INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

2 May 2014 (pm)

Subject ST6 – Finance and Investment
Specialist Technical B

Time allowed: Three hours

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You have 15 minutes before the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*
3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all seven questions, beginning your answer to each question on a new page.*
6. *Candidates should show calculations where this is appropriate.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

NOTE: In this examination, you are never required to prove the use of an arbitrage-free methodology unless clearly stated in the question.

1 In normal circumstances, a yield curve derived from swap rates in a particular currency will lie above a similar yield curve derived from the corresponding government guaranteed bonds.

(i) Explain why this is the case. [2]

However, during the financial crisis, examples of yield curve inversions were observed where the swap yield curve was below the government yield curve.

(ii) Set out reasons why investors might be prepared to enter into swaps (as the receiver of the fixed payments) when the yield curves are inverted. [3]

(iii) Describe the theoretical arbitrage opportunity that the yield curve inversion presented. [1]

(iv) Set out possible reasons why banks could not eliminate the yield curve inversion by taking advantage of this opportunity. [3]
[Total 9]

2 A life insurance company is not comfortable with the level of longevity risk it is taking and wishes to hedge out some of this risk. The insurer has been in discussions with a reinsurer to reinsure a block of annuities on an original terms basis. This means that, in return for a one-off reinsurance premium, the reinsurer would pay the insurer each month a reinsurance claim equal to the insurer's total annuity outgo.

The use of longevity derivatives has been suggested as an alternative to reinsurance.

(i) Describe:

(a) a survivor index.
(b) longevity bonds.
(c) principal-at-risk longevity bonds.
(d) mortality swaps.
(e) survivor caps.

[5]

(ii) Discuss the extra risks that the insurer would be taking if it chose to hedge using any of these hedging instruments rather than to reinsure. [5]
[Total 10]

3 Let the stochastic process $\{W_t\}_{t\geq 0}$ be a $\mathbb{P}$-Brownian motion, with respect to a probability measure $\mathbb{P}$.

(i) State the necessary and sufficient conditions for $\{W_t\}_{t\geq 0}$ to be a $\mathbb{P}$-Brownian motion. [2]

(ii) (a) Show that $\mathbb{E}_{\mathbb{P}}\left[|W_t|\right] = \sqrt{\frac{2t}{\pi}}$ (where |.| is the absolute value function).

(b) Hence or otherwise show that $\mathrm{Var}_{\mathbb{P}}\left[|W_t|\right] = t\times(1-2/\pi)$. [4]

Let $a, b \in \mathbb{R}$ and $0 < s < t < u$.

(iii) Show that $\mathrm{Var}_{\mathbb{P}}[aW_s + bW_t] = (a+b)^2 s + b^2(t-s)$. [2]

(iv) Show that $\mathbb{E}_{\mathbb{P}}[W_s \times W_t \times W_u] = 0$. You may use the fact that the odd moments of W_r for $r \geq 0$ are 0. [3]

[Total 11]

4 A Bermudan option is similar to an American option, but early exercise is restricted to certain dates.

(i) Explain how a binomial tree can be used to value a Bermudan option on a non-dividend paying underlying asset. [5]

A Bermudan call option has a strike price of *K* and an expiry date in one year's time. It has possible exercise dates in three months, six months, nine months and twelve months from now.

(ii) Sketch the following variables against the underlying asset price, showing the situation at the current date (i.e. one year from expiry), in three months and at expiry (ignoring any transaction costs and initial premium):

(a) profit/loss
(b) delta
(c) gamma

[You should produce three graphs, one for each of the above variables.] [9]

[Total 14]

5 A Euro-based retail company is considering hedging its US dollar currency exposure using a forward contract.

Let:

- r be the constant US dollar interest rate (continuously compounded);
- v be the constant Euro area interest rate (continuously compounded);
- K be the delivery price agreed to in the forward contract; and
- C_0 be the current (i.e. time $t = 0$) spot exchange rate in Euros of the US dollar.

(i) Define a forward foreign exchange contract. [1]

(ii) Derive the forward price (K) for the Euro-based investor wanting to agree the cost in dollars of one Euro at time T, entering into the contract at time $t = 0$. [4]

The Euro-based retail company receives several imports from a country which has its currency (x) pegged to the US dollar (i.e. the exchange rate is fixed) at a rate of 10x to the dollar.

(iii) Derive the one month forward price of x in terms of the dollar. [2]

(iv) Comment on the interest rate implications for these countries. [1]

The retail company is considering purchasing more complex derivatives to hedge its foreign currency exposure. In order to do this, it is looking at the Black-Scholes currency model.

Let C_t be the exchange rate model reflecting the spot exchange rate in Euros of the US dollar, D_t the price of a US dollar cash bond holding ($D_t = e^{rt}$) and E_t the price of a Euro cash bond holding ($E_t = e^{vt}$).

The exchange rate model, C_t, follows a geometric Brownian motion with respect to a probability measure $\mathbb{P}$.

(v) Derive an expression for the Euro-discounted value of the dollar cash bond. [2]

(vi) Show, using the Cameron-Martin-Girsanov theorem, that the Euro-discounted value of the dollar cash bond can be transformed into a martingale under a new measure $\mathbb{Q}$. [4]

It can be shown that the value of a claim X based on the processes described in this question is given by:

$$V_t = E_t \mathbb{E}_Q\left[E_T^{-1} X \mid \mathcal{F}_t\right],$$

where $\mathcal{F}$ is the filtration.

(vii) Hence derive the forward price (K) of the contract considered in part (ii), using the results from parts (v) and (vi). [4]

[Total 18]

6 (i) (a) Define a binary option.

(b) Outline the key features of a cash-or-nothing binary option. [2]

(ii) Derive the put-call parity equation for cash-or-nothing binary options, stating any assumptions made. [3]

(iii) (a) Show how a cash-or-nothing binary call option can be replicated approximately from a spread on plain vanilla call options.

(b) Hence show that, in the limit, the price of a cash-or-nothing binary call option is theoretically equal to the negative derivative of the price of a plain vanilla call option with respect to the strike price.

(c) Explain the practical difficulties in exactly replicating the binary call option using a spread.

[6]

(iv) Outline the advantages and disadvantages for a speculator trading in these options. [3]

(v) Show how a plain vanilla European call option can be decomposed in terms of cash-or-nothing and asset-or-nothing binary options. [2]

[Total 16]

7 A bank with a diverse range of derivative positions has recently written a large and unusual over-the-counter (OTC) one-year put option on a basket of two equities and is considering hedging out the market risks within this deal.

(i) Describe in detail how the bank could price this option using a formula. [8]

(ii) List:

(a) the Greeks that the bank will be most keen to hedge.

(b) possible hedging instruments.

(c) the signs of the Greeks for the OTC put and for each of the suggested hedging instruments.

[9]

(iii) Describe how the bank would hedge out the risks associated with this deal. [5]

[Total 22]

END OF PAPER