INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2015

Subject ST6 – Finance and Investment Specialist Technical B

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

F Layton
Chairman of the Board of Examiners
December 2015

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Finance and Investment Technical B subject is to instil in successful candidates the ability (at a higher level of detail and ability than in CT8) to value financial derivatives, to assess and manage the risks associated with a portfolio of derivatives, including credit derivatives and to value credit derivatives using simple models for credit risk.

2. This subject deals with mathematical techniques for the valuation and risk management of derivatives, together with some aspects of their practical application.

3. Different numerical answers may be obtained to those shown in these solutions, depending on whether figures were obtained from tables or from calculators. Allowance was made for these minor differences. However, candidates may have been penalised for using excessive rounding or showing insufficient working.

4. Derivative theory is an interesting but very exacting subject which needs to be tackled in a precise and disciplined manner. Questions that appear unfamiliar can, with some clear thinking at the start of the exam, be found to be straightforward examples of well-known techniques. Thorough preparation of the course material is essential, however. This subject also requires quite a varied approach in writing solutions. Algebraic and numerical content needs to be supplemented with well-reasoned but brief arguments where the question demands. Candidates should always try to provide several distinct relevant points for the discursive questions, not lengthy paragraphs developing a single idea.

B. General comments on *student performance in this diet of the examination*

1. The overall performance of students was good and broadly consistent with the standard displayed in the April 2015 session. Although candidates found some questions relatively challenging, as is normally the case, well-prepared candidates scored well above the pass mark.
2. In terms of areas for improvement, candidates struggled more with those questions that required an element of application of Core Reading to situations that were not immediately familiar, so did not demonstrate the desired depth of knowledge and understanding. In some questions there were key details in the question which candidates missed, which had a significant influence on the direction of their answers. Furthermore, the two parts of questions which involved sketching of graphs (6(vii) and 7(iv)(b)) were either poorly attempted or not attempted at all by a significant number of candidates. These types of questions appear on most papers and emphasises the need for practising past examples of these questions. The final point is to emphasise the need to be familiar with the Core Reading in order to be able to apply it in the questions.

3. The comments that follow the questions concentrate on areas where candidates could have improved their marks, as an attempt to help future candidates to revise accordingly and to develop their ability to apply the Core Reading to related situations. This skill is likely to be valuable both in exams and in using this theory in their professional careers.

C. Comparative pass rates for the past 3 years for this diet of examination

Year	*%*
September 2015	36
April 2015	40
September 2014	28
April 2014	41
September 2013	30
April 2013	30

Reasons for any significant change in pass rates in current diet to those in the past:

It should be noted that the number of candidates sitting this exam is low and so a reasonably stable pass rate should not be expected.

Solutions

Q1 (i) A stochastic process M_t is a martingale with respect to a probability measure $\mathbb{P}$ if and only if:

$$\mathbb{E}_{\mathbb{P}}[|M_t|] < \infty, \text{ for all } t, \text{ and}$$

$$\mathbb{E}_{\mathbb{P}}[M_t \mid F_s] = M_s, \text{ for all } s \le t,$$

where F is the filtration.

(ii) In order to show that Y is a martingale, it will need to be shown that it satisfies the conditions identified in (i).

As the Radon-Nikodym derivative exists for $\mathbb{P}$ and $\mathbb{Q}$ then it follows that $\mathbb{P}$ and $\mathbb{Q}$ are equivalent.

That is, since $\mathbb{P}(X_t > 0, \forall t) = 1$ then it must also be the case that $\mathbb{Q}(X_t > 0, \forall t) = 1$. From the properties of X being a martingale which is positive: $0 < X_t < \infty$ for all t. Hence $0 < Y_t < \infty$ and $\mathbb{Q}(Y_t > 0, \forall t) = 1$, i.e. Y is positive.

Further, the first condition of being a martingale is also satisfied as X is positive and finite which means that $Y = 1/X$ is positive and remains finite. Hence the expectation of Y under $\mathbb{Q}$ is bounded for all t.

