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THE TAXATION OF ANNUITY FUNDS

By G. V. BAYLEY, F.I.A.

Assistant Actuary of The Equitable Life Assurance Society

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'Income tax is another subject on which it is difficult for me to speak freely, but I cannot refrain from noting that the general taxation position, as it now affects life assurance and pensions business, has given rise to serious anomalies: ...one has only to consider... that the terms quoted by offices for the purchase of life annuities vary to a quite remarkable extent, depending on the distribution of the company's business between various funds, as it exists at the moment, and the consequential incidence of tax;...'

(*J.I.A.* Vol. LXXV, p. 7.)

WITH these words Sir George Maddex, in his Presidential Address, directed our attention to the present system of taxation of annuity funds. M. E. Ogborn (1), on 27 October 1947, touched on the subject of uneven incidence of tax upon differing funds, and some uneasiness about the consequences was manifest in the discussion on that paper.

Much has been written in recent years upon the general subject of taxation of annuities. S. J. Rowland and F. H. Wales (2), A. H. Shrewsbury (3), and M. E. Ogborn (1) have fully described and commented upon the present method. Relatively little attention, however, has been devoted to the effects of the system upon annuity funds, probably because it is only recently that we have become aware of its full implications. There is some need therefore to review the consequences and to investigate more closely the actuarial problems which follow.

It is proposed here to analyse the effects of the system and, proceeding from some simple basic principles, to point to a method of solution of the actuarial problems connected with valuation, investment and the terms for new business and withdrawals.

RELATIONSHIP BETWEEN SURPLUS AND TAXABLE PROFIT

In this paper the notation used by Rowland and Wales (2) will be followed as closely as possible. The Annuity Taxation Account provides a convenient starting point, and for all types of fund, whether Type A, B or C, it will appear as follows:

Annuity Taxation Account—Types A, B and C

Actuarial liability at beginning of year ...	V_0	Annuities paid (gross)	A
Premiums and considerations	C	Surrenders	W
Interest (gross)... ...	I	Taxable profit	P
Profit on investments realized	R	Actuarial liability at end of year	V_1
	$V_0+C+I+R$		$A+W+P+V_1$

Thus $$P = V_0 + C + I + R - A - W - V_1.$$

I shall assume that the life fund as a whole is taxed on an 'Interest less Expenses' basis. Income tax is effectively suffered on the 'taxable profit', P, by deduction from the management expenses claim, but P itself may be reduced in certain circumstances as we shall see later. Expenses attributable to the annuity fund are excluded from the taxation account by the operation of Finance Act, 1923, s. 16(2)(*b*), but may form part of the normal management expenses claim. The profit on realization of an investment is the excess of the selling (or redemption) price over its original price.

Type A funds

A Type A fund is defined as one where the annual annuity payments exceed the annual interest income. The annuity revenue account may be drawn up in the following manner:

Annuity Revenue Account—Type A

Actuarial liability at beginning of year	V_0	Annuities paid (gross)	A
Premiums and considerations	C	Surrenders	W
Interest (gross)...	I	Expenses	E
		Valuation surplus (before tax adjustments)	S
		Actuarial liability at end of year ...	V_1
	V_0+C+I		$A+W+E+S+V_1$
Valuation surplus brought down	S	Income tax on taxable profit	tP
Income tax relief on expenses	tE	Net surplus credited to participators ...	S'
	$S+tE$		$S'+tP$

Interest and annuities have been brought into account gross because the tax deducted from interest at source is compensated by an equal deduction from annuities due to the operation of the General Rule 21(3).

It is convenient to assume that the net surplus S' is carried each year to a 'participators' fund', so that each year's revenue account commences with the valuation liability. The second part of this account shows how the valuation surplus S is adjusted for tax* before being passed to the participators.

By the operation of Finance (No. 2) Act, 1940, s. 9(3), $t = \cdot 375$ when the standard rate of income tax exceeds this rate. I have assumed, for simplicity, that the whole of the management expenses attributable to the annuity fund are allowable for relief.

* In the rare cases where the life fund is taxed on profits under Case I of Schedule D, the second part of the annuity revenue account would evidently be debited with tax on surplus as modified by Finance Act, 1923, s. 16(1). In these circumstances tax is not charged on profit on investments realized (see §§ 12 and 20 of Ref. (3)). Apart from this detail the main argument is not affected.

