INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

22 September 2021 (am)

Subject SP6 - Financial Derivatives
Specialist Principles

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

1 (i) Define three broad categories of derivative traders. [3]

(ii) State the main differences between exchange-traded contracts and over-the-counter traded contracts. [3]

A large US-based pharmaceutical company generates income by selling medicine in a wide range of developed and emerging markets and is concerned about the impact of currency fluctuations relative to its home currency (the US Dollar).

(iii) Describe the payoff of a currency forward. [3]

(iv) Describe how the company could use currency forwards to hedge its exposure to currency fluctuations. [3]

(v) Discuss sources of basis risk the company may encounter when implementing a hedge using currency forwards. [4]

(vi) Write down two arguments against hedging foreign currency risk for this company. [2]

[Total 18]

2 (i) Describe how index-linked bonds can be used by pension schemes to hedge certain types of liabilities. [3]

The government of a European country that already issues a wide range of conventional bonds decides to issue index-linked bonds. The pay-outs of the index-linked bonds will be linked to a measure of European inflation, EU CPI.

(ii) Describe why the government may issue index-linked bonds in addition to conventional bonds. [3]

In order to consider the pricing of these new index-linked bonds, the government observes current spot rates on conventional bonds and estimates the future values of EU CPI. The EU CPI Index relating to year 0 (base) is 105.69. The results are shown below:

Year	*Spot rate*	*EU CPI Index (estimated)*
1	–0.5099	106.61
2	–0.4823	107.57
3	–0.4402	108.57
4	–0.3854	109.61
5	–0.3212	110.71
6	–0.2507	111.84
7	–0.1769	113.00
8	–0.1020	114.20
9	–0.0280	115.43
10	0.0438	116.70

(iii) Suggest three ways that the government might have estimated the future CPI index rates. [3]

(iv) Propose, with reasons, an appropriate coupon rate for a newly issued 10-year index-linked bond. [3]

[Total 12]

3 (i) Describe the uses of interest rate floors. [2]

Consider a floorlet that floors interest rates for 3 months starting in 1.5 years based on a principal value of amount 15,000 and a floor interest rate of 2% quarterly compounded.

(ii) Give the payoff from the floorlet at expiry, defining any notation used. [2]

It is assumed that the implied volatility of the forward interest rate is 19% p.a., the zero-coupon bond yield curve is currently flat at 3.0% p.a. (continuously compounded) at all tenors and the forward interest rate for the period covered by the floorlet is 1.89% p.a. (quarterly compounded).

(iii) Calculate the price of this floorlet. [4]

There are two main methods of estimating volatility for use in pricing a floor:

- spot volatilities: the use of a different implied volatility rate for each floorlet depending on the expiry of the floorlet
- flat volatilities: the use of a single implied volatility for a floor.

(iv) Compare the use of spot and flat implied volatility curves in the pricing of floors. [4]

[Total 12]

4 An Actuary defines a new Greek, PerCent, to be the percentage change in the option value per percentage change in the price of the underlying asset.

(i) Write down PerCent in terms of the option value and underlying asset price only, defining any symbols used. [2]

(ii) Write down PerCent in terms of the delta (Δ) of the option. [1]

(iii) Determine the lower bound of PerCent for a long vanilla call option, where the underlying asset is non-income paying. [3]

(iv) Explain, using a basic numerical example of an option, why PerCent can be considered as a Greek measuring the leverage of an option. [4]

Let σ_{option} be the volatility of the call option's price, $\sigma_{underlying}$ the volatility of the call option's underlying asset price and $PerCent$ the value of PerCent of the call option.

(v) Justify, using a one-step binomial model or otherwise, why $\sigma_{option} = \sigma_{underlying} \times PerCent$. [4]

[Total 14]

5 (i) Describe why a life insurance company might use derivatives. [5]

(ii) Give an example of how basis risk may arise when a life insurance company uses derivatives. [1]

A life insurance company has a large portfolio of assets. The Board of the insurance company has developed a system to assign credit ratings to assets in their portfolio.

(iii) Explain why the Board might have adopted this approach. [3]

(iv) Discuss how the approach taken by the insurance company may compare to the approach taken by external rating agencies. [3]

[Total 12]

6 (i) State the Binomial Representation Theorem (BRT) for discrete time stochastic processes. [1]

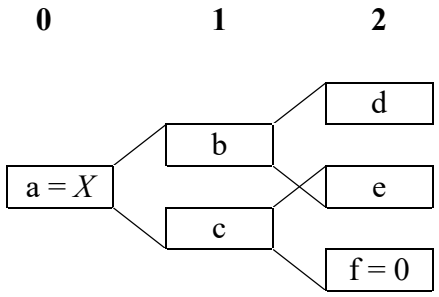
(ii) Explain the relevance of the BRT when pricing derivatives using binomial trees. [3]

A claim, X, is contingent on the process, S. The payoff from X at time $t = 2$ is

$$X = \begin{cases} (S - 5) & \text{if } S \geq 10, \\ 0 & \text{if } S < 10. \end{cases}$$

The process, S, is assumed to follow a discrete random walk under probability measure, Q, where its value either increases by 10% or decreases by 5% at each time step i ($i = 1, 2$). The starting value of S is 10 and the risk-free rate is 0.

(iii) Determine the probability measure, Q, and the values of nodes 'a' to 'e' in the following tree to value claim, X. [4]



(iv) Verify that the expected value of X under Q satisfies the BRT for times $i = 1, 2$. [3]

[Total 11]

7 (i) Describe how implied volatility, σ, affects the price of a European call option. [1]

A volatility surface is a three-dimensional representation of the implied volatility as a function of its strike price and term to maturity.

(ii) Outline why the volatility surfaces for European put options and European call options on a given asset are the same. [3]

(iii) State the typical shape of the volatility surface for equity index options. [2]

(iv) Explain why the shape is typically that given in part (iii). [1]

The VSTOXX index is a European volatility benchmark index that measures the implied volatility of 30-day options on the EuroStoxx index.

(v) Suggest why investors may wish to use derivatives based on the VSTOXX. [3]

The VSTOXX index increases from 12 to 80 following a severe natural catastrophe in mainland Europe.

(vi) Discuss the impact of this event on the shape of the volatility surface for EuroStoxx options. [4]

[Total 14]

8 A stock, S, is assumed to follow the stochastic price process:

$$dS = \mu S dt + \sigma S dW,$$

where

- μ is the stock's expected rate of return.
- σ is the volatility.
- dW is Brownian motion.

Let G be a derivative on stock S that follows the price process $g(S,t)$.

(i) Explain why a portfolio that consists of $\frac{dg}{dS}$ units of stock S and one short position in derivative G is considered to be riskless. [4]

(ii) Explain the relevance of the riskless portfolio in part (i) in the derivation of the Black–Scholes partial differential equation for G. [3]

[Total 7]

END OF PAPER