Using the Radon-Nikodym properties:

Let $\zeta_t = \mathbb{E}_{\mathbb{P}}\left({}^{d\mathbb{Q}}\!/\!{}_{d\mathbb{P}} \middle| F_t \right)$, so $\zeta_t = \mathbb{E}_{\mathbb{P}}\left(X_T \middle| F_t \right) = X_t$.

$$\mathbb{E}_{\mathbb{Q}}\left(Y_t \middle| F_s \right) = \zeta_s^{-1}\mathbb{E}_{\mathbb{P}}\left(\zeta_t Y_t \middle| F_s \right), \ s \le t \le T,$$

$$= \frac{\mathbb{E}_{\mathbb{P}}\left(X_t Y_t \middle| F_s \right)}{X_s},$$

$$= \frac{1}{X_s},$$

$$= Y_s.$$

Y is therefore a martingale, as required, as it meets all of the criteria.

(iii) Using Ito's lemma on Y:

$$dY_t = dX_t^{-1},$$

$$= -\sigma_t X_t^{-1} dW_t + X_t^{-1}\sigma_t^2 dt$$

$$= -\sigma_t Y_t \left(dW_t - \sigma_t dt \right).$$

Using the Cameron-Martin-Girsanov converse theorem,

and since W is $\mathbb{P}$-Brownian motion and $\mathbb{P}$ and $\mathbb{Q}$ are equivalent:

The process $d\tilde{Z}_t = dW_t - \sigma_t dt$ defines a $\mathbb{Q}$-Brownian motion.

From the basic properties of Brownian motion, $dZ_t = -d\tilde{Z}_t$, is also $\mathbb{Q}$-Brownian motion.

Part (i) was generally well answered.

Part (ii) was incorrectly or incompletely answered by most students with many unable to recall the Radon-Nikodym properties as given on page 81 of Baxter and Rennie. In particular, many just carried out the analysis at *T* and not for general *t*. Very few candidates considered the boundedness requirement for a martingale, despite most writing it in part (i).

Part (iii) was attempted by most candidates and it was generally recognised that Ito's lemma and the Cameron-Martin-Girsanov theorem should be used, although only a minority applied these correctly to obtain full marks.

In general this question relied on some of the key bookwork of chapter 3 of Baxter and Rennie. In only a few cases was this knowledge applied sufficiently effectively to score very well.

Q2 (i) From the definition of the first order Greeks, each Greek measures the sensitivity of the change in price of a portfolio of derivatives to a small change in a chosen variable.

Typical variables for a given derivative include:

the underlying asset price (delta);
the volatility of the underlying asset price (vega);
the passage of time (theta);
the interest rate (rho).

Out of these, the sensitivity to the underlying asset price (delta) is usually the most significant…

… and so the most likely to be managed.

In general, portfolios are not made theta-neutral as the passage of time is inevitable.

The Greeks characterise the market and other risks associated with the portfolio of derivatives.

The aim of risk management in this case is that the value of the portfolio can be immunised to the factors affecting it (i.e. one or more of the ones identified above).

Consider an unhedged portfolio of derivatives:

$$V = \sum_{i=1}^{n} w_i A_i \,,$$

where:

V is the value of the portfolio;
w_i is the number of units of derivative *i*, and
A_i is the value of derivative *i*.

Let

$$H = V + \sum_{j=1}^{m} y_j B_j \,,$$

where:

H is the value of the hedged portfolio (i.e. the unhedged portfolio plus hedging instruments);

y_j is the number of units of financial instrument *j* and

B_j is the value of financial instrument *j*.

The sensitivity of the hedged portfolio to an arbitrary factor *x* is then:

$$\frac{\partial H}{\partial x} = \sum_{i=1}^{n} w_i \frac{\partial A_i}{\partial x} + \sum_{j=1}^{m} y_i \frac{\partial B_j}{\partial x} \,.$$

The objective of *x*-hedging is to pick the y_j such that the value of the hedged portfolio stays constant when x changes.

That is, pick $\left\{ y_j \right\}$ so that $\dfrac{\partial H}{\partial x} = 0$

Then the value of the portfolio stays approximately constant when x changes by a small amount:

$$dH = \frac{\partial H}{\partial x} dx \approx 0.$$

The hedging portfolio $\left\{ y_j \right\}$ typically needs to change over time to ensure the portfolio remains hedged.

(ii) **Advantages**

It is theoretically correct to use the first order Greeks in this way (for example to expand the price of the portfolio of derivatives using a Taylor expansion), assuming the payoff is sufficiently smooth and all required trades can be made frictionlessly.

It enables the effect of each variable to be isolated and investigated.

Using certain models (e.g. Black-Scholes) can lead to closed form solutions for the first order Greeks.

This enables them to be calculated easily and …

… it makes hedging easier to implement.

