INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2016

Subject ST6 – Finance and Investment
Specialist Technical B

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

F Layton
Chair of the Board of Examiners
July 2016

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Finance and Investment Technical B subject is to instil in successful candidates the ability (at a higher level of detail and ability than in CT8) to value financial derivatives, to assess and manage the risks associated with a portfolio of derivatives, including credit derivatives and to value credit derivatives using simple models for credit risk.

2. This subject deals with mathematical techniques for the valuation and risk management of derivatives, together with some aspects of their practical application.

3. Different numerical answers may be obtained to those shown in these solutions, depending on whether figures were obtained from tables or from calculators. Allowance was made for these minor differences. However, candidates may have been penalised for using excessive rounding or showing insufficient working.

4. Derivative theory is an interesting but very exacting subject which needs to be tackled in a precise and disciplined manner. Questions that appear unfamiliar can, with some clear thinking at the start of the exam, be found to be straightforward examples of well-known techniques. Thorough preparation of the course material is essential, however. This subject also requires quite a varied approach in writing solutions. Algebraic and numerical content needs to be supplemented with well-reasoned but brief arguments where the question demands. Candidates should always try to provide several distinct relevant points for the discursive questions, not lengthy paragraphs developing a single idea.

B. General comments on *student performance in this diet of the examination*

1. The overall performance of students was broadly in line with the standard observed in the past few years. Candidates generally found this paper challenging, but well-prepared candidates scored well above the pass mark.

2. In terms of areas for improvement:
   - Some candidates were unable to demonstrate a breadth of knowledge across the whole syllabus and so did not score all of the bookwork marks that were available.
   - Many candidates did not appear to tailor their answer to the command words in the questions, particularly the higher-order commands such as "Assess" or "Test". Some candidates used their time providing basic descriptions or calculations relating to the general area in question, rather than focusing their answer to the question posed.
   - A number of candidates provided a significant amount of detail on relatively narrow arguments when responding to the discursive questions. This appeared to mean that they had insufficient time to tackle the remaining sections of the paper to an appropriate depth.

- Candidates struggled significantly with questions that required an element of application of Core Reading to situations that were not immediately familiar, so did not demonstrate the desired depth of knowledge.

3. The comments that follow the questions concentrate on areas where candidates could have improved their marks, as an attempt to help future candidates to revise accordingly and to develop their ability to apply the core reading to related situations. This skill is likely to be valuable both in exams and in using this theory in their professional careers.

C. Comparative Pass Rates for the past 3 years for this diet of examination

Year	*%*
April 2016	32
September 2015	36
April 2015	40
September 2014	28
April 2014	41
September 2013	30

Reasons for any significant change in Pass Rates in current diet to those in the past:

The pass rate is broadly in line with the range of pass rates observed historically.

D. Pass Mark

The Pass Mark for this exam was 55%.

Solutions

Q1 (i) The waterfall is the set of rules by which tranches receive their return in order of seniority.

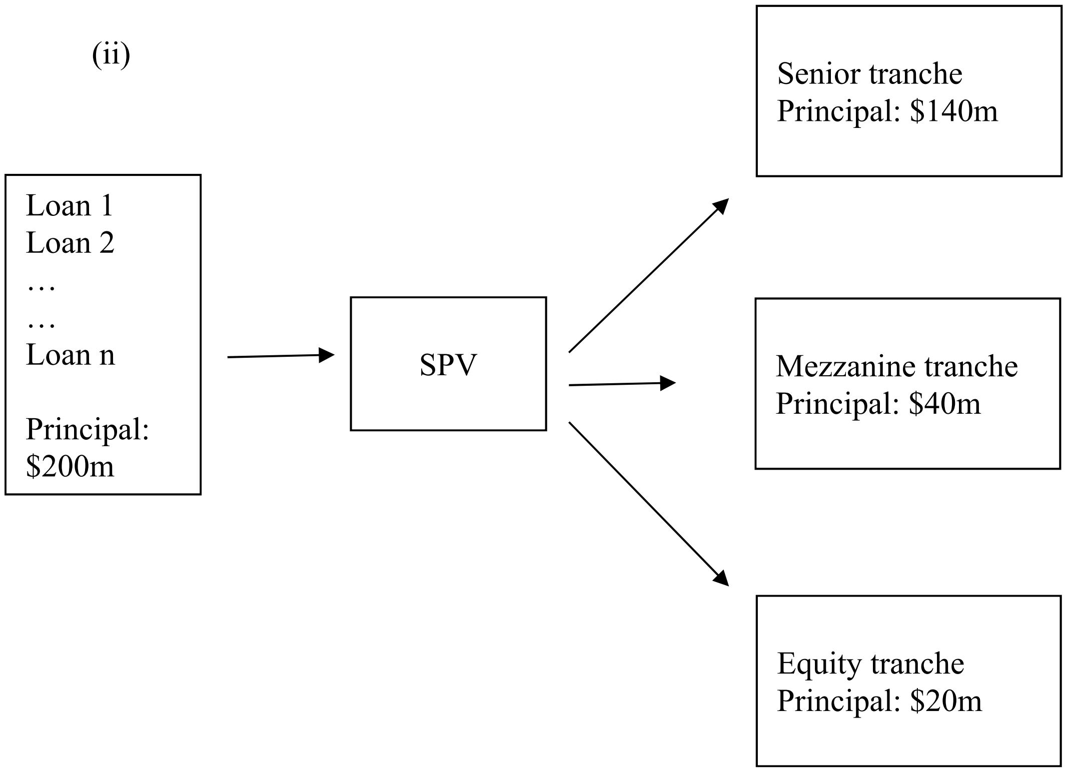
The cashflows from the portfolio of assets backing the ABS are first used to pay the investors in the senior tranche the return that has been promised to them.

