INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

27 September 2012 (pm)

Subject ST6 – Finance and Investment Specialist Technical B

Time allowed: Three hours

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You have 15 minutes before the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*
3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all eight questions, beginning your answer to each question on a separate sheet.*
6. *Candidates should show calculations where this is appropriate.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

NOTE: In this examination, you are never required to prove the use of an arbitrage-free methodology unless clearly stated in the question.

1 Consider a stochastic process Z_i that evolves over discrete time i, $0 \le i \le T$.

(i) Define the following terms as they relate to this process:

(a) binomial probability measure **P**

(b) filtration F_i at time i

(c) previsible process φ_i

(d) conditional expectation $E_{\mathbf{P}}(Z_j \mid F_i)$, $j \ge i$

(e) **P**-martingale

[5]

(ii) (a) State the Binomial Representation Theorem (BRT) for a binomial probability measure **P** and a **P**-martingale process M.

(b) Outline how the BRT can be applied to the pricing of contingent claims on a stock.

[3]
[Total 8]

2 A UK manufacturer of aircraft equipment is set to deliver a US$1.5 million sales order to a firm in the United States for settlement in 12 months. It is considering hedging the currency exposure associated with this order, by either:

- Shorting exchange-traded US$/sterling futures for one year; or
- Shorting a liquid US$ asset (such as a US equity) for one year

(i) Describe how each of these hedges would be constructed and maintained. [5]

(ii) (a) Compare the risks associated with the two hedges.

(b) Propose, with reasons, appropriate modifications to the target asset for each hedge that would improve effectiveness and/or reduce market risk. [5]

[Total 10]

3 The price S of a non-dividend paying asset follows a geometric Brownian motion process:

$$dS = \mu\, S\, dt + \sigma\, S\, dW_t$$

where μ, σ are positive constants and W_t is a standard Brownian motion.

(i) Show that the process $X = S^n$, for $n \neq 0$, also follows a geometric Brownian motion. [3]

Consider two currencies, A and B. The price of currency A expressed in terms of currency B follows the same geometric Brownian motion process as S above, with growth rate $\mu = r_B - r_A$, the difference between the interest rates in currencies B and A respectively.

(ii) Show that the process followed by the price of currency B expressed in terms of currency A is also a geometric Brownian motion but with growth rate equal to $r_A - r_B + \sigma^2$. [3]

A UK insurance company pays an annual licence fee for a major suite of software to a European software vendor. The licence fee was set last year at €265,000, and the vendor has just communicated that this year's renewal, due in six months' time, will be significantly higher to allow for inflation and additional functionality.

As a concession, the vendor is offering the insurance company a choice of deciding on the payment date whether to settle in Euros at €300,000 or in sterling at £250,000. The vendor is able to offer this choice since it has budgeted for sterling transactions during that book-keeping year at a rate of €1.20 per £1.

(iii) Discuss this payment arrangement as a currency transaction from the viewpoint of both the insurance company and the software vendor. [4]

[Total 10]

4 (i) Show that, at time t, the theta of a European Call option on a non-dividend paying stock of price S_t is:

$$\theta_C = -S_t \varphi(d_1) \frac{\sigma}{2\sqrt{T-t}} - rKe^{-r(T-t)} N(d_2)$$

where the strike is K, expiry time is T, risk-free interest rate is r, volatility is σ, $\varphi(x) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}x^2)$ and the terms $N(..)$, d_1 and d_2 are as defined in the Black-Scholes formula at time t, $0 \le t < T$. [4]

[*Hint: You may use without proof the equation* $Ke^{-r(T-t)}\varphi(d_2) = S_t\varphi(d_1)$.]

(ii) Derive, using Put-Call parity or otherwise, the equivalent expression for the theta of a European Put option on a non-dividend paying stock. [2]

(iii) (a) Sketch a graph showing how the theta of a European Call varies with the stock price.

(b) Sketch a graph showing how the theta of a European Call varies with time to maturity, comparing options that are in-the-money, at-the-money and out-of-the-money.

[4]

(iv) Discuss the significance to a trader of the theta of a portfolio of options. [2]

[Total 12]

5 Consider a 4% annual coupon corporate bond with five years to expiry which is convertible at any point into a non-dividend paying equity at par, i.e. £1 nominal of bond is convertible into one share. An exactly equivalent bond from the same issuer but without the conversion option yields 7% per annum (annually compounded).

The following current market conditions apply:

- price of the equity is 100 pence (i.e. £1)
- volatility of the equity is 20% per annum
- risk-free money market rates are 5% per annum (continuously compounded) for all maturities
- government par bond yields are 5% per annum (annually compounded) for all maturities

(i) (a) Explain why conversion before maturity is not optimal.

(b) Estimate the value of the convertible bond using the Black-Scholes formula for a European option.

(c) Discuss the factors that might make the true value of the convertible different from this estimate.

