INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2016

Subject ST6 – Finance and Investment
Specialist Technical B

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Luke Hatter
Chair of the Board of Examiners
December 2016

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Finance and Investment Technical B subject is to instil in successful candidates the ability (at a higher level of detail and ability than in CT8) to value financial derivatives, to assess and manage the risks associated with a portfolio of derivatives, including credit derivatives and to value credit derivatives using simple models for credit risk.

2. This subject deals with mathematical techniques for the valuation and risk management of derivatives, together with some aspects of their practical application.

3. Different numerical answers may be obtained to those shown in these solutions, depending on whether figures were obtained from tables or from calculators. Allowance was made for these minor differences. However, candidates may have lost marks for using excessive rounding or showing insufficient working.

4. Derivative theory is an interesting but very exacting subject which needs to be tackled in a precise and disciplined manner. Questions that appear unfamiliar can, with some clear thinking at the start of the exam, be found to be straightforward examples of well-known techniques. Thorough preparation of the course material is essential, however. This subject also requires quite a varied approach in writing solutions. Algebraic and numerical content needs to be supplemented with well-reasoned but brief arguments where the question demands. Candidates should always try to provide several distinct relevant points for the discursive questions, not lengthy paragraphs developing a single idea.

5. Candidates who give well-reasoned points, not in the marking schedule, are awarded marks for doing so.

B. General comments on *student performance in this diet of the examination*

1. The overall performance of students was broadly in line with the standard observed in the past few years. Candidates generally found this paper challenging, but well prepared candidates scored above the pass mark.

2. In terms of areas for improvement:

   - Some candidates were unable to demonstrate a breadth of knowledge across the whole syllabus and so did not score all of the available knowledge marks from the Core Reading.

   - Many candidates did not appear to tailor their answer to the command words in the questions, particularly the higher-order commands such as "Assess" or "Suggest".

- Some candidates used their time providing basic descriptions or calculations relating to the general area in question, rather than focusing their answer to the question posed.

- A number of candidates provided a significant amount of detail on relatively narrow arguments when responding to the discursive questions. This appeared to mean that they had insufficient time to tackle the remaining sections of the paper to an appropriate depth.

- Candidates struggled significantly with questions that required an element of application of Core Reading to situations that were not immediately familiar, so did not demonstrate the desired depth of knowledge.

- The comments that follow the questions concentrate on areas where candidates could have improved their marks, in an attempt to help future candidates to revise accordingly and to develop their ability to apply the core reading to related situations. This skill is likely to be valuable both in exams and in using this theory in their professional careers.

C. Pass Mark

The Pass Mark for this exam was 57.

Solutions

Q1 (i) From a real world viewpoint, a tradable asset is an asset where there exists a market in which it can be traded either directly or indirectly. [1]

From a theoretical viewpoint, within a complete market of tradable securities a process represents a tradable asset if its discounted price is a martingale under the risk-neutral measure. [1]

[Maximum 1]

(ii) The market price of risk, γ, is defined as: $\gamma = \frac{\mu - r}{\sigma}$. [1]

Where μ is the expected growth rate of the stock with the dividends reinvested … [1]
…and σ is the volatility process, and r is the risk-free rate of interest. [1]

[Alternatively: the market price of risk can be defined as:

$$\gamma = \frac{m + q - r}{\sigma}.$$

Where m is the expected growth rate of the stock net of dividends, q is the dividend yield and σ is the volatility process as above.]

[Maximum 2]

(iii) For $S(t)$, the market price of risk, $\gamma(S(t))$, is:

$$\gamma(S(t)) = \frac{\mu - 0.05}{0.1}.$$ [½]

Solve for $R(t)$ using Ito's lemma. Let $Y(t) = \ln R(t)$. [½]

Differentiating $R(t) = \exp(Y(t))$ gives:

$$dR(t) = 0.25\exp(Y(t))dW(t) + (0.04\exp(Y(t)) + 0.5 \times 0.25^2 \times \exp(Y(t)))dt$$ [2]

$$dR(t) = 0.07125 \times R(t)dt + 0.25R(t)dW(t)$$ [½]

The process for a stock price, $Z(t)$, with a constant dividend yield is of the form:

$dZ(t) = (return - q)Z(t)dt + S(t)\sigma(t)dW(t)$, where q is the constant dividend yield and the *return* is the constant total expected return of the stock. [1]

Therefore, for $R(t)$, the total expected return of this stock is $0.07125 + 0.00125 = 0.0725$. [½]

Assuming that there is no arbitrage… [1]
…implies that the market price of risk is the same for $S(t)$ and $R(t)$. [1]

Equating the market price of risk for each asset gives:

$$\frac{\mu - 0.05}{0.1} = \frac{0.0725 - 0.05}{0.25} = 0.09.$$ [1]

So $\mu = 0.059$. [½]

[Maximum 6]

[Total 9]

The first two parts were well answered by most candidates. A reasonable number of candidates produced good attempts at part (iii), noting that the market price of risk would be the same for S and R. However, many candidates missed out on scoring full marks due to calculation errors, for example not taking into account the constant dividend yield for R.

