INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

3 October 2017 (pm)

Subject ST6 – Finance and Investment
Specialist Technical B

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all six questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 (i) Describe the key benefits of trading derivatives under an International Swaps and Derivatives Association (ISDA) master agreement. [3]

A financial institution will soon trade equity derivatives for the first time. The derivatives will not be subject to central clearing and fixed interest securities will be used as collateral.

(ii) Outline four restrictions that could be placed on the fixed interest securities received as collateral, in order to reduce counterparty risk. [2]

The financial institution is looking to agree a Minimum Transfer Amount (MTA). If the difference between the market value of the outstanding derivatives and the collateral account is greater than the MTA at the end of a business day, then collateral will be exchanged. If the difference is less than the MTA at the end of the day, no collateral will be exchanged.

(iii) Explain the main advantage and the main disadvantage of having a (non-zero) MTA. [2]

The institution expects to trade \$100 million notional of equity futures at the start of the first trading day and the position will be fully collateralised at the time of trading. It wishes to set the MTA level such that there is a less than 10% chance of exchanging collateral at the end of that day.

(iv) Estimate, stating your assumptions, the MTA level that would be needed to meet the institution's requirement. [Hint: you may assume that equity returns are normally distributed.] [3]

[Total 10]

2 (i) Compare the key features of a European and an American option. [2]

(ii) Explain why American options are hard to value. [1]

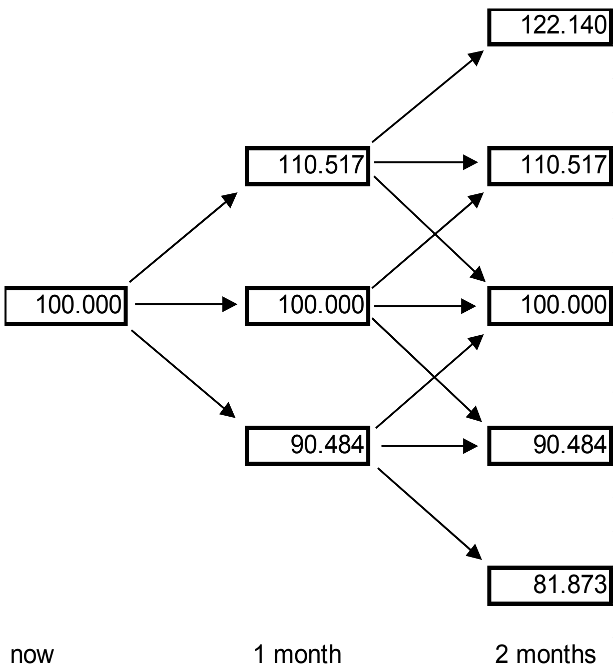
An insurer is reviewing its pricing models for American options on non-dividend-paying equities.

(iii) Describe how Monte Carlo simulations, using the method of least squares, can be used to price American options. [8]

The insurer is instead considering using a trinomial tree. As an assessment of this possibility, the insurer is valuing the following American put option using the following two-period recombining trinomial tree and parameters:

- Period to expiry = 2 months
- Strike price = 98
- Current price of the underlying asset = 100
- Continuously compounded risk-free rate = 3% per annum
- Risk-neutral probability of an up movement of the price of the underlying over a month = 0.255
- Risk-neutral probability of a down movement of the price of the underlying over a month = 0.245
- Risk-neutral probability of no movement of the price of the underlying over a month = 0.500

The trinomial tree of the possible price movements of the underlying is as follows:



(iv) Calculate the value now of the option, using this trinomial tree. [5]

[Total 16]

3 (i) State the requirements for a probability measure P to be equivalent to another probability measure Q. [1]

Let $X(t)$ be a non-zero stochastic process with volatility $\sigma(t) = \sigma X(t)$ under P, where σ is a positive constant. Under Q, $X(t)$ has a volatility of σ.

