu_1\sigma_2 - \mu_2\sigma_1)dt = rf_1 f_2(\sigma_2 - \sigma_1)dt$ [0.5]

Cancelling and dividing by $f_1 f_2 \sigma_1 \sigma_2$ gives:

$\dfrac{\mu_1}{\sigma_1} - \dfrac{\mu_2}{\sigma_2} = \dfrac{r}{\sigma_1} - \dfrac{r}{\sigma_2}$. [0.5]

Hence; $\dfrac{\mu_1 - r}{\sigma_1} = \dfrac{\mu_2 - r}{\sigma_2}$ so $\lambda_1 = \lambda_2$ [0.5]

[Total 2]

[Total 7]

All parts of this question were well attempted. A few candidates made small errors in their algebraic steps but generally the solutions seen were well set out.

Q2

(i) $\frac{\partial c}{\partial t} = Bre^{rt}$, [0.5]

$\frac{\partial c}{\partial x} = A$, and [0.5]

$\frac{\partial^2 c}{\partial x^2} = 0$. [0.5]

Substituting theses into the Black-Scholes differential equation gives:
$Bre^{rt} + rx(A) + 0 = r(Be^{rt} + Ax) = rc$, as required. [0.5]

[Total 2]

(ii) This derivative pays the value of A of the underlying asset at time t, and [1]

an amount B invested in a risk-free bond [0.5]

which is continuously compounded at an interest rate r. [0.5]

[Total 2]

(iii) Recall Ito's lemma:
$df(X) = \left(uf'(X,t) + \dot{f}(X,t) + 0.5s^2 f''(X,t)\right)dt + sf'(X,t)dW$. [0.5]

In order for this to be a martingale the dt term should be zero. [0.5]

This leads to the differential equation satisfied by f:

$u(t)\frac{\partial f}{\partial x} + 0.5s(t)^2\frac{\partial^2 f}{\partial x^2} + \frac{\partial f}{\partial t} = 0$. [1]

(Note, this is subject to the technical condition as described in Baxter and Rennie but it is not considered here.)

[Total 2]

(iv) Let $f(X(t),t) = c(X(t),t)$.

$\partial f / \partial x = A$. [0.5]

$\partial^2 f / \partial t^2 = 0$. [0.5]

$\partial f / \partial t = Bre^{rt}$. [0.5]

The differential equation in part (iii) becomes $a(t)A + 0 + Bre^{rt} = 0$, [0.5]

and the condition for $c(x,t)$ to be a martingale is $a(t) = -{}^{B}\!/_{A}\, re^{rt}$, with $A \neq 0$. [1]

In the case $A = 0$ then there is the trivial solution $B = 0$, [0.5]
or the solution with $r = 0$ (and B any value). [0.5]

[Total 4]

[Total 10]

The only part where candidates did not in general score full marks was in part (iv). Most candidates were able to differentiate the relevant terms and form a differential equation. The more difficult part was interpreting this. A potential approach to these types of question is to break down the solution into cases and ensure special cases are considered.

Leniency was shown by the examiners in awarding marks for any relevant conditions stated in part (iv).

Q3

(i) The company will issue ESOs to motivate employees to act in the best interests of the company's shareholders… [1]
… by providing a financial return to employees that aligns to that received by shareholders… [0.5]
.. and given that if employees perform their duties effectively, then this would be expected to lead to an increase in the share price. [0.5]
ESOs may also be a favourable way to remunerate employees due to accounting and/or tax treatment. [1]
ESOs provide a way to recruit talented employees … [0.5]
… and to retain employees as ESOs are an attractive company benefit. [0.5]
[Max 2]