Q2 (i) An index-linked bond is a bond under which coupon … [½]

… and redemption payments are increased in proportion to increases in a specified inflation index. [½]

There may be a lag between the time used to determine the increases and when the increase is actually paid. [½]

LPI bonds are index-linked bonds that increase coupon and redemption payments in line with an index subject to some maximum … [½]

… and typically a minimum (e.g. zero). [½]

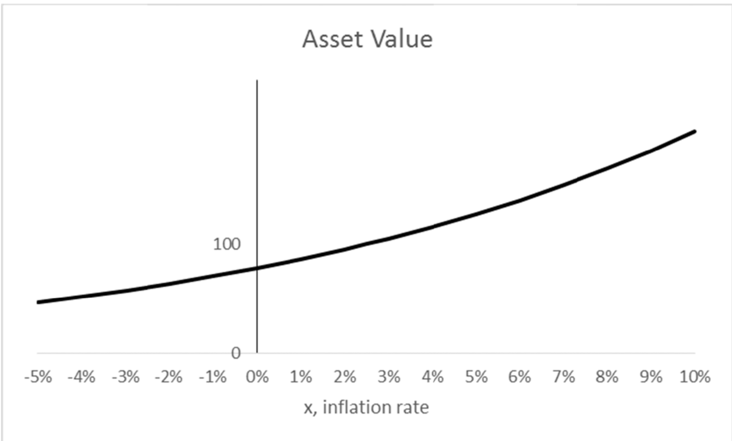
LPI swaps are OTC contracts typically between banks and institutions. [½]

The institution will typically receive payments in line with an inflation index subject to a cap and floor and pay, for example, fixed cash flows. [1]

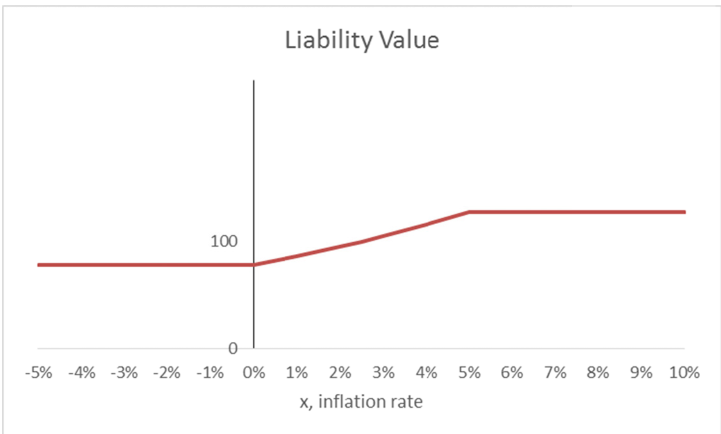
They are typically traded on a zero coupon basis. [½]

[Maximum 3]

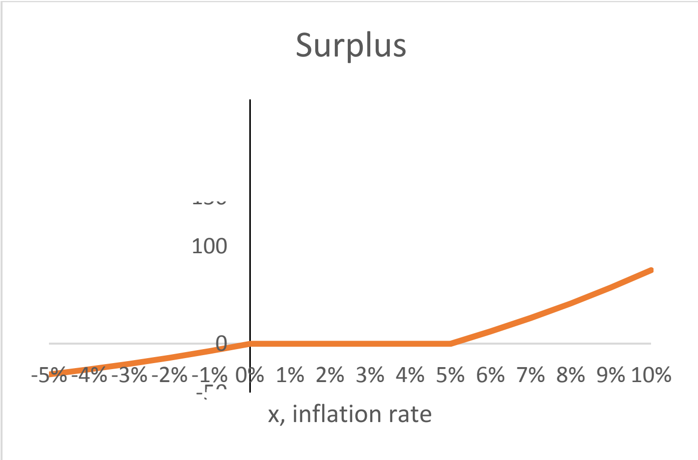
(ii)



[1½]



[2]



[1½]

[Maximum 4]

(iii) (a) Vega risk in this context would be the change in value of the scheme's surplus arising from changes in inflation volatility. [1]

A deterministic projection will not allow for the stochastic nature of inflation over time… [½]
… so will not allow for causing the 0% liability floor (or the 5% cap) to be breached in future…. [½]
… even if current inflation expectations always sit within the range of 0% to 5%. [½]

(b) The surplus is likely to be overstated when x is lower than the expectation of 2.5%... [1]
… as an increase in inflation volatility would likely cause a greater chance of the liabilities being floored at 0% rather than capped at 5%. [½]
The vega sensitivity will be highest when an increase in volatility will cause the 0% liability floor to be breached most significantly…. [½]
… so the overstatement will be largest around $x = 0\%$. [½]

The exact values of x for which the surplus is overstated would be dependent on the stochastic model assumed for inflation. [½]

[Maximum 4]

(iv) **Sustained Negative Inflation**

The hedge is likely to be very effective as the payoff from the swap will not be reduced during the negative inflation due to the max(….,1) condition… [1]
… which will match the liabilities which will have been floored at 0%. [½]

Sustained Inflation Above 5%

The hedge is likely to be very effective at immunising the surplus from inflation changes as the payoff from the swap will be capped during the high inflation due to the min(….,1.05^T) condition… [1]
… which will match the liabilities which will have been capped at 5%. [½]

Volatile periods of high and negative inflation

The liability payment increases are determined in each calendar year, whereas the payoff on the LPI is based on the cumulative increase in inflation over the period *T*. [1]