[8]

(ii) Derive the positions in the underlying equity and other risk-free instruments required to hedge a £10 million nominal holding of the convertible bond. You may assume that a five-year government bond and the non-convertible bond have the same modified duration. [3]

[Total 11]

6 (i) (a) Define a mean-reverting stochastic process.

(b) Explain why, in the context of derivative pricing, a mean-reverting process is not appropriate for modelling bond prices, but is appropriate for modelling short-term interest rates.

[3]

A market risk specialist wishes to model a number of more complex interest rate options, including Bermudan swaptions. He is considering various choices of one-factor models as follows:

- Cox-Ingersoll-Ross
- Hull & White
- Heath Jarrow Morton (HJM)
- LIBOR Market Model

(ii) Explain the significance of using a single stochastic factor when modelling Bermudan swaptions. [2]

(iii) Outline the main features of <u>three</u> of the above models, giving the main risk-neutral process adopted and briefly discussing their advantages and disadvantages. [8]

An interest rate r follows the Cox-Ingersoll-Ross stochastic process:

$$dr = a(b-r)dt + c\sqrt{r}dz$$

but in the real world with market price of risk λ, where a, b and c are constants and z is a standard Brownian motion.

(iv) Derive the process that the interest rate follows in the risk-neutral world. [3]

[Total 16]

7 A trader on the equity option desk of a small investment bank is attempting to derive the market implied volatility for a one-year European Put option with a strike of 800 on a dividend bearing share index. Current market conditions are as follows:

- Current index value = 1,000
- One-year risk-free rate = 1% per annum continuously compounded
- Dividend yield on the share index = 2.5% per annum continuously compounded
- Market price of Put option = 14.8

(i) Show that the price of the Put option would be 13.5 if a volatility of 20% per annum were assumed. [3]

Using 20% per annum volatility, the trader has calculated that the vega of the Put option is 203, or 2.03 per percentage point in volatility.

(ii) (a) Estimate the implied volatility of the Put option at its market price.

(b) Discuss how to assess the value in (a) against other potentially available implied volatility data. [3]

The trader extends the calculation to determine the implied volatilities for a range of strikes and terms and has produced the following volatility matrix of values:

	European Put on stock			**European Put on currency**		
	1 year	3 year	5 year	1 year	3 year	5 year
Out-of-the-money	a_1	a_3	a_5	d_1	d_3	d_5
At-the-money	b_1	b_3	b_5	e_1	e_3	e_5
In-the-money	c_1	c_3	c_5	f_1	f_3	f_5

(iii) Explain under what conditions you would expect:

(a) $a_5 > b_5 > c_5$
(b) $d_3 > e_3$ and $f_3 > e_3$
(c) $b_1 > b_3 > b_5$

[4]

BSM, a rival bank with a much larger capital base, is considering using an alternative pricing approach whereby, for pricing purposes, volatility is estimated from historical data rather than inferred from option prices.

(iv) (a) Describe how such a volatility might be derived, how much data to use and how to allow for price jumps/crashes. You may assume that the index pays dividends smoothly and continuously.

(b) Describe the potential consequences were BSM to adopt this pricing policy.

[7]
[Total 17]

8 An insurance company has purchased a large amount of credit protection on a five-year reference entity issued by Country A and has sold the same amount of protection on a five-year reference entity issued by Country B. The countries are of similar size and are in the same economic trade area. Both are expected to deteriorate in credit quality over time, with Country B initially less likely to default than Country A but with the possibility of deteriorating much faster in the future.

The transactions have been arranged with a major international bank using five-year credit default swaps (CDS), and have been undertaken at a time of rising CDS spreads and some instability in the credit markets. The company does not own either of the reference bonds, or indeed any sovereign bonds issued by Country A or Country B.

Published research by a major credit rating agency gives the default probability (conditional on no earlier default) for Country A as 2% during the first year, increasing by 0.5% each year thereafter. For Country B, the default probability is 1% during each of the years 1 and 2, then 5% for each year thereafter. The agency suggests a likely constant recovery rate of 20% for both countries, with any default taking place at the end of a year. The risk-free interest rate can be assumed to be zero for all maturities.

(i) (a) Suggest reasons why the insurance company might hold this CDS spread position.

(b) Outline the main risks that the insurance company is taking on with this transaction, excluding legal and operational risks.

[4]

(ii) Determine the theoretical spread for the CDS on Country A, fully using the information given. [5]

(iii) (a) Show that the value in part (ii) is almost exactly equal to the average default rate multiplied by the loss given default (LGD).
[*LGD = 1 – expected recovery rate*]

(b) Hence estimate the spread for the CDS on Country B.

[2]

After a few months the spread on each CDS moves higher still, but with the difference between them remaining similar. Liquidity has declined in the market, and there are rumours that Countries A and B might ban speculative buying of CDS protection by companies who do not own the underlying bonds.

(iv) (a) Explain how liquidity risk could affect CDS prices.

(b) Suggest why the market value of the CDS spread could be different to its theoretical price.

(c) Describe the impact on the insurance company’s position that a ban on speculation might have.

[5]
[Total 16]

END OF PAPER