EXAMINATION

1 October 2010 (pm)

Subject ST6 — Finance and Investment Specialist Technical B

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You have 15 minutes before the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*

3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all nine questions, beginning your answer to each question on a separate sheet.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

NOTE: In this examination, you are never required to prove the use of an arbitrage-free methodology unless clearly stated in the question.

1 (i) (a) Explain the use and significance of Ito's lemma for the valuation of derivatives based on stochastic processes.

(b) Explain what it means for a stochastic process X_t to be a martingale under a probability measure **P**. [3]

Consider a standard **P**-Brownian motion W_t.

(ii) Obtain the differential equation for the stochastic process $Y_t = W_t^2 - t$, and state with reasons whether it is a martingale. [2]

(iii) Show for what value(s) of the constant *a*, if any, the stochastic process $Z_t = W_t^4 - 6W_t^2 t + at^2$ is a martingale. [3]

[In parts (ii) and (iii) you may assume that both processes are sufficiently bounded.]

[Total 8]

2 Consider a floating-rate note (FRN) which pays semi-annual floating coupons of *m*% per annum above six-month LIBOR. LIBOR interest rates are *r*% per annum for each period t = ½, 1, 1½ etc. The FRN trades in the market at a discount yield of *y*% per annum. Both *r* and *y* are semi-annually compounded.

(i) (a) Derive a valuation formula for a five-year FRN, expressing it in simplest terms.

(b) Calculate the value of the FRN in the case where $m = 1$, $r = 5$ and $y = 7$.

(c) Write down a formula for the duration of the FRN (you do not need to simplify this).

(d) Estimate (but do not calculate precisely) the duration of the FRN using the same variables as in (b).

[6]

You work in the risk management department of a bank that has FRN holdings on its books. A colleague has asked why your risk systems are showing some unhedged market risk for these FRNs despite the fact that all floating LIBOR payments are recorded in the system as having zero interest rate risk.

(ii) Explain to your colleague why the risk system is correct. [2]

[Total 8]

3 (i) State the Cameron-Martin-Girsanov Theorem for a **P**-Brownian motion process W_t, defining any symbols you use. [3]

(ii) State the Martingale Representation Theorem for a **Q**-martingale process, defining any symbols you use. [2]

(iii) Explain the significance of the Cameron-Martin-Girsanov and the Martingale Representation Theorems for the martingale approach to valuing derivatives. [5]

[Total 10]

4 Consider a bespoke "trading range" option. This option pays a fixed amount X at the end of three months if the daily trading range of a particular market is larger than a given range for at least m days during the three-month period. [X and m are agreed before the option is traded.]

Assume that there are 20 trading days in each month, and that the probability of breaking the given trading range on any given day is a constant p. Five years' history of full market data is available.

(i) Explain how you could use a Poisson distribution to model this option. [2]

[<u>Note</u>: *For a Poisson distribution with parameter λ, the probability of there being exactly k occurrences is* $p(k) = \frac{\lambda^k e^{-\lambda}}{k!}$ $k = 0, 1, 2 \ldots$]

The trader believes that the probability of breaking the trading range on any given day is 5%, i.e. $p = 0.05$.

(ii) (a) Calculate the expected number of range breaks over a three-month period.

(b) Calculate (in terms of X) the trader's expected value of the option for a strike equal to this number of range breaks. You may assume interest rates are zero.

(c) Explain whether the result in (ii)(b) is a "risk neutral" price. [4]

An investment fund has acquired this option and has asked for advice on certain aspects of the transaction.

(iii) Comment on:

(a) the valuation from the fund's perspective
(b) any other risk management matters that might be relevant [5]

[Total 11]

5 Over many years the FTSE 100 index volatility has traded (both in implied and historic terms) in the range of 10% to 15% per annum. In the midst of a financial sector crisis in late 2008, option price implied volatility rose to levels of around 30%. In March 2009, with the FTSE 100 index well below the 4,000 level, the managers of a pension fund became concerned about the downside risk in the market, having previously been positive and expecting a long-term rise in the market. They purchased a large number of short-dated Puts on the index, and part financed them by selling one year out-of-the-money Calls at a strike of 4,000.

It is now two months later (May 2009) and the market, instead of falling, has in fact risen. The Puts are of negligible value, but the short Call position is becoming potentially problematic. A broker has suggested that the fund should buy back the short Calls and fund the cost by selling twice as many Calls on the index at a strike of 4,400 with the same expiry date.

You have been asked to assess the likely performance of this switch transaction, i.e. the purchase of the 4,000 Calls and sale of twice as many 4,400 Calls. (You do not need to review the rationale behind the original trades two months ago.)

(i) Sketch on one graph (against values of the FTSE index) the likely profit and loss curve for the switch transaction for three different time points:

(a) on the day of the transaction
(b) one month before expiry
(c) immediately before expiry

[3]

(ii) Sketch similarly the gamma profile of the switch transaction for the same time points as in (i). [3]

(iii) Comment on the broker's proposal and its potential attractiveness to the fund. [4]

[Total 10]

6 (i) Define market risk and credit risk as they apply to derivatives. [2]

A bank sells a foreign exchange rate (FX) forward contract to a customer, and decides to hedge this trade by buying a forward contract with identical terms from another bank.

