INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2011 examinations

Subject ST6 — Finance and Investment
Specialist Technical B

Introduction

The attached subject report has been written by the Principal Examiner with the aim of helping candidates. The questions and comments are based around Core Reading as the interpretation of the syllabus to which the examiners are working. They have however given credit for any alternative approach or interpretation which they consider to be reasonable.

T J Birse
Chairman of the Board of Examiners

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QUESTION 1
Syllabus section: (a)-(d),(f)
Core reading: 1-4, 6

(i)

A property swap is a type of total return swap which aims to replicate the financial consequences of a physical property transaction.

Most property swaps are constructed to give the return on a property index, not an actual physical property, although the principles are the same.

The buyer of a property swap receives the total return on the property or index, i.e. all the benefits (e.g. rent, capital repayments) associated with owning the property or properties underlying the swap, just like a physical buyer would.

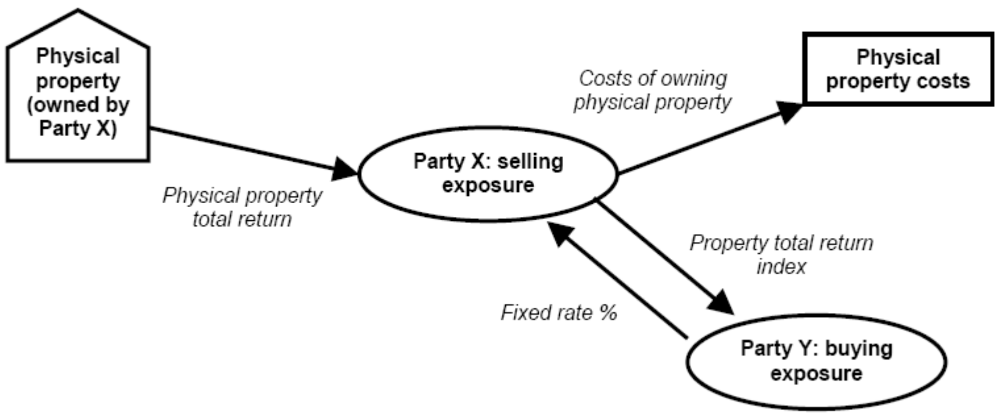
For this set of cash flows, the swap buyer pays a (usually) fixed coupon to cover the cost of the above benefits, such as financing (e.g. mortgage) and other outgoings (e.g. stamp duty, legal fees, agency fees).

The swap is normally constructed so there is no net capital payment at outset, i.e. the fixed rate is chosen to match the value of the benefits.

The swap seller has the opposite set of payments, i.e. receives fixed and pays the total return on the property.

A diagram may be helpful to explain the mechanism [*taken from the Core Reading*]:

Diagrammatic Representation of a Property Swap



[*Credit was also given for describing the above with, instead of a fixed payment on the non-property leg, a floating (e.g. LIBOR) payment plus a margin which together match the value of the benefits.*]

(ii)

Benefits for a swap <u>buyer</u>:

- participation in the returns from physical property market, without the transaction costs or overheads of direct investment
- low cost diversification across different sectors or countries using an index

Benefits for a swap <u>seller</u>:

- reduction in exposure to the property market for property owners, without the cost or effort of physically selling individual properties
- hedging against a market downturn
- portfolio re-balancing and/or reduction of concentration
- alpha retention (i.e. hold physical property and short a general index synthetically to lock in specific sector outperformance)

Benefits for both:

- speed and efficiency of entry to / exit from the market
- use of related options and other structured solutions
- potential to avoid direct property transaction taxes (e.g. stamp duty)

(iii)

Main risks inherent in property derivatives (<u>excluding</u> market performance):

- Basis – between the property index and the underlying physical property; can work out in favour of derivatives (e.g. buying the index when it is cheap) or against.

- Liquidity – whether the market can sustain the volume required, both at outset and in the future; this especially applies to options. [The property derivative market has not been as liquid as some had hoped.]

- Counterparty credit risk – whether the swap counterparty can be relied upon if the market moves against them. [New Basel III standards are proposing to introduce central counterparties for many OTC contracts, which may help in this respect.]

- Volatility – whether property derivatives follow the underlying market closely or are more volatile.

- Transparency – whether property derivative prices are readily available and consistent from one dealer to another.

- Legal – whether the policy wording or contract is enforceable in the event of a dispute, particularly on close-out or default.

- Operational – the need to operate sufficiently robust mechanisms in the firm to support derivatives trading, such as accounting, risk management, IT systems etc.

[*Other valid points could be made.*]

This was a straightforward question covered property derivatives, a relatively new part of the syllabus. The three parts all required repetition of Core Reading bookwork and were designed to ensure that candidates had understood the material.

In part (i), almost all candidates were able to define a property swap correctly, but in several cases the requested mechanics were not explained. In part (ii), candidates could have structured their answers better, as the question specifically asked for a comparison with direct property investment, rather than simply an analysis of the general benefits of derivatives. Part (iii) is typical of several list-type questions, where for two marks a good response would list say four main risks with a very brief explanation against each (the solution given below is more comprehensive than is needed and reflects the range of responses possible).

