INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2012 examinations

Subject ST6 – Finance and Investment Specialist Technical B

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

D C Bowie
Chairman of the Board of Examiners

December 2012

General comments on Subject ST6

This subject deals with mathematical techniques for the valuation and risk management of derivatives, together with some aspects of their practical application.

Different numerical answers may be obtained to those shown in these solutions, depending on whether figures were obtained from tables or from calculators. Allowance was made for these minor differences. However, candidates may have been penalised for using excessive rounding or showing insufficient working.

General comments on the September 2012 paper

Derivative theory is very exacting subject which needs to be tackled in a precise and clear manner, but is also very interesting and rewarding. Questions can appear daunting, but with good preparation most will be seen to be straightforward examples of familiar techniques. Those who take ST6 for the algebraic and numerical content also need to be able to produce well reasoned brief arguments where the question demands. For the discursive questions, candidates should always try to think clearly before writing, and provide several distinct relevant points in short paragraphs, not one or two points made at length. This approach not only helps attract more marks, but also makes it quicker to write.

Some questions contained fairly familiar bookwork, such as Questions 1 and 6, whilst other required application of basic principles learned during the course to unfamiliar situations. In the latter category, Questions 2, 5 and 8 were typical, but were not generally tackled well by candidates. Question 5 in particular was disappointing in that the set-up of the question clearly hinted at the need to price an equity option using Black-Scholes, yet few candidates really pressed on and did this despite being given the required parameters. Question 8 addressed credit default swaps, now well established in the syllabus, but covered their use as well as pricing. This required some thought about how these instruments operate and what risks they hedge. Such practical insight is a key component of any study of derivatives.

In this session, although applicant numbers were on the low side historically, the overall standard improved from previous sessions, with the percentage of passes reaching nearly 40%.

The solutions below have been partly written to aid future candidates. Hence, as well as outlining a correct answer, they also often add an explanation relating to the course material from a practical perspective. As such, a study of these solutions is always beneficial to candidates preparing for ST6.

QUESTION 1

(i) (a)

The probability measure **P** is a set of probabilities, covering all possible future paths, that governs the evolution of the process Z_i.

P is binomial if at each time step i and value Z_i, there are only two possible values that Z_{i+1} can take at the next time step.

(b)

A filtration is the history of values of Z_i up until a particular time (i). The process Z_i, $0 \le i \le T$, generates a filtration F_i, $0 \le i \le T$, where F_i is the collection of all the events that depend only on $Z_0, Z_1, \ldots, Z_i$.

(c)

A process φ_i, $0 \le i \le T$ is *previsible* if φ_i depends only on the filtration $\mathbf{F}_{i-1}$, i.e. up to the previous time step.

(d)

The conditional expectation $\mathrm{E}_{\mathbf{P}}(Z_j | F_i)$, $j \ge i$, is the expected value of the variable Z at time j given the history up to time i under measure **P**.

(e)

A process Z_i is a **P**-martingale if Z_i is bounded for all i and

$$\mathrm{E}_{\mathbf{P}}(Z_j | F_i) = Z_i \text{ for all } j \ge i$$

where F_i is the filtration to time i.

(ii) (a)

Let M be a process that is a martingale under binomial measure **P**.

The Binomial Representation Theorem states that, for every other process N that is a martingale under measure **P**, there exists a previsible process φ_i such that:

$$N_i = N_0 + \sum_{k=1}^{i} \phi_k (M_k - M_{k-1})$$

or, equivalently, $\Delta N_i = \phi_i \Delta M_i$.

(b) To see how this could be used to price contingent claims on a stock, consider a stock with discounted asset price process *M*.

Provided we can find a measure **P** that makes *M* a martingale, then for any contingent claim *X* on *M*, the process N_i, which represents the present value of the claim, is a martingale ...

... and further we can find a previsible process φ from which to construct a replicating portfolio from *M* ...

... and so we can price *N* and hence the claim.

This was a standard bookwork question asking for definitions of stochastic concepts, familiar enough to those who have studied the syllabus and previous ST6 exam papers. It was well answered. Part (i) asked for some basic definitions, and part (ii) asked the candidate to present a familiar result relating to martingales.

QUESTION 2

(i)

<u>Hedge A – Currency futures</u>

Traded currency futures are (by definition) actively traded at a derivative clearing house.

The seller instructs a market maker at the clearing house to sell the currency future (i.e. sell US dollars, buy sterling).

The seller is required to maintain a margin account with the clearing house ...
… with both initial (up front) margin and variation (valuation adjusted) margin posted.

