INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

25 September 2014 (pm)

Subject ST6 – Finance and Investment
Specialist Technical B

Time allowed: Three hours

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You have 15 minutes before the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*

3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all seven questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

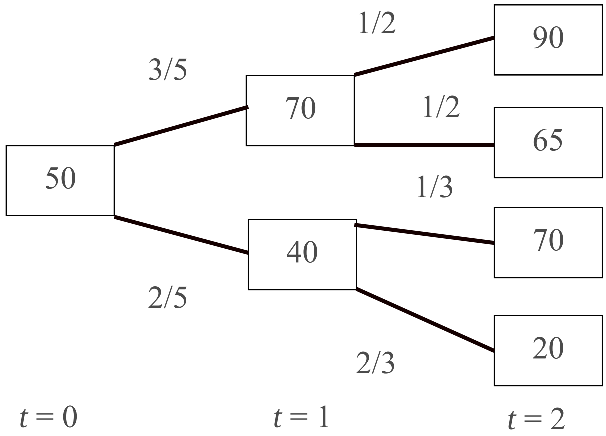
In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Let $\mathbb{P}$ be a probability measure equivalent to a different probability measure $\mathbb{Q}$ on a non-recombinant binomial tree.

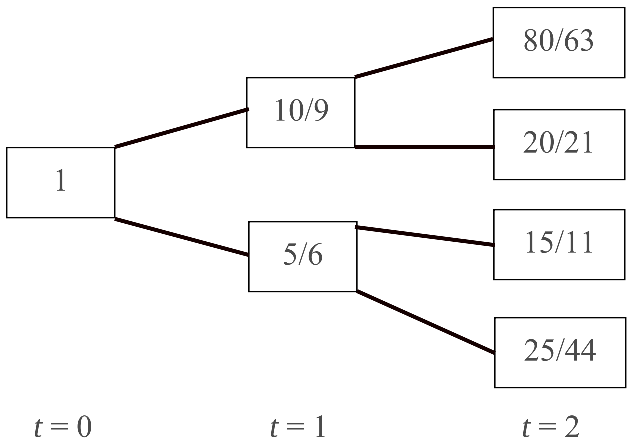
(i) Define the Radon-Nikodym derivative of $\mathbb{Q}$ with respect to $\mathbb{P}$ over the time horizon T, written as $\dfrac{d\mathbb{Q}}{d\mathbb{P}}$ and with respect to a random variable X defined on the nodes of the tree. [4]

(ii) Explain the purpose of the Radon-Nikodym derivative and how it can be used in option pricing. [2]

Consider a stochastic process that evolves in discrete time t according to a binomial tree. The set of possible values of the process are shown below for $t = 0$, 1 and 2, together with a probability measure $\mathbb{P}$.



Consider an equivalent measure $\mathbb{Q}$ on the above tree and the Radon-Nikodym derivative $\dfrac{d\mathbb{Q}}{d\mathbb{P}}$. The associated Radon-Nikodym process is given below:



(iii) Calculate the measure $\mathbb{Q}$ for this tree. [4]

[Total 10]

2 (i) Describe the following interest rate derivatives:

(a) an interest rate floor
(b) an interest rate floorlet
(c) a plain vanilla swap
(d) a European receiver swaption
(e) a forward rate agreement

[5]

A financial institution has committed to paying one of its clients a minimum rate of interest of 3% p.a. convertible half yearly and payable half yearly on an agreed notional for five years, starting in five years' time. At that point (i.e. in five years' time), the client chooses whether to receive this guaranteed 3% p.a. throughout the next five years or to earn LIBOR on the notional amount.

(ii) Discuss the effectiveness of the financial institution attempting to hedge out the guarantee, focussing on the interest rate implications only, using each of the following:

(a) a 3% p.a. five-year interest rate floor, starting in five years' time

(b) ten floorlets to pay 1.5% over the six month periods ending at times 5.5, 6, 6.5, ..., 10

(c) a swaption to enter into a swap to receive 3% p.a. for five years starting in five years' time

(d) a five year call option to buy a five-year 3% p.a. coupon bond in five years' time

(e) a forward rate agreement to earn 3% p.a. over five years, starting in five years' time and expected to be cash settled in five years' time

[For parts (a), (c), (d) and (e) the 3% p.a. is convertible and payable half-yearly.] [7]

[Total 12]

3 Suppose that, in a developed market, the most popular caps have reset times $t = 1, 2, 3, \ldots$ (in years).

A company which operates within that market is developing and calibrating a LIBOR Market Model (LMM) to use for pricing interest rate derivatives.

Define the following notation:

- $F_k(t)$ = forward rate between times k and $k+1$ as seen at time t, annually compounded
- $\zeta_k(t)$ = volatility of $F_k(t)$ at time t
- $\nu_k(t)$ = volatility of the zero-coupon bond price $P(t,k)$ at time t
- $P(t,k)$ = price at time t of zero-coupon bond with principal 1 maturing at time k.

You may assume that:

$$dF_k(t) = \zeta_k(\mathrm{t})[\nu_{\mathrm{int}(t)}(t) - \nu_{k+1}(t)]F_k(t)dt + \zeta_k(t)F_k(t)dz$$

in the rolling forward risk-neutral world, where $\mathrm{int}(t)$ is the smallest integer such that $\mathrm{int}(t) \geq t$ (i.e. it is the time of the next reset).

