INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2018

Subject ST6 – Finance and Investment
Specialist Technical B

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Luke Hatter
Chair of the Board of Examiners
July 2018

A. General comments on the *aims of this subject and how it is marked*

1. The aim of the Finance and Investment Technical B subject is to instil in successful candidates the ability (at a higher level of detail and ability than in CT8) to value financial derivatives, to assess and manage the risks associated with a portfolio of derivatives, including credit derivatives and to value credit derivatives using simple models for credit risk.

2. This subject deals with mathematical techniques for the valuation and risk management of derivatives, together with some aspects of their practical application.

3. Different numerical answers may be obtained to those shown in these solutions, depending on whether figures were obtained from tables or from calculators. Allowance was made for these minor differences. However, candidates may have lost marks for using excessive rounding or showing insufficient working.

4. Derivative theory is an interesting but very exacting subject which needs to be tackled in a precise and disciplined manner. Questions that appear unfamiliar can, with some clear thinking at the start of the exam, be found to be straightforward examples of well-known techniques. Thorough preparation of the course material is essential, however. This subject also requires quite a varied approach in writing solutions. Algebraic and numerical content needs to be supplemented with well-reasoned but brief arguments where the question demands. Candidates should always try to provide several distinct relevant points for the discursive questions, not lengthy paragraphs developing a single idea.

5. Candidates who give well-reasoned points, not in the marking schedule, are awarded marks for doing so.

B. General comments on *student performance in this diet of the examination*

Most candidates were able to score marks across all the questions in the paper and the best prepared candidates scored well above the pass mark. Candidates generally scored higher marks on questions 1, 2 and 5.

In terms of areas for improvement:

- Some candidates were unable to demonstrate a breadth of knowledge across the whole syllabus and so did not score all of the available knowledge marks from the Core Reading (e.g. Q2(i)).
- Many candidates did not appear to tailor their answer to the command words in the questions, such as "Determine" or "Assess" (e.g. Q6(ix)).
- Many candidates could have scored more marks if they demonstrated their working more clearly when answering questions requiring calculation or derivation (e.g. Q3(iii)).

- Many candidates provided a significant amount of detail on relatively narrow arguments when responding to the discursive questions (e.g. Q6(iii)), rather than discussing a greater range of relevant points.
- Some candidates struggled with questions that required an element of application of Core Reading to situations that were not immediately familiar, so did not demonstrate the desired depth of knowledge (e.g. parts of Q6).

The comments that follow the questions concentrate on areas where candidates could have improved their marks, in an attempt to help future candidates to revise accordingly and to develop their ability to apply the Core Reading to related situations. This skill is likely to be valuable both in exams and in using this theory in their professional careers.

C. Pass Mark

The Pass Mark for this exam was 60.

Solutions

Q1 (i) An arbitrageur carries out an arbitrage trade… [½]

… i.e. takes offsetting positions in two or more instruments to make a certain and risk-free profit from mispricings. [1]

[*Alternatively, i.e. takes offsetting positions in two or more instruments to have positive probability of a profit with zero probability of making a loss, due to mispricings*] *[1]*

[Maximum 1]

(ii) Consider 1 GBP. There are two ways in which this can be converted into USD dollars at time *T*:

1. We can invest 1 GBP at r_{GBP} and enter into a forward contract to buy F_0 USD per unit GBP at time *T*. [1]

2. Alternatively, we can trade 1 GBP for S_0 USD in the spot market and then invest at r_{USD}. [1]

Both these portfolios have the same starting value… [1]

… since the forward contract has zero value… [½]

… and there are no external cashflows… [½]

… and hence in the absence of arbitrage opportunities, the two strategies must have the same value at *T*. [1]

i.e.: $1 \times e^{r_{GBP}T} \times F_0 = S_0 e^{r_{USD}T}$, hence, $F_0 = S_0 e^{(r_{USD} - r_{GBP})T}$ [½]

[Maximum 4]

(iii) No transaction costs [½]
Market participants are subject to the same tax rate on all net trading profits. [½]
Market participants can borrow and lend at the same risk-free rate of interest. [½]
Market participants take advantage of arbitrage opportunities as they occur. [½]
No counterparty/credit risk (on the forward) [½]

[Maximum 2]

(iv) Central clearing would likely require posting of both variation margin… [½]
… and initial margin... [½]
… which may not be available to the arbitrageur… [½]
…or would increase the cost of trading FX forwards. [½]
Even a small proportional cost is likely to be very material given the small mispricing the arbitrageur is likely trading on. [1]
Such costs would likely erode the apparent arbitrage opportunity in (ii). [½]

Central clearing also places further operational requirements on the arbitrageur, which may slow the speed at which the arbitrageur can trade on an arbitrage opportunity. [1]
For example, reporting of trade details / timely confirmation / back-office risk monitoring. [½]

Central clearing may result in standardisation of the FX market, which may remove the arbitrage opportunities all together. [1]
Conversely, the increased liquidity may mean that the arbitrageur can trade using a bigger notional or more quickly, increasing the risk-free profit. [1]

[Maximum 3]

[Total 10]

Overall, this question was well answered.