Every day when the dollar futures price is recalculated, any increase/decrease in the price is debited/credited to the seller's margin account.

The future will be cash settled at maturity if it is not closed out beforehand.

The hedge may need to be rolled over between current contracts as time passes.

<u>Hedge B – Shorting US Equity</u>

The shorting party ("shorter") borrows the security from the portfolio of another client ...

… and sells the share. [*Either or both of these actions can be done via a broker.*]

The shorter will then in this example need to convert the sale proceeds (which will be in US$) into sterling.

If using a broker, the shorter is required to maintain a margin account with the broker to protect the broker against the shorter defaulting. If the shorter fails to do this, the shorted position is closed out.

The shorter is responsible for paying back to the lender of the stock all dividends/interest associated with the security (via the broker if appropriate).

If at any time the shorter is no longer able to borrow the shares, the short must be closed.

When the shorting is closed out, the shorter swaps back sterling to US$, buys back the security and returns it to the portfolio from which it was borrowed.

Any capital gain/loss on the security is a credit/debit to the shorter.

(ii) (a)

The main objection to the second hedge is that there is open equity risk if the stock price rises.

Also … selling futures through a clearing house avoids counterparty risk.

But there is roll-over risk with futures (rolling to the next quarterly series) …
… as well as futures basis risk when the trade is put on and taken off.

For stock borrowing, there is the need to roll-over or possibly having to close out a short because the stock cannot be borrowed …
… unless borrowing has been pre-agreed for a one-year term (which is quite long)

There is a cost attached to borrowing the stock which is not linked to any currency risk.

Either approach could potentially lead to cashflow problems if prices change and margin is called.

(b)

Suggestions for improved hedges are:

- Instead of using a currency future, sell the currency forward for a year …
  … it is a more liquid market and precisely matches the cash flows (although incurs counterparty risk unless using an OTC clearing house).

- Instead of shorting a stock, prefer a sale and repurchase agreement on a one-year Treasury bond or bill …
  … because this removes price risk – no equity risk, no interest rate risk (although still rollover risk unless the repo can be for a 12 month term).

- Instead of the future, perhaps use a suitable option strategy such as a synthetic future.

This question asked for a practical assessment of two currency hedges: to describe how the hedges would be set up in part (i), then assess the effectiveness of the hedges in part (ii).

In part (i), the examiners were looking for a simple description of the steps taken to set up each hedge, which should have been straightforward enough using bookwork from an early part of the course. However, several candidates strayed into answering a different question, *such as why the firm would take the position described, or how to cope with any dynamic aspects of the hedge.*

In part (ii), mention should be made of the unsuitable nature of a US equity stock as a hedge for a certain outcome, due to its volatility. Replacing this with an equity index does not really improve things – a much less volatile instrument should be suggested, such as a bond.

QUESTION 3

[*Note: answer to parts (i) and (ii) can also be derived using a Taylor expansion.*]

(i)

Ito's Lemma for a function $X(S, t)$ based on the geometric process given is:

$$dX = \left(\frac{\partial X}{\partial t} + \mu S \frac{\partial X}{\partial S} + \tfrac{1}{2}\sigma^2 S^2 \frac{\partial^2 X}{\partial S^2} \right) dt + \sigma S \frac{\partial X}{\partial S} dW_t$$

Putting $X = S^n$ into Ito

$$\Rightarrow \frac{\partial X}{\partial t} = 0, \quad \frac{\partial X}{\partial S} = nS^{n-1}, \quad \frac{\partial^2 X}{\partial S^2} = n(n-1)S^{n-2}.$$

$$\Rightarrow dX = \left(\mu\, S.nS^{n-1} + \tfrac{1}{2}\sigma^2 S^2.n(n-1)S^{n-2} \right) dt + \sigma\, S.nS^{n-1}.dW_t$$

$$\Rightarrow dX = \left(n\mu + \tfrac{1}{2}n(n-1)\sigma^2 \right) Xdt + n\sigma X dW_t$$

which is geometric Brownian like S.

(ii)

If S is the value of A in terms of B, then $X = 1/S$ is the value of B in terms of A.

Putting $X = 1/S$ into Ito

$$\Rightarrow \frac{\partial X}{\partial t} = 0, \quad \frac{\partial X}{\partial S} = -\frac{1}{S^2}, \quad \frac{\partial^2 X}{\partial S^2} = \frac{2}{S^3}.$$

$$\Rightarrow dX = \left(-\mu S \frac{1}{S^2} + \tfrac{1}{2}\sigma^2 S^2 \frac{2}{S^3} \right) dt - \sigma S \frac{1}{S^2} dW_t$$

$$\Rightarrow dX = \left(-\mu + \sigma^2 \right) Xdt - \sigma X dW_t$$

Alternatively, simply use part (i) with n = -1.

