INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2018

Subject ST6 – Finance and Investment
Specialist Technical B

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Mike Hammer
Chair of the Board of Examiners
December 2018

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Finance and Investment Technical B subject is to instil in successful candidates the ability (at a higher level of detail and ability than in CT8) to value financial derivatives, to assess and manage the risks associated with a portfolio of derivatives, including credit derivatives and to value credit derivatives using simple models for credit risk.

2. This subject deals with mathematical techniques for the valuation and risk management of derivatives, together with some aspects of their practical application.

3. Different numerical answers may be obtained to those shown in these solutions, depending on whether figures were obtained from tables or from calculators. Allowance was made for these minor differences. However, candidates may have lost marks for using excessive rounding or showing insufficient working.

4. Derivative theory is an interesting but very exacting subject which needs to be tackled in a precise and disciplined manner. Questions that appear unfamiliar can, with some clear thinking at the start of the exam, be found to be straightforward examples of well-known techniques. Thorough preparation of the course material is essential, however. This subject also requires quite a varied approach in writing solutions. Algebraic and numerical content needs to be supplemented with well-reasoned but brief arguments where the question demands. Candidates should always try to provide several distinct relevant points for the discursive questions, not lengthy paragraphs developing a single idea.

5. Candidates who give well-reasoned points, not in the marking schedule, are awarded marks for doing so.

B. General comments on *student performance in this diet of the examination*

The paper was well attempted by many candidates. Suitably prepared candidates were able to score highly across all questions demonstrating the ability to apply their ST6 knowledge and techniques to unfamiliar situations. The better attempted questions were 4, 5 and 6.

In terms of areas for improvement:

- Some candidates were unable to demonstrate a breadth of knowledge across the whole syllabus and so did not score all of the available knowledge marks from the Core Reading, for example question 1 parts (i) and (ii).
- Many candidates did not appear to tailor their answer to the command words in the questions, such as "Justify", "Assess", or "Comment on", for example questions 3 (iv) or 5 (v).

- Many candidates made a number of small errors in algebraic steps, or failed to explain the change from one line to the next in the algebra, for example questions 4 (v) or 5 (iv).
- A number of candidates provided a significant amount of detail on relatively narrow arguments when responding to the discursive questions.
- Some candidates struggled with questions that required an element of application of Core Reading to situations that were not immediately familiar, so did not demonstrate the desired depth of knowledge.

The comments that follow the questions concentrate on areas where candidates could have improved their marks, in an attempt to help future candidates to revise accordingly and to develop their ability to apply the Core Reading to related situations. This skill is likely to be valuable both in exams and in using this theory in their professional careers.

C. Pass Mark

The Pass Mark for this exam was 60

Solutions

Q1

(i) When a stochastic process is simulated a large number of simulations is needed to model accurately the distribution of the underlying process. [1]

This requires considerable and expensive computational time and resources. [0.5]

Variance reduction techniques can be used to reduce the amount of computation time and resources. [1]
[Max 2]

(ii) There is no analytic formula for the arithmetic average Asian option so a simulation (or other numerical method) is required to value the option. [0.5]

As a simulation (or other numerical method) is required variance reduction procedures may be applicable (and useful). [0.5]

The control variate technique can be used as a variance reduction procedure in this case as there is a geometric average Asian option, which is similar to the arithmetic average Asian option, with … [1]

… the geometric average Asian having an analytic formula available. [0.5]

A random number stream is generated to run the simulation of the arithmetic average Asian option (or another appropriately described numerical method), obtaining an estimate F_A^* of the option. [0.5]

The same random number stream and time interval is also used to run a simulation of the geometric average Asian option (or the same numerical method set-up as for the arithmetic average Asian option), obtaining an estimate F_G^* of the option. [0.5]

The geometric average Asian option also has an analytic solution: F_G. [0.5]

The error in running the simulation (or other numerical method) for calculating the value of the geometric average Asian option is: $F_G - F_G^*$. [0.5]

This is assumed to be equal to the error in calculating the actual value of the arithmetic average Asian option (F_A) against the one calculated from the simulation (or other numerical method). [0.5]

This means $F_A - F_A^* = F_G - F_G^*$ or $F_A = F_A^* + \left(F_G - F_G^*\right)$. [0.5]

This is a better estimate for F_A than just F_A^* under the particular model used.

[0.5]

[Max 4]

(iii) Using part (ii), a better estimate is $10.30+(9.57-9.41)=10.46$. [1]

[Total 1]

[Total 7]

Candidates who were familiar with this area of the syllabus scored highly. This demonstrates the importance of studying the whole syllabus. Successful candidates often score well in less examined parts of the syllabus and are able to apply the concepts developed in these areas to a range of questions.