(ii) Determine whether P and Q are equivalent measures. [3]

An asset has a price process $S(t)$ satisfying $dS(t) = \sigma S(t)dW(t) + \mu S(t)dt$. $W(t)$ is standard Brownian motion under a probability measure P, and both σ and μ are constants. Let r be the continuously compounded risk-free rate.

(iii) Determine a probability measure Q such that the discounted asset price $\overline{S}(t) = e^{-rt}S(t)$ is a Q-martingale. [7]

An option on this asset matures at time T.

(iv) Express as a mathematical formula (defining any symbols used) the payoff at time T for this option if it is:

(a) an Asian put option with fixed strike price K and arithmetic averaging over the whole period.

(b) a European lookback put option.

[2]

(v) Justify whether or not the options in part (iv) can be priced using the Black-Scholes model. [3]

[Total 16]

4 (i) Explain how a survivor cap can reduce longevity risk. [2]

A bank is about to launch a new product called "LifeCap". This longevity derivative is purchased at time $t = 0$ and provides the following payoff at time T to the purchaser:

$$\text{Payoff}(T) = \text{Max}[L - K, 0]$$

where:

- L is the expected future lifetime of a 65-year-old male at time T.
- K is the strike.

The expected future lifetime of the 65-year-old male, L, will be calculated independently from the bank by the local office for statistics at time T based on the average population.

The bank will offer contracts with a range of maturities at times $T = 1$, 2, 5 and 10 years.

(ii) Contrast the LifeCap product and a survivor cap. [2]

The bank is proposing to use the Black model to value the LifeCap derivative.

(iii) Write down the relevant Black model formula for the value of LifeCap, defining all additional terms used. [2]

(iv) (a) Outline the assumptions the bank is making by using the Black model.

(b) Comment on how suitable these assumptions are for the LifeCap product.

[4]

(v) Outline how the bank can use the formula in part (iii) to estimate the vega of the derivative. [2]

A large closed pension fund with a small deficit has only deferred and pensioner members. The trustees of the pension scheme wish to reduce exposure to longevity risk as much as possible throughout the remaining life of the scheme's members.

(vi) Assess the suitability of LifeCap for this pension fund. [7]

[Total 19]

5 A financial institution has a single fixed liability of \$1,000 million payable at the end of year 20 (i.e. at time $t = 20$, where t is measured in years). Every year-end, the company has to disclose a Statutory Value of this liability based on a solvency methodology prescribed by the regulator. Under the Old Solvency method, liabilities are discounted at a single fixed rate of 4% per annum continuously compounded.

(i) Show that the value of the liability under the Old Solvency method would have been \$549 million at time $t = 5$. [1]

The regulator has decided to replace the Old Solvency method with a New Solvency method at time $t = 0$. Under the New Solvency method, the single discount rate used will be 4% per annum at all terms for the year-end just passed (i.e. $t = 0$), continuously compounded. However, at each year-end over the next ten years this single discount rate will transition linearly to the prevailing swap rate, i% per annum, continuously compounded. For the valuations taking place at the end of years 10 to 20, the discount rate will be the prevailing swap rate, continuously compounded.

The New Solvency method discount rate for each year-end, t, continuously compounded, is therefore:

$$NewSolvencyDiscountRate = 4\% \times \text{Max}\left(1-\frac{t}{10},0\right)+i\%\times\text{Min}\left(\frac{t}{10},1\right)$$

where $0 \le t \le 20$.

(ii) (a) Show that the Statutory Value of the liability at time $t = 5$ is \$638 million, assuming the swap rate, i%, is 2.0% per annum.

(b) Show that the Statutory Value of the liability at time $t = 5$ would increase from \$638 million by around \$5 million if the swap rate decreased by 0.1% per annum, i.e. from 2.0% per annum to 1.9% per annum.