QUESTION 2
Syllabus section: (h)(i)-(iii)
Core reading: 8, 9

(i)

(a)

Ito's Formula (or Lemma) states that, if $dx_t = \mu(x,t)dt + \sigma(x,t)dz_t$, where *zt* is a standard Brownian motion, then any function $G(x, t)$ follows the process:

$$dG = \left(\mu \frac{\partial G}{\partial x} + \frac{\partial G}{\partial t} + \tfrac{1}{2}\sigma^2 \frac{\partial^2 G}{\partial x^2} \right) dt + \sigma \frac{\partial G}{\partial x} dz$$

[*Ito's Lemma forms the basic extension of differential calculus to variables which are stochastic in nature.*]

(b)

In non-stochastic calculus, the formula for a function dependent on two variables would be as follows:

$$\begin{aligned} dG &= \frac{\partial G}{\partial x} dx + \frac{\partial G}{\partial t} dt + \tfrac{1}{2} \frac{\partial^2 G}{\partial x^2} (dx)^2 + \tfrac{1}{2} \frac{\partial^2 G}{\partial t^2} (dt)^2 + \ldots \\ &= \left(\mu \frac{\partial G}{\partial x} + \frac{\partial G}{\partial t} \right) dt + \sigma \frac{\partial G}{\partial x} dz + \ldots \end{aligned}$$

where the additional terms in $(dx)^2$ and $(dt)2$ are negligible in the limit.

However, Brownian motion has a particular fractal form whereby it does not diminish linearly as the time interval for observation gets smaller – the randomness is still present even at the smallest time intervals.

Brownian motion has mean 0 and variance t, which $\Rightarrow E(z2) = t$ ...
... so $(dz)2 \rightarrow dt$, which instead of disappearing in the limit leaves a first order differential term.

Hence the additional term $\frac{1}{2}\sigma^2 \frac{\partial^2 G}{\partial x^2} dt$ is caused by the stochastic nature of x.

[*There are other valid ways of explaining this.*]
(ii)

Applying Ito's Lemma to $f(z_t) = z_t^2 \;\Rightarrow\; d(z_t^2) = 2z_t dz_t + dt$

So $z_t^2 = 2\int_0^t z_t dz_t + t$

(iii)

(a)

[*Note: in this part, though not explicitly stated in the question, μ and σ are taken to be constant. Most candidates made this assumption, but those who assumed dependence on x_i and t were not penalised.*]

We have $A_t = \frac{1}{n}\sum_{i=1}^{n} x_{t_i} = \frac{1}{n}(x_1 + x_2 + ... + x_n)$ using a simplified suffix notation.

Now, $x_i = x_{i-1} + \Delta i$, where each Δi is distributed as dx over an increment of $\frac{t}{n}$.

These Δ_i are identically distributed, with mean $\mu\frac{t}{n}$ and variance $\sigma^2\frac{t}{n}$.

So we can re-write At as:

$$A_t = \frac{1}{n}\left(x_0 + \Delta_0 + x_0 + \Delta_0 + \Delta_1 + ... + x_0 + \Delta_0 + \Delta_1 + ... + \Delta_{n-1}\right)$$
$$= \frac{1}{n}\left(nx_0 + n\Delta_0 + (n-1)\Delta_1 + ... + \Delta_{n-1}\right)$$

Variance (At) = $\frac{1}{n^2}\sigma^2 \frac{t}{n}\sum_{i=1}^{n}\left(n^2+(n-1)^2+...+1^2\right)$

which, using the formula given:

$$= \frac{1}{n^2}\sigma^2 \frac{t}{n}\frac{1}{6}n(n+1)(2n+1) = \sigma^2 t.\frac{(n+1)(2n+1)}{6n^2}$$

(b)

Hence, in the limit $n \to \infty$, Var(At) $\to \frac{\sigma^2 t}{3}$

This question was based around Ito's Lemma, containing some elements one would find in CT8 but also with some more advanced applications.

Parts (i) and (ii) did not generally cause any problems, but part (iii) was harder, although as the solution shows the algebra is not that lengthy The key to part (iii) was to consider the variance of the Brownian motion for each increment.

QUESTION 3
Syllabus section: (h)(iv)-(ix), (i)
Core reading: 10-12

(i)

(a)

Using the definition in the question, the at-the-money strike is $X = S\exp[(r-q)T]$.

[*We can easily see this is the case because the value of the stock at time 0 excluding dividends up to time T is* $\tilde{S} = S\exp(-qT)$. *At time T, this has forward value* $\tilde{S}\exp(rT) = S\exp[(r-q)T]$.]

