INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2021

SP6 – Financial Derivative Principles
Specialist Principles

Introduction

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Sarah Hutchinson
Chair of the Board of Examiners
December 2021

A. General comments on the *aims of this subject and how it is marked*

The aim of Financial Derivatives Principles (SP6) is to develop a candidate's ability to understand different types of financial derivatives and their uses, the markets in which they are traded, methods of valuation of financial derivatives, and the assessment and management of risks associated with a portfolio of derivatives. It builds on material covered in earlier subjects, particularly Loss Reserving and Financial Engineering (CM2).

Candidates are reminded to ensure that their answers are sufficiently detailed to demonstrate their understanding, as well as to make sure that more obvious points are still made to gain full marks. The model solutions are intended to reflect the level of detail that a high scoring candidate might be able to produce. For many questions there are more marks available than the question requires to achieve full marks. This reflects that the examiners will give credit for valid alternative solutions, particularly in questions focussed on higher level skills.

Candidates who give well-reasoned points, not in the marking schedule, are awarded marks for doing so.

B. Comments on *candidate performance in this diet of the examination.*

The majority of candidates made a good attempt at the exam and were able to score at least some marks in each of the 7 questions.

In general, candidates demonstrated good knowledge of the core reading material and its application to familiar situations. This was evident in nearly all of the questions as most candidates managed to pick up at least half of the available marks. Additional comments are provided after each question below.

C. Pass Mark

The Pass Mark for this exam was 60
38 presented themselves and 15 passed.

Solutions for Subject SP6 – September 2021

Q1
(i)
Three broad categories of derivative traders are:

Hedgers	[½]
using derivatives to reduce a risk they face	[½]
Speculators	[½]
seeking exposure to a risk they desire	[½]
Arbitrageurs	[½]
take offsetting positions to lock in an arbitrage profit	[½]

(ii)
Exchange traded contracts are
Contracts traded via an exchange [½]
With the contracts specified by the exchange [½]
That are standardized [½]
Over-The-Counter (OTC) contracts are:
Contracts traded between two parties over the counter [½]
Contracts are as agreed between parties [½]
Contracts can be customized [½]
Once the contract is agreed it can be cleared bilaterally or presented to a CCP [½]
Furthermore, there are differences in marketability / ability to close our early [½]
As well as differences in how margin works [½]
[Marks available 4½, maximum 3]

(iii)
The pay-off of a long position of a currency forward is $S_T - K$ [1]
In relation to a currency forward S_T and K would be the then prevailing spot
exchange rate and the strike agreed at the outset [1]
The pay-off of a long position would be multiplied by the notional amount agreed [1]
The pay-off is linear in nature and zero for $S_T = K$ [1]
[Marks available 4, maximum 3]

(iv)
The company receives profit in foreign currencies which exposes it to currency
fluctuations [½]
Currency forwards allow an investor to gain exposure to currencies [½]
The company should choose currency forwards that move in the opposite direction
of the exposure that it has, so that it can hedge that back to its home currency USD [½]
Therefore the company should sell / short the foreign currency forward [1]
in exchange buying / long its home currency the USD [½]
The company would need to use a range of forwards contracts that correspond to
each of the currencies that it has exposure to [½]
The company would have to think about the size of contracts, to match its
expected cash flows [½]
The company would have to think about the expiry of the contracts [½]
[Marks available 4½, maximum 3]

(v)
Sources of basis risk
The future profit steams will not be exactly known in advance [½]
Business and other risks of pharmaceutical selling medicine may influence income [½]
therefore timing [½]
and size of forwards needed can only be estimated [½]
Modelling might be needed and assumptions to be made [½]
Some currencies might not be available to hedge in the market [½]
Or be prohibitively expensive [½]
Or can only be approximately hedged with another (but hopefully correlated) liquid
currency forward [½]
This can be particularly relevant for Emerging Market currencies [½]

There will be costs involved in hedging [½]
Collateral implications lead to the hedge not being perfect [½]
There may already be a natural hedge, e.g. costs in the foreign countries [½]
The price of medicine (and hence profit) might be set relative to the USD equivalent
so that the currency risk is already naturally dampened [½]
Issues related to counterparty credit risk [½]
Marks to be awarded for other suitable answers [½]
[Marks available 7½, maximum 4]

