

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2016

Subject ST2 – Life Insurance Specialist Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Luke Hatter
Chair of the Board of Examiners
December 2016

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Life Insurance Specialist Technical subject is to instil in successful candidates principles of actuarial planning and control, and mathematical and economic techniques, relevant to life insurance companies. The student should gain the ability to apply the knowledge and understanding, in simple situations, to the operation, on sound financial lines, of life insurance companies. The life insurance products covered by this subject exclude health and care insurance products covered by the Health and Care Specialist Technical subject.
2. The Examiners' Report covers more points than would be expected to get full marks. This is so that alternative approaches to questions by different candidates can be accommodated. Candidates are expected to show knowledge of the relevant content of the Core Reading, but those who tailor their answer to the specifics mentioned in the question will score more highly than those who answer in a more generic way.
3. Candidates who give well-reasoned points, not in the marking schedule, are awarded marks for doing so.
4. In this diet the scoring for the exam was done out of 200 and therefore the mark scheme shows a total of 200 marks available for the paper.

B. General comments on *student performance in this diet of the examination*

As with previous papers, questions that focussed on knowledge of the Core Reading were well answered by well-prepared students. In the higher mark application questions, candidates tended to restrict themselves by generating only a narrow range of points rather than thinking more widely, e.g. questions 2 part (ii), 3 and 5. Stronger candidates considered the specifics of the question and used these in their answers e.g. in question 4 part (i) and 5 part (iv).

C. Pass Mark

The Pass Mark for this exam was 59%.

Solutions

- Q1**
- (i) Data at previous investigation
 PLUS business that has come onto the books
 LESS business gone off the books
 EQUALS data at current investigation. [2]
 [Maximum 2]
- (ii) Number of contracts [1]
 Sum assured (or policy benefit relevant for the policy type) [1]
 Premium [1]
 Attaching bonus (for with profits business) [1]
 Number of units split by unit fund (for unit-linked business) [1]
 [Maximum 4]
- (iii) The average sum assured should be appropriate for the business [1]
 and consistent with the previous investigation. [1]
 The average premium should be appropriate for the business [1]
 and consistent with the previous investigation. [1]
 The ratio of policy benefit to the premium should be appropriate for the [1]
 business and consistent with the previous investigation. [1]
 The value of units purchased by premiums should be consistent with the [1]
 corresponding revenue account items. [1]
 The value of units encashed to pay benefits should be consistent with the [1]
 corresponding revenue account items. [1]
 The internal unit movements, for example charge encashments, should be [1]
 consistent with the surplus emerging in the year. [1]
 The persistency rates should be consistent with the previous investigation as a [1]
 check on the movements data. [1]
 Ratio of bonus to sum assured should be fairly consistent with the previous [1]
 investigation. [1]
 [Maximum 6]
- (iv) Benefit values (e.g. sum assured or unit values) are zero or negative for sum [1]
 assured. [1]
 Benefit values (e.g. sum assured or unit values) are very large. [1]
 Premiums are very large. [1]
 Premiums are negative [1]

Date of birth is not possible.	[1]
Benefit date is not possible.	[1]
Unusual retirement age or date	[1]
Contract start date is not possible.	[1]
For any of these sets of dates, there is an unusual distribution.	[1]
For example, dates of birth have a large proportion in a particular month.	[1]
Unknown or inappropriate value...	[1]
...e.g. smoker indicator.	[1]

[Maximum 6]

[Total 18]

This question was largely knowledge-based and was answered fairly well by most candidates, although part (iii) was not as well answered as the other parts. The strongest candidates considered both averages and consistency with previous periods in part (iii), as well as generating a wide range of distinct points in part (iv).