Q2

(i) When interest rates are high, there is low demand for funds from borrowers… [0.5]

… and so interest rates decline. [0.5]

When interest rates are low the demand for funds on the part of borrowers increases... [0.5]

… and so interest rates tend to rise. [0.5]

[Hull p684]

Central banks may pursue inflation targets via monetary policy, which could lead to interest rates mean reverting. [1]

An excessively high interest rate may dampen economic growth which may cause a cyclical decline of the interest rate. [0.5]

If interest rates turn negative investors will hold cash instead, reducing the demand for borrowing and also the effectiveness of monetary policy. [0.5]

[Max 3]

(ii) $P(t,T) = \exp(-R(t,T)(T-t))$ [0.5]

$$R(t,T) = -\frac{1}{T-t}\ln\left[P(t,T)\right]$$

[1]

$$R(t,T) = -\frac{\ln A(t,T)}{T-t} + \frac{\ln\left[\exp(B(t,T)r(t))\right]}{T-t}$$

[0.5]

$$R(t,T) = \frac{B(t,T)r(t)}{T-t} - \frac{\ln A(t,T)}{T-t}$$

[0.5]

[Max 2]

(iii) $R(t,T)$ is linearly dependent on $r(t)$. [1]

$r(t)$ determines the level of the term structure of the continuously compounded spot curve at time t. [1]

The general shape of the term structure at time t is independent of $r(t)$. [0.5]

[Max 2]

(iv) In a risk neutral world, all asset prices must have an expected return of the risk free rate. [0.5]

Allowing for mean reversion of bond prices would prevent free stochastic movement… [0.5]

… meaning the expected return could deviate from the risk free rate… [1]

… and imply arbitrage opportunities… [1]

… as bonds are tradable instruments (as opposed to interest rates). [1]

[Max 2]

[Total 9]

The arguments relating to mean reversion of financial variables are an important consideration in financial modelling. A minority of candidates scored highly in part (i) *and those that did typically gave more consideration to the role of central banks. The key to scoring highly was to consider the extremes of interest rates; what happens when they are low or high in terms of the economy.*

Part (ii) was a basic result relating to interest rates and these types of results should be familiar to a well prepared candidate. Only the top candidates were generally able to then use the result in (iii) and analyse the equation.

Questions have been asked in previous papers about the risk-neutral world directly. An understanding of what risk-neutral means, beyond the equations, is a fundamental concept in ST6 and it is important candidates spend time familiarising themselves with this. Those candidates who had done this were able to successfully tackle part (iv).

Q3

(i) Using Ito's lemma,

$\frac{dS^n(t)}{dS(t)} = nS^{n-1}(t)$, [0.5]

and $\frac{d^2S^n(t)}{dS(t)^2} = n(n-1)S^{n-2}(t)$. [0.5]

Combining these using Ito's lemma gives:

$$d(S^n(t)) = \left(nS^{n-1}(t) \times \mu S(t) + 0.5 \times (\sigma S(t))^2 \times n(n-1) \times S^{n-2}(t)\right)dt + \sigma S(t) \times nS^{n-1}(t) dW(t).$$

[0.5]

This can be simplified by noting the common factor of $S^n(t)$:

$d(S^n(t)) = S^n(t) \times \left(\left(\mu n + 0.5 \times \sigma^2 \times n(n-1)\right)dt + \sigma n dW(t)\right)$. [0.5]

This is also a geometric Brownian motion with respect to P satisfying:

$d(S^n(t)) = S^n(t)\sigma_n dW(t) + S^n(t)\mu_n dt$,

where: $\mu_n = \mu n + 0.5 \times \sigma^2 \times n(n-1)$, and [0.5]

$\sigma_n = \sigma n$. [0.5]

(There are alternative methods to getting the solution, for example looking at d (log(S^(n)(t)).)

[Total 3]

(ii) From the result in part (i), $S^n(t)$ also is geometric Brownian motion.

This can be integrated to give:

$S^n(t) = S^n(0)e^{\left(\mu_n - 0.5\sigma_n^2\right)t + \sigma_n W(t)}$. [1]

Taking the expectation:

$E_P\left[S^n(t)\middle|F(0)\right] = S^n(0)E_P\left[e^{\left(\mu_n - 0.5\sigma_n^2\right)t + \sigma_n W(t)}\right]$. [0.5]

The log of the distribution inside the expectation is a normal distribution with mean $(\mu_n - 0.5\sigma_n^2)t$ and variance $\sigma_n^2 t$. [0.5]

Using the formula for the expectation of a lognormal distribution gives:

$E_P\left[S^n(t)\middle|F(0)\right] = S^n(0)e^{\left(\mu_n - 0.5\sigma_n^2\right)t + 0.5\sigma_n^2 t}$. [0.5]

$= S^n(0)e^{\mu_n t}$

This simplifies to give: $E_P\left[S^n(t)\middle|F(0)\right] = S^n(0)e^{\left(\mu+0.5\times\sigma^2(n-1)\right)nt}$. [0.5]

(As in part (ii), alternative correct derivations are given full credit.)