For $T = 1$, the LPI will be very effective as the cumulative increase will equal the annual increase by construction. [½]

As *T* increases however, the effectiveness will reduce… [½]
… as there is a significant chance that there is an annual instance of negative inflation or inflation in excess of 5% without causing the cumulative inflation rate to exceed the 0% floor or 5% cap…. [1]
… which would mean the flooring or capping of the liabilities is not matched by any flooring or capping on the LPI swap. [½]

This could be beneficial if there were annual instances of very high inflation without significant cumulative inflation…. [½]
… but overall the hedge is likely to perform better than if only RPI instruments were held but poorly compared to holding LPI instruments that apply caps and floors annually rather than cumulatively. [1]

[Maximum 4]

[Total 15]

Part (i) was straightforward for most candidates.

Many candidates struggled to produce the graphs required for part (ii). There is a general trend of questions requiring graph sketching not being well answered. One approach to this type of question is to consider which points can be plotted on the graphs with a degree of certainty, and then allowing approximately for the general shape of the graph around those points.

For example in this case for both the assets and the liabilities the point 100 at 2.5% is known, which leads to a point of 0 at 0% on the surplus graph. Other important points are at 0% and at 5% which can be solved for. Candidates who worked through these points, and noted the fact that the graph is increasing due to assets increasing with inflation and the liabilities increasing between 0% and 5%, were able to score highly.

Part (iii) was an application of the familiar concept of vega risk in an unfamiliar context, with which many candidates struggled. For these situations it is useful to start with the definition of what is being tested and try to apply it in the current situation.

Part (iv) was generally well answered, with many candidates able to generate a wide range of valid points.

Q3 (i) (a) Consider the following strategy:

Borrow an amount of cash equal to: $S_0\, e^{uT}$ and use it to purchase one unit of the consumption asset and to pay storage costs. [½]

Short a futures contract on one unit of the underlying consumption asset. [½]

$F_0 > S_0 \exp\left[(r+u)T\right]$ can therefore be ruled out as a tradeable arbitrage opportunity. [½]

If $F_0 < S_0 \exp\left[(r+u)T\right]$, then to exploit any potential arbitrage opportunity someone would need to go long on futures contract and short the consumption asset…. [½]

… and shorting a consumption asset is typically deemed neither possible nor desirable. [1]

(b) $F_0 = S_0 \exp\left[(r+u-y)T\right]$ where y is the convenience yield. [1]

[Maximum 3]

(ii) There may be significant basis risk… [½]
… as the price of jet fuel will not be perfectly correlated with the price of oil… [1]
… or a forward may not be available on the particular grade of oil that is used to make fuel. [½]

The convenience yield of oil is often large [½]
and volatile. [½]

As a result shorter dated forward prices and spot prices may move significantly even if longer terms forward rates do not change. This results in an even lower correlation with jet fuel over the long term. [1]

And the airline will not know exactly how much fuel will be used. [½]

Rolling the hedge once a year would likely involve closing the contract out prior to maturity… [½]
… and so the company will incur basis risk on each roll. [½]
There will also be costs incurred each year in the governance of the forward rolling of the hedge. [½]

There may be insufficient liquidity on the 1-year forward… [½]
… so the hedge may become expensive. [½]
Using a forward gives away the upside if oil prices fall. [½]
The company may have to post collateral if the oil price falls… [½]
… which could cause cash flow problems … [½]
… due to the unknown timings and amounts of the margin. [½]

The company may also face counterparty risk. [½]

[Maximum 5]

(iii) The airline may have to report the details of all open positions… [1]
…resulting in extra reporting burdens. [½]

An oil forward may be considered a "standardised OTC derivative"… [½]
… so the airline may have to clear the forwards through a central counterparty (CCP) [½]
… which would require the provision of margin… [½]
… specifically both initial and variation margin [½]

If the forward is not considered "standardised", then there are additional operational risk management requirements… [½]
… and as the airline would likely be classed as a "non-financial" counterparty and is using the derivatives for risk management, collateralisation would only be required if the positions are large. [1]
As fuel is a major component of operating costs for an airline the position is likely to be large. [½]
The benefits of EMIR include reducing the risk, including counterparty risk, of using this hedging approach. [1]
[*Other reasonable points could be made.*]

[Maximum 3]

[Total 11]

Many candidates produced inconsistent solutions to part (i)(a). In many cases, unjustified adjustments were made to portfolio strategies which appeared to lead to the required result but which were not in fact correct. The implicit hint in this question was that there would be storage costs associated with the consumption asset, which should suggest that money needs to be borrowed to pay for the asset and the storage costs. This then leads to the futures contract being shorted.

Part (i)(b) was well answered.

Part (ii) required detailed answers for 5 marks, but most focussed on basis risk relating to the price and considered issues relating to frequent rolling of the hedge only briefly. These types of questions are more typical of non-numerical based question in the STs which require depth to the answers. Candidates who scored highly were able to demonstrate their familiarity with the practical aspects of derivatives as well as the mathematics.

Few questions have been asked on EMIR so far. Those candidates who had studied and understood the relevant areas of the course made good attempts at part (iii), which was a relatively straightforward application question.

Although not every part of the course can be examined at each sitting, all parts of the Core Reading are examined over time and candidates should be prepared for this.