- Q2**
- (i) To increase the marketability of its products [1]
 And so increase sales [1]
 And hence make greater profits (due to higher volumes sold) [1]
 Distributors may have demand for them. [1]
 Competitors might offer them... [1]
 ...and so the company needs to also offer them to remain competitive and not lose market share [1]
 Or if competitors do not offer them, it could be a niche product design [1]
- It believes it can make additional profit from the guarantee/option by offering them at an additional premium that remains attractive. [2]
 e.g. disagrees with market view on mortality but can charge market price for it (or other relevant example) [1]
 It could be trying to improve customer retention ... [2]
 ...as it makes the policy more valuable and therefore more to give up [1]
 To meet a specific customer need. [1]
 e.g. changing needs or offering flexibility. [1]
 To provide policyholder protection and hence reduce potential reputational risk / negative publicity. [2]
 It may be a statutory/regulatory requirement. [1]

[Maximum 8]

- (ii) There is a risk that the cost of the guarantee is higher than expected... [1]
 ...and so the premium charged to the customer is not sufficient... [1]
 ...and so the company makes a loss [1]
 This may be due to:
 Derivatives (if used for hedging) cost more than expected [1]

Investment performance is unfavourable... [2]
 ... or interest rates are unfavourable (depending on the nature of the
 guarantee)... [2]
 ... and the guarantee is not matched by either investment or derivatives [2]
 This could be due to the long duration of the guarantee and so investments are
 not available to match this in the market (or other example) [1]
 For guaranteed annuity options on unit-linked business, there is a risk that
 investment performance is higher than expected when the guarantee bites... [1]
 ... so that the guarantee applies to a greater unit fund value [1]
 More customers than expected may take up the guarantee [2]
 This may be due to mortality being lighter than expected and so more
 policyholders are alive at the guarantee date [1]
 Or may be due to better persistency than expected and so more policies are in-
 force at the guaranteed date [1]
 Withdrawals may be selective (in relation to investment markets)... [1]
 ... i.e. are lower than expected when investment performance is poor and the
 guarantee looks like it will "bite". [2]
 Or withdrawals are higher than expected when investment performance is low
 if there is a guarantee on withdrawal. [2]
 There is an anti-selection risk from the policyholder's choice of investment
 fund. [1]
 The policyholder may choose a more risky fund knowing that there is the
 under-pin from the guarantee. [2]
 GAO also gives exposure to mortality/longevity risk. [1]
 There may be additional operational risks relating to derivative use [1]
 Or due to the added complexity of administering the guarantee [1]
 There is increase reputational risk. [1]
 There is also increased model risk in relation to valuing it. [1]
 Derivative use introduces potential counterparty risk [2]
 There may be additional mis-selling risk if the guarantee is not well explained
 or can be misinterpreted [2]
 There may be additional data risk in relation to obtaining sufficient data with
 which to price the guarantee... [1]
 ... e.g. appropriate correlations or volatilities for stochastic modelling [1]
 The administration or development expenses incurred when a policyholder
 takes up the guarantee could be greater than expected [1]
 There is additional risk from selling more business with the investment
 guarantee than expected, [1]
 Particularly in relation to the need to set up higher than expected reserves and
 capital requirements. [1]
 There is a risk that new business volumes are lower than expected due to the
 price being high. [1]
 There is the risk of regulatory changes, e.g. new methodology for guarantee
 reserving. [1]

There is also a risk of lapse and re-entry. [1]

[Maximum 14]

[Total 22]

Part (i) was generally answered well, with most candidates covering the key points. In part (ii), the strongest candidates generated the required breadth of points covering the additional risks on investment guarantees, including the related selection and data risks. However, some candidates wrote about guarantees in general, without tailoring their answer specifically to investment guarantees as required.

Q3 The risk profile of the company and experience will change for the business written through the new approach. [1]

The changes will become increasingly more significant as the proportion of business written through the revised distribution approach increases compared to the overall total business. [1]

The company's target market will change with the move from the direct salesforce to a direct marketing approach. [2]

Typically products sold through a direct marketing approach tend to be less sophisticated as advice is not being provided as part of the sales process. [2]

This may lead to the target market being less financially sophisticated. [1]

Direct marketing targets those who are less affluent / have a lower level of income. [1]

The level of underwriting used by the company is likely to reduce (and be simplified) compared to that employed for business sold through a direct salesforce. [2]

The level of risk exposure for the company will therefore be increased in relation to under-disclosure or non-disclosure. [2]