Disadvantages

The partial derivatives of the portfolio depend on the underlying distribution of price movements, and on price volatilities of the underlyings.

A weakness arises in using statistical models to derive both sets of these values.

For example, in the Black-Scholes framework it has been shown that certain asset returns do not follow the lognormal model.

This model risk means that the hedging strategy is not correct, hence hedging errors occur.

Transaction costs can be substantial for larger portfolios in this form of hedging, thus reducing the effectiveness of the hedge if they are not taken into account.

Shorting assets may not be possible or require significant capital…

… or the required asset is not available in the required volume or at competitive prices.

Hedging using first order Greeks assumes that the portfolio can be changed over time. This may not be possible to do continuously. As a result the hedge will drift in effectiveness between the hedging portfolio being rebalanced.

The use of higher order Greeks can help to mitigate the previous point.

(iii) The ability to hedge perfectly using the first order Greeks is not realistic using the arguments of part (ii).

The use of higher order Greeks in hedging can result in hedges which are sufficiently effective over wider ranges of underlying variables.

This should lead to less rebalancing of the hedging portfolio…

… and potentially lower costs.

For example, delta and gamma hedging can result in the hedge being effective over a wider range of the underlyings' prices compared to just delta hedging.

Higher order Greeks can be important if the underlying pricing model allows for extra stochastic terms.

Some higher order cross Greeks are also important as they are also representative of risks to a portfolio.

For example, to deal with non-constant volatility it may be modelled stochastically. This would result in higher order derivatives with respect to volatility becoming important.

[Marks can also be awarded for using other suitable examples.]

Most candidates were able to score a good proportion of the available marks for this question. For each of the three parts, the core points were set out by many candidates but only a few generated the wider points required to get full marks, especially in part (ii).

In part (iii) most candidates mentioned gamma but failed to mention higher order Greeks more generally in line with what the question asked.

Q3 (i) (a) The holder of a credit default swap (CDS) has the right to sell the reference entity (bond) for face value (to the seller of the CDS) when a credit event occurs (i.e. the reference entity defaults).

The CDS therefore provides insurance for the holder against the risk of a default by a particular company.

The holder makes periodic payments to the seller until the end of the life of the CDS or until a credit event occurs.

These payments are typically made in arrears but sometimes are made in advance.

The settlement in the event of a default involves either physical delivery of the bonds or a cash payment.

(b) A basket default swap is a product with which the investor gains either long or short exposure to a relatively small basket of credits.

Baskets typically consist of up to a dozen credit names.

The holder of a third-to-default basket swap is protected against the third default of the basket, and the seller is exposed to it.

No protection is provided for the first or second defaults

The holder has the right to sell the reference entity (bond) for face value when the third credit event occurs.

At this point, the swap is terminated.
The holder makes periodic payments to the seller until the end of the life of the third–to-default basket swap or until a credit event occurs.

Third-to-default baskets can be traded in funded or unfunded form.

(ii) One could sell CDS protection….

… and short the corporate bond…

…and invest in a risk-free instrument with the proceeds.

If the bond defaults, then the loss on the CDS should be exactly offset by the profit on the short corporate bond.

Therefore this portfolio would always return greater than the risk-free rate, if the CDS-bond basis is positive.

(iii) The CDS may provide greater protection in that it could pay out on credit events that are technical defaults which would not fully impact a cash bondholder.

There may be other minor contractual differences in the CDS contract such as provisions allowing delivery of a range of bonds…

… or how accrued coupons are dealt with.

The CDS spread may include a premium or …

… a discount for counterparty risk….

… as if the protection buyer defaults, the CDS will terminate and the protection seller will no longer receive the premiums…

…and the protection seller may default if the credit event occurs.

The CDS spread may incorporate a premium to reflect the greater liquidity of the CDS compared to the bond, and this liquidity premium can only be harvested if the position is held to term.

It may be either impractical or costly to short the underlying bond.

The transaction costs may also remove the arbitrage.

(iv) *Impact on cost*

The basket swap will be much cheaper…

… as no protection is given to the first or second defaults, whereas the CDS would be expected to cover defaults on half the portfolio.
The CDS transaction would be expected to reduce the credit spread on the portfolio by around half, whereas the basket swap will reduce it by much less.

Impact on credit risk

Both transactions will partially reduce credit risk in the portfolio, as required.