(ii) K would typically be set at, or slightly below, the current share price… [0.5]
… to ensure that any marginal improvement in the company's share price leads to a financial reward for employees. [0.5]
K would not typically be set at a deep discount to the current share price as this is akin to providing free shares and could be costly to the company. [0.5]
Alternatively, K could be set significantly above the current share price to set a stretching target for employees … [0.5]
… but it is unlikely that all employees will be able to influence the performance of this company to such an extent… [0.5]
… and therefore this is more suitable for ESOs issued to senior management. [0.5]
T would typically be set in line with the business planning horizon of the business… [0.5]
… such as 3 to 5 years… [0.5]
… as this is the period over which employees' actions can reasonably influence the share price. [0.5]
Short term movements in the share price may be more associated with wider market movements and therefore a figure of T less than 1 year should be avoided. [0.5]
Employees may not place much value on ESOs if T is set too long into the future (e.g. 10 years). [0.5]
[Max 3]

(iii) When ESOs are exercised, the company issues more shares of its own stock and sells them to the option holder for the strike price… [1]

… which will dilute the proportion of the company that existing shareholders own… [0.5]
…unless the company issues 'shadow shares' to avoid this dilution. [0.5]
ESOs can also lead to significant immediate and deferred costs… [1]
… depending on the performance of the share price. [0.5]
In particular, certain types of ESOs have been associated with excessive pay packages when the share options have been exercised, leading to negative media coverage. [0.5]
ESOs may not fully align the incentives of shareholders and employees due to the optionality in the ESO giving the holder upside but no downside. [0.5]
[Max 2]

(iv) The market risk is that the ESOs might fall due to an adverse change in the level of the Company's share price. [1]
This could be due to Company specific factors or wider market changes, e.g. a stock market crash. [0.5]
Given that the share price is now double the strike price, the ESOs will likely represent similar market risk to that of the underlying Company's share price… [0.5]
… so the employees will be exposed to falls in the market price of the Company's shares… [0.5]
… with a Delta approximately equal to 1. [0.5]
The ESOs are generally non-transferrable from an individual with no secondary market. [0.5]
Therefore, the market risk will be present up until expiry of the ESOs. [0.5]
Employees are unlikely to place any value on the time value of the option. [0.5]
[Max 2]

(v) One of the key drivers of the market risk of the ESOs will be the financial performance of the Company, which may also influence the employees' future earnings potential (i.e. "human capital") and/or pensions… [1]
… so the employee may have a significant concentration risk in the Company's financial performance. [0.5]

The Company's share price appears highly volatile given the significant increase in such a short space of time, suggesting that future large falls could occur. [1]

There are limited ways in which most employees can practically hedge the market risk, given most employees may not have significant expertise in understanding the risks associated with ESOs and mitigating them. [1]
[Max 2]
[Total 11]

It was in the last three parts of the question where candidates did not score full marks. There was a wide range of possible solutions, but few candidates went beyond the basic points. Market risk is an important risk to consider with derivatives and understanding how it applies to derivatives and the ways of mitigating should be studied.

For reference, alternative and appropriately justified suggestions for K and T were allowed for in part (ii). Further, there was cross-marking between parts (iv) and (v) to ensure any valid points made were awarded marks even if written in the wrong part.

Q4

(i) Bootstrapping involves building up the pattern of zero coupon yields that is consistent with market prices of conventional bonds. [1]
The first zero-coupon yield is populated using the market price of the shortest dated instrument… [0.5]
… and then the second shortest instrument, combined with the first zero-coupon yield, is used to determine the next zero-coupon yield and so on. [0.5]
Linear interpolation is usually between observed values where necessary. [1]
An approach is also required for the points on the curve prior to the first quoted market price and post the last quoted market price of conventional bond… [0.5]
… such as assuming the curve is horizontal at these points. [0.5]
[Max 2]

(ii) Take the 4-year bond priced at 109.12. The price of coupons from t=1 to t=3 are: $\pounds 5\times\left[e^{-4\%}+e^{-2\times 3\%}+e^{-3\times 2.75\%}\right]=\pounds 14.117$ [1]
The equation of value is therefore:
$14.117+105\times e^{-4\times X\%}=109.12$ [1]
Hence $X=-\frac{1}{4}\ln\left[\frac{109.12-14.117}{105}\right]=2.501\%\sim 2.50\%$ [1]
(Note: full marks if just 2.50% is written down without any derivation.)
[Total 3]

