INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

2 October 2013 (am)

Subject ST6 – Finance and Investment Specialist Technical B

Time allowed: Three hours

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You have 15 minutes before the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*
3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all eight questions, beginning your answer to each question on a separate sheet.*
6. *Candidates should show calculations where this is appropriate.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

NOTE: In this examination, you are never required to prove the use of an arbitrage-free methodology unless clearly stated in the question.

1 Consider a non-dividend-paying stock whose price S_t at time t (in years) follows the geometric random walk process:

$$dS_t = \mu S_t dt + \sigma S_t dz$$

where μ and σ are constants and z is a standard Brownian motion. Assume that the continuously compounded risk-free rate of interest r is constant for all maturities.

During the first two years $\mu = 1.5$ and $\sigma = 3$, but thereafter $\mu = 2$ and $\sigma = 4$. $S_0 = 10$.

(i) Derive the probability distribution of S_4. [4]

Consider a forward contract for purchasing the stock at time T, whose price is $F_t(T)$.

(ii) (a) Write down a formula for $F_t(T)$ in terms of r and S_t.

(b) Derive the process followed by $F_t(T)$.

[4]
[Total 8]

2 (i) Explain the purpose of a central counterparty (CCP) clearing over-the-counter (OTC) derivative trades. [2]

Individual Clearing House (ICH), a CCP, allows its members to transact OTC swaps and options under a fixed schedule of initial margins by instrument. The margins are calculated separately for each OTC instrument based on 99% ten-day Value at Risk (VaR). For a portfolio of instruments, the margins are added together on a gross basis, i.e. without netting longs and shorts.

Portfolio Clearing House (PCH), another CCP, similarly allows its members to transact OTC swaps and options but with a daily initial margin requirement based on 99% ten-day VaR for the whole of a member's portfolio, i.e. all positions taken together. In other respects, including data sources and observation periods, the VaR methodologies for PCH and ICH are identical. Variation margin calculations are also identical.

(ii) Estimate ICH's initial margin requirement for an equity index swap based on a price volatility of 24% per annum for the index. [2]

(iii) Discuss:

(a) the issues that a bank would consider when choosing between ICH and PCH as its clearing house for OTC derivatives.

(b) the shortcomings of ICH's and PCH's initial margin calculations.

[5]

(iv) Outline the problems that a CCP might encounter in clearing property derivatives. [2]
[Total 11]

3 An up-and-in put is a European barrier put option that can only be exercised if the spot price of the underlying has at some point before expiry exceeded a specified barrier price H. Conversely, an up-and-out put is one that becomes worthless if the spot price of the underlying ever exceeds the barrier price H.

Consider an up-and-in put, an up-and-out put and a vanilla put option, all with identical strike K which is lower than the barrier price H. The price of the underlying asset is S, and the barrier price has not yet been breached.

(i) Sketch **two** graphs showing, for these options:

(a) the value
(b) the delta

against S shortly after the options are issued. All three options should appear on each graph. [6]

(ii) Explain the circumstances under which the vega of a barrier option could be negative. [2]

A bank is reviewing its pricing calculations for barrier options.

(iii) Outline the advantages and disadvantages of valuing the barrier options using:

(a) algebraic formulae.
(b) binomial or trinomial trees.
(c) Monte Carlo techniques.

You may assume that formulae exist for pricing up-and-in calls and puts based on continuous monitoring for barrier breaches.

[4]
[Total 12]

4 An investment bank is considering setting up a hedging service for companies to manage risk in their defined benefit pension funds. The service includes offering hedges for longevity and inflation risks.

(i) Describe the main derivatives-based products (i.e. swaps, options or structured bonds) that the bank could offer in these two areas of risk, explaining in each case why the hedges would be important to a pension fund. [8]

In considering how to price these products, there has been a debate within the bank about whether to use risk-neutral or real-world valuation methods and parameters.

(ii) Outline the main points that should be considered in this debate. [6]
[Total 14]

5 The ABC managed fund has £100m of assets. It has always invested 50% of its funds to track a total return equity index, holding the remaining 50% in cash.

In the light of uncertain equity markets, it has been proposed that the fund protects its entire equity holding with a one-year put option on the tracked index with a strike 20% below the current index level. The purchase cost would be funded by the selling of a call option on the same index for the same notional, with a strike chosen to make the prices of the call and the put equal.

The risk-free rate is 2% per annum continuously compounded at all durations and the implied volatility of the put is 25% per annum.

(i) Show that the price of the put option is approximately £966,500. [4]

It has been determined that the appropriate call option has a strike equal to 124% of the current index level. The implied volatility of the call is 20% and its price is equal to the put price.

ABC's risk department is content that the fund's mandate allows the use of such derivatives. However, it is questioning whether the fund managers have fully understood the impact of the transactions being proposed.

As a starting point, the risk department wishes to calculate the short-term sensitivity of the overall portfolio to equity prices (assuming that the put and call transactions take place) and to express this as an "effective equity exposure" comparable to the current 50%.

(ii) Calculate the fund's effective equity exposure. [3]

(iii) Explain how the effective equity exposure would change if:

(a) equity prices suddenly increase.

(b) market implied volatilities increase.

(c) the fund managers choose to use options with a shorter term.

(d) the fund managers choose to use a lower strike for the put and a higher strike for the call.

[5]

The risk department is also interested in the fund's sensitivity to interest rates, which it would like to be expressed as an "effective bond duration". This duration is defined to be the term of a zero-coupon bond that, if it replaced the 50% cash in a fund with no derivatives, would result in the same overall interest rate sensitivity.

