INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

29 April 2015 (am)

Subject ST6 – Finance and Investment
Specialist Technical B

Time allowed: Three hours

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You have 15 minutes before the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*

3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all seven questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 An asset manager has previously offered an investment fund which provides a broad exposure to corporate bonds and which hedges out exposure to interest rates using swaps. The hedge aims to reduce the interest rate duration of the funds to close to zero.

(i) Outline the likely constituents of the fund and the type of investor that the asset manager is targeting. [3]

(ii) (a) Explain the problems that this fund faces given the move to the use of clearing via central counterparties.

(b) Suggest alternative approaches that could be taken to achieve the same broad exposure to credit, without material exposure to interest rates. [3]

(iii) Describe the risks that the asset manager and investors face in respect of this fund. [4]
[Total 10]

2 Let $\mathbb{P}$ be a probability measure and let M_t be a stochastic process.

(i) State the necessary and sufficient conditions for M_t to be a martingale with respect to $\mathbb{P}$. [1]

Let B_t be a $\mathbb{P}$-Brownian motion.

(ii) (a) Demonstrate, using expectations, that $M_t = B_t^2 - t$ is a martingale with respect to $\mathbb{P}$, stating any assumptions made.

(b) Express this equation as a stochastic differential equation, using Ito's lemma. [4]

Let $S_t = B_t^2 - (2\gamma B_t + 1)t + \gamma^2 t^2$ be the price process assumed to be followed by an asset in a market, where B_t is a $\mathbb{P}$-Brownian motion and γ^2 is a constant characteristic of the asset.

(iii) Demonstrate that a measure $\mathbb{Q}$ exists under which S_t is a martingale. [4]

(iv) Describe how a derivative with a payout X at time T and an underlying asset with price S_t (which follows the above process) could be priced in a zero interest rate world. [5]

(v) Explain the difficulties in using this asset price process to price a derivative in a non-zero interest rate world. [2]
[Total 16]

3 An executive has been offered a position as Chief Executive Officer (CEO) of a large listed organisation and is attempting to calculate the value of the total remuneration package being offered to him.

Part of the package is an executive share option scheme. Details of the scheme are as follows:

- On joining, the CEO receives a fixed number of options to buy shares in the organisation on one particular date each year.
- The exercise price of the options is equal to the organisation's share price on the date at which the options are awarded.
- The options cannot be exercised for the first three years and if the CEO leaves (for whatever reason) within three years the options expire with no value.
- If the CEO leaves after three years, the options are automatically exercised or expire with no value.

The number of share options is small compared to the number of shares currently issued, so the dilution effect of exercising options can be ignored.

The organisation pays annual dividends.

One valuation methodology that the CEO is considering is to assume that he would only exercise the options on leaving the company and to assume a fixed set of probabilities for his exit from the organisation on the specified date each year.

(i) Outline how the CEO could value the share options in this case. [4]

Instead, the CEO could develop a model where the probability of his exit depends on the share price and where he always chooses to exercise the option early if the share price reaches 200% of the exercise price.

(ii) Recommend, with reasons, whether the share options should be valued in this case using a tree or Monte Carlo method. [4]

[Total 8]

4 The current risk-free yield curve is as follows, with all interest rates expressed as continuously compounded per annum.

Term (years)	*Risk-free spot rate*
0.5	0.54%
1.0	0.61%
1.5	0.73%
2.0	0.85%
2.5	1.00%
3.0	1.15%
3.5	1.32%
4.0	1.48%
4.5	1.64%
5.0	1.80%
5.5	1.94%
6.0	2.08%

All instruments considered below are payable semi-annually and the respective rates are convertible half-yearly.

(i) Show that the forward swap rate for a three year swap starting at time 3 is 3.02% p.a. [4]

A financial institution is committed to paying 3.5% p.a. interest (convertible half-yearly and payable semi-annually) on a $100m nominal amount for three years from time 3. The investor is likewise committed to depositing $100m with the financial institution and earning 3.5% p.a. during that period.

The financial institution is considering hedging this deal against interest rate falls. It is considering two alternative hedging deals:

- Entering into a forward rate agreement (FRA) to receive 3.5% p.a. on the $100m nominal over the three year period from time 3.

- Purchasing an <u>at-the-money</u> receiver swaption on a three-year swap with an exercise date in three years' time.

The Black volatility for such an at-the-money "three into three" swaption is 20% p.a.

(ii) (a) Calculate the price of the FRA.

(b) State whether the financial institution would pay or receive this amount.

[3]

(iii) Calculate the price of the swaption. [5]

(iv) Compare the two hedging approaches. [4]

[Total 16]

5 A model is being developed to value derivatives with a payoff dependent on the six-month interest rate. The model will be based on the underlying assumption that the six-month annualised, continuously compounded interest rate R follows a Hull-White process:

$$dR = [\theta(t) - aR]dt + \sigma dz.$$

As a starting point, a trinomial tree is being built for a variable R^* which starts at zero and follows the process:

$$dR^* = -aR^* dt + \sigma dz.$$

The tree has time steps equal to six months and the spacing between different values of R^* will be $\Delta R = \sqrt{1.5}\sigma$.

Define (i,j) as the node on the tree where $t = i/2$ years and $R^* = \sqrt{1.5}\sigma j$. Assume that node (i,j) has a standard branching pattern, where R^* can increase, decrease or stay fixed with probabilities p_u, p_d and p_m respectively.

