INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

14 April 2016 (pm)

Subject ST6 – Finance and Investment
Specialist Technical B

Time allowed: Three hours

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You have 15 minutes before the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*

3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all seven questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 A simple securitisation arrangement is used to turn loans into an asset-backed security, which has a senior, mezzanine and equity tranche.

(i) Describe the “waterfall” feature of such an asset-backed security. [2]

The portfolio consists of $200m nominal of loans, split 70% into the senior tranche, 20% into the mezzanine tranche and 10% into the equity tranche.

(ii) Draw a simple diagram to illustrate how this asset-backed security may have been structured. [3]

The mezzanine tranche is repackaged with the mezzanine tranches of other identically structured asset-backed securities on the same type of loans into an asset-backed security collateralised debt obligation (CDO). This CDO is split 55% into a senior tranche, 35% into a mezzanine tranche, and 10% into an equity tranche.

(iii) Calculate the total proportion of the original loans that will now form senior tranches. [1]

Assume that loss given default on any loan is 70%.

(iv) Calculate the proportion of the loans that would need to default for the mezzanine tranche of the CDO to receive no repayment of principal. [2]
[Total 8]

2 (i) Explain the term risk-neutral. You should include a basic example. [2]

(ii) Explain the term risk-neutral probability measure. [2]

(iii) Compare the risk-neutral and real-world probability measures. [2]

(iv) Describe the limitations of the risk-neutral approach when pricing options. [3]
[Total 9]

3 Assume that a stock S goes ex-dividend at time t with a dividend D due at that time.

An investor holds a plain vanilla American call option on S with strike price K which expires at $T > t$. The stock will not pay any other dividends before time T.

(i) State the times at which it may be optimal to exercise this option. [1]

(ii) Derive the inequality that D must satisfy for it not to be optimal to exercise early.

[Hint: You may use the fact that a lower bound for the price of an American call option on a non-dividend paying stock with price S, exercise price K, continuously compounded interest rate r and time to expiry T is $S - Ke^{-rT}$.] [3]

Another stock $\tilde{S}$ goes ex-dividend at times $t_1, t_2, \ldots t_n$ with $t_1 < t_2 < \ldots t_n$. The dividends corresponding to these times are denoted $D_1, D_2, \ldots D_n$.

(iii) State the condition for it not to be optimal to exercise immediately prior to t_i for any $i < n$. [1]

The price of a particular stock is \$60. The stock is expected to pay dividends of \$1 in two and five months' time, and these are also the ex-dividend dates. The time to maturity of an American call option is six months, the exercise price is \$60, the volatility is 25% per annum and the continuously compounded risk-free rate is 8% per annum.

(iv) Determine the value of the American call option using Black's approximation. [10]

[Total 15]

4 (i) Define foreign exchange risk. [1]

A small, but growing, construction company in an Asian country has won a contract to build a road in India, where the domestic currency is the Rupee. The Asian country uses the currency of Fiagold. The current spot exchange rate is 0.1 Fiagolds per Rupee.

The net payoff from the construction contract will be 1,000,000 Rupees, paid in one year's time.

The company is concerned about the foreign exchange risk and is using Rupee futures to hedge against Rupee depreciation.

The company hedges 900,000 Rupees using one-year future contracts with an exchange rate on delivery of 0.11 Fiagolds per Rupee.

(ii) Determine the net payoff in Fiagolds from the construction contract at the time at which it is paid, if the exchange rate at that time is 0.07 Fiagolds per Rupee. You can ignore interest and should state any other assumptions made. [2]

The company is also considering bidding on a contract to build a train line in another part of India. It is required to quote a contract price in Rupees now, in June, while the exchange rate is still 0.1 Fiagolds per Rupee. The company is concerned about the foreign exchange risk if it wins the bid and is paid in Rupees in September.

The company wants to hedge this risk using 3-month European options. The following options are available:

- a put option with a strike price of 0.1 Fiagolds per Rupee and a cost of 0.001 Fiagolds per Rupee; and

- a call option with a strike price of 0.1 Fiagolds per Rupee and a cost of 0.002 Fiagolds per Rupee.