(vi)
Shareholders of the company may be able to hedge themselves, so no need for
the company to do this [1]
Hedging may not be the norm in the markets and competitive pressures [1]
Hedging may lead to a worse outcome and career risk for the treasurer [1]
[Marks available 3, maximum 2]
[Total 18]

> *Most candidates scored well on this question. For each part there were generally more answers which also scored marks not shown in the marking schedule. Candidates that provided complete answers were able to score well on this question. Most candidates were able to achieve at least half, and often more, of the available marks.*

Q2
(i)
An index-linked bond is a bond where the coupon and final redemption payments are
increased with a certain index, typically inflation [1]
Bonds could also be linked to another index, for example a survivor index [½]
For example, the UK government issues index-linked bonds linked to RPI inflation [½]
Index-linked bonds typically have a lag with respect to the inflation index, in the
UK this is 8-months for old style bonds and 3 months for newer style bonds [½]
Index-linked bonds can be used by investors that have liabilities that are increasing
with inflation [1]
Such as for example life annuity or defined benefit pension schemes [½]
For liabilities that increase with LPI, using an index-linked bond will still give an
approximate hedge [½]
[Marks available 4½, maximum 3]

(ii)
To diversify its sources of debt financing [½]
To help investors seeking to hedge inflation linked liabilities [½]
To offer more choice to investors and hence become more appealing [½]
To signal that inflation is under control in this country [½]
To diversify the cost of debt between fixed and real bonds [½]
Because issuing index-linked bonds might be cheaper [½]
Award ½ mark for other reasonable explanations [½]
[Marks available 3½, maximum 3]

(iii)
The government might have looked at the following markets:
CPI swaps [½]
Looked at market observable European CPI swap curves and project the current
base CPI rate forward [½]
CPI bonds [½]
Looked at other European countries that issue bonds linked to CPI rates and use
inflation implied in those bond prices [½]
Economic scenario generator / Monte Carlo simulations / stochastic model [½]
Used an economic forecast model to project CPI rates [½]
Historical data [½]
The government might have used past experience, inflation trends and projected
this forward [½]
[Marks available 4, maximum 3]

(iv)
Any reasonable proposed coupon rate that is zero or slightly positive
(certainly less than 1%) [½]
The current real yield over the term of the bond would be a good starting point to
consider the coupon [½]
The current real yield is negative over all maturities [½]
It would be impractical to issue a bond with a negative coupon [½]
As this would imply a requirement for the government to collect (rather than pay)
coupons [½]
It might therefore be more reasonable to have a zero coupon rate [½]
Or a slightly positive coupon rate [½]
[Marks available 3½, maximum 3]
[Total 12]

> *Part (i) proved straightforward for most candidates.*
>
> *Parts (ii) and (iii) were generally answered well by better prepared candidates.*
>
> *Most candidates struggled with part (iv) which was a question focused on higher order skills, as indicated by the command verb used.*

Q3
(i)
Interest rate floors are designed to provide insurance [½]
against the rate of interest on a floating-rate note falling below a certain level
known as the floor rate [½]
The floor provides a payoff [½]
equal to the excess of the interest determined at the floor rate on the floating rate
over the interest [½]

(ii)
The payoff is: $0.25 \times 15{,}000 \times \max\{0.02 - R, 0\}.$ [1]

Here R is the interest rate relevant to the floor expressed as a quarterly compounded rate   [½]
R may be at the end of the expiry period in a standard floorlet as part of a floor (for a floating rate note as an example) or   [½]
Some form of average interest rate over the period   [½]
Other suitable examples should be awarded   [½]

[Marks available 3, maximum 2]

(iii)
Using Black's equation:
Price = $15,000 \times 0.25 \times P(0,1.75)\left[K\Phi(-d_2) - R\Phi(-d_1) \right].$ [½]
Here P(0,1.75) is the discount factor of the zero coupon bond and is

equal to $e^{-1.75 \times 0.03} = 0.94885$ [½]

