INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

21 September 2018 (am)

Subject ST6 – Finance and Investment Specialist Technical B

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all seven questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 (i) Describe why variance reduction procedures might be used in a stochastic simulation. [2]

Asian options often come in two types: one where the average price of the underlying asset is based on the geometric mean and the other where it depends on the arithmetic mean. There is an analytic formula for the price of the geometric average Asian option using the Black Scholes framework, but there is not a closed form analytic formula for the arithmetic average Asian option.

(ii) Describe how the control variate technique can be used as a variance reduction procedure for valuing an arithmetic average Asian option. [4]

A simulation produces an estimate for the price of an arithmetic average Asian option of 10.30 and an estimate for a geometric average Asian option of 9.41. The Black Scholes analytic value for the geometric average Asian option is 9.57.

(iii) Write down a better estimate under the Black Scholes model for the arithmetic average Asian option than 10.30 by using the control variate technique. [1]

[Total 7]

2 The Cox-Ingersoll-Ross (CIR) model is a one-factor interest-rate model of the short rate $r(t)$ of the following form:

$$dr(t) = a(b - r(t))dt + \sigma\sqrt{r(t)}dz(t)$$

where a, b, σ are constants and $z(t)$ is Brownian motion.

(i) Describe the economic arguments in favour of mean reversion of interest rates. [3]

Under the CIR model, the price $P(t, T)$ of a zero coupon bond of maturity T valued at time t is given by:

$$P(t,T) = A(t,T)\exp\left[-B(t,T)r(t)\right]$$

(ii) Show that the continuously compounded interest rate at time t for a term of $T - t$, $R(t, T)$, is given by:

$$R(t,T) = \frac{B(t,T)r(t)}{T-t} - \frac{\ln A(t,T)}{T-t}$$ [2]

(iii) Comment on the relationship between $r(t)$ and the term structure of the continuously compounded rate of interest $R(t, T)$. [2]

(iv) Justify, in the context of derivative pricing, why it would NOT be appropriate to model bond prices as mean reverting in a risk neutral world. [2]

[Total 9]

3 Let P be a probability measure with associated filtration $F(t)$ and expectation denoted by $E_P[.]$. Consider a process $S(t)$ satisfying $dS(t) = \sigma S(t)dW(t) + \mu S(t)dt$.

(i) Show that $dS^n(t)$ also follows a geometric Brownian motion with respect to P, where $n > 0$. [3]

(ii) Derive an expression for $E_P\left[S^n(t)\middle|F(0)\right]$. [3]

(iii) Justify why any value of $n > 0$ may be suitable for modelling a stock price. [1]

An investment management team is carrying out some model governance of its stock option pricing model. One area being reviewed in depth is the underlying stochastic stock model. This model is currently based on geometric Brownian motion fitted to a given stock. The team also uses a standard Black Scholes option pricing model. Most other market participants use similar models.

(iv) Assess the advantages and disadvantages of using the current choice of stock pricing model and option pricing model compared to updating to a new model. [6]

[Total 13]

4 An investor seeks to maximise his total return from fixed income securities and has been offered a Complex Bond from the local government. The investor may sell the Complex Bond back to the local government for a Strike Price at a single Option Date before the final maturity of the security. At the Option Date:

- If the investor decides to sell the bond, the Strike Price is paid to the investor and no further cashflows are paid.
- If the investor decides NOT to sell the bond, the local government will continue to pay any scheduled coupons and the final redemption payment.

The investor has been offered a Complex Bond with the following features:

- No coupons are payable throughout the life of the security, the bond is redeemable at a par of £100 million, with a final maturity term of 15 years.
- The investor has an option to sell the security back to the local government at an Option Date in 10 years' time for a Strike Price of £90 million.

Risk-free rates are set equal to the yield on the local government bonds which do NOT have options and are currently 2% p.a. at all terms continuously compounded.

From the investor's perspective, the Complex Bond can be decomposed into a long vanilla bond with a maturity term of 15 years and a bond option.