(b)

The Put option price P is given by the Black-Scholes formula including dividends:

$$P = X\exp(-rT)N(-d_2) - S\exp(-qT)N(-d_1)$$

where N is the cumulative Normal distribution and

$$d_1 = \frac{[\ln(S/X) + (r-q+\tfrac{1}{2}\sigma^2)T)]}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

For the at-the-money option, $\ln(S/X) = -(r-q)T$, so these simplify to:

$$d_1 = \tfrac{1}{2}\sigma\sqrt{T} \text{ and } d_2 = -\tfrac{1}{2}\sigma\sqrt{T}$$

and hence

$$\begin{aligned} P &= X\exp(-rT)N(\tfrac{1}{2}\sigma\sqrt{T}) - S\exp(-qT)N(-\tfrac{1}{2}\sigma\sqrt{T}) \\ &= S\exp(-qT)[N(\tfrac{1}{2}\sigma\sqrt{T}) - N(-\tfrac{1}{2}\sigma\sqrt{T})] \\ &= S\exp(-qT)[2N(\tfrac{1}{2}\sigma\sqrt{T}) - 1] \end{aligned}$$

Rearranging terms:

$$N(\tfrac{1}{2}\sigma\sqrt{T}) = \tfrac{1}{2}\left(\frac{P}{S\exp(-qt)} + 1\right)$$

so

$$\sigma = \frac{2}{\sqrt{T}} N^{-1}\left[\tfrac{1}{2}\left(\frac{P}{S\exp(-qt)} + 1\right)\right]$$

where N–1 is the inverse cumulative Normal function.

[*Note that this formula has no direct dependency on r.*]

(ii)

(a)

Total dividends = 2 and T = 1.

Therefore $\tilde{S} = S\exp(-q)$ = 80 – 2 = 78,

so $-q = \ln(78/80) = \ln(0.975)$, or q = 2.532%.

[*No credit in part (a) for making q* = ln(82 / 80) = 2.469%*, but allowance for following it through into part (b) and (c), giving 81.234 and 33.88% respectively.*]

(b)

At-the-money strike $X = S\exp[(r-q)]$ = 80 × exp(0.04 – 0.02532) = 81.183

(c)

Using the formula in (i), $\frac{1}{2}\left(\dfrac{P}{S\exp(-qt)}+1\right)$ = 0.5 [1 + 10.5 / 78] = 0.56731

Hence implied volatility σ = 2 × *N*–1(0.56731) = 33.91%.

(iii)

The existing Put option will rise in value if *r* falls, due to the Rho effect. A portfolio of forward delivered Stock + European Put will always equal the strike price at time 1, so if the Stock is cheaper to fund the Put value must rise to compensate.

The formula in (i)(b) does not have a direct dependency on *r*, so it is true that if the calculation is re-done from the start, the volatility calculated will be the same.

However, because *r* has changed, the at-the-money strike will be different, hence the apparent anomaly.

This question considered a dividend paying stock and showed how the Black-Scholes formula simplifies for an at-the-money option. It was generally well answered.

In part (ii)(a), there were some differences in interpretation of how to allow for dividends. The main point to note is that dividends paid before option expiry reduce the value of the forward price at expiry – several candidates produced analysis that would imply an increase.

Part (iii) introduced an apparent anomaly relating to the risk-free rate, but most candidates were able to explain the true effect.

QUESTION 4
Syllabus section: (g) + (i)
Core reading: 7, 12

(i)

(a)

Start from Delta = $N(d_1)$ where $d_1 = \dfrac{[\ln(S/X) + (r + \frac{1}{2}\sigma^2)T)]}{\sigma\sqrt{T}}$.

Differentiate wrt *S*:

Gamma $\Gamma = \dfrac{\partial\Delta}{\partial S} = \dfrac{\partial\Delta}{\partial d_1}\dfrac{\partial d_1}{\partial S} = \dfrac{1}{\sqrt{2\pi}} e^{-d_1^2/2} . \dfrac{1}{\sigma S\sqrt{T}}$

(b)

To get the "speed", differentiate wrt S again:

$$\frac{\partial\Gamma}{\partial S} = \frac{1}{\sqrt{2\pi}}.\frac{1}{\sigma\sqrt{T}}\left[e^{-d_1^2/2}\frac{\partial(1/S)}{\partial S} + \frac{1}{S}\frac{\partial(e^{-d_1^2/2})}{\partial S}\right]$$

$$= \frac{1}{\sqrt{2\pi}}.\frac{1}{\sigma\sqrt{T}}\left[-e^{-d_1^2/2}\frac{1}{S^2} - \frac{1}{S}e^{-d_1^2/2}\frac{d_1}{\sigma S\sqrt{T}}\right]$$