As far as is possible, they are then used to provide the investors in the mezzanine tranche with a return of up to the amount that has been promised to them.

Residual cashflows, if any, are used to provide the equity investors with a return.

Principal repayments are also subject to the waterfall.

(ii)



(iii) $55\% \times 20\% = 11\%$
$70\% + 11\% = 81\%$ (or \$162m)

(iv) As 81% of the ABS is senior tranches, losses greater than $100\% - 81\% = 19\%$ will wipe out the mezzanine tranche.

[Alternatively, this can be determined as $10\% + (35\%+10\%) \times 20\% = 19\%$.]

With an assumed loss given default of 70%, this requires a proportion of loans defaulting of 19%/70% = 27.143%.

> Generally well answered with many candidates scoring highly. In parts (iii) and (iv) many candidates missed out on scoring full marks due to calculation errors or misunderstanding how the defaults affected the payoffs of the different tranches.

Q2 (i) A market participant is *risk-neutral* if they are indifferent to risk, i.e. they are neither risk averse nor risk seeking.

The focus of a risk-neutral investor is their expected return and not the risk taken on.

[Alternatively, the above can be expressed as not requiring a greater expected return from an investment with increased risk.]

For example, consider an investor with a choice of winning \$100 with certainty or, the chance of winning \$200 or \$0 with equal probability of 0.5 each.

The expected returns are the same, but the risks are different. A risk-neutral investor would be indifferent to either of these choices.

(ii) The *risk-neutral measure* is an artificial mathematical concept…

… which can be used in pricing derivatives.

It represents a probability distribution (or equivalently a set of probabilities).

The condition of no arbitrage is equivalent to the existence and uniqueness of the risk-neutral measure for a complete market. *[Alternatively: the measure exists in, and only in, a complete no arbitrage market.]*

The expected return of an asset under the risk-neutral measure is the risk-free rate of return.

It is the measure that results in price processes (discounted at the risk-free rate) being martingales.

It does not assume that market participants are risk-neutral.

It does not assume that the market is risk-free. In fact, the market is generally assumed to have an unpredictable (or stochastic) future.

(iii) Both are probability measures that can be defined on the same set of scenarios (i.e. sample space) and filtration (sigma algebra).

At a simplistic level they only differ in terms of the probabilities they assign to a given scenario.

The risk-neutral measure does not have any direct meaning in terms of the real-world probability of an event happening.

However, the risk-neutral measure is equivalent to the real-world measure.

The measures are equivalent if and only if every positive probability outcome for the risk-neutral measure has an equivalent positive probability outcome under the real-world measure.

In certain cases, e.g. where the underlying asset is driven by a geometric Brownian motion, the Radon-Nikodym derivative and the CMG theorem can be used to transform between the two measures.

Under the real-world measure, the expected return of an asset is not necessarily equal to the risk-free rate...
… and there is subjectivity in determining the expected return.

Under the real-world measure, discounted asset price processes are not generally martingales.

(iv) The risk-neutral approach is only useful for arbitrage-free pricing.

An arbitrage-free price is not necessarily a fair price,…

… or the correct price; it is only a market consistent price…

The market value could be different to the market consistent price due to reasons such as….
- Tax
- Incompleteness of the market
- Illiquidity
- Transaction costs

There is an explicit assumption that the market is complete, which is not always the case.

Incomplete markets can lead to non-unique risk-neutral measures, which complicates any resulting pricing model.

There are further explicit assumptions relating to frictionless trading, short selling and the infinite divisibility of assets, which are generally not true.

Any implementation of a risk-neutral approach will be model dependent…

… for example Black-Scholes, Monte Carlo or a GARCH model.

This can result in inappropriate prices due to simplifying assumptions…

… and model risk.

There can be difficulties in determining the risk-free rate

Risk-neutral approaches do not allow you to calculate real-world probabilities which can be helpful for risk management.

