INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2017

Subject ST6 – Finance and Investment
Specialist Technical B

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Luke Hatter
Chair of the Board of Examiners
July 2017

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Finance and Investment Technical B subject is to instil in successful candidates the ability (at a higher level of detail and ability than in CT8) to value financial derivatives, to assess and manage the risks associated with a portfolio of derivatives, including credit derivatives and to value credit derivatives using simple models for credit risk.

2. This subject deals with mathematical techniques for the valuation and risk management of derivatives, together with some aspects of their practical application.

3. Different numerical answers may be obtained to those shown in these solutions, depending on whether figures were obtained from tables or from calculators. Allowance was made for these minor differences. However, candidates may have lost marks for using excessive rounding or showing insufficient working.

4. Derivative theory is an interesting but very exacting subject which needs to be tackled in a precise and disciplined manner. Questions that appear unfamiliar can, with some clear thinking at the start of the exam, be found to be straightforward examples of well-known techniques. Thorough preparation of the course material is essential, however. This subject also requires quite a varied approach in writing solutions. Algebraic and numerical content needs to be supplemented with well-reasoned but brief arguments where the question demands. Candidates should always try to provide several distinct relevant points for the discursive questions, not lengthy paragraphs developing a single idea.

5. Candidates who give well-reasoned points, not in the marking schedule, are awarded marks for doing so.

B. General comments on *student performance in this diet of the examination*

1. The overall performance of candidates was better than the average standard observed in the past few years. Most candidates were able to score marks across all the questions in the paper and well prepared candidates scored well above the pass mark.

2. In terms of areas for improvement:

   - Some candidates were unable to demonstrate a breadth of knowledge across the whole syllabus and so did not score all of the available knowledge marks from the Core Reading.

   - Many candidates did not appear to tailor their answer to the command words in the questions, such as "Determine" or "Derive".

   - Many candidates made a number of small errors when deriving standard results.

- A number of candidates provided a significant amount of detail on relatively narrow arguments when responding to the discursive questions.
- Some candidates struggled with questions that required an element of application of Core Reading to situations that were not immediately familiar, so did not demonstrate the desired depth of knowledge.
- The comments that follow the questions concentrate on areas where candidates could have improved their marks, in an attempt to help future candidates to revise accordingly and to develop their ability to apply the Core Reading to related situations. This skill is likely to be valuable both in exams and in using this theory in their professional careers.

C. Pass Mark

The Pass Mark for this exam was 60.

Solutions

Q1 (i) $\lambda = \dfrac{\mu - r}{\sigma}$ [½]

where r is the risk-free rate [½]
[Total 1]

(ii) $dg = rgdt$ [1]

(iii) $df = (\sigma\lambda + r) fdt + \sigma f dz$ [½]

$d \ln f = \left(\sigma\lambda + r - \dfrac{\sigma^2}{2}\right) dt + \sigma dz$ by Ito [1]

$d \ln g = rdt$ by Ito [½]

$d\left(\ln \dfrac{f}{g}\right) = d \ln f - d \ln g$ [½]

Therefore $d\left(\ln \dfrac{f}{g}\right) = \left(\sigma\lambda - \dfrac{\sigma^2}{2}\right) dt + \sigma dz$ [½]

(iv) By Ito's lemma applied to the exponential of ln f/g,

$$d\left(\frac{f}{g}\right) = \sigma\lambda\left(\frac{f}{g}\right)dt + \sigma\left(\frac{f}{g}\right)dz$$ [1]

(v) $\lambda = 0$ [½]
… as this is the market price of risk that ensures there is no drift in the stochastic differential equation for f/g [½]
[Total 1]

(vi) (a) The equivalent martingale measure result states that if we use the price of a traded security g as the unit of measurement (i.e. numeraire) then assuming that there are no arbitrage opportunities there is a particular market price of risk for which all security prices follow martingales.[1]
… and that particular market price of risk is the volatility of g. [½]

(b) The volatility of g in part (ii) is zero… [1]
… and we have determined that the market price of risk in (v) is zero, hence this in line with the equivalent martingale measure result. [½]

(c) The valuation of interest rate derivatives can be complicated as interest rates are used for discounting the payoff as well as defining the payoff. [1]
By setting the market price of risk in line with the volatility of the numeraire, the equivalent martingale measure result ensures that the security prices follow martingales. [1]

This then allows standard risk-neutral pricing techniques to be used to value interest rate derivatives. [½]

Different numeraires are used to value particular derivatives [½]

For example, an annuity factor as a numeraire may be used to value an interest rate swap. [½ max for any relevant example]
[Max 4]
[Total Max 11]

Candidates performed well on the earlier parts of the question but found the later parts more difficult.

A number of candidates incorrectly included a sigma term in the SDE for risk free cash. In parts (iii) and (iv), some candidates did not score full marks as they simply stated results rather than deriving results as required by the command verb.

Many candidates found part (vi) of the question challenging. Some answers showed a general mis-understanding of the EMM result or confusion between

> this and other results. Well prepared candidates were able to explain clearly how EMM may be used practically for valuing interest rate derivatives.