The first part of the revenue account shows that

$$S = V_0 + C + I - A - W - E - V_1.$$

But it was shown earlier that

$$P = V_0 + C + I + R - A - W - V_1,$$

so that

$$S = P - R - E, \tag{1}$$

which is a general equation expressing the valuation surplus in terms of the taxable profit or vice versa. For the present, P is taken to be positive.

The second part of the revenue account gives

$$S' = S - t(P - E),$$

and substituting for S from equation (1),

$$S' = (1 - t)(P - E) - R, \tag{2}$$

or, alternatively,

$$S' = S - t(S + R). \tag{3}$$

It follows that tax may be regarded as being levied upon

(i) the valuation surplus as defined in the above revenue account, and

(ii) profit on investments realized.

It is clear that, whatever steps may be taken in valuation and in the treatment of investments in order to stabilize surplus from year to year, there is no escape from the fluctuating element of income tax on investment profits, tR.

If one investment is exchanged for another, for example, to produce a larger yield, the 'capital profit' on the former investment is not normally brought into revenue. The only effect upon the business is to vary the annual income and hence the taxable profit in subsequent years. In these circumstances it does not seem logical to tax the apparent 'profit' when the exchange takes place. When the second investment is exchanged there might well be an apparent 'loss' of the same order which, other things being equal, will rank for tax relief.

Capital profit can, of course, be brought into revenue; indeed this course is ultimately inevitable for a closed fund. It would be logical to charge the profit when it is actually brought into account. The precise treatment of assets and capital appreciation is intimately associated with the valuation of liabilities and an insurance company makes its own decisions on both matters. Since the company's decision on the basis for valuation of liabilities is accepted by the Inland Revenue it is bewildering to find that the closely related decision on the treatment of assets cannot also be accepted as a basis for taxation.

To this extent, the computation of taxable profit seems divorced from reality and it seems to me that an unnecessary and fluctuating element is inflicted upon the business which can only be described as arbitrary.

Since tax is levied on surplus, and on redemption profit of investments, any excess of the gross redemption yield, over what is required to build up the valuation liability, will be taxed at some time or another.

Where a 'taxable loss' is incurred (P negative), tax relief is not obtained at once but the taxable loss is carried forward to be set against future profits. The net surplus becomes

$$S' = P - R - (1 - t)E, \tag{4}$$

namely the valuation surplus defined earlier (equation 1) with the addition of tax relief on management expenses. This is less than the expression for S′ in (2) above by the positive quantity $(-tP)$.

The taxable loss (−P) is carried forward in what may be described* as the 'notional loss fund' and without the benefit of interest accumulation. If and when a taxable profit emerges in a subsequent year, the fund is drawn upon to write down the profit and tax is not levied until the fund has been exhausted. It follows that tax relief on a loss on redemption will be secured only if there emerge corresponding profits, whether in the form of valuation surplus or profits on realizations of investments.

Where a high coupon stock is purchased at a premium the annual income is usually greater than interest at the valuation rate on the sum invested so that tax is levied on the excess of income over interest at the valuation rate. There will be a loss on redemption but such loss may not lead to a relief from tax and even where relief is obtained it may be somewhat postponed.

In these circumstances it is not possible to draw the unqualified conclusion that any excess of the gross redemption yield over the valuation rate of interest is taxed at the appropriate rate, for that might seriously understate the position. However, where a fund consists of a reasonably balanced portfolio of premium and discount investments, there seems a good prospect of securing the full gross redemption yield on the premium investments.

The foregoing remarks lead to the general rule that an annuity fund of Type A can be regarded as though all interest income, expenses and annuity payments are gross without tax deduction. The valuation surplus together with redemption profit then suffer tax at the appropriate rate. If there is a taxable loss, however, there is no corresponding tax relief immediately although it may be obtained effectively at a later date. For the present this practical complication will be largely ignored and it will be sufficient, whilst pursuing the main principles, merely to remember that it exists.