[3]

(iii) Sketch a graph showing approximately how the Statutory Value of the liability would be affected by a 0.1% per annum instantaneous reduction in the swap rate (i%) at each year-end t, up until the liability is due at time $t = 20$ (i.e. for $0 \le t \le 20$), assuming the swap rate (i%) is 2.0% per annum at all terms at each year end.

[3]

The financial institution is now interested in hedging the instantaneous movement of the Statutory Value of the liabilities from movements in the swap curve. It does not currently invest in any assets that are sensitive to movements in interest rates, but has been advised that it could use Treasury bond futures.

(iv) Describe the key features of a Treasury bond future. [2]

(v) Explain the basis risk involved in using Treasury bond futures to hedge the Statutory Value of the liability. [3]

(vi) Explain why the Treasury bond futures would likely need to be rolled. [1]

(vii) Discuss the challenges involved in rolling the Treasury bond futures and in determining the number of futures required at each roll. [3]

(viii) Suggest other reasons why the financial institution may choose not to hedge the instantaneous interest rate sensitivity implied by the New Solvency method. [3]

[Total 19]

6 (i) Describe the purposes of the foreign exchange market. [3]

A business based in Europe receives income primarily in Euros. It has a small known future payment to make in US dollars of $\$D$ which it wants to hedge back to Euros. It has therefore decided to purchase D call options on the exchange rate of 1 US dollar to Euros, in order to hedge its exposure to US dollars.

(ii) (a) Explain how this strategy would hedge the exposure.

(b) Outline the three aspects of these options which the business will need to select in order to meet its needs.

(c) Outline the key advantage, other than hedging the US dollars exposure, and key disadvantage of this chosen approach.

[5]

A US bank with an office in Europe provides an over-the-counter European call option to meet the business's needs. The business purchases D of these options for a premium of P Euros per option, which is paid directly to the bank. The policy of the bank is to exchange this premium directly into US dollars (at the spot rate) when it is received.

The delta of the (long) option is given by:

$$\Delta = e^{-r_{US}\tau}\Phi(d_1),$$

where:

- r_{US} is the continuously compounded risk-free US interest rate.
- τ is the time to expiry of the option.
- Φ is the cumulative distribution function of the standard normal distribution.
- $d_1 = \dfrac{\ln(Se^{(r_{EURO}-r_{US})\tau} / K) + 0.5\sigma^2\tau}{\sigma\sqrt{\tau}}$.
- r_{EURO} is the continuously compounded risk-free Eurozone interest rate.
- S is the current spot exchange rate of 1 US dollar into Euros.
- K is the strike exchange rate of 1 US dollar into Euros.
- σ is the volatility of the spot exchange rate.

The gamma of this option can be assumed to be negligible.

(iii) Explain what Δ represents in terms of delta-hedging to the bank. [2]

Let C be the current price in Euros of one of the options above, which was sold to the business to buy a notional amount of 1 US dollar. This price is calculated based on the bank's internal model.

The bank holds an amount $E \times D$ of Euros. It wishes to make the value of its combined portfolio of these Euros and the D short options sold to be delta-neutral in US dollar value terms (i.e. independent of exchange rate S to first order).

(iv) Show that the bank should choose E to satisfy the equation:

$$\Delta - C/S + E/S = 0.$$

[Hint: Start by converting the value of the portfolio to US dollars.]

[4]

The bank decides to define a new delta: $\delta = \Delta - C/S$.

(v) State how the new delta can be used by the bank in delta-hedging the above option. [1]

Under the Black-Scholes model, the current price of this option is given by:

$$C = e^{-r_{US}\tau} S\Phi(d_1) - e^{-r_{EURO}\tau} K\Phi(d_2),$$

where $d_2 = d_1 - \sigma\sqrt{\tau}$.

(vi) Show that $\delta = K/S \, e^{-r_{EURO}\tau}\Phi(d_2)$. [1]

(vii) Sketch on the same graph delta and new delta against spot price, for a given time to expiry τ. You can assume that $\delta < \Delta$ for all values of the spot price.

[4]

[Total 20]

END OF PAPER