The CDS will substantially reduce the credit risk by covering defaults on five of the reference entities…

… whereas the basket swap covers only the third default event.

The basket swap will provide some protection from credit risk across the whole portfolio…

… whereas the CDS will not cover any credit risk for five of the holdings.

Both provide protection against increased default correlation.

The basket swap is particularly designed to protect against highly correlated default events.

As both the CDS and basket swap would be unwound on a credit event, the manager would need to purchase new derivatives to ensure the protection is maintained…

… this will be more important for the basket swap, as immediately following a relevant credit event there would be no protection in place against the remainder of the whole portfolio.

(v) Given the large cost of the CDS and consequent reduction in exposure to credit yield, it is likely that the second transaction would be most suitable (given the requirement for "limited cost") and so the basket swap is recommended.

> Part (i) was standard Core Reading and the majority of students scored highly here.
>
> Part (ii) was generally well answered, although some candidates thought that the arbitrage opportunity would involve holding the corporate bond rather than shorting it. Many candidates also incorrectly stated that, as the corporate bond was trading at par, it was risk-free.
>
> Part (iii) was not so well answered generally, as most did not consider the basic points around contractual differences, which is an application of the Core Reading.
>
> Parts (iv) and (v) were generally well answered.

Q4 (i) Equilibrium models usually start with assumptions about economic variables and derive a process for a particular interest rate (e.g. the short rate *r*).

Equilibrium models then explore what the particular interest rate process (and the interest rates output by the model) implies about bond and option prices.

Equilibrium models do not therefore automatically fit the initial term structure of interest rates and therefore may imply arbitrage opportunities.

A no-arbitrage model is designed to be exactly consistent with the initial term structure of interest rates …

… and does this by using it as an input to the model.

In an equilibrium model, the drift of the short rate is not usually a function of time, whereas in a no-arbitrage model it generally is.

(ii)
$$V(t,T) = E_Q\left[\exp\left(-\int_t^T r(u)du\right)X(T)\middle|F(t)\right]$$

where:

$V(t, T)$ is the value of the derivative at time t with a payoff of $X(T)$ at time T;

$F(t)$ is the filtration at time t;

E_Q is the expectation under the risk-neutral measure;

$r(t)$ is the short rate at time t.

(iii) $E_Q[r_1 \mid r_0] = 0.05 + e^{-0.04}[0.07 - 0.05] = 0.069216$

$$Var_Q[r_1 \mid r_0] = \frac{0.03^2}{2 \times 0.04}\left[1 - e^{-2\times 0.04\times 1}\right] = 0.000864941 = 0.02940988^2$$

$$\frac{0.01 - 0.069216}{0.02940988} = -2.013$$

Therefore probability is $\Phi(-2.013) = 2.20\% \sim 2\%$

(iv) The probability will increase.

This is because the standard deviation will naturally increase over time as there is a greater possibility for more extreme movements.

In addition, the short rate is expected to mean revert down…
… from the initial figure of 7% to 5% (parameter b).

As T goes to infinity, the probability will converge in distribution to $N(b,\ \sigma^2 / 2a)$

(v) **Advantages**

The Vasicek model is an equilibrium model that will not replicate the initial term structure of interest rates.

In contrast, the alternative model will replicate the initial term structure and so will be arbitrage free.

The greater number of parameters in the alternative model will allow a closer calibration to current market data (e.g. swaption prices) than the Vasicek model.

The alternative model would also be expected to fit historical interest rate data more accurately, given the greater number of parameters.

The Vasicek model will allow negative interest rates.

This could be a weakness for valuing this derivative, as the value is driven by the extreme "tail" of the distribution.

It is therefore possible that the Vasicek model will potentially overstate the value of the derivative substantially.

In contrast, the alternative model will ensure that the short rate is always positive.

This is because the alternative model SDE considers the logarithm of the short rate, hence by construction the short rate is positive.

The tail of the normal distribution in the Vasicek model is likely to be too simplistic for the range between 0% and 1%.

For example, it may not capture the volatility associated with central bank action that may be taken if interest rates were to drop near zero.

However, the alternative model is a two factor model and so should produce more realistic dynamics.

In particular, the alternative model will capture medium term falls in the long term average short rate via the second factor.

For example, as witnessed currently in many developed economies) leading to more realistic prices of the derivative.

Disadvantages

The Vasicek model provides an analytical formula to analyse the derivative, assisting quick and easy pricing.