Q4 (i) The payoff to the option holder at time T is:

$$\max\{0, S(T) - K\}. \qquad [1]$$

[Maximum 1]

(ii) For an expectation to be meaningful a probability measure $\mathbb{P}$ is assumed to exist. [½]

Further, it is assumed that a distribution of the random variable S(T), the terminal stock price, exists with respect to $\mathbb{P}$. [½]

The expected present value is therefore:

$$\mathbb{E}_{\mathbb{P}}[e^{-rT} \max\{0, S(T) - K\}]. \qquad [1]$$

[Maximum 2]

(iii) Let n be the integer number of loops in the algorithm ($n > 1$). [½]

Assume that independent samples of the price of the underlying at expiry, *S(T)*, can be generated. [1]

Let S_i be a sample drawn from the distribution for <u>*S(T)*</u>. [½]

Algorithm:

For each $i = 1, \dots, n$ [½]

- generate a sample S_i, [½]
- set $V_i = e^{-rT} \max\{0, S_i - K\}$. [½]

Set $\overline{V_n} = (V_1 + \ldots + V_n)/n$. [½]

Using the hint given in the question: as $n \to \infty$,
$\overline{V_n} \to V = \mathbb{E}_{\mathbb{P}}[e^{-rT} \max\{0, S(T) - K\}]$ with probability 1. [1]

[Maximum 3]

(iv) $p(S)$ has been defined as the probability density function of $S(T)$. Writing S for $S(T)$ and noting that $p(S)$ and S are continuous, then using the definition of the expectation of a continuous function:

$$\mathbb{E}_{\mathbb{P}}[e^{-rT} \max\{0, S-K\}] = \int_{-\infty}^{\infty} [e^{-rT} \max\{0, S-K\}] p(S) dS,$$ [½]

$$= e^{-rT} \int_{-\infty}^{\infty} [\max\{0, S-K\}] p(S) dS$$ (as e^{-rT} does not depend on S),

$$= e^{-rT} \int_{K}^{\infty} (S-K) p(S) dS$$ (as $\max\{0, S-K\}$ is 0 for $S \le K$, so the lower limit can be changed).

[½]

[Maximum 1]

(v) Note that: $f(x) = \dfrac{f(x)}{g(x)} g(x)$. [½]

We need to ensure this is well defined for all $x \in \mathbb{R}$. [1]

One approach is to assume that $f(x)$ and $g(x)$ are equivalent measures. [½]

A more detailed argument from first principles is to assume that g is such that:

$f(x) > 0 \Rightarrow g(x) > 0$ for all $x \in \mathbb{R}$. [½]

This is required for the integral to be well defined. [½]

Then, $h(x) f(x) = h(x) \dfrac{f(x)}{g(x)} g(x)$ and $h(x) \dfrac{f(x)}{g(x)}$ is a function from $\mathbb{R}$ to $\mathbb{R}$.

[½]

Hence $\int_{\mathbb{R}} h(x) f(x) dx = \int_{\mathbb{R}} h(x) \dfrac{f(x)}{g(x)} g(x) dx$. [½]

The right-hand side can be interpreted as $\mathbb{E}_{\mathbb{Q}}[h(X)f(X)/g(X)]$ and therefore $\mathbb{E}_{\mathbb{P}}[h(X)] = \mathbb{E}_{\mathbb{Q}}[h(X)f(X)/g(X)]\ldots$ [½]

…where $\mathbb{E}_{\mathbb{Q}}[.]$ is an expectation with respect to the probability density function $g(x)$ and $\mathbb{E}_{\mathbb{P}}[.]$ is an expectation with respect to the probability density function $f(x)$. [½]

[Maximum 3]

(vi) The change in representation can be identified as a change in measure from $\mathbb{P}$ to $\mathbb{Q}$. [1]

The ratio $\dfrac{f(x)}{g(x)}$ is the Radon-Nikodym derivative at x. [1]

[Maximum 1]

(vii)
$$\mathbb{E}_{\mathbb{Q}}\left[\left(h(X)\frac{f(X)}{g(X)}\right)^2\right] = \int_{\mathbb{R}} h^2(x)\frac{f^2(x)}{g^2(x)}g(x)dx, \quad [½]$$

$$= \int_{\mathbb{R}} h^2(x)\frac{f(x)}{g(x)}f(x)dx,$$

$$= \mathbb{E}_{\mathbb{P}}\left[h^2(X)\frac{f(X)}{g(X)}\right]. \quad [1]$$

[Maximum 1]

(viii) Changing the measure is often used in financial mathematics, for example in the pricing of options… [½]
… and changing the numeraire. [½]

A change in the measure can result in obtaining a more convenient representation of the expectation for use in calculations or numerical estimation. [1]

$\mathbb{Q}$ is typically a risk-neutral probability measure… [½]
…which is useful for pricing calculations … [½]
… and easier as the discounted price process is a martingale. [½]

As the actual expected values are the same under each measure … [½]
… an important consideration is the second moments. [½]

The previous part shows that the second moments are different: $\mathbb{E}_{\mathbb{P}}\left[h^2(X)\right]$ and $\mathbb{E}_{\mathbb{P}}\left[h^2(X)\dfrac{f(X)}{g(X)}\right]$. [½]

A suitable choice of $g(X)$ may result in $\mathbb{E}_{\mathbb{P}}\left[h^2(X)\dfrac{f(X)}{g(X)}\right]$ being smaller than $\mathbb{E}_{\mathbb{P}}\left[h^2(X)\right]$. [1]

As a result this may be useful in the numerical estimation of integrals. [½]
For example, in a Monte Carlo simulation this may result in more efficient estimates, as … [½]

…fewer simulations will be needed to produce an estimate with lower variance compared to one in not changing measures. [½]

This could be done by selecting an appropriate distribution which emphasises "important" values, for example in the tails of distributions. [1]

[Maximum 3]

[Total 15]

This question was quite well answered by most candidates. Candidates typically found parts (i), (ii), (iv) and (vi) reasonably straightforward.

