INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS’ REPORT

September 2017

Subject ST6 – Finance and Investment
Specialist Technical B

Introduction

The Examiners’ Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners’ preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Luke Hatter
Chair of the Board of Examiners
December 2017

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Finance and Investment Technical B subject is to instil in successful candidates the ability (at a higher level of detail and ability than in CT8) to value financial derivatives, to assess and manage the risks associated with a portfolio of derivatives, including credit derivatives and to value credit derivatives using simple models for credit risk.

2. This subject deals with mathematical techniques for the valuation and risk management of derivatives, together with some aspects of their practical application.

3. Different numerical answers may be obtained to those shown in these solutions, depending on whether figures were obtained from tables or from calculators. Allowance was made for these minor differences. However, candidates may have lost marks for using excessive rounding or showing insufficient working.

4. Derivative theory is an interesting but very exacting subject which needs to be tackled in a precise and disciplined manner. Questions that appear unfamiliar can, with some clear thinking at the start of the exam, be found to be straightforward examples of well-known techniques. Thorough preparation of the course material is essential, however. This subject also requires quite a varied approach in writing solutions. Algebraic and numerical content needs to be supplemented with well-reasoned but brief arguments where the question demands. Candidates should always try to provide several distinct relevant points for the discursive questions, not lengthy paragraphs developing a single idea.

5. Candidates who give well reasoned points, not in the marking schedule, are awarded marks for doing so.

B. General comments on *student performance in this diet of the examination*

1. Candidates generally found this paper challenging, but well prepared candidates scored above the pass mark.

2. In terms of areas for improvement:

   - Some candidates were unable to demonstrate a breadth of knowledge across the whole syllabus and so did not score all of the available knowledge marks from the Core Reading.
   - Many candidates did not appear to tailor their answer to the command words in the questions, such as "Justify", "Assess", or "Comment on".
   - Many candidates made a number of small errors in algebraic steps, or failed to explain the change from one line to the next in the algebra.
   - A number of candidates provided a significant amount of detail on relatively narrow arguments when responding to the discursive questions.
   - Some candidates struggled with questions that required an element of

application of Core Reading to situations that were not immediately familiar, so did not demonstrate the desired depth of knowledge.

- The comments that follow the questions concentrate on areas where candidates could have improved their marks, in an attempt to help future candidates to revise accordingly and to develop their ability to apply the Core Reading to related situations. This skill is likely to be valuable both in exams and in using this theory in their professional careers.

C. Pass Mark

The Pass Mark for this exam was 60.

Solutions

Q1 (i)

An ISDA master agreement ensures "close-out netting" is used… [1]
… this ensures that all derivatives are amalgamated together for offset when there is a credit event (such as a default). [½]
Even if close-out netting is agreed, there could still be a financial loss on default. [½]
To help mitigate the credit risk the ISDA also offers a credit support annex ("CSA") to cover the posting of collateral. [½]
If the contracts were not traded under an ISDA, a liquidator could choose to honour contracts with positive value to the counterparty yet default on those with positive value to other firms. [½]

An ISDA master agreement is the most efficient way of preparing to trade derivatives… [1]
… since every subsequent transaction can sign a single document that has been signed once… [½]
… and each product type is standardised in key aspects… [½]
… such as the protocol on the expiry of options / how to cope with valuation differences after default *(please award for any appropriate example)* [½]
Trade confirmations are therefore quite brief, so are easier to produce and check … [½]
… thus reducing operational risk. [½]
ISDA agreements have been tested in court, giving market participants greater confidence in their legal effectiveness. [½]

[Maximum 3]

(ii)

The fixed interest securities could be restricted to only …
… include high credit ratings / investment grade… [½]
… or just include government bonds… [½]
… include securities up to a particular duration… [½]
… include securities in certain currencies/local currency… [½]
… include certain amounts in particular sectors… [½]

… exclude securities with non-standard features (callable/floating rate note/ABS) [½]
Impose maximum exposure to a single issuer [½]

In all cases, a haircut could be applied to the valuation of the collateral to allow for the potential for fluctuations in market value of the collateral. [½]
The size of the haircut could depend on the liquidity of the fixed interest securities. [½]

[Maximum 2]

(iii) The main advantage is reduced operational complexity… [½]
… as fewer exchanges of collateral would be expected. [½]
Alternative advantage: the non-zero MTA acts to reduce the unnecessary costs of small transfers. [½]

The main disadvantage is a potentially higher counterparty exposure… [½]
… up to a maximum of the MTA… [½]
… particularly if the MTA is set at a relatively high level. [½]

Alternative disadvantage: the MTA operates in both directions… [½]
… so could result in the financial institution being over-collateralised. [½]

[Maximum 2]

(iv) Assume a normal distribution for equity returns as per the hint with…
… 20% annualised equity volatility. [½]
… mean return of zero over a one day horizon (on grounds of materiality) [½]
… 260 trading days in a year. [½]
… any collateral already held is not sensitive to equity market movements .[½]
(Please accept plausible alternative assumptions such as lognormal distributions, equity volatility in the range 10%–40%, non-zero average returns and trading days between 200–300 days)

As equity movements in either direction will cause a breach of the MTA, there is a 10%/2 = 5% chance of the financial institution posting collateral and a 5% chance of the institution receiving collateral. [½]

5% corresponds to $x = 1.6449$ on the standard normal distribution. [½]
The daily volatility is $20\% \div \sqrt{260} = 1.2403\%$ [½]
Hence the total market value movement and MTA is
$\$100m \times 1.2403\% \times 1.6449 = \$2.04m \sim \$2m$ [½]

[Maximum 3]

[Total 10]

Candidates were generally able to score well on the first two parts, although in part (i) some candidates misinterpreted the question and focussed on the benefits of a central counterparty.