[Total 3]

(iii) It was shown in part (i) that for all $n > 0$, $S^n(t)$ is geometric Brownian motion. This is the underlying model for stocks in the Black-Scholes model. [0.5]

Therefore for suitable chosen parameters μ, n and σ one can obtain a suitable stock model for all $n > 0$ (in as far as it suitable for the Black Scholes model). [0.5]

[Total 1]

(iv) **Pricing stock options**

By adopting a stock price model which is used by most other market participants then it is likely that it will price options closely to most other market participants. [1]

This is beneficial especially for over the counter trades in agreeing a price for any contract. [0.5]

Any differences in option prices will likely be as a result of calibration of the stock model to market conditions,… [0.5]
… for example implied volatility. [0.5]

The ability to calculate the Greeks in the new model is also important, as they will be useful in hedging and managing risks. [0.5]

Another aspect relates to the prevalence of the geometric Brownian motion stock model. Clearly, other stock models exist but whether they offer any advantages to the geometric Brownian motion model is not clear in this case. [0.5]

Although the geometric Brownian motion stock model and Black Scholes pricing model are not perfect they offer advantages such as ease of understanding the theory, analytical formulas for some option prices and ease of implementation. [0.5]

Any alternative stock and pricing model will need to provide strong advantages relative to the current model or address some of the weaknesses of the current model,… [0.5]

… for example the log-normal distribution assumed or volatility remaining constant over time. [0.5]

Any new stock model which might be considered more theoretically accurate, or better in some way to the current model, will still be calibrated to the markets. [0.5]

As a result of adjusting assumptions in any new model the likelihood is that the new model will price very close to the market prices, and hence be closely aligned to the current model. [0.5]

The reason for holding options is important, for example if the reason is speculation then identifying mispricing through the use of a new model could be hugely beneficial, … [0.5]
… although many investment managers use stock options for hedging and the model choice may be less of a concern as risk management is more important. [0.5]

From this argument it appears that there will an element of model convergence between market participants. [0.5]

It also suggests that any benefits of a new model are likely to be marginal and they need to be considered in the wider context of the investment manager company. [0.5]

The alternative to this argument is that the investment manager has confidence in any new model and can understand any deviation from market prices. [0.5]

General
For any new model proposed there are several factors which may also need to be considered:

- researching and testing the new model relative to market prices will incur time and costs;
- the cost and resources required for building and implementing any new model;
- the significance of stock options within the portfolio, if there is only a small proportion of stock options in the portfolio then updating a model may not be the best use of resources;
- the amount of stock forecasting done by the investment manager; and
- the risks involved in using a model out of line with other market participants and how this fits in with the risk tolerance of the investment manager.

[0.5 marks for each, max 1.5]
[Max 6]
[Total 13]

Most candidates scored highly in part (i). Many candidates were then able to apply part (i) to parts (ii) and (iii).

Part (iv) was answered well by very few candidates. There was a wide range of possible solutions but few candidates went beyond some basic advantages and disadvantages of the *current model, for example, it was typical to ignore the points made in the mark scheme under General. The implementation of financial models is an important consideration for many financial institutions. Candidates may want to think about the benefits and risks to a stakeholder in a financial institution in answering these types of question in order to score highly.*

Q4

(i) It is a long… [0.5]
… European… [0.5]
… put option on the vanilla bond… [0.5]
…with a term of 10 years, … [0.5]
… a nominal value of £100m [0.5]
… and a strike price of £90m [0.5]
[Sub-Total 2]

(ii) $100\exp\left[-r\times 5\right]=90$ [0.5]

$r=-\ln\left[\frac{90}{100}\right]/5=2.1072....\%\approx 2.1\%$ [0.5]

[Sub-Total 1]

(iii) It provides some protection to the investor from rising interest rates… [1]
… as if interest rates are higher than 2.1% after 10 years the investor can exercise the bond option… [0.5]
… and reinvest in other securities with higher interest rates. [0.5]
The bond option supports the value of the Complex Bond [0.5]
[Max 2]

(iv) $V=P(0,T)\left[KN(-d_2)-F_0N(-d_1)\right]$ [0.5]

Where $d_1=\frac{\ln(F_0/K)+\sigma^2T/2}{\sigma\sqrt{T}}$ and $d_2=d_1-\sigma\sqrt{T}$ [0.5]

Where: T = payment time of option, σ = volatility of the forward bond price, K = Strike Price. $P(0,T)$ is the price of a zero coupon bond at time zero that matures at time T. [0.5]
F_0= forward bond price observed at time zero [0.5]
$F_0=S_0e^{rT}$ [0.5]
S_0= current price of the bond [0.5]
[Max 2]

(v) V(Complex Bond) = V(Vanilla Bond) + V(Bond Option) [0.5]

V(Vanilla Bond) = $100e^{-0.02\times 15}=74.082$ [0.5]

V(Bond Option):
$F_0=100e^{-0.02\times 5}=90.484$ [0.5]

$d_1=\frac{\ln(90.484/90)+0.1^2\times 10/2}{0.1\sqrt{10}}=0.17507$ [0.5]

$d_2=0.17507-0.1\sqrt{10}=-0.14116$ [0.5]

$N(-d_1)=0.43051$ [0.5]

$N(-d_2) = 0.55613$ [0.5]

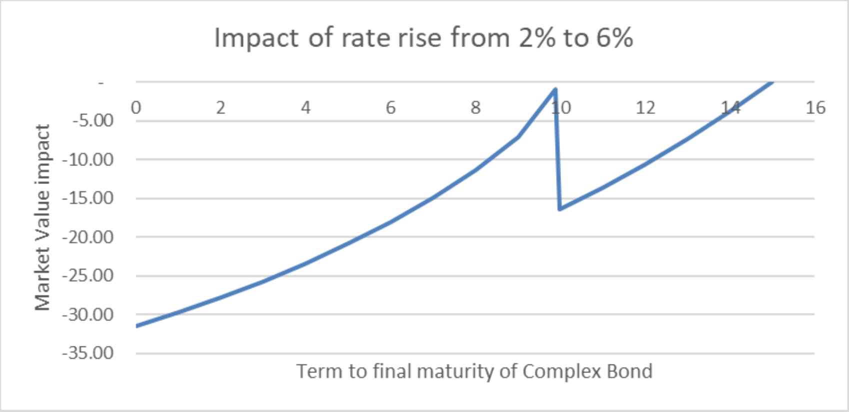
$P(0,T) = e^{-0.02\times 10} = 0.81873$ [0.5]