(i) Outline the features of this bond option. [2]

(ii) Calculate the flat level of the yield curve in 10 years' time that would mean the bond option was at the money. [1]

(iii) Explain the key benefit to the investor of investing in the Complex Bond rather than the vanilla bond. [2]

The investor now seeks to value the Complex Bond using Black's formula. Risk-free interest rates are still 2% p.a. continuously compounded at all terms and the volatility of the forward bond price is 10%.

(iv) Write down the Black Formula for valuing the bond option, defining all terms used. [2]

(v) Calculate the value of the Complex Bond. [5]

The investor seeks to understand how the sensitivity of the Complex Bond to interest rates could evolve over the maximum 15-year period. The investor has calculated that at time zero, the price of the Complex Bond will fall by about £30 million if interest rates rise from 2% to 6% p.a. at all terms.

(vi) Sketch a chart which shows how an increase in interest rates from 2% to 6% p.a. would affect the market value of the Complex Bond in each of the next 15 years from now. For the period beyond 10 years, you should assume that the Bond Option was not exercised. [4]

[Total 16]

5 A trader is considering investing in the shares of a company which has a current share price, $S(0)$, of 10.2 per share. The trader is considering two different investment strategies over the following year:

Strategy 1: short sell 1,000 shares for a year and invest in a risk-free one year bond with a fixed return of 1.7% on the initial investment.

Strategy 2: short sell 1,000 shares for a year and purchase 1,000 call options with an expiry date in one year and a strike price of 14. The current price of the call option is 2. Invest the remaining cash in a risk-free one year bond with a fixed return of 1.7% on the initial investment.

Any transaction costs can be ignored and it is assumed that no dividends are paid during the year.

(i) Determine the values of the share price in one year's time such that Strategy 2 provides the higher profit. [4]

(ii) Describe the effect of the announcement of a previously unexpected dividend on the value of a call option and a put option on the underlying stock. [2]

A bank is writing an over-the-counter product tailored to an individual investor. Upon investment the bank deducts a fee of $x\%$ of the investment; the remainder is then invested in the product with it maturing after one year. The product provides a payoff of 99% of the value of the share price after one year (based on a share of the investor's choice) with a minimum guaranteed return of 2%.

(iii) Show that the amount paid to the investor at the end of the year is: $P \times (1 - x\%) \times Max\left\{0.99S(1)/S(0), 1.02\right\}$, where P is the amount invested, $S(t)$ is the share price at time t (measured in years from the investment date), assuming no dividends are paid during the year. [2]

The trader above makes an investment of P in the bank's product based on the shares the trader currently holds. The bank models this share price based on a volatility of 25% per annum and a one year risk-free interest available to the bank of 1.5% continuously compounded.

(iv) Determine the fee deducted of $x\%$ which the bank should set for this product in order to make an expected risk-neutral overall profit of 3% of P at time 0, stating any assumptions made.
[Hint: Consider how the payoff to the trader as given in part (iii) can be rearranged to include a term which can be valued as a call option.] [6]

(v) Assess the effectiveness of the product as a form of profit generation to the bank, including considering a suitable hedging strategy. [3]

[Total 17]

6 An investor is monitoring the price (C) of a long European call option on a company's stock in her portfolio. This stock does not pay dividends. The investor is modelling the stock by assuming it follows geometric Brownian motion. Let δx denote a change in a quantity x.

(i) Derive an expression in terms of the Greeks for δC over a time period δt by using a Taylor expansion of δC with respect to the underlying variables, defining any symbols used and assuming that higher order Greeks aside from gamma can be ignored. [3]

The investor is investigating the effect on C if the value of the underlying variables independently increase or decrease. The investor is preparing a table to record these effects. The table is of the following form:

Effect on C	*[Variable 1]*	…	*[Variable n]*
Variable increases			
Variable decreases			

Here n represents the number of underlying variables that C is sensitive to.
The investor would like to complete the missing entries in the table using either a +, – or a ? sign as follows:

- a + sign should be entered if the value of the call option increases;
- a – sign should be entered if the value of the call option decreases; and
- a ? sign should be entered if there is insufficient information to determine if the value increases or decreases.