Risk-neutral approaches may be harder to explain to management

This was one of the less well answered questions on the paper. Many candidates did not appear to have a full understanding of risk neutral probabilities and how they can be used in practice. This is fundamental to derivative pricing theory. Many associated risk neutral probabilities solely with the Black Scholes model and associated logarithmic returns.

Candidates struggled to articulate their knowledge of risk-neutral measures to the specific part of question, with some resorting to duplicating responses. This is unnecessary, as the Examiners will "cross-mark" answers to closely related or overlapping question parts where students demonstrate the required understanding.

In part (i), many candidates did not include a basic example as requested and so did not score full marks.

Q3 (i) Times t or T.

(ii) If the option is exercised at time t, the holder of the call option receives $S(t) - K$.

If it is not exercised at time t, the stock price drops to $S(t) - D$.

So using the hint in the question the value of the option is greater than or equal to $S(t) - D - K \exp(-r(T - t))$.

The option should not be exercised if the value not exercised is greater than or equal to the value exercised.

It follows that it cannot be optimal to exercise early if

$S(t) - D - K \exp(-r(T - t)) \geq S(t) - K$
i.e. if $D \leq K[1 - \exp(-r(T - t))]$

(iii) It is not optimal to exercise immediately prior to time t_i if

$$D_i \leq K[1 - \exp(-r(t_{i+1} - t_i))]$$

(iv) Start by valuing the equivalent European call option.

The present value of the dividends is

$\exp(-0.08 \times 2/12) + \exp(-0.08 \times 5/12) = 1.9540$

The European option can therefore be calculated from the Black-Scholes-Merton formula with the given parameters, $T = 0.5$ and with

$S_0 = 60 - 1.9540 = 58.0460$

$d_1 = [\ln(58.046/60) + (0.08{+}0.25^2/2) \times 0.5]/0.25\sqrt{0.5} = 0.1274$

$d_2 = d_1 - 0.25\sqrt{0.5} = -0.0494]$

so $\Phi(d_1) = 0.5507$
and $\Phi(d_2) = 0.4803$
and $c = 58.046 \times 0.5507 - 60\exp(-0.08/2) \times 0.4803 = 4.277$
Now $1 < 60\,[\,1 - \exp(-0.08 \times (5 - 2)/12)] = 1.188$

So the option should not be exercised immediately before the first ex-dividend date.

Now $1 > 60\,[1 - \exp(-0.08 \times (6 - 5)/12)] = 0.399$

So it may be optimal to exercise the option immediately before the second ex-dividend date.

Therefore need to revalue the option as above, but with expiry at the second ex-dividend date, i.e. $T = 5/12$ and

$S_0 = 60 - \exp(-0.08 \times 2/12) = 59.0134$

$d_1 = [\ln(59.0134/60) + (0.08{+}0.25^2/2) \times 0.41667]/0.25\sqrt{0.41667}$
$= 0.1845$

$d_2 = d_1 - 0.25\sqrt{0.41667} = 0.0231$

so $\Phi(d_1) = 0.5732$

and $\Phi(d_2) = 0.5092$

and finally $c = 59.0134 \times 0.5732 - 60 \times 0.5092\exp(-0.08 \times 5/12)$
$= 4.274$

Black's approximation is to take the maximum of these two values as the value of the American call option.

So the determined value is 4.277

> This was the least well answered on the paper. In parts (i) to (iii), a number of well-prepared candidates were able to score highly but most struggled to understand the impact of a dividend being paid and how to incorporate the hint.
>
> Responses to part (iv) were very mixed – some students scored full marks but a significant majority of candidates did not appear to be aware of Black's approximation. Some candidates appeared to be daunted by this part and did not attempt it, despite having answered the earlier parts of the question well. Candidates are reminded that a significant number of marks are available for partial solutions and interim calculation stages, even if the final answer is not obtained.

Q4 (i) Foreign exchange risk is the risk that the value of an investment or financial transaction …]

… changes due to changes in foreign exchange rates.

(ii) Assume that the company can manage the in and out flow of money into the margin account at no cost…
… and that there are no other costs (e.g. transaction costs, taxes..)

The company converts the 1,000,000 net payoff into Fiagolds at the then spot exchange rate of 0.07 Fiagolds per Rupee to give $0.07 \times 1{,}000{,}000 = 70{,}000$ Fiagolds.
There will be a futures profit on the amount that was hedged of $(0.11 - 0.07) \times 900{,}000 = 36{,}000$ Fiagolds.

[Alternatively: unhedged 100,000 gives $0.07 \times 100{,}000 = 7{,}000$ F plus hedged 900,000 gives $0.11 \times 900{,}000 = 99{,}000$ F]

The overall cashflow is therefore a net profit of 106,000 Fiagolds.