Parts (i), (ii) and (iii) were well answered.

In part (iv) relatively few candidates developed enough distinct points. Most candidates identified the requirement to post margin but did not consider either wider implications or the consequences margin would have for any potential arbitrage.

Q2 (i) First, we introduce a variable R, which is a discrete-time representation of the continuous rate process for r. [½]

R follows the SDE: $d\ln R = \left[\theta(t) - a\ln R\right]dt + \sigma dz(t)$ [1]

Now propose a variable $x = \ln R$ [1]

Build a tree for variable x^* which follows the SDE:
$dx^* = -ax^*dt + \sigma dz$ (i.e. x^* follows the same process as x except $\theta(t)$ is zero and the initial value is zero). [1]

σ can be determined from implied caplet volatilities. [½]

Now construct a trinomial tree for x^* which will have three states: up, down and midway between. [½]

The spacing between successive nodes on the tree can be set to: $\Delta x^* = \sigma\sqrt{3\Delta t}$ [1]

… which is found to give the best numerical efficiency / to optimise error minimisation. [½]

Start with a symmetrical branching pattern… [½]

… and then use non-symmetrical branching patterns when certain maximum or minimum values are breached… [1]

… to ensure the probabilities on all three branches are always positive... [½]

…while maintaining the recombining structure. [½]

The probabilities of each of the three branches are derived by solving three equations for …

… the probabilities summing to 1 [½]

… the expected change in x^* over Δt [½]

… the variance of in x^* over Δt [½]

The nodes $i\Delta t$ are then displaced by a set of values for α_i … [½]

… to provide an exact fit to the initial term structure. [½]

This is done by defining a factor $Q_{i,j}$ as the present value of 1 unit of cash payable if node (i,j) on the tree is reached and zero otherwise... [½]

… and then using forward induction… [½]

… to replicate the required zero-coupon bond prices. [½]

This provides values for x at each node, which can then be simply converted to values for R by $R = e^x$ [½]

[Maximum 6]

[Note to markers: Hull Reference: Ch 30.7, p698]

(ii) The Black-Karasinski SDE models the natural logarithm of the short rate *r*, so *r* cannot be negative as there is no natural logarithm of a negative number. [1]

[Equivalently, when converting from ln(r) to r by using exp, the result will always be positive.]

[Total 1]

(iii) The Hull-White (HW) SDE models the absolute value of the short rate *r*… [½]
with a fixed volatility $\tilde{\sigma}$… [½]
… and hence there is a non-zero probability of *r* being negative [½]
… as the distribution of *r* is normal. [½]

[Maximum 1]

(iv) Negative interest rates under the HW model are generally seen as an undesirable consequence… [½]
… of the HW model being so tractable… [½]
… so the HW model is unlikely to demonstrate robust modelling of negative interest rates. [1]

However, assuming that the bank uses a plausibly large value for the constant volatility in the SDE… [½]
… there should be a large probability of negative interest rates, as the bank wishes. [1]
Interest rates would also mean revert back to positive values… [½]
… which may be expected given the desire of policymakers to avoid negative interest rates. [1]

The HW model can be calibrated to fit the current term structure, even if some or all of the yields at different terms are negative. [1]

In any case, given that negative interest rates have never occurred previously, it is hard to determine whether a model robustly allows for negative interest rates.

[1]

[Maximum 3]

(v) Using the hint, substituting $\tilde{\theta}(t)$ in gives:

$$dr = \left[\frac{dF(0,t)}{dt} + \tilde{a}(F(0,t) - r)\right]dt + \ldots$$ [½]

This shows that on average, r, follows the slope of the initial instantaneous forward rate curve… [1]
…. and if r deviates from this curve, it reverts back to it at rate $\tilde{a}$. [1]

[Maximum 2]

[Total 13]

Answers to part (i) varied considerably in depth. A substantial proportion of candidates were able to explain most of the key steps and scored full marks or close to full marks. Some candidates focused their answer on the trinomial tree for the Hull-White model but still scored highly given the common steps involved. Some candidates did not outline many of the key steps and so only scored some of the marks. This question illustrated the need to prepare across the whole syllabus.