The customer may not disclose information at the point of sale either through mis-interpreting a question or by deliberately withholding information. [1]

The risk of mis-estimating the level of anti-selection is increased. [1]

The risk of accepting policies on standard terms when they should be rated or declined is increased. [2]

However, the additional underwriting risk will be mitigated by the fact that policies would expect to have lower levels of benefits provided than those offered through a direct salesforce. [2]

The assumptions used to price and value the business will need to be reviewed with the change in the target market. [1]

For example:

- Mortality

With the likely change in the target market to less financially sophisticated customers / reduction in underwriting... [1]

... the mortality experience would be expected to be heavier than for those lives using the direct salesforce, or may be lighter mortality if more affluent customers are targeted. [2]

- Persistency

The level of lapses and surrenders under the new approach is likely to be different from the new sales approach compared to that from the direct salesforce. [1]

Persistency experience may be worse as no advice is being offered on the directly marketed products compared to the advice received from the direct salesforce. [1]

This may lead to more customers taking out an inappropriate policy as a result of not understanding the costs or benefits. [1]

Persistency experience may improve as the policies will tend to be smaller and more affordable than a policy sold through a direct salesforce leading to a lower level of customer not being able to afford the premiums. [1]

However this may be offset by the target market being less affluent than that for a direct salesforce. [1]

Persistency experience may improve as the policyholder is more likely to have initiated the sale. [1]

However, persistency may worsen if the level of financial underwriting used reduces. [1]

- Expenses

In the longer term expenses would be expected to be lower than for a direct sales force... [2]

... as no commission is likely to be paid for these products. [1]

And because direct marketing should involve a lower level of underwriting. [1]

In the short term, expenses will be higher as the company will incur development costs for setting up new approach. [2]

For example there will be administration system development, literature printing and marketing. [1]

Or there may be redundancy costs [1]

Per policy expenses will differ due to the expected differences in persistency and mortality experience. [1]

The smaller policy size for directly marketed policies will mean that fixed expenses will be a greater proportion of the per policy expense. [2]

The company will have no data on which to base the new pricing assumptions... [1]

...especially with the reduction in the level of underwriting [1]

- ... so will have to rely on either industry data available or seek assistance from reinsurers... [1]
- ... which introduces additional mis-pricing risk. [2]
- There is also a new risk of mis-estimating the costs of developing and implementing the new channel. [1]
- There is the increased risk of lapse and re-entry from existing policyholders... [2]
- ... if similar levels of benefit are available through the direct marketing approach to those available from the direct sales force but at cheaper premiums. [1]
- There is an increased risk of the salesforce contacting their clients and persuading them to switch their policies to the new company they are working for... [2]
- ... leading to poor persistency from the existing block of business. [2]
- Due to the completely new target market and distribution approach, there is a greater risk of mis-estimating the volume of new business... [1]
- ... and mix of new business. [1]
- Competition risk may be higher or lower.... [1]
- There is the potential for lower volumes if the price is higher than competitors, e.g. due to margins being included in pricing assumptions [1]
- ... depending on the extent to which other companies use direct marketing... [1]
- ... and the existence of platforms such as internet comparison sites [1]
- The risk of mis-selling by the direct sales force is reduced by the move to direct marketing,... [1]
- ... however poor literature wording may lead to higher risks. [1]
- Similarly, there will be changes in the related level of reputational risk. [1]
- Direct marketing may involve the use of third parties e.g. internet hosting sites [1]
- Which would increase counterparty risks [1]
- Regulatory risk exposure may change if the regulator is more active in regulating direct marketing *[Or any sensible example]* [1]
- Overall, the small average expected policy size will reduce the exposure to risk per policy. [1]
- However, total risk exposure could increase if large volumes are sold. [1]
- There is greater operational risk due to changes made to systems. [1]

[Maximum 28]

The question was reasonably well answered overall. The majority of candidates identified the change in the target market and the associated risk characteristics. Stronger candidates expanded their answers to consider the impact of the change in the distribution channel on the experience and linked the risk characteristics with the experience impact.