In part (iii) candidates regularly either scored 0 or 2. Those scoring 0 often wrongly focussed on the advantages and disadvantages of collateral in general rather than having an MTA. This part and part (i) highlight the importance of clearly understanding the question being asked.

Part (iv) was well started with most candidates able to score at least some marks. Few were able to continue their solutions, with many not setting-out their assumptions clearly.

Q2 (i) Both entitle the holder to buy or to sell an asset at a specified price at a future time [1]

However, under a European option that future time is a specified date, and under an American option that future time is on or at any time before the specified expiration date. [1]

[Maximum 2]

(ii) American options are hard to value as there is no closed-form solution for the price of a general American option. [1]

This is due to the holder having to make decisions prior to the expiration date. [1]

Further, the optimal exercise policy needs to be known when trying to solve any resulting potential differential equation, and in general it is not known. [½]

[Maximum 1]

(iii) **Monte Carlo Simulations**

The starting point is to model the underlying asset or assets using a stochastic model… [1]
… e.g. a geometric Brownian motion… [½]

… and correlations if there are multiple assets. [½]

The stochastic model also needs to be calibrated to the market data of the assets or assets. [1]

A decision has to be made relating to the discrete time step to use in the simulation, taking into account potential exercise dates given that the option is American. [1]

Random sampling takes places to determine the values for the underlying random process or processes. [½]

This sampling generates a simulated path for the asset (or assets) and can be repeated to generate the required number of paths. [½]

At the final exercise date the optimal exercise strategy for an American option is to exercise the option if it is in the money. [½]

At other dates the optimal strategy is to choose between the value obtained from exercising the option immediately and the expected value in continuing to hold the option. [1]

The value of exercising the option at a given time is usually easy to determine. [½]

The more difficult part is calculating the expected value in continuing to hold the option. [½]

The Longstaff-Schwartz method takes the cross-sectional information in the simulated paths of the Monte Carlo simulation… [½]

… to determine the value of continuing to hold the option (the continuation value). [½]

The approach uses a least-squares regression analysis… [½]
… to determine the best-fit relationship between … [½]

… the value of continuing and the values of the relevant state variables (e.g. the value of the underlying asset) … [½]

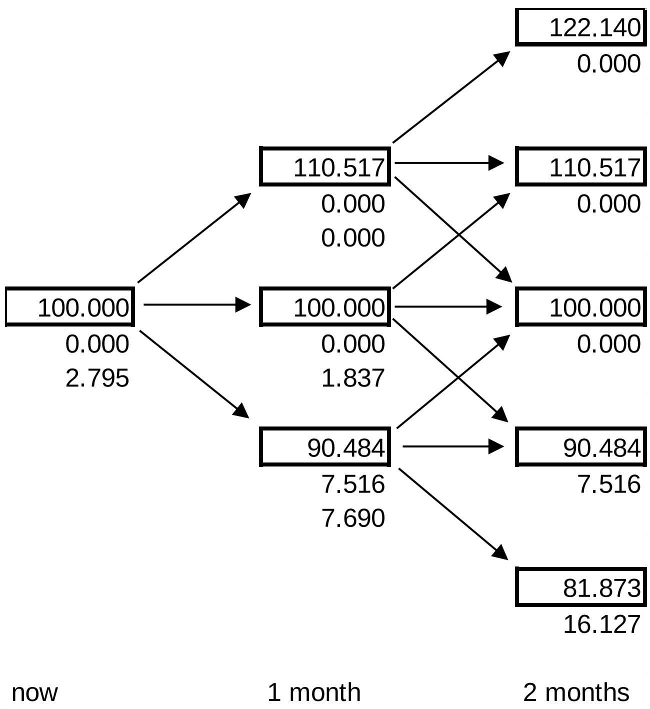
… at each time a decision regarding the optimal strategy has to be made. [½]

For example, an approximate relationship may be assumed between V (the value of continuing) and S (the stock price)… [½]
… such as $V = a + bS + cS^2$ [½]
Values of a, b and c are determined that minimise the sum of the squares $(V - a - bS - cS^2)^2$… [½]
… across observations from sampled paths… [½]
… which are in-the-money at that particular time. [½]
This gives an approximation for the continuation value that can then be compared to the early exercise value and the option value at each point in time on each path. [½]

The values are then discounted within the model in order to determine the price… [½]
… using the risk-free rate. [½]

[Maximum 8]

(iv)



[up to 1½ marks for the correct values of exercising]

The first row after the boxed underlying value shows the value of exercising, and the second row after the boxed underlying value shows the value of continuing.

Values of continuing at time 1 month:

Middle node = exp(–.03/12) × 7.516 × 0.245 = 1.837 [1]
Lower node = exp(–.03/12) × { 7.516 × 0.5 + 16.127 × 0.245} = 7.690 [1]

In this case it is not optimal to exercise the option early at any node. [1]

The value of the option now is

exp(–.03/12) × { 1.837 × 0.5 + 7.690 × 0.245} = 2.795 (*or 2.796 depending on rounding*). [1]

[Maximum 5]

[Total 16]

Parts (i), (ii) and (iv) were straightforward for most candidates. There were a few numerical errors in part (iv), like forgetting to discount in the relevant places, but nothing significant.