(iv) Calculate the effective bond duration of the fund after the derivative transactions.

You may use the results that the rho of a European call is $KTe^{-rT}N(d_2)$ and that the rho of a European put is $-KTe^{-rT}N(-d_2)$.

[3]

(v) Outline two other main changes to the risk profile of the fund that would arise as a result of the proposed derivatives strategy. [2]

[Total 17]

6 A stock pays dividends at a continuous rate δ. Its price S_t at time t (excluding dividends) follows the Black-Scholes stochastic process:

$$S_t = S_0 \exp(\sigma W_t + \mu t)$$

where μ and σ are constant and W_t is a standard Brownian motion. In addition, a risk-free bond exists with price $B_t = \exp(rt)$ at time t, where r is the risk-free rate.

Consider a claim X based on the value of the stock at time T.

(i) (a) Write down a formula for the value $\tilde{S}_t$ of the total return index that consists of the stock together with reinvested dividends.

(b) Describe, using formulae and allowing for dividends, how a replicating strategy for X can be constructed.

[6]

Consider the following statement: "The Black-Scholes assumption of lognormality in the distribution of asset price changes is seldom experienced in practice in financial markets. This introduces major errors into option pricing when using the Black-Scholes formula."

(ii) (a) Explain why lognormality of price changes might be a good but imperfect approximation to future asset price movements.

(b) Explain, using examples, how variations in the underlying price movement distribution from lognormal can impact option pricing.

[6]

[Total 12]

7 As part of a new solvency regime, the regulatory authorities have deemed that all insurers should report profits based on a market consistent valuation of liabilities.

Within the regulations is the definition of an official yield curve to be used for the calculations. The yield curve is to be constructed as follows:

- Up to 10 years, the official yield curve is to be one that replicates swap rates traded within the market, converted into continuously compounded spot rates. It is assumed that the swap rates are liquid and prices are always available.

  This section of the official yield curve is given in the table below.

Term (years)	*Spot Rate %*	*Term (years)*	*Spot Rate %*
1	0.61	6	1.15
2	0.65	7	1.35
3	0.71	8	1.54
4	0.82	9	1.73
5	0.97	10	1.90

- From 10 to 50 years, the one-year forward rates are linearly interpolated between the [9, 10] one-year forward rate and the [49, 50] one-year forward rate, the latter being fixed at 4%.

- Beyond 50 years, all one-year forward rates are 4%.

(i) Calculate the fixed rate in a three-year swap with annual payments. [2]

(ii) Show that the 50-year continuously compounded spot rate on the official yield curve is 3.3577%. [4]

(iii) Find the term at which the continuously compounded spot rate on the official yield curve reaches 3.5%. [2]

A life insurance company has a liability comprising fixed cashflows at durations 1, 2, ….., 50. It wishes to derive a hedge against movements in the official yield curve and has access to risk-free zero coupon bonds with terms 1, 2, …, 50 that are priced consistently with swaps.

(iv) Describe how the insurance company could construct the hedge. [3]

The regulator has announced that it retains the right to adjust the 4% ultimate forward rate in future to reflect economic conditions.

(v) Outline the possible impact of this announcement on the insurance company’s risk profile and hedging strategy. [2]

[Total 13]

8 Heath Jarrow Morton (HJM) is a model of the yield curve using one stochastic risk factor. Its starting point is to define zero-coupon bond prices $P(t, T)$ maturing at time T in the traditional risk-neutral world, which at time t must yield $r(t)$, the risk-free short rate. The generalised process for P is therefore:

$$dP(t, T) = r(t)\, P\, dt + v(t, T)\, P\, dz$$

where $v(t, T)$ is a time-dependent representation for the volatility of P, and $z(t)$ is a standard Brownian motion driving term structure movements.

[*In its most general form, the volatility factor in HJM can also depend on past and future interest rates and bond prices. The term volatility is here (and in other literature) used slightly loosely since, strictly, v(t, T) is a standard deviation.*]

(i) Write down formulae, in terms of P, for:

(a) the forward rate $f(t, T_1, T_2)$ between times T_1 and T_2 observed at time t.

(b) the instantaneous forward rate $F(t, T)$ for time T observed at time t. [2]

Let the process followed by the instantaneous forward rate $F(t, T)$ under HJM be expressed in a similar general form as follows:

$$dF(t, T) = m(t, T)\, dt + s(t, T)\, dz$$

for some time-dependent functions of drift $m(t, T)$ and volatility $s(t, T)$.

(ii) Show how $m(t, T)$ can be expressed only in terms of $s(t, T)$. [6]

[<u>*Hint*</u>*: Observe the process followed by* $f(t, T, T+\Delta T)$ *in the limit as* $\Delta T \to 0$.]

Suppose that the instantaneous forward rate volatility is given by $s(t,T) = \sigma e^{-a(T-t)}$ for some constants σ and a.

(iii) Demonstrate, by deriving and comparing the dz terms in the stochastic differential equations for $P(t, T)$, that in this special case the HJM model is intrinsically the same as the Hull-White model.

You may use the Hull-White result that $P(t,T) = A(t,T)e^{-B(t,T)r(t)}$ for some time-dependent functions A and B, where $B(t,T) = \dfrac{1-e^{-a(T-t)}}{a}$ with $a > 0$, and the standard deviation of $r(t)$ is a constant σ. [5]

[Total 13]

END OF PAPER