Type B funds

A Type B fund is defined as one where the gross interest income is greater than the gross annuities paid but less than the annuities paid and taxable profit combined. The annuity taxation account is unaltered but income tax on taxable profit is reduced by the excess of tax on interest income over tax on annuities paid. Alternatively, it is permissible to regard the taxable profit as fully chargeable to tax and the tax paid on interest as limited to the amount of tax withheld from annuities. This is essentially the same as the Type A situation; tax on interest and annuities compensate exactly so that both can be brought into account gross. Finally by charging tax on taxable profit in full, the expression for the net surplus accruing to the participators becomes the same also.

The definition of a Type B fund requires the taxable profit to exceed the excess of interest over annuities. Because profits in general cannot be predicted with any degree of certainty, conclusions on the tax position of Type B funds would seem to possess inherent uncertainty and must be postponed until Type C funds have been considered.

Type C funds

A Type C fund is defined as one where the gross interest income exceeds the annuities paid and taxable profit combined. The annuity taxation account is

* Strictly, the use of the expression 'notional loss fund' should be limited to the Case I loss of a Type C fund. Since the treatment of the loss is the same in both cases it is convenient to describe the 'loss fund' in this way.

unchanged, but since the tax deducted at source on interest income is recovered only to the extent of the tax withheld on annuities, the net surplus passing to the participators becomes

$$S' = P - R - (1-t)E - t(I-A). \tag{5}$$

No income tax on profits has to be deducted from this sum and a 'notional loss' is transferred to the notional loss fund amounting to $I - A - P$.

The expression in (5) may be put into the form

$$S' = (1-t)(P-E) - R - t(I-A-P), \tag{6}$$

and a comparison with formula (2) shows the last term as an additional tax burden compared with Type A. Recognition of this situation is given in Finance Act, 1923, s. 16(2) which permits the notional loss of $(I - A - P)$ to be carried forward indefinitely to be set against future taxable profits.

Annuity Revenue Account—Type C

Actuarial liability at beginning of year			V_0	Annuities paid (gross) ...	A	
Premiums and considerations			C	less income tax ...	tA	
Interest (gross)	I				—	$(1-t)A$
less income tax	tI			Surrenders		W
	—	$(1-t)I$		Expenses	E	
				less income tax ...	tE	
					—	$(1-t)E$
				Net surplus		S'
				Actuarial liability at end of year		V_1
		$V_0 + C + (1-t)I$				$(1-t)A + W + (1-t)E + S' + V_1$

Hence $\quad S' = V_0 + C + I - A - t(I-A) - W - (1-t)E - V_1.$

But $\quad P = V_0 + C + I + R - A - W - V_1,$

so that $\quad S' = P - R - (1-t)E - t(I-A).$

This is, of course, the same as expression (5) deduced earlier for the net surplus carried to the participators' fund. Tax relief on expenses has been brought into account immediately in this instance, whereas for the Type A fund it was allowed for as one of the tax adjustments to the gross valuation surplus. The difference is more than one of convenience. Not only does it show

(i) that the Type A surplus is gross and fully chargeable to tax, and

(ii) that the Type C surplus is net and free from any tax adjustment;

it permits a Type C fund, in contrast to Type A, to be regarded on a net basis in which net interest is received and net annuities and expenses are paid. At the same time there exists a notional loss fund that can be regarded as an intangible credit. The fund is available to protect future profits from being charged to tax if the fund ever becomes Type A (or B) and taxable profits then emerge.

Returning to Type B funds, it is evidently permissible to look upon the position there also on a net basis. The Type C equation for net surplus must here be adjusted for the additional tax payable, because the taxable profit has exceeded the difference $(I - A)$. The deduction from formula (5) is therefore $t(P - \overline{I - A})$ and the net surplus available to the participators becomes

$$S' = P - R - (1-t)E - t(I-A) - t(P - \overline{I-A}).$$

It is worth noticing that this reduces to formula (3), the net surplus from a Type A fund, a result which confirms an earlier impression that Types A and B can be regarded as identical.

It is more convenient however to associate Types B and C because any rule is then unqualified by any assumption about the level of taxable profits. The rule is simply that if interest income exceeds annuity payments both these items together with expenses of management can be brought into account net after deduction of tax. Whether a fund is Type B or C merely determines whether tax is paid on part of the taxable profit and is therefore chargeable to the participators.

THE HISTORY OF A FUND

It is now possible to trace the tax position of certain types of fund over a period of time. For simplicity, each fund will be assumed to consist of ordinary immediate annuities and deferred annuities with return of the surrender values on death.