This is potentially a big benefit for the bank, given that it is planning on issuing the product across a number of different maturities.

In contrast, the alternative model may require a more complicated method such as Monte Carlo simulation or an interest rate lattice to value the derivative.

And therefore is more time-consuming to implement.

Overall, this question was well answered by many candidates. Many scored full marks on the first two parts and made good attempts for parts (iii) and (iv).

Part (v) was more challenging and this was a good differentiator of candidates. The key point to answering this part well was to link the answer back to the original derivative being described (as explicitly asked for in the question). The better candidates expanded on the basic bookwork of comparing two interest rate models to the specific derivative.

Q5 (i) A property swap is a type of total return swap which aims to replicate the financial consequences of a physical property transaction.

Most property swaps are constructed to give the return on a property index, not an actual physical property, although the principles are the same.

The buyer of a property swap receives the total return on the property or index, in return for paying a (usually) fixed coupon.

The swap is normally constructed so there is no net capital payment at outset, i.e. the fixed rate is chosen to match the expected total return payments under the swap.

The swap seller has the opposite set of payments, i.e. receives fixed and pays the total return on the property.

A property linked note is a bond whose value moves in line with property returns.

A property linked note can be created by the addition of a principal cash investment in a low-risk security (e.g. government bond) to a property swap.

A property index future / forward will replicate the return of a property index.

Property options may also be available, usually structured as an option on a future or forward.

(ii) In the context of the property derivatives in part (i), it is the risk that the reference properties or index underlying the derivatives do not correlate perfectly with such houses.

(iii) Basis risk is likely to be significant as most liquid property derivatives are constructed on property indices which may not move in tandem with the house.

In particular:

…most property derivatives are based on commercial property rather than residential property.

…even a residential property derivative is unlikely to specifically have Sam's preferred type of house as the sole underlying.

…most property derivatives are based on a large geographical area rather than Sam's preferred area.

…many property derivatives are based on the total return, whereas Sam may be more focused on capital appreciation.

Basis risk would be lower if the property derivative transacted was based on residential property.

(iv) It may be very costly for Sam to hedge such a small exposure.

Similarly, liquidity may be poor on such contracts.

Property derivatives may not be available for a two year period, meaning Sam may have to roll over the contract…

… potentially incurring further basis risk and/or cost.

Sam may be required to use the deposit to post collateral on the derivatives.

Furthermore, if the property derivative moves against Sam, then he faces the prospect of posting his entire $20,000 deposit as margin and having the position closed out. He would then be exposed should prices subsequently rise.

Collateral requirements could be high if the property derivatives are required to be centrally cleared.

It is unlikely that Sam will have the expertise to trade such derivatives.

Transparency – valuations for the property derivatives may not be readily available.

Counterparty risk – if the property derivative is in the money to Sam, a default of the counterparty may lead to losses.

(v) Invest the deposit in equities correlated with the housing market…

e.g. property management companies or real estate investment fund

e.g. a homebuilder or mortgage lender would likely provide a more "geared" or higher beta exposure.

House prices would be *expected* to increase very broadly in line with inflation on average, so could invest in inflation-linked assets such as index-linked gilts

Accelerate the purchase time of the house by either taking a bridging loan / saving more.

Agree a "forward price" with the vendor for the sale.

Invest in a smaller, affordable property now to access future changes in the housing market.

> This question was reasonably well answered by candidates, particularly the first three parts, although relatively few candidates gave sufficiently detailed and varied answers to score full marks on part (i).
>
> In part (iv), again only the better candidates generated a sufficient range of distinct points to score highly, with many answers being repetitive.
>
> Many candidates generated creative solutions to part (v) and marks were awarded where these solutions were realistic.

Q6 (i) Consider two portfolios:

A: Long forward + long cash holding of F_0e^{-rT}
B: One unit of the bond, S_0, and a short holding of I in cash.

At time *T*, both of these portfolios have the same value of S_T , and hence by the principle of no-arbitrage the portfolios must have the same value at time zero.

At time zero, the equated value of the two portfolios can be expressed as $F_0e^{-rT} = S_0 - I$, which can be rearranged to give $F_0 = (S_0 - I)e^{rT}$ as required.

(ii) It is a short call option on the vanilla bond.

It has a term of 10 years and a strike price of 100.