$V = 0.81873 \times [90 \times 0.55613 - 90.484 \times 0.43051] = 9.086$ [0.5]

V(Complex Bond) = $9.086 + 74.082 = 83.167 \sim £83m$ [0.5]

[Sub-total 5]

(vi) Actual chart below:



Appropriately labelled axes. [0.5]

Y-axis intercept at -c£30m. [0.5]

Stress impact reducing materially (close to zero) as we approach T=10 [1]

… with a concave shape. [0.5]

Stress impact spiking out once T=10 is passed following expiry of the option [0.5]

… specifically to around $-£100m \times (\exp[-0.02\times 5] - \exp[-0.06\times 5])$ ~-£16m [1]

… and then decaying to zero at T=15… [0.5]

…but an overall increasing shape. [0.5]

[Max 4]

[Total 16]

Parts (i) to (iii) were generally well answered. There were many mistakes made in answering part (iv); ranging from numerical errors in Black's formula to not valuing the bond.

These errors carried through to part (v) but were not penalised and hence part (v) was well answered.

There were some good attempts at sketching the chart but few candidates captured all the relevant features. A methodical approach is usually required to score well on these questions: plot any known points, look at extremes or asymptotes, consider the general shape (for example concave or linear), look for any turning points or points of inflexion and finally add labels to the axes.

Q5

(i) Both strategies short sell 1,000 shares at the current time for an income of $10.2 \times 1,000 = 10,200$. [0.5]

Let $S(1)$ be the value of the share price in one year.

For strategy 1, the 10,200 is invested in the risk-free one year bond,…

… which returns $10,200 \times 1.017 = 10,373.4$ in one year. [0.5]

The total profit is $10,373.4 - 1,000S(1)$. [0.5]

For strategy 2, $1,000 \times 2 = 2,000$ is invested in the call options,

… leaving $10,200 - 2,000 = 8,200$ to be invested in the risk-free one year bond,

… which returns $8,200 \times 1.017 = 8,339.4$ in one year. [0.5]

The call options return $1,000 \times \max(S(1) - 14, 0)$. [0.5]

If $S(1) \leq 14$ then from strategy 2, 0 return from the call options and a total profit of $8,339.4 - 1,000S(1)$, which is less than strategy 1. [0.5]

If $S(1) > 14$ then from strategy 2, a return of $1,000 \times (S(1) - 14)$ from the call options and so a total profit of $8,339.4 - 14,000 = -5,660.6$. [0.5]

Therefore strategy 2 produces the greater profit when $-5,660.6 > 10,373.4 - 1,000S(1)$ or $S(1) > 16.0$, rounded to 1 decimal place.
[0.5]
[Total 4]

(ii) Dividends have the effect of reducing the stock price on the ex-dividend date.
[0.5]

This is bad news for the value of call options as the delta of a call option is positive, … [0.5]

… that is, a reduction in stock price will reduce the value of the call option.
[0.5]

This is good news for the value of put options as the delta of a put option is negative, … [0.5]

… that is, a reduction in stock price will increase the value of the put option.
[0.5]

If the stock goes ex-dividend after expiry of the options it has no impact on their price. [1]

[Max 2]

(iii) The net amount invested in the product after the deduction of the bank's fee is: $P \times (1 - x\%)$. [0.5]

The share return over the year is $S(1)/S(0)$ and only 99% of this is returned to the investor, hence an adjusted share return of $0.99S(1)/S(0)$. [0.5]

This is subject to a minimum guaranteed return of 2%, hence the percentage return is $Max\{0.99S(1)/S(0), 1.02\}$. [1]

This results in the stated equation.

[Total 2]

(iv) The payoff to the trader at time 1 can be written as follows:

$$P \times (1 - x\%) \times Max\{0.99S(1)/S(0), 1.02\} = \frac{P \times (1 - x\%) \times 0.99}{S(0)} \times Max\left\{S(1), {}^{1.02 \times S(0)}\!\big/_{0.99}\right\}.$$

[0.5]

Letting $K = {}^{1.02 \times S(0)}\!\big/_{0.99}$ and some further manipulation gives:

$$P \times (1 - x\%) \times Max\{0.99S(1)/S(0), 1.02\} = \frac{P \times (1 - x\%) \times 0.99}{S(0)} \times \left(K + Max\{0, S(1) - K\}\right).$$

[0.5]

The term $Max\{0, S(1) - K\}$ is the payoff at time 1 of a European call option with strike price K. [0.5]

The expected payoff to the trader discounted to time 0 of:

$$e^{-r}E\left[\frac{P \times (1 - x\%) \times 0.99}{S(0)} \times \left(K + Max\{0, S(1) - K\}\right)\right].$$ [0.5]

This simplifies to: $\frac{P \times (1 - x\%) \times 0.99}{S(0)} \times \left(e^{-r}K + e^{-r}E\left[Max\{0, S(1) - K\}\right]\right)$. [0.5]

Here, under the risk neutral valuation: $e^{-r}K = E\left[e^{-r}K\right]$, … [0.5]

… and the second term is the Black Scholes price of the call option described above, which will be denoted by c. [0.5]

Using the formula for the Black Scholes valuation of the call option with the following parameters:

$S(0) = 10.2$, $K = {}^{1.02 \times 10.2}\!/_{0.99} = 10.509...$.