In parts (ii) and (iii), some candidates began to assume a distribution for the stock price process. This was not required anywhere in the question.

This raises a general point in that it is not always necessary to assume a lognormal or normal (or indeed any other) distribution for a stock process. In some cases this can make a question more difficult as it can lead to the candidate focussing on the Black-Scholes world, when actually more general points are required.

Relatively few candidates made use of the hint in part (iii) to demonstrate that their algorithm would actually work. The hints are deliberately put in to make the question easier. This part was comparing the simulation approach to the analytical approach.

In parts (v) and (vi), many candidates recognised that this was related to the Radon-Nikodym derivative and scored highly.

Part (vii) introduced the second moment but only the better candidates were able to produce the required result. The final part of the question then built on this by looking at why changing the distribution may help through making the second moment different. Most candidates made points around the risk-neutral measure in part (viii).

Q5 (i) A panel of banks is appointed to set the LIBOR rate… [1]
…for each of the ten major international currencies, [½]
…and for several funding periods. [½]

The panel of banks reflects the balance of the market, by country and by type of institution [1]
The constituents of this panel are reviewed annually. [½]

The LIBOR administrator assembles the interbank borrowing rates from the contributor panel banks… [½]
… in the morning of each business day and…. [½]
… discards the top and bottom quartile of the rates…. [½]
… and calculates the average of the middle two quartiles… [½]
This process is repeated for all maturities and currencies. [½]

LIBOR is not a compounded rate, but is calculated on the basis of actual days in funding period… [½]
… divided by 360 (or 365 for Sterling). [½]

Rates for periods for which LIBOR is not set are obtained by linear interpolation. [½]

[Note to candidates: the number of currencies and other details of how LIBOR has been calculated has changed in recent years, e.g. it is now only calculated for 5 currencies. The core reading students were revising from was the one dated for 2016 exams, and the answer above is consistent with the factual details in that. It should be noted that marks were awarded for factual comments that differ from the above but which were correct in September 2016.]

[Maximum 4]

(ii) It is unlikely that an individual bank, acting alone, would be able to materially influence the LIBOR rate…. [1]
… as if the bank wanted a significantly different LIBOR rate from other banks, it would get discarded due to the use of the middle quartiles. [½]
Even if the rate disclosed provided was within the middle two quartiles, the impact would be reduced due to the averaging process. [½]

However, a small change in the LIBOR rate could have a large absolute impact, depending on the type and size of position. [½]

The publication of all quotes on screen would also highlight any odd quotes from a specific bank. [½]

In order to significantly affect the LIBOR rate, a bank would likely need to influence other banks to similarly distort their LIBOR submissions, … [½]
… which has happened in the past. [½]

[Maximum 2]

(iii) (a) The quarterly coupon amount is $\frac{(i+x)}{4}\times\$100m$. [½]

So, the present value of the FRN in \$m is:

$$100\frac{(i+x)}{4}\sum_{t=1}^{t=4T} v^t + 100v^{4T}$$ [½]

where $v=\left(1+\frac{i+s}{4}\right)^{-1}$ [½]

Which simplifies to:

$$100\frac{(i+x)}{4}\left(\frac{1-v^{4T}}{(i+s)/4}\right)+100v^{4T}$$ [½]

$$=100\left[\frac{(i+x)}{(i+s)}(1-v^{4T})+v^{4T}\right]$$ as required. [½]

(b) $$v=\left(1+\frac{0.03+0.01}{4}\right)^{-1}=0.99010$$ [½]

$$PV=100\left[\frac{(0.03+0.02)}{(0.03+0.01)}\left(1-0.99010^{40}\right)+0.99010^{40}\right]=\$108.2m$$ [½]

[Maximum 3]

(iv) (a) *By calculation:*

$$v=\left(1+\frac{0.04+0.01}{4}\right)^{-1}=0.98765$$ [½]

$$PV=100\left[\frac{(0.04+0.02)}{(0.04+0.01)}(1-0.98765^{40})+0.98765^{40}\right]=\$107.8m$$ [½]

Hence the change in value is £0m to the nearest £1m [½]

By reasoning:

£0m [½]

… as the FRN will have very limited interest rate exposure given that the coupons float in line with LIBOR [½]
… and the spread payments are of a similar magnitude to the discount spread. [½]

(b) *By calculation:*

$$PV = 100\left[\frac{(0.03+0.02)}{(0.03+0.02)}(1-v^{40})+v^{40}\right] = \$100m \qquad [1]$$