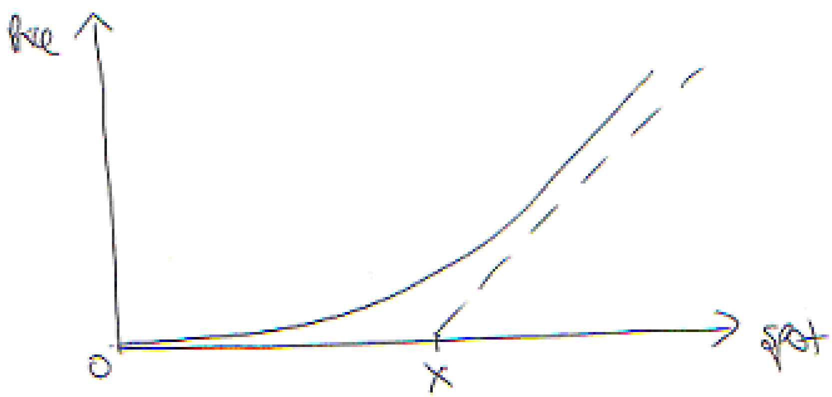
$$= \frac{1}{\sqrt{2\pi}}.\frac{1}{\sigma S\sqrt{T}}e^{-d_1^2/2}\left[-\frac{1}{S} - \frac{d_1}{\sigma S\sqrt{T}}\right]$$

$$= -\frac{\Gamma}{S}\left[1 + \frac{d_1}{\sigma\sqrt{T}}\right] \qquad \text{as required}$$

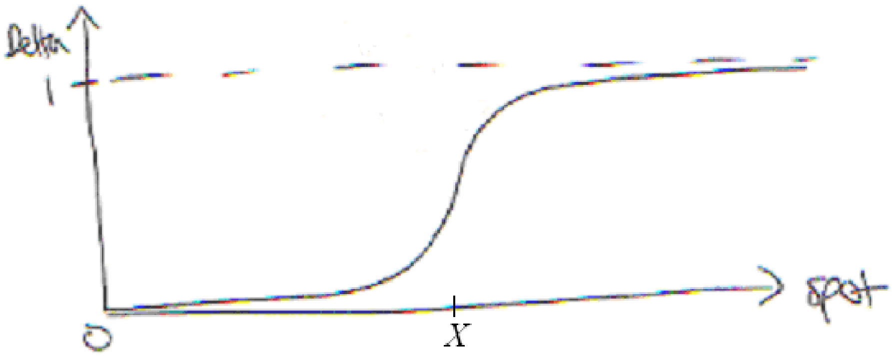
(ii)

[*The sketches shown below are typical of what the examiners would expect. In part (a), equal credit was given if the candidate included the option premium (i.e. drew a P&L curve). Full credit was not given, however, for presenting values at expiry, as the question clearly states there is time T* <u>*to*</u> *expiry.*]

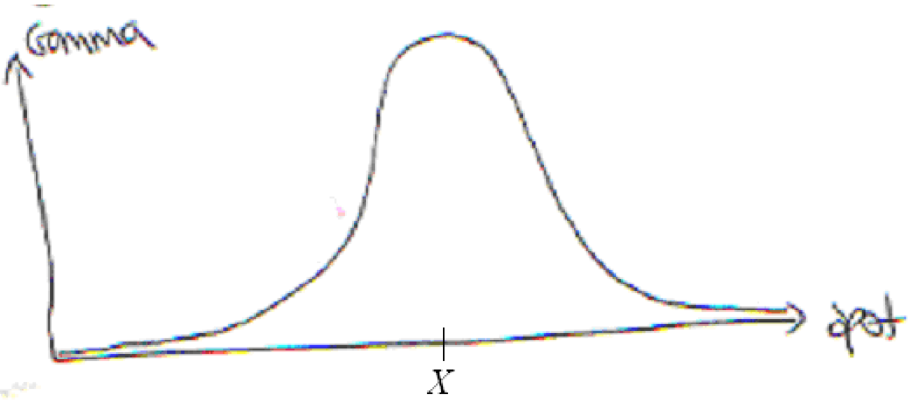
(a)



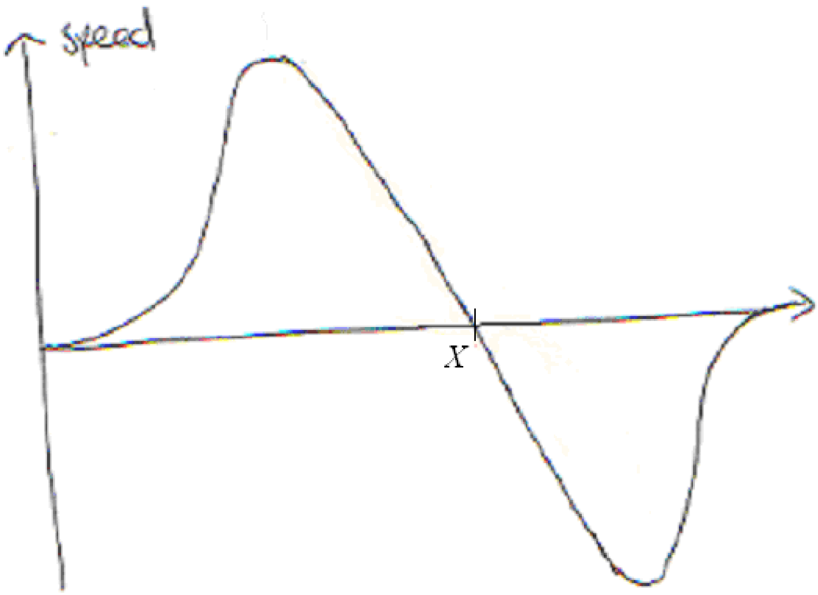
(b)