Q2 (i) (a) A speculator is an individual who takes a position in the market. [1]

Speculators use derivatives to take positions on the future direction of a market variable. [½]

They are either betting that the price will go up or they are betting that it will go down. [½]

They are informed traders, which distinguishes them from gamblers. [½]

Speculators may use derivatives because they can speculate using leveraged positions (or speculate more quickly). [½]

Other definitions are acceptable, for example:

Speculators are market participants who take considerable business risk in expectation of commensurate gain, in which commensurate gain refers to a positive return in excess of a benchmark risk-free alternative. [1]

(b) The three types of speculator are …..
…scalpers, [½]
…day traders, and [½]
... position traders. [½]

Scalpers are watching for very short-term trends in the markets. [½]

They attempt to profit from small changes in the contract price. [½]

They usually hold their positions for only a few minutes. [½]

In recent years high frequency trading has become commonplace in many markets, which could be viewed as an extreme form of scalpers. [½]

High frequency traders often hold positions for small fractions of a second. [½]

Day traders hold their positions for less than one trading day. [½]

They are unwilling to take the risk that adverse news will occur overnight. [½]

Position traders hold their positions for much longer periods of time. [½]

They hope to make significant profits from major movements in the markets. [½]

[Max 4]

(ii) **Advantages**

Speculation usually increases liquidity in the market. [1]

This is partly through making it possible for market participants to transact more quickly for a given size of trade. [½]

Further, due to the large number of speculators in the market it enables the market to be resilient (i.e. individual transactions do not cause large shifts in prices). [½]

As speculators are generally informed investors they have a critical role of bringing changing information into the price of a derivative. [1]

Therefore via their trading, speculators help move prices closer to more fundamental values. [½]

This helps in creating more efficient markets. [½]

In some cases, speculative short selling can have the effect of identifying underperforming companies. [½]

Speculation enables market completeness, i.e. market participants who wish to hedge a risk can more easily find an interested party to take the other side (usually in the form of a speculator). [½]

Speculation can generate profits for investors. [½]

Disadvantages

Speculation may lead to higher market volatility… [½]
… especially in the form of short selling. [½]

Speculation can cause unjustified changes in prices of underlying assets. [½]

There are cases of speculation leading to market manipulation and illegal profits. [½]

As a result, excessive speculation can have a destabilizing effect on the market… [½]
… depending on the regulations. [½]

It can also be argued that the additional liquidity that speculators bring to markets is missing when it is needed most, e.g. when the market is undergoing extreme movements. [1]

[Up to 1 further mark can be awarded for citing and explaining relevant examples, such as the flash crash or the fall fall.] [Max 4]

[Total Max 8]

This question largely tested knowledge of the Core Reading. In part (i)(b), few candidates were familiar with the specific titles for the types of speculator, but more were able to describe the different attributes of the types of speculator. Although not every part of the course can be examined at each sitting, all parts of the Core Reading are examined over time and candidates should be prepared for this.

Most candidates were able to identify the key advantage and disadvantage in part (ii). However, many candidates did not list enough distinct points. These candidates often put too much detail into individual points or listed minor variations on the same point multiple times.

Q3 (i) $$f(t,T_1,T_2) = -\left[\frac{\ln P(t,T_2) - \ln P(t,T_1)}{T_2 - T_1}\right]$$ [1]

(ii) $$F(t,T) = -\left[\frac{\partial \ln P(t,T)}{\partial T}\right]$$

Or alternatively $\lim_{T_2 \to T_1} f(t,T_1,T_2)$ [1]

(iii) (a) $$df(t,T_1,T_2) = -\left[\frac{d\ln P(t,T_2) - d\ln P(t,T_1)}{T_2 - T_1}\right]$$ [½]

Now using Ito's lemma and that $\frac{d}{dP}\ln P = \frac{1}{P}, \frac{d^2}{dP^2}\ln P = -\frac{1}{P^2}$ [1]

$$d(\ln P) = \left[\frac{1}{P}rP - \frac{1}{2P^2}v^2P^2\right]dt + \frac{1}{P}vPdz$$ [½]

$$\therefore d(\ln P) = \left[r - \frac{1}{2}v^2\right]dt + vdz$$ [½]

Inserting this into $df(t,T_1,T_2)$ gives:

$$df\left(t,T_1,T_2\right)=\frac{1}{2}\left[\frac{v\left(t,T_2\right)^2-v\left(t,T_1\right)^2}{T_2-T_1}\right]dt-\left[\frac{v\left(t,T_2\right)-v\left(t,T_1\right)}{T_2-T_1}\right]dz \qquad [½]$$

(b) Letting $T_2 \to T_1$, then $df\left(t,T_1,T_2\right) \to dF(t,T)$ [½]

$$\therefore dF\left(t,T\right)=\frac{1}{2}\frac{\partial}{\partial T}\left[v\left(t,T\right)^2\right]dt-\frac{\partial}{\partial T}\left[v\left(t,T\right)\right]dz \qquad [1]$$

By comparison of terms:

$$m\left(t,T\right)=\frac{1}{2}\frac{\partial}{\partial T}\left[v\left(t,T\right)^2\right]=v\left(t,T\right)\frac{\partial v\left(t,T\right)}{\partial T} \text{ by the chain rule} \qquad [1]$$

$$s\left(t,T\right)=-\frac{\partial v\left(t,T\right)}{\partial T} \qquad [½]$$

Now, integrating, $v\left(t,T\right)=\int_t^T \frac{\partial v\left(t,\tau\right)}{\partial \tau}\partial\tau$ since $v\left(t,t\right)=0$ [½]

Hence $m\left(t,T\right)=s\left(t,T\right)\int_t^T s\left(t,\tau\right)\partial\tau$ [1]

[Max 6]

(iv) In the general HJM, the short rate is non-Markov… [½]
… so the model is typically implemented using Monte Carlo simulation… [½]
… as there is no general closed form solution. [½]