Q4 (i) Profitability

- Premiums need to be sufficient to cover expenses and benefits provided [1]
- And to provide sufficient profit margin... [1]
- ...could be on NPV, IRR or DPP... [1]
- ... to meet the requirements of shareholders. [2]
- The extent to which the product will contribute to overheads will need to be considered. [1]
- And the recovery of the product development costs. [1]
- Including the costs of setting up the new distribution channel. [1]
- The volume of new business expected to be sold will therefore also be important. [1]
- The mix of business is important to set risk factors and model points [1]
- Assumptions will be required for pricing... [1]
- ...including longevity, expense, expense inflation and investment return. [2]
- The backing assets chosen to back the annuities will be important for pricing. [1]
- These are likely to be bonds. [1]
- May choose to invest in corporate rather government bonds... [1]
- ... in order to be able to offer more attractive rates / gain more profit. [2]
- The availability of sufficiently long duration assets to avoid reinvestment risk is a consideration. [1]

Marketability

- Consider the target market and how this will change moving from a direct salesforce to direct marketing. [1]
- The product needs to be attractive to the intended market. [2]
- Given that this is a new product and new distribution channel, the company will need to research the likely market. [1]
- Company will need to consider what direct marketing approach to take. [1]
- May want to offer additional features e.g. impaired life annuity. [2]
- e.g. index-linked alternative. [2]
- e.g. different benefit payment frequencies. [1]
- Consider if surrender values are to be offered. [1]
- Consider offering both single life and joint life. [1]
- However, the proposed distribution channel suggests that the product variations may need to be kept relatively simple. [2]

Competitiveness

- Need to consider competitor products and premium rates. [1]
- Immediate annuity business is typically a competitive market... [1]
- ... with frequent and published league tables. [1]
- Rates need to be lower than or consistent with those of competitors to attract business. [2]
- Particularly if sold via the internet and there are comparison sites. [1]

Financing requirements

- Need to consider the capital needed for writing the new business... [1]
- ... including to cover any solvency capital requirements. [1]
- Should not have particularly large financing requirements on day 1 since is single premium. [1]
- However, need to bear in mind any differing capital requirements for a conventional product compared to existing unit-linked products. [2]
- The cost of capital should be factored into the price. [2]
- Some designs (e.g. guaranteed payment periods) may be more capital intensive than others. [1]
- The total capital available in the company to support the product... [1]
- ... may affect the volume that can be written... [2]
- ... which may have implications for the chosen pricing level. [1]

Risk characteristics

- Need to consider the relative risk characteristics of each proposed design. [1]
- E.g. greater longevity risk plus inflation risk for an index-linked version. [2]
- E.g. greater anti-selection risk depending on the extent to which underwriting is used and whether impaired life annuities are offered. [2]
- The overall amount of risk will impact the pricing margin or setting the risk margin in the discount rate... [2]
- ... particularly the level of longevity risk. [1]
- Also need to allow for counterparty risk if corporate bonds are used as investments. [1]
- There is also increased data/pricing risk due to not having written business for this type of product before... [1]
- ... so the company does not have its own experience to use. [1]
- It also is unlikely to have experience in estimating future longevity improvements. [1]
- Could use external data [1]
- It is therefore likely to need to price using higher risk margins... [1]
- ... at least until it gains sufficient credible own experience. [1]
- There is potential to use reinsurance to help reduce any capital requirements... [1]
- ... or to mitigate some of the longevity or expense risks or to gain technical assistance. [1]
- The cost of reinsurance will also affect the pricing. [1]
- Consider any other mitigation techniques that could be used. [1]

Guarantees

- Need to consider whether to offer any guarantees. [1]
- For example a return of premium on death within a certain period or a guaranteed payment period. [1]
- This would increase the attractiveness of the product. [2]

And reduce the likelihood of complaints from recipients of the estate following the early death of an annuitant. [1]