Parts (ii) and (iii) were generally well answered.

In part (iv), those candidates who scored highly provided reasoned analysis of the advantages and disadvantages of the Hull-White model in the context of a negative interest rate environment. Other candidates stated features of the model but did not tailor their answer to the higher-order "assess" command word.

In part (v) most candidates were able to perform the substitution appropriately per the hint but only a minority successfully explained the implications.

Q3 (i) A position in the underlying asset cannot be used to make the portfolio gamma-neutral. [1]

The gamma of the underlying asset is zero. [½]

This is because the value of the asset **is** the asset, and hence there is no second order dependency of the asset price to itself. [½]

Therefore any position in it will not change the gamma of the portfolio. [½]

[Maximum 2]

(ii) The gamma of a vanilla option is positive for an investor who is long the option [1]

The gamma of the hedged portfolio is linear in its constituents, so the total gamma is the (weighted) sum of the gamma of the constituents, including the options. [½]

The gamma-hedged portfolio will have zero gamma (by definition). [½]

Therefore the gamma of the delta-hedged portfolio (before gamma-hedging) must be negative (since the investor is going long vanilla options to hedge it). [½]

For example, it might include holding only short options, [½]

Or the absolute value of options with negative gammas are greater than the sum of options with positive gammas. [½]

[Maximum 2]

(iii) Consider a Taylor series expansion near the initial price of the underlying asset: S_0. [½]

Let S be the current price of the underlying asset and V the option value.

$$\begin{aligned} V(S) &\approx V(S_0) + \frac{\partial V(S_0)}{\partial S}(S - S_0) + \frac{1}{2!}\frac{\partial^2 V(S_0)}{\partial S^2}(S - S_0)^2 + \cdots + \frac{1}{n!}\frac{\partial^n V(S_0)}{\partial S^n}(S - S_0)^n \\ &\approx V(S_0) + \frac{\partial V(S_0)}{\partial S}(S - S_0) + \frac{1}{2!}\frac{\partial^2 V(S_0)}{\partial S^2}(S - S_0)^2 + o(S - S_0)^2. \end{aligned}$$

[1.5]

Ignore terms of $o(S - S_0)^2$. [½]

$$\begin{aligned} V(S) - V(S_0) &\approx \frac{\partial V(S_0)}{\partial S}(S - S_0) + \frac{1}{2!}\frac{\partial^2 V(S_0)}{\partial S^2}(S - S_0)^2 \\ &\approx \Delta(S - S_0) + \frac{1}{2}\Gamma(S - S_0)^2 \end{aligned}$$

[½]

Changes in other variables which influence V are assumed to have negligible effect (e.g. time or volatility).

[½]
[Maximum 3]

(iv) Using the result of part (iii) and the linear nature of the components of the portfolio gives:

$$F(S) - F(S_0) \approx \Delta_{Fund}(S - S_0) + \frac{1}{2}\Gamma_{Fund}(S - S_0)^2 .$$ [1]

Here, F is the value of the portfolio, S is the current value of the underlying asset, Δ_{Fund} is the delta of the portfolio and Γ_{Fund} is the gamma of the portfolio. [½]

As the portfolio is delta-hedged at the start of the business day then by definition delta = 0, and letting S_0 be the value at the start of the business day:

$$F(S) - F(S_0) \approx \frac{1}{2}\Gamma_{Fund}(S - S_0)^2 .$$ [½]

$$F(S) - F(S_0) \approx -1{,}853.1 \times (S - S_0)^2$$ [½]

Therefore, any change in value of the underlying asset is going to lead to a large loss to the portfolio. [1]

The change in value can be either up or down. [½]

[Maximum 3]

(v) The portfolio can be made gamma-neutral by buying $3{,}706.2 / 1.74 = 2{,}130$ of the traded options. [1]

As a result of buying these the delta of the hedged portfolio increases by $0.70 \times 2{,}130 = 1{,}491$. [1]

To regain delta-neutrality, 1,491 units of the underlying asset are required to be sold. [1]

[Sub-Total 3]

(vi) The delta-hedging portfolio could use options which have smaller negative gammas yet also provide the required delta hedging. [1]

Use options with different characteristics, e.g. different maturity dates… [½]
… or non-vanilla options. [½]

[Maximum 1]

(vii) In future, set risk tolerances on how low negative gamma can reach before taking action. [1]

[Maximum 1]

[Total 15]

Part (i) was well answered. A wide range of responses were provided in response to part (ii) with only some candidates identifying that the gamma of a long vanilla option was positive.