(i) Write down, with brief justifications, the basic equations that need to be satisfied by the three probabilities. [You do not need to solve these equations.] [3]

After constructing the tree, it is found that at the top and bottom extremes the solutions to the equations for the probabilities do not lie in [0,1].

(ii) Describe how the tree structure should be revised in order to eliminate this problem. [2]

(iii) Write down the new equations that the probabilities need to satisfy. [You do not need to derive the new probabilities.] [5]

(iv) Describe how the tree for R^* would be converted into a tree for R. [3]
[Total 13]

6 A global technology company has an overseas subsidiary which develops the batteries for its laptops and smartphones. The batteries require the use of the metal lithium. There are currently no futures traded for this commodity, so the company has put in place a trade agreement with one of the lithium suppliers, which is subject to frequent renegotiations.

(i) (a) Define a consumption asset, the cost of carry and the convenience yield.

(b) Explain why the convenience yield exists.

(c) Write down an expression for the futures price for a consumption asset, defining all terms used.

[6]

Due to the volatility of the current trade agreement with the lithium supplier (and lithium prices in general), the technology company is considering cross hedging its lithium position.

The company has identified oil futures as a possible hedge, due to a high correlation between changes in the prices of oil and lithium in recent years.

In order to determine the optimal number of oil futures to buy, the company is considering using the Sharpe ratio for the portfolio instead of the more common minimum variance hedge ratio ($\mathrm{Var}(R_h)$), where R_h is defined below. The Sharpe ratio is defined as:

$$\frac{E(R_h)-r}{\sqrt{\mathrm{Var}(R_h)}},$$

where R_h is the return on the hedged portfolio, $E(R_h)$ is the expected return on the hedged portfolio, $\mathrm{Var}(R_h)$ is the variance of return on the hedged portfolio and r is the constant risk-free rate.

(ii) (a) Explain what the Sharpe ratio shows and how it differs from the minimum variance hedge ratio.

(b) Outline the disadvantages of using the Sharpe ratio.

(c) State the condition that should be imposed on the Sharpe ratio in order to determine the optimal number of futures to buy.

[3]

(iii) Define the term optimal hedge ratio. [1]

It can be shown that the optimal hedge ratio for the Sharpe ratio is:

$$-\frac{\sigma(R_s)}{\sigma(R_f)}\left\{\frac{\left[\frac{\sigma(R_s)}{\sigma(R_f)}\frac{E(R_f)}{[E(R_s)-r]}\right]-\rho}{1-\frac{\sigma(R_s)}{\sigma(R_f)}\frac{E(R_f)}{[E(R_s)-r]}\rho}\right\}$$

where $\sigma(R_s)$ is the standard deviation of the return on the underlying asset, $\sigma(R_f)$ is the standard deviation of the return on the future, $E(R_s)$ is the expected return on the underlying asset, $E(R_f)$ is the expected return on the future and ρ is the correlation coefficient between R_s and R_f.

The optimal hedge ratio for the minimum variance hedge ratio is $\rho\sigma(R_s)/\sigma(R_f)$.

(iv) (a) State the condition under which the two optimal hedge ratios are equal.

(b) Comment on this condition.

[2]

The company has the following information relating to the spot price of lithium: $E(R_s)$ = 3% p.a. and $\sigma(R_s)$ = 13% p.a. For the oil future: $E(R_f)$ = 2.7% p.a. and $\sigma(R_f)$ = 15% p.a. The relevant correlation coefficient is 0.8 and the risk-free return is 2% p.a.

(v) (a) Calculate the optimal hedge ratio for the Sharpe ratio and for the minimum variance hedge ratio.

(b) Comment on these results.

[2]

(vi) Suggest other methods that the company could implement to mitigate the risks associated with purchasing lithium. [3]

[Total 17]

7 (i) Show that, using the standard Black-Scholes notation:

$$Se^{-qT} N'(d_1) = Ke^{-rT} N'(d_2)$$

where $N'(x) = e^{-x^2/2} / \sqrt{2\pi}$. [2]

(ii) Show, using part (i), that the sensitivity of a call option price (c) to changes in the exercise price (K) is

$$\partial c / \partial K = -e^{-rT} N(d_2).$$ [3]

A life insurance company is considering putting in place a zero cost collar to hedge its \$10m equity portfolio. The hedge consists of:

- a one-year European put option on an equity price index with strike \$8m, and
- a shorted one-year European call option on the same index with strike K.

Market implied volatility for one-year options is given by 25% – (A – 80%) × B, where A is the exercise price as a proportion of the spot price and B is a fixed constant.

(iii) (a) State whether B would be expected to be positive or negative under typical market conditions.

(b) Justify your statement, by reference to a normal feature of option markets.

(c) Suggest possible reasons for this feature of option markets.

[2]

The one-year risk-free rate is 3.3% p.a. continuously compounded. Dividend income on the index is 1% p.a. payable continuously. B is currently 0.1.

(iv) Show that a strike of K = \$12.55m for the call would result in a zero cost collar. [9]

Under a small change in market volatilities of the form

$$\sigma_{new} = 25\% - (A - 80\%) \times (0.1 + \varepsilon)$$

it is necessary to change the strike K of the call in order for the collar to remain zero cost.

(v) Derive a formula to estimate to first order the required change in strike K of the call under these conditions.

Hint: The vega of a call on a dividend paying stock is $Se^{-qT} N'(d_1)\sqrt{T}$. [4]

[Total 20]

END OF PAPER