The HJM model is expressed in terms of instantaneous forward rates which are not directly observable. [½]

It is also difficult to calibrate the model to prices of actively-traded instruments… [½]
…and you cannot calibrate the volatility and risk-free rate separately. [½]
[Max 2]

(v) The LIBOR Market Model (LMM) is based on forward rates over discrete time periods… [1]
… which are directly observable. [½]

The LMM assumes a rolling forward risk-neutral world… [1]
… i.e. a world that is always forward risk-neutral with respect to a bond maturing at the next reset date. [½]

By using a rolling certificate of deposit as the numeraire, a SDE can be derived for the discrete forward rate. [½]

Using simplifying assumptions, the key model parameter Λ_i can be calibrated from the following equation:

$$\sigma_k^2 t_k = \sum_{i=1}^{k} \Lambda_{k-i}^2 \delta_{i-1} \qquad [1]$$

Where σ_k are the spot volatilities for caplets…. [½]
… which could be estimated from the Black model. [½]
The distribution of the forward rate at any given time is lognormal… [½]
.. and so the LMM can then be used to produce an exact formula for valuing vanilla caps…. [½]
… and can also be used to value non-standard caps such as rachet caps, sticky caps, flexi caps. [½]

[Max 4]
[Total Max 14]

The algebraic parts (i) to (iii) were typically very well answered. Occasional marks were lost through sign errors and failing to state assumptions –e.g. Ito's lemma. Candidates are reminded that in order to score full marks for a "derive" or "show" question they need to show all their working and reasoning.

Candidates did not score as high a proportion of marks on parts (iv) and (v) which required wider knowledge of the HJM and LMM. Few recognised how the HJM model is constrained by having volatility linked to the level of the risk free rate.

Q4 (i) (a) This Greek is the percentage change in derivative value per percentage change in the underlying price. [1]

From economics this is termed *elasticity*, i.e. the elasticity of the derivative price with respect to the underlying. [½]

(b) In terms of delta, Δ, $\lambda = \Delta \times {}^{S}\!/\!{}_{V}$. [1]

[Max 2]

(ii) (a) Out-of-the-money options are riskier in percentage terms than in-the-money options because you obtain greater leverage (i.e. the same in-the-money payoffs for a lower option premium). [1]

A given reduction in value of the underlying asset will cause a smaller reduction in the absolute value of the option… [½]
… but this will be a larger percentage change, and hence larger λ . [½]

In the limit as $S \to 0$, $\lambda \to \infty$. [½]

(b) Similarly, all else being equal, options closer to expiry are riskier in percentage terms because they have greater leverage. [1]

For example, a small movement in the underlying price close to expiry can move the option into or out of the money, and hence a large change in the value of the option. [½]

[Max 2]

(iii) The price of a call option, V, is $V = SN(d_1) - K\exp(-r\tau)N(d_2)$. [½]

Hence lambda is:

$$\lambda = \frac{\partial V}{\partial S} \times \frac{S}{V}$$

$$= \left(N(d_1) + S\frac{\partial N(d_1)}{\partial S} - K\exp(-r\tau)\frac{\partial N(d_2)}{\partial S} \right) \times \frac{S}{SN(d_1) - K\exp(-r\tau)N(d_2)}$$ [½]

$$= \left(N(d_1) + S\phi(d_1)\frac{\partial d_1}{\partial S} - K\exp(-r\tau)\phi(d_2)\frac{\partial d_2}{\partial S} \right) \times \frac{S}{SN(d_1) - K\exp(-r\tau)N(d_2)}$$

[1]

Now,

$d_1 = \dfrac{\ln(S/K) + (r + 0.5\sigma^2)\tau}{\sigma\sqrt{\tau}}$ and $d_2 = \dfrac{\ln(S/K) + (r - 0.5\sigma^2)\tau}{\sigma\sqrt{\tau}}$ therefore $\dfrac{\partial d_1}{\partial S} = \dfrac{\partial d_2}{\partial S} = \dfrac{1}{S\sigma\sqrt{\tau}}$. [½]

Substituting this into the above expression for λ gives:

$$\lambda = \left(N(d_1) + S\phi(d_1)\frac{1}{S\sigma\sqrt{\tau}} - K\exp(-r\tau)\phi(d_2)\frac{1}{S\sigma\sqrt{\tau}} \right) \times \frac{S}{SN(d_1) - K\exp(-r\tau)N(d_2)}$$

[½]

$$= \left(N(d_1) + \frac{\phi(d_1)}{\sigma\sqrt{\tau}} - (S\phi(d_1))\frac{1}{S\sigma\sqrt{\tau}} \right) \times \frac{S}{SN(d_1) - K\exp(-r\tau)N(d_2)},$$

using the assumption in the question, [½]

$$= \left(N(d_1) + \frac{\phi(d_1)}{\sigma\sqrt{\tau}} - \frac{\phi(d_1)}{\sigma\sqrt{\tau}} \right) \times \frac{S}{SN(d_1) - K\exp(-r\tau)N(d_2)}$$

$$= \frac{SN(d_1)}{SN(d_1) - K\exp(-r\tau)N(d_2)}.$$ [½]

[Max 4]

(iv) Rearranging the expression found in (ii) for λ:

$$\lambda = \frac{1}{1 - {K\exp(-r\tau)N(d_2)}\big/{SN(d_1)}}.$$ [½]

As $N(d_1)$ and $N(d_2)$ are both cumulative probability functions they lie in the range [0,1]. [½]