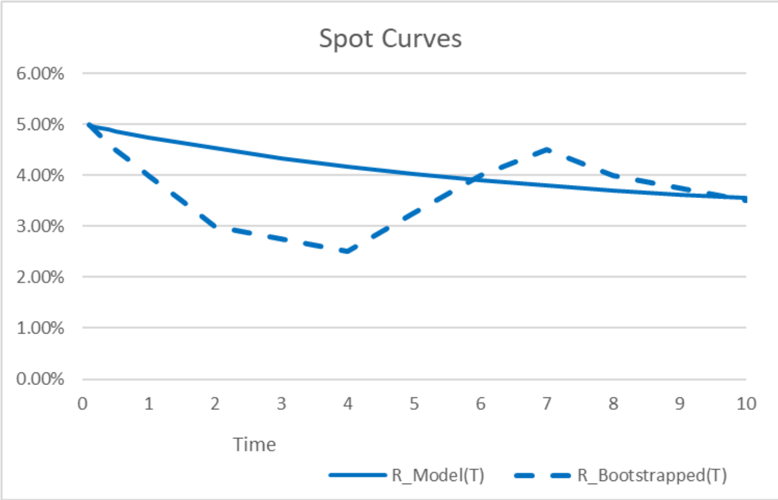
(iii) Given that no bond prices are quoted at this maturity, linear interpolation is used as an approximation between the nearest available points. [0.5]
Hence $R_{Bootstrap}(t=5)=average(2.50\%,4.00\%)=3.25\%$. [1]
[Note: an allowance is made for errors carried forward from (ii) and credit is given for a linear interpolation between t=3 and t=6 if (ii) is not attempted. As in part (ii), full marks are awarded for just writing down 3.25%]
[Total 1]

(iv) An equilibrium model for interest rates is one that is derived from a model of the economy. [1]

Then what this process for interest rates implies about bond prices and option prices is explored. [0.5]

The Vasicek model starts with assumptions about economic variables and derives a process for the short rate, and as such is an equilibrium model. [1]
[Max 2]

(v) Actual chart below:



Suggested mark scheme for chart (noting command word is *Sketch* not *Plot*):

- Axes appropriately labelled [0.5]
- R_Model values at t=0 and t=10 correct. [0.5]
- R_Model convex shape and decaying between t=0 and t=10. [1.5]
- R_Bootstrapped plotted broadly in line with figures provided in the question. [1]
- "Hump" of R_Bootstrapped above R_Model at t=7. [1]

[Max 4]

(vi) In order to investigate the impact of future scenarios on the value of the bond portfolio, the key requirement is to have a model that can generate plausible and appropriate dynamics in future periods. [1]

The current fit of $R_{Model}(t)$ to $R_{Bootstrap}(t)$ could be argued to be of secondary importance to how the model predicts interest rates will evolve in the future. [1]

Nonetheless, if the model cannot correctly calculate the current term structure of interest rates, it is questionable whether the future evolution of interest rates (and therefore bond prices) will be accurate. [0.5]

The Vasicek model parameterised is not consistent with the current term structure of interest rates… [0.5]

… and therefore will misprice the bond traders current holdings / imply arbitrage. [0.5]

There are some material divergences in the curves, particularly around t=2 to t=4 and t=7 which could coincide with large bond holdings the trader may have given the portfolio is so diverse. [0.5]

The Vasicek model may not produce a broad enough range of scenarios to capture the full risks in the portfolio in the future… [0.5]

… given it is a one factor model… [0.5]

… and can only produce a narrow range of possible yield curves. [0.5]

The shape of the current yield curve ($R_{Bootstrap}(t)$) implies that the future evolution of the yield curve and the behaviour of interest rates may be very complex, which would not be captured by the Vasicek model. [1]

The Vasicek model is easy to use and bond prices can be calculated analytically, making it more suitable for initial analysis. [1]

[Max 3]

(vii) Switching to the Hull and White model would involve changing the parameter *b* for a mean reversion level $\mu(t)$ which is not constant and is a function of time. [1]