Putting $d\tilde{W}_t = -dW_t$ still is a Wiener process, so:

$$dX = \left(-\mu + \sigma^2\right) Xdt + \sigma X d\tilde{W}_t$$

so X is a geometric Brownian motion like S, but with growth $r_A - r_B + \sigma^2$.

(iii)

The licence fee has an embedded six month currency option.

From the insurance company viewpoint, this is a free option which could even be sold (hedged) for a cash premium.

Assuming that the current rate of € to £ is around 1.20, it could be quite valuable.

If sterling appreciates against the Euro in six months, the company will choose to pay in Euros; if sterling falls, it will pay in sterling ...
… although as a UK insurance company in general it will want to pay in sterling.

The software vendor considers it will not show a loss whatever happens, because the internal budgeting level is the rate at which the option is struck.

However, in reality there will be a loss if e.g. sterling falls because converting the sterling to Euros will realise a lower value than assumed.

The outcome is slightly less clearly demonstrated if the insurance company pays in Euros when the rate is below 1.20, i.e. the option expires out of the money, but a value was initially granted nonetheless.

Parts (i) and (ii) of this question invited the candidates to use Ito's Lemma on two different functions of the underlying stochastic process. Part (ii) could either be solved separately or as a special case of part (i): both methods received the full allocation of marks.

Part (iii) was a real life example of a hidden embedded option (in this case, a Euro currency call) that has been "sold" for no cost. A few candidates appeared to get themselves confused into thinking there was a more exotic option involved than there was. The situation was simply that the vendor did not recognize the free option it was granting because it chose to value everything on a non-stochastic exchange rate. Of course, the insurance company would not benefit from the option if the Euro were to weaken against sterling (payment was due in Euros and the company would ultimately choose to pay in Euros), but a zero outcome at expiry does not mean the option is worthless at outset.

QUESTION 4

(i)

Price of call $C_t = S_t N(d_1) - Ke^{-r(T-t)} N(d_2)$

Hence theta is

$$\theta_C = \frac{\partial C_t}{\partial t} = S_t \varphi(d_1)\frac{\partial d_1}{\partial t} - Ke^{-r(T-t)}\varphi(d_2)\frac{\partial d_2}{\partial t} - rKe^{-r(T-t)}N(d_2) \qquad (*)$$

Now

$$d_{1 \text{ or } 2} = \frac{\ln(S_t / K) + (r \pm \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

hence $\dfrac{\partial d_{1 \text{ or} 2}}{\partial t} = \dfrac{\ln(S_t / K)}{2\sigma(T-t)^{3/2}} - \dfrac{(r \pm \frac{1}{2}\sigma^2)}{2\sigma(T-t)^{1/2}}$ (A)

and we know (hint) that $Ke^{-r(T-t)}\varphi(d_2) = S_t\varphi(d_1)$ for these d_1 and d_2. (B)

Substituting expressions (A) and (B) into (*) gives:

$$\begin{aligned}
\theta_C &= S_t\varphi(d_1)\frac{\partial d_1}{\partial t} - Ke^{-r(T-t)}\varphi(d_2)\frac{\partial d_2}{\partial t} - rKe^{-r(T-t)}N(d_2) \\
&= S_t\varphi(d_1)\left[\frac{\ln(S_t / K)}{2\sigma(T-t)^{3/2}} - \frac{(r + \frac{1}{2}\sigma^2)}{2\sigma(T-t)^{1/2}}\right] - S_t\varphi(d_1)\left[\frac{\ln(S_t / K)}{2\sigma(T-t)^{3/2}} - \frac{(r - \frac{1}{2}\sigma^2)}{2\sigma(T-t)^{1/2}}\right] - rKe^{-r(T-t)}N(d_2) \\
&= -S_t\varphi(d_1)\frac{\sigma}{2\sqrt{T-t}} - rKe^{-r(T-t)}N(d_2)
\end{aligned}$$

as required.

