INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

15 April 2011 (pm)

Subject ST6 — Finance and Investment Specialist Technical B

Time allowed: Three hours

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You have 15 minutes before the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*

3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all eight questions, beginning your answer to each question on a separate sheet.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

NOTE: In this examination, you are never required to prove the use of an arbitrage-free methodology unless clearly stated in the question.

1 (i) Explain the mechanics of a property swap. [3]

(ii) Describe the potential benefits that could be obtained from transacting a property swap based on a property index, compared with direct investment in the property market. [3]

(iii) Outline the risks involved in buying and selling property derivatives, other than those directly related to the performance of the property market. [2]
[Total 8]

2 Consider a standard Brownian motion z_t and a stochastic process x_t based on z_t, where $dx_t = \mu(x,t)dt + \sigma(x,t)dz_t$.

(i) (a) State Ito's Lemma for a function of x_t.

(b) Explain why an additional term arises that is not present in non-stochastic calculus. [3]

(ii) Show that $\int_0^t z_s dz_s = \frac{1}{2}(z_t^2 - t)$. [2]

(iii) (a) Derive the variance of a discrete average A_t of observations of x_t:

$$A_t = \frac{1}{n}\sum_{i=1}^{n} x_{t_i} \text{ where } t_i = t.\frac{i}{n} \text{ for } i = 1, 2, ..., n$$

(b) Evaluate this variance in the limit of continuous sampling ($n \to \infty$). [6]

[*Hint: Consider the distribution of the discrete increments* $\Delta_i = dx_{t_i}$. *The following formula may be useful:* $\sum_1^n r^2 = \frac{1}{6}n(n+1)(2n+1)$.]

[Total 11]

3 Consider an at-the-money European Put option with strike X, maturity T, price P and market implied volatility σ, based on a dividend-paying stock with spot price S that releases income at a continuously compounded rate q per annum. The T-year risk-free rate is r per annum, also continuously compounded.

[*Note: In this context, an at-the money strike is one that is equal to the corresponding forward price of the underlying asset.*]

(i) (a) Write down a formula for X.

(b) Derive a formula for the market implied volatility σ in terms of P, S, q, r and T, defining any notation you use.

[5]

Over the next year, a particular equity, currently priced at 80, will pay two cash dividends: one of 0.9 in three months and one of 1.1 in nine months. The one-year risk-free rate is 4%, continuously compounded.

(ii) (a) Calculate the equivalent continuously compounded dividend rate over a one-year horizon. You may ignore the time value of the dividends paid within the year.

(b) Calculate the at-the-money strike for a one-year European Put option on this equity, using the same definition of "at-the-money" as in (i).

(c) Calculate the implied volatility if this at-the-money Put option is priced at 10.5.

[4]

If the risk-free interest rate now falls to 2% per annum, the above Put option will have a higher value. However, if the calculation performed in (ii) is re-done from the outset with a risk-free rate of 2% per annum, the at-the-money option priced at 10.5 has exactly the same implied volatility as calculated in (ii)(c).

(iii) Explain these effects and the apparent anomaly. [3]

[Total 12]

4 The "speed" of a derivative is defined as its third order sensitivity to movements in the price of the underlying, i.e. $\frac{\partial \Gamma}{\partial S} = \frac{\partial^3 V}{\partial S^3}$, where S is price of the underlying asset, Γ is the gamma of the option and V is the option price.

Let

$$d_1 = \frac{[\ln(S/X) + (r + \tfrac{1}{2}\sigma^2)T)]}{\sigma\sqrt{T}},$$

where X is the strike and r is the risk-free rate (continuously compounded).

(i) (a) Derive a formula for the gamma of a European Call option of maturity T years based on a non income bearing security S with volatility σ.

[*Note: You may use the result that the delta of the option is* $N(d_1)$.]

(b) Hence show that the "speed" of the option is given by:

$$-\frac{\Gamma}{S}\left[1 + \frac{d_1}{\sigma\sqrt{T}}\right]$$

[4]

(ii) Sketch graphs for this European Call option at time T from expiry, showing how the following vary with the price of the underlying asset:

(a) price
(b) delta
(c) gamma
(d) "speed"

[6]

A dealer, who is in charge of hedging a portfolio of options on the US$-Euro exchange rate, has just discovered the formula for "speed" and thinks it would be a good indicator to monitor, on the basis that reducing the absolute level of portfolio "speed" would result in a more stable gamma hedge.

(iii) Discuss this proposal. [4]

[Total 14]

5 A bond market consists of the following four government bonds, with prices (per 100 nominal) and redemption yields (in % per annum) as per the table below:

Term	*Annual coupon %*	*Price*	*Redemption yield % p.a.*	*Discount factor*
1 year	3.00	101.03	1.95	0.98087
2 year	4.00	102.05	2.93	
3 year	4.75	103.21	3.60	
4 year	5.50	105.36	4.02	0.85153

In each case, coupons are paid annually and the next one is due to be paid in a year's time. Redemption yields are annually compounded.

Discount factors are shown for years 1 and 4. The discount factor for year n is the decimal price of a zero coupon bond of duration n.

(i) Complete the table above by calculating the discount factors applicable to the yield curve for years 2 and 3. [2]

Consider a 2-year European Put option on the 4-year government bond shown above. The strike price of the option is 100 (per 100 nominal) and the applicable forward yield volatility is 25% per annum. The option is exerciseable immediately after the bond pays its second annual coupon.

(ii) (a) Show that the 2-year forward price of the 4-year government bond is 100.45.

(b) Calculate, using a forward modified duration of 1.85 and a forward yield of 5.25%, the volatility of the price in (a).

(c) Estimate, using the Black model, the price of the option.

