

EXAMINATION

6 September 2006 (pm)

Subject ST6 — Finance and Investment Specialist Technical B and Certificate in Derivatives

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*
3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 8 questions, beginning your answer to each question on a separate sheet.*
6. *Candidates should show calculations where this is appropriate.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

NOTE: In this examination, you are never required to prove the use of an arbitrage-free methodology unless clearly stated in the question.

- 1**
- (i) Explain arbitrage and the role of arbitrageurs in the context of securities and derivatives trading. [3]
 - (ii) A simplistic market consists of a riskless bond B and a risky security S . A claim X is based on the price of security S_T at some time T .
 - (a) State what is meant by a self-financing strategy in this market.
 - (b) Explain why it is useful to construct a replicating strategy for X in the form of a self-financing strategy. [5]
- [Total 8]

- 2** As an option trader, you are considering some strategies based upon buying and selling options on a natural gas futures contract with current price 400. The volatility of the futures contract is currently around 45%.

You have two potential strategies to choose from:

Strategy A: An at-the-money “straddle” consisting of a simultaneous purchase of a call and a put, both with strike price 400.

Strategy B: An at-the-money “strangle” consisting of a simultaneous purchase of a call with strike price 500 and a put with strike price 320.

Assume that all the options expire three months from now, and the nominal amounts for each purchased option would be the same.

- (i) Draw approximate profit & loss diagrams **for each strategy** against the price of the underlying future, at the following points in time:
 - (a) now
 - (b) at expiry [6]

[**Note:** Sketch both curves for a strategy on the same graph, plotted against the futures price using a suitable scale. You should show the effect of the purchase price of the options. Any curves may of course be drawn freehand, and no calculations are required.]
 - (ii) Discuss the factors that you might take into account in choosing either strategy. [6]
- [Total 12]

3 Consider the basic Black-Scholes formula as it applies to an option on a non-dividend paying stock.

- (i) Show how the basic formula would need to be modified to cope with the following alternative situations:
 - (a) The underlying stock has a known continuously compounded dividend yield.
 - (b) The underlying stock pays a known discrete dividend during the period to option expiry.
 - (c) The underlying stock is instead a futures contract. [6]
- (ii) Show how the basic formula would need to be modified to price a one-year option on a five-year zero coupon bond. [3]
- (iii) Show how the basic formula would need to be modified for a non-dividend paying stock where interest rates are stochastic but independent of stock price processes. [**Hint**: consider the effect of the additional volatility of the interest rate.] [3]

[Total 12]

- 4 A retail investment institution offers a tranche of single premium unit linked investment bonds that track an equity total return index, currently standing at 2,500. The bonds are issued at time $t = 0$, with a total aggregate investment of £1.1 billion.

The product provides to the customer, four years after the initial investment, a benefit equal to the greater of the accumulated equity linked funds, net of tax and management charges, and a guaranteed minimum benefit being a return to the customer of the amount invested. Management charges are levied each year at the rate of $1\frac{1}{2}\%$ of the fund value. In addition, the assets under management suffer tax on the total investment return at the rate of 25% per annum over the life of the product.

[Note: hence, if the total return index has value E_t at time t , the amount subject to tax at maturity T is $\frac{E_T}{E_0} - 1$.]

The institution is considering purchasing put options on the equity index to ensure that it can meet the guaranteed minimum benefit. The purchase would be made out of the institutions own resources, i.e. without selling any unit linked assets.

- (i) Derive general formulae for the strike price X and total amount N of put options needed at time 0, expressed in terms of the index value E_t , unitised assets under management U_t , tax rate τ and management charge c , such that the payoff of the put options at maturity T exactly matches the guarantee at that date. Define the terms you use. [6]
 - (ii) If equity index volatility is 20% and risk free interest rates are 4% per annum:
 - (a) Show that the optimal strike price for each option is approximately 2,700, and
 - (b) Using a strike price of 2,700, calculate the cost to the institution of purchasing the suggested put options. [4]
 - (iii) Suggest a different way the institution could deliver the exactly same benefits. [2]
- [Total 12]

- 5** You have been asked to assess a portfolio containing a number of different types of options. The main part of the portfolio consists of vanilla currency, equity index and interest rate options. The options are valued using appropriate variants of Black-Scholes option formula. The portfolio also contains underlying cash positions, plus futures and options on futures.