Part (iii) was a question which gave candidates an opportunity to demonstrate a thorough understanding of Monte Carlo simulations and of the method of least squares. Many candidates could provide a high level description but fewer were able to describe the two areas in sufficient depth to score much above half marks. The ability to understand the material in ST6 in detail is important in understanding how to apply it in unfamiliar situations.

Q3 (i) The two measures P and Q are equivalent if and only if:
they operate on the same sample space; and [½]
they agree on what is possible (and what is impossible). [½]

Alternative answer:
If A is any event in the common sample space in which P and Q operate, [½]
then P and Q are equivalent if and only if $P(A) > 0 \Leftrightarrow Q(A) > 0$. [½]

[Maximum 1]

(ii) The Cameron-Martin-Girsanov theorem [½]

allows the drift of a stochastic process to change [½]

by changing the probability measure. [½]

It should be noted that by this theorem only the drift can change and not the volatility. [1]

In the case given in the question the change highlighted is the change in volatility. [½]

As $X(t)$ has a different volatility under P and Q …. [½]
… then these measures are not equivalent. [½]

[Note: full marks can also be awarded for providing a counter-example to show that the measures are not equivalent.]

[Maximum 3]

(iii) The first step is to change the measure… [½]
… using the Cameron-Martin-Girsanov theorem. [½]

This will change the drift to a value ν, say. [½]

Rewriting $dS(t): dS(t) = \sigma S(t)dW(t) + (\mu - \nu)S(t)dt + \nu S(t)dt$ [½]

So $dS(t) = \sigma S(t)\left(dW(t) + {(\mu-\nu)}\big/_{\sigma}\, dt\right) + \nu S(t)dt$. [½]

Let $\gamma = {(\mu-\nu)}\big/_{\sigma}$. [½]

As γ is constant, it is a previsible process. [½]

Also, as γ is constant, $E_P\left[\exp\left(0.5\int_0^T \gamma^2 dt\right)\right] < \infty$, where $E_P[.]$ is the expectation with respect to P. [½]

The conditions of the Cameron-Martin-Girsanov theorem are satisfied. [½]

So, there exists a measure Q such that $Z(t) = W(t) + \int_0^t \gamma dt$ is Q-Brownian motion. [½]

This can be written in the form: $dZ(t) = dW(t) + \gamma dt$. [½]

Substituting this equation into the equation for $dS(t)$ gives:
$dS(t) = \sigma S(t)dZ(t) + \nu S(t)dt$ or $dS(t) = \sigma S(t)dZ(t) + (\mu - \gamma\sigma)S(t)dt$. [½]

By picking $\nu = r$, gives $\gamma = {(\mu - r)}\big/_{\sigma}$. [½]

In this case γ is the market price of risk. [½]

With respect to Q associated to $\gamma = {(\mu - r)}\big/_{\sigma}$: $dS(t) = \sigma S(t)dZ(t) + rS(t)dt$.
[½]

The discounted asset process is given in the question by $\bar{S}(t) = e^{-rt}S(t)$.
Using the product rule:

$$
\begin{aligned}
d\bar{S}(t) &= d(e^{-rt}S(t)), \\
&= e^{-rt}dS(t) - re^{-rt}S(t)dt, && [½] \\
&= S(t)e^{-rt}\, {dS(t)}\big/_{S(t)} - r\bar{S}(t)dt, \\
&= \bar{S}(t)\, {dS(t)}\big/_{S(t)} - r\bar{S}(t)dt, && [½] \\
&= \bar{S}(t)\left[\sigma dZ(t) + rdt\right] - r\bar{S}(t)dt && [½]
\end{aligned}
$$

$$= \sigma\bar{S}(t)dZ(t).$$ [½]

Further, as σ is constant: $E_Q\left[\left(\int_0^T \gamma^2 ds\right)^{0.5}\right] < \infty$, where $E_Q[.]$ is the expectation with respect to Q. [½]

The discounted asset process is a martingale with respect to the probability measure Q as there is no drift term and the technical condition above is also satisfied. [1]

Alternative solution: $\bar{S}(t)$ can be considered instead of $S(t)$.

The discounted asset process is given in the question by $\bar{S}(t) = e^{-rt}S(t)$.

$$d\bar{S}(t) = d(e^{-rt}S(t)),$$
$$= d\left(e^{-rt+\ln S(t)}\right),$$ *[½]*

Using Ito's lemma, $d\ln(S(t)) = \sigma dW(t) + \left(\mu - 0.5\times\sigma^2\right)dt$. *[1]*

Let $X(t) = -rt + \ln S(t)$, *then*

$$dX(t) = \sigma dW(t) + \left(\mu - 0.5\times\sigma^2 - r\right)dt,$$ *[½]*

Using Ito's lemma again

$$d\bar{S}(t) = d(e^{-rt}S(t)),$$
$$= d\left(e^{X(t)}\right),$$
$$= e^{X(t)}\left(\sigma dW(t) + \left(\mu - 0.5\times\sigma^2 - r + 0.5\times\sigma^2\right)dt\right),$$ *[1]*
$$= \bar{S}(t)\left(\sigma dW(t) + \left(\mu - 0.5\times\sigma^2 - r + 0.5\times\sigma^2\right)dt\right),$$
$$= \bar{S}(t)\left(\sigma dW(t) + (\mu - r)dt\right).$$ *[1]*

In order for this to be a martingale the drift term needs to be removed.