Hence a fall in market value of £8m (to the nearest £1m) [½]

By reasoning:

£8m fall in market value [½]
… as the coupon rate and discount spread is now the same, so the value will be par [1]

[Maximum 3]

(v) As x increases:

The interest rate sensitivity will increase … [1]
… materially… [½]
… as the coupon spread will now significantly exceed the discount spread… [½]
… so the extra coupons will be similar to interest-sensitive fixed payments.[½]

The spread sensitivity will also increase… [1]
… as the higher coupon payments will increase the present value of the FRN ... [½]
… although the spread sensitivity from the redemption payment at maturity is unchanged. [½]

The spread sensitivity will still significantly exceed the interest rate sensitivity though, even for very high rates of x. [½]

[Maximum 3]

(vi) This feature will help mitigate credit risk… [½]
… as the increased coupons following a downgrade will compensate for the widening spread as the credit quality deteriorates. [1]

[Maximum 1]

(vii) The credit rating represents one simple comparator statistic… [½]
… that may not fully reflect the credit risk of the FRN. [½]

Credit rating agencies have also suffered reputational risk since the credit crunch. [½]

For example ratings may not fully allow for:
The recovery amount on default [½]
The liquidity of the FRN [½]
[1 maximum for two relevant examples]

Responsiveness – the credit rating may not be refreshed frequently … [½]
… so the spread could widen, or there could even be default, before the FRN was downgraded. [½]
This could also cause significant spikes in the price of the FRN when the bond is finally downgraded and the coupons increased. [½]

Scope – it is possible that the FRN may cease to be rated by the agency. [½]

Consistency – the downgrade is triggered using a single credit rating rather than gaining wider perspective from a group of credit rating agencies. [½]

Transparency of fee structure – this may not be sufficiently transparent… [½]
… and consequently the credit rating agency may have too familiar a relationship with the FRN issuer… [½]
…, possibly opening themselves to undue influence or being misled. [½]
Competition – the credit rating agency may have a monopoly position in the market… [½]
… which may reduce its incentive to continue to develop and adjust to changing market conditions. [½]

[Maximum 3]

[Total 19]

Many candidates did not demonstrate a good understanding of the Core Reading on LIBOR in parts (i) and (ii) of this question, despite it featuring in several areas of the course and being quite topical in the years before the exam.

The calculation questions of parts (iii) and (iv) produced a wide spectrum of answers. Many candidates made good attempts and calculated the answers correctly. Others did not reflect the coupon being paid quarterly in their answers and errors were introduced early on.

Part (v) gave candidates the opportunity to demonstrate deeper understanding of the underlying principles. It was not as well answered, suggesting that candidates may be going through mathematical processes without thinking fully about what the answers mean.

Parts (vi) and (vii) were generally attempted well and many scored highly.

Q6 (i) Minimum $X = 0$ [½]
Maximum $X = 0.20$ [½]

[Maximum 1]

(ii) Let $P(Y)$ = the probability of default of Y and $P(\cancel{Y})$ = the probability of survival of Y.

The following probabilities of default in year 1 are as follows (where strike through indicates survival):

$P(A \cup B) = P(A) + (B) - P(A \cap B) = 0.45$ [1]
[or could calculate this as the sum of the two values below plus $P(A \cap B) = 0.15$ as stated in the question, i.e. 0.25 + 0.05 +0.15 = 0.45]

$P(A, \cancel{B}) = P(A) - P(A \cap B) = 0.25$
$P(\cancel{A}, B) = P(\mathrm{B}) - P(A \cap B) = 0.05$ [1]

There are four possibilities that would lead to payoff of the derivative at the end of the second year with the following losses:
Both default in Y1: $P(A \cap B) \times 100 \times 0.7 = 10.5$ [½]
No default Y1, both default in Y2:

$(1 - P(A \cup B)) \times P(A \cap B) \times 100 \times 0.7 = 5.775$ [1]
[or $(1 - P(A \cup B)) \times P(A \cap B) = 0.0825$]

A defaults Y1, B defaults Y2:

$P(A, \cancel{B}) \times P(B) \times 100 \times 0.7 = 3.5$ [1]
[or $P(A, \cancel{B}) \times P(B) = 0.05$]

B defaults Y1, A defaults Y2:

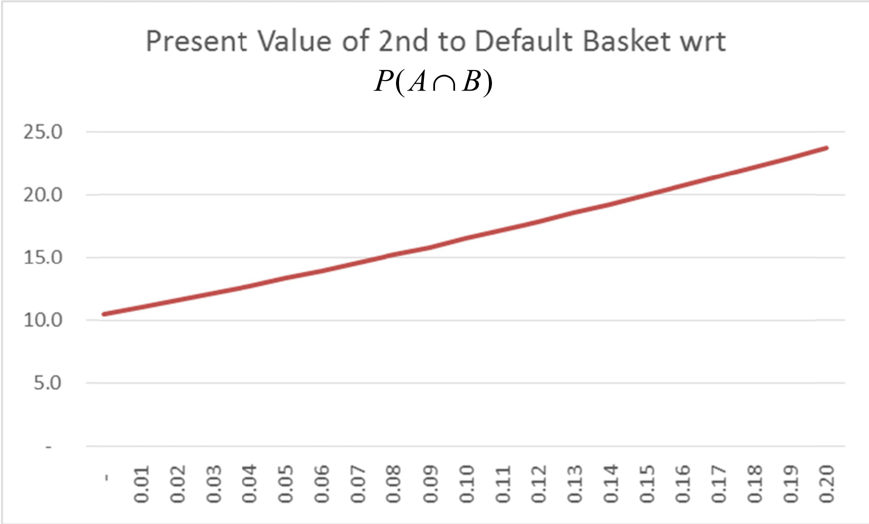
$P(\cancel{A}, B) \times P(\mathrm{A}) \times 100 \times 0.7 = 1.4$ [1]
[or $P(\cancel{A}, B) \times P(\mathrm{A}) = 0.02$]