(iii) $$PV(\text{Coupons}) = \frac{(1-e^{-0.03\times 20})}{e^{0.03}-1}\times 5\%\times 100 = 74.08$$

$$PV(\text{Redemption}) = 100\times e^{-0.03\times 20} = 54.88$$

Hence vanilla bond price = 74.08+54.88 = 128.96

(iv) $$I = \frac{(1-e^{-0.03\times 10})}{e^{0.03}-1}\times 5\%\times 100 = 42.55$$

Therefore $F_0 = (128.96 - 42.55)e^{0.03\times 10} = 116.63$

$$C = e^{-rT}\left[F_0N(d_1) - KN(d_2)\right]$$

where $d_1 = \dfrac{\ln(F_0 / K) + \sigma^2 T/2}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$

$$d_1 = \frac{\ln(116.63/100) + 0.3^2 \times 10/2}{0.3\sqrt{10}} = 0.6365$$

$$d_2 = 0.6365 - 0.3\sqrt{10} = -0.3121$$

$N(d_1) = 0.7378$ and $N(d_2) = 0.3775$

$$C = e^{-0.03\times 10}\left[116.63 \times 0.7378 - 100 \times 0.3775\right] = 35.78$$

(v) Hence price of callable bond is 128.96 – 35.78 = 93.17

(vi) The Black model assumes the forward bond price is lognormally distributed which may not be appropriate.

In particular:

… bond prices returns may display fatter tails

… particularly fatter left hand tails due to defaults

… and bond prices may display discontinuous jumps due to rating upgrades / downgrades

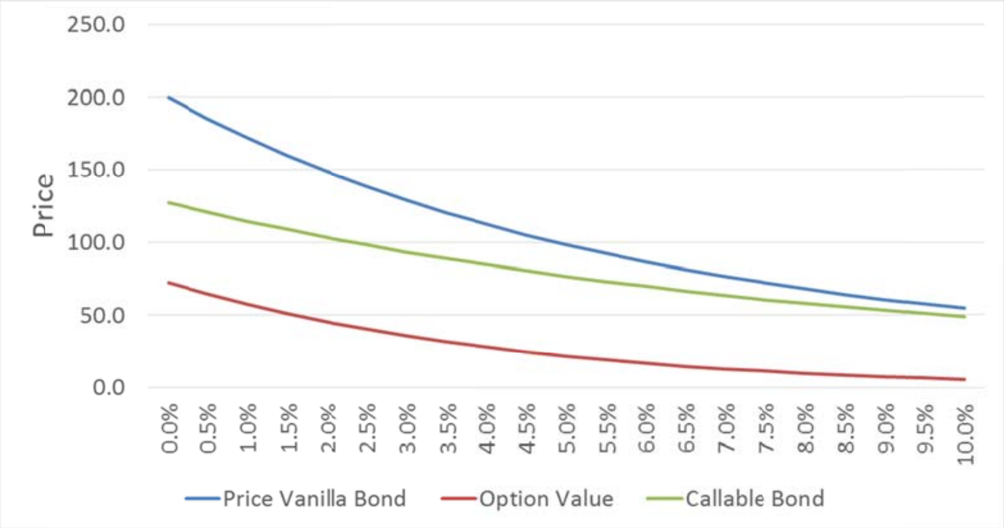
The stochastic behaviour of interest rates is not taken into account

The expected value of the bond at the option exercise date is assumed to be equal to F_0.

The underlying bond is likely to have positive systematic risk and hence the expected bond price at the option exercise date will likely be greater than F_0.

The volatility of the forward bond price is assumed to be constant.
In practice, the volatility of a forward bond price will decay as the term to maturity decreases.

(vii)



Vanilla bond price (correct curvature etc.)
Option value price (correct shape, tending to zero)

Callable bond price (shape, tending to the vanilla price at high rates)

Correct labelling of axes

(viii) The callable bond will be proportionally less sensitive to both falls and rises in interest rates compared to the liabilities.

As such, assuming the pension fund purchases the same value of callable bonds as the liabilities, the pension fund will gain profit on interest rate rises, and lose out on interest rate falls.

As the option term approaches, the embedded option will increasingly act to cap any price appreciation of the callable bond beyond the call price.

At the option term, the issuer will likely call if the vanilla bond is priced >100.

This would mean the pension fund would be refunded the call price and be "out of the market", but would still need to hedge the liabilities.

The pension fund would then face significant reinvestment risk to find a suitable investment that provided a similar return to the original callable bond and also hedges the liabilities.