(ii) Write down the missing entries in the above table for each underlying variable used in part (i). Please use the +, – and ? notation described above. [4]

The investor has noted that the stock price has decreased compared to ten days ago but the call option price has increased compared to ten days ago.

(iii) Justify how this may be possible. [3]

The investor is concerned about how to hedge against the time to expiry. The formula for the theta of this option under the Black-Scholes-Merton model is:
$\Theta = -\dfrac{S_t N'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2)$, where S_t is the stock price at the current time t, T is the time to expiry, σ is the volatility of the stock, K is the strike price, r is the risk-free rate, $N(.)$ is the cumulative density function of the standard normal distribution, $N'(.)$ is the probability density function of the standard normal distribution, $d_1 = \dfrac{\ln(S_t / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$.

(iv) Determine the behaviour of Θ as $T \to 0$ and $T \to \infty$ when the option is at the money.
[Hint: use the formula for theta given above and simplify the formula by setting variables which are not relevant to your analysis to 1.] [3]

(v) Draw the graph of Θ against time to expiry when the option is at the money. [3]

(vi) Comment on whether the investor is likely to hedge against time to expiry when the option is at the money. [2]

[Total 18]

7 (i) Describe the main benefit of credit ratings for investors in fixed interest investments. [1]

A bank owns a risky vanilla 10-year corporate bond ("Corporate Bond") of par value £100 million and a current market value of £100 million which it has been struggling to sell in the market. The bank is therefore proposing to raise cash by restructuring the Corporate Bond into a senior tranche ("Senior Note") and an equity tranche using a bankruptcy-remote Special Purpose Vehicle (SPV). The bank is proposing that the Senior Note will be £55 million notional, £55 million market value and will be sold to individual investors. The Senior Note will be structured so that it has no interest rate sensitivity on issue. The equity tranche will be retained by the bank.

(ii) Draw a simple diagram showing both the cashflows that will arise when the securitisation is set up and throughout the life of the SPV. [3]

(iii) Explain how investing in the Senior Note reduces the default risk for an investor compared to investing in the underlying Corporate Bond. [1]

A credit rating agency (CRA) is determining the rating of the Senior Note by carrying out scenario analysis. The CRA calculates the loss to the Senior Note under two scenarios:

- an immediate default of the Corporate Bond; and
- an increase in interest rates of 2% and immediate default of the Corporate Bond.

The CRA selects the highest rating possible for the Senior Note that satisfies both of the thresholds in the table below. The CRA assumes a recovery value of 40% of the market value of the Corporate Bond just prior to default. The CRA calculates the impact of interest rate movements on the Corporate Bond by using a duration of 10 years (with no allowance for convexity).

Rating of Senior Note	*Criteria 1: Senior Note market value loss on immediate default of Corporate Bond must be no greater than:*	*Criteria 2: Senior Note market value loss if interest rates rise by 2% and then Corporate Bond defaults must be no greater than:*
AAA	£6m	£10m
AA	£16m	£20m
A	£26m	£30m
BBB	£36m	£40m

(iv) (a) Calculate the loss on the Senior Note on immediate default of the Corporate Bond.

(b) Calculate the approximate loss on the Senior Note if rates rise by 2% and then the Corporate Bond defaults.
[Hint: the market value of the Senior Note is not sensitive to changes in interest rates.]

(c) Show that the rating agency will select a rating of A, using the results from parts (a) and (b), or otherwise.

[4]

(v) Outline the weaknesses of the CRA's approach to rating the Senior Note. [3]

The bank seeks to improve the attractiveness of the Senior Note by adding a pay fixed interest rate swap with duration 5 years to the SPV.

(vi) Comment on the implications of adding this interest rate swap to the SPV. [4]

(vii) Determine the minimal notional of the interest rate swap required to improve the rating to AA. [2]

(viii) Suggest ways that the bank could alter the securitisation to achieve a AAA rating from the CRA for the Senior Note. [2]

[Total 20]

END OF PAPER