INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

24 April 2013 (am)

Subject ST6 – Finance and Investment
Specialist Technical B

Time allowed: Three hours

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You have 15 minutes before the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*

3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all eight questions, beginning your answer to each question on a separate sheet.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

NOTE: In this examination, you are never required to prove the use of an arbitrage-free methodology unless clearly stated in the question.

1 The current annually compounded risk-free yield curve is given as follows:

Term (years)	*Spot Rate %*	*Term (years)*	*Spot Rate %*
0.5	0.65	4.5	0.95
1.0	0.67	5.0	1.03
1.5	0.70	5.5	1.11
2.0	0.72	6.0	1.20
2.5	0.75	6.5	1.30
3.0	0.78	7.0	1.39
3.5	0.82	7.5	1.49
4.0	0.88	8.0	1.58

Interest rate swaps have semi-annual payments.

(i) Show that the current forward swap rate for a three year swap starting in five years' time is 2.48% per annum. [4]

(ii) Calculate the value of a European five year payer swaption to enter into a three year swap with £1m notional and a 2% per annum swap rate, using a Black volatility of 20% per annum. [4]

(iii) Determine the value of the equivalent five year receiver swaption using put-call parity. [2]

A trader is offering to buy or sell a derivative with a payout in five years' time equal to £1m multiplied by the three year swap rate on that date. He is quoting a price of £1,000,000 × 2.48% × 1.0103^{-5} = £23,561.

(iv) Explain (without calculation) the arbitrage opportunity that this presents. [2]

[Total 12]

2 Consider an interest rate derivative that pays f_T at time T in a world where the instantaneous short rate r is stochastic, following a random walk.

(i) Show that the value of the derivative at time t is:

$$f_t = \hat{E}(e^{-\bar{r}(T-t)} f_T)$$

where $\hat{E}$ means "expected value in the risk-neutral world", and $\bar{r}$ is the average short rate in the time interval between t and T. [4]

[*Hint: consider a money market account as the numeraire.*]

(ii) Hence show that the entire term structure of interest rates can be derived from the risk-neutral process for r. [2]

In the Vasicek model, r_t is governed by the stochastic differential equation:

$$dr_t = a(b - r_t)dt + \sigma\, dW_t$$

where a and b are positive constants and W_t is a standard Brownian motion.

(iii) Solve this differential equation to find the exact distribution followed by r_t. [5]

[*Hint: consider the process* $s_t = e^{at} r_t$.]

[Total 11]

3 The European Space Agency (ESA) will be launching a manned rocket to Mars and is expecting to order 100,000 barrels of rocket fuel in three months' time. It is concerned about the risk that fluctuating fuel prices pose to the project's total cost.

ESA has decided to hedge this risk using crude oil futures, for which a contract is available which expires in three months' time.

You are given the following information:

- The spot price of one barrel of crude oil is \$101.2.
- The relevant futures price for delivery of one barrel of crude oil is \$106.3.
- The standard deviation of changes in the spot price of a barrel of rocket fuel over the next three months is \$4.2.
- The standard deviation of changes in the relevant futures price for crude oil over the next three months is \$2.5.
- The coefficient of correlation between changes in crude oil futures prices and rocket fuel spot prices is 0.75.
- One crude oil future represents 1,000 barrels of crude oil.

(i) Define the following terms:

(a) cross hedging
(b) hedge ratio
(c) minimum variance hedge ratio
(d) tailing the hedge

[4]

(ii) Calculate the number of crude oil futures that ESA should buy to minimise the variance of the project's total costs. [3]

(iii) Discuss in general terms what can be deduced about crude oil storage costs from crude oil futures prices. [3]

[Total 10]

4 Consider an option on a dividend paying stock whose price S follows a continuous random walk. A traded asset X, whose value depends only on S, is being valued by the binomial tree method with n discrete time steps and a risk-neutral probability p.

(i) (a) Explain how, for sufficiently large n, the binomial tree method evaluation achieves a good estimate of the true value of X.

(b) Draw a simple diagram to illustrate how the binomial tree would be adjusted to allow for the payment of a known dividend amount D at time interval $t = 2$.