Part (iii) was well answered by many candidates. Candidates who scored most highly included the initial steps in their derivation and framed their approach in the context of a Taylor series expansion. This highlights the importance of explaining what is being done at the start of an approach. Some candidates used Ito's Lemma rather than Taylor's theorem, and were awarded marks accordingly.

In part (iv), many candidates correctly determined the algebraic impact but only a minority went further and explained in detail what this meant, as implied by the "identify" command word. Candidates are encouraged to pay close attention to the command words.

Part (v) was well answered by those who attempted it.

Parts (vi) and (vii) were only answered well by a small proportion of candidates. Those that did answer well successfully identified a distinct alteration or another action that would have the required effect.

Q4 (i) $X(t)$ is a stochastic process… [½]

…as Itô's lemma is specifically developed to extend calculus to stochastic processes [½]

f must be a deterministic function [½]

… and it must be twice continuously differentiable function. [1]

This ensures that $Y(t) = f(X(t))$ is also a stochastic process [½]

And that the first and second derivatives with respect to $X(t)$ exist… [½]
… these forming the main inputs to Itô's formula. [½]

[Maximum 2]

(ii) Itô's lemma forms the basic extension of differential calculus to variables which are stochastic in nature. [½]

Given a functional expression for a stochastic process, Itô's lemma can be used to generate a stochastic differential equation. [½]

In the context of valuing derivatives, Itô's lemma can derive stochastic differential equations (SDEs) for derivatives whose payoffs depend on the evolution of a stochastic process. [1]

Often, the need is to go in the other direction, from a stochastic differential equation to a stochastic process. [½]

This is much more difficult and it cannot be done in general. [½]

Itô's lemma can be used to check that an inspired guess is a solution of a stochastic differential equation. [½]
Examples from valuing derivatives include the geometric Brownian motion process. [½]

Itô's lemma is used in the Black-Scholes framework. [½]

In particular it is used to derive the Black-Scholes differential equation for pricing basic call or put options. [½]

Itô's lemma provides the theoretical framework for the use of Delta and Gamma for hedging. [½]

[Maximum 3]

(iii) $X(t)Y(t) = 0.5((X(t)+Y(t))^2 - Y(t)^2 - X(t)^2)$ [½]

Then using the linear properties of the differential:

$d(X(t)Y(t)) = 0.5(d(X(t)+Y(t))^2 - d(Y(t)^2) - d(X(t)^2))$. [½]

Using Itô's lemma with the square function gives:

$d(X(t)^2) = 2X(t)dX(t) + \sigma^2(t)dt.$ [1]

$d(Y(t)^2) = 2Y(t)dY(t) + \rho^2(t)dt.$ [½]

$d((X(t)+Y(t))^2) = 2(X(t)+Y(t))(dY(t)+dX(t)) + (\rho+\sigma)^2(t)dt.$ [1½]

Expanding:

$$d((X(t)+Y(t))^2) = 2X(t)dX(t) + 2Y(t)dY(t) + 2X(t)dY(t)$$
$$+2Y(t)dX(t) + (\rho+\sigma)^2(t)dt.$$

[½]

Putting these into the second equation above gives the solution as the $X(t)dX(t)$ and $Y(t)dY(t)$ terms cancel, [¼]

along with the $\sigma^2(t)dt$ and $\rho^2(t)dt$ terms. [¼]

[Maximum 4]

(iv) Let $X(t) = B + C\int_0^t e^{Ds}dW(s)$ [½]

Then in differential form, $dX(t) = Ce^{Dt}dW(t)$. [1]

Let $Y(t) = e^{-At}$.
Then, as this is not stochastic, $dY(t) = -AY(t)dt$. [½]

Using the solution provided in the question $S(t) = e^{-At}\left(B + C\int_0^t e^{Ds}dW(s)\right)$,
$S(t)$ can then be written in the form $S(t) = Y(t)X(t)$. [½]

In differential form this becomes: $dS(t) = d(Y(t)X(t))$. [½]

As $Y(t)$ is not stochastic, $d(Y(t)X(t)) = X(t)dY(t) + Y(t)dX(t)$. [½]

Using the result of part (iii) gives:

$d(X(t)Y(t)) = -AX(t)Y(t)dt + Ce^{(D-A)t}dW(t) + 0$. [1]

