2010 Examinations

SPECIMEN SOLUTIONS

Subject ST9 — Enterprise Risk Management

1 (i) Interest rate risk is the risk arising from changes in interest rates which includes the potential changes in customer behaviour as well as the financial impact. A wealth of historic data are available and a range of forecasting models have proved to be useful predictors of future interest rates meaning that the risk is relatively quantifiable.

Foreign exchange rate risk is the risk arising from movements in foreign exchange rates. A wealth of historic data are available and a range of forecasting models have proved to be useful predictors of future interest rates meaning that the risk is relatively quantifiable.

Credit risk is the risk that a counterparty to an agreement is unable or unwilling to make payment(s) due under an agreement. It can also include changes to the value of an asset due to changes in the perceived creditworthiness of the underlying obligor(s). The inability to pay is relatively quantifiable given the wealth of historic data and the resultant range of credible forecasting models. The unwillingness to pay is not particular quantifiable depending as it does on socioeconomic behaviour.

Basis risk is the risk arising from differences in the movements of two comparable indices: That is the extent to which a particular position reflects the position required. It is quantitative by its nature because it is the measurement of the potential difference between two other quantified indices.

[Note: other appropriate risk types would be acceptable, such as insurance risk and other forms of market (financial) risk.]

(ii) Social risk is the risk of a change in society creating a change in demand and/or the opening of new market opportunities and/or the alteration of a business's responsiveness to demand, and as a consequence the characteristics of its workforce. The risk is society's impact on business and not vice versa. This risk cannot be reliably measured because it is behavioural and reliant on people. Changes in society can arise for a myriad of reasons making the past a relatively poor guide and certainly no guide as to the timing of change.

Legal risk is the risk of failing to operate within the law, failure to show evidence of operating within the law and failure to recognise and manage legal threats. This risk cannot be reliably measured because it is behavioural and reliant on people. Effective and strictly observed business processes will of course mitigate the risk. Provided the business processes and observance levels remain up to date then the past may be some guide to the future but at best this will only ever be a very approximate guide.

Political risk is the uncertainty that stems, in whole or part, from the actions of governments and/or non-governmental groups. This risk cannot be reliably measured because it is behavioural and reliant on people. Because people change in government and because economic/social/religious circumstances change over time, the past is an unreliable guide to the future.

Technology risk is the risk of events that would lead to insufficient, inappropriate or mismanagement of investment in technology. The risk cannot be reliably measured because insufficiency, inappropriateness and mismanagement are all qualitative concepts.

[Note: other appropriate risk types would be acceptable, such as regulatory risk, agency risk, reputational risk.]

2 (i) Include risk skills as a valuable aspect in future hiring decisions to help to embed risk awareness in the firm's culture.

Form a risk committee made up of management from risk, audit, financial and compliance. Ensure that the committee is for the most part separated from business personnel and that the committee has significant terms of reference, reporting lines and regular access to the chief risk officer.

Inform management and personnel of enterprise risks on a regular basis and encourage feedback.

Incentivise business management to identify risks, report risks to more senior management and to produce risk-adjusted profits, through use of an appropriate reward/bonus system. That is, modify the current remuneration system without adding to the overall cost.

Include promoting ERM, ensuring its effectiveness and implementation of its recommendations in the CEO's own performance evaluation.

Produce simple ERM management reports for both senior management and the board. The reports must include the risks, an estimation of their frequency and severity and recent measures taken to mitigate them.

Introduce a common risk language to the firm including risk rating systems and standard templates for risk reports to improve efficiencies and to improve comprehension.

Include risk discussion as an integral part of staff/management meetings.

Develop and communicate clear organisational responsibilities for the identification and management of risks.

Develop a culture that encourages open and wide communication around the company.

[Note: The solution only requires eight practices for full marks.]

(ii) Introduce constraints on business practices in order to reduce risk through diversification. For example, set additional portfolio constraints on investments.

Introduce counterparty constraints in order to reduce risks through not trading with more risky counterparties. For example, a bank could introduce lending guidelines which targeted only investment grade companies.

Increase corporate governance within the firm. For example and using available internal resources only, introduce frequent internal audit to all areas of the firm and increase the volume of reporting in the firm. Ban overtime in the firm and require all personnel to take at a continuous period of at least two weeks off each year. People who are less tired may make fewer mistakes and corporate fraud, if existing, is often discovered when the perpetrator is away on leave.

Sell fewer "high risk" products. For example, withdraw from the immediate annuity market (thereby reducing the potential to make profit from it) if it is believed that the company has reached its longevity risk capacity.

(iii) Transfer risk, e.g. buy insurance and/or buy protection in the financial markets.

Sell risky assets and replace them with less risky assets (or which better match the liabilities).

Close down or reduce in size divisions or product lines which are felt to be particularly risky.

Centralise operations in order to reduce operational risks.

