EXAMINATION

21 April 2010 (pm)

Subject ST6 — Finance and Investment Specialist Technical B

Time allowed: Three hours

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You have 15 minutes before the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*

3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all eight questions, beginning your answer to each question on a separate sheet.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

NOTE: In this examination, you are never required to prove the use of an arbitrage-free methodology unless clearly stated in the question.

1 (i) Define the following terms in the context of a binomial model:

(a) stochastic process
(b) filtration
(c) previsible process
(d) probability measure
(e) contingent claim
(f) risk-neutral valuation

[6]

(ii) Outline how risk-neutral valuation and a change of measure are used in valuing derivatives. [2]

[Total 8]

2 Consider a European Call option on a futures price *F*, with time to maturity *t* and strike price *K*.

(i) Write down a general formula for the current value of the European Call in terms of the probability density function (p.d.f.) of *F* and the risk-free rate *r*. [1]

The most common representation of this p.d.f. is a lognormal distribution, with volatility parameter σ. Another possible representation is a gamma distribution of the form $f(x) = \dfrac{x^{\lambda-1}\exp(-x/\beta)}{\beta^{\lambda}\Gamma(\lambda)}$.

The gamma distribution has a higher kurtosis than the lognormal distribution.

(ii) Show that, for this gamma distribution representation, the current value of the European Call is equal to

$$e^{-rt}\left[\frac{\beta\Gamma(\lambda+1)}{\Gamma(\lambda)} - P(\lambda,\beta,K)\right],$$

where $P(\lambda,\beta,K) = \displaystyle\int_0^K \frac{x^{\lambda}\exp(-x/\beta)}{\beta^{\lambda}\Gamma(\lambda)}dx$. [3]

Whilst using the above gamma distribution to model the futures price, you have observed that the β parameter is very stable when fitting actual at- and out-of-the-money option prices. This compares with the lognormal distribution, where the volatility parameter, σ, varies significantly when fitting the same option prices.

(iii) Set out briefly the advantages and disadvantages of choosing a gamma distribution to model *F* instead of the lognormal distribution. [3]

[Total 7]

3 Consider an economy where the short rate of interest r follows the lognormal stochastic process:

$$dr = \mu(r,t)rdt + \sigma(r,t)rdz$$

where z is a standard Brownian motion.

In this economy, the prices of all traded assets depend only on the short rate r and its evolution over time.

(i) (a) Describe the “market price of risk”, λ, in the context of the stochastic process.

(b) Show (either by argument or by algebra) that λ cannot be a function of any traded asset.

[5]

The price of a traded asset $V = V(r, t, T)$ satisfies the following partial differential equation (PDE):

$$\frac{\partial V}{\partial t} + r\left(\mu - \lambda\sigma\right)\frac{\partial V}{\partial r} + \frac{1}{2}\sigma^2 r^2 \frac{\partial^2 V}{\partial r^2} = rV$$

The finite difference method can be used to obtain a numerical solution to the above PDE. The binomial tree valuation method is also a widely used alternative to the finite difference method.

(ii) (a) Discuss how each of these two methods achieves a solution to the PDE.

(b) Comment on the relative suitability of each method for the valuation of options.

[5]
[Total 10]

4 An insurance company with substantial domestic pensions business is reviewing its policy with respect to derivatives across various asset classes.

The actuarial risk function is concerned about the amount of longevity and Limited Price Indexation (LPI) risk inherent in the pensions business. It has noted that most of the OTC contracts available to manage these risks tend to be arranged with counterparties in the banking sector, which has been under stress recently.

(i) Describe the main types of derivative instruments that can be used to hedge longevity risk. [5]

(ii) Explain how counterparty risk originates in OTC contracts and how the insurance company can mitigate it. [4]

(iii) (a) Discuss the nature of the insurance company's LPI risk in a low inflation environment, commenting on potential ways of reducing it.

(b) Identify institutions that might be natural hedgers of inflation risk in the financial markets, commenting on how likely it is that they would want to use inflation derivatives.

[5]
[Total 14]

5 A financial institution has written £1 billion of five-year equity linked bonds with maturity guarantees. Rather than buying a five-year Put option as a hedge, it has been delta hedging the implicit option dynamically using equity index futures.

Eighteen months from the outset, you have been asked to compare retrospectively the effect of the futures based strategy against what the institution would have experienced had it purchased a five-year option. You have broken down the difference between the two strategies into the main option sensitivities (Greeks) and separated the figures into three time periods:

- Months 1–6 ("Market fall"), during which equity prices fell steadily.
- Months 7–12 ("Volatile period"), during which equity prices were extremely volatile but finished at levels similar to those at the start of the period.
- Months 13–18 ("Market recovery"), during which equity prices rose steadily.

Your analysis assumes that market implied volatilities remained at 30% throughout the eighteen month period, this being the same volatility as was assumed throughout in the delta hedging calculations.