Sensitivity of profits

Need to consider the sensitivity of profits. [2]
E.g. to different mix of business expected – by size of policy [1]
and by mix of policyholders. [1]
Also need to consider the impact of experience item movements on profit, e.g. inflation, interest rate, [1]
longevity, expense movements on profit. [1]

Cross-subsidies

Ideally should reduce cross-subsidies... [1]
... by pricing according to several rating factors. [1]
Need to look at the profit profile of smaller and larger policies and consider any cross-subsidies between them. [1]
Need to consider if there are any cross-subsidies between this product and the unit-linked product. [1]

Administration

This is a new product and hence administration systems... [1]
... and admin processes will need to be amended to cope with annuities. [1]
Consider whether to outsource. [1]
As the company is starting from nothing, there should not be administrative constraints on the chosen design. [1]
However, it may be that certain design features are more difficult to implement than others and this should therefore be considered when choosing a suitable design. [1]
The cost of these changes will need to be factored into the pricing for the product. [1]

Regulatory requirements

Need to consider regulatory requirements in terms of treating customers fairly. [1]
And in terms of any limits on design or price. [1]
Need to consider any taxation implications. [1]

[Maximum 40]

- (ii) The company should target the marketing at those who are close to retirement. [2]
- Could use mailshots. [1]
e.g. to a mailing list for members of organisations or clubs that are typically joined by those close to retirement. [1]
e.g. any sensible example of activities (gardening/golf/cruise holidays etc.). [1]
And could use adverts in specific venues or events that are frequented by those close to retirement. [2]
e.g. any sensible example (doctors' surgeries/club meeting halls/shows or events). [1]
- Could use telephone sales. [1]
e.g. by using lists of potential customers provided by external companies. [1]
- Could use press advertising. [1]
e.g. in magazines that are specifically targeted at the above market. [1]
- Could use television advertising... [1]
...or radio advertising... [1]
... during programmes that are targeted at the above market. [1]
e.g. possibly during daytime to attract those who may be partly retired and have not yet fully purchased their annuities. [1]
- Likely to use internet selling. [1]
e.g. advertising on websites that are targeted at the above market. [1]
The company is likely to ensure that rates are available on product comparison websites. [1]
- The company could use social media [1]
- Could use direct marketing to employers... [1]
... particularly those with a significant number approaching retirement. [1]
Or provide workplace visits. [1]
- Could target existing customers [2]
- The company could provide educational sessions on retirement planning. [1]
or sponsor events that will be of interest to the above market. [1]
e.g. gardening shows/golf events/bowling competitions (any sensible example) [1]

The company could offer promotions for early purchasers, [1]
e.g. discounted rates or free gifts (any sensible example). [1]

[Maximum 12]

[Total 52]

This question was answered fairly well by most candidates. In part (i), most well-prepared candidates considered a reasonable range of factors and expanded suitably on these. Candidates who scored well in part (ii) considered a wide range of potential direct marketing methods. However, some candidates also covered other marketing methods, e.g. direct sales force, which was not appropriate to the scenario described.

- Q5** (i) At inception the aggregate asset share will equal the total single premiums received as new business [1]
Less commission of 5% of these single premiums. [1]

[Maximum 2]

- (ii) The aggregate asset share at the end of year t will equal the aggregate asset share at the end of year $t - 1$ [1]
Less annuity benefits paid out at the start of year t [1]
All accumulated at the actual rate of return earned on the backing investments during year t ... [2]
... reduced by 0.2% for expense charges. [1]

The annuities paid are the actual annuities paid [1]
as opposed to the expected, since the asset share takes account of actual mortality [1]
However, if the guarantee bites then the annuity deduction should be reduced to remove the additional guarantee cost [2]

There are no further shareholder transfers to deduct since the shareholder profit is allowed for within the deduction of charges rather than actual expenses. [1]

[Maximum 6]

- (iii) The product may not meet some customer needs. [2]

There is a risk that the annuity benefit income on this product turns out to be insufficient or less than expected. [2]
...especially with the uncertain bonus amounts [1]
This could be due to low investment returns... [1]
... meaning that the annuity payable has decreased. [1]