Also, as *K*, *S* and $\exp(-r\tau)$ are all positive then it follows that ${K\exp(-r\tau)N(d_2)}\big/{SN(d_1)}$ is also positive [½]

Therefore, $1 - {K\exp(-r\tau)N(d_2)}\big/{SN(d_1)} \le 1.$ [1]

This expression must also be positive (i.e. >=0) since the delta of a call is always positive (and using the part (i) result) *(or any alternative justification)* [½]

Hence $\lambda \ge 1$. [½]

[Max 3]

(v) As $S \to \infty$ the call option is heavily in the money… [½]
… assuming that the strike price is finite. [½]

The price of the call option tends to the value of *S*, as $S \to \infty$ … [½]
… due the option behaving like the actual underlying asset (i.e. the option is almost certain to be exercised). [½]

It can therefore also be assumed that the delta of the option tends to 1 as $S \to \infty$. [½]

Using the definition from part (i)(b), $\lambda = \Delta \times {S}/{V}$, it follows that as $S \to \infty$, $\lambda \to 1$. [½]

This is a non-rigorous determination of the asymptotic behaviour and other approaches were given credit. For example the following alternative solution:

Starting with the expression from part (iii):

$$\lambda = \frac{1}{1 - {K\exp(-r\tau)N(d_2)}/{SN(d_1)}}$$

The component ${K\exp(-r\tau)N(d_2)}/{SN(d_1)}$ *tends to 0 as* $S \to \infty$ *[1]*

… assuming a finite strike price K *[½]*
… since $N(d_2)$ *and* $N(d_1)$ *will converge* *[1]*

Hence $\lambda \to 1$. *[½]*
[Max 2]

(vi) As $\lambda \geq 1$ it follows that the sensitivity of the call option price relative to the asset price is therefore greater than or equal to 1. [1]

So, the call option is more risky in terms of percentage changes in its value than the underlying asset. [½]
And prices available in the market for investing in the call option will be more volatile than the underlying asset price. [½]

This is partially as a result of the gearing provided by the option. [½]
[Max 2]
[Total Max 15]

Few candidates scored highly with this question. Part (iii) was a slight modification on a standard bookwork proof and candidates generally scored well here. However, few candidates scored highly on the other parts of the question which required the application of knowledge and techniques within the course to a new situation.

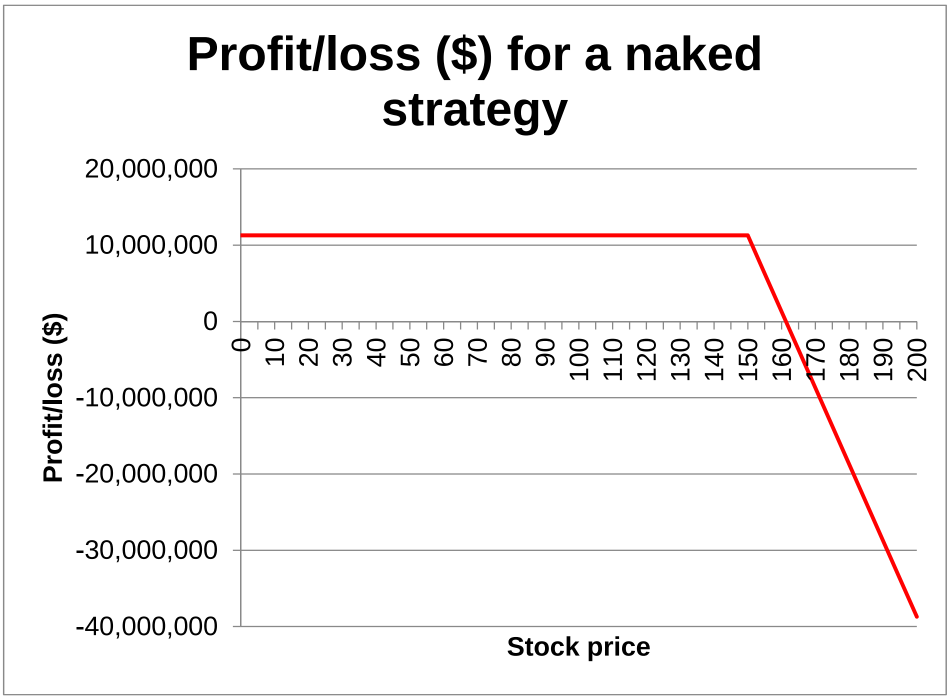
Candidates were, nonetheless, often able to make relevant observations around the behaviours of the underlying components in the limits, but few scored full marks. Very few candidates attempted the final part of the question.

Q5 (i) (a) If the option is not exercised then the profit at expiry for the bank is: $\$11,000,000 \times \exp(0.5 \times 0.05) = \$11,300,000$ (rounded).

If the option is exercised then the profit/loss at expiry for the bank is $\$11,300,000 - 1,000,000 \times (S - \$150)$, where S is the stock price at expiry.

The bank breaks even when:

$\$11,300,000 = 1,000,000 \times (S - \$150)$, or
$S = \$161.3$.



[3½ marks: 1 for correct profit when the option is not exercised, 1 for correct breakeven point, 1 mark for correct shape of line including corner lined up with strike price, ½ marks for labelling]

(b) By adopting a covered position the bank will purchase 1,000,000 shares when the option is sold.

The cost to the bank when the shares are purchased is: $1,000,000 \times \$145 = \$145,000,000$.