$\mu(t)$ is derived to ensure the model exactly replicates the current yield curve (i.e. the modelled curve at time zero would equal $R_{Bootstrap}(t)$). [1]

[Total 2]

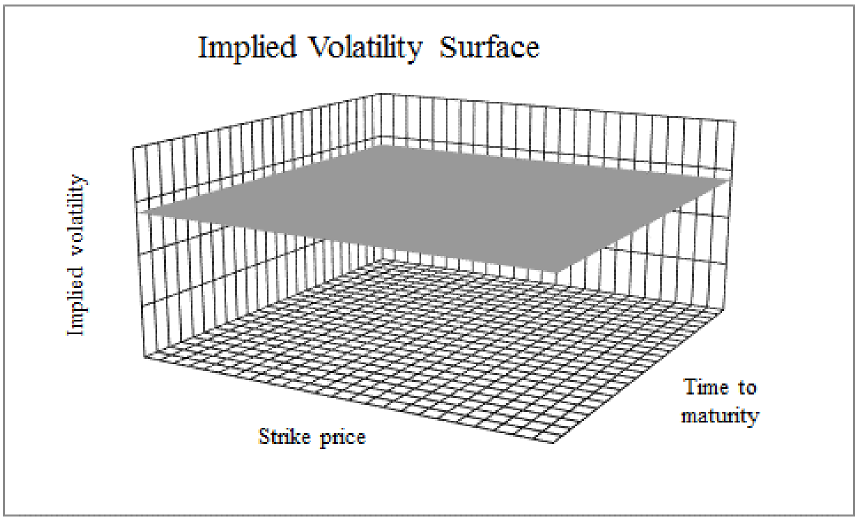
[Total 17]

Most candidates had clearly studied bootstrapping and interest rate models. As a result the first four parts were well answered. Candidates began to struggle to score well with the remaining parts.

The ability to compare models is an important consideration, not just within interest rates, and it is an area which has been examined before and will be again. In their studies candidates may benefit from critically thinking about the suitability of different models and indeed different derivatives.

Q5

(i)



[1 mark for the surface being constant for all times to maturity and strike price, 1 mark for the axes being labelled.]

[Total 2]

(ii) As implied volatilities for short dated terms to maturity are at a historic low it is likely that implied volatilities at longer dated terms to maturity will increase. [1]

This results in implied volatility being an increasing function of time to maturity. [0.5]

The implied volatility of an equity index generally decreases as the strike price increases. [0.5]

For a single stock the implied volatility is more of a balanced concave shape. [0.5]

This is referred to as a volatility skew. [0.5]

Possible reasons for the volatility skew include:
As a company's equity declines in value, the company's leverage increases. This means that the equity becomes more risky and its volatility increases. The opposite happens when a company's equity increases in value.

Option market participants are concerned of crashes in the equity market and price options accordingly.

Assumptions underlying the Black-Scholes do not hold in this market (*note: include an example for the full mark)*

[Max 1 mark for any of these reasons]
[Max 3]

(iii) In both cases the risk-neutral binomial tree has up/down values which depend on $e^{\sigma(t,S)\sqrt{\Delta t}} / e^{-\sigma(t,S)\sqrt{\Delta t}}$ where σ is the volatility at time t and equity price S, and Δt is the time interval.

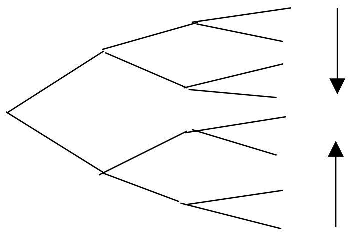
(a)
As the volatility strictly decreases with time the change in up and down jumps at each time interval decreases (apart from the first one which stays the same), hence the resulting binomial tree will be narrower in height. [1]

Also the resulting tree will generally not recombine as the volatility is different over each time interval. [1]

(b)
As the volatility strictly increases with equity price the change in up and down jumps at each time interval increases with increased equity price and decreases with decreased equity price, hence the resulting binomial tree will be narrower for smaller equity values and wider for larger equity values. [1]