The final term is 0 as $Y(t)$ is not stochastic. [½]

Hence,

$$dS(t) = -AX(t)Y(t)dt + Ce^{(D-A)t}dW(t)$$
$$= -AS(t)dt + Ce^{(D-A)t}dW(t).$$ [1]

Comparing this to the equation given in the question of
$dS(t) = -5S(t)dt + 4dW(t)$, gives: [½]

$A = 5$, [½]
$D = 5$, and [½]
$C = 4$. [½]

[Alternative solution for markers:

The process S(t) is an Ornstein–Uhlenbeck process. *[1]*

Using an integrating factor of e^{5t}. *[1]*
Then,

$$d\left(e^{5t}S(t)\right) = d\left(e^{5t}\right)S(t) + e^{5t}dS(t), \quad [1]$$
$$= 5e^{5t}S(t) + e^{5t}(-5S(t)dt + 4dW(t)), \quad [1]$$
$$= 5e^{5t}S(t) - 5e^{5t}S(t)dt + 4e^{5t}dW(t), \quad [½]$$
$$= 4e^{5t}dW(t). \quad [½]$$

This can be integrated to give:

$e^{5t}S(t) = B + 4\int_0^t e^{5s}dW(s)$, *where B is a constant.* *[1]*

$$S(t) = e^{-5t}\left(B + 4\int_0^t e^{5s}dW(s)\right) \quad [½]$$

Comparing this to the solution given in the question gives

$A = 5$, *[½]*
$D = 5$, *and* *[½]*
$C = 4$. *[½]]*

[Maximum 7]

(v) $B = S(0)$. [1]

[Maximum 1]

[Total 17]

Overall, there was a wide variety of performances between candidates, with only the strongest candidates scoring well.

A wide range of marks were awarded in part (i). Many candidates identified one or two of the key constraints but were not able to recall the majority of points. Most candidates stated that the function must be twice differentiable, rather than twice continuously differentiable (i.e. the first two derivatives exist and are continuous).

In part (ii), most candidates identified at least one key point, but many would have scored higher by considering the wider relevance of Ito's lemma rather than focusing on Black Scholes.

Many candidates were able to pick up a good proportion of marks by following the hint in (iii), but only the best prepared candidates were able to arrive at the final answer.

Many candidates gained full marks in (iv) and (v), which was pleasing to see, although a number of candidates did not score any marks at all. Candidates were awarded equal credit for deriving the answer either via the hint in the question or the integrating factor approach.

Q5 (i) The pension scheme is taking equity risk in the expectation that over time the risk will be rewarded. [½]

Small deviations in equity prices are expected, but large falls can lead to a large deterioration in the scheme's balance sheet, [½]

i.e. a large gap between assets and liabilities of the pension scheme. [½]

In the extreme it may threaten the solvency of the pension scheme, [½]

and that of the company, [½]

… which may be unable to make additional contributions to restore the resultant deficit. [½]
Therefore, the pension scheme may want to hedge this risk to obtain higher confidence that it is partially protected against a large fall in equity prices. [1]

[Maximum 2]

(ii) Possible hedging objectives include:

The pension fund could hedge against the equity investments losing a certain % of value… [½]

…over a given time period. [½]

The pension fund could aim to reduce the fluctuations in the value of the equity investments… [½]

…over a given time period. [½]

For example the time period could relate to the duration of the liabilities or the regularity of a funding valuation of the scheme. [½]

It could aim to minimise the cost of putting in place the hedge... [½]

…or minimise the cost of maintaining it. [½]

It could aim to minimise the cost of additional contributions for the company [½]

It could aim to ensure the funding level doesn't fall below a certain level. [½]

[Maximum 2]

(iii) The advantages/suitable features of the hedging strategy are as follows:

The strategy provides a broad hedge against the equity investments losing 30% or more of their current value over the next five years… [½]
… particularly if basis risk is low. [½]

This is suitable as it provides a healthy downside protection level… [½]
… without being overly expensive… [½]
… and retains benefits of the upside should equities rise. [½]

The nominal amount is suitable as it covers all equities held. [½]

It is a simple strategy for the pension scheme to understand given that they or the company may not have experience in derivatives. [½]

The pension scheme has only one decision to make following the implementation of the strategy: that is if or when to exercise the option. [½]
The American nature of the option means the execution is flexible… [½]
… e.g. it could be more optimally exercised during the 5 year period should markets fall. [½]

Therefore the ongoing governance involved should be low. [½]

The transaction costs should be relatively small… [½]