Hence the total present value of the losses, is equal to:

$21.175 \times e^{-0.03 \times 2} = \$19.9m$ [1]
[or (0.15+0.02+0.05+0.0825) $\times 100 \times 0.7 \times e^{-0.03 \times 2} = \$19.9m$]

[Maximum 5]

(iii)



[Maximum 2]

(iv) Sample random values … [½]
… from a $N(0,1)$ distribution. [½]

Need two sets of such random values, one for each bond. [½]
Need to sample from a multivariate distribution using a correlation of ρ_{AB} .[½]
ρ_{AB} could be estimated from the correlation between equity returns for the two companies. [½]
This produces correlated pairs of values x_A, x_B with the correct correlation structure, but still with a $N(0,1)$ distribution. [½]
Carry out a percentile-to-percentile transformation from these values… [½]
… to simulate the times to default of Chain A and Chain B, t_A, t_B . [1]

To do so, first estimate the cumulative probability distributions, Q_A, Q_B , for these times to default … [½]
… from data produced by credit rating agencies… [½]
…or from bond prices. [½]

Simulated pairs of default times t_A, t_B are then derived by comparing the x_A, x_B to the cumulative probability distributions, Q_A, Q_B using the Gaussian copula relationship $x_i = N^{-1}[Q_i(t_i)]$, where N^{-1} is the inverse of the standard cumulative normal distribution. [1]

The simulated default times across all sampled pairs can then be used to determine simulated numbers and hence probabilities of default of each bond within the two years. [½]

Allowance would then also need to be made for the loss given default and discounting. [½]

[Maximum 4]

(v) The financial institution (FI) may wish to hedge existing exposure to credits of Chain A and Chain B… [1]
… and using a Second-to-default basket would be cheaper than buying separate protection on both chains. [½]

Or it would allow a reduction in concentration risk to global supermarkets… [½]
… even if the FI had a positive view on both credits. [½]
The FI may speculate that the correlation implied by the banks pricing is too low… [½]
… and hence wants to go long on credit correlation (which buying the second-to-default basket effectively is). [½]
The basket swap also provides protection against default tail risk. [½]

[Maximum 2]

(vi) The equity price of Chain C is volatile, and so market convention is now likely to require the swap to be over-collateralised [1]

Crucially, the equity of Chain C will be likely be negatively correlated with the price of the derivative… [1]
… given that Chain C is also a global supermarket. [½]
So as the likelihood of a combined default of Chain A and Chain B increases, the bank would have to purchase more and more equity of Chain C to ensure the swap remains fully (or over) collateralised… [½]
… which would be unattractive as this the equity may not be available in the market…. [½]
… and it could lead to the bank being selected against by market participants. [½]
In the extreme scenario when Chain A and B default and the derivative is required to be settled, it is likely that the equity of Chain C will have fallen dramatically, or even be worthless. [½]

Even if the equity of Chain C did have some residual value, the liquidity of the equity may be limited in a stressed environment… [½]
… and it may not be possible to liquidate the holding sufficiently quickly to settle the derivative [½]

So on settlement the holder of the derivative would likely have to pursue a claim against the bank once the collateral has been exhausted. [½]

It could be argued that post the default of Chain A and Chain B, Chain C may face less competition and so it shares might perform well. [½]

This would be most likely if the default of Chain A and Chain B were due to specific factors only affecting these Chains (e.g. aggressive expansion into new markets). [½]

Equity is not good to use as collateral due to the volatility of its value. [½]

Overall, using the equity of Chain C would likely be an ineffective source of collateral. [½]
[Other valid points were awarded marks.]

[Maximum 4]

[Total 18]

This question was the least well answered on the paper. It required candidates to work methodically through the several parts and apply their general understanding to this specific case.

Parts (i) and (ii) were mainly probability based and required care in working out exactly which probabilities were required. The key to solving part (ii) was to look at each year separately and then to work out which possibilities lead to default. The final steps are working out the probabilities and calculating the present value.

Part (iv) was an area of the course which has not been examined before. Despite this, the better prepared students scored well on this.

The final two parts were attempted by most students although few were able to generate sufficient ideas to score highly, particularly in part (vi).