(iii) The company will receive the revenues from the contract in Rupees in the future, so is at the risk of the Rupee depreciating in the three month period relative to the Fiagold (i.e. being able to buy fewer Fiagolds per Rupee).

It should therefore buy puts.

Possible approach to determine the number of option contracts:
The effective exchange rate in September if the puts are exercised needs to incorporate the premium. This results in an effective exchange rate of 0.1 Fiagolds – 0.001 Fiagolds = 0.099 Fiagolds per Rupee.

The minimum expected revenue of 1,000,000 Fiagolds would equate to 1,000,000 / 0.099 = 10,101,010 Rupees.

The number of option contracts required is the amount at risk divided by the size of a put option contract. In this case it is 10,101,010 / 2,000,000 = 5.05 contracts.

As fractional contracts are not available, the number of contracts to be bought is either 5 or 6.

If 5 contracts are bought then part of the risk will not be hedged, which does not meet the requirements of the company – so the company therefore should buy 6 options, even though this does over hedge.

Alternative approach to determine the number of option contracts:

If the puts are exercised, the minimum expected revenue of 1,000,000 Fiagolds would equate to 1,000,000 / 0.1 = 10,000,000 Rupees, ignoring hedging costs.

The number of option contracts required for this payoff is the amount at risk divided by the size of a put option contract. In this case it is 10,000,000 / 2,000,000 = 5 contracts.
However, the contract also has to pay out enough to meet the hedging costs and 5 contracts would not be sufficient to hedge the currency risk in relation to this additional amount.

As fractional contracts are not available, the number of contracts to be bought must therefore be 6 or more.

Each option contract hedges 2,000,000 Rupees and the cost of purchasing 6 options is materially less than this – so the company therefore should buy 6 options, even though this does over hedge.

(iv) The cost of hedging is 6 × 2,000,000 x 0.001 = 12,000 Fiagolds.

So the required contract size is (1,000,000 + 12,000) / 0.1 = 10,120,000 Rupees.

The four scenarios to consider are:

Bid lost and Rupee appreciates to 0.2 Fiagolds per Rupee

The bid is lost so construction revenue is zero …
… and also the put options are not worth exercising.

The only cashflow is therefore a loss of the premium paid of 12,000 Fiagolds.

Bid won and Rupee appreciates to 0.2 Fiagolds per Rupee

With the bid won, the expected revenue of 10,120,000 Rupees can be exchanged at the current rate of 0.2 Fiagolds per Rupee to give 10,120,000 × 0.2 = 2,024,000 Fiagolds.

The put options would not be worth exercising so these will expire …
and there will be a loss of the premium paid of 12,000 Fiagolds.

The net cashflow is 2,024,000 – 12,000 = 2,012,000 Fiagolds.

Bid won and Rupee depreciates to 0.05 Fiagolds per Rupee

With the bid won, the expected revenue of 10,120,000 Rupees can be exchanged at the current rate of 0.05 Fiagolds per Rupee to give 10,120,000 × 0.05 = 506,000 Fiagolds.

The put options would be worth exercising …
… and there will be a profit of 6 × 2,000,000 × (0.1 – 0.05) – 12,000 = 588,000 Fiagolds.

The net cashflow is 506,000 + 588,000 = 1,094,000 Fiagolds.

Bid lost and Rupee depreciates to 0.05 Fiagolds per Rupee

The bid is lost and the company would not receive the contract revenue.
It would, however, exercise the put options.

The put options will generate a profit of 6 × 2,000,000 × (0.1 – 0.05) – 12,000 = 588,000 Fiagolds.

(v) It could use FX futures for the currency pair, maturity and amount required, if traded.

In cases where a future with the correct maturity, amount and currency is unavailable the company could cross hedge using another (correlated) future.

It could use FX forward contracts.

The company could use a suitable currency swap.

The company may have the ability to invoice other projects or clients in the currencies required.

The company may purchase any inputs needed for its construction work in the local currency.

The company may be able to enter contractual agreements with input providers or on projects to share or split any foreign exchange movements which affect payments which pass between them.

The company could be able to set up a reinvoicing centre, which is a separate corporate subsidiary that manages all transaction exposure from intracompany trade. *[If this is feasible then it centralises foreign exchange risk, and diversifies the exposure of the company (or group) to foreign exchange risk.]*

The company could diversify its operations.

This could mean diversifying:

the countries in which it has subsidiaries;
the countries in which it carries out construction projects;
the source of raw materials or other inputs.

The company could also diversify its financing: by raising funds in more than one capital market and in more than one currency.

The company could change its strategy to only complete construction work in its own country.

The company could request payment in Fiagolds.

The company could ask to be paid upfront (or in instalments).

Candidates achieved a wide range of marks for this question and it appeared to stretch almost all candidates.