$K = 0.02$ is the floorlet strike, $R = 0.0189$ is the current year forward price of the interest rate over the period of the floorlet and $\sigma = 0.19$ the volatility   [½]
$d_1 = \left[\ln(0.0189/0.02) + (1.5 \times 0.19^2 / 2) \right] / \left(0.19 \times \sqrt{1.5} \right) = -0.12675$,   [1]

$d_2 = d_1 - 0.19 \times \sqrt{1.5} = -0.35945$.   [½]

$\Phi(-d_1) = 0.55043$ and $\Phi(-d_2) = 0.64037$.   [½]

The value of the call option is
$15,000 \times 0.94885 \times 0.25 \times \left[0.02 \times 0.64037 - 0.0189 \times 0.55043 \right] = 8.55$ [½]

(iv)
Flat volatilities are how floor volatilities are usually quoted in the market   [½]
There is an advantage in keeping prices in line with market convention   [½]
They are simple to use,   [½]
in that only one volatility is needed to value a cap or floor,   [½]
and the value of the cap of floor at a later date is easy to recalculate as only one volatility is needed to be obtained from the market   [1]
Spot volatilities are how traders like to see their volatility surface,   [½]
as it enables them to spot any pricing anomalies or arbitrages   [½]
Traders also can directly compare spot volatilities with those from options on the short interest rate futures (e.g. Eurodollar, Euribor, Eurosterling)   [½]
Traders can also convert from flat to spot volatilities   [½]
Both volatility curves often show a hump in the short-dated values, which is thought to be due to greater trading activity taking place in the tenors just after the very short ones   [1]
which are controlled by the central banks   [½]
It is not clear whether there is a best approach - the spot volatilities approach should be theoretically more correct   [1]
but flat volatilities are more stable and heavy supply/demand for specific floor tenors sometimes means they are more representative of market prices   [1]

[Marks available 8½, maximum 4]
[Total 12]

Most candidates scored well on this question. Some candidates could have gained more marks on part (iv) by ensuring that their answers were sufficiently detailed to demonstrate their understanding.

Q4

(i)

$$\text{PerCent} = \frac{\partial V / V}{\partial S / S} = \frac{\partial V}{\partial S}\frac{S}{V} = \frac{\partial \ln V}{\partial \ln S}.$$ [1]

Here V is the price of the option, [½]

and S is the price of the underlying [½]

(ii)

$$\text{PerCent} = \Delta \frac{S}{V}.$$ [1]

(iii)

The Black Scholes formula for the price of a call option, $C_t = S_t N(d_1) - Ke^{-r(T-t)}N(d_2)$,

here C_t is the price of the call at time t, S_t is the underlying price at time t, K is the strike price, r is the risk-free rate, T is the maturity time and N, d_1, d_2 are the standard definitions [½]

Recall that $\Delta_{call} = N(d_1)$, so $C_t = S_t \Delta_{call} - Ke^{-r(T-t)}N(d_2)$. [1]

The second term is positive due to the properties of K, the exponential function and N being a cumulative distribution function, so $C_t \leq S_t \Delta_{call}$. [1]

Rearranging gives $PerCent_{call} \geq 1$. [½]

(iv)

From the basic properties of investing in an option there is a leverage effect, that is profits or losses can be magnified

Consider a stock currently trading at a price of 100. Suppose an at the money call option has a strike price of 100 on this stock with price 9.41 [½]

Further assume it has a delta of 0.60 [½]

The value of the PerCent is $= 0.60 \times (100 / 9.41) = 6.38$ [½]

If an investor holds 5 of these shares then the investor's stake is 500 [½]

Further, if the price of the share increases by 1% (100 to 101) then the value of the investor's shares increases to 505, an increase of 5 [½]

Instead if the investor has 5 of the options on this share then the investment is worth $5 \times 9.41 = 47.05$ [½]

If the share price increases by 1% (100 to 101) then the value of the option increases approximately to $9.41 + delta = 9.41 + 0.60 = 10.01$ [½]

