INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

9 October 2015 (am)

Subject ST6 – Finance and Investment
Specialist Technical B

Time allowed: Three hours

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You have 15 minutes before the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*
3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all seven questions, beginning your answer to each question on a new page.*
6. *Candidates should show calculations where this is appropriate.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 (i) State the conditions for a continuous process to be a martingale with respect to a probability measure $\mathbb{P}$. [1]

Let X be a continuous martingale process with respect to a probability measure $\mathbb{P}$, with $X_t > 0$ for all time t. Fix time $T > 0$ and let the Radon-Nikodym derivative of $\mathbb{Q}$ with respect to $\mathbb{P}$ up to time T be $\frac{d\mathbb{Q}}{d\mathbb{P}} = X_T$, where $\mathbb{P}$ and $\mathbb{Q}$ are probability measures.

Let $Y_t = 1/X_t$.

(ii) Demonstrate that $(Y_t)_{0 \le t \le T}$ is a martingale under $\mathbb{Q}$, with $Y_t > 0$. [4]

Suppose that X also satisfies the following equation: $dX_t = X_t \sigma_t dW_t$, where W is Brownian motion under $\mathbb{P}$.

(iii) Demonstrate that there exists a $\mathbb{Q}$-Brownian motion Z such that $dY_t = Y_t \sigma_t dZ_t$. [3]

[Total 8]

2 A first order Greek is a first order partial derivative of the price of a portfolio of derivatives, with respect to a variable which determines the price.

(i) Describe how the first order Greeks are used in the risk management of a portfolio of derivatives. [5]

(ii) Discuss the advantages and disadvantages of using the first order Greeks in the risk management of a portfolio of derivatives. [4]

Higher order Greeks are second and higher order partial derivatives of the price of a portfolio of derivatives, with respect to variables which determine the price.

(iii) Describe the benefits of using higher order Greeks in the risk management of a portfolio of derivatives. [2]

[Total 11]

3 (i) Describe the following credit derivatives:

(a) credit default swap (CDS)
(b) third-to-default basket swap

[4]

The "CDS-bond basis" for a particular corporate bond is defined as the CDS spread minus the excess of the bond yield over the risk-free rate.

(ii) Describe the arbitrage opportunity that should theoretically exist if the CDS-bond basis is positive and the bond is trading at par. [2]

(iii) Suggest reasons why this arbitrage opportunity may not exist in practice. [3]

A corporate bond asset manager has a fund that currently holds ten different corporate bonds in similar proportions, a small amount of cash and no other assets.

The manager wishes to introduce credit derivatives into the fund, with the objective of partially reducing credit risk in the portfolio for a limited cost.

She is considering two alternative derivative transactions:

- purchasing credit default swaps for five of the corporate bonds

- purchasing a third-to-default basket swap, where the basket comprises the ten holdings

(iv) Compare the likely effectiveness of these transactions in meeting the manager's objective. [4]

(v) Recommend one of these transactions, with justification. [1]

[Total 14]

4 A bank has developed a binary interest rate floor product which provides a £1m payoff to the purchaser if the short rate of interest is below 1% at maturity in T years, and nothing otherwise. The bank intends to sell separate products for a number of different maturity terms T.

The bank wishes to use a no-arbitrage interest rate model that will accurately and efficiently calculate the value of the products.

(i) Contrast a no-arbitrage interest rate model with an equilibrium interest rate model. [2]

(ii) Write down a general formula that the bank can use to value an interest rate derivative that provides a payoff of $X(T)$ at time T, defining all terms. [2]

The bank is considering using the Vasicek model to value the interest rate floor product. This model has the following stochastic differential equation for the short rate r_t:

$$dr_t = a(b - r_t)dt + \sigma dz$$

where r_t is the short rate at time t and a, b and σ are parameters of the model which have been calibrated by the bank.

Using this model, the distribution of the short rate after t years, r_t, is normal with a mean and variance under the risk-neutral measure of:

$$E_Q[r_t \mid r_0] = b + e^{-at}[r_0 - b]$$

$$Var_Q[r_t \mid r_0] = \frac{\sigma^2}{2a}\left[1 - e^{-2at}\right]$$

The current values of the parameters are:

$r_0 = 0.07$ and $a = 0.04$, $b = 0.05$, $\sigma = 0.03$.