The most straightforward example is a closed fund of Type A and for the present it is assumed that annuity payments will always exceed the interest income until the last annuitant dies. It is legitimate to regard all interest, annuities and expenses on a gross basis. The gross valuation surplus will then, in general, be subject to tax each year, with the minor qualification that losses do not necessarily secure tax relief, at least immediately.

Broadly speaking, a deferred annuity withdrawal, before the annuity commences, causes a reduction in interest income thereafter. Although the total annuities will also be smaller at some later date the fund will usually remain, *a fortiori*, Type A.

If the fund is opened at any time to new immediate annuities it will also remain Type A; but a large increase in deferred annuity policies, which increases interest income alone at first, may cause an alteration sooner or later to Type B or C.

The position of a Type C closed fund is interesting because with the passage of time the balance of the business on the books will normally swing towards annuities in possession. Although interest at first exceeds annuities, a time should arrive when the position is reversed. For the present, it is simpler to assume only one such change but it is of course possible for more than one change to occur.

During the period whilst interest exceeds annuities both items together with expenses can be brought into account net after deduction of tax. The valuation surplus shown in the annuity revenue account passes to the participators without further tax deduction and a 'tax credit' is stored for a future date.

When the annuities eventually exceed the interest a fund of Type A* takes the place of Type C and thereafter both items together with expenses may be brought into account on a gross basis. The gross surplus, if any, will not however suffer tax deduction so long as the notional loss fund contains a balance. Deferred annuity withdrawals usually have the effect of advancing the date when the fund switches from Type C to Type A, and new immediate annuity business has the same effect. New deferred annuity business may advance or postpone it.

* Possibly the fund will be of Type B for an intervening period in the circumstances considered here. For present purposes, however, Type B and Type C funds can be regarded as identical.

Returning to the Type A closed fund it is evident that the fund may remain Type A for a period, then switch to Type C and finally revert to Type A. This would happen, for example, if there were a relatively large number of annual premium policies for deferred annuities. The interest income might increase more rapidly for a time than the amount of deferred annuities entering into possession.

HYPOTHETICAL MODELS

Consideration of the actuarial problems connected with new business terms, investment policy, and so on, is simplified by the construction of a hypothetical model fund corresponding to the business on the books and the investments held at the date of investigation.

Ideally, the emerging income and outgo in each subsequent year is first calculated by making suitable estimates of mortality, interest and expenses. This fixes the estimated future course of the fund provided there are no subsequent entrants or withdrawals.

The existing business is assumed to have been adequately dealt with and valued on assumptions consistent with the tax position of the fund from time to time as disclosed by the estimates of emerging income and outgo. It is assumed that the net surplus or deficiency disclosed at the date of investigation and at later valuations is borne by the participators. Thus the existing business must be treated as a closed fund, so far as the tax position is concerned.

Theoretically, new business is then considered to be self-supporting, not independently but as an integral part of the fund of which it becomes a part. Similarly withdrawals and deaths among deferred annuity policies, when they occur, are assumed to take cash surrender values which leave no strain upon the fund to be borne by the existing business.

Thus new business and surrenders are assumed to be dealt with on terms which introduce no surplus or strain, yet nevertheless involve a continuous adjustment of the technical basis of the business to the new circumstances.

Where the existing business forms a Type A fund the terms for new business must take account of whether that new business will cause a switch to Type B/C at some later date. So long as the fund remains Type A, interest annuities and expenses may be dealt with on a gross basis.

Where the existing business forms a Type B/C fund, consider how the position is affected by new business. Let n be the period of years (called the taxed period) at the end of which the existing business, treated as a closed fund, will become Type A. During this period interest, annuities and expenses must be treated on a net basis, thereafter on a gross basis.

As soon as any new business has been written on, the taxed period, n, will alter to conform to the new circumstances. It follows that the terms for new business will theoretically be subject to continuous adjustment on account of changes in the value of n. Strictly, to establish terms, n should be computed at each stage on the assumption that one half of the expected new business is written. By so doing, no strain will be placed on the fund for a given volume of new business.

The underlying principle in the foregoing remarks is of fundamental importance and a more rigorous demonstration has been prepared in the appendix to this paper.