This is done by changing the measure. *[½]*

Let λ be the constant process: $\lambda = (\mu - r)/\sigma$. *[1]*

The conditions of the CMG theorem are satisfied as λ is finite. *[½]*

By the C-M-G theorem… *[½]*

… there exists a measure R *such that* $V(t) = W(t) + \int_0^t \lambda dt$ *is* Q*-Brownian motion.* *[½]*

This can be written in the form: $dV(t) = dW(t) + \lambda dt$. *[½]*

$$d\bar{S}(t) = \bar{S}(t)\left(\sigma\left(dV(t) - \lambda dt\right) + (\mu - r)dt\right),$$ *[½]*

$$= \sigma\bar{S}(t)dV(t) + \bar{S}(t)\left(\mu - r - \lambda\sigma\right)dt$$ *[½]*

$$= \sigma\bar{S}(t)dV(t).$$ *[½]*

Further, as σ *is constant:* $E_R\left[\left(\int_0^T \lambda^2 ds\right)^{0.5}\right] < \infty$*, where* $E_R[.]$ *is the expectation with respect to* R. *[½]*

The discounted asset process is a martingale with respect to the probability measure R *as there is no drift term and the technical condition above is also satisfied.* *[1]*

[Maximum 7]

(iv) (a) $\max\left\{0, K - \frac{1}{T}\int_0^T S(t)dt\right\}$. [1]

Full marks can also be awarded if a summation is used instead of an integral.

(b) $\max\left\{0, \max_{t\in\{0,T\}}\{S(t)\} - S(T)\right\}$. [1]

Full marks can also be awarded if a different type of lookback option is used instead of above, e.g. with a fixed strike of K instead of S(T).

[Maximum 2]

(v) Both of the options payoffs are dependent on the asset price throughout the period $t \in \{0,T\}$ [½]

and they cannot be expressed as a function of $S(T)$, [½]

i.e. there is no terminal value pricing. [½]

Even so, the Black-Scholes model's only constraint on the payoff is that it is known by time T. [½]

The no-arbitrage price can be found using the Black-Scholes model: $V(t) = e^{-r(T-t)}E_Q\left[X \mid F_t\right]$. [1]

Here, $V(t)$ is the price of the option at time t, Q is the probability measure found in part (iii), X is the payoff from the option and F the filtration. [½]

Therefore, it works for path-dependent options. [½]

However, it is not always possible to obtain a closed-form solution for the price of a path-dependent option in the Black-Scholes model. [1]

One example where a closed-form is known is for an Asian option with geometric averaging. [½]

If explicit solutions do exist for either of the options in part (iv) then they can be directly used for pricing. [½]

If explicit solutions do not exist for either of the options in part (iv) then numerical methods, such as Monte Carlo simulation, are useful techniques to determine the price of such options in the Black-Scholes model. [1]

[Maximum 3]

[Total 16]

Parts (i) and (iv) were well answered.

Part (ii) produced a variety of answers. Some candidates produced one of the solutions given in this report but often did not know where to start; those who did not make material headway did not appear to realise that the volatility would remain unchanged and that this was a key part of the change in measure result.

The better solutions to part (iii) were logically and clearly laid out, and these candidates were sufficiently familiar with, for example, the Cameron-Martin-Girsanov theorem and the product rule to apply them accurately.

Part (v) was testing the application of the Black-Scholes model to pricing path dependent options. The key observation is that the only constraint is for the payoff to be known at expiry. From this the options can then be priced. Many candidates did not consider this and focussed purely on closed-form solutions. The Black-Scholes model can be applied in a variety of ways and to many derivative problems; most do not involve closed-form solutions and instead rely on numerical techniques.

Q4 (i) Longevity risk is the risk that aggregate mortality rates are lower than expected… [½]
… in the long run / due to mortality rates improving more quickly than expected. [½]

A survivor cap is a collection of survivor caplets. [½]

A survivor caplet will pay at time $t = Max\left[S(t) - K, 0\right]$ … [½]
… where $S(t)$ is the actual survivor index at time t, and K is the cap rate agreed. [½]

A cap therefore provides protection from high survival rates above the cap rate…. [½]
… and so reduces longevity risk. [½]

[Maximum 2]

(ii) The LifeCap product has a payoff based on future expected longevity… [1]
…whereas a survivor cap has a payoff based on realised mortality up until time t. [1]
The LifeCap provides a single payoff, whereas a survivor cap will have a series of caplets. [1]

[Maximum 2]

(iii) $N \times P(0,T)\left[F \times \Phi(d_1) - K \times \Phi(d_2)\right]$ [½]

$$d_1 = \frac{\ln(F/K) + \frac{\sigma^2}{2}T}{\sigma\sqrt{T}}, d_2 = d_1 - \sigma\sqrt{T}$$ [½]

Where: N = notional, σ = volatility of the future expected lifetime, Φ is the standard normal cumulative distribution function. [½]
F = forward expected future lifetime of a 65-year-old male at T. [½]
$P(0,T)$ is the price of a zero-coupon bond at time zero that matures at time T. [½]

[Maximum 2]