… as such an option should be available in the domestic derivatives markets… [½]
… although long dated options tend to be more expensive. [½]

The ongoing costs of monitoring the option should also be small [½]

as only one option is being monitored relative to the country's main domestic equity index. [½]

The disadvantages/less suitable features of the hedging strategy are as follows:

There needs to be a large fall in the index value for the intrinsic value of the hedge to increase … [½]
… although the market value of the option (i.e. including the time value) could increase materially following a fall. [½]
The value of the option will decay over time (i.e. theta decay), everything else being equal. [½]

There is likely to be basis risk between the underlying equity index and the equity investments of the pension scheme… [½]
… due to differences between the equities actually held and the constituents of the index [½]
… and also potentially to whether it is a price only or dividend-reinvested index [½]

As the time to maturity is five years, there is no protection beyond five years. [½]

The pension scheme may be able to roll the option to a new one after five years. [½]

but the price to be paid for this may be more expensive than currently. [½]

Because the option's strike references the current equity values... [½]

… if there are significant returns on the equity portfolio over the early period of the investment then these will not be protected if there is a later fall in equity prices. [½]

The hedging is therefore highly sensitive to both the strike price and maturity date. [½]
The hedging strategy introduces counterparty risk. [½]

[Maximum 6]

(iv) It could buy:

- put options with a variety of different maturities and same strike prices [1]

- put options at the same maturity but with different strike prices [1]

- a combination of the above (i.e. both different maturities and different strike prices) [1]
- put options on other indices, if available and relevant to the equities held [1]

[Maximum 2]

(v) It is an index of implied volatility, [½]

of short-dated (30-day) options [½]

on the S&P 500 Index, [½]

calculated from a wide range of calls and puts. [½]

It is the most popular volatility index. [½]
It is published by the CBOE. [½]

[Maximum 2]

(vi) When there is a fall in US equity values it is expected that the S&P 500 Index will also fall. [½]

It would also be expected that investors would increase their purchase of options [½]

to hedge their portfolios. [½]

In particular, there will be a large increase in the purchase of put options to protect the downside risk. [½]

The increase in demand of put options increases their price, [½]

and as a result increases the implied volatility of the options. [½]

Therefore the increase in implied volatility will increase the VIX. [1]

It should also be noted that due to the put-call parity the demand in puts also affects the call options and hence the implied volatility for call options. [½]
[Note to markers: 1 mark can be given to the statement that the VIX is expected to rise but with no justification or a wrong justification.]

[Maximum 3]

(vii) By purchasing VOIX futures the pension scheme is taking a view on the future volatility of the underlying domestic equity index, [1]

and not on the future level of the domestic equity index. [½]

It is important to recall that implied volatility is a measure of future volatility and not historical volatility. [½]
As a result the VOIX is a measure of investors' expectations of future market volatility. [½]

The pension scheme would purchase VOIX futures at the current level…
[½]

…and at a maturity date in line with the time horizon of the hedging (where possible). [½]

In the case of a fall in the domestic equity market, the VOIX levels will tend to rise, as explained in part (vi) for the VIX. [½]

In this case, the gain in the value of the futures contract will offset some of the losses to the equity investments resulting from a fall in the domestic equity market. [1]

The pension scheme will need to calculate the number of futures required based on the expected movements in the VIX and the equity exposure in the scheme (e.g. using scenario testing or historical analysis of different movements). [1]

The hedge may then need to be rolled forward at the expiry of the contracts to provide continuous hedging over the desired time horizon. [½]

[Maximum 3]

[Total 20]

This question was generally well answered.

Part (i) was well answered.

In part (ii), many candidates outlined effective objectives, but only some candidates generated sufficiently many distinct points.

Part (iii) was generally well answered. Better answers were those that were developed in sufficient breadth instead of focusing on specific points such as basis risk. Many marks were awarded to those candidates that covered a broad range of topics.

Part (iv) was well answered with the majority of candidates achieving full marks. In part (v), a number of candidates were not familiar with the SPX VIX.

Part (vi) was only answered well by a minority of candidates. The stronger answers logically explained a number of steps that would lead to an increase in implied volatility. More limited answers gained a mark for stating that the VIX is expected to rise but provided incorrect, or no, justification for this. Candidates are encouraged to use the number of marks awarded in each question as a guide to the number of distinct points needed to score full marks.

Some candidates scored highly in part (vii) but overall it was not well answered. Only a small number of candidates explained clearly how the VOIX future would act as a hedge against a large fall in equity prices.