$\sigma = 0.25, q = 0, r = 1.5\%, t = 1$ where σ, q, t are the volatility, dividend yield and time to expiry respectively. [0.5]

The Black Scholes formula gives: $c = S(0)N(d_1) - Ke^{-r}N(d_2)$.

with $d_1 = \dfrac{\ln(10.2/10.509...) + (0.015 + 0.25^2/2)}{0.25} = 0.06558...$, and [0.5]

$d_2 = 0.06558... - 0.25 = -0.18441...$. [0.5]

$N(.)$ is the standard normal cumulative distribution function.
$N(d_1) = 0.52614...$ and $N(d_2) = 0.42684...$. [1]

Putting these values in c gives
$c = 10.2 \times 0.52614... - 10.509... \times e^{-0.015} \times 0.42684...$ and $c = 0.9477...$. [0.5]

From the bank's perspective the cashflows are:
$+P \times x\%$ at time 0 as its fee,
$+P \times (1 - x\%)$ is invested in the product with an expected return in time 0 money of $\dfrac{P \times (1 - x\%) \times 0.99}{S(0)} \times \left(e^{-r}K + c\right) = 1.0967... \times P \times (1 - x\%)$, which is paid back to the investor.

The equation of value is:
$3\% \times P = P \times x\% - (1.0967... - 1) \times P \times (1 - x\%)$. [1]

This gives $x\% = 11.6\%$ to 1 decimal place. [0.5]
[Total 6]

(v) The product has the potential to generate an expected risk-neutral profit for the bank of around 3% of the initial investment. [0.5]

This is because the bank can put in place a hedging strategy. [0.5]

This can be determined from part (iv). Writing the payoff to the trader as:

$$\frac{P \times (1 - x\%) \times 0.99}{S(0)} \times Max\left\{S(1), {}^{1.02 \times S(0)}\!/_{0.99}\right\} = 0.9027P + 0.0858P \times Max\{S(1) - 10.509, 0\}.$$

[0.5]

This is a zero coupon paying $0.9027P$ in one year and $0.0858P$ European call options with a strike price of 10.509. This is an obvious hedging strategy the bank can adopt. [0.5]

This has a cost of $e^{-0.015}0.9027P + 0.0858P \times 0.9411 = 0.97P$, which is as expected due to the condition of 3% profit. [0.5]

The expected risk-neutral profit will be less than 3% when expenses are allowed for. [0.5]

There are alternative hedging strategies which could be adopted but may carry more downside risk. [0.5]

An example of a risky hedging strategy is given below:

The bank makes an immediate profit of 11.5% of the initial investment. [0.5]

However, if a basic hedging strategy of using the remaining 88.5% to buy the shares would be profit neutral if the share price rises by more than 2% as it only pays back 99% of this rise with the remaining 1% probably being used to cover transaction costs of the buying and selling of the shares in the open market. This leaves the 11.5% of the initial investment as profit. [1]

This hedging strategy does not cover any significant fall in share price which could result in overall large losses. [0.5]

Other factors which may impact on profitability include:

- the marketability of the product;
- competitors and the depth of the market for this product;
- the cost of capital;
- the capital requirements;
- the state of the economy; and
- the sales method (e.g. the cost of any commission). [0.5 each, max 1.5]

[Max 3]

[Total 17]

Parts (i), (ii) and (iii) were some of the best answered question parts on the paper.

Part (iv) was attempted by many candidates. Most were able to value an option but sometimes this was the wrong option due to mistakes in algebraic manipulation. Candidates often found it challenging to correctly set up the equation of value. Some application questions involve an equation of value. Taking time to work on this sets the foundations for answering the question.

Very few candidates scored many marks on part (v). An approach to answering these type of questions is to consider some of the key words in the question: profit generation and hedging strategy. These can then be used to generate ideas. For example this could include the type of hedging strategy, the costs of it or the risks associated with it. It is important to generate lots of diverse points to score well.