Parts (i) and (ii) were well answered. In part (iii), many candidates struggled to incorporate the transaction costs but were still able to score highly for reasoned arguments of why put options should be used. In part (iv), most candidates were similarly unable to fully incorporate the revenue of the construction contract. However, it was pleasing to see a number of these candidates still scoring highly by making sensible assumptions for the contract revenue and proceeding.

In part (v) few candidates provided a wide range of risk management strategies and most candidates narrowly focused on using other types of derivatives, which scored fewer marks. The command verb "Suggest" also indicated that a wide range of relatively brief suggestions was expected, rather than detailed descriptions or justifications.

Q5 (i) For $n = 1$ to $n = 5$; the price of the annuity will be always be lower using the Regulatory Curve.

This is because the spot rates are higher for the payments at times 1 and 2 and the spot rates are equal thereafter.

For n = 6 comparing the present value differences:

For times 1 & 2, the present value of the difference in the coupons between the Swap Curve and Regulatory Curve is:

$$(1+e^{-0.02\times 2})-(e^{-0.03\times 1}+e^{-0.03\times 2})=0.049\,.$$

For time 6, the PV difference in the coupons is:

$$(e^{-0.06\times 6})-(e^{-0.05\times 6})=-0.043\,.$$

As 0.049 – 0.043 > 0, the Regulatory Curve valuation will still be lower overall for n = 6.

For n = 7, compare the present value of the difference in the coupons at time 7:

$$(e^{-0.07\times 7})-(e^{-0.05\times 7})=-0.092\,.$$

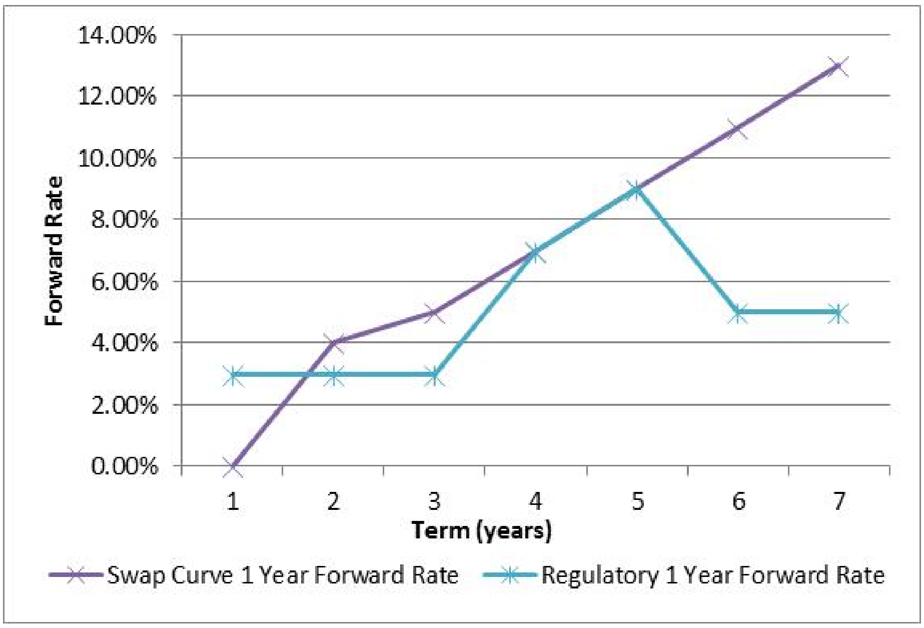
As 0.049 – 0.043 – 0.092 < 0…

… the Regulatory Curve valuation will now be higher overall at n = 7.

(ii) $F_{t,t+1}=(t+1)R_{t+1}-tR_t$

Where $F_{t,t+1}$ is the (continuously compounded) 1 year forward rate from t to t + 1, R_{t+1} is the (continuously compounded) spot rate at term t + 1.

(iii)



(iv) For short terms (up to around 3 years), small increases in swap rates and any fall in swap rates will have no impact on the liabilities…
… and so the institution will be fully exposed to the change in market value of the swaps (i.e. the position is completely over-hedged).
In particular, the institution will experience net losses when swap rates increase slightly (swaps value falls but no change in liabilities)…
... and will experience net gains when swap rates fall (swaps value rises but no change in liabilities).
The swaps will only provide a partial hedge for rises in swap rates that breach the 3% floor.

For medium terms (around 3 to 5 years), the existing swaps will provide a good hedge for small changes in swap rates, as the changes in Swap Curve will be matched by changes in the Regulatory Curve.
If the changes in swap rates breach the 3% floor or 5% cap, then the liabilities will move by a proportionally smaller amount than the market value of the swaps (i.e. the position will be partially over-hedged).
In particular, the institution will experience net losses when swap rates increase beyond the 5% cap (swaps value falling but a smaller reduction in liabilities)…
… and will experience net gains when swap rates fall below the floor (swaps value rising but there is a smaller rise in liabilities).