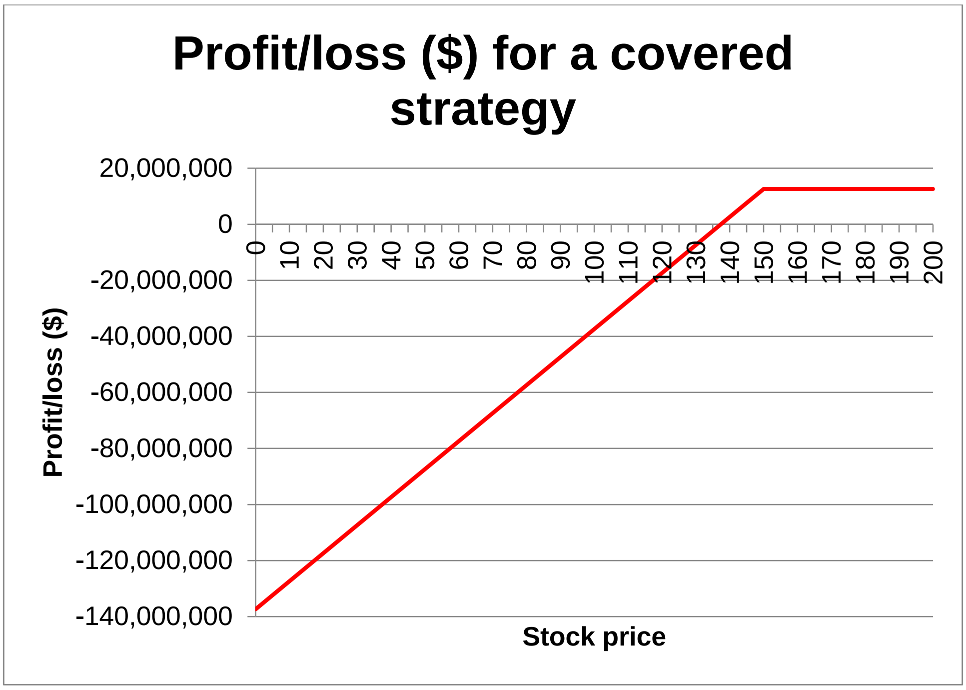
If the option is not exercised then the (book) profit/loss to the bank is: $\$11,300,000 - \$145,000,000 \times \exp(0.5 \times 0.05) + S \times 1,000,000$.

The breakeven point is when
$\$11,300,000 - \$145,000,000 \times \exp(0.5 \times 0.05) + S \times 1,000,000 = 0$,
or $S = \$137.4$.

If the option is exercised then the bank can deliver the shares without having to buy them in the open market at expiry.

The profit/loss to the bank is:

$$\$11,300,000 - \$145,000,000 \times \exp(0.5 \times 0.05) + \$150 \times 1,000,000$$
$$= \$12,600,000.$$



[3½ marks: 1 for correct profit when the option is exercised, 1 for correct breakeven point, 1 mark for correct shape of line including corner at strike and ½ marks for labelling]

[Total 7]

(ii) By inspection of the graph for the covered strategy, the pay-off for the bank is equivalent to a naked short put option position… [½]
… with strike \$150. [½]
[Total 1]

(iii) As the option is out-of-the-money at the time the option is sold there is no immediate action taken by the bank, i.e. it adopts a naked position. [1]

If the option remains out of-the-money up until expiry then the bank also takes no action … [½]
… and hence the profit is equal to the option premium paid by the buyer. [½]

If the stock price rises above the strike price… [½]
… then the bank immediately adopts a covered position, buying the required number of stocks to cover the option for the strike price. [½]
All of these stocks are sold immediately for the strike price… [½]
… if the stock price drops below the strike price again. [½]

There are likely to be practical problems in making transactions at the actual strike price and this introduces a cost to this hedging procedure. [1]

This should be taken into account as part of the hedging strategy, even if there are no dealing costs for the bank. [½]
[Max 3]

(iv) The bank could use dynamic delta hedging in this case. [1]

For example, it could hold the underlying stock / a call with a different strike / a stock correlated to the underlying. *(Please give credit for any reasonable example).* [½]
[Max 1]
[Total Max 12]

> Candidates scored highly for the graphs in part (i) although a common oversight was a failure to incorporate the interest earned/paid by the bank. The command word of "draw" meant the examiners were looking for more than a sketch chart. The best answers included supporting calculations at certain share prices.
>
> In part (iii) of the question few candidates sufficiently tailored their answer to the strategy in the context of this question. For this question the examiners were looking for candidates to explain clearly how the strategy might work in the real world given the starting position.

Q6 (i) (a)

The following table displays the payoff at expiry calculations:

Sample path	*Time = 3 (expiry)*	*Payoff at expiry*
1	102.2	17.8
2	111.1	8.9
3	124.7	0.0
4	157.9	0.0
5	130.8	0.0
6	88.7	31.3
7	140.2	0.0
8	92.2	27.8

[1 for correct payoff at expiry calculations]

(b) The optimal strategy is to exercise the option if it is in-the-money at expiry. [½]
The optimal strategy is to exercise the option for sample paths 1, 2, 6 and 8. [1]
[Max 2]

(ii) (a) The sample paths where the option is in-the-money at time 2 months is shown in the following table.