Q6

(i) C depends on the value of the underlying stock (denoted by S), the volatility of the underlying stock (denoted by σ), the calendar time (denoted by t) and the risk-free interest rate (denoted by r). [1]

Assuming that Δt is small then expanding ΔC using a Taylor expansion gives: [0.5]

$$\delta C = \frac{\partial C}{\partial S}\delta S + \frac{\partial C}{\partial t}\delta t + \frac{\partial C}{\partial \sigma}\delta\sigma + \frac{\partial C}{\partial r}\delta r + 0.5\frac{\partial^2 C}{\partial S^2}\left(\delta S\right)^2. \quad [1]$$

Writing this in terms of the Greek symbols: $\Delta = {\partial C}/{\partial S}$, $\Gamma = {\partial^2 C}/{\partial S^2}$, $\Theta = {\partial C}/{\partial t}$, $\nu = {\partial C}/{\partial \sigma}$ and $\rho = {\partial C}/{\partial r}$. [1]

$$\delta C = \Delta\delta S + \Theta\delta t + \nu\delta\sigma + \rho\delta r + 0.5\Gamma\left(\delta S\right)^2. \quad [0.5]$$

[Max 3]

(ii) Part (i) showed that C depends on four variables and hence there are four columns to be completed in the table. [0.5]

	Value of stock	Volatility	Time to expiry	Risk-free rate
Variable increases	+	+	+	+
Variable decreases	-	-	-	-

[0.5 marks for each correct entry]
[Max 4]

(iii) From the table in part (ii) if the stock price decreases then C should decrease. [0.5]

The table is based on only one variable changing and not several, … [0.5]

…therefore other variables must have had a total effect of increasing C more than the effect of the stock price decreasing. [0.5]

The time to expiry has decreased over this period and this will have further decreased C. [0.5]

As a result the volatility and risk-free rate are the only variables which can increase C. [0.5]

A possible explanation is that as the stock price has decreased the volatility of the stock has increased by a sufficient amount to offset the fall in stock price. [1]

Another possible explanation is that there has been a rise in the risk-free rate, … [0.5]

… which may have reduced the demand for stocks in general. [1]
[Max 3]

(iv) Using the hint in the question, r, K, S_t and σ are set to 1. These simplify the formula for theta to:

$$\Theta = -\frac{N'(3\sqrt{T}/2)}{2\sqrt{T}} - e^{-T}N(\sqrt{T}/2).$$ [0.5]

Using the Formula and Tables book the expression for N' is:
$N'(3\sqrt{T}/2) = 1/\sqrt{2\pi}\, e^{-9T/8}$.

This gives an expression for theta: $\Theta = -\frac{e^{-9T/8}}{2\sqrt{2\pi T}} - e^{-T}N(\sqrt{T}/2)$. [0.5]

The following analysis is not intended to be mathematically rigorous but rather to provide an understanding of the shape of the graph.

As $T \to \infty$, $e^{-T}N(\sqrt{T}/2) \to 0$ as $N(\sqrt{T}/2)$ has an upper bound of 1 and $e^{-T} \to 0$, … [0.5]

… and $\frac{e^{-9T/8}}{2\sqrt{2\pi T}} \to 0$. Hence, $\Theta \to 0$. [0.5]

As $T \to 0$, $e^{-T}N(\sqrt{T}/2) \to 0.5$ as $N(\sqrt{T}/2) \to 0.5$ and $e^{-T} \to 1$. [0.5]

In contrast, $-\frac{e^{-9T/8}}{2\sqrt{2\pi T}}$ tends to $-\infty$ as it is dominated by the $1/\sqrt{T}$ term. [0.5]

Hence, as $T \to 0$, $\Theta \to -\infty$. [0.5]
[Max 3]

(v) In general the shape of the graph is approximately of the form $-e^{-T}$. [0.5]

An immediate observation is that using part (iv), Θ is negative for all T. [0.5]

Further, the asymptotic behaviour of Θ can be determined from (iii).

Differentiating the expression for Θ with respect to T gives:

$$\frac{d\Theta}{dT} = \frac{9e^{-9T/8}}{16\sqrt{2\pi T}} + \frac{e^{-9T/8}}{4\sqrt{2\pi T^3}} + e^{-T}N(\sqrt{T}/2) - e^{-T}\frac{dN(\sqrt{T}/2)}{dT}. \qquad [0.5]$$

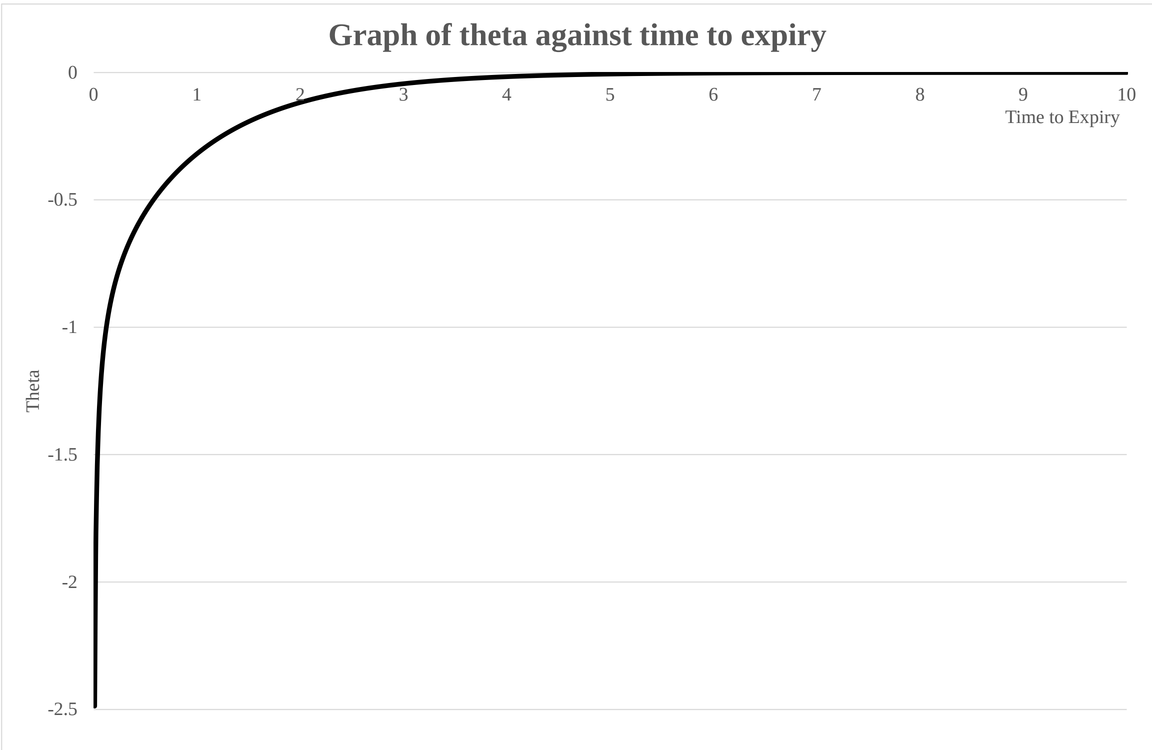
This can be simplified using the expression for N' and the product rule:

$$\frac{d\Theta}{dT} = \frac{9e^{-9T/8}}{16\sqrt{2\pi T}} + \frac{e^{-9T/8}}{4\sqrt{2\pi T^3}} + e^{-T}N(\sqrt{T}/2) - \frac{e^{-9T/8}}{4\sqrt{2\pi T}}. \qquad [0.5]$$

This simplifies to: $\dfrac{d\Theta}{dT} = \dfrac{5e^{-9T/8}}{16\sqrt{2\pi T}} + \dfrac{e^{-9T/8}}{4\sqrt{2\pi T^3}} + e^{-T}N(\sqrt{T}/2)$. [0.5]

As each term is positive, $\dfrac{d\Theta}{dT} > 0$ and Θ is monotonically increasing. [0.5]

This enables the graph to be drawn:



[0.5 marks for theta being negative, 0.5 marks for the general shape of $-e^{-T}$, 0.5 marks for asymptotic behaviour for small T, 0.5 marks for the correct labelling of axes, 0.5 marks for asymptotic behaviour for large T and 0.5 marks for monotonically increasing shape.]

[Max 3]

(vi) Using the graph in part (iv) shows that the option will lose money the closer it gets to expiry (and in fact accelerates in loss if it stays at the money). [0.5]

If the investor chose to hedge against theta then effectively the investor would be hedging against the passage of time. [0.5]

In one sense there is no uncertainty in the underlying variable of time to expiry as it will get smaller at a constant rate, … [0.5]
… where as an underlying variable like the underlying asset price has uncertainty. [0.5]

Therefore it is a foreseeable risk and nothing can stop time elapsing. The investor is unlikely to hedge against theta,… [0.5]

but instead will likely monitor the theta risk exposure. [0.5]
[Max 2]
[Total 18]

Parts (i) and (ii) were well answered and candidates demonstrated an understanding of the Greeks, even though they were in a slightly different setting.

There were some good initial statements for part (iii) but these were not generally developed to score more than around half the available marks. Often in questions previous parts can be useful to generate ideas. In this case part (ii) was helpful in understanding some of the possible independent factors and the limitations (for example joint movements of variables). A similar situation occurs in using the graph in part (v) to answer part (vi).

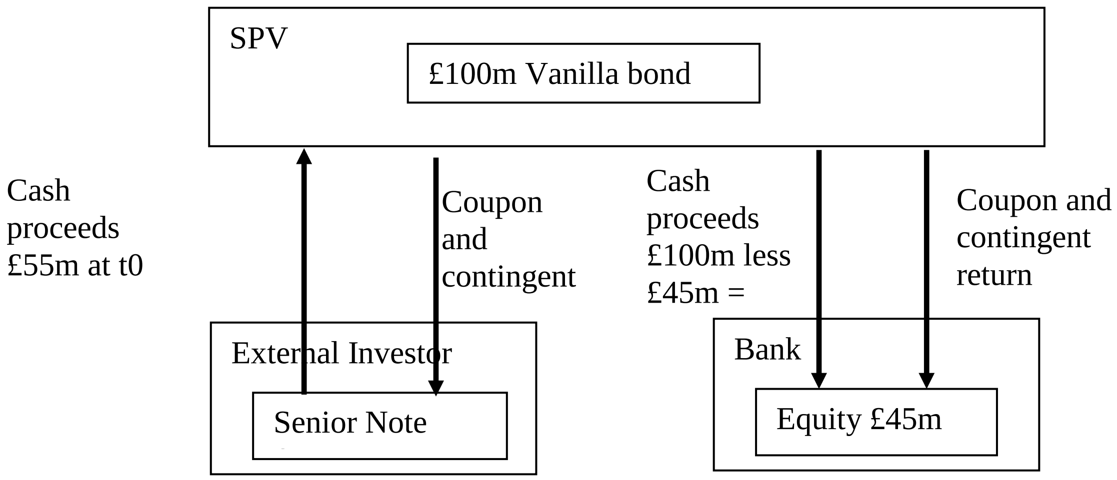
Parts (iv) and (v) were well attempted. It should be noted that the above solutions to (iv) and (v) provide more detail than was required to score full marks. Full marks for (v) could be obtained by just drawing the graph.