The value of the investor's investment increases to $5 \times 10.01 = 50.05$, an increase of $50.05 - 47.05 = 3$ [½]

However, $(1 + PerCent / 100) \times 47.05 = (1 + 6.38\%) \times 47.05 = 50.05$ [½]

Therefore the PerCent is a measure of the leverage [½]

[Marks available 5, maximum 4]

(v)

Using the hint in the Q, let p be the real world probability of the up state (u) in the binomial model and (1-p) the real world probability of the down state (d). Interest rates are assumed to be 0 [1]

The volatility of the underlying asset is by definition: $\sigma_{underlying} = \sqrt{p(1-p)}\,(u-d)$ [1]

The volatility of the call option value is by definition:

$\sigma_{option} = \sqrt{p(1-p)}\,\dfrac{c_u - c_d}{c}$, where c_u is the value of the call in the up state, c_d is the Value of the call option in the down state and c is the value of the option at the previous node [1]

From the definition of PerCent:

$PerCent = (S/c)\Delta,$

$= (S/c)\dfrac{c_u - c_d}{(u-d)S},$

$= \dfrac{c_u - c_d}{c(u-d)}.$

[1]

Here S is the value of the underlying at the previous node

Substituting in the expressions for σ_{option} and $\sigma_{underlying}$ gives, $PerCent = \sigma_{option} / \sigma_{underlying}$

or rearranging $\sigma_{option} = \sigma_{underlying} \times PerCent$. [1]

[Marks available 5, maximum 4]

[Total 14]

Parts (i) and (ii) were well answered, but the remaining parts of the question proved to be more challenging. The algebraic derivations and underlying mathematical theory are an important part of SP6 and are well covered in Baxter and Rennie's: Financial calculus: an introduction to derivative pricing. Candidates do need to be familiar with these and to have practised these derivations under exam conditions.

Q5

(i)

The main use of derivatives in a life insurance company is for hedging [½]

They use these derivatives to manage and mitigate risks [½]

Such high-level risks include [½]

Interest rate risk [½]

Credit risk [½]

Currency risk [½]

Longevity risk and [½]

Equity risks [½]

These risks arise in both the asset portfolio and the liabilities [½]
There are also more specific risks which require the use of derivatives, depending
on the lines of business and objectives of the company: [½]
Improving the matching of assets and liabilities; [½]
Reducing the risk of options and guarantees; [½]
Solvency management; [½]
Improving the predictability of earnings; and [½]
Reducing the level of any risk-based capital requirements [½]
Derivatives might also be used for tax management purposes [½]
Also, they could be used in enhancing returns, for example through creating
synthetic assets [½]
[Marks available 9½, maximum 5]

(ii)
Risk is likely to arise in longevity and mortality hedging [½]
using longevity derivatives that often do not precisely match the insured population. [½]
Other valid examples, such as basis risk arising from interest rate or equity risk, are
acceptable with half mark for stating an example and half mark for a brief description [½]
[Marks available 1½, maximum 1]

(iii)
One of the roles of the board is to understand risks within company. The use of
credit ratings can aid in understanding the risks of assets within the portfolio, and [1]
hence on the balance sheet [½]
The board also sets the risk appetite and using these internal credit ratings can help
the board understand its risk appetite, and [1]
making sure it stays within its limits [½]
These ratings can also assist in capital management [½]
The ratings can help the board in understanding the downside risks associated with
the assets in the portfolio [½]
With a roust methodology the credit ratings can be used to objectively monitor the
asset portfolio over time [½]
They can reduce the reliance on external credit rating agencies [½]
[Marks available 5, maximum 3]

(iv)
External rating agencies have complex methodologies and vast experience and
expertise in credit ratings [½]
It is unlikely that a life insurer will have the same depth of experience and data as
an external rating agency [½]
As a result the methodologies may be much simpler [½]
This may not be a problem as the life insurance company may be able to update their
ratings and methodologies much quicker [½]
This is due to the internal ratings being only required for the life insurance company [½]
whereas for external rating agencies there are whole markets and financial
institutions which use their ratings. Any changes to methodologies or ratings can
have a major impact [1]
[Marks available 3½, maximum 3]
[Total 12]

