INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

18 April 2018 (pm)

Subject ST6 – Finance and Investment
Specialist Technical B

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all six questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 (i) Describe the type of trade which is undertaken by an arbitrageur. [1]

An arbitrageur is considering trading a foreign exchange (FX) forward on the United States Dollar (USD) / Great British Pound (GBP) exchange rate.

Let F_0 be the forward price in USD of one GBP and S_0 be the spot price in USD of one GBP.

Let r_{USD} and r_{GBP} be the risk-free continuously compounded interest rates in USD and GBP respectively, and let T be the term of the forward contract.

(ii) Show, by the principle of no arbitrage, that the forward price is given by:

$$F_0 = S_0 e^{(r_{USD} - r_{GBP})T}.$$ [4]

(iii) State any assumptions used to derive the relationship in part (ii). [2]

The market price deviates from the forward price given in part (ii).

(iv) Assess whether the arbitrageur will be able to achieve risk-free profits if they are required to clear their FX forward trades through a central counterparty. [3]

[Total 10]

2 A bank sells a range of interest rate derivatives and uses the Black-Karasinski model for the risk-free short rate, $r(t)$, at time t. The bank uses the following stochastic differential equation (SDE) for $r(t)$ under the Black-Karasinski model:

$$d \ln r = \left[\theta(t) - a \ln r\right] dt + \sigma dz(t)$$

where $\theta(t)$ is a time-dependent function, a and σ are constants and $z(t)$ is standard Brownian motion.

(i) Describe the key steps the bank needs to follow in building a recombining trinomial tree for the short rate under the Black-Karasinski model. Your answer should include the main equations, but does not need to include detailed equations on the derivation of probabilities at each node. [6]

(ii) Explain how the Black-Karasinski model prevents negative values for $r(t)$. [1]

Following falls in interest rates to new historical lows, the bank is concerned that negative interest rates will occur in future with a high probability. The bank wants to ensure that its valuation models will still be robust as interest rates fall. It is therefore now considering using the Hull-White model, with the following SDE:

$$dr = \left[\tilde{\theta}(t) - \tilde{a}r\right]dt + \tilde{\sigma}d\tilde{z}(t)$$

where $\tilde{\theta}(t)$ is a time-dependent function, $\tilde{a}$ and $\tilde{\sigma}$ are constants and $\tilde{z}(t)$ is standard Brownian motion.

(iii) Explain why the Hull-White model can lead to negative values for $r(t)$. [1]

(iv) Assess whether using the Hull-White model is likely to meet the bank's requirement of having a robust interest rate model if interest rates go negative. [3]

The bank has now decided to switch to using the Hull-White model, with $\tilde{\theta}(t)$ approximately given by:

$$\tilde{\theta}(t) \approx \frac{dF(0,t)}{dt} + \tilde{a}F(0,t)$$

where $F(0,t)$ is the instantaneous forward rate for a maturity t as seen at time zero and $\tilde{a}$ is the same constant as in the SDE for the short rate $r(t)$.

(v) Explain what this equation implies about the expected drift of the short rate over time. [Hint: Substitute the above equation into the main SDE.] [2]

[Total 13]

3 An investment manager has a portfolio of options on a single underlying asset.

(i) Explain whether a position in this underlying asset could be used to make the portfolio gamma-neutral. [2]

The investment manager ensures that the portfolio is delta-hedged at the start of each business day. It would require the investment manager to purchase vanilla options to gamma hedge the portfolio.

(ii) Explain why the gamma of the delta-hedged portfolio must be negative. [2]

(iii) Derive the following approximation for a vanilla option:

$$\text{change in option value} \approx \Delta \times \text{change in underlying price}$$
$$+\frac{1}{2}\Gamma \times (\text{change in underlying price})^2 ,$$

where Δ is the delta of the option and Γ is the gamma of the option. [3]

The portfolio, on a given day, is delta-neutral at the start of the business day but has a gamma of –3,706.2.

(iv) Identify the potential variation in the value of the portfolio on this day, using your answer to part (iii) to justify this. [3]

The delta and gamma of a particular traded option on the underlying asset are 0.70 and 1.74 respectively on the given day.

(v) Determine the transactions in the traded option and/or underlying asset which are required to make the portfolio both delta-neutral and gamma-neutral. [3]

(vi) Propose alterations to the original portfolio to make it less gamma negative. [1]

(vii) Recommend another action that the company should take to mitigate the risk of a large negative gamma position occurring in future. [1]
[Total 15]

4 (i) Explain the constraints on $f(X(t))$ in order for Itô's lemma to apply. [2]

(ii) Discuss the relevance of Itô's lemma in the valuation of derivatives. [3]

Let $X(t)$ and $Y(t)$ be two stochastic processes adapted to the same Brownian motion $W(t)$:

$$dX(t) = \sigma(t)dW(t) + \mu(t)dt,$$
$$dY(t) = \rho(t)dW(t) + \nu(t)dt.$$

(iii) Prove, using Itô's lemma, that:

$$d(X(t)Y(t)) = X(t)dY(t) + Y(t)dX(t) + \sigma(t)\rho(t)dt.$$

[Hint: Consider the expression $(X(t)+Y(t))^2$.] [4]

An asset has a price at time t of $S(t)$, which satisfies the following stochastic differential equation:

$$dS(t) = -5S(t)dt + 4dW(t),$$

where $W(t)$ is standard Brownian motion.