[5]

"Finite difference" (FD) methods can be used to obtain an approximate numerical solution to the stochastic partial differential equation for X.

(ii) (a) Describe and briefly compare two distinct FD methods.

(b) Give the initial and boundary conditions for the FD valuation in the case where X is a two-year American call option on the stock.

[6]
[Total 11]

5 A trader is considering pricing options on a total return equity index whose value at time t is $S(t)$. The continuously compounded risk-free rate is r.

She has suggested an alternative approach to estimating an implied volatility matrix as an input in the Black-Scholes pricing formula. She proposes to derive the risk-neutral probability distribution for future values of S directly and use this as an input in an adapted version of the formula.

Denote by $f(S, t)$ the risk-neutral probability distribution of S at time t, and by $P(X, t)$ the price of a European put option on the index with expiry time t and strike X.

(i) (a) State (in integral form) a formula for $P(X, t)$ in terms of $f(S, t)$.

(b) Hence demonstrate that $f(X,t) = e^{rt} \dfrac{\partial^2 P}{\partial X^2}$.

[3]

(ii) (a) Discuss how in practice the expression in part (i)(b) could be used to derive the risk-neutral probability distribution $f(S, t)$.

(b) Assess the suitability of this approach for the daily pricing of the trader's options.

[7]

A life insurance company has written policies linked to the equity index. These policies have investment guarantees with a wide range of terms and strikes.

(iii) Discuss whether the risk-neutral probability distribution might be suitable for use in the insurance company's year end financial reporting. [2]
[Total 12]

6 A "chooser" option is an option where, after a specified period of time, the holder can choose whether the option is a call or a put.

Consider a chooser option on a non-income bearing security of price S with a choice at time T_1 between two European options both with maturity T_2 and strike K.

(i) Show how this chooser option can be considered as being the sum of two components:

- a call with strike K and maturity T_2, and
- a put with strike $Ke^{-r(T_2-T_1)}$ and maturity T_1.

[2]

(ii) Sketch **six** graphs of the chooser option and its two components, showing the movement of option value, delta and gamma against S:

(a) when the option is first written.
(b) shortly before time T_1.

Each graph should have three lines on it, one corresponding to the chooser option and one for each of its two components. [12]

[Total 14]

7 (i) (a) Define Value at Risk (VaR) in the context of a 99% one-day measure.

(b) Assess how this might differ numerically from a 95% ten-day VaR.

[3]

(ii) Describe three different methods of calculating VaR, indicating the situations in which they might best be used. [6]

The treasury department of a fund management firm holds a liquidity portfolio of high grade bonds, mainly from sovereign issuers. A risk manager in this firm uses VaR to calculate the market risk on the portfolio based on an historical observation period of one year.

Credit markets have been stable for many months, but just recently there has been a minor currency crisis and a small country is now in danger of default. Fortunately, the treasury department does not own any bonds issued by this country. However, the firm is concerned about nearby countries, where it does have exposure. In recent weeks credit spreads have already risen for these neighbouring countries.

(iii) (a) Outline the limitations of VaR as a market risk measure in this situation.

(b) Suggest, with reasons, additional risk measures that would address these limitations.

[5]

[Total 14]

8 (i) State the Binomial Representation Theorem (BRT) for discrete time stochastic processes, defining any terms you use.

[3]

A process S follows a discrete random walk under probability measure **Q** where, at each time step i (i = 1, 2, …), its value either increases by 20% or decreases by 30%. Initially its value is 2.5.

A claim X exists on S at time i = 3 such that $X = \ln(S)$ if this is positive, or zero otherwise. The risk-free rate is zero.

(ii) (a) Find the probability measure **Q**, confirming that **Q** is constant for all i.

(b) Verify for the first **two** time steps that the BRT applies to the expectation of X under **Q**.

[8]

(iii) (a) Explain briefly why taking expectations, as in part (ii)(b), is a useful step in the context of the BRT.

(b) State the continuous time equivalent of the BRT.

(c) Show from this theorem why one common test for a process being a martingale is whether or not it has zero drift.

[5]
[Total 16]

END OF PAPER