(ii)

Put-Call parity states that $P_t = C_t + Ke^{-r(T-t)} - S_t$

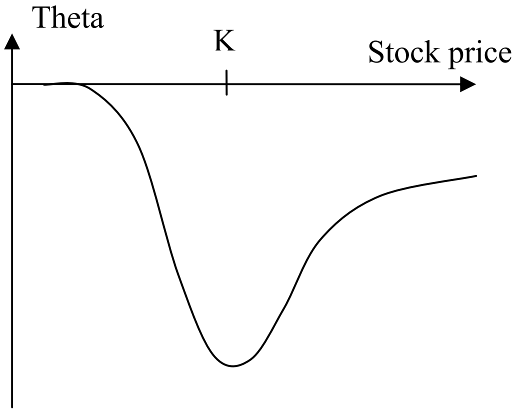
Differentiating with respect to t:

$$\frac{\partial P_t}{\partial t} = \frac{\partial C_t}{\partial t} + rKe^{-r(T-t)}$$

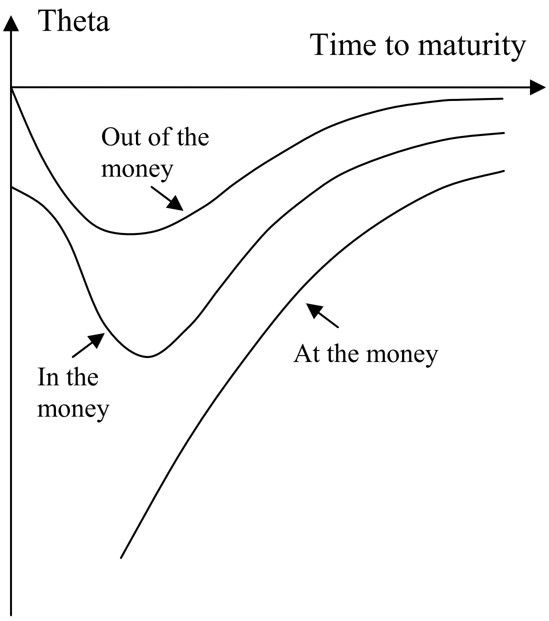
So theta of Put = theta of Call + $rKe^{-r(T-t)}$, i.e.

$$\theta_P = -S_t \varphi(d_1) \frac{\sigma}{2\sqrt{T-t}} + rKe^{-r(T-t)}(1 - N(d_2))$$
$$= -S_t \varphi(d_1) \frac{\sigma}{2\sqrt{T-t}} + rKe^{-r(T-t)} N(-d_2)$$

(iii) (a)



(b)



(iv)

Theta of a portfolio gives the trader information about how much time decay will be experienced …
… i.e. theta will be positive if overall short of options, or negative if overall long.

Using classic delta-hedging, the time decay is offset against the profits (or losses) made on the delta trades.

Theta is not usually hedged directly: traders usually prefer to hedge against gamma and vega instead.

However, theta can be a useful descriptive statistic for a portfolio as a rough proxy for gamma and vega.

Parts (i) and (ii) asked for an algebraic derivation of the formulae for theta of a call and put option respectively, the latter using put-call parity. These were generally handled well.

Part (iii) was a now familiar request for graphical sketches of option parameters, in this case the evolution of theta against stock price and time to maturity for a call. These appear in a similar form in Hull's chapter on "the Greek letters".

Part (iv) asked about the use of theta, which is not often hedged directly but thought of as a proxy metric for overall gamma and vega. It was generally quite well answered.

QUESTION 5

(i) (a)

Early exercise is not optimal since:
the income on the convertible bond exceeds that on the equity …
… and the remaining option premium would be lost.

(b)

Value of convertible bond = value of bullet bond + value of conversion option

Conversion option valued by Black-Scholes has:

S = 100, K = 100, σ = 20%, r = 5%, T = 5 years

Option value = $SN(d_1) - Ke^{-rt}N(d_2)$

where $d_1 = \dfrac{\ln(S_0 / K) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$.

Then d_1 = 0.78262 and d_2 = 0.33541,

So conversion option = 100 N(0.78262) – 100. 0.77880. N(0.33541) = 29.14

The value of a 5-year non convertible bond is:

$$B = 4\left(\frac{1-v^5}{i}\right) + 100v^n \quad \text{where } i = 7\% \text{ and } v = \frac{1}{1.07}$$

Thus B = 87.70 in pence (or £ per £100 nominal).

Hence estimated convertible bond value = 29.14 +87.70 = 116.84.

(c)

Black-Scholes assumes that interest rates are deterministic
… but they are not since the underlying bond is itself risky
… hence there will be a correlation between the equity and bond price.

However, the lack of early exercise means that as the bond approaches maturity it will become less volatile, so approaching the Black-Scholes assumption.

Also, on exercise the bond itself is replaced by an equity …
… so we would need to model this if it occurred before the bond maturity
… but we know that will not happen as there will not be early exercise.