Q6 (i) The holder of a CDS has the right to sell the reference entity (bond) for face value to the seller of the CDS, when a credit event occurs (i.e. the reference entity defaults). [1]

The CDS therefore provides insurance for the holder against the risk of a default by a particular company. [1]

The holder makes periodic payments to the seller until the end of the life of the CDS or until a credit event occurs. [1]

The settlement in the event of default involves either physical delivery of the bonds or a cash payment. [½]
… but is usually cash settled. [½]

CDS prices are usually quoted based on the CDS spread… [½]
… which is defined as the total amount paid per year, as a percentage of the notional. [½]

The CDS is generally the most liquid type of credit derivative. [½]

CDS are typically traded over the counter (OTC). [½]

[Maximum 3]

(ii) The CDS-bond basis is defined as the CDS spread on a bond minus the excess of the bond yield over the risk-free rate. [1]
Arbitrage arguments suggest that the CDS-bond basis should be close to zero. [½]

The CDS-bond basis was slightly positive prior to the 2007 crisis… [½]
… before becoming highly negative during the crisis. [½]

If the CDS-bond basis is positive, then a risk-free profit can in theory be made by selling CDS protection and shorting the corporate bond. [1]

[Maximum 2]

(iii) **Advantages of TRS (vs CDS)**

The TRS provides a hedge for both market (price) risk and for the credit (default) risk of the bonds as required by the hedge fund… [1]
… whereas the CDS only provides protection for default risk. [½]
Therefore, the CDS will only be effective in hedging the market (price) risk to the extent that the CDS-bond basis remains constant. [1]
In crisis, the CDS-bond basis has historically become very negative, implying the CDS will not be a good hedge in extreme events for falls in bond prices.[1]
For example, if credit spreads widened significantly with no expectation of additional defaults, bond prices would fall leading to a market value rise on the TRS, but no market value change on the CDS. [1]

Using a single TRS allows the hedge fund to take a large position with minimal outlay… [1]
… whereas entering into up to 100 CDS would be highly complex… [½]
… CDS may not be available/highly illiquid on some names… [½]
… and would likely require central clearing. [1]

Disadvantages of TRS (vs CDS)

The TRS may not be as liquid as the underlying CDS (on average). [½]
The TRS is limited to a one-year tenor… [½]
…whereas CDS are typically traded for longer terms (e.g. 5 years), affording longer credit protection. [½]

The CDS positions will be more flexible (for example if the hedge fund has a favourable view on a credit it could choose not to hedge). [½]

The TRS (in isolation) introduces interest rate risk as falls in interest rates would lead increases in bond prices… [1]
… whereas the CDS will display limited sensitivity to interest rate movements. [½]

There may be substantial basis risk between the bonds in the iBond index and the 100 bonds currently held… [½]
.. whereas the reference entities under CDS will correspond exactly/more closely with the required credits. [½]

The TRS will introduce material counterparty risk with one single counterparty… [½]
… whereas the CDS contracts are more easily spread around other counterparties… [½]
… and would be more likely to be collateralised or cleared. [½]

[Maximum 5]

(iv) The hedge fund could…

Carry out historical regression/correlation analysis based on the performance of the iBond index versus the aggregate 100 bonds currently held (Current Bonds). [1]
The hedge fund may want to consider the correlation in the extreme or tail events as well as the "belly" of the distribution… [½]
... for example, it could consider how the hedge would have performed over the financial crisis. [½]

It could analyse the underlying constituents of the iBond index versus the Current Bonds held… [½]
For example by rating, sector, currency, duration...
[Note to markers: please award ½ mark for each example, to a maximum of 1]
[1]

It could carry out scenario analysis to understand how different economic conditions cause divergences in the performance of the iBond index versus the Current Bonds. [1]

[Maximum 3]

(v) The receive leg of the TRS is not expected to contribute any material interest rate sensitivity… [1]
… as the payments are linked to LIBOR, which would be expected to "float" in line with movements in interest rates, and its market (or discounted) value would be unchanged. [½]

On the pay leg, if interest rates fall, then all other things equal the value of the bonds in the iBond index would rise… [1]
… meaning the hedge fund will have to pay a larger amount under the TRS…
[½]
… hence the market value to the hedge fund will decrease/become more negative. [½]
The interest rate risk would be expected to be approximately symmetric (for small movements in interest rates) in that if interest rates rise the mark to market for the hedge fund will increase. [1]

[Maximum 3]

(vi) $\$1,000m \times 7 \times 0.01\% = \$0.7m$ [1]