Many candidates scored good marks on this question, and those who worked methodically through the calculations paying attention to the detail in the question scored highly. Only around half of the candidates correctly identified the fact that the callable bond was a call option for the bond issuer. This typically led to challenging logic for the subsequent calculations and consequently a number of errors.

Relatively few candidates gave good answers to part (vii), and many made no attempt at all.

Part (viii) was probably the hardest question on the paper and only the better candidates scored marks. Candidates should expect there to be a mix of difficulty across the paper, as this demonstrates.

Q7 (i) (a) Barrier options are options where the payoff depends on whether the underlying asset's price reaches a certain level during a certain period of time.

Single barrier options can be classified as either knock-out options or knock-in options.

A knock-out option ceases to exist when the underlying asset price reaches a certain barrier.

A knock-in option comes into existence only when the underlying asset price reaches a barrier.

Single barrier options come in four different types: up & in, up & out, down & in, down & out.

For "down" barriers the spot price starts above the barrier level and has to move down for an event to occur.

For "up" barriers the spot price starts below the barrier level and has to move up for an event to occur.

Barriers can also come in other forms: double barriers, Parisian options, partial timed barrier options, window barriers, fluffy barriers, Edokko options and several more.

Barrier options can be calls or puts.

Barrier options can be exercised as American, European or Bermudan options.

Barrier options are path-dependent options.

Barrier option prices are generally only monitored relative to daily close prices rather than inter-day trading; this is set out in the contract.

(b) Barrier option payoffs may more closely match views about the future behaviour of the market.

For example, an investor buying a barrier option can eliminate paying for scenarios which are deemed unlikely by the investor.

Or, alternatively an investor selling a barrier option can enhance the return by selling a barrier option which only pays on scenarios which are deemed improbable.

Barrier options may be more appropriate for hedging particular exposures than other options, because the payouts are limited to particular scenarios.

Barrier options premiums are generally lower than those of similar vanilla options.

This is because when a barrier option is compared to an equivalent vanilla option with the same strike price, period to expiry and the same underlying, it is identical apart from paying out in fewer scenarios.

Barrier options are easy to understand given knowledge of the equivalent vanilla derivative.

(ii) Consider a portfolio of one up & in single barrier option and one up & out single barrier option.

Both of which should be European call in nature.

Both of which should have a strike and barrier equal to the strike price of the European vanilla call option to be replicated.

At expiry just one of the barrier options will be active as the other will be worthless due to the barrier being crossed.

As the payoff from either option when it is active is identical to a European call option, it follows that at expiry the payoff from the portfolio will be identical to the European call option.

If the option starts out of the money, only the up & in single barrier option with strike price = barrier price is needed.

(iii) If the barrier is not hit during the lifetime then the payout is the same as for the vanilla put option, i.e. the payout is the strike price less the price of the underlying asset if this is greater than zero.

It should be noted that the payoff is limited due to the barrier…

…unlike with the standard put option which has an unlimited payout.

If the barrier is hit (i.e. the price falls below the barrier level at any time during the year) …

… then the underlying option is knocked out and there is no payout.

(iv) (a) The graph of the delta of the barrier option can be segmented into four regions.

1. When the exchange rate is much larger than the barrier of 0.7, the impact of the barrier is negligible.

   Therefore in this region the delta is negative like that of a vanilla put option.

   Asymptotically, the delta will tend towards zero from below.

2. When the exchange rate is lower than the barrier the barrier option is knocked-out.

   The delta in this region is therefore zero.

3. In the region just above the barrier, a small increase in the exchange rate will reduce the probability of the barrier option being knocked-out.

   This would therefore lead to a large increase in the barrier option's value.

   Hence, the delta of the barrier option will be large and positive.