(iv) (a) The Black model assumes a lognormal distribution…. [½]
… for the expected future lifetime, L, at time T. [½]

The expected lifetime, L, at time T, is assumed to be equal to the current forward price F. [1]

The stochastic behaviour of interest rates is not taken into account in the way the discounting is done, or equivalently is not co-dependent with future lifetimes. [½]

The volatility of the forward price is assumed to be constant and equal to σ. [½]

(b) A lognormal distribution for the expected lifetime may not be entirely appropriate as future life expectancies may exhibit fatter tails… [1]
… due to medical advances… [½]
… or new illnesses. [½]
Similarly, the expected lifetime may show non-stationary statistical behaviour when medical or societal changes start impacting different cohorts of individuals. [½]

The volatility of the future expected lifetime would likely decrease as time T approaches, i.e. as the time to the ultimate death of the 65-year-old reduces. [½]

Given the excess number of participants looking to sell longevity risk in the market, a risk premium will likely exist…. [½]
… and so adjustments would be required to any market quotes to derive the value F that will equal the expected future lifetime at time T. [½]

The bank may view the Black model inappropriate given they are unlikely to be able to adopt a corresponding hedging strategy, and so may set the price based on some other basis (e.g. suitably low probability of loss under a VaR event, cost of capital,…) [1]

[Maximum 4]

(v) Vega is the change in value of the LifeCap derivative arising from changes in the implied volatility of the forward expected future lifetime. [½]

The bank should calculate the value of the LifeCap under the expected volatility… [½]
… and then revalue the LifeCap derivative under a shocked volatility… [½]
… which would typically be a small amount (e.g. 1%). [½]
The vega of the derivative can then be estimated as the change in the market value of LifeCap under stress, divided by the shock to the volatility. [½]

[Maximum 2]

(vi) The pension fund will be exposed to increases in life expectancy as it will have to pay the benefits for longer… [½]
… and the exposure would be expected to be broadly symmetric (i.e. life expectancy decreases would benefit in a similar fashion to the detrimental impact of increasing life expectancy). [½]

LifeCap would increase in value should longevity expectations increase and so should help reduce longevity risk. [½]
The strike of the LifeCap derivatives will determine the degree of protection offered… [½]
… and only an at-the-money contract would satisfy the trustees' desire to hedge longevity risk as much as possible. [½]

Basis risk is likely to be material… [½]
… as LifeCap is based on a 65-year-old, but the pensioner and deferred members will be of many different ages... [½]
... and the average age will increase over time, increasing the basis risk. [½]
… and the retirement age of the scheme may be different from 65… [½]

… Life Cap is based on a male but the scheme will likely have male and female members. [½]
… the pension scheme members will likely be from a different (possibly concentrated) geographic region/socio-economic background than the average population. [½]
… the maximum terms of 10 years may mean that the longevity risk in respect of the youngest deferred members cannot be accurately hedged. [½]
… LifeCap is based on expected longevity rather than realised longevity… [½]
… so there is no protection for actual longevity being longer than expected beyond age 65. [½]

As the LifeCap payoff is a cash settled contract at expiry, the pension fund will only realise cash gains right at the end of the contract… [½]
… and so the hedges will need to be structured carefully to avoid liquidity issues as pension payment fall due if longevity increases… [½]
… which may involve a series of hedges with one expiring in each year, for example. [½]

The LifeCap derivative is an option so will incur a cost… [½]
… which is likely to be large given the excess desire to sell longevity risk in the market… [½]
… and the large amount of notional required to hedge the exposure for the large pension fund… [½]
… and so would likely increase the deficit of the fund. [½]
However, LifeCap will not require payment over and above the initial option cost if life expectancy decreases… [½]
… so may be more attractive to the trustees than a longevity swap. [½]

LifeCap will introduce Vega risk… [½]
… as reductions in the implied volatility will reduce the mark to market of the hedge but not reduce the liabilities (unless there was guarantees) [½]

LifeCap will introduce counterparty risk… [½]
… which will always be present as the option is purchased by the pension scheme… [½]
… but this risk would be reduced by collateralisation. [½]

LifeCap will not hedge the cross-gamma risk from changes in real/nominal yields (or other demographic assumptions/experience) which change the amount of notional required to be expected to delta hedge the longevity exposure. [1]

LifeCap would only be suitable if the pension scheme has the operational capability… [½]
… or is permitted by regulations to trade derivatives. [½]
Any operational risk will be heightened by LifeCap being a new derivative. [½]

[Maximum 7]

[Total 19]

Parts (i) – (iii) were well answered.

The aim of this question was to explore Black's model in the setting of a longevity derivative. Candidates who understood Black's model were able to apply it in this unfamiliar setting well. Part (iv) allowed some candidates to demonstrate this well by being able to think about expected lifetime as an underlying asset and the factors influencing it.

The large majority of candidates struggled with part (v) with most not getting past the definition of vega.

Answers to part (vi) were generally not detailed or distinct enough to score highly. This emphasises the need to use knowledge to generate points from various areas of the ST6 course in application type questions. Good candidates will think methodically through areas like risks, practicalities, costs and benefits in context of the specific question asked and develop these to form detailed answers.

