INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

26 April 2017 (am)

Subject ST6 – Finance and Investment Specialist Technical B

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all seven questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Consider a derivative with price f that follows the process given by the stochastic differential equation:

$$df = \mu f dt + \sigma f dz$$

where μ is the expected return, σ is the price volatility, z is standard Brownian motion and t is time.

(i) Write down an expression for the market price of risk, λ, for the derivative, defining any additional notation used. [1]

Consider an asset which is a money market account with value g. The asset earns the constant instantaneous risk-free rate r at any given time.

(ii) Write down the stochastic process followed by g. [1]

(iii) Derive a stochastic differential equation followed by $\ln\left[\frac{f}{g}\right]$, in terms of λ and σ. [3]

(iv) Derive a stochastic differential equation followed by $\frac{f}{g}$, in terms of λ and σ. [1]

(v) Determine the market price of risk, λ, under which $\frac{f}{g}$ is a martingale. [1]

(vi) (a) State the equivalent martingale measure result.

(b) Verify that your answer to part (v) is consistent with the equivalent martingale measure result.

(c) Explain how the equivalent martingale measure result can be used in the valuation of interest rate derivatives.

[4]
[Total 11]

2 (i) (a) Define a speculator in the context of the derivatives market.

(b) Describe the three types of speculators.

[4]

(ii) Discuss the advantages and disadvantages of allowing speculators to trade in the derivatives market. [4]
[Total 8]

3 Under the Heath Jarrow Morton (HJM) model, the stochastic differential equation for $P(t, T)$, the price at time t of a zero-coupon bond maturing at time T, is given by:

$$dP(t,T) = r(t)P(t,T)dt + v(t,T)P(t,T)dz(t)$$

where $r(t)$ is the risk-free short rate at time t, $v(t, T)$ is a time-dependent representation of the volatility of $P(t, T)$ and $z(t)$ is standard Brownian motion.

(i) Write down a formula, in terms of $P(t, T)$, for the continuously compounded forward rate $f(t,T_1,T_2)$ between times T_1 and T_2 observed at time t. [1]

(ii) Write down a formula for the instantaneous forward rate $F(t, T)$ for time T observed at time t. [1]

Let the process followed by the instantaneous forward rate $F(t, T)$ under the HJM model be expressed in a general form as:

$$dF(t,T) = m(t,T)dt + s(t,T)dz(t)$$

where $m(t, T)$ and $s(t, T)$ are time-dependent instantaneous drift and volatility functions respectively.

(iii) (a) Derive the stochastic differential equation followed by $df(t,T_1,T_2)$.

(b) Show how $m(t, T)$ can be expressed in terms of $s(t, T)$.

[Hint: Consider the stochastic differential equation derived in part (a) in the limit as $T_2 \to T_1$.]

[6]

A bank wishes to use an interest rate model to value a number of plain vanilla and complex interest rate caps.

(iv) Outline the weaknesses of using the HJM model for this purpose. [2]

(v) Explain how the LIBOR Market Model overcomes some of these weaknesses.
[4]
[Total 14]

4 For a derivative with price V and an underlying asset with price S, a further Greek, λ, is defined as:

$$\lambda = \frac{\partial V}{\partial S} \times \frac{S}{V}.$$

(i) (a) Explain what this Greek represents.

(b) Express this Greek in terms of the Greek delta. [2]

(ii) Explain why λ increases in value when an option moves:

(a) further out-of-the-money.

(b) closer to expiry. [2]

Consider a European call option on a non-dividend paying stock with price S, in a market where the assumptions underlying the Black-Scholes model apply.

Let τ be the time to expiry of the option, r the constant continuously compounded risk-free interest rate, K the strike price and σ the volatility of the underlying.

N and ϕ represent the cumulative probability distribution function and probability density function of the standard normal distribution respectively.

d_1 and d_2 are as defined in the Black-Scholes formula.

(iii) Show that:

$$\lambda = \frac{SN(d_1)}{SN(d_1) - Ke^{-r\tau}N(d_2)}.$$

[Hint: It can be assumed without proof that $Ke^{-r\tau}\phi(d_2) = S\phi(d_1)$.] [4]

(iv) Determine a numerical lower bound for λ. [3]

(v) Determine the asymptotic behaviour of λ as $S \to \infty$, stating any assumptions made. [2]

(vi) Comment on what this lower bound means for someone investing in a European call option. [2]

[Total 15]

5 A bank sells a European call option to buy 1,000,000 shares of a non-dividend paying stock with a strike price of \$150 per share, for an option price of \$11,000,000. The stock price when the option was sold was \$145 and there were six months to expiry. The continuously compounded risk-free interest rate is 5% per annum.

The bank is considering strategies to manage the risks in selling this option.

It can be assumed that the bank has sufficient financial resources to purchase shares in the open market and that transaction costs in the open market can be ignored.

(i) Draw two graphs showing the profit/loss for the bank at expiry against the stock price for the following situations:

(a) The bank adopts a naked position (i.e. the bank has not taken any action to hedge the downside risk of writing this call option).

(b) The bank adopts a covered position (i.e. the bank buys, or already owns, the shares on which the call option is written).