Or it could be due to longevity being worse than expected or mortality being better than expected... [1]
 ... which reduces the asset shares and hence the bonuses and benefits. [1]
 Or inflation could be higher than expected... [1]
 ... and higher than the returns on the investments backing the asset shares. [1]
 This would erode the value of the benefits [1]

There is a risk that the policyholder does not understand the product as it is complex. [1]

The customer may not understand that there is no surrender value / may be disappointed not to be able to access their investment if their circumstances change. [1]
 There is a risk that the policyholders' dependants will not have sufficient financial security on early death. [1]

There is a risk that the company becomes insolvent... [1]
 ... and thus unable to meet the guaranteed benefits in full. [1]
 The policyholder may mismanage the money they receive and run out of cash between annual payments. [1]
 There is a risk that tax changes erode the benefit value. [1]

[Maximum 10]

(iv) Longevity risk remains if asset shares fall below the value of the guarantees. [2]

But typically will be lower since the risk is passed on to the policyholder via the asset shares [2]

There is a risk of anti-selection... [2]
 ... in that more healthy lives than expected may take the contract out... [1]
 ... which may result in the asset shares not supporting the guaranteed annuity [1]

If investment returns are lower than expected... [1]
 ... then this will reduce bonuses payable to the policyholder / result in a reduction in annuity amounts... [1]
 ... which may result in complaints from policyholders or reputational risk... [2]
 ... and lower new business volumes. [1]

Low investment returns will also reduce the expense charges taken. [1]

Much lower than expected investment returns will result in the guarantee biting ... [2]
 ... and therefore significant potential losses to shareholders. [1]
 There may be difficulty in finding matching assets for the guarantee [1]

There is a similar risk of counterparty default...	[1]
... if corporate bonds are used for investments	[1]
The insurer is exposed to mis-selling risk...	[2]
... if the policyholder did not understand that their annuity could reduce....	[2]
Or if the policyholder did not understand that they were exposed to longevity risk.	[1]
This could also lead to reputational risk.	[1]
And potentially regulatory intervention.	[1]
There could be high capital requirements due to the level of guarantees.	[1]
There is therefore a risk of higher than expected new business,...	[1]
... assuming that there is no limit on the tranche,...	[1]
... which could lead to significant capital strain.	[1]
High new business volumes may also generate administration strain.	[1]
There is a risk of lower than expected new business...	[1]
... resulting in fixed costs not being met.	[1]
There is a risk of competitors offering better bonus rates...	[2]
... or better guarantee levels.	[1]
There is a risk of actual expenses being higher than expected...	[1]
... or expense inflation being higher than expected...	[1]
... which would generate direct losses for shareholders.	[1]
Particularly since this is the only method of shareholder participation.	[1]
There is a risk of inadequate actual experience data for the determination of asset shares...	[2]
... for example not having the most recent investment return information	[1]
Or for pricing the product...	[1]
... particularly if it has not been written before.	[1]
There is a risk of writing smaller than expected policy sizes ...	[2]
... since charges are expressed as a % of asset share ...	[1]
... and these need to cover fixed expenses	[1]
There is a risk that management puts pressure on to declare higher bonuses than are sustainable...	[2]
... in order to maximise new business volumes	[1]
There are operational risks relating to the ongoing payment of the annuities	[1]
And relating to getting the launch of the tranche done on time for the 1 January payment date	[1]
And relating to the bonus setting process	[1]
The company is exposed to the risk of fraud in terms of not being informed of the death of an annuitant	[2]
There may be regulatory risk in terms of the changes to annuity purchase legislation,	[1]
Or from changes to the capital requirements.	[1]

There may be a risk of tax changes, i.e. no longer tax-free [1]
 ... to the company, which would impact profits [1]
 ... to the policyholder, which would impact new business sales [1]

[Maximum 34]

(v) Let i be the required investment return

First considering each individual policyholder:

We need $A(2) = A(1)$,
 i.e. $(A(1) / 1.03) \times (1 + B(2)) = A(1)$
 So we need $B(2) = 3\%$
 So we need to solve for $A(2) = A(1)$ or $B(2) = 3\%$ [2]

$$AS(1) = 0.95 \times G \times (1 + i - 0.002) - A(1) \times (1 + i - 0.002) \quad [2]$$

$$\text{And since } 0.95 \times G = A(1) \times \ddot{a}_x$$

We get

$$AS(1) = A(1) \times \ddot{a}_x \times (1 + i - 0.002) - A(1) \times (1 + i - 0.002) \quad [2]$$

Across the tranche we want to have $A(2) = A(1)$ such that the total asset shares at the end of the year = total present value of annuities for those left in force:

$$\begin{aligned} \text{So } AS(1) &= \sum \{A(2) \times \ddot{a}_{x+1} \times (1 - q_x)\} \\ &= \sum \{A(1) \times \ddot{a}_{x+1} \times (1 - q_x)\} \end{aligned} \quad [2]$$

where q_x is the expected mortality rate over the first year

Substituting across totals we get

$$\begin{aligned} AS(1) &= A(1) \times \ddot{a}_{x+1} \times (1 - q_x) \\ &= A(1) \times \ddot{a}_x \times (1 + i - 0.002) - A(1) \times (1 + i - 0.002) \end{aligned}$$

Using the recursive formula for an annuity factor on an individual basis:

$$\ddot{a}_{x+1} = (\ddot{a}_x - 1) / (1 - q_x) \times 1.028 \quad [1]$$

We therefore get:

$$\begin{aligned} A(1) \times [(\ddot{a}_x - 1) / (1 - q_x) \times 1.028] \times (1 - q_x) \\ = A(1) \times \ddot{a}_x \times (1 + i - 0.002) - A(1) \times (1 + i - 0.002) \end{aligned}$$

Or rearranging:

$$A(1) \times [(\ddot{a}_x - 1) \times 1.028] = A(1) \times [(\ddot{a}_x - 1) \times (1 + i - 0.002)] \quad [2]$$

$$\text{So } i = 3\% \quad [4]$$

i.e. the company needs to earn 3% for the customer to receive the same annuity in 1 year's time.

Alternative solution (more explanatory):

We require $A(2)$ to equal $A(1)$ [2]

From the question, we know that the asset share at the end of year 1 needs to be equal to the summation of $A(2)$ ($= A(1)$) multiplied by an annuity due factor at the end of year 1 at 2.8% [1]

for the policies left in-force at the end of that year i.e. need to multiply by $(1 - q_x)$ [1]

The asset share at the end of time 1 equals the premium paid less $A(1)$ less commission, all rolled up at the net of expenses interest earned i.e. at $(i - 0.2\%)$ [1]

The premium is given as $A(1)$ times an annuity factor at time zero (\ddot{a}_x) grossed up for commission [1]

It follows (by substituting the premium for the above equivalent expression) that the asset share at the end of year 1 is $A(1)$ multiplied by an annuity in advance at time 0 at 2.8% less $A(1)$ all rolled up at $i - 0.2\%$ [1]

So, cancelling out $A(1)$, we need

$$[\ddot{a}_x \text{ at } 2.8\% - 1] \times (1 + i - 0.2\%) = \ddot{a}_{x+1} \text{ at } 2.8\% \times (1 - q_x). \quad [2]$$

Since actual q_x is assumed to be as expected... [1]

... this equation works if $1 + i - 0.2\% = 1.028$ [1]

Hence $i = 3\%$ [4]

[Maximum 10]

(vi) $B(2)$ is calculated by equating across the total in force:

$$AS(1) = 10,000 / 1.03 \times (1 + B(2)) \times \ddot{a}_{x+1} \times (1 - q_x) \quad [1]$$

$$AS(1) = 0.95 \times G \times 1.048 - 10,000 \times 1.048 \quad [1]$$

And since $0.95 \times G = 10,000 \times \ddot{a}_x$

We get

$$AS(1) = 10,000 \times \ddot{a}_x \times 1.048 - 10,000 \times 1.048 \quad [1]$$