[7]

(iii) Discuss whether the Black model is an appropriate model for valuing this particular bond option. [3]

[Total 12]

6 An analyst is attempting to value a three-year American Put option with strike 1,000 on a non income bearing security with a spot price of 1,000. Risk-free interest rates are 5% per annum at all durations, continuously compounded. The option is to be priced using a volatility of 25%.

She has already constructed a recombining risk-neutral binomial tree for the price of the underlying (with time in years):

Time 0	Time 1	Time 2	Time 3
			2,210.3
		1,696.8	
	1,302.6		1,302.6
1,000.0		1,000.0	
	767.7		767.7
		589.3	
			452.4

At each node of the tree, the risk-neutral probabilities are 0.53 for an upward movement and 0.47 for a downward movement. The tree has not been calibrated using the standard Cox-Ross-Rubenstein (CRR) formulae.

(i) (a) Explain why the CRR formulae might not be appropriate for this tree.

(b) Demonstrate that the tree has been calibrated appropriately by deriving the expectation and standard deviation of the security price at Time 1, both from the tree and theoretically.

[4]

[Note: The variance of a Lognormal distribution with parameters μ and σ is: $\{exp(\sigma^2)-1\}*exp(2\mu+\sigma^2)$.]

(ii) Use the tree to determine:

(a) the price of a three-year European Put option with a strike of 1,000

(b) the price of a three-year American Put option with a strike of 1,000

[6]

(iii) (a) Determine a more accurate price for the American option using the control variate technique.

(b) Explain whether you would expect your answer to (iii)(a) to overstate or understate the price of an American Put option.

[6]
[Total 16]

7 (i) Describe the concept of a long-term credit rating for a company or individual security, as assigned by a credit rating agency (CRA). [3]

During the global credit crisis of 2008, many residential mortgage-backed securities (RMBS) and collateralised debt obligations (CDOs) that had initially been given the highest ("triple A") rating were downgraded sharply by the CRAs over the space of only a few months, some even to C grade.

(ii) (a) Discuss possible reasons for this action by the CRAs.

(b) Assess the immediate significance of these sudden downgrades to the various users of ratings in RMBS and CDOs.

(c) Describe three changes in the way CRAs operate that market regulators have proposed as a result of the 2008 crisis.

[8]

Prior to the credit crisis, when banks lent floating rate money to small or medium sized corporate customers they often insisted that these corporates should hedge their exposure to rising interest rates by using a fixed-floating swap. The swap would take the form of an over-the-counter contract between the bank and the corporate, transacted under an ISDA Master Agreement but without an accompanying Credit Support Annexe.

Since the credit crisis, interest rates have fallen very sharply in many Western economies.

(iii) (a) Describe the hedge that the corporate customer would have undertaken in respect of a £25 million 5-year loan when interest rates were 6% per annum.

(b) Explain why the banks would have insisted on such a hedge.

(c) Explain in credit risk terms (but without performing any calculations) the impact of the swap to the lending bank when interest rates fell from 6% per annum to 1% per annum.

(d) Discuss briefly possible ways in which the bank could mitigate its credit exposure arising from the swap.

[5]
[Total 16]

8 A market consistent economic scenario generator (ESG) is being built for yield curves that will be used to calculate prices for interest rate derivatives. The LIBOR market model has been chosen as the underlying model, implemented using a single stochastic factor with discrete time steps. The numeraire for the model is a rolling deposit, i.e. one where £1 is invested at time zero in a bond maturing at time t_1, with the proceeds at time t_1 being invested in a bond maturing at time t_2, and so on.

The underlying stochastic differential equation (SDE) within the model is:

$$\frac{dF_k(t)}{F_k(t)} = [v_{m(t)}(t) - v_{k+1}(t)]\zeta_k(t)dt + \zeta_k(t)dz$$

where

- $F_k(t)$ is the forward rate between times t_k and t_{k+1} as seen at time t, expressed with a compounding period of $\delta_k = t_{k+1} - t_k$ using actual/actual day count
- $P(t, t_k)$ is the price at time t of the zero-coupon bond that pays 1 at t_k
- $\zeta_k(t)$ is the volatility of $F_k(t)$ at time t
- $v_k(t)$ is the volatility of $P(t, t_k)$ at time t
- $m(t)$ is an index for the next reset date at time t, so:

  $m(t) = 1$ for $t \le t_1$
  $m(t) = 2$ for $t_1 < t \le t_2$ etc.

- dz is a standard Brownian motion

(i) (a) Describe how a derivative price would be calculated from the set of derivative payoffs produced by the Monte Carlo runs using this ESG.

(b) Discuss the advantages and disadvantages of using the LIBOR market model for this task compared with simpler models of the yield curve.

[5]

(ii) (a) Write down a formula that expresses $P(t, t_i) / P(t, t_{i+1})$ in terms of the forward rates.

(b) Show, by applying Ito's Lemma to the logarithm of your formula in (a), or otherwise, that the underlying SDE can be rewritten as:

$$\frac{dF_k(t)}{F_k(t)} = \left[\sum_{i=m(t)}^{k} \frac{\delta_i \zeta_i(t)\zeta_k(t)F_i(t)}{1+\delta_i F_i(t)}\right]dt + \zeta_k(t)dz$$

[4]

The sequence of ζ_i for $i = 1, 2, 3$ ... forms a step function of volatilities, and the model now needs to be calibrated to market prices. Caplet volatilities $\sigma_1, \sigma_2, \sigma_3$, ... have been supplied, where each σ_i is the Black volatility for the caplet that corresponds to the period between t_i and t_{i+1}.

(iii) Demonstrate how to derive the values of $\zeta_1, \zeta_2, \zeta_3$, ... etc from the σ_i. [2]

[Note: For ease of notation, you may assume for this part that $t = t_0 = 0$.]

[Total 11]

END OF PAPER