Q5 (i) After 5 years, the fixed liability will be due in 20 – 5 = 15 years [½]

Therefore, the value under Old Solvency is

$\$1000m \times \exp(-0.04 \times 15) = \$548.81... \simeq \$549m$ [1]

[Maximum 1]

(ii) (a) After 5 years, the *NewSolvencyDiscountRate* is

$= 4\% \times \text{Max}\left(1 - \frac{5}{10}, 0\right) + 2\% \times \text{Min}\left(\frac{5}{10}, 1\right) = 3\%$ [1]

Hence the value is $\$1000m \times \exp(-0.03 \times 15) = \$637.628... \simeq \$638m$ [½]

(b) If the swap rate drops to 1.9% then the *NewSolvencyDiscountRate* is

$= 4\% \times \text{Max}\left(1 - \frac{5}{10}, 0\right) + 1.9\% \times \text{Min}\left(\frac{5}{10}, 1\right) = 2.95\%$ [1]

The value of the liability is therefore:

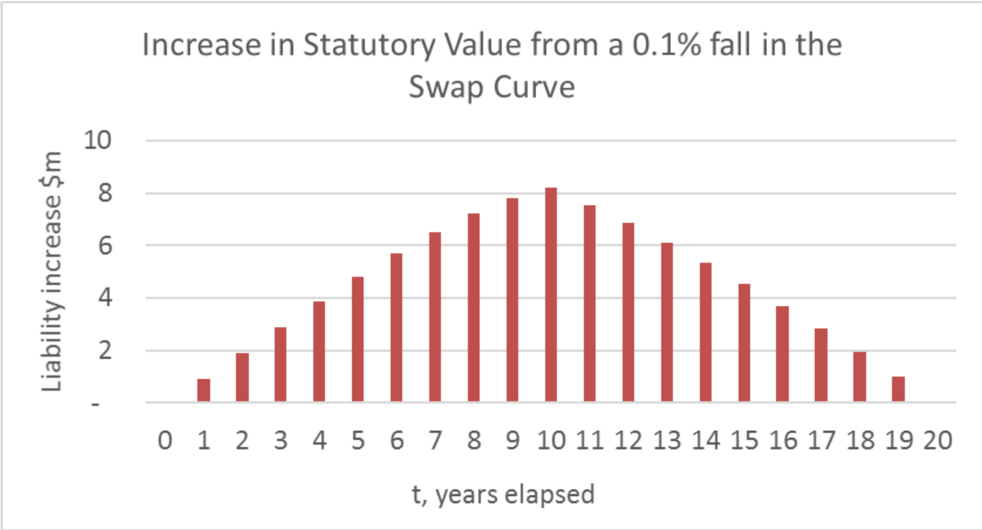
$\$1000m \times \exp(-0.0295 \times 15) = \$642.428$ [½]

.. and the increase in the value of the liability compared to a 2% swap rate is:

$\$642.428m - \$637.628m = \$4.800m \simeq \$5m$ [½]

Alternatively, $637.628 × 0.1% × 15 × 5/10 ~ $4.782 ≃ $5m [2]

[Maximum 3]



(iii)

Suggested marks:

Zero sensitivity at $t = 0$	[½]
Zero sensitivity at $t = 20$	[½]
\$5m sensitivity at $t = 5$	[½]
Peak at $t = 10$	[½]
General "Triangular" shape	[1]
Labelled Axes	[1]

[Maximum 3]

(iv) It is a futures contract on a Treasury or government bond [½]
Settlement is made when the party with the long position receives a bond in return for cash paid to the party with the short position. [1]
A range of bonds can be delivered that satisfy the restrictions on the remaining term of the bonds. [½]
A conversion factor defines the price received by the party with the short position… [½]
… who can choose which of the available bonds is "cheapest-to-deliver". [½]

[Maximum 2]

(v) Basis risk is the uncertainty associated with the future basis… [½]
… where the basis is defined as the spot price of the asset or liability to be hedged less the futures price of the contract used to implement the hedge. [½]

Basis risk in this case can arise from a number of sources:

The change in the Statutory Value of the liability may not be matched in the change in the value of the Treasury bond future… [1]

… as the liabilities are calculated using a swap rate, whereas changes in the Treasury bond future will be driven by movements in government bond yields… [1]
… and the term of the bond underlying the Treasury bond future will likely not match the remaining term of the liability. [1]
The change in the Statutory Value will therefore not be immunised against nonparallel shifts in the yield curve. [½]
The cheapest-to-deliver bond could change over time, introducing further basis risk… [1]
… although this is mitigated by a conversion factor which aims to reduce the differences in pricing between bonds. [½]

The Treasury bond future may need to be closed out before its delivery month. [1]
As the liability is fixed, there is not expected to be basis risk arising from a change in the exact date when the liability falls due. [½]

[Maximum 3]

(vi) The futures would likely need to be rolled as the delivery date of the latest Treasury bond future will likely be well in advance of the 20 year term of the liability. [1]

(vii) The institution needs to ensure the number of Treasury bond futures matches the required interest rate sensitivity of the Statutory Value of the liability until the next roll. [½]

It may be challenging to estimate the interest sensitivity of the Statutory Value of the liability because:
The institution needs to allow for the effect of the discount rate transitioning over the first 10 years… [1]
… which will act to increase the number of contracts required quite materially. [½]
Therefore, it will be optimal to roll the hedge at least at each year end when the discount rate changes. [½]
Similarly, as the liability term contracts beyond 10 years, the required number of contracts will reduce materially. [½]
However, the change in the Statutory Value sensitivity is known in advance so this shouldn't pose too much of a challenge. [1]