(i) Define the rolling forward risk-neutral world. [1]

(ii) Show that:

$$\nu_i(t) - \nu_{i+1}(t) = F_i(t)\zeta_i(t)/(1+F_i(t))$$

by applying Ito's lemma to $\ln P(t,i) - \ln(P(t,i+1) = \ln(1+F_i(t))$. [4]

(iii) Derive an expression for $dF_k(t)$ in terms of the forward rate volatilities in the rolling forward risk-neutral world, using the result from part (ii). [5]

Suppose now that $\zeta_k(t)$ is only a function of the number of whole years between the next reset date and time k.

So $\zeta_k(t) = \Lambda_{k-\mathrm{int}(t)}$.

On a particular day, the Black volatilities of a series of one-year caplets are as follows:

Year	*1*	*2*	*3*	*4*	*5*
Black volatility	15%	18%	18%	18%	17%

(iv) Derive the $\Lambda_0, \Lambda_1, \ldots, \Lambda_4$ parameters necessary to calibrate the LMM. [4]

[Total 14]

4 A financial institution is developing a new guaranteed bond. The terms of the bond are that:

- The client makes a single investment P.
- After five years, the client receives back the greater of P and $P(1 + xR)$.
- R is the proportionate change over five years in an equity price index (i.e. an index that does not allow for the reinvestment of dividends).
- x is a pricing parameter, to be set by the company.

Current market conditions are as follows:

- The five-year risk-free rate is 8% p.a. continuously compounded.
- The five-year market implied volatility on the equity index is 22% p.a.
- The dividends payable on the market index are estimated to stay fixed at 3% p.a. continuously compounded.

Tax can be ignored.

(i) Show that the guaranteed bond can be decomposed into a portfolio comprising holdings of a zero-coupon bond and a European call option on the equity price index. [3]

(ii) Hence determine:

(a) the value of x which would allow the company to take a 5% contribution to expenses and profit from each investment.

(b) the reduced expenses and profit loading available to the company if market competition forces it to set $x = 115\%$.

[8]

(iii) Explain how changes in the following could result in a lower pricing parameter x (ignoring the market influence described in part (ii)(b)):

- risk-free rates
- market implied volatilities
- the dividends payable on the index

[4]

A popular consumer magazine has suggested that it might be desirable for a version of the product to be introduced where R is based on the average of the index values on a number of dates close to the five year maturity date, rather than being based on a single date.

(iv) Describe the possible impact of this suggestion on the pricing of the guaranteed bond. [2]

[Total 17]

5 (i) (a) Define Value at Risk ("VaR").

(b) Give an example of VaR with respect to a portfolio. [2]

A risk officer at a large company is concerned about a future stock market crash.

(ii) Outline typical features of a stock market crash. [3]

The risk officer currently uses historical simulation to assess the VaR, but this is under review. One alternative approach she is comparing against is a "real-time" VaR methodology, as outlined below. In the first instance she is investigating this for a single stock.

Consider a portfolio P consisting of long positions in call options on a single underlying stock with price S.

(iii) (a) Derive an expression for the change in the price of the portfolio (δP) due to a change in the stock price (δS) in terms of the portfolio's Greeks (ignoring δS^3 terms and higher). [Hint: consider using Taylor's expansion.]

(b) Plot δP in terms of δS.

(c) Determine the worst change which could happen to the value of the portfolio instantaneously. [Hint: consider the minimum of δP.]

(d) Explain how δP varies in terms of δS if the coefficient of the δS^2 term is very small.

(e) Comment on the implications of the situation described in part (d) for the instantaneous worst possible outcome from the portfolio. [8]

Suppose that there is a contract available with which to hedge the above portfolio. Further this contract has a bid-offer spread of value C, delta Δ^* and gamma Γ^*.

Let a new portfolio P^* consist of original portfolio P plus λ of the hedging contracts.

(iv) Express the total change in the price of the new portfolio (δP^*) due to a change in the stock price (δS) in terms of the portfolio's Greeks (ignoring δS^3 terms and higher). [Note that the situation described in part (iii)(d) should be ignored.] [2]

(v) Outline how λ would be determined in order to hedge the portfolio P^* against a market crash, and how this process may be extended to other situations. [2]
[Total 17]

6 (i) Describe the volatility "smile" (or sometimes termed "skew" for equities), volatility term structure, volatility surface and their uses. [5]

A trader is considering taking a speculative position with regard to the current volatility skew.

(ii) Suggest, with reasons, a trading strategy to be adopted where a change in the volatility skew is expected, but with no clear indication of the direction of movement. [2]

(iii) Explain how the graph of the volatility smile may change over time, assuming that the fundamental shape does not change (i.e. it remains in the shape of a smile). [3]

(iv) (a) Give an expression for the delta of a European call option on an underlying equity in terms of partial differentials with respect to the equity price and the implied volatility.

(b) Explain how to delta hedge a portfolio of European options using a volatility smile. [3]

[Total 13]

7 A company has written a large investment guarantee that can be exactly replicated by a five year at-the-money plain vanilla put option on an equity index.

The Board of the company would like to hedge the guarantee but, because of the current high levels of market implied volatilities, is undecided between:

(a) buying the option that would exactly match the guarantee, and

(b) setting up a delta hedging process with the objective of replicating the payoff of the option at expiry.

The Board wishes to understand better these alternative strategies and their associated risks, and has requested numerical analysis of the possible cost of the delta hedging.

It can be assumed that there is a deep and liquid market in futures on the equity index.

(i) List the expert judgements that need to be made, including the parameters that need to be set, in specifying the mechanics of the delta hedging strategy. [3]

(ii) Describe the risks that the company would be taking in choosing to delta hedge rather than buying the option. [5]

(iii) Describe how the possible costs of the delta hedging over the lifetime of the guarantee would be determined. [5]

(iv) Suggest additional information that should be provided to help the Board make an informed decision. [4]

[Total 17]

END OF PAPER