(c)



(d)



(iii)

Theoretically, the dealer is correct that reducing the rate of change of gamma sensitivity would result in a more stable hedge, however ...
... speed is not necessarily the only or best second order sensitivity value to monitor (there are others that could be considered, such as $d\Gamma/d\sigma$) ...
... the formula would need to be adjusted to apply to currency rates (not too hard – it's like adding a dividend rate to a stock) ...
... liquidity and transaction costs will have a big impact, i.e. the availability of options at tight prices for the strikes and maturities required.
... taking differences to such a degree will not be accurate, as the Black-Scholes formula only applies approximately ...
... allowing for changes in implied volatility and smile effects (different volatilities for different strikes) makes gamma hedging difficult, let alone hedging its derivative ...
... time decay changes the "speed" (and gamma), requiring a rebalancing – this is particularly noticeable at short maturities ...
... gamma is unstable at short maturities, so "speed" will be even more so – either side of strike near maturity it will fluctuate dramatically.

[*Other valid points could be made.*]

This question dealt with "speed", essentially the rate of change of gamma. It is not part of the syllabus so was introduced as an extension application. It is a more advanced "Greek" whose usefulness is tempered by accuracy and estimation errors.

Part (i) involved some straightforward algebra. The examiners were alert to cases where the algebra was not working out but the candidates magically managed to obtain the answered given.

Part (ii) asked for graphs of the position at the current date, not at expiry – too many candidates produced expiry charts (for which partial credit was given). For the first time, the model solution has included hand-drawn sketches, to emphasise the fact that very precise graphs are not required.

Part (iii) is typical of a type of question that is often asked in ST6, essentially asking for the evaluation of an unfamiliar proposal. Candidates should always try to provide several distinct points in short paragraphs, not one point made at length, and through those points aim to demonstrate their understanding of as many different relevant issues as the marking schedule would seem to require. Also, it is usually helpful to give an assessment of whether the proposal is sensible or not.

QUESTION 5

Syllabus: (e) & (j)
Core reading: Units 5 and 13

(i)

Let g_j be the coupon of bond maturing in year j, and B_t^j its price (present value).

The bond formula is $B_t^j = g_j \sum_{k=1}^{t} d_k + 100 d_t$, with B_t^j and gj expressed in %.

Hence $d_t = \dfrac{B_t^j - g_j \sum_{k=1}^{t-1} d_k}{100 + g_j}$, which can be solved iteratively, starting at t = 1.

So $d1$ = 101.03 / 103 = 0.98087 but we are given this.

Then

$d2$ = (102.05 – 4 $d1$) / 104 = 0.94352
$d3$ = (103.21 – 4.75 ($d1$ + $d2$)) / 104.75 = 0.89803

We are also given $d4$ = (105.36 – 5.5 ($d1$ + $d2$ + $d3$)) / 105.5 = 0.85153.

[*Note that redemption yields are not needed.*]

(ii)

(a)

Take present value of the two coupons due at the end of years 1 and 2:

PV(coupons) = 5.5 ($d1$ + $d2$) = 5.5 (0.98087 + 0.94352) =10.584

Then deduct from the current bond price and roll up to end of year 2 using the relevant discount factor ($d2$):

Fwd price (4 year bond at end year 2) = (105.36 – 10.584) / 0.94352 = 100.45.

(b)

To convert forward yield volatility to forward price volatility, multiply by forward modified duration and (forward) yield:

Fwd price vol = Fwd yield vol × fwd modified duration × yield
= 25% × 1.85 × 5.25% = 2.43%

(c)

Use the above calculated forward price and price vol as inputs into the Black model.
$F = 100.45$, $K = 100$, $T = 2$, volatility $\sigma = 2.43\%$

$$N_1 = \Phi\left[-\frac{\ln(F/K) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}\right] = \Phi[-(0.00449 + 0.00059) / 0.034365]$$

$$= \Phi[-0.14782] = 0.44124$$

$$N_2 = \Phi\left[-\frac{\ln(F/K) - \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}\right] = \Phi[-(0.00449 - 0.00059) / 0.034365]$$

$$= \Phi[-0.11348] = 0.45482$$

Hence

Put price $= d2\ (K\,N2 - F\,N1)$
$= 0.94352\ (45.482 - 44.323) = 1.09$

(iii)

In creating a forward bond price, the Black model assumes that the risk-free rate is constant and independent of the underlying bond price.

It also assumes Lognormal bond price movements (i.e. geometric Brownian motion).

The Lognormal assumption may not be correct, but we don't have any information to make an assessment.

However, we can be reasonably sure that the 4-year bond price will be correlated to some extent with the 2-year interest rate, i.e. their movements are not independent.