So the estimate is probably fairly accurate.

The other usual assumptions around Black-Scholes (such as constant volatility) are less important in this context but could be mentioned here.

(ii)

Delta of the option from (i)(b) is $N(d_1) = 0.783$.

Let the modified duration of the 5-year government bond $= w$.

The government bond will be priced at 100 as its coupon is at par.

Hence:

PV01 of a 5 year government bond is 100 w / 100 = w

PV01 of a 5 year non-convertible bond is 87.70 w / 100 = 0.877w

So to hedge one unit of the 5 year non-convertible bond requires a short position in 0.877w / w = 0.877 units of 5 year government bond.

Similarly to hedge the conversion option on one unit of bond requires a short position in 0.783 units of equity and a long position in 0.783 units of 5-year money market or government bonds.

Thus, £10m of the convertible bond requires

- Short position in £7.83m equity
- Short position in £10m . (0.877 – 0.783) = £0.94m units of 5-year government bonds

This question proposed a bond that was convertible into an equity at par. In part (i), the candidate was led through the concept of approximating the value of the option by a simple Black-Scholes equity call. This approach seemed to elude many candidates, which was a shame. It might have been an unfamiliar application in relation to the Core Reading, but the basic principles of Black-Scholes were readily applicable. As with Question 1, the key insight is that an equity is generally more volatile than a bond close to its maturity, so ignoring the latter's stochastic nature is not too far-fetched an assumption.

Part (ii) was short but a little harder. Even if confused by the concept of a convertible, candidates should have been able to recognise that in the replicating portfolio the part relating to the target instrument, in this case the equity, was the delta of the option. The complexity of the question lay in calculating the part relating to the numeraire, in this case the underlying bond. The examiners were careful to give marks for understanding the approach as well as correctly calculating the answer. The balancing cash amount did not need to be calculated.

QUESTION 6

(i) (a)

A stochastic process can be defined as a mathematical model of a variable whose value over time is influenced by random factors.

A mean-reverting process is one where the drift makes the process tend back to a predefined level (which is not necessarily constant over time, but is not random).

(b)

Mean-reverting processes cannot be used to model prices of tradable instruments as these do not allow free stochastic movement.

Bonds are tradable instruments, hence if their prices were mean-reverting this would mean the market were inefficient.

Interest rates are not tradable instruments, so there is no market inefficiency in using any form of mean-reverting or path-dependent process.

In a normal economic cycle, when interest rates are high, borrowing is curtailed and economic growth is slowed, leading to lower rates …
… and when rates are low, borrowing increases and economic growth picks up, so rates tend to rise to cap off demand inflation.

(ii)

With one factor models, all discount rates (and forward rates) along the curve are perfectly correlated – they move in the same direction every time, but not necessarily by the same amount.

They are not adequate for pricing options where payoffs are affected by:

- changing yield curve shapes, or
- imperfect correlation between different rates.

Bermudan swaptions have multiple exercise periods which need to be valued across the yield curve.

(iii)

Only THREE of the following four solutions were required

Cox-Ingersoll-Ross (CIR)

- Equilibrium model, i.e. follows economic parameters but not calibrated to fit the yield curve exactly.

- It is analytically tractable for obtaining prices and forward rates.

- The risk-neutral process for short rate r is:

  $$dr = a(b-r)dt + \sigma\sqrt{r}dW_t \text{ with } a, b, \sigma \text{ constant}$$

- Mean reverting, with pull-back towards b.

- It is possible to model some simple yield curve shapes, e.g. upward sloping, downward sloping or slightly humped.

- The impact of the damped volatility term as r decreases means that rates are always non-negative.

- CIR is Markovian …

- … and uses the non-central chi-squared distribution.

Hull-White

- No arbitrage model, calibrated to fit the initial yield curve exactly as an input.

- It is analytically tractable for obtaining prices and forward rates.

- The risk-neutral process for short rate r is:

  $dr = a[\theta(t) - r]dt + \sigma dW_t$ with a, σ constant

- Initial yield curve structure can be modelled explicitly using $\theta(t)$.

- Mean reverting, with pull-back towards initial instantaneous forward rate curve $\theta(t)$.

- Unlike CIR, Hull-White can lead to negative rates …

- … but like CIR it is Markovian.

<u>Heath Jarrow Morton (HJM)</u>

- No arbitrage model, calibrated to fit the initial yield curve and volatility structure exactly as an input.