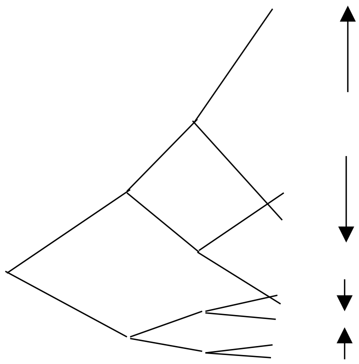
As in (a) the resulting tree will generally not recombine as the volatility is different for each equity price. [1]
[Total 4]

(iv) (a)



[1]

(b)



[1]
[Total 2]

(v) The trees do not recombine which means that the number of nodes which need to be evaluated is liable to become very large, and computationally intensive. [1]

The impact of modelling dividends and the interaction with variable volatility becomes more complex. [0.5]

Model calibration of the deterministic volatility function to the market implied volatility values is required. This will likely involve interpolation and some subjective judgement. [1]
[Max 2]

(vi) The option value is not given by the Black-Scholes formula due to stochastic volatility. [0.5]

This means there is unlikely to be a closed analytic formula and some form of numerical calculation method will be required. [0.5]

There are now two stochastic factors: one for the equity model itself and one for the volatility. [0.5]

There are many more parameters to estimate, and to calibrate to the market. [0.5]

This includes the correlation between the two stochastic factors. [0.5]

There are many forms that the stochastic differential equation for the volatility could take… [0.5]

… for example by looking at the similarities to interest rates

- geometric Brownian motion (Hull-White);
- CIR type equation; or
- Ornstein Uhlenbeck.

[0.5 marks each, max 1 mark]

Introducing a second stochastic term makes it more difficult to form riskless portfolios. In a Black Scholes model with constant volatility the one source of randomness can be hedged using the equity price and a riskless bond. As volatility is not traded another asset would be required to set-up a riskless portfolio in a stochastic volatility model, another option could be an example. [1]

Option prices are risk-neutral but volatility is not, a market price of risk is required for volatility. [0.5]

This makes hedging difficult in having to hedge stock and another asset within this model. [0.5]

Ultimately, there is no correct model and users will have to choose an appropriate model by weighing the pros and cons of each. [0.5]

[Max 4]
[Total 17]

There were many credible attempts to part (i), the key was recalling that the Black-Scholes volatility remains constant. In general the concepts of historical and implied volatility are important in the Black-Scholes model. The ability to understand these concepts in the context of an economy or a given asset class demonstrate a sound understanding of the Black-Scholes framework.

Parts (iii), (iv) and (v) covered binomial trees in a slightly different setting. Candidates made good attempts.

The considerations in choosing an appropriate (stochastic) model in an unfamiliar situation is one which does occur in actuarial modelling. Part (vi) is a good example of this. Candidates would do well to study the answer to this part and questions in previous years to understand the areas which are covered in model selection.

Q6

(i) For the two events it is clear that $P(A) = P(A \cap B) + P(A \cap \bar{B})$ [1]

Using the formula for conditional probability: $P(A \cap B) = P(A|B)P(B)$ gives the required solution: $P(A) = P(A|B)P(B) + P(A|\bar{B})P(\bar{B})$ [1]

[Total 2]

(ii) If $T_x > t$ then the Brownian motion W_t has yet to reach a value of x by time t. [1]

Therefore $W_t < x$, [0.5]

and hence $P(W_t \geq x | T_x > t) = 0.$ [0.5]

[Total 2]

(iii) If $T_x \leq t$ then $W_{T_x} = x$ (from the definitions). [1]

It is assumed that the independent increments of Brownian motion also holds true when one of the times is a random variable, resulting in: $W_{T_x+s} - W_{T_x} = W_{T_x+s} - x$ and $W_{T_x+s} - W_{T_x} \sim N(0,s)$. (For reference these are non-trivial results and the proofs are beyond the syllabus.) [1]

Once the Brownian motion reaches x, and by using the independent increments assumption above, the underlying normal distribution has equal chance of going up or down after the time interval s. [1]

[Total 3]

(iv) Using the formula from part (i) with the results from parts (ii) and (iii) gives:

$P(W_t \geq x) = P(W_t \geq x | T_x \leq t)P(T_x \leq t) + P(W_t \geq x | T_x > t)P(T_x > t)$, [0.5]

$P(W_t \geq x) = 0.5P(T_x \leq t)$.