There could also be problems estimating forward volatility so far ahead.

Hence the model should measure a suitable forward volatility and choose an appropriate correlation – neither of these is easy to find.

There appear to be only four bonds, and we don't know if there are other instruments, so estimating anything in this market might be difficult.

A more representative pricing model would be a two-factor model including correlated 2-year and 4-year risk factors ...
... and preferably including a full yield curve representation.

This question, based on fixed income theory, asked the candidate to calculate the value of a bond option based on a given yield curve implied by government bond (clean) prices. It was generally well answered.

Part (i) was straightforward, and many candidates did well in part (ii). The formula for converting modified duration between price and yield is in the Core Reading (section 3.6). Should such a formula not be recalled, but a value is required for a later part, a candidate could make a sensible estimate and use that in the next part, thereby gaining marks for method if not for numerical results.

The comment above for part (iii) of Question 4 would also apply to part (iii) of this question.

QUESTION 6

Syllabus section: (h)(iv)-(ix), (i)
Core reading: 10-12

(i)

(a)

[*The calibration of a binomial tree involves finding the solution to some simultaneous equations that ensure that the expectation and standard deviation of the security price is appropriate at each point in the tree.*]

The CRR solution to the equations is approximate and is only accurate for small time steps. In this example, we are using yearly time steps, so the CRR solution is likely to be inaccurate.

(b)

Expected price at Time 1 from the tree = 0.53 × 1302.6 + 0.47 × 767.7 = 1051.2
Check: the expectation should be 1000 exp(0.05) = 1051.3 ✓

Standard deviation of price at Time 1 from the tree
= √[0.53 (1302.6 – 1051.2)2 +0.47 (767.7 – 1051.2)2] = 267.0

Check: We expect to Standard deviation to be 1000*√[(exp(σ2) – 1)*exp(2μ + σ2)],
where volatility σ = 25% and drift μ = *r* – ½ σ2 = 1.875%,
i.e. 1000*√[(exp(0.252) – 1)*exp(0.1)] = 267.0 ✓

(ii)

(a)

Maturity payoffs are max(1000 – *S*, 0).

Working backwards through the tree with $V = \exp(-0.05)[0.53V_{\text{up}} + 0.47V_{\text{down}}]$:

Time 0	Time 1	Time 2	Time 3
			0
		0	
	46.5		0
119.2		103.9	
	214.2		232.4
		362.0	
			547.6

Hence tree value of European option = 119.2

(b)

Payoffs at maturity are still max(1000 – S,0).

But in working backwards through the tree, we need to allow for early exercise option with $V = \max\{\exp(-0.05)[0.53V\text{up} + 0.47V\text{down}], 1000 - S\}$.

Time 0	Time 1	Time 2	Time 3
			0
		0	
	46.5		0
129.0		103.9	
	236.0		232.4
		410.7	
			547.6

Hence tree value of American option = 129.0

(iii)

(a)

Using the European option as control variate, a more accurate value for the American option will be:

	value of American option on tree
minus	value of European option on tree
plus	accurate value of European option

The accurate value of the European option can be found using Black-Scholes:

$S = 1000$, $T = 3$, $r = 5\%$, $\sigma = 0.25$, so $\sigma\sqrt{T} = 0.43301$

$$d_1 = \frac{\ln(S/1000) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} = (0.05 + 0.5 \times 0.252) \times 3 / 0.43301 = 0.56292$$

and $d_2 = d_1 - \sigma\sqrt{T} = 0.12991$

Then $N(-d1) = 0.28674$ and $N(-d2) = 0.44832$
so Put value = 1,000 [exp(–3 * 0.05) × 0.44832 – 0.28674] = 99.1

Hence a more accurate value for the American option would be:

129.0 – 119.2 + 99.1 = 108.9

(b)

The revised estimate probably <u>understates</u> the value of the option.

The option price can be thought of as being in two components: the European option plus the extra value from the American feature.

The European component is valued using Black-Scholes, so is accurate, but the American feature is valued using the tree.

In reality, the American option can be exercised continuously but the tree only allows it to be exercised at the end of the first and second years – rather like a Bermudan option.

This means the tree is allowing the option holder less flexibility, hence probably undervaluing the option.

This question asked the candidate to value a European and American Put on a binomial tree. This should have been familiar territory and led to strong responses, but disappointingly this was not the case. Possibly the fact that a tree calculation has not been asked in ST6 for a while might explain the less confident attempts at parts (i)(b) and (ii). This was particularly noticeable where candidates did not clearly set out how (or even whether) they were using the CRR methodology outlined in part (i)(a).

Part (iii) was generally well answered, showing that many candidates had reviewed the September 2010 paper that also referred to control variate techniques.

QUESTION 7
Syllabus section: (l) & (m)
Core reading: 15, 16

(i)

The long-term credit rating for a company or security is a single generic indicator that reflects their credit worthiness ...
... namely, its ability to pay principal and interest over a three- to five-year horizon.