NEW BUSINESS

Hereafter it is assumed that the projection of existing business as a closed fund has been carried out. It will show, for a Type A fund, whether it will remain Type A. For a Type B/C fund it will disclose the taxed period of n years before the annuities paid exceed the interest income. It may happen that the fund will revert to Type B/C for a further period before finally settling down to Type A.

Possessed of this information we may proceed to the calculation of premiums and considerations on the assumptions

(i) that, whilst the fund is Type A, interest, annuities and expenses can be brought into account gross without allowing for tax,

(ii) that, whilst the fund is Type A, any margin in the premium bases which is not required will emerge as surplus chargeable to income tax,

(iii) that, whilst the fund is Type B/C, during that (taxed) period interest, annuities and expenses can be brought into account net after deduction of tax, and

(iv) that during a taxed period any margin in the premium bases will emerge as surplus not chargeable to income tax.

When a fund becomes Type B after being Type C, the notional loss fund will protect the taxable profit from being partly charged to tax for some while. Indeed this protection may well extend to an ensuing Type A state, but for how long it is impossible to predict. If, however, Type B follows Type A this special protection would not normally be afforded.

These considerations only affect the profit margins in the premiums which ought logically to be reduced during the 'tax-free' periods. Assumptions (ii) and (iv) therefore are not strictly valid in all circumstances, but for the purpose of calculating premiums they are logical and, moreover, safe premises.

If the fund is Type A at the outset, then, for purposes of numerical illustration, I shall assume that the (closed) fund either remains Type A, or will become Type B/C so soon that it may be regarded as Type B/C from the outset, reverting to Type A after n years and remaining so.

If the fund is Type B/C from the outset, again, for simplicity, I shall assume that the (closed) fund reverts to Type A after n years and remains so. During this period interest, annuities and expenses will be allowed for after deducting tax.

The following basis will be adopted for calculating premiums and is put forward without comment as illustrative only.

Mortality for annuities in possession:	$a(f)$ ult. with a deduction from the age
Income tax	6*s.* 8*d.* in the £, i.e. $t = \frac{1}{3}$
Interest	Gross 3 % (i), net 2 % (i')
Commission	1 % of gross premiums or considerations
Expenses	3 % of gross premiums or considerations and $1\frac{1}{2}$ % of annuity payments
Formulae	It will be sufficient to give the formulae for a Type B/C fund with a taxed period of 10 years. Formulae for other cases follow the same principles.

Accented symbols are calculated at rate i'.

Immediate annuity per £100 consideration at age x, payable by half-yearly instalments without proportion to date of death:

$$\text{Rate } \% = \frac{100\,[1 - \cdot 04\,(1-t)]}{(1-t)\,(1\cdot 015)\,a^{(2)}_{x-1:\overline{10}|} + (1\cdot 015)\,a^{(2)}_{x+9}\,(D'_{x+9}/D'_{x-1})}.$$

Deferred annuity of £10 from age 60 payable as above, with return of the surrender value on death or withdrawal:

$$\text{Annual premium} = \frac{10\,[v'^{10} v^{60-10-x}\,(1\cdot 015)\,a^{(2)}_{60-x}]}{[1 - \cdot 04\,(1-t)]\,\ddot{a}'_{\overline{10}|} + \cdot 96 v'^{10}\,\bar{a}_{\overline{60-10-x}|}}.$$

Table 1. Rates of immediate annuity per £100 consideration—females

Age at entry x	Taxed period of fund in years, n		
	0 (Type A)	10	20
	£ s. d.	£ s. d.	£ s. d.
50	5 5 2	5 14 11	6 3 5
60	6 10 10	7 8 4	8 2 1
70	9 3 6	10 19 9	11 15 11
80	15 6 3	18 18 2	19 4 2

Table 2. Annual premium for a deferred annuity of £10 from age 60—females

Age at entry x	Taxed period of fund in years, n		
	0 (Type A)	10	20
	£ s. d.	£ s. d.	£ s. d.
20	2 0 5	2 1 1	2 3 0
30	3 4 1	3 5 3	3 8 11
40	5 13 5	5 16 2	6 4 11
50	13 5 11	13 17 3	12 5 3
55	28 14 1	27 8 4	24 7 3

The extent of the variation in terms, purely on account of the incidence of taxation on different funds, is remarkable.*

It is evident that Type A funds must perforce quote the least attractive terms for immediate annuities and the advantage of Type B/C funds here is very striking, particularly at the older ages. On the other hand Type B/C funds must presumably quote the worst terms for deferred annuities with a deferred period in the neighbourhood of the taxed period.