Rearranging gives: $P(T_x \leq t) = 2P(W_t \geq x)$. [1]

As $W_t \sim N(0,t)$ the probability distribution function of the normal distribution can be used. [1]

$$P(T_x \leq t) = \frac{2}{\sqrt{2\pi t}} \int_x^\infty e^{-z^2/2t} dz,$$

$$= \frac{2}{\sqrt{2\pi}} \int_{x/\sqrt{t}}^\infty e^{-y^2/2} dy$$ (change of variables). [0.5]

[Total 3]

(v) The probability that the Brownian motion attains the value x in a finite amount of time is $P\left(T_x \le \infty\right)$. [0.5]

$P\left(T_x \le \infty\right) = \lim_{t\to\infty} P\left(T_x \le t\right)$, [1]

$$= \lim_{t\to\infty} \frac{2}{\sqrt{2\pi}} \int_{x/\sqrt{t}}^{\infty} e^{-y^2/2}\, dy,$$

$$= \frac{2}{\sqrt{2\pi}} \int_0^{\infty} e^{-y^2/2}\, dy,$$ [0.5]

$$= \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{-y^2/2}\, dy + \frac{1}{\sqrt{2\pi}} \int_0^{-\infty} e^{-(-y)^2/2}\, (-dy)$$ ($y \to -y$ in the second integral),

$$= \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{-y^2/2}\, dy + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} e^{-y^2/2}\, dy,$$ [0.5]

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-y^2/2}\, dy,$$

$= 1$ (the pdf of a N(0,1) is $e^{-y^2/2}/\sqrt{2\pi}$).

[0.5]
[Total 3]

(vi) In a Black-Scholes type model the underlying asset is modelled as geometric Brownian motion. By using Ito's lemma the distribution T_x can be used in understanding the first time geometric Brownian motion hits a certain value.

[1]

This distribution can then be used in pricing only certain types of exotic options, not all of them. [0.5]

The distribution is important for types of barrier options and lookback options. [0.5]

A good example is for knock-in options as they enter into existence when the stock price hits for the first time a certain barrier before option maturity. [0.5]

If a different stochastic model (for example a non Black-Scholes model) is used for the underlying asset price then the distribution T_x will be different to the distribution found above … [1]

… but it will still be useful in pricing exotic options. [0.5]
[Max 2]

(vii) Barrier options are expected to have a lower premium than a similar vanilla option (same: expiry date, exercise style, underlying asset and strike price). [0.5]

The payoffs will be identical if they do payout, but barrier options have the possibility of not paying out anything (i.e. expiring worthless despite being in the money when the similar vanilla option would have paid out). This is as a result of the knock-out barrier being activated making these knock-out types of options worthless or the knock-in barrier not being activated making these knock-in type of options worthless too. [1]

A suitable barrier option does not value future scenarios for the underlying asset which may be considered unlikely by an investor. [0.5]

This is not directly possible with a vanilla option as a vanilla option considers all future scenarios. [0.5]

This can also make the use of derivatives cheaper (based on the above logic of barrier options being cheaper than the vanilla option). [0.5]

This can also be viewed as an opportunity for an investor to sell barrier options that pay off only for unlikely future scenarios for the underlying asset. The investor would expect to make a profit from the premium and the option expiring worthless. [0.5]

As a result barrier options enable an investor to reflect their views on the behaviour of the future underlying asset. [1]

The arguments above can be combined to suggest that barrier options may be more useful in hedging than vanilla options for certain investors. [0.5]

This may be particularly relevant for investors who have strong views on future economic scenarios. [0.5]
[Max 3]
[Total 18]

The first four parts were generally well answered, and candidates were able to apply their knowledge of probability and the normal distribution to this situation.