For longer terms (beyond around 5 years), small decreases in swap rates and any increase in swap rates will have no impact on the liabilities and so the institution will be fully exposed to the change in market value of the swaps (i.e. the position is completely over-hedged).
In particular, the institution will experience net losses when swap rates increase (swaps value falling but no change in liabilities) and will experience net gains when swap rates fall (swaps value rising but no change in liabilities).

The swaps will only provide a partial hedge for falls in swap rates that breach the 5% cap.

(v) The institution is exposed to a loss if yields rise above 5%.
It can hedge this risk by going <u>long</u> on a series of caplets…
… with strikes of 5%...
… such that the payoff is $Max(i_t - 5\%, 0)$ for each future year.

The institution makes a gain if yields fall below 3%.
It can hedge this out by going <u>short</u> on a series of floorlets,
…with a strike of 3%...
… such that the payoff is $Max(3\% - i_t, 0)$ for each future year.
The floorlets and caplets should be transacted to match the terms and notional corresponding to the vanilla swaps in the existing hedge.

A number of candidates scored highly on this question, but many candidates missed out on full marks due to calculation errors or from simple misunderstandings.

In part (i), many candidates incorrectly assumed a flat yield curve structure to value all the cashflows rather than using the spot curve rates at different terms. It was disappointing to see that almost no candidates realised that for n= 1 to 5 no calculations were necessary, leading to many candidates spending significant time on calculations which were not required. This illustrates the importance of paying attention to the command word – "Test." In part (iv), many candidates appeared to understand the high level principle, but did not break this down by term as required and so did not score as highly as they might have. Part (v) was generally well answered, although some candidates were not specific on the terms of caps/floors and so missed marks.

Q6 (i) A basket option is an option where the payoff is dependent on the value of a portfolio of assets.

(ii) $dS_i(t) = \mu_i S_i(t)dt + \sigma_i S_i(t)dW_i(t),\ i = 1, 2$

where μ_i is the drift of S_i and σ_i is the volatility of S_i.

(iii) $G(S_1, S_2) = \max\{\max\{S_1(T), S_2(T)\} - K, 0\}$.

(iv) (a) The finite difference method cannot be used to solve the above equation numerically for $S_1 \in [0, \infty)$ and $S_2 \in [0, \infty)$.
Therefore reasonable upper bounds need to be chosen for S_1 and S_2.

Hull suggests choosing $S_{\max}$ such that the option time value at $S_{\max}$ is virtually zero …
… and so that when the state space is discretised (as in part (b) below), the current values of S_1 and S_2 are included.

To help with the implementation, the upper bound $S_{\max}$ is often chosen to be the same for S_1 and S_2.
Time is modelled from 0 to T

As there are two asset prices and time being modelled, the space is 3-dimensional.

A space for numerically solving the equation is given by:

$[0, S_{\max}] \times [0, S_{\max}] \times [0, T]$,

where T is the time to expiry.

(b) The finite difference method approximates the space of the equation by setting up a discrete cuboid grid of the asset price changes $\Delta S_1, \Delta S_2$ and time steps Δt.

These span all possible outcomes of the asset prices and evolution over the time 0 to final expiry *T*.

These steps must be small enough to make the approximation accurate…

… but not so small that the number of steps is too computationally intense…

… leading to rounding errors…

… or to equations not solving in reasonable time.

The space can therefore be described by a cuboid grid:

$$(m\Delta S_1, n\Delta S_2, k\Delta t) \in [0, M\Delta S_1] \times [0, N\Delta S_2] \times [0, K\Delta t],$$

where $m = 0,\ldots,M; n = 0,\ldots,N$ and $k = 0,\ldots,K$ and $S_{\max} = M\Delta S_1 = N\Delta S_2$ and $T = K\Delta t$.

(c) To reduce the length of formulae let the value of $V(S_1, S_2, t)$ at the mesh point $(m\Delta S_1, n\Delta S_2, k\Delta t)$ be $V_{m,n}^k = V(m\Delta S_1, n\Delta S_2, k\Delta t)$. Using a forward difference approximation of the time derivative:

$$\frac{\partial V}{\partial t}(m\Delta S_1, n\Delta S_2, k\Delta t) \approx \frac{V_{m,n}^{k+1} - V_{m,n}^k}{\Delta t}.$$

Using a central difference approximation of the first order S_1 and S_2 derivatives:

$$\frac{\partial V}{\partial S_1}(m\Delta S_1, n\Delta S_2, k\Delta t) \approx \frac{V_{m+1,n}^k - V_{m-1,n}^k}{2\Delta S_1}, \text{ and}$$

$$\frac{\partial V}{\partial S_2}(m\Delta S_1, n\Delta S_2, k\Delta t) \approx \frac{V_{m,n+1}^k - V_{m,n-1}^k}{2\Delta S_2}.$$