3 (i) **The validation process must include:**

Quantitative Methods

The use of effective statistical processes to evidence the goodness of fit of key aspects of the model to past data. Goodness of fit techniques will likely include maximum likelihood estimates and method of moments.

The use of sensitivity analysis to changes in key assumptions to test the stability of the model.

The use of back-testing the model to observed experience. Back-testing is important because depending on how volatile past experience has been, it may be possible to test the relative reasonableness across the quantile range of the modelled distributions and not just at, say, extreme results.

Qualitative Methods

The use of scenario analysis and stress testing to assess the apparent reasonableness of the model output to expectations.

Most ERM models will be employed to estimate the potential cost of extreme results. By their nature extreme results don't happen very often (if ever depending on how extreme). Most people accept that appropriate loss distribution models will be monotonically decreasing in the tails; it is the relative weight of the tail over which people will have some differences of opinion. Extreme event stress testing with input from experienced persons is a very useful way of trying to validate the relative weight in the tail of the model.

Internal Validation

The model must be interrogated by appropriately skilled persons at every level of detail from the individual calculations to the overall output being considered by the board.

External Validation

The model will be interrogated at different levels of detail by the auditors, the stock exchange, the credit rating agencies and the regulator. These entities will be expected to compare and contrast the design/input/processes/output of the model with their own internal model ideas and with other ERM models which have been submitted to them by similar firms. As an aside, external parties will not be formally validating the model. They are not in the business of helping firms to build ERM models. They will merely be corresponding with the bank until such time that they are satisfied that the model is sufficient for their own particular oversight purposes.

Data

The model is only as good as the data upon which it is based. The data must be reviewed in order to assess the appropriateness of the population of data employed, the accuracy of the data and the completeness of the data. If model points have been used, the appropriateness of the groupings chosen can be tested by comparison with selective full data runs.

Assumptions

The model assumptions should be presented to all appropriate persons including documentation supporting the derivation/reasonableness of the assumptions. Assumptions made on the basis of nothing more than judgement can be expected to be fully interrogated.

The validation process:

Is iterative

Both internal and external parties will review the model/model output and comment on all aspects of the model. The comments must be addressed and responded to. Agreed model changes should be regularly released to all parties to review and make any further comments.

Requires clear reporting lines and ultimate responsibility

Requires clearly designated responsibilities so that all parties inputting to the validation process know who is responsible for what aspects of the model and who is ultimately responsible for the validation.

Requires Documentation

Of course the model will be fully documented. The validation process must also be fully documented so that any interested party can review the comments made, the responses, the work that was done, the key points of the discussions, the resultant changes to the model and the final agreements from all parties on the acceptability of the model.

Output

Particularly senior management will not wish to review the model itself. Instead they will rely on output from the model. This output must be straight-forward, consistent over time, not too voluminous and sufficient to describe both the key model results and the work undertaken in the validation process.

(ii) The ongoing re-validation process is in many ways a real time process along the lines of the actuarial control cycle of model specification, parameterisation and construction, testing to new data and unfolding experience, redesign and re-parameterisation. It will involve:

- Testing actual experience to modelled expected experience
- Re-fitting distributions as new data unfolds
- Considering introducing new data types/processes to improve the model
- Corresponding with internal and external parties as necessary over major and minor changes
- Calculating the impact of the changes
- Updating the model documentation

4 (i) Interest rate risk and more particularly the spread between the average coupon on the bonds and the cost of the debt.

Investment market risk and more particularly the risk of overall capital loss resulting from sales of the corporate bonds in the market at the end of year 5 and during the term as the credit ratings change.

Foreign exchange rate risk which could be significant if the currency of the debt is different from the currency or currencies of the bonds. It could also be significant if the currency of the equity investment is different from the currency or currencies of the bonds.

Default losses risk on the BB corporate bonds, being the risk that default losses are greater than forecast.

(ii) Assuming that the bank is not the debtholder then the largest risk facing the bank is reputational risk in the case that the fund's returns are significantly less than forecast.

(iii) **Assumptions**

The BB bonds will be sold once their ratings change and new BB bonds bought as immediate replacements.

1.06% of the held bonds will default each year without transitioning.

The fund does not recapitalise for default losses. However it does receive coupons from the bonds and net capital gains from the bond sales.

The fund will pay small commissions to sell the corporate bonds as necessary.

The fund will pay the interest on the debt from time to time.

Assume that capital gains net of selling costs are zero each year.

Assume the fund makes 7% p.a. coupon on the bonds and pays 5% p.a. interest on the debt throughout the investment period.

Assume coupons received at year end and defaults all occur at year end and after receipt of the coupons.

Assume net cash receipts are all reinvested.

Assume no principal is recovered on defaulted bonds.