Sample path	*Time = 2 months stock value*	*Value of option payoff at 2 months*	*In-the-money?*
1	111.9	8.1	Yes
2	118.2	1.8	Yes
3	149.9	0.0	No
4	159.7	0.0	No
5	152.9	0.0	No
6	79.0	41.0	Yes
7	119.5	0.5	Yes
8	89.3	30.7	Yes

[1 for correct intrinsic value calculations]

The sample paths which are in-the-money at time 2 months are 1, 2, 6, 7 and 8. [½]

(b) The discount rate over 1 month is $10\% / 12 = 0.833\%$. [½]

For sample paths 1, 2, 6, 7 and 8 the discounted value of the payoff at time 3 months to time 2 months is shown in the table below:

Sample path	*Time = 3 months stock value*	*Payoff at expiry*	*Discount payoff at expiry to time 2 months*
1	102.2	17.8	$= 17.8 \times \exp(-0.00833) = 17.65$
2	111.1	8.9	$= 8.9 \times \exp(-0.00833) = 8.83$
6	88.7	31.3	$= 31.3 \times \exp(-0.00833) = 31.04$
7	140.2	0.0	$= 0 \times \exp(-0.00833) = 0$
8	92.2	27.8	$= 27.8 \times \exp(-0.00833) = 27.57$

[1½ for correct discounting calculations]
[Max 3]

(iii) (a) Standard statistical regression techniques are used to determine these values. [1]

In particular the values of a, b, c, and d are determined such that they minimise the following expression:

$$\sum_{i=1,2,6,7,8}\left(V_i - a - bS_i - cS_i^2 - dS_i^3\right)^2, \qquad [1½]$$

[1½ = 1 mark for the quadratic expression + ½ marks for summing over the correct paths]

where:

i is the sample path;

V_i is the payoff at expiry discounted to time 2 months for sample path *i* where the option is in-the-money at time 2 months; and

S_i is the stock price at time 2 months for sample path *i*. [1]

(b) Only in-the-money sample paths are used in order to better estimate the function *V* in the region where early exercise is a relevant possibility. [1]

From an implementation perspective it should improve the efficiency of any algorithm written … [½]
… as there are fewer points in the regression calculations. [½]
[Max 4]

(iv) The values for *a*, *b*, *c*, and *d* determine the function *V*. The decision of whether or not to exercise early depends on whether or not this payoff is greater than continuing to hold the option to expiry. [1]

These calculations are illustrated in the table below.

Sample path	*Intrinsic value of option at time 2 months*	V_i
1	8.1	$=1138-35.8\times(111.9)+0.385\times(111.9)^2-0.00138\times(111.9)^3=19.2$
2	1.8	$=1138-35.8\times(118.2)+0.385\times(118.2)^2-0.00138\times(118.2)^3=6.4$
6	41.0	$=1138-35.8\times(79.02)+0.385\times(79.0)^2-0.00138\times(79.0)^3=32.2$
7	0.5	$=1138-35.8\times(119.5)+0.385\times(119.5)^2-0.00138\times(119.5)^3=2.8$
8	30.7	$=1138-35.8\times(89.3)+0.385\times(89.3)^2-0.00138\times(89.3)^3=28.5$

[2½]

For paths 6 and 8 the intrinsic value of the option at time 2 months is worth more than the estimate option value of continuing to hold the option. Therefore the option should be exercised for these paths. [1]

[Max 4]

(v) The option should be exercised immediately … [½]

… as the estimated option value from not exercising the option is 19.0 while the payoff from the option if it is exercised immediately is 120 – 100 = 20.0. [½]

[Total 1]

(vi) The test simulation has only used 8 samples compared to 100,000 to be used in the full run. [½]

This means the test simulation will have too few runs to produce a stable and convergent value. [½]

The regression is also based on a very small set of samples, resulting in relatively inaccurate and unstable values for *a*, *b*, *c*, and *d* for the expression *V*. [½]

In turn the approximations produced for *V* can be inaccurate. [½]

In the full simulation a time step of 1 day is used compared to 1 month in the test simulation. [½]

This will also produce greater accuracy in the result. [½]

[Max 2]

[Total Max 16]

> This was a well answered question. Many candidates scored well on the calculation questions and demonstrated a good overall knowledge of the algorithm underlying the Longstaff Schwartz method. Few candidates scored full marks as typically they did not provide complete explanations of the steps they were taking in arriving at their conclusions.
>
> In part (vi), few candidates provided sufficient distinct points to score highly.

Q7 (i) Floating leg:

$$\$500m \times (1+2\%)^{-5} = \$452.87m$$ [½]

Fixed leg:

$$\$500m \times (1+2\%)^{-5} \times \left[\frac{(1-(1+2\%)^{-5})}{2\%} \times 3\% + (1+2\%)^{-5} \right] = \$474.21m$$

[1]

Hence $\$474.21m - \$452.87m = \$21.34m \sim \$21m$ [½]

[Total 2]

(ii) When interest rates = 0% p.a.