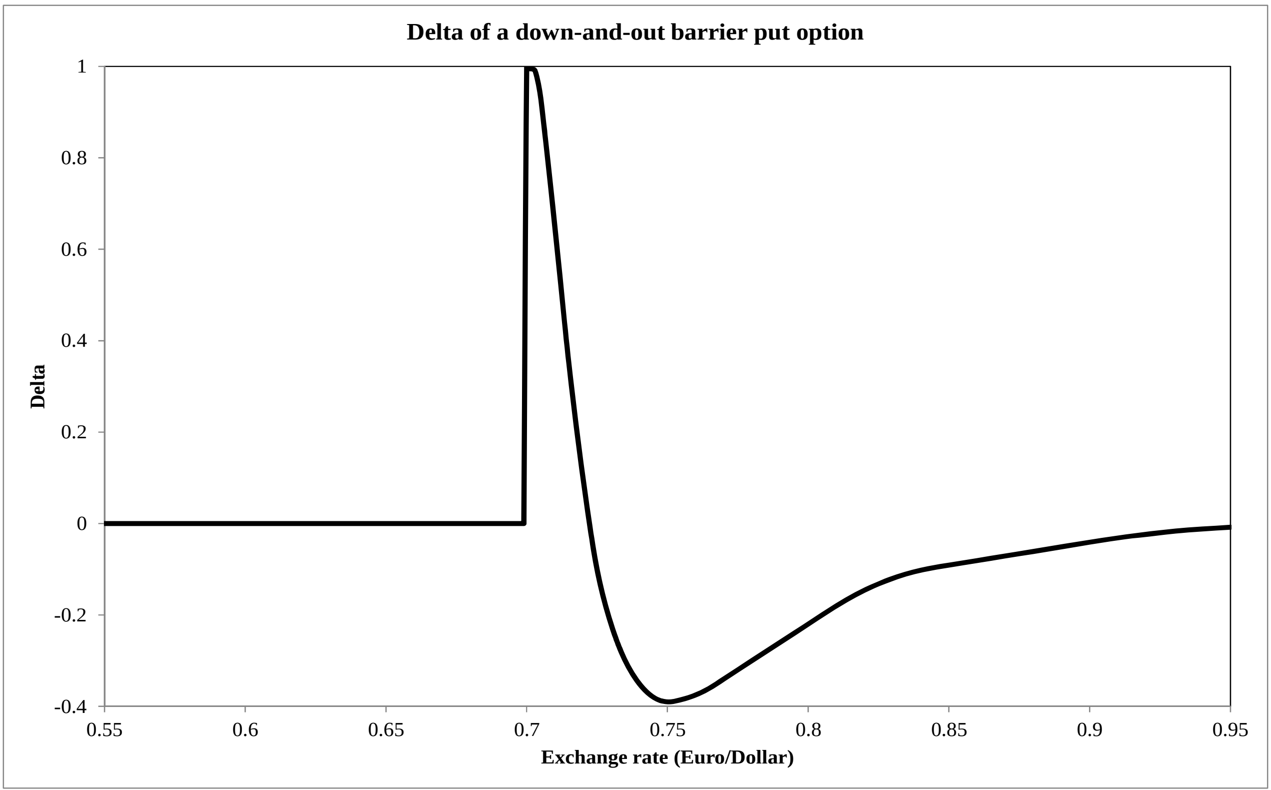
4. At the barrier itself, there is a discontinuity in the payoff.

   The delta becomes infinitely large.

   Overall, the fluctuation of the barrier option's delta from positive to negative is as a result of a decrease in the value of the barrier option but with increasing probability of it not being knocked out.

   Invoking the continuity of the payoff (and delta) away from the barrier enables the complete graph to be sketched.

(b) *[For simplicity the discontinuity at the barrier has not been shown to scale to enable the rest of the chart to remain in scale.]*



(v) The bank is delta hedging a long position with nominal 20 million Euros.

It therefore needs to sell 20 million times the delta of the barrier option in Euros.

If the spot price approaches the barrier then, based on the chart in (iv), the delta becomes larger.

This requires the bank to sell more Euros to maintain the delta hedge.

This can influence the currency market, as gradually selling Euros will accelerate the movement of the spot price towards the barrier (in isolation from other market influences).

In extreme cases this could lead to the barrier being breached.

A more likely scenario is that the bank will have insufficient capacity to continue purchasing Euros.

If the option is knocked-out then there is no barrier option to hedge.

As a result of the option being knocked-out, there may be less or no downward movement as fewer Euros are being sold in the market.

In extreme cases the bank may need to unwind the hedge quickly, resulting in a large upward movement in the spot price.

Part (i) was basic bookwork but only a minority provided sufficient detail to score well.

Parts (ii) and (iii) were more challenging, and answers often reflected the lack of detailed knowledge of the Core Reading identified in part (i).

Those who took a methodical approach to part (iv) scored well.

Candidates generally found part (v) difficult and only the strongest candidates scored marks in this part.

END OF EXAMINERS' REPORT