# INSTITUTE AND FACULTY OF ACTUARIES

# EXAMINATION

7 October 2011 (pm)

## Subject ST6 — Finance and Investment
## Specialist Technical B

*Time allowed: Three hours*

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You have 15 minutes before the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*

3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all eight questions, beginning your answer to each question on a separate sheet.*

6. *Candidates should show calculations where this is appropriate.*

***Graph paper is required for this paper.***

*AT THE END OF THE EXAMINATION*

*Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.*

*In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.*
*NOTE: In this examination, you are never required to prove the use of an arbitrage-free methodology unless clearly stated in the question.*

**1** (i) Explain arbitrage and the role of arbitrageurs in the context of securities and derivatives trading. [3]

An arbitrageur, a hedger and a speculator all hold positions in the same oil futures contract.

(ii) (a) Define basis risk in this context.

(b) Describe briefly the differing attitudes that these three market participants are likely to have to basis risk.

[4]

An arbitrageur has just sold 50 Brent Crude Oil July futures at \$86.20 and has simultaneously purchased the same amount of physical oil (50,000 barrels) at a spot rate of \$85.50 per barrel. The July futures expire in 47 days. Financing costs over this period (including storage costs) are 2% per annum annually compounded.

(iii) (a) Calculate the arbitrageur's expected profit from this trade.

(b) Assess the likelihood of achieving this profit in practice.

[4]
[Total 11]

**2** (i) List necessary and sufficient conditions for a stochastic process $\{W(t): t \geq 0\}$ to be a standard Brownian motion with respect to a probability distribution **P**. [2]

(ii) Show that one of the conditions for Brownian motion can be alternatively expressed as:

$$\text{covariance}[W(s), W(t)] = \min\{s, t\} \text{ for } s \geq 0 \text{ and } t \geq 0.$$ [2]

Let $B_t$ be a standard Brownian motion.

(iii) Demonstrate that the following processes are also standard Brownian motions:

(a) $-B_t$

(b) $t\,B_{1/t}$ [5]

[Total 9]

**3** Consider the Cox-Ingersoll-Ross (CIR) interest rate model:

$$dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}\,dW_t$$

where $\kappa$, $\theta$, $\sigma$ are positive constants and $W_t$ is a standard Brownian motion.

(i) Explain how the possibility of negative interest rates is dealt with in the CIR model. [2]

(ii) Prove, by differentiating the function $e^{\kappa t}r_t$, or otherwise, that a solution to the CIR stochastic differential equation is given by:

$$r_t = \theta + (r_0 - \theta)e^{-\kappa t} + \sigma e^{-\kappa t}\int_0^t e^{\kappa u}\sqrt{r_u}\,dW_u \qquad [3]$$

(iii) Hence calculate the variance of $r_t$, evaluating any integrals. [4]

[Total 9]

**4** (i) Write down the formula for the value (based on Black's model) of a European interest rate floor, defining any terms you use. [3]

Interest rates have recently fallen sharply in many economies, but in certain countries like Australia they have held steady or even slightly risen. An investor is concerned that this trend may reverse and is looking to lock in the higher rates that apply in Australia.

This investor is considering two alternatives: buying a five-year interest rate floor, or purchasing a five-year "reverse floater" bond which redeems at par and pays quarterly coupons equal to a fixed value of 2.5% <u>less</u> a quarter of the value of LIBOR at each fixing (so, for example, if LIBOR is 6% per annum at the next fixing, the next coupon becomes 1%).

(ii) Calculate the price of a one-year 5% floorlet based on the 3-month LIBOR rate for Australian dollars with an A$10,000,000 notional. You may assume that 3-month forward rates are currently flat at 5% per annum (quarterly compounded), and that the implied volatility of the forward rates is 18%. [3]

(iii) (a) Show how the cashflows of the reverse floater can be decomposed into those of a quarterly paying fixed-floating interest rate swap and a zero-coupon bond.

(b) Calculate the value of the five-year reverse floater on the same 5% per annum flat yield curve as in (ii).

[4]

(iv) Outline the merits of the two investments being considered. [2]

[Total 12]

**5** Funds of With Profits Mutual Life Insurance Company are invested 55% in UK equities, 20% in property and 25% in fixed income/cash, and the company has in the past written significant maturity benefit guarantees.

In highly volatile market conditions, the Investment Manager has initially proposed that the free assets be protected by purchasing one unit of 90% (of current market level) six-month FTSE100 Put option contracts, financed by selling two units of 80% six-month FTSE Put option contracts at no net cost.

The Investment Manager acknowledges that there is a risk the market could fall significantly, but has suggested that, if it does, the company could have a contingency plan to repurchase the two units of 80% Put options and sell instead four units of 70% Put options, continuing to "roll" this strategy forward until the market recovers.