The CRA assesses the probability of default of the issuer or security, and ascribes a code from a range as follows:
... from AAA (on the S&P scale, almost invulnerable to any future conditions) ...
... to BBB (adequately strong at present, but could weaken in some adverse economic conditions) ...
... to CCC (already vulnerable, needs a favourable economic climate to meet financial commitments)
and finally D (defaulted).

Ratings do not however generally take into account the amount of loss expected.

CRAs who calculate these ratings try to make them independent and objective, as well as consistent across the whole range of global ratings.

(ii)

(a)

The assumptions underlying the rating methodology were clearly wrong, e.g. correlation assumptions were too low. If the original rating of AAA/Aaa had been correct, the bonds would not have suffered through the credit crisis (AAA or Aaa $\Rightarrow$ invulnerable).

Normally, credit migration would have been much more gradual.

Many of the bonds had performed very badly after the high levels of defaults seen on US mortgages.

Also, there was higher future default risk as the underlying economics had deteriorated, e.g. house prices had fallen sharply, ability to pay had reduced due to rising unemployment.

The lack of trust in the accuracy of the ratings had led to criticism of the CRAs.

The CRAs were determined to try to regain credibility through making their ratings more realistic.

(b)

Looking at the various users of ratings:

- <u>Investors</u> had lost considerable amounts of money as their bonds had fallen in value ...
- ... also, their capital charges were based on these ratings so rose considerably.
- Investors with minimum credit rating mandates might find themselves in breach of mandate and have to sell bonds at distressed prices.
- Investors could lose trust in the CRAs whose ratings they had relied on.
- <u>Issuers</u> were unable to issue new bonds and the primary market closed.
- <u>Regulators</u> were suspicious that the CRAs had been too closely linked to the bond issuers and therefore not objective enough ...
- ... also, the credit crisis had caused considerable dislocation to economies and regulators were wanting to address weaknesses in the system.
- Regulators have become concerned about the impact on capital adequacy of institutions which held assets that were downgraded.

(c)

<u>Three</u> of the following should be discussed:

- responsiveness – CRAs should take action more quickly when conditions deteriorate
- surveillance – maintenance of existing ratings should become a higher priority, with more senior staff engaged
- consistency – ratings need to be more stable; CRAs should have declined to rate certain exotic structures that did not fit their ratings framework
- transparency of fee structure – rating agencies have been criticized for having too familiar a relationship with issuers of structured bonds, possibly opening themselves to undue influence or being misled
- scope – non-credit risks should be addressed in the rating process, e.g. liquidity or market risk
- different types of rating for structured bonds – a single indicator is too limited, so include more information on stability (quite hard to achieve)

- regulation and oversight – given their critical importance in the financial world, ensure CRAs are carefully monitored to enhance the wider good of the markets they serve

(iii)

(a)

The corporate would pay fixed rate on a fixed-floating interest rate swap of nominal £25m and term 5 years.

(b)

The bank is concerned that, if rates rise sharply, the corporate will not be able to afford the interest rate payments on the loan (which are floating).

The hedge would ensure that, should rates rise, the corporate's net liability is only the fixed rates on the swap, which the bank has assessed the corporate is currently able to finance.

(c)

If rates fell sharply, from the perspective of the bank the valuation of the swap would increase dramatically, from approximately zero to a large amount (say 25% of the total loan value).

This creates a large additional (unexpected?) credit risk that the bank now has to the corporate, as well as the risk of the loan itself.
(d)

The bank could purchase credit default swap (CDS) protection on the corporate names for which it has the most exposure ...
... but there may not be a good price for this in the market ...
... or it might have to be on a macro basis, since CDS prices do not exist for small to medium sized corporates.

Alternatively, the bank could ask for a Credit Support Annexe ...
... but this would then require a large cash margin from the corporate, which they might refuse as it could affect their credit worthiness.

The loan could be restructured.

This question drew on another recent addition to the Core Reading, this time relating to credit rating agencies (CRAs), a topical subject in global market conditions prevailing in 2011. Given that this was largely a bookwork question, responses to parts (i) and (ii) were surprisingly patchy, suggesting that candidates were familiar with the broad ideas behind the CRA debate but not all of the detail. A section that ascribes eight marks for bookwork really requires quite a large number of separate points.

The final part (iii) dealt with risk management issues surrounding the use of interest rate swaps to hedge corporate loans, also a topical subject. Generally this was well tackled and showed good understanding.