(iii) Calculate the risk-neutral probability (to the nearest one percent) of the derivative being in the money at maturity at $T = 1$ year, using the above parameter values and showing all workings. [3]

(iv) Explain how the probability of the derivative being in the money at maturity will change as the maturity of the product, T, increases beyond one year. [2]

The bank is considering using an alternative interest rate model with the following stochastic differential equations:

$$d\ln(r_t) = \left[\theta(t) + u_t - k_1\ln(r_t)\right]dt + k_2 dz_1$$

$$du_t = -k_3 u_t dt + k_4 dz_2$$

where $k_1, ..., k_4$ are constants and $\theta(t)$ is chosen to make the model consistent with the initial term structure of interest rates.

(v) Discuss the advantages and disadvantages of using this alternative model for valuing the derivative, compared to using the Vasicek model. [5]

[Total 14]

5 Sam is planning to purchase his first house in two years' time. The type of house that he wishes to buy currently costs around $250,000 in his preferred area. Sam has already saved $20,000 to use in the purchase and this is held in cash.

Following recent house price gains in the country, Sam is concerned that the price will rise above $250,000 and so become unaffordable. He is therefore considering using property derivatives to hedge the risk of not being able to afford what he wishes to buy.

(i) Describe the property derivatives which might be available. [4]

Sam is concerned that using property derivatives would introduce basis risk.

(ii) Define basis risk in this context. [1]

(iii) Explain the extent to which basis risk would be introduced if Sam were to use the property derivatives outlined in part (i). [2]

(iv) Outline other potential disadvantages for Sam of using property derivatives.[4]

(v) Suggest ways, other than using property derivatives, in which Sam could mitigate property price rises over the next two years. [3]

[Total 14]

6 You should ignore credit risk throughout this question.

Let F_0 be the forward price of a plain vanilla bond with current price S_0. The bond provides a known income with present value I during the life of the forward contract. Let r be the risk-free rate (continuously compounded) and T the term of the forward contract.

(i) Show, by the principle of no arbitrage, that the forward price is given by:

$$F_0 = (S_0 - I)e^{rT}\,. \qquad [3]$$

A callable bond provides the issuer of a bond with the option to redeem or "call" the bond at par at a single call date before the final maturity of the security. At the call date:

- If the issuer decides to call the bond, the issuer pays the par value to the investor and no further cashflows are paid.
- If the issuer decides not to call the bond, the issuer will continue to pay the scheduled coupons and the final redemption payment.

Bank ABC has recently issued a callable bond with the following features:

- 5% coupon payable annually in arrears, redeemable at par, with a final maturity term of 20 years.
- The issuer has the option to call the security at time t = 10 years by paying the par value of 100 back to the investor.

From the holder's perspective, the callable bond can be decomposed into a long position in a vanilla bond with a maturity term of 20 years and a bond option.

(ii) Outline the features of this bond option. [1]

Risk-free interest rates are 3% p.a. continuously compounded at all terms and the volatility of the forward bond price is 30%.

(iii) Show that the price of the vanilla bond is 128.96.

[Hint: You may use the fact that an annuity of term n payable annually in arrears and discounted at a continuously compounded rate of r is given by $a_{\overline{n}|} = \dfrac{1-e^{-rn}}{e^{r}-1}$.] [1]

(iv) Calculate the price of the bond option using Black's formula. [5]

(v) Calculate the price of the callable bond. [1]

(vi) Describe the limitations of the Black model used in part (iv) in determining the price of the bond option. [3]

(vii) Sketch a graph which shows how the prices of the following depend on interest rates:

- the vanilla bond
- the bond option
- the callable bond

[2]

A pension fund's liabilities consist of cashflows that exactly match the profile of the vanilla bond. The pension fund is considering investing in the callable bond issued by Bank ABC.

(viii) Discuss how effective the callable bond would be at hedging the pension fund against changes in interest rates over time. [3]
[Total 19]

7 (i) (a) Describe the properties of barrier options.

(b) Explain why an investor would consider using barrier options.
[6]

(ii) Determine a portfolio of barrier options which is equivalent to a European plain vanilla call option. You should ignore transaction costs. [2]

A bank has purchased a one-year down-and-out barrier put option which is European plain vanilla on expiry but which has an American barrier.

(iii) Describe the possible payouts from this option during its lifetime. [2]

This barrier option is written on the Euro/dollar exchange rate. The strike price is 0.77 Euros and the barrier is 0.70 Euros. The nominal is 20 million Euros and the remaining time to expiry is one month.

(iv) (a) Explain how to construct a graph of the delta of this barrier option against the exchange rate at this duration.

(b) Sketch this graph of the delta of the barrier option.
[6]

The bank decides to delta hedge its position.

(v) Discuss the problems the bank may encounter in maintaining this hedge, including the effect on currency markets. [4]
[Total 20]

END OF PAPER