- The risk-neutral process for zero coupon bond $P(t, T)$ is:

  $dP(t,T) = r(t)P(t,T)dt + \sigma(t,T)P(t,T)dW_t$

  [*Alternatively, the SDE can be expressed in terms of the instantaneous forward rate, as in Hull.*]

- The volatility term may also depend on past values.

- It is calibrated from instantaneous forward rates, which are not observable.

- HJM is generally non-Markovian, i.e. generates non-recombining ("bushy") trees …

  … so has to be solved using Monte Carlo methods.

<u>LIBOR Market Model (LMM)</u>

- Developed to overcome calibration problems with HJM …
  … by implying a formula for caplets that has the same structure as the Black formula, thereby enabling calibration to Black pricing formula for a caplet.

- Hence LMM implies lognormal processes for the forward LIBOR rates under the equivalent martingale measure.

- The volatility function is time-dependent but deterministic, which means forward LIBOR rates can have different volatilities.

- The risk-neutral process for zero coupon bond $P(t, T)$ is

  $dP(t,T_i) = \mu_n^P(t)P_n(t,T_i)dt + \sigma_n^P(t,T_i)P(t,T_i)dW_t$ for $i = 1, 2, \ldots, N$

[*Alternatively, the SDE can be expressed in terms of the forward rate under the risk-neutral measure, as in Hull.*]

- Calibrates from Black model of caps & floors (or swaptions) ...

  … but the implantation is complex, requiring Monte Carlo simulation.

(iv)

We know that $\lambda = \dfrac{\mu - r}{\sigma}$, where μ is the drift and σ is the volatility in the real world.

Moving to the risk-neutral world sets the instantaneous return to $r = \mu - \lambda\sigma$, hence

by Girsanov and substituting, the risk neutral SDE is: $dr = [a(b-r) - \lambda c\sqrt{r}]dt + c\sqrt{r}d\bar{z}$.

This was the first of three longer questions that concluded the paper. On a proportionate basis, half the exam time should have been dedicated to these three questions.

Parts (i), (ii) and (iii) of this question focused on definitions and features of yield curve models, an area of the syllabus which candidates seemed to know well. What was disappointing was that answers were often too brief to obtain anywhere near the 13 marks on offer, so it is likely that quite a few marks were dropped unnecessarily. For example, for part (i), six to eight precise features would have secured all of the three marks available, but most candidates only wrote two or three short sentences. Similar considerations applied in parts (ii) and (iii). Also, in part (ii) some candidates were content just to supply the definition of a Bermudan swaption, whereas the question invited them to explore the modelling challenges.

Part (iv), asking for an algebraic manipulation involving the market price of risk, was less familiar, but many candidates found the result readily enough.

QUESTION 7

(i)

Put option formula p = $Ke^{-rt}N(-d_2) - Se^{-qt}N(-d_1)$

where $d_1 = \dfrac{\ln(S/K) + (r - q + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$

d_1 = [ln(1000/800) + (0.01 – 0.025+ ½(0.2)^2)*1] / 0.2 = 1.1407

$N(-d_1)$ = 0.1270

d_2 = 1.1407 – 0.2 = 0.9407

$N(-d_2) = 0.1734$

Put value = 800. $e^{-0.01}$. 0.1734 – 1000. $e^{-0.025}$. 0.1270 = 13.5 as required

(ii) (a)

Since vega = $\dfrac{dV}{d\sigma}$, where V is the Put value, then to first order $\Delta\sigma = \dfrac{\Delta V}{vega}$

So for price to be 14.8, σ = 20% + (14.8 – 13.5) / 203 = 20.64%

(b)

We are using the market implied volatility but should also look at other different maturity options …
… since the 1-year value may be out of line or less liquid.

Also look at history of implied volatilities …
… which will give some idea of how volatility is trending and will help in deciding whether volatility is moving away from or back to its historical average for the period of the option.
We could look at the volatility at different strikes ...

… and/or consider call options via put-call parity e.g. for in-the-money put strikes.

(iii) (a)

Implied volatility decreases as strike increases, so is skewed.

This is typical of <u>normal</u> conditions for stocks where:

- moves downward are likely to be more volatile due to panic selling …
- … and there is greater demand for downside protection
- moves upward are likely to be less volatile as investors are more orderly

(b)

Implied volatility is lowest at the money and higher for in–the-money and out-of-the money options, i.e. has a "smile".

This is typical of <u>normal</u> conditions for currencies where:

- moves up or down have equal impact to investors, as it is a currency pair

- but premiums are lower away from the current spot level, so the additional jump risk / leverage makes them proportionally higher in volatility

(c)

Implied volatility increases as term to expiry decreases.