Additionally, a small percentage of the portfolio consists of more complex (“exotic”) interest rate options and structured bonds, including such features as barriers and American expiries. These are valued using a Vasicek model. [Note: Barrier options are options where the payoff depends on whether the underlying asset price reaches a certain level during a given time period.]

- (i) Describe in detail the basic market risks that would apply to this portfolio, noting especially how the risks of the options on futures and “exotic” options would differ from those of the vanilla options. [9]
 - (ii) Outline the advantages and disadvantages of moving to a Hull-White model of the interest rate yield curve for this portfolio. [4]
- [Total 13]

- 6** Let S_t be a stock price process governed by two parameters, μ and σ , that follows the nodes of a recombining binomial tree, such that the process moves from value s at some particular node along the up/down branch to a new value:

$$\begin{cases} s \exp(\mu\delta t + \sigma\sqrt{\delta t}) & \text{if up} \\ s \exp(\mu\delta t - \sigma\sqrt{\delta t}) & \text{if down} \end{cases}$$

where δt is the small interval of time it takes for the stock to jump from one node to the other, and up and down moves are equally likely.

Let X_n be the total number of up jumps out of the first n jumps.

- (i) If there have been n jumps by time t , write down the relationship between n and t . [1]
- (ii) Show that the value of S_t at time t is given by:

$$S_t = S_0 \exp\left(\mu t + \sigma\sqrt{t}\left(\frac{2X_n - n}{\sqrt{n}}\right)\right) \quad [3]$$

- (iii) By considering the probability distribution of X_n , explain what happens to the distribution of S_t as $\delta t \rightarrow 0$. [4]
- (iv) Derive values for the martingale measure up and down probabilities, approximate to first order in δt . [3]

[Total 11]

- 7** Consider two securities, F and G , that are both positively dependent on the same underlying source of uncertainty.

Security F has price f that follows the process $df = \mu f dt + \sigma f dw$, where w is a standard Brownian motion. Security G has price g that follows the process $dg = \nu g dt + \theta g dw$, with the same underlying Brownian motion.

It has been found that the volatility of G is a damped version of the volatility of F , as defined by the formula $\theta = \sigma e^{-\beta t}$ for some constant β . The risk-free rate r is 4% per annum.

- (i) Explain the significance of the “market price of risk” $\lambda = \frac{\mu - r}{\sigma}$. [2]

- (ii) If F has a constant expected return of 7% per annum, i.e. $\mu = 7\%$, and if $\beta = 0.05$, calculate the expected return of G at time $t = 10$. [3]

A squared payoff call on a security with strike price K is an option where the payoff at exercise is the excess, if any, of the square of the stock price over K .

- (iii) (a) By creating a suitable risk-free portfolio, or otherwise, derive the differential equation for the price of an American style squared payoff call based on security F .

- (b) State the boundary conditions required to solve the equation.
(A solution to the equation is not required.)

[6]

- (iv) Explain how you would adapt the differential equation derived in (iii)(a) to price an American style squared payoff call based on G . [3]

[Total 14]

- 8** A fixed interest swap market has a term structure defined by the following discount factors (i.e. the values in decimal of a zero coupon bond of that maturity):

<i>Year</i>	<i>Discount</i>
0	1.000
1	0.960
2	0.920
3	0.879
4	0.840
5	0.804

Based on this information:

- (i) (a) Calculate the first three annual forward rates that apply in this market.
- (b) Calculate the fixed coupon on a 3-year annual coupon fixed-to-floating swap.
- (c) Show that the fixed coupon on a 3-year annual coupon fixed-to-floating swap that commences in 2 years' time is 4.598%.

[6]

The owner of a European payer swaption exercisable in T years has the right on exercise to pay, in exchange for floating rate funds, annual fixed rate funds of $X\%$ per annum in arrears from time T for a further n years.

- (ii) Using the Black option formula, derive in terms of the forward swap rate the formula for the value of the European payer swaption. Define any terms you use. [5]
- (iii) Calculate the value of a €50,000,000 European payer swaption exercisable after 2 years into a 3-year annual LIBOR swap paying 4.5% fixed. Use a volatility for the forward swap of 15% per annum. [4]
- (iv) Describe how you would expect volatilities on swaptions to compare with those on bond options of equivalent maturity. [3]

[Total 18]

END OF PAPER