In calculating the sensitivity of the bond future, the institution will need to allow for the cheapest-to-deliver bond… [½]
… and the difference between the swap and government bond yield curves. [½]
These inputs may be subjective and will vary over time. [½]

It will be challenging to roll the hedge into the Treasury bond future with the longest available delivery date… [1]
… which would usually be up to a year away… [½]

… as liquidity is usually greatest in the contract next to be delivered. [½]
However, using the contract with the next delivery date would necessitate more rolls in the future… [½]
… which would increase the rollover basis risk… [½]
… and be more onerous/complex to manage. [½]
Treasury bond futures are one of the more liquid futures contracts available, which should ease liquidity concerns. [½]

It may be challenging to roll large numbers of futures over a long period of time without being selected against by market participants. [½]
It may be disproportionately costly to keep rolling the hedge…. [½]
… especially when the sensitivity of the liabilities is low at $t = 0$ or $t = 20$. [½]

[Maximum 3]

(viii) The financial institution may consider that the NewSolvency metric is not the most economically rational measure of interest rate sensitivity… [1]
For example, it may decide to hedge full swap rate sensitivity / accounting / earnings sensitivity. *(Any sensible example can be awarded this mark.)* [½]

The interest rate sensitivity of the financial institution may be expected or valued by the shareholders. [1]

The financial institution may have a naturally offsetting exposure elsewhere in the business. [1]

The interest rate sensitivity is not deemed material. [½]
The financial benefits of hedging are not deemed sufficient to outweigh the operational complexity…. [½]
… or cost… [½]
… or additional counterparty risk. [½]

There may be regulatory constraints on derivative use. [1]
The financial institution may want to retain the upside benefit if interest rates rise [½]

The financial institution may not have sufficient liquidity to manage the margins, e.g. to make any required variation margin payments. [½]

Other sensible suggestions were also credited.

[Maximum 3]

[Total 19]

Overall this question was well answered.

Candidates did not score highly with the graph in part (iii). One

> *approach would be to start with the known points from earlier parts and work out, or calculate, other factors needed for the shape. Many candidates did not plot the known points.*
>
> *The higher order questions in parts (vii) and (viii) also proved challenging despite the large amount of possible marks available. The comments at the end of question 4 are relevant here.*

Q6 (i) One of the main purposes is to facilitate international investment and trade, [½]

by allowing market participants to convert from one currency to another currency. [½]

For example, a Belgium chocolate maker can convert Euros into the West African CFA Franc to buy cocoa beans from the Ivory Coast. [½]

[Other suitable examples can be awarded ½ mark.]

Another purpose is to enable risk management, [½]

by transferring foreign exchange risk to another party. [½]

Foreign exchange risk is the financial risk of an investment's value changing due to the changes in currency exchange rates. [½]

Financial derivatives exist in the foreign exchange market to help with risk management. [½]

For example, a company is due to receive a cashflow denominated in a foreign currency on a given future date. The company can lock into the current exchange rate by entering into a futures position that expires on the date of the cashflow. [1]

[Other suitable examples can be awarded ½ mark.]

The foreign exchange market allows for speculation. [½]

It also lets speculators borrow in low yielding currencies and invest in, or lend, in higher yielding currencies. [½]

[Maximum 3]

(ii) (a) This hedging strategy guarantees that the cost of the US dollars will not be greater than a certain amount of Euros, [½]

… since if the exchange rate (Euros per $1) increases… [½]

… the business can exercise the call option and purchase $ at a (lower) fixed exchange rate. [½]

(b) European option. [½]
Expiry date: matures at the time of the payment. [½]
Strike price: to meet the business' needs. [½]
Nominal: matches the known amount of payment. [½]

(c) It allows the company to benefit from favourable exchange-rate movements. [1]

There is a cost in Euros associated with this form of hedging: the premium of buying the options. [1]

[Maximum 5]

(iii) In general, the delta is a measure of the ratio of option contracts to the underlying asset in order to establish a neutral hedge. [1]

It expresses the sensitivity of the option to the underlying asset. [½]

In this case the delta is the amount of US dollars to buy … [½]

in order to make the combined position (of selling one option and buying US dollars) insensitive to exchange rate movements, in Euro terms. [½]

[Maximum 2]

(iv) The amount of Euros expressed in US dollars is ${(E \times D)}/{S}$. [½]

The bank sold D options for a total premium of $D \times P$ Euros. [½]

The bank converted this amount into US dollars at the time it was received from the business. This has a value of ${(D \times P)}/{S(0)}$ US dollars, where $S(0)$ is the exchange rate at the time the premium was received. [½]

This is a fixed amount in US dollars and it does not depend on the current exchange rate. [½]

The current value of the short option to the bank, expressed in US dollars is $-{(C \times D)}/{S}$. (This is negative as the bank holds the short position.) [½]

The total value of the portfolio in US dollars is:

$${(E \times D)}/{S} + {(D \times P)}/{S(0)} - {(C \times D)}/{S}.$$ [½]

Therefore for the portfolio to have constant value in US dollars, we need:

$$\frac{\partial}{\partial S}\left({(E\times D)}/{S} + {(D\times P)}/{S(0)} - {(C\times D)}/{S}\right) = 0. \qquad [1]$$

Expanding this gives: $0 = -\frac{E}{S^2} - \frac{1}{S}\frac{\partial C}{\partial S} + \frac{C}{S^2}$. [½]