Candidates struggled with the remaining three parts. Improvements to marks could have been obtained through better understanding of the exotic options. Questions have been asked previously about exotic options in different situations. Most of these questions use the core reading definition of a specific exotic option and then use this definition to explore some of its properties or uses in risk management. This, and previous, questions may be useful for candidates to study to understand the type of questions examined in this area.

Q7

(i) Arbitrage involves taking offsetting positions in two or more instruments to make a certain and risk free profit from mis-pricings. [1]

[*Alternatively, i.e. Arbitrage involves takes offsetting positions in two or more instruments to have positive probability of a profit with zero probability of making a loss, due to mis-pricings* *[1]]*

[Hull p15]

[Max 1]

(ii) *P, R* and *SwapR* have no cashflows before T_1. [0.5]

Let F_{12} = the forward swap rate between T_1 and T_2 calculated at T_1. [0.5]

At time T_1, consider what would happen to *R* and a combined portfolio of P + *SwapR* if F_{12} is greater or less than s_K …

$F_{12} > s_K$:

R would not be exercised and therefore generate no future cashflows. [0.5]

P would be exercised (as the option holder will receive a higher rate than they pay)… [0.5]

… but the cashflows would be equal and opposite to the cashflows on *SwapR*, so *SwapR* + P generates no cashflow overall. [0.5]

$F_{12} < s_K$:

… *R* would be exercised and therefore generate cashflows for each future time period identical to those of *SwapR*. [0.5]

P would not be exercised … [0.5]

… but the cashflows will still arise on *SwapR*. [0.5]

R therefore generates the same cashflows as *P* + *SwapR* in all future circumstances and therefore under the principle of no arbitrage, they must have the same value. [0.5]

This assumes that the detailed terms of the swaptions and swaps are consistent (e.g. similar day count convention, tenor)… [0.5]

… and that there are no arbitrage opportunities. [0.5]

[Max 3]

(iii) $V_P + V_{SwapR} = V_R$, therefore for $V_P = V_R$ then $V_{SwapR} = 0$ [1]

For $V_{SwapR} = 0$, s_K must equal the forward swap rate …. [1]

… between T_1 and T_2 … [0.5]

… with a compounding frequency equal to the tenor of the payments on the underlying swap. [0.5]

[Max 2]

(iv) $v\left(V_P + V_{SwapR}\right) = v\left(V_R\right)$. [0.5]

$v\left(V_P\right) + v\left(V_{SwapR}\right) = v\left(V_R\right)$. [0.5]

Now, $v\left(V_{SwapR}\right) = 0.$ [1]

Hence $v\left(V_P\right) = v\left(V_R\right).$ [0.5]

[Max 2]

(v) $LA \times \left[s_K \times \Phi(-d_2) - F_0 \times \Phi\left(-d_1\right)\right].$ [1]

$$d_1 = \frac{\ln(F_0 / s_K) + \frac{\sigma^2}{2} T_1}{\sigma\sqrt{T_1}},$$ [0.5]

$$d_2 = d_1 - \sigma\sqrt{T_1}.$$ [0.5]

Where:

L = principal, s_K is the -fixed rate on the underlying swap [0.5]

T_1 = payment time of option, σ = volatility of the forward swap rate, [0.5]

F_0 = the forward swap rate between T_1 and T_2 calculated at time zero. [0.5]

A is an annuity value $A = \frac{1}{m}\sum P(0,t_i)$ where *m* is the number of payments per year under the swap and t_i are the payment times of the underlying swap.