QUESTION 8
Syllabus section: (k)
Core reading: 14

(i)

(a)

Within each Monte Carlo run:

- the derivative payoff at each time t needs to be divided by the value of the numeraire asset at time t within that run to give a discounted value
- this is summed over all time-steps t
- the resulting figures are then averaged over all the runs

(b)

Compared with other simpler market models:

- LMM gives the user more flexibility in the choice of the volatility structure and its future evolution
- so should be more accurate
- it will cope well if there are complex interest rate options to value, especially those with path dependency ...
- ... or correlation-dependent options, for which it can easily be extended to multiple factors (sources of uncertainty)
- BUT it can only realistically be implemented using Monte Carlo simulation, which requires expert quantitative resource to implement
- so it needs more computer time than simpler models
- it is slower to run ...
- ... and more complex to calibrate
- the better fit of LMM can make the modeller complacent – when a simpler model does not cope so flexibly with unusual shapes in the yield curve, the modeller is driven to inquire <u>why</u> these are appearing

- hence LMM is often used for development / research rather than pricing

[*Other valid points may be made.*]

(ii)

(a) $\dfrac{P(t,t_i)}{P(t,t_{i+1})} = 1 + (t_{i+1} - t_i)F_i(t) = 1 + \delta_i F_i(t)$

[*The continuous version is usually an acceptable alternative, but here it was specifically not asked for.*]

(b)

Take logs of both sides in (a) to get:

$$\ln P(t,t_i) - \ln P(t,t_{i+1}) = \ln[1 + \delta_i F_i(t)] \qquad (*)$$

Since we have only one factor, the individual stochastic variation of each $P(t, ti)$ will be expressed in terms of the same underlying Brownian motion as the $Fi(t)$.

Differentiating, consider only the stochastic term for each of the three parts of (*).

(1) From the definition of $v_i(t)$, $dP(t,t_i)$ = (drift term)dt + $v_i(t)P(t,t_i)dz$

Using Ito's Lemma:

$$d \ln P(t,t_i) = \text{(drift term)}dt + v_i(t)P(t,t_i)\frac{\partial \ln P(t,t_i)}{\partial P(t,t_i)}dz = \text{(drift term)}dt + v_i(t)dz$$

(2) Similarly, $d \ln P(t,t_{i+1}) =$ (drift term)dt + $v_{i+1}(t)dz$

(3) $$d \ln[1+\delta_i F_i(t)] = \text{(drift term)}dt + \frac{\partial \ln[1+\delta_i F_i(t)]}{\partial F_i(t)}\zeta_i(t)F_i(t)dz$$

$$= \text{(drift term)}dt + \frac{\delta_i \zeta_i(t)F_i(t)}{1+\delta_i F_i(t)}dz$$

Hence equating the coefficients of dz in (*):

$$v_i(t) - v_{i+1}(t) = \frac{\delta_i \zeta_i(t)F_i(t)}{1+\delta_i F_i(t)}$$

The term $[v_{m(t)}(t) - v_{k+1}(t)]$ in the underlying SDE is the summation of successive $v(t)$ differences, starting from the next reset time with index value $m(t)$.

Hence $v_{m(t)}(t) - v_{k+1}(t) = \sum_{i=m(t)}^{k} \frac{\delta_i \zeta_i(t) F_i(t)}{1+\delta_i F_i(t)}$

which gives the required re-statement of the underlying SDE.

(iii)

The caplet volatilities can be thought of as the volatility of forward rates between today and the exercise date of the cap, whereas the $\zeta_k(t)$ can be thought of as the volatilities of the forward rates over each individual δ_i time-step.

Consider the time period t_k.

The variance of the forward rate according to the caplet price is $\sigma_k^2 t_k$.

The variance of the forward rate according to the LIBOR market model is the sum of the variances of each individual forward time step, i.e. $\sum_{i=1}^{k} \zeta_i^2(t)\delta_i$.

Equating variances: $\sigma_k^2 t_k = \sum_{i=1}^{k} \zeta_i^2 \delta_i$

and this set of equations is solved iteratively, starting with σ_1.

[*An alternative way of expressing this is to look at each time period in succession. Then* $\sigma_1^2 t_1 = \zeta_1^2 t_1$, *so* $\sigma_1 = \zeta_1$; $\sigma_2^2 t_2 = \zeta_1^2 t_1 + \zeta_1^2 (t_2 - t_1)$ *which implies* ζ_2; *and so on.*]

This question on use of the LIBOR market model was often left to the end, where time was short. It appeared difficult, perhaps due to the unfamiliarity of notation in parts (ii) and (iii), although it did actually consist of a large amount of bookwork. Overall it was not well answered.

Part (i) was a very familiar type of question, comparing one model approach with others, and should have led to a relatively easy set of marks. Candidates need not fear these questions – in preparation, they should map out the key features of each interest rate model in the syllabus and then draw on these to compare and contrast as required. (The same applies to techniques such as Monte Carlo, binomial and trinomial trees and finite differences.)

Part (ii) was challenging but, as the solution shows, the required result (PDE for forward rates in a rolling forward risk neutral world based on bond prices) can be derived using the stochastic terms only, ignoring the drift terms which are much more complicated. This is also covered by the Hull textbook in the section on the LIBOR market model, although Hull derives the result by using a change of numeraire.

Part (iii) was looking for a description of the technique of bootstrapping based on equating variances at each time step. Both parts (ii) and (iii) were marked generously if the right approach was attempted.

END OF EXAMINERS' REPORT