Your results in £ millions are as follows:

£m	*Delta*	*Gamma*	*Theta*	*Rho*	*Vega*	*Total*
Market fall	–1	–11	+5	–5	—	–12
Volatile period	–2	–22	+3	–22	—	–43
Market recovery	+1	–5	+5	—	—	+1
Total	–2	–38	+13	–27	—	–54

You now need to consider how to report these figures to the Board.

(i) Describe the likely causes of these figures and the lessons that could be learnt from them. [10]

The Board receives your initial report, but responds that it is concerned about the assumption that implied volatility stayed at one level throughout the eighteen months. It therefore asks you to adjust your analysis to allow for implied volatilities that actually occurred in the market during that time.

You repeat your calculations with these actual market implied volatilities, and derive the following table of results:

£m	*Delta*	*Gamma*	*Theta*	*Rho*	*Vega*	*Total*
Market fall	–1	–11	+5	–5	—	–12
Volatile period	—	–13	+13	–25	–42	–67
Market recovery	+1	–5	+5	—	+26	+27
Total	—	–29	+23	–30	–16	–52

(ii) Explain how this new information might change your answer to (i). [4]

[Total 14]

6 You have been asked to fit a yield curve to the benchmark swap rates published in the *Financial Times* (FT). The FT publishes swap rates for annual maturities from 1 year up to 10 years, then 12, 15, 20, 25 and 30 years.

You have decided to fit the yield curve by formula using a spreadsheet. You have chosen a family of functions with a number of parameters and will vary these parameters to achieve an optimal fit.

(i) Describe the considerations involved in the choice of formula and the number of variable parameters. [5]

(ii) Compare the relative merits of the above method of obtaining a yield curve with deriving yield curves via bootstrapping. [3]

(iii) Explain why a swap dealer might be interested in the impact on their swap book of changes in individual benchmark swap rates. [3]

A Euro swap dealer wishes to buy (i.e. receive fixed payments on) a three-year €1 million annual coupon fixed-floating interest rate swap. The annually compounded one-year forward rates used to price the swap are as follows:

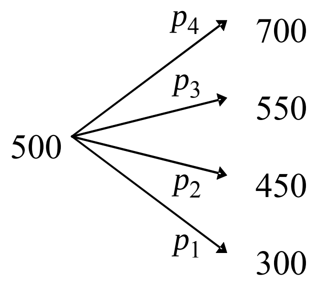
Time 0 to 1	0.50%
Time 1 to 2	1.25%
Time 2 to 3	1.50%

(iv) (a) Calculate the annual fixed payments on this swap, assuming it is transacted with zero mark-to-market value.

(b) Calculate the impact on the mark-to-market value of the swap if the "Time 2 to 3" forward rate (only) falls immediately by a quarter of a percent, i.e. to 1.25%.

[4]
[Total 15]

7 The price of a non-dividend paying equity worth 500 today has four possible values in a year's time: 300, 450, 550 or 700.



Consider the task of calculating appropriate risk-neutral probabilities for this equity using a single step quadrinomial (four branch) tree. The approach to be used is to calibrate the tree by calculating the four risk-neutral probabilities (p_1, p_2, p_3, p_4) so that the tree correctly prices:

- the equity
- one-year risk-free bonds
- a Call option on the equity with a strike of 600
- a Put option on the equity with a strike of 400

Assume that risk-free interest rates are 5% continuously compounded and market implied volatilities are 22.5% for both options.

(i) (a) Show that the current values of the Call and Put using the Black-Scholes formulation are 20.60 and 5.33 respectively.

(b) Calculate the four risk-neutral probabilities.

[9]

Now assume that risk-free interest rates remain at 5% continuously compounded but that market implied volatilities are such that a volatility of 20% applies to options with a strike of 600 and 25% applies to options with a strike of 400. This is called a "volatility skew".

(ii) Explain how the Black-Scholes values of the Call and Put, and hence the four risk-neutral probabilities, will differ relatively from those found in (i).
[Note: You do not need to calculate any values.] [4]

(iii) Illustrate the impact of the volatility skew by sketching the continuous risk-neutral probability distribution against equity prices both with the skew and without it. [3]

[Total 16]

8 A life insurance company is considering entering into a zero-cost collar on a £1bn portfolio of equities that it is holding within its long term fund. This will be achieved by:

- buying a Put option on £1bn of equities with strike X
- selling a Call option on £1bn of equities with strike Y, where $Y > X$

This collar is zero-cost because the premiums paid on the Put and received on the Call are constructed to be the same (by varying X and Y).

(i) Explain why the company might be considering hedging its equity holdings and why it is considering a collar rather than a standalone Put. [3]

The company has chosen a term of one year for the collar, with strikes of £0.8bn for the Put and £1.1bn for the Call.

(ii) Explain why the company might have chosen these particular terms. [4]

(iii) Sketch three graphs showing:

- the value of the collar
- the Delta of the collar
- the Gamma of the collar

against the value of the equity portfolio, with each graph showing the position shortly after outset and shortly before maturity.

[9]
[Total 16]

END OF PAPER