Calculation

The equity portion of the fund is growing at the rate of (7−1.06=) 5.94%p.a. and incurring 1.06% in default losses each year. The debt portion of the fund

is growing at the rate of (7–5–1.06=) 0.94%p.a. and incurring 1.06% in default losses each year. Hence,

$$£5.68\text{m} = 20 \times 0.0106 \times 1.07 \times \sum_{n=1}^{5} (1.0594)^{n-1} + 80 \times 0.0106 \times 1.02 \times \sum_{n=1}^{5} (1.0094)^{n-1}$$

(iv) The investment performance of the fund is largely determined by the spread between the redemption yield on a basket of BB bonds and the interest rate that the debt is tied to e.g. 3 month LIBOR. The positive differential pays for the expected costs of the default and provides the investors with a leveraged return.

(v) The spread is the difference between two interest rate series. It will likely be best to develop a simulation model which separately models the two different time series and then seeks to allow for auto-correlations between the two time series. The resultant spread can then be calculated for any given iteration. The alternative of developing a single model of the spread volatility using ARCH or GARCH processes should be tested but will probably result in a less stable model.

The forecast period will be 5 years.

The model will be based on time series going back over many years and paying particular regard to the range of BB bonds that the fund manager expects to invest in.

Whilst time series could be fitted directly from the two underlying data sets, the forecasting capability of the model might be improved if other time series were introduced which were correlated with the two required time series. This would only be helpful where the predicting powers of the additional time series were considered to be superior to the predicting powers of the two required time series on their own.

(vi) Assume that there are m models $M_1, \ldots, M_m$ with model j having k_j parameters denoted by $\theta_j = (\theta_{j1}, \ldots, \theta_{jk_j})'$ and a likelihood function $L_j\left(\theta_j; X\right)$. Let $\hat{\theta}_j$ denote the MLE of θ_j.

The Akaike Information Criterion (AIC) selects the model that minimises:

$$AIC(M_j) = -2 \ln L_j\left(\hat{\theta}_j; X\right) + 2k_j .$$

The Bayes Information Criterion (BIC) selects the model that minimises:

$$BIC(M_j) = -2 \ln L_j\left(\hat{\theta}_j; X\right) + k_j \ln N$$

where N is the number of observations in the dataset.

5 (i) Insurance and underwriting risk, being the risk of unexpected loss arising from the property insurance risks and in this case the risk of natural catastrophes resulting in unexpected losses to ABC. Such losses may be the result of experience fluctuations or of inappropriate selection and pricing of the risks. They write £120m of premium income each year. The average loss has been £77m and the maximum loss has been £168m. The risk of unexpected loss could be about a 20% chance (1 in 5) of a £91m loss.

Financial risk being the risk of loss resulting from the £100m equity holding. The equity holding is likely to have incurred a loss of more than £30m last year because the total investment income was a loss of £30m and the remaining investments were cash and government securities. In rough terms share prices generally collapse by say 30% every 7 years or so before eventually recovering ground. The size of this risk is therefore circa £30m.

Foreign exchange risk could be large if the cash and government securities are not held in US dollars. 90% of the £240m (£216m) outstanding claims liabilities is likely USD denominated and only the £100m in equity is clearly USD denominated. Exchange rate volatility can be dramatic. It is possible that a 20% reduction in GBP relative to the USD could occur. The size of the risks could be the order of say 20% of £116m, i.e. £23m.

There is a possibility that the underwriting risks and the risk of market value loss to the equity holding will be partly negatively correlated. 90% of the underwriting losses are anticipated to arise from US windstorms. US housebuilders should benefit from natural catastrophes as they will be needed to rebuild the homes.

(ii) The model does not require random variates because its purpose is to perform sensitivity analysis and stress testing. The model will generate future balance sheets (so will need to generate both assets and liabilities) and profit & loss statements. A three year time horizon would likely be sufficient.

Whilst the model itself will not require random variables some analysis of the potential range of each of the key business risks and their interdependencies should be done for the purpose of keeping the sensitivity analysis plausible and for the purpose of testing a range of remotely possible but extreme outcomes during stress testing.

The balance sheets and profit & loss statements will need to be internally consistent over time. For example:

- the balance sheet balancing and consistency between assets and liabilities
- unpaid claims moving to the outstanding claims reserves
- the outstanding claims reserve being run down as old claims are paid
- profits/losses moving across to shareholders' funds
- carried forward tax losses

(iii) The purpose of sensitivity analysis is to see which of the assumptions are most material on the business. In this case the two key outcomes to track are the net profits and the capital.

Create a base case balance sheet and profit & loss statement which has average expected returns and claims. Test the relative movement in net profits and capital:

- Changing the incurred claims up and down 20% and holding all else constant
- Changing the interest rates on the cash and government securities up and down by 20%
- Changing the value of the equity holdings up and down by 20%
- Changing direct acquisition costs up and down by 20%

Whilst the probabilities of the 20% movements actually happening for the different variables will be different all are reasonably likely to occur and it will give management a feel for the most important contributors to both profit and capital.