(i) Draw three out of the following four charts for the initial hedge proposal on a standalone basis (i.e. the option positions only), showing on the same diagram the situation both shortly after initiation and just before expiry:

(a) profit / loss
(b) delta
(c) gamma
(d) vega

[6]

(ii) Calculate the (unrealised) profit or loss made by the initial hedge if the market falls by 15% in the first three months, and implied volatility is 25% for in-the-money options and 35% otherwise. You may assume zero interest and dividend rates. [5]

(iii) (a) Comment on the appropriateness of the proposed strategy, including the contingency plan and "roll" aspects.

(b) Explain whether the company should accept or reject the proposal.

[6]

[Total 17]

**6** Retirico is a UK life insurance company which specialises in writing business directly to annuitants. It has a stable book of immediate annuity insurance business, with around £10bn of liabilities at an average interest rate duration of 10 years. Assets backing the annuity business are mainly invested in high quality corporate bonds, also at an average interest rate duration of 10 years. Working capital is held in short-term UK government debt. The company anticipates making a profit of £10m per annum after operating costs of £10m per annum (i.e. the gross margin on the assets is around 20bp, i.e. 0.20%).

Under the current regulatory regime, the company is required to hold a minimum working capital of £800m, of which 75% is due to asset based credit risk.

The company is considering the implications of a new regulatory solvency regime that greatly increases capital requirements for holdings of corporate bonds, particularly those of longer maturity. It has come to the conclusion that, unless it takes appropriate action, the charge for its asset based credit risk will double, resulting in a total capital requirement of around £1.4bn.

The company is therefore interested in undertaking financial market transactions that would reduce its credit capital requirements under the new regime. The following relevant information has been provided:

- Average corporate bond spreads are 100bp at short maturities, rising to 200bp at 10 years and then falling to 50bp at 20 years.

- Northern European government bonds are available at 40bp spread and Southern European government bonds at 300bp spread.

- Collateralised funding via repurchase agreements is available from banks at 150bp spread.

- The premium required for credit protection on corporate bonds is significant.

Discuss four appropriate strategies that might be used to reduce the asset based credit risk capital requirements.

[*You may assume that, under the new regulatory solvency regime, holdings of European government bonds incur no capital charge. When assessing potential financial market transactions, you may ignore transaction (dealing) costs. Note that the question is not looking for suggestions on capital raising activities.*]

[12]

**7** In a Black-Scholes economic world where risk-free interest rates are a single level $r$ per annum for all maturities (continuously compounded), consider Call and Put options on a non-dividend paying equity.

(i) (a) Derive the relationship of Put-Call parity for European options of strike $K$ and time to maturity $T$ years.

(b) Derive a formula for rho, the sensitivity of a European Call option price to changes in the risk-free rate.

[6]

(ii) (a) Demonstrate, by showing that it is never optimal to exercise an American Call option early, or otherwise, that American and European Call options of the same strike and maturity have the same fair value price.

[*Hint: consider a portfolio of Cash plus a European Call.*]

(b) Hence prove the following bounds at time $t$ for the difference between an American Call option (fair value price $C_t$) and American Put option (fair value price $P_t$) with the same strike $K$ and maturity $T$:

$$S_t - K \leq C_t - P_t \leq S_t - Ke^{-r(T-t)}$$

where $S_t$ is the stock price at time $t$ and $0 \leq t \leq T$.

[*Hint: consider two portfolios, one containing Cash plus a European Call, the other Stock plus an American Put.*]

[7]

(iii) Sketch the fair value prices of American and European Put options against varying prices of the underlying stock, illustrating the relationship between price and intrinsic value. [3]

[Total 16]

**8** Let $S_t$ be a stock price process which follows a geometric Brownian motion according to the following stochastic differential equation (SDE):

$$dS_t = (\mu + \tfrac{1}{2}\sigma^2)S_t dt + \sigma S_t dW_t$$

where $\mu$ and $\sigma$ are constants, and $W_t$ is a standard Brownian motion.

Let $B_t$ be a risk-free bond whose price grows deterministically at the risk-free rate $r$, continuously compounded, according to the formula $B_t = e^{rt}$.

(i) Derive the SDE for the discounted stock price process $Z_t = B_t^{-1}S_t$. [5]

Let $X = f(S_T)$ be a path-independent claim on $S_T$ for some horizon $T$.

(ii) Describe what is meant by a self-financing and replicating strategy for $X$. [3]

(iii) Explain the steps needed to construct a replicating strategy for $X$ that enables the claim to be valued, referencing (but not proving) any general results you use. [6]

[Total 14]

## END OF PAPER