The size of each option contract is 2,000,000 Rupees.

The company requires a minimum net revenue from the construction contract of 1,000,000 Fiagolds <u>after allowing for hedging costs</u>. Interest, tax and any other costs relating to the construction contract can be ignored.

(iii) Determine a hedging strategy using one, or both, of the options above that will allow the company to hedge the risk of the net revenue being below the required level after hedging costs. You should bear in mind that the revenue in Rupees will need to cover the hedging costs paid in Fiagolds. [3]

The company decides to buy six put options and is carrying out a scenario analysis to understand the impact of this hedge. It is considering the scenario where the Rupee appreciates to 0.20 Fiagolds per Rupee in September and one where the Rupee depreciates to 0.05 Fiagolds per Rupee in September. The company also realises that it needs to consider that it might not win the bid, so there are four scenarios to consider.

For each scenario, the company wishes to calculate the total net profit/loss in Fiagolds arising from the combination of the hedging strategy and, for those scenarios where the construction contract is assumed to be won, the revenue from that contract.

It can be assumed that the company has quoted a contract price in Rupees which is exactly sufficient to meet its minimum revenue requirements after allowing for hedging costs. If the construction contract is won, it can be assumed that this price is paid to the company in Rupees in September.

Interest, tax and any other costs relating to the construction contract can again be ignored.

(iv) Determine the profit/loss in September for each of the four scenarios. [5]

(v) Suggest other risk management strategies the company could adopt to manage its foreign exchange risks. [4]

[Total 15]

5 In an emerging economy, swap yields (the “Swap Curve”) have historically been used to value liabilities. A new regulator is proposing to introduce an official Regulatory Curve which all financial institutions will now have to use to value their liabilities.

The Regulatory Curve will be set equal to the spot rates of the Swap Curve, subject to a minimum of 3% p.a. and a maximum of 5% p.a. The current continuously compounded spot rates of the Swap Curve and Regulatory Curve are shown below:

Term (years)	*Swap Curve*	*Regulatory Curve*
1	0%	3%
2	2%	3%
3	3%	3%
4	4%	4%
5	5%	5%
6	6%	5%
7	7%	5%

Consider an n-year annuity certain, payable annually in arrears.

(i) Test whether the price of the annuity certain is higher, equal or lower when valued using the Regulatory Curve compared to when valued using the Swap Curve, for each of the years n = 1 to 7. [4]

(ii) Write down the formula for deriving the continuously compounded one-year forward rate from the relevant spot rates, defining your notation. [1]

(iii) Plot a graph showing the implied one-year continuously compounded forward rates of both the Swap Curve and the Regulatory Curve. [4]

A financial institution has a range of liabilities that are payable over the next seven years. These liabilities are currently valued using the Swap Curve and are very closely hedged, using plain vanilla receiver (i.e. receive the fixed payment) interest rate swaps.

Following the introduction of the new regulations, the financial institution will value the liabilities using the Regulatory Curve. The swaps will continue to be priced using the Swap Curve.

(iv) Assess, based on the current Swap Curve, how effective the existing hedge will be at matching the change in the value of the liabilities that are payable at different terms following both small and large movements in swap rates, once the Regulatory Curve has been introduced. You can ignore the balance sheet change caused by the introduction of the Regulatory Curve. [5]

(v) Comment on how interest rate caplets and floorlets could be added to the existing hedge to improve the matching position against the Regulatory Curve. [2]

[Total 16]

6 (i) Define a basket option. [1]

Consider a basket option dependent on two assets: S_1 and S_2. It is assumed that S_1 and S_2 are correlated geometric Brownian motions: S_1 is a function of the $\mathbb{P}$-Brownian motion W_1, S_2 is a function of the $\mathbb{P}$- Brownian motion W_2 and $\mathbb{E}_{\mathbb{P}}[W_1(t)W_2(t)] = \rho t$, where $\mathbb{P}$ is a probability measure, $\mathbb{E}_{\mathbb{P}}[.]$ the expectation operator and ρ is the non-zero correlation parameter.