> *Most candidates scored well on this question. They were able to generate a wide range of acceptable answers to the different parts.*

Q6
(i)
Given a binomial price process S which is a Q martingale. If there exists another process N which is also a Q martingale, then there exists a previsible process φ such that:

$$Ni = No + \sum_{k=1}^{i} \varphi k \Delta Sk \qquad [1]$$

(ii)
Pricing derivatives using binomial trees requires the construction of a replicating portfolio which provides the same pay-off as the derivative [1]
The BRT provides a link between the discounted asset price process S and the expected value of the discounted derivative claim , under the martingale measure Q for the discounted asset price process S via φ , the previsible process [1]
The previsible process φ, is the ratio of the branch widths between the discounted asset price process S and the expected value of the discounted derivative claim [1]
In the replicating portfolio, φ is the amount of the stock which should be held at each node
[1]
[Marks available 4, maximum 3]

(iii)
q = (1-0.95)/(1.1-0.95) = 0.33 for all nodes [1]
1-q = 1-0.33 = 0.67 for all nodes [½]

Price process S

t=	0	1	2
			12.10
		11.00	
	10		10.45
		9.50	
			9.03

[2]

E(X|Fi)

t=	0	1	2
			7.10
		6.00	
	3.21		5.45
		1.82	
			-

Therefore,
f = 0
e = 5.45

d = 7.10
c = 1.82
b = 6.00
a = 3.21   [2]
[Marks available 5½, maximum 4]

(iv)
Because r=0, the BRT is satisfied if the ratio of branch widths between process S and
E(X|Fi) are consistent at each node ie φ is the same for an upwards shift and
downwards shift   [1]

Using the naming convention given in the Q.

Node	Si-Si-1	E(X|Fi)-E(X|Fi-1)	Ratio
b	1	2.79	2.79
c	-[½]	-1.39	2.79
d	1.1	1.1	1
e - down	-[½]5	-[½]5	1
e - up	0.95	3.63	3.82
f	-0.48	-1.82	3.82

[1 mark for each column correct]
[Marks available 4, maximum 3]
[Total 11]

Question 6 was reasonably well answered by most candidates with most scoring more than half marks.

Part (iv) proved to be most challenging.

Q7
(i)
The value of a European call option increases as the volatility increases and vice versa   [½]
This is because the higher volatility means the price of the underlying is more likely
to reach extreme values, either on the upside which would increase the payoff from
the option   [½]
or the downside which has little or no impact due to the asymmetric payoff of
the option   [½]
[Marks available 1½, maximum 1]

(ii)
Put-call parity provides a relationship between the prices of European puts and calls
of the same term and strike price within the Black-Scholes model   [½]
As it based on the principle of no-arbitrage, put-call parity also holds for the
market prices of puts and call   [½]
Comparing the put-call parity for the BS prices and the market prices shows that
the pricing error given by the BS model should theoretically be the same for puts
and calls   [½]

As all other terms used in the pricing formula are consistent, we can deduce that the
implied volatility for a European call is the same for an equivalent European put option [½]
Therefore,
the relationship between the implied volatility and strike price should be the same
for equivalent puts and calls [½]
the relationship between the implied volatility and term to maturity should be the
same for equivalent puts and calls [½]
the volatility surface is then shown to be the same for equivalent puts and calls [½]
[Marks available 3½, maximum 3]

(iii)
The implied volatility of equity index options as a function of the strike price is
typically downward sloping (aka the volatility skew) [½]
The implied volatility is generally:
an increasing function of the term to maturity if volatility is considered to be lower
than average [½]
a decreasing function of time to maturity if the level is higher than average [½]
The volatility skew becomes flatter as a function of time, creating a downward
sloping surface both as a function of time and strike price [½]

(iv)
With respect to strike:
It is suggested that this arises because of
the leverage effect - as the value of equity decreases the overall leverage of the
company increases. This makes the company riskier and hence more volatile [½]
Crashophobia - investors are more concerned about the prospect of a market crash
and therefore attach additional value to downside protection [½]
With respect to term:
 It is generally assumed that the level of volatility will return to average levels over time [½]
[Marks available 1½, maximum 1]