For higher order questions the development of basic facts by considering items like reasons why, how or implications from an investor's perspective enable full marks to be scored. For example, a candidate may state that an investor is unlikely to hedge against time to expiry but then does not examine why or the implications of not hedging.

Q7

(i) Credit ratings represent a simple comparator statistic that summarises a diverse set of input data. [1]
They provide an indicator of the financial strength of an institute issuing a fixed interest security. [0.5]
They can also provide a comparison of risk involved in an investment. [0.5]
It can potentially reduce the resources of an investor in researching an issuing institute by relying on the research of the credit rating agency. [0.5]
[Max 1]

(ii)



Suggest mark scheme:
0.5 marks for each of the SPV, External Investor and Equity boxes (i.e. up to 1.5 marks).
0.5 marks for each cashflow line and description (i.e. up to 2 marks).
[Max 3]

(iii) If the bond defaults, then the equity tranche will be absorb the first £45m of losses… [0.5]
… and the equity tranche is held by the bank. [0.5]
For example, if the bond default, with a £70m loss, then the Senior Note will only lose £25m. [0.5]
[Max 1]

(iv) (a) On immediate default, the losses on the Vanilla Bond will be (1-40%) x £100m = £60m. [0.5]
The equity tranche will absorb £45m of these… [0.5]
… so the losses on the Senior Note will be £60m - £45m = £15m. [0.5]

(b) If interest rates rise by 2%, then the Vanilla Bond will fall in value to approximately £100m x (1-10y*2%) = £80m (not allowing for convexity as given in the question). [0.5]
On subsequent immediate default the Vanilla Bond will be worth 40%*£80m = £32m… [0.5]

… resulting in losses of £68m, the equity tranche will absorb £45m of these… [0.5]
… and so the loss on the Senior Note will be £68m - £45m = £23m. [0.5]

(c) The Senior Note passes the "AA" threshold on Criteria 1… [0.5]
… but only passes the "A" threshold on Criteria 2… [0.5]
… hence the lower of the two ratings is selected, i.e. "A" [0.5]
[Max 4]

(v) The rating approach does not explicitly consider the probability of default of the Vanilla Bond. [1]
This could be assessed, for example, by forward-looking analysis of the issuer of the Vanilla Bond… [0.5]
… and the macroeconomic environment. [0.5]

No allowance is made for a number of risk factors including…
…liquidity risk. [1]
…operational risk. [0.5]
…variability in the recovery rate [0.5]

The rating agency does not appear to provide granular "notched" ratings (e.g. A- to A+) [0.5]
No allowance is made for convexity when calculating the interest rate stress. [0.5]
A "AAA" rated Senior Note can still suffer significant losses on immediate default. [0.5]
[Max 3]

(vi) Operational changes will need to be made to allow the execution of an interest rate swap within the SPV… [0.5]
… for example, an ISDA Master Agreement will need to be set up. [0.5]

Interest rate swaps are required to be cleared as they are standardised OTC derivatives… [0.5]
… so the SPV will need to post initial margin and variation margin. [0.5]
This will have material liquidity implications for the SPV… [1]
… as it doesn't currently have any readily available collateral… [0.5]
… especially not the cash and gilts likely to be required by a CCP. [0.5]

The addition of an interest rate swap will introduce counterparty risk… [0.5]
… although this may be mitigated by the posting of collateral above. [0.5]

The interest rate swap has a shorter duration than the Vanilla Bond, which means that the net interest rate sensitivity of the structure may increase over time... [0.5]
… which could cause the Senior Note to be downgraded… [0.5]
… unless the hedge was rebalanced. [0.5]

[Max 4]

(vii) The Senior Note already passes the "AA" threshold on Criteria 1… [0.5]
… so from (iv)(c) one just needs to reduce the losses in the 2% interest rates "up" scenario by £23m - £20m = £3m. [0.5]
The notional of the swap should therefore be £3m / 2% / 5 years = £30m. [1]
This makes no allowance for convexity, consistent with the CRA's approach [0.5]
The provider requires a profit, so there will be lower expected payouts. [0.5]
If the bond defaults, the swap will make unwinding the SPV more complex as the swap payments will still need to be made. [0.5]
[Max 2]

(viii) In order to achieve a AAA rating, the losses on the Senior Note need to be reduced under both criteria. [0.5]
This could be achieved by reducing the size of the Senior Note. [1]
From (iv), the losses on the Senior Note are £15m and £23m under the current structure under criteria 1 and 2, which are £9m and £13m respectively in excess of the AAA loss thresholds… [0.5]
…therefore the Senior Note should be reduced in size by at least £13m, i.e. to at most £42m. [1]
[Max 2]
[Total 20]

Parts (i) to (iv) were generally well answered. There were areas for improvement, for example in part (i) candidates summarised the uses of credit ratings rather than their benefits.

Parts (v) to (viii) were generally poorly answered. Partly this appears to reflect lack of time at the end of the exam, as most left this question to the end, and partly due to candidates finding this part of the question challenging. Many of the approaches in answering these types of question have been covered in previous examinations.

END OF EXAMINERS' REPORT