This is an <u>unusual</u> condition for stocks where:

- stock prices have most likely become volatile in a period of uncertainty
- the market is expecting a sudden move soon, but longer out volatility is expected to return to move normal levels

(iv) (a)

<u>Calculation</u>

Daily data should be used.

Let S_i be the closing price of the index on day i
and let $u_i = \ln(S_i / S_{i-1})$

[*<u>Note</u>: $u_i = (S_i - S_{i-1}) / S_{i-1}$ is an acceptable approximation. The difference is small when compared to statistical sampling error) and either u_i can have an extra $+q/N$ term where N is number of business days in a year.*]

Then daily variance = $\sigma_{daily}^2 = [\ \Sigma(u_i - \bar{u})^2\ ] / (n - 1)$
where n is number of historical data points and $\bar{u}$ is average of the u_i
and $\sigma_{annual} = \sigma_{daily} \sqrt{N}$, where N is the number of <u>business</u> days in a given year.

<u>How much data to use</u>

It is important to match the data periods to the option periods.

Using all available data reduces sampling error in your estimate provided volatility is stationary.

In practice it is not stationary, particularly over short periods.

Use enough data to reduce sampling error to a reasonable level whilst keeping the data relevant to the intended <u>future</u> forecast.

Historical supply/demand situation may have created a variability profile that is not relevant to the immediate future.

<u>Jumps and crashes</u>

Black-Scholes assumes no jumps or crashes, i.e. all movements are samples from the same lognormal distribution.

It is good to be aware of significant distortions caused by one-off events.

Future jumps would theoretically need a jump diffusion model, but these are hard to build and difficult to parameterise.

The market allows for some future jump effects by adding a smile to the volatility of out-of-the-money options, particularly on puts.

(b)

BSM will initially suffer arbitrage losses …
… because where the BSM is underpricing, other banks will buy from it and sell the options in the market; where it is overpricing, other banks will sell options to BSM.

BSM has large capital resources. Hence, notwithstanding the arbitrage potential, over time BSM's prices could become the recognised market price.

The normal pattern of market implied volatilities is partly a result of banks' capital requirements. BSM's pricing ignores the capital implications of writing options.

If the BSM price becomes the recognised market price, they may find themselves writing the options with the most onerous capital requirements and giving their shareholders a lower return on capital than those of the other banks.

BSM's pricing methodology ignores the possibility of market crashes. They could run into problems if they are delta hedging options using futures.

BSM is slow to adapt to changing market conditions.

This question covered implied volatility calculations for a put option. Part (i) was a Black-Scholes calculation, which is easy but tedious to do. It gives the examiners an indication of the candidate's numeric precision under time pressure: most candidates perform well.

Then followed part (ii), asking for a calculation of implied volatility using Newton-Raphson interpolation with vega as the first derivative. This calculation could be done by trial and error (and marks were still awarded), but that route is a lot more time-consuming.

Part (iii) set up a series of scenarios involving implied volatility variations across strike and term, looking for descriptions of skews, smiles and maturity effects. This was not well answered, which is disappointing because the scenarios were not complicated. Several candidates chose to consider the changes against the stock price, not strike or term. This is where a pause for some "thinking time" could be valuable.

Part (iv) considered the advantages and disadvantages of using historic volatility as a proxy for implied volatility. The main disadvantages are its lack of forward-looking view and inconsistency with market prices. Most candidates wrote too little here. For seven marks, a lot more than a brief paragraph was required.

QUESTION 8

(i)

(a) Reasons for using CDS

- The company is expecting the sovereign credit spread of Country A to widen.

and/or

- It is expecting its sovereign credit spread of Country B to narrow,
  e.g. perhaps the company thinks the rise to 5% default rate in year 3 is unlikely to happen and the rate will stay nearer the initial 1%.

- The company may be using derivatives to help it switch exposure from Country A's bond market to Country B's bond market synthetically to avoid dealing costs.

- It may have exposure to other bonds in Country A and Country B, just not any sovereign bonds.

- Or it may be wanting to have a neutral cash position, i.e. use the premiums from Country B to subsidise the payment of premiums for Country A.

- Whatever the practicalities, the company must think that Country A will deteriorate in default expectations vs Country B.

(b) Main risks in the position

- Market risk – this is an outright spread trade, so the company is exposed if the CDS for Country B rises in value, or for Country A falls in value.

- Counterparty credit risk – whether the swap counterparty can be relied upon, especially if the market moves against them ...
  … although posting of margin will mitigate this.