Swap has a value equal to gross cash flows from receive leg
$\$500m \times 5 \times 3\% = \$75m$ [½]

Hence $\mathit{ShiftSensitivity} = \$75m - \$21m = \$54m$ [½]

[Total 1]

(iii) (a) Delta, Gamma, Vega [1½]

(½ for each)

(b) Vega is not captured by the ShiftSensitivity… [½]
… as it doesn't consider changes in implied volatility… [½]
… so the bank would need another metric to analyse Vega risk. [½]

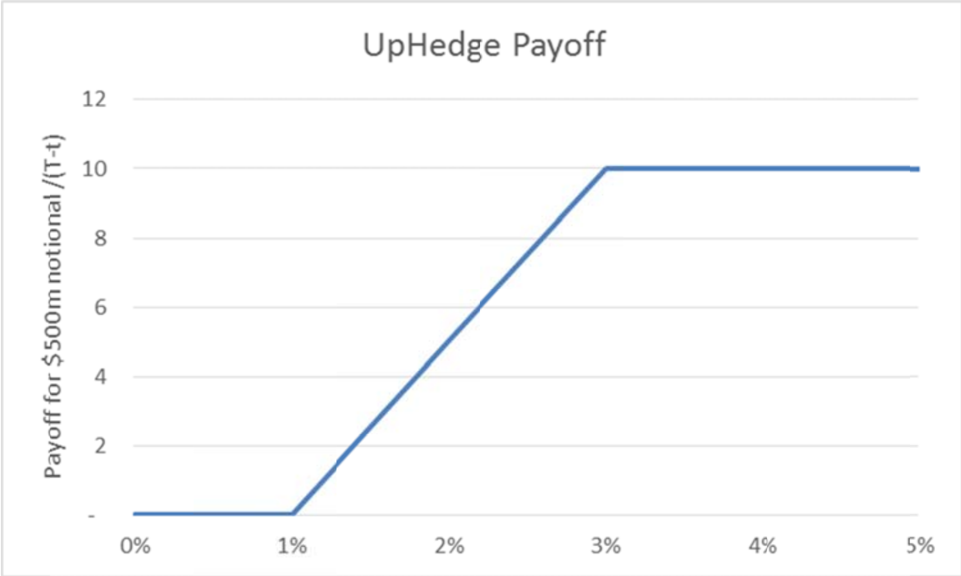
Delta is usually defined as the impact of a one basis point movement in the zero curve… [½]
… so the ShiftSensitivity metric would not allow the management of Delta in isolation… [½]
… as the ShiftSensitivity metric also includes the market value impact of the change in Delta as the zero curve is stressed through −1% to 1%... [½]
… i.e. it partially includes Gamma risk. [½]

But again, the bank cannot manage the Gamma risk explicitly as it is aggregated in the one statistic. [½]

However, a strength of the ShiftSensitivity metric is that it does allow approximate hedging of larger interest rate shifts from one single statistic. [½]

[Max 4]

(iv)



(v) The payoff for a caplet is given by $L\delta_t \max(I - \text{CapRate}, 0)$ [1]
where L is the notional and δ_t is the period covered by the cap [½]

Hence the UpHedge can be decomposed into going long a caplet with a CapRate of a … [½]
… and then going <u>short</u> another caplet... [½]
… with a CapRate of b. [½]

Both caplets have a notional of $500m… [½]
… and pay at time T based on the forward interest rate I observed at time t over period $T - t$. [½]
[Max 3]

(vi) $L \times \delta_t \times P(0,T)\left[I_0\Phi\left(d_1\right) - \text{CapRate} \times \Phi(d_2)\right]$ [1]

$$d_1 = \frac{\ln(I_0 / \text{CapRate}) + \dfrac{\sigma^2}{2}t}{\sigma\sqrt{t}}, d_2 = d_1 - \sigma\sqrt{t}$$ [½]

Where: L = notional, t = time at which caplet starts, T = payment time of option, σ = volatility of the forward interest rate, δ_t is the period covered by the caplet (in years) and Φ is the standard normal cumulative distribution function. [1]

[*need only define* σ *and* Φ *here if defined other notation in earlier parts of the question*]

I_0 = forward interest rate at time zero between t and T, compounded with frequency $T - t$. [½]

$P(0,T)$ is the price of a zero coupon bond at time zero that matures at time T. [½]

[Max 3]

(vii) $I_0 = \frac{(1+1\%)^5 - 1}{5} = 1.02020\%$ [½]

$P(0,T) = (1+1\%)^{-10} = 0.90529$ [½]

Caplet *a*

$$d_1 = \frac{\ln(0.0102020/0.01) + \frac{0.3^2}{2}5}{0.3\sqrt{5}} = 0.36522$$ [½]

$$d_2 = 0.36522 - 0.3\sqrt{5} = -0.30560$$ [½]

$$\Phi(d_1) = 0.64253$$ [½]

$$\Phi(d_2) = 0.37996$$ [½]

Therefore value is:

$$\$500m \times 0.90529 \times 5 \times [0.0102020 \times 0.64253 - 0.01 \times 0.37996] = \$6.24m$$ [½]

Caplet *b*

$$d_1 = \frac{\ln(0.0102020/0.03) + \frac{0.3^2}{2}5}{0.3\sqrt{5}} = -1.27249$$ [½]

$$d_2 = -1.27249 - 0.3\sqrt{5} = -1.94331$$ [½]

$$\Phi(d_1) = 0.10160$$ [½]

$$\Phi(d_2) = 0.02599$$ [½]

Therefore value is:

$$-\$500m \times 0.90529 \times 5 \times [0.0102020 \times 0.10160 - 0.03 \times 0.02599]$$
$$= -\$0.581m$$ [½]

Hence total value is $\$6.24m - \$0.58m = \$5.66m$ [½]

[Max 6]

(Candidates must get the correct final answer to (vii) for full marks.)