As the premium is fixed in US dollar terms and D is fixed, these have a derivative of 0, and do not appear in the previous equation. [½]

Recalling that $\Delta = \frac{\partial C}{\partial S}$ and multiplying through by $-S$ gives the result. [½]

[Maximum 4]

(v) The definition of the new delta follows exactly from part (iv). [½]

Therefore, the bank should hold $-\delta$ US dollars for each option it has sold to the business in order to make its position delta-neutral in US dollar terms. [1]

[Maximum 1]

(vi) From the definitions:

$$\delta = \Delta - {C}/{S},$$

$$= e^{-r_{US}\tau}\Phi(d_1) - \frac{e^{-r_{US}\tau}S\Phi(d_1) - e^{-r_{EURO}\tau}K\Phi(d_2)}{S}, \qquad [½]$$

$$= {K}/{S}\, e^{-r_{EURO}\tau}\Phi(d_2). \qquad [½]$$

The last equality follows from cancelling-out the $\Phi(d_1)$ terms.

[Maximum 1]

(vii) Explanation of graph shape:

The graph of the delta is standard as found in the core reading. The only difference is that there is a factor of $e^{-r_{EURO}\tau}$ compared to the delta in the core reading.

As a result the delta here reaches a maximum of $e^{-r_{EURO}\tau}$ and not 1.

New delta:

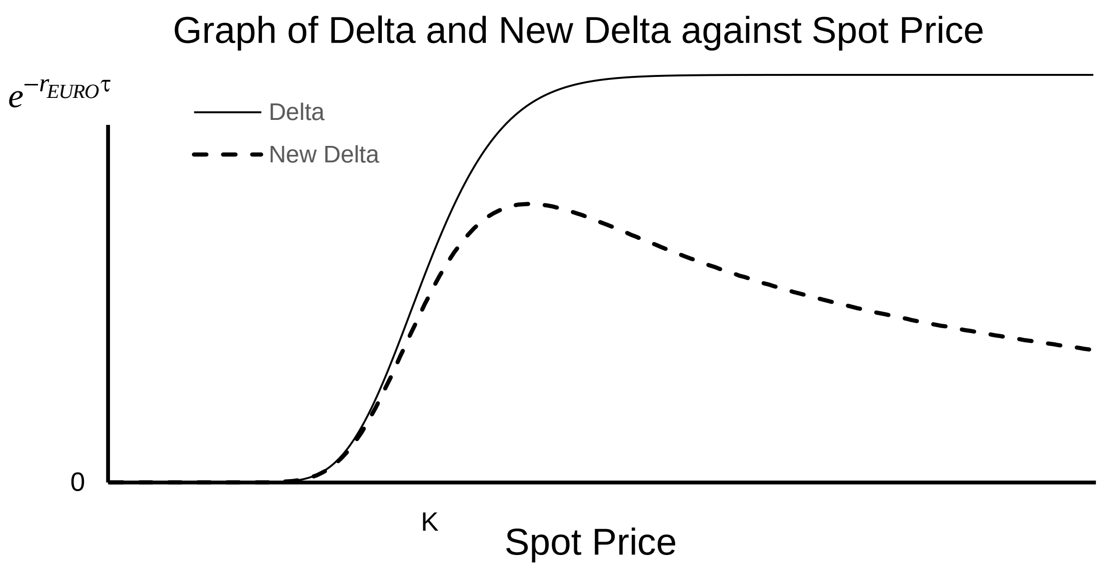
Using the hint in the question, and noting that $\Delta \to 0$ as $S \to 0$, implies that $\delta \to 0$ as $S \to 0$.

In the region around $S \approx K$, $\delta \approx e^{-r_{EURO}\tau}\Phi(d_2)$.

Further, as $d_1 - d_2 = \sigma\sqrt{\tau}$, a fixed amount, it follows that the graph of $\Phi(d_2)$ is a small translation to the right of $\Phi(d_1)$.

Hence, in the region around $S \approx K$, $\delta \approx \Delta - \varepsilon$, where ε is a fixed quantity.
As $S \to \infty$, $\delta \to 0$. This is because $\Phi(d_2) \to 1$ as $S \to \infty$ and
$K/S\, e^{-r_{EURO}\tau} \to 0$ as $S \to \infty$.

Based on the behaviour at $S = 0$ and as $S \to \infty$ and as δ is continuous, there must be at least one maximum.



[½ mark for labels of the vertical axis, 0 and $e^{-r_{EURO}\tau}$ need to be included]
[½ mark for labelling the spot price axis]
[½ mark for labelling K]
[½ mark for labelling the delta graph and new delta graph]
[1 mark for correctly plotting the delta graph]
[½ mark for the new delta graph's behaviour near $S = 0$]
[½ mark for the new delta graph's behaviour at large S]
[½ mark for the new delta graph's behaviour near $S = K$]
[½ mark for the new delta graph having a maximum]
[½ mark for the new delta graph always being below the delta graph]

[Maximum 4]

[Total 20]

Overall this question was well answered.
The algebraic parts were well attempted but candidates often struggled

in explaining the logic between steps in the algebra or in interpreting a formula. These are required in order to show to examiners evidence of understanding, which is even more important where candidates make an error in their algebra as it allows markers to award marks for demonstrating that understanding.

This question demonstrates the need for understanding practical applications of ST6 beyond the underlying technical content.

END OF EXAMINERS' REPORT