- Liquidity risk – if the market value of trade moves against the company, cash margin will need to be posted.

(ii)

Let the spread on the Country A CDS be S_A.

Risk-free rates are zero, so the discounting factors are all 1.

Periodic premium payment (*times* S_A)

Note: The default rate is always applied to the surviving population.

Years	Default rate	Defaults per 100	Periodic Premium Payment Survival per 100	Discount factor	Present value
1	2.0%	2.000	98.000	1.0000	0.9800
2	2.5%	2.450	95.550	1.0000	0.9555
3	3.0%	2.867	92.683	1.0000	0.9268
4	3.5%	3.244	89.439	1.0000	0.8944
5	4.0%	3.578	85.861	1.0000	0.8586
				Total	4.6153

So periodic premium payment = 4.6153 S_A.

[*Although the above is the simplest approach, technically it is more correct to build the table with the premium being paid by the population at the start of the year, rather than end year, since on default the premium is still due. Both approaches were considered acceptable.*]

Principal payment on default, assumed to take place at year end

Years	Defaults per 100	Recovery	Principal Payment on default Payoff	Discount factor	Present value
1	2.000	0.2	1.600	1.0000	0.0160
2	2.450	0.2	1.960	1.0000	0.0196
3	2.867	0.2	2.294	1.0000	0.0229
4	3.244	0.2	2.595	1.0000	0.0260
5	3.578	0.2	2.862	1.0000	0.0286
				Total	0.1131

So PV of principal payment on default = 0.1131.

There is no need for an accrual adjustment, as each payment is made at year end.

Hence 4.6153 S_A = 0.1131, so S_A = 0.1131 / 4.6153 = 0.02451 or 245 bps.

(iii) (a)

Average default rate for Country A $\bar{u}_A = 3\%$

So approximate spread for Country A CDS = $\bar{u}_A \cdot (1 - 0.2) = 0.024$, or 240 bps.

This is very similar to the result in part (ii), as the question suggests.

(b)

Average default rate $\bar{u}_B = (1\% + 1\% + 5\% + 5\% + 5\%) / 5 = 3.40\%$

Hence for Country B CDS, $S_B \sim \bar{u}_B\,.\,(1 - 0.2) = 0.0272$, or 272 bps.

[*Note: the actual value using the full information is 276 bps.*]

(iv) (a)

Liquidity risk could impact if writers of CDS lose money and have to post substantial amounts of collateral – this could create a cashflow or funding problem.

Also, there may be problems closing out the sold CDS position if trading losses cause market participants to reduce their capacity to offer CDS prices (even marking the position correctly to market might be difficult in these circumstances).

(b)

Why would the market quote a different spread from the theoretical price?

- Supply/demand – maybe some speculation, buying of CDS generally, particularly during times of stress
- The rating agency's default analysis may be out-of-date, or perhaps not reflecting the market's current expectations
- Risk aversion – a systematic bias against selling CDS due to the possibility of a very large loss on default of a country.
- Capacity issues – the banks offering the CDS may not have enough capital or risk appetite to arbitrage CDS spreads with the underlying cash bond market
- The CDS was valued in the real world, not as an arbitrage free price – this is unlikely to make a huge difference, but there are other better models for obtaining market-consistent CDS prices [*e.g. Jarrow Turnbull*]

(c)

The company is not "speculating", but it does not own the underlying bonds so is affected by the ban.

The ban might suddenly require the company to sell its purchased CDS.

This might possibly cause a loss (as many others will be similarly affected) …
… but, more concerningly, the company will lose the ability to keep the protection as a macro hedge.

Even the rumour of a ban could cause CDS bid-offers to widen.

This question dealt with two sovereign credit default swaps (CDS). Part (i) asked candidates to discuss the sovereign default risk that the examples hedged and reasons for taking on a particular long-short strategy. Perhaps some candidates were running out of time at this stage, but generally there was not enough thought given to the precise features of the *situation described. The basic rationale for the hedging strategy was understood, but there were several other points to make.*

Part (ii) was a calculation of the price of a CDS under some slightly simplified assumptions (no accrual adjustment, for example). Part (iii) showed how the spread on the CDS relates to expected default value, which of course is the main reason why these instruments are used as a default hedge. Where the candidates had covered this area of the syllabus, these parts were answered accurately.

Part (iv) covered further applications of risk management theory. As in part (i), the basic concept was understood, in this case illiquidity, but there were other points to make. As is often mentioned in these reports, short responses that make a few clear well-chosen points are preferable to a paragraph long exposition on only one point.

END OF EXAMINERS' REPORT