So substituting we get:

$$\begin{aligned} 10,000 / 1.03 \times (1 + B(2)) \times \ddot{a}_{x+1} \times (1 - q_x) \\ = 10,000 \times \ddot{a}_x \times 1.048 - 10,000 \times 1.048 \end{aligned} \quad [1]$$

Recursive approach across the totalled tranche (i.e. not just per policy) gives, as for part (iv):

$$\ddot{a}_{x+1} = (\ddot{a}_x - 1) / (1 - q_x) \times 1.028$$

Substituting gives

$$\begin{aligned} 10,000 / 1.03 \times (1 + B(2)) \times (\ddot{a}_x - 1) / (1 - q_x) \times (1 - q_x) \times 1.028 \\ = 10,000 \times \ddot{a}_x \times 1.048 - 10,000 \times 1.048 \end{aligned} \quad [1]$$

Rearranging/cancelling out we get:

$$(1 + B(2)) / 1.03 = 1.048 / 1.028 \quad [2]$$

$$B(2) = 1.048 / 1.028 \times 1.03 - 1 = 0.05004 \text{ or } 5.004\%$$

$$\begin{aligned} \text{And so } A(2) &= 10,000 / 1.03 \times (1 + B(2)) = 1.048 / 1.028 \times 10,000 \\ &= 10,195 \end{aligned} \quad [2]$$

Alternative solution:

From part (v), if the company earned 5%, then the asset share at the end of year 1 is 10,000 multiplied by an annuity in advance at time 0 at 2.8% less 10,000 all rolled up at 4.8%. [2]

The asset share at the end of year 1 can therefore be re-expressed as:
 $10,000 \times 1.048 / 1.028 \times [\ddot{a}_x \text{ at } 2.8\% - 1] \times 1.028$ [2]

And since:

$$\begin{aligned} 10,000 \times 1.048 / 1.028 \times [\ddot{a}_x \text{ at } 2.8\% - 1] \times 1.028 &= 10,000 \times 1.048 / 1.028 \times \\ [\ddot{a}_{x+1} \text{ at } 2.8\%] \times (1 - q_x) \end{aligned} \quad [2]$$

We can deduce that $A(2) = 1.048 / 1.028 \times 10,000$ [2]

So $A(2) = 10,195$ [1]

[Maximum 8]

- (vii) If returns are negative, then the guarantee kicks in and the annuity payable is
 $A(1) / 1.03 = 10,000 / 1.03 = 9,709.$ [2]

[Maximum 2]

- (viii) The guarantees could be onerous. [1]
 At present only the shareholders pay for the guarantee. [1]
 Therefore charges could be made to asset shares in respect of the guarantee. [2]
 However, this would reduce the bonuses payable to policyholders [2]
 Which would reduce the attractiveness of future tranches of the product... [1]
 Depending on what competitors do... [1]
 And may generate complaints from existing policyholders. [1]
 It would also make it more likely that the guarantee would bite. [2]

The company could similarly consider making a charge for the cost of capital... [1]
 ... to support contracts in the early years [1]
 Particularly since the guarantee will make this higher. [1]

The company could also consider making a charge to contribute to free assets. [1]

There is no smoothing of bonuses here so there is less need for such a contribution. [2]
 However, it may be needed in order to allow greater investment flexibility. [1]
 Explicit charge for initial expense [1]
 Take into account any regulatory constraints on charge levels [1]

[Maximum 8]

[Total 80]]

This question was only answered well by a minority of candidates. In part (ii) stronger candidates recognised that no further premiums are paid past time zero, actual investment returns would be used, and utilised the information in the question. Parts (iii) and (iv) were the better answered parts of the question, although not all candidates tailored the risks closely enough to the provided details of the product. Parts (v) and (vi) proved challenging for candidates, with only the strongest building on the answer from part (ii) and using the hint in the question. Part (vii) proved more difficult for students, although was answered correctly by those few who were able to step back from the mathematical detail and demonstrate understanding of the underlying nature of the product. Part (viii) was generally not answered very well, with only the stronger candidates thinking about the various different charges that could be applied to asset shares.

END OF EXAMINERS' REPORT