[Note that 5 marks maximum were awarded if candidates used $I_0 = 1\%$. This gives the following figures:

$P(0,T) = (1+1\%)^{-10} = 0.90529$ *[as before]* *[½]*

Caplet a

$$d_1 = \frac{\ln(0.01/0.01) + \frac{0.3^2}{2}5}{0.3\sqrt{5}} = 0.33541$$ *[½]*

$d_2 = 0.33541 - 0.3\sqrt{5} = -0.33541$ *[½]*

$\Phi\left(d_1\right) = 0.63134$ *[½]*

$\Phi\left(d_2\right) = 0.36866$ *[½]*

Therefore value is:

$\$500m \times 0.90529 \times 5 \times \left[0.01 \times 0.63134 - 0.01 \times 0.36866\right] = \$5.95m$ *[½]*

Caplet b

$$d_1 = \frac{\ln(0.01/0.03) + \frac{0.3^2}{2}5}{0.3\sqrt{5}} = -1.30230$$ *[½]*

$d_2 = -1.30230 - 0.3\sqrt{5} = -1.97312$ *[½]*

$\Phi\left(d_1\right) = 0.09641$ *[½]*

$\Phi\left(d_2\right) = 0.02424$ *[½]*

Therefore value is:

$$-\$500m \times 0.90529 \times 5 \times \left[0.01 \times 0.09641 - 0.03 \times 0.02424\right] = -\$0.54m$$ *[½]*

Hence total value is $\$5.95m - \$0.54m = \$5.41m$ *[½]*

[The alternative solution has a Max 5. If candidates made an attempt to convert 1% but dis not get the correct final answer, a maximum of 5.5 for the question was allowed.]

(viii) As interest rates increase from 1% the ShiftSensitivity of the swaps will continually decrease… [1]
… as the fixed cash flows on the swaps will be discounted at ever increasing rates, so the absolute value of a market value shift of a 1% move in rates will reduce. [½]

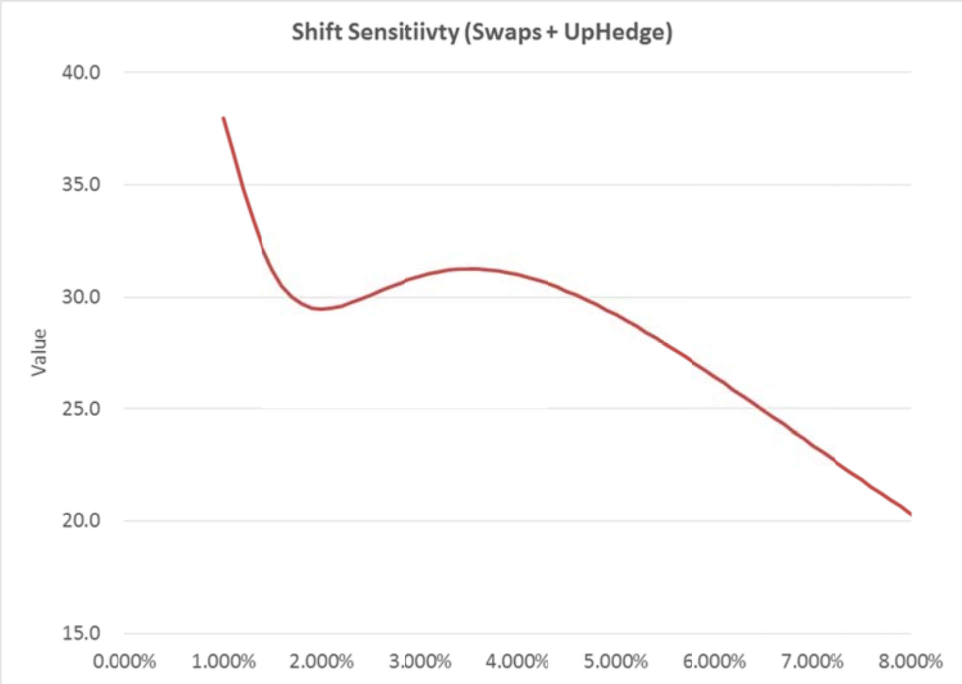
The UpHedge is more complicated because as interest rates increase from 1% the likelihood of exercising the derivative will increase… [1]
… until interest rates are well past 3%, when the exercise of the UpHedge will be almost guaranteed… [½]
… so this will act to increase the shift sensitivity as rates approach 3%, but will only have limited impact beyond this level… [½]
… as the UpHedge will begin to act like a zero coupon payment at time 10. [½]

The impact of this effect will depend on the volatility parameter assumed… [½]
… with the increase in ShiftSensitivity of the UpHedge being more rapid with lower assumed volatility. [½]

In addition, the discounting effect will have an opposing effect to reduce the ShiftSensitivity of the UpHedge as interest rates increase. [½]

Overall, it is likely that the ShiftSensitivity will initially reduce… [½]
… then reduce less or possibly increase as 3% is approached… [½]
… before continuing to reduce once more as 3% is passed. [½]

(for info, graph of ShiftSensitivity is included below)



[Max 3]
[Total Max 24]

Some candidates struggled with sections of this question, whilst well prepared candidates scored highly.

Parts (i) and (ii) were well answered by many candidates. In part (iii), many candidates demonstrated a reasonable understanding of the link to the Greeks, but typically not in enough detail to score full marks. A high number of candidates incorrectly identified theta as an important Greek, but the theta of the forward starting receiver swaps in question was zero.

Candidates were generally comfortable with the concepts of caplets, but many failed to score full marks as they did not recall the pricing formula correctly. For example, many candidates did not convert the frequency of the interest rate appropriately.

Only a handful of candidates were able to assess the impact of the UpHedge in part (viii).

END OF EXAMINERS' REPORT