Maximum 1]

(vii) The value of the swap when interest rates are 3% p.a. is zero as the swap is at the money. So we just need to value the derivative when interest rates are 2.99%... [1]

Floating Leg

$$B_{Float} = PV_{FirstCoupon} + DiscountedNotional_{t=1month}$$ [½]

$$B_{Float} = \left[\$1,000m \times \frac{2.96\%}{12} + \$1,000m\right] \times \left(1 + 3\% - 0.01\%\right)^{-\frac{1}{12}} = \$1,000.01m$$ [1]

Or alternatively:

The fixing of the first payment under the floating leg would not be expected to be material given payments are made monthly and the 12-year term, hence $B_{Float} \sim \$1,000m$ [1.5]

Fixed Leg

$$B_{Fixed} = PV_{Coupons} + PV_{Notional\,@\,Maturity}$$ [½]

We need to work out the interest rate convertible monthly to value the PV of the coupons:

$$i^{(12)}_{Stress} = 12\left[(1 + 2.99\%)^{\frac{1}{12}} - 1\right] = 2.9498\%$$ [½]

$$B_{Fixed} = 2.96\% \times \$1,000m \times \left[\frac{1 - (1 + 2.99\%)^{-12}}{2.9498\%}\right] + \$1,000m \times (1 + 2.99\%)^{-12}$$

[½]

$$B_{Fixed} = \$1,001.03$$ [½]

$$V_{Swap} = B_{Fix} - B_{Float} = \$1,001.03m - \$1,000.01m = \$1.02m \sim \$1.0m$$ [½]

[Maximum 4]

(viii) $$N = \frac{Delta_{TRS}}{Delta_{Swap}} \times \$1,000m = \frac{\$0.7m}{\$1.0m} \times \$1,000m = \$700m$$

Or (using more exact figures):

$$N = \frac{Delta_{TRS}}{Delta_{Swap}} \times \$1,000m = \frac{\$0.7m}{\$1.02m} \times \$1,000m = \$686m$$

[Note to markers: please ensure Error Carried Forward is applied on this question and award full credit for any sensible rounding of the $Delta_{Swap}$ figure – for example, figures between $675m and $725m can be correctly calculated using slightly different calculation methods and/or rounding conventions.] [1]

(ix) (a) Increasing the term of the TRS would not affect the interest sensitivity of the receive leg materially as these coupons are still linked to LIBOR… [½]
… nor would it affect the interest rate sensitivity of the pay leg which is determined by the duration of the iBond index… [½]
… hence no change to the required notional. [½]

(b) Receiving fixed amounts under the TRS would mean that the receive leg of the TRS increases in value when interest rates fall… [½]
…and so reduces the net overall exposure from the TRS… [½]
… and so this will reduce the required notional. [½]
Given the short term of the receive leg, the reduction in the required notional would not be expected to be that significant. [½]

(c) If credit spreads increase, then other things being equal, the duration of the bonds underlying the iBond index will contract…. [½]
… so the overall sensitivity of the TRS will reduce … [½]
… reducing the notional required. [½]

[Maximum 3]

[Total 25]

Part (i) was well answered. Many candidates scored highly on part (ii), and did so by expanding their answer beyond the definition of the bond basis.

A range of marks was achieved in part (iii). Many candidates scored highly where they considered a wide variety of advantages / disadvantages of the TRS vs the CDS. Some answers lacked depth and narrowly focused on the operational aspects. The best prepared candidates identified the key benefit of the TRS was the hedging of spread as well as default risk and scored highly.

Part (iv) was well answered by many. Candidates often focused on several different ways to assess the constituents in the various hedges, while only a minority considered wider scenario testing or regression analysis.

Only a minority of candidates scored highly in part (v). Those that did correctly identified that the key interest rate exposure was on the pay, *rather than the receive, leg. This demonstrates the need for understanding practical applications of ST6 beyond the underlying technical content.*

Part (vi) was well answered.

Part (vii) was only answered well by a small minority of candidates. Some candidates spent considerable time on extensive algebra and exhaustive calculations which could have been simplified by application of simple annuity formulae. A substantial number of marks were available for qualitative explanations of why the LIBOR leg would not have contributed material interest rate sensitivity.

Part (viii) was answered well by those that attempted it.

Few candidates provided any justification for their answers in part (ix) and so did not score high marks. Candidates are encouraged to pay close attention to the command word ("determine" in this case). Those candidates that scored highly were able to provide a clear justification for the impacts.

END OF EXAMINERS' REPORT