[1]

[Unit 13, Section 3.8, also Hull pp660-661]

[Max 3]

(vi) V_R should be determined in the current market conditions using (v), call this $V_{R,0.00\%}$. [0.5]

Then determine V_R when interest rates decrease by 0.01% by recalculating the inputs in equation (v), notably F_0, A, d_1 and d_2, and evaluating the formula, call this $V_{R,-0.01\%}$. [1]

Then determine Delta as $V_{R,-0.01\%} - V_{R,0.00\%}$ [0.5]

Alternatively, Delta could be defined as $\dfrac{V_{R,-0.01\%} - V_{R,0.00\%}}{0.01\%}$ [0.5]

[Max 1]

(vii) Exact chart shown below when volatility = 20% and defining Delta as $V_{R,-0.01\%} - V_{R,0.00\%}$ per (vi). Suggested mark scheme:

Labelled axes. [0.5]

Delta swap starting at a high positive number… [0.5]

… and decaying away as interest rates increase. [0.5]

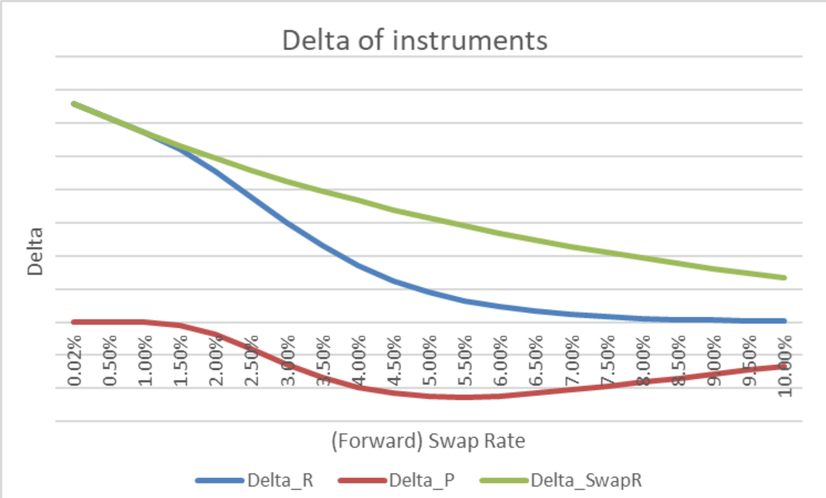
Delta_R starting at a similar figure to Delta_Swap.. [0.5]

… then decaying at a faster rate than Delta Swap to zero beyond S_K. [1]

Delta_P starting at zero… [0.5]

… then becoming negative as S_K is approached… [0.5]

… with a "hump" around S_K … [0.5]
… before decaying towards -Delta_SwapR [0.5]



[Max 3]

(viii) The risk department could request:
Information on the Greeks such as … [0.5]

- Delta
- Gamma
- Vega
- Rho [0.5 each, Max 1.5]

The Value at Risk (VaR) of the portfolio… [1]
… at a different confidence intervals and time horizons… [0.5]
… and details of the underlying model used to calculate VaR. [0.5]
Stress testing of the portfolio to understand the movements in certain scenarios… [1]
… for example, very extreme market movements or historical events. [0.5]

Detailed information on the contractual terms of the trades… [0.5]
… such as the trade confirmations / ISDA agreements… [0.5]
… and how the swaption and swap arbitrage trades are "paired". [0.5]

(Note: up to 1 mark can be awarded for providing details about what is contained within the ISDA agreements.)
[Unit 16, Section 1]

[Max 4]

(ix) The arbitrage trade requires all three transactions in *P*, *R* and *SwapR* to be executed at the same time. Liquidity could deteriorate part way through execution which could mean that execution in one or more instruments is completed at unattractive prices. [1]

The bank will need to post collateral in liquid form on all the contracts. If there are timing differences in collateral payment/receipt on the contracts underlying the arbitrage trade, the bank may have to liquidate other assets, which could be unattractive if liquidity is constrained. [1]

[Max 1]

[Total 20]

[Paper Total 100]

Part (vi), (vii) and (viii) proved the most challenging to candidates in this question, and they were some of the most difficult parts on the paper. The use of derivatives for risk management is an important part of the course. Understanding how market, credit and liquidity risk can arise in the use of derivatives is a key concept in answering some of the more difficult questions in a paper.

END OF EXAMINERS' REPORT