(v)
Future and options based on the VSTOXX index allow investors to take a position
on the future level of volatility in European equity market [1]
Investors could take a speculative position on the level of volatility in the market
by buying (selling) VSTOXX futures [½]
Or options [½]
they would do this if the if they thought that the level of volatility will increase
(decrease) more than is currently anticipated [½]
They may also use the volatility derivatives as part of a hedging programme [1]
For example, volatility indices are typically inversely correlated to equity prices
and therefore their derivatives can be an effective hedge when equity markets fall
significantly [½]
They may also be used a portfolio diversifier or to provide protection to multi-asset
portfolios [1]
Cross market positions on volatility can also be set up to take positions on the relative
levels of volatility in different markets [½]
eg a VIX vs VSTOXX position would give a payoff which would be dependent on
the relative levels of implied volatility in S&P options and EuroStoxx options [½]

[Marks available 6, maximum 3]

(vi)

The significant increase in the level of the VSTOXX is reflective of a significant and sharp fall in the stock market [1]

As the VSTOXX is based on 30 day options [½]

we can deduce that the volatility surface at near terms to maturity would increase in level following a spike in the VSTOXX [½]

Given the high degree of uncertainty among investors, the volatility skew could be expected to steepen [½]

as investors price in the probability of further falls in the equity market [½]

The initial VSTOXX level of 12 suggests that implied volatility was lower than average before the catastrophe [½]

suggesting that the volatility surface was upward sloping a function of term to maturity [½]

After increasing to 80, it could be assumed that the level would fall back to more normal levels in the future. Therefore, the surface could be expected to now be downward sloping [1]

However, given the high degree of uncertainty in the days following the catastrophe, the downward slope may not be very steep initially [½]

[Marks available 5½, maximum 4]

[Total 14]

Question 7 was overall one of the more challenging questions. Part (i) was relatively straightforward and well answered.

For the remaining parts only better prepared candidates were able to gain full marks

Q8
(i)
Ito's lemma states that the function g(S,t) follows the process

$$dG = \left(\frac{dG}{dS}\mu S + \frac{dG}{dt} + \frac{1}{2}\frac{d^2G}{dS^2}\sigma^2 S^2\right)dt + \frac{dG}{dS}\sigma S dW \qquad [1]$$

From the Q, the stock S follows the process

dS = μSdt + σSdW

and $V = \frac{dg}{dS}$ x S - G

Then consider a change in the value of the portfolio, V, over short period of time

$$dV = \frac{dg}{dS}dS - dG \qquad [½]$$

$$= \frac{dg}{dS}(\mu Sdt + \sigma SdW) - \left(\left(\frac{dG}{dS}\mu S + \frac{dG}{dt} + \frac{1}{2}\frac{d^2G}{dS^2}\sigma^2 S^2\right)dt + \frac{dG}{dS}\sigma S dW\right)$$

[1]

Importantly, the dW terms are the same and therefore cancel in the above equation to give [½]

$$dV = -\left(\frac{dG}{dt} + \frac{1}{2}\frac{d^2G}{dS^2}\sigma^2 S^2\right)dt \qquad [1]$$

As there is no volatility term in the above equation, we can see that the portfolio is indeed riskless [½]

[Marks available 4½, maximum 4]

(ii)
As the portfolio is riskless, its change in value during this infinitesimally small period
of time will be the risk free return it generates [1]

$$dV = V \times rdt = (\frac{dg}{dS} \times S - G.) \times rdt$$ [½]

where r is the risk-free rate [½]

Comparing the two equations for the change in the value of the portfolio, reordering
and cancelling dt, gives us the BS PDE

$$\frac{d}{dS}\mu S + \frac{dG}{dt} + \frac{1}{2}\frac{d^2G}{dS^2}\sigma^2 S^2$$ [1]

[Total 7]

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*This question was one of the trickier ones, with more of a binary outcome across
candidates. Candidates tended to either score close to full marks or just one or two marks
in each part.*

[Paper Total 100]

END OF EXAMINERS' REPORT