The solution for $S(t)$ is of the form:

$$S(t) = e^{-At}\left(B + C\int_0^t e^{Ds}dW(s)\right),$$

where A, B, C and D are integer constants.

(iv) Calculate A, C and D.
[Hint: Let $Y(t) = e^{-At}$ and find a suitable $X(t)$ to enable the use of the result of part (iii).] [7]

(v) Give an expression for the value of B. [1]

[Total 17]

5 A company has a defined benefit pension scheme. The pension scheme has an investment strategy which includes a 50% allocation to domestic high quality equity. The company is legally obliged to make annual contributions into the pension scheme as the liabilities of the pension scheme are greater than its assets.

The pension scheme is concerned about the risk of a large fall in equity prices.

(i) Explain why the pension scheme could want to hedge this risk. [2]

The pension scheme is planning to use derivatives to hedge this risk.

(ii) Outline possible hedging objectives which might be put in place. [2]

The pension scheme is considering the following hedging strategy: purchasing an out-of-the-money American put option on the country's main equity index, with a strike price of 70% of the current domestic equity market level and a time to maturity of five years. The notional value of the option is equal to the current market value of the equity portfolio.

(iii) Assess the suitability of this hedging strategy. [6]

(iv) Propose improvements that could be made to this hedging strategy, involving purchasing several put options. [2]

The pension scheme is also considering equity market volatility.

(v) Describe the key features of the SPX VIX. [2]

(vi) Determine what is expected to happen to the SPX VIX when there is a fall in US equity values. [3]

The pension scheme has decided to invest in futures on the VOIX, which is the domestic country's equivalent of SPX VIX and which is based on the country's main equity market index.

(vii) Explain how the pension scheme can hedge the risk of a large fall in equity prices using VOIX futures. [3]
[Total 20]

6 (i) Describe the key features of a credit default swap (CDS). [3]

(ii) Describe the key features of the CDS–bond basis. [2]

A hedge fund has long positions in around 100 corporate bonds and wants to hedge the risk of bond prices falling due to changes in credit spreads and defaults.

The hedge fund is considering entering into the following total return swap (TRS):

- Notional: $1,000 million
- Term: one year

- Hedge fund pays: total return on the iBond index over the one-year term (so the counterparty pays the hedge fund if the total return is negative).

- Hedge fund receives: one month LIBOR returns accumulated monthly over the one-year term and payable as a single amount at expiry.

The iBond index is the main corporate bond index for the local market and contains around 1,000 investment grade bonds of terms between 1 to 20 years, with an average term of 10 years.

(iii) Discuss the advantages and disadvantages to the hedge fund of using the TRS compared to using CDSs on the 100 bonds currently held. [5]

(iv) Suggest analyses that the hedge fund could perform in order to assess the basis risk between the 1,000 bonds in the iBond index and the 100 bonds currently held. [3]

The hedge fund does not wish to alter the interest rate sensitivity of the fund following the introduction of the TRS and has therefore decided to consider delta-hedging any additional interest rate exposure. The average duration of the iBond index is currently seven years.

(v) Explain the source and direction of the interest rate exposure for the hedge fund of the TRS in isolation. [3]

(vi) Show that the sensitivity of the \$1,000 million notional TRS to a 0.01% per annum absolute change in interest rates is approximately \$0.7 million. [1]

The hedge fund is considering using a receiver swap to delta-hedge the additional interest rate exposure. The swap under consideration is a 12-year at-the-money interest rate swap with fixed monthly coupons in arrears of 2.96% per annum. Interest rates are 3% per annum annually compounded at all terms.

(vii) Show that the sensitivity of \$1,000 million notional of the swap to a 0.01% per annum absolute fall in interest rates is approximately \$1.0 million. [4]

(viii) Calculate the notional of the receiver swap that the hedge fund should transact. [1]

(ix) Determine, without performing any further calculations, whether the required notional calculated in part (viii) would increase, decrease or remain approximately unchanged if each of the following occurred separately:

(a) The term of the TRS was increased to two years.

(b) The hedge fund received fixed amounts under the TRS, rather than LIBOR payments.

(c) Credit spreads on the corporate bonds in the iBond index increased.

[3]
[Total 25]

END OF PAPER