Using a symmetric central difference approximation of the second order S_1 and S_2 derivatives:

$$\frac{\partial^2 V}{\partial S_1^2}(m\Delta S_1, n\Delta S_2, k\Delta t) \approx \frac{V_{m+1,n}^k - 2V_{m,n}^k + V_{m-1,n}^k}{(\Delta S_1)^2},$$

$$\frac{\partial^2 V}{\partial S_2^2}(m\Delta S_1, n\Delta S_2, k\Delta t) \approx \frac{V_{m,n+1}^k - 2V_{m,n}^k + V_{m,n-1}^k}{(\Delta S_2)^2},$$

$$\frac{\partial^2 V}{\partial S_1 \partial S_2}(m\Delta S_1, n\Delta S_2, k\Delta t) \approx \frac{V_{m+1,n+1}^k - V_{m+1,n-1}^k - V_{m-1,n+1}^k + V_{m-1,n-1}^k}{4\Delta S_1 \Delta S_2}$$

(v) Using more than two different assets is a difficulty in itself as there is increased complexity in the modelling.

There is often instability in using the explicit finite difference methods

i.e. the difference between the numerical solution and the actual solution does not remain bounded as the number of time steps tends to infinity.

There are often convergence problems.

As a result, explicit finite differences methods put severe constraints on the size of the time step.

There may be high computational run times, particularly with a large number of random factors.

(vi) Implicit finite difference method.

Monte Carlo simulation.

Analytically

This question was mainly bookwork and differentiated those students who had prepared well on this area of the syllabus. Parts (i) to (iii) were well answered. Those candidates who were able to articulate the key concepts in (iv) scored highly. Other candidates struggled and appeared not to have studied this section of the syllabus in detail.

Q7 (i) **Interest rate risk**

ABC Insurance is exposed to falls in interest rates…
…which increase the value of the guarantee.

The low risk cash funds will provide limited benefit, if any, to offset the above exposure should interest rates fall.

The exposure is asymmetric due to the nature of the guarantee, in that interest rate falls will lead to a larger increase in the cost of the annuity guarantee compared to the decrease observed if interest rates rise by the same amount.

The main interest rate risk relates to changes in the forward rate of interest from the policyholder's retirement date to their expected death.

Longevity risk

ABC Insurance is exposed to increases in the expected longevity of policyholders (i.e. lower expected mortality)…

… at higher (i.e. post retirement) ages.

Again, the exposure is asymmetric due to the nature of the guarantee, in that longevity increases will lead to a larger increase in the cost of the annuity guarantee compared to the decrease observed if longevity expectations decrease by the same amount.

Both the longevity risk and interest rate risk are interrelated in that the increase in one risk will lead to an increased exposure to changes in the other risk.

For example, if people are expected to live for longer, the exposure to a 1% interest rate fall will increase.

Due to the underlying optionality in the annuity guarantee, ABC Insurance is also exposed to an increase in the expected volatility of interest rates…
… or in the expected volatility of longevity improvements.

(ii) (a) ABC should go long on receiver swaptions.

(b) Option term – the option periods should be set in line with the term to retirement of the policies.

Swap term – the underlying swap term or tail of the swaption should be set in line with the expected life of the policyholders, assuming they reach retirement.

Strike – The strike of the receiver swaptions should be set at the zero cost guarantee interest rate, i.e. the rate at which the annuity guarantee exactly bites under the current longevity assumption.

As the annuity guarantees are deeply in the money, the strike rate would be expected to be quite high relative to current market conditions.

Notional – the notionals of the hedge could be set in line with the expected amount of funds at retirement, taking account of deaths pre-retirement.

A more accurate approach could involve adjusting the notionals of the receiver swaptions until the interest rate delta of the swaptions matches that of the annuity guarantee.]

ABC Insurance would want to use a range of receiver swaptions to cover the various different characteristics of the policies outlined, but would want to ensure that the number of swaptions is small enough to manage.

(iii) $LA\left[K\Phi\left(-d_2\right)-F_0\Phi(-d_1)\right]$

$$d_1=\frac{\ln(F_0/K)+\dfrac{\sigma^2}{2}t}{\sigma\sqrt{t}} \text{ and } d_2=d_1-\sigma\sqrt{t}$$

A is the present value of the annuity representing the cashflow payoff of the underlying swap.

(Or, equivalently $A=1/m.\sum P(0,t_i)$ where m is the no. of payments per year)

Where: L = notional, t = term of option period, K = Strike rate, F_0 = forward swap rate, σ = volatility of the forward swap rate.