Q7 (i) Without loss of generality let $t > s$. [½]

From the definition of covariance:

$$\mathrm{cov}[W_s, W_t] = \mathbb{E}_{\mathbb{P}}[\{W_s - \mathbb{E}_{\mathbb{P}}[W_s]\}\{W_t - \mathbb{E}_{\mathbb{P}}[W_t]\}] \ .$$ [½]

From the definition of $\mathbb{P}$-Brownian motion,

$$\mathbb{E}_{\mathbb{P}}[W_s] = \mathbb{E}_{\mathbb{P}}[W_t] = 0 \ .$$ [½]

Therefore, $\mathrm{cov}[W_s, W_t] = \mathbb{E}_{\mathbb{P}}[W_s W_t]$. [½]

The corresponding time intervals $[0,s]$ and $[0,t]$ are overlapping. Expressing W_t as the sum of independent random variables W_s and the increment $W_t - W_s$ gives:

$$\begin{aligned}\operatorname{cov}[W_s,W_t] &= \mathbb{E}_{\mathbb{P}}[W_s(W_s + W_t - W_s)]\ ,\\ &= \mathbb{E}_{\mathbb{P}}[W_s{}^2 + W_s(W_t - W_s)],\\ &= \mathbb{E}_{\mathbb{P}}[(W_s)^2] + \mathbb{E}_{\mathbb{P}}[W_s(W_t - W_s)].\end{aligned}$$ [1]

Due to the independence of the terms in the second expectation it follows that

$$\begin{aligned}\operatorname{cov}[W_s,W_t] &= \mathbb{E}_{\mathbb{P}}[(W_s)^2] + \mathbb{E}_{\mathbb{P}}[W_s]\mathbb{E}_{\mathbb{P}}[(W_t - W_s)],\\ &= s + 0,\\ &= s.\end{aligned}$$ [½]

Similarly, if $s < t$ then $\operatorname{cov}[W_s,W_t] = t$. [½]

Therefore, for any s and t:

$$\operatorname{cov}[W_s,W_t] = \min\{s,t\}\ .$$ [½]

[Maximum 3]

(ii) Using Taylor's theorem for $V(S_t,R_t,t)$:

$$dV = \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial S_t}dS_t + \frac{\partial V}{\partial R_t}dR_t + 0.5\frac{\partial^2 V}{\partial t^2}(dt)^2 + 0.5\frac{\partial^2 V}{\partial S_t{}^2}(dS_t)^2 + 0.5\frac{\partial^2 V}{\partial R_t{}^2}(dR_t)^2 +$$

$$\frac{\partial^2 V}{\partial S_t \partial t}(dS_t)(dt) + \frac{\partial^2 V}{\partial R_t \partial t}(dR_t)(dt) + 0.5\left(\frac{\partial^2 V}{\partial S_t \partial R_t}(dS_t)(dR_t) + \frac{\partial^2 V}{\partial R_t \partial S_t}(dR_t)(dS_t)\right) +$$

higher order terms. [2]

$(dt)^2 = 0$. [½]
$(dt)(dW_t) = 0$. [½]
$(dt)(dZ_t) = 0$. [½]
$(dW_t)^2 = dt$.
$(dZ_t)^2 = dt$, as both W and Z are Brownian motions. [1]

Using the definition for Z:

$$\begin{aligned}dZ_t &= \rho dW_t + \sqrt{1-\rho^2}\,dB_t \text{ and so}\\ dW_t dZ_t &= \rho(dW_t)^2 + \sqrt{1-\rho^2}\,dW_t dB_t,\end{aligned}$$

$= \rho dt + \sqrt{1-\rho^2}\, dW_t dB_t$ [using the work above),

$= \rho dt$ [using the result given in the question $dW_t dB_t = 0$]. [1]

Using the results above gives:

$(dS_t)^2 = \sigma^2 S_t^2 dt$,

$(dR_t)^2 = \alpha^2 R_t^2 dt$,

$(dt)(dS_t) = 0$,

$(dt)(\mathrm{d}\, R_t) = 0$,

$(dS_t)(dR_t) = (dR_t)(dS_t) = \rho\sigma\alpha S_t R_t dt$. [2]

Putting these into the Taylor expansion gives the required result (by ignoring the higher terms as stated in the question). [½]

[Maximum 6]

(iii) The equation is Ito's lemma generalised to two assets… [1]
with different Brownian motions. [½]

[Maximum 1]

(iv) **Construction of Z_t**

The correlated Brownian processes can be used in the simulation of two correlated assets. [1]

For example, equity prices generally move together in the same direction. [1]

Such assets may also arise in the case of an asset which is dependent on another asset. [½]
This may be useful in determining the price of derivatives based on such assets, such as a basket option. [1]

For example by using a Monte Carlo simulation approach. [½]

The method set out in the question for the construction of Z_t from two independent standard Brownian motions can instead be used in reverse, to take two correlated Brownian motions and create independent Brownian motions. [½]

Independent Brownian motions can be easier to work with. [½]

Equation in (ii) (Ito's lemma for 2 assets)

This can be used for modelling processes which involve two assets. [½]

For example, it can be used as a basis for deriving a Black-Scholes equation using two assets. [1]

This can lead onto pricing options whose payoff is dependent on two assets. [½]

[Maximum 3]

[Total 13]

Overall, this was the best answered question on the paper.

In general candidates were quite comfortable with this mathematical area of the course. They were able to apply their knowledge in part (ii) and in part (iii), with typically only some small algebraic errors in part (ii).

It was only in part (iv) that most candidates struggled to score highly. This more challenging part was a good differentiating question in terms of the standard of candidates who passed the exam.

END OF EXAMINERS' REPORT