[7]

(ii) Suggest an equivalent portfolio to adopting a covered position, using the graph in part (i)(b) or otherwise. [1]

(iii) Outline how the bank could set up a stop-loss hedging strategy for this option by switching between naked and covered positions. [3]

(iv) Suggest an alternative hedging strategy that the bank could adopt. [1]

[Total 12]

6 An analyst at a bank is part of a team building a Monte Carlo simulation. The analyst is using the Longstaff-Schwartz approach for pricing the following American put option on a non-dividend paying stock:

- time to expiry = three months
- risk-free interest rate = 10% per annum continuously compounded for all durations
- current stock price = 100.0
- strike price = 120.0
- number of simulated paths = 100,000
- simulated time step = 1 day

The analyst is currently testing and calibrating the full simulation using a simplified model which includes the following eight sample paths and a time step of one month for the generated stock price under the risk-neutral measure:

Sample path	*Time = 0 (now)*	*Time = 1 month*	*Time = 2 months*	*Time = 3 months (expiry)*
1	100.0	76.4	111.9	102.2
2	100.0	110.5	118.2	111.1
3	100.0	116.4	149.9	124.7
4	100.0	133.1	159.7	157.9
5	100.0	123.5	152.9	130.8
6	100.0	85.0	79.0	88.7
7	100.0	93.3	119.5	140.2
8	100.0	88.6	89.3	92.2

(i) (a) Calculate the payoff at expiry for each sample path, assuming that the option has not been exercised at an earlier time.

(b) Determine the optimal strategy (for the buyer of the option) at expiry for each sample path, assuming that it has not been exercised at an earlier time.

[2]

(ii) Determine, for each sample path where the option is in-the-money at time two months:

(a) the intrinsic value of the option at time two months

(b) the payoff at expiry discounted to time two months

[3]

Let V represent the value of the payoff at expiry discounted to time two months, for sample paths where the option is in-the-money at time two months.

The analyst is currently using the following function to approximate this value:

$$V = a + bS + cS^2 + dS^3$$

where S is the value of the stock at time two months.

(iii) (a) Explain how the values for a, b, c, and d would be determined, including any formulae that would be used.

(b) Suggest reasons why only sample paths where the option is in-the-money at time two months have been used.

[4]

The analyst has determined the following values to be used for the function V:

Constant	*Value*
a	1,138.0
b	–35.8
c	0.385
d	–0.00138

(iv) Determine which sample paths would lead to early exercise at time two months. [4]

The analyst continues this process to the one month time step and then determines a price for the option at time 0 of 19.0.

(v) Recommend, with justification, whether or not the option should be exercised immediately, based on this estimate. [1]

(vi) Explain why the full simulation is likely to give a different value for the price of the option at time 0. [2]

[Total 16]

7 A bank has a total of \$500m notional forward-starting receiver swaps, under which it receives a fixed 3% coupon and pays LIBOR. Coupons are paid annually in arrears, with the swap commencing in five years' time and maturing ten years from now (i.e. the swaps will have five payment dates in total).

Following large reductions in LIBOR interest rates to 1% per annum (annually compounded) at all terms since the swaps were sold, the bank has made mark-to-market gains and is concerned about interest rates rising back to previous levels.

The bank measures the interest rate risk via the ShiftSensitivity metric, which is defined as:

$$\text{ShiftSensitivity} = MV(-1\%\,\text{Parallel}) - MV(+1\%\,\text{Parallel})$$

where MV (+/−1% Parallel) is the mark-to-market value of the position under a parallel interest rate movement at all terms of +/−1% respectively, where this movement is in absolute terms (e.g. 1% per annum plus a 1% per annum shift is 2% per annum). For example, MV (−1% Parallel) is the mark-to-market value of the position assuming the yield curve is 1% lower at all terms.

(i) Show that the market value of the swaps would be \$21m if LIBOR interest rates were to rise to 2% per annum at all terms. [2]

(ii) Show that the current ShiftSensitivity of the swaps is \$54m. [1]

(iii) (a) State the three Greeks that the bank would typically want to analyse for interest rate derivatives.

(b) Assess how effectively the ShiftSensitivity metric will allow the bank to manage exposure to these three Greeks for interest rate derivatives. [4]

The bank is now considering overlaying an UpHedge derivative.

Such a derivative has the following payoff at time T (years), based on the interest rate I observed at time t (years) for the period between times t and T, where I, a and b are expressed as per annum rates with compounding frequency $T - t$:

$$\begin{array}{ll} 0 & \text{if } I < a \\ \$500m \times (T-t) \times (I-a) & \text{if } a < I < b \\ \$500m \times (T-t) \times (b-a) & \text{if } I > b \end{array}$$

(iv) Sketch a graph showing the payoff of the UpHedge with respect to I. [2]

(v) Describe how the UpHedge can be decomposed into two caplets, stating the key terms of each caplet. [3]

(vi) Write down the formula for the price of an interest rate caplet using Black's model, defining all additional notation used. [3]

(vii) Calculate, showing your workings, the current value of the UpHedge under the following parameter values:

Parameter	*Value*
a	0.01
b	0.03
t	5
T	10
Interest rate volatility	30% per annum

[6]

(viii) Assess how the ShiftSensitivity of a combined portfolio of the bank's swaps and the UpHedge in part (vii) would change as interest rates increase from 1% per annum. [3]

[Total 24]

END OF PAPER