(iv) $$A=e^{-5\times0.03}\frac{1-e^{-10\times0.03}}{0.03}=7.4360$$

$$d_1=\frac{\ln(0.03/0.10)+\dfrac{0.2^2}{2}5}{0.2\sqrt{5}}=-2.4686$$

$$d_2=-2.4686-0.2\sqrt{5}=-2.9158$$

$$\Phi\left(-d_1\right)=0.99322$$

$$\Phi\left(-d_2\right)=0.99823$$

Therefore,

$$100m\times7.4360\left[0.10\times0.99823-0.03\times0.99322\right]=52.071m\Rightarrow52.1m$$

(v) Under the long receiver swaption: if swap rates are greater than 10% p.a. in five years' time, the swaption will not be exercised and there are no cashflows generated.
However, if swap rates are below 10% in five years' time the insurer would receive 10% p.a. on 100m (i.e. 10m p.a.) and pay the (floating) swap rate on 100m, both being for the ten year period from time 5 to time 15.

In both hedges there are no net cashflows up to year 5.

From year 5, if swap rates are below 10% the swaption would expire worthless.

However, under the forward starting receiver swap, the insurer would pay the swap rate on 100m and receive 3m p.a. for ten years.

And since the STRIPS pay 7m p.a. for the same ten year period, the insurer would receive 10m p.a. in total for the ten year period.

If swap rates are above 10%, the insurer would exercise the swaption.

This means that it would need to pay out £10m p.a. on the payer swaption to then receive the swap rate on 100m.

The swap rate payments received would offset the payments due under the forward starting swap, and the 10m p.a. payable would offset the fixed amounts received from the forward starting swap and the STRIPS. There is therefore no net cashflow.

In both cases, the cashflow position is the same as under the receiver swaption.

(vi) ABC Insurance should consider the relative return (or equivalently cost) of each hedging strategy.
This will depend on the relationship between STRIP rates and swap rates.
It will also depend on the amount of collateral posted and the amount earned on the collateral.

It is likely that the alternative hedge will involve a reduced amount of collateral posting…
… which may reduce the relative cost of the alternative hedge…
… and may reduce potential issues relating to liquidity (in terms of finding the capital required)…
…and may lead to different costs of unwinding the hedge (i.e. the liquidity of the underlying hedge itself).

ABC should consider the relative liquidity of each hedge.
For example, the market in gilt STRIPS may not be especially liquid.

ABC Insurance should consider how complex each strategy is to implement.

ABC Insurance should consider all relevant regulation.
For example, EMIR could have an impact on the hedges.

It may not be easy to obtain the necessary STRIPS, depending on the jurisdiction.
It should consider the transaction costs relating to each option.

ABC Insurance should consider the counterparty default risk posed by each hedge.

The receiver swaption is likely to have a greater counterparty risk than the derivatives under the alternative hedge since it is heavily in-the-money. However, under the alternative hedge the insurance company also needs to consider the risk of default of the government.

ABC Insurance should consider the (intra-year) timing of the payments.

(vii) (a) Given that the annuity guarantee is deeply in-the-money, a longevity swap will most likely be an approximate match for small changes in the longevity experience from that currently assumed…..
… but will not provide a good match for the asymmetric risk posed by the annuity guarantee.
In particular, ABC insurance would be exposed to deteriorations of longevity experience beyond the "breakeven" longevity guarantee rate i.e. if the annuity guarantee moves out-of-the-money.
The swap may introduce counterparty risk

(b) A principal-at-risk longevity bond issued by ABC insurance could be quite effective as the redemption payment would likely be reduced if the survival experience was greater than a threshold, hence providing the asymmetric payoff required.

However, ABC Insurance may have to issue a number of securities to ensure it can hedge the longevity risk across all policies …
… this may be expensive.

(c) A survivor cap would be an appropriate hedge as the underlying caplets would provide protection from longevity increases beyond the strike rates.

The strike rate of the caplets could be set to match closely the "breakeven" rate of longevity that would reduce the guarantee cost to zero.
The cap would introduce counterparty risk

In all of the above options…
….it is likely that basis risk will be present between the longevity experience of the policyholders and the reference population.
… ABC Insurance will need to ensure the maturities on the longevity hedges match the timing of payments of the underlying guarantee costs.
… the capital treatment of the instruments should be considered.

In addition, ABC Insurance will need to manage the risks of movements in both interest rates and longevity (the cross-gamma risk).

This was one of the most challenging questions for students. In parts (i) and (ii), many students appeared to understand the impact of a guarantee at a high level but struggled to identify the more detailed implications and so did not score full marks. Few candidates noted the interaction between longevity and interest rate risk and the asymmetry of these risks. Parts (iii) and (iv) were better answered but few candidates achieved full marks due to formula or calculation errors.

Many students failed to attempt part (v), but those candidates who methodically considered the cashflows of each structure scored highly. In part (vi), many students provided a range of factors and scored highly, with full marks being awarded where candidates were able to develop these points in detail.

In part (vii), the majority of candidates did not consider the "Assess" command word, and instead described each longevity hedging product. Candidates who focussed on these narrative descriptions scored few marks.

END OF EXAMINERS' REPORT