(ii) (a) Describe how the hedge affects market and credit risk from the perspective of the bank selling the contract to the customer.

(b) Show how the credit risk on a matched pair of forward contracts resembles a straddle. (A straddle is a combination of a Put and a Call with the same strike price and expiry date.)

(c) If the underlying currency rate for the FX forward is S, and the forward is struck at X at time T, find the strike of this "credit straddle". [5]

CDOs (Collateralised Debt Obligations) were originally developed as a means of selling packages of corporate bonds in a single instrument. In the last few years, a new type of CDO has been marketed based on packages of residential mortgaged-backed securities (RMBS).

(iii) (a) Explain the concept of default correlation, why it may occur and how it affects the value of CDOs.

(b) Discuss how default correlation might differ between CDOs of corporate bonds and CDOs of RMBS during an economic downturn.

[4]

[Total 11]

7 The Hull-White one-factor model of interest rates projects the short-term rate r according to the following stochastic differential equation:

$$dr = [\theta(t) - ar]\, dt + \sigma dz$$

where z is a standard Brownian motion and the other variables are time-dependent or constant parameters.

A common practical method of implementing this model is in the form of a trinomial recombining tree, with discrete time steps of Δt (for dt) and a transformed interest rate process R that follows the simplified stochastic differential equation:

$$dR = -aR\, dt + \sigma dz$$

(i) Derive the three simultaneous equations that need to be solved to determine the risk-neutral probabilities for this trinomial tree at each node. [4]

You work for a financial institution that has written a variety of fixed interest derivatives. You are attempting to value them using an economic scenario generator (ESG) that uses Monte Carlo techniques on an underlying Hull-White one-factor model. The output from the ESG consists of:

- cash accumulations $C(t, n)$, being the amount that £1 would have accumulated to by time t in the nth economic scenario if continuously invested at short term rates
- bond prices $B(t, T, n)$, being the prices at time t in the nth economic scenario of bonds that will pay £1 at time $T > t$

for 1,000 economic scenarios (i.e. $n = 1$ to 1,000).

(ii) Write down a formula for the price of a five-year zero coupon bond in terms of the $C(t, n)$ outputs. [1]

(iii) Express in terms of the ESG outputs a formula for the value of a receiver swaption giving the holder the option after three years to enter into a five-year swap with annual payments, receiving 5% per annum fixed and paying LIBOR on a notional of £1 million. [3]

The ESG has been calibrated to bond and swaption prices, and you are checking the calibration by comparing market yield curves and swaption prices to those that the model produces. Some of the bond and swaption prices calculated by the ESG are close to market prices but not exact. A colleague has suggested that this could simply be the result of sampling error.

(iv) Describe how you could determine the statistical significance of the differences. [3]

(v) Discuss whether you would expect the model to be able to value accurately a range of bonds and swaptions provided a large enough number of runs were used. [2]

[Total 13]

8 (i) Outline the main differences between futures and forwards. [2]

A large bakery has entered into major contracts with a number of domestic supermarket chains to supply them with a specified number of loaves of wheat bread at fixed prices for the next five years. It is concerned about the risks to its financial health if the cost of producing wheat bread increases, and is considering hedging using wheat futures or forward trades that lock in the cost of wheat at a future purchase date.

(ii) Define basis risk in the context of the futures hedge described. [1]

(iii) Define the minimum variance hedge ratio for the futures hedge and derive a formula for it, explaining any notation used. [4]

(iv) (a) Describe the cashflow risks facing the bakery in using futures to hedge long-term contracts of this type.

(b) Discuss whether these risks could be avoided by the use of forwards. [4]

(v) Discuss the relative merits of forwards and futures as suitable hedging instruments for the bakery. [4]

[Total 15]

9 You work for a unit trust manager that is launching a unit trust that will hold over-the-counter American Puts on a total return equity index $F(t)$. You have been asked to set up a process to verify the prices that the option provider provides each day to be used in unit pricing.

The option has term T and strike X. On the valuation date at time 0:

- the total return index is $F(0)$
- the continuously compounded T-year risk-free rate is r
- the implied volatility of the option is σ

You have decided to carry out your verification by developing a spreadsheet calculator that values American Puts using a 30-step Cox-Ross-Rubinstein binomial tree.

(i) Explain why a binomial tree (as opposed to an algebraic or Monte Carlo approach) is an appropriate valuation mechanism for this derivative. [2]

(ii) Specify each of the steps that the calculator would perform. [5]

(iii) Outline how you might introduce a control variate technique to improve the accuracy of the valuation. [2]

The spreadsheet has been developed, but there are differing views about how the calculator should be used:

- Manager A is expecting σ to be "derived from the market" and used to calculate a price to be compared to that quoted by the option provider.

- Manager B is expecting the spreadsheet to be used to backsolve for the volatility implied in the price quoted by the option provider and for the volatility to be "sense checked".

(iv) (a) Demonstrate by means of an example how an appropriate volatility might be derived under Manager A's approach.

(b) Describe how you might "sense check" the implied volatility for reasonableness under Manager B's approach.

(c) Outline the key differences and relative merits of the two approaches. [5]

[Total 14]

END OF PAPER