(iv) The purpose of scenario analysis or stress testing is to test different possible states of the world.

Create scenarios showing:

- A base case with average performance of all variables
- A case showing extremely good claims experience for three years with all other variables performing in line with average levels
- A case showing one average year of claims experience (average everything else) followed by two bad years of both claims experience and investment performance
- A case showing one year of "worst foreseeable case" of claims performance followed by two average years. All other variables performing in line with average levels

Testing scenarios such as these would help the company to understand its return on equity in average years, good years and bad years. It would also help the company to estimate the amount of capital it should be holding against the risks it is writing.

6 (i) **Potential justifications for the actuary's approach**

The approach is simple.

If the 97.5% quantile VaR is only used as a broad guide internally then it may be sufficient for purpose.

Model error, parameter error and changing business mix (data heterogeneity) can make any approach potentially inaccurate and misleading.

The potential shortcoming of VaR

VaR is a simple measure but it does not give any indication of the extent to which losses might potentially exceed the 97.5% quantile.

VaR has poor aggregation properties. Specifically it fails the important subadditive property of a coherent risk measure. This means that the VaR for the overall distribution will not necessarily be less than the sum of the VaR for the three separate loss distributions.

The VaR estimate can be misleading in the sense that it takes no account of model error which in this case could be far larger than normal because of the subjectivity of the assumptions.

Because VaR is a point estimate, at the extreme ends of the distribution it will often be necessary to run the simulation for a relatively long time before the VaR estimate becomes statistically constant.

Other potential shortcomings of the actuary's approach

The initial best estimates of the three classes of business are just that, estimates. They may not be the mean. For example, they could be the median.

The goodness of fit of the lognormal distribution assumption does not appear to have been tested in any way.

The judgmentally selected coefficients of variation could be wrong. This will likely cause the biggest error to the eventual VaR calculation because standard deviations of loss reserves can vary extremely widely.

The adopted coefficients of variation are judgmentally selected. They might be materially wrong. The company has been in business for many years

suggesting that the data should be available to estimate the correlation coefficients directly from the data.

(ii) **Translation invariance**

The Risk Measure should show that the amount of capital required supports the perceived variability of a loss and not its expected amount. Adding or subtracting a fixed amount from a loss leaves the capital (being the amount excess of the expected loss) unchanged.

Subadditivity

Compounding loss distributions should create a diversification benefit. Even if the distributions were 100% correlated the Risk Measure of the compounded distribution should not exceed the sum of the Risk Measures of the individual distributions.

Positive homogeneity

Also known as positive scalability the Risk Measure should show that the capital required to support "*n*" identical losses is equal to "*n*" times the capital need to support one loss.

Monotonicity

The Risk Measure should show that the capital needed to support a smaller loss (with the same distribution) is less than the capital needed to support a larger loss.

(iii) Let $L_1,\ldots,L_n$ be a generic sequence of random variables with associated order statistics $L_{1,n} \geq \ldots \geq L_{n,n}$ and note that for arbitrary m satisfying $1 \leq m \leq n$ we have:

$$\sum_{i=1}^{m} L_{i,n} = \sup\{L_{i_1} + \ldots + L_{i_m} : 1 \leq i_1 < \ldots < i_m \leq m\}.$$

Consider two random variables L and $\tilde{L}$ with joint distribution function F and a sequence of iid bivariate random vectors $(L_1, \tilde{L}_1), \ldots, (L_n, \tilde{L}_n)$ with the same distribution function F. Writing $(L+\tilde{L})_i := L_i + \tilde{L}_i$ and $(L+\tilde{L})_{i,n}$ for an order statistic of $(L+\tilde{L})_1, \ldots, (L+\tilde{L})_n$ then we must have:

$$\sum_{i=1}^{m} (L+\tilde{L})_{i,n} = \sup\{(L+\tilde{L})_{i_1} + \ldots + (L+\tilde{L})_{i_m} : 1 \leq i_1 < \ldots < i_m \leq m\}$$

$$\leq \sup\{(L_{i_1} + ... + L_{i_m} : 1 \leq i_1 < ... < i_m \leq m\} + \sup\{(\tilde{L}_{i_1} + ... + \tilde{L}_{i_m} : 1 \leq i_1 < ... < i_m \leq m\}$$
$$= \sum_{i=1}^{m} L_{i,n} + \sum_{i=1}^{m} \tilde{L}_{i,n} .$$

By setting $m = n(1-p)$ and letting $n \to \infty$, the "law of large numbers for expected shortfall Lemma" gives us that $ES_p(L + \tilde{L}) \leq ES_p(L) + ES_p(\tilde{L})$.

END OF SOLUTIONS