(ii) Write down an expression for the stochastic processes followed by S_1 and S_2, defining any additional symbols used. [1]

Let $V(S_1, S_2, t)$ be the value of the European basket option at time *t* with payoff function $G(S_1, S_2)$. The payoff for this option is defined to be a call option on the maximum of S_1 and S_2, with strike price *K*.

(iii) Write down an expression for the payoff for this option. [1]

It can be shown that the value of this option satisfies the following Black-Scholes partial differential equation:

$$\frac{\partial V}{\partial t} + 0.5\sigma_1^2 S_1^2 \frac{\partial^2 V}{\partial S_1^2} + 0.5\sigma_2^2 S_2^2 \frac{\partial^2 V}{\partial S_2^2} + \rho\sigma_1\sigma_2 S_1 S_2 \frac{\partial^2 V}{\partial S_1 \partial S_2} + rS_1 \frac{\partial V}{\partial S_1} + rS_2 \frac{\partial V}{\partial S_2} - rV = 0,$$

where *r* is the risk-free interest rate and *t* is the elapsed time.

This equation is to be solved numerically using an explicit finite difference method.

(iv) (a) Describe an appropriate continuous space for solving this numerically using a finite difference method.

(b) Describe how this space can be discretely approximated by a grid.

(c) Express the six derivatives in the Black-Scholes equation given above as finite difference approximations.

[8]

(v) Outline the problems that can arise from using the explicit finite difference method to value a basket option with more than two different assets. [2]

(vi) Suggest two alternative approaches that could be used. [1]

[Total 14]

7 ABC Insurance holds a large portfolio of retirement policies that were issued around 20 years ago. The policies are invested in a mixture of low risk cash funds prior to retirement and each policy is completely converted to a continuously payable immediate annuity at retirement.

Each policy includes a minimum annuity guarantee. On conversion at retirement, this guarantee ensures that the policyholder receives an income payable for life based on the greater of (a) their fund divided by the prevailing market annuity rate and (b) 10% of their fund at retirement each year.

None of the policyholders has yet reached retirement. When the policies were first issued, the annuity guarantee was out of the money to policyholders. However, due to recent market movements, the annuity guarantee is now deeply in the money.

(i) Describe the interest rate risk and longevity risk faced by ABC Insurance in relation to the value of the annuity guarantee. [3]

ABC Insurance wishes to use a portfolio of receiver swaptions as a broad hedge for the interest rate risk on the annuity guarantee.

(ii) (a) State whether it should go long or short on receiver swaptions.

(b) Outline how the key features of the swaptions should be set.

[3]

ABC Insurance is now considering using a single European receiver swaption.

(iii) Write down the Black formula for valuing a receiver swaption, defining all terms used. [2]

The chosen swaption is for £100m notional, exercisable after five years into a ten year swap, receiving 10% p.a. payable continuously. Forward swap rates are 3% p.a. continuously compounded at all terms, with volatility of 20% p.a.

(iv) Calculate the value of this receiver swaption.

[Hint: You may use the fact that an annuity of term n payable continuously and discounted at a continuously compounded rate of r is given by

$a_{\overline{n}|} = \dfrac{1-e^{-rn}}{r}$.] [3]

ABC Insurance is also considering an alternative hedging strategy which would use:

- A £100m notional long <u>at-the-money</u> forward starting receiver interest rate swap, starting in five years' time for a further ten years.

- A £100m notional long <u>out-of-the-money</u> payer swaption, exercisable after five years into a ten year swap with a 10% p.a. strike rate.

- A series of gilt STRIPS paying £7m p.a. from years 6 to 15 inclusive.

This alternative hedge would be structured to replicate the cashflows of the receiver swaption valued in part (iv).

(v) Show that the cashflows of the receiver swaption in part (iv) can be replicated using the alternative hedge. [4]

(vi) Discuss the main factors ABC Insurance should consider when deciding whether to use the alternative hedge, compared to the receiver swaption in part (iv). [4]

ABC Insurance is also considering hedging the longevity risk of the annuity guarantee.

(vii) Assess how suitable the following longevity derivatives would be for hedging this risk:

(a) longevity swaps
(b) principal-at-risk longevity bonds
(c) survivor caps

[4]
[Total 23]

END OF PAPER