In practice, because new business itself continuously alters the technical basis of the fund it would be advisable to incorporate a margin in the assumed taxed period, n. This enables substantial blocks of business to be undertaken before a review is needed. For example, in the case of a fund of Type B/C with a taxed period of 20 years, it would be wise to assume only 10 or 15 years for immediate

* In this section I have assumed that a Type B fund is treated in the same way as a Type C fund. Strictly, if sufficient profits could be confidently expected during the taxed period so that the fund would remain Type B, the terms for new business could be either those shown in the tables for Type A or those for Type C. The difference does not become a charge upon the fund: it is reflected in the tax received by the Inland Revenue.

annuities. It is perhaps worth noticing that such an adjustment to the taxed period does not necessarily give a margin for deferred annuities: it may increase or decrease the premiums. The margin would have to be varied according to the circumstances.

VALUATION

Theoretically, the business on the books should be valued according to the foregoing principles. Net annuity payments and net expenses should be valued at a net rate of interest whilst interest income exceeds annuities, and conversely gross annuity payments and gross expenses at a gross rate when the position is reversed. A prospective gross premium valuation seems suitable on theoretical grounds although distinctly troublesome in practice for funds other than Type A.

An alternative method of viewing funds of Type B/C is mentioned briefly in the appendix whereby each block of business, according to date of entry, is regarded as having its respective taxed period fixed at the outset. This approach might be adopted for valuation but, apart from practical difficulties, the method does not automatically adjust the original estimate of n for each block. Each 'n' would have been based on the anticipated experience for a closed fund of business and the experience may have altered in some respects, for example as a result of withdrawals.

In the valuation of assets and liabilities no credit can be taken for a balance in the notional loss fund. The balance can only be used to save tax on a future taxable profit when the fund becomes Type A or B. To take any credit is therefore tantamount to valuing future profits and this is hardly the way to anticipate them. If future profits are expected and it is desired to anticipate them the proper course is to weaken the valuation basis suitably to 'take up some of the slack'. The surplus discharged in the process will certainly reduce the current balance in the notional loss fund—legitimately—but credit cannot be taken for any part of the remainder. In short, I think it is a helpful picture to regard any balance in the notional loss fund as corresponding directly to those regions of profit 'outside' the valuation bases.

The practical effect of dismissing any credit now is to withhold contingent surplus from the participators and to reward them at a later date by way of tax relief on profits otherwise chargeable.

DEATHS AND WITHDRAWALS

It is unfortunate for the actuary that there should be such strong pressure for guaranteed surrender values of deferred annuities with return on death.

The valuation reserve for a particular policy is an uncertain quantity if the method of valuation follows the first course I have suggested. It depends upon events taking place after the policy has been effected, because the technical basis of the method is continually altering to take account of new circumstances.

The alternative method of valuation suggests a more equitable basis for surrender values because it seeks to deal correctly with individual policies. Thus the tax position would be fixed for any given policy at the outset and the other factors in calculation would follow established surrender value practice. The method has the merit of shedding light upon surrender terms that should, if necessary, be guaranteed at the outset. Evidently it would not be unreasonable to allow a return of the premiums paid less expenses and accumulated with

interest: expenses and interest would be allowed for either both net or both gross in a manner consistent with the tax position from time to time as estimated at the date of the contract.

For reasons considered later* it is doubtful whether this solution would be followed in practice.

The use of two different rates of interest in calculating surrender values creates some obvious practical difficulties.

INVESTMENT POLICY

In practice the choice to be made is whether one investment is more profitable than another and for this purpose it is usual to calculate the expected yield for each as an obvious test.

It has been seen that, where a fund is and will remain Type A, it is immaterial whether the investment produces interest income at the expense of capital profit or *vice versa*, apart from the minor reservation mentioned on p. 240. It is therefore clearest to base calculations on the gross redemption yield.

A practical consideration is that, other things being equal, stocks standing at a premium usually show a higher gross redemption yield than stocks at a discount—but it has been seen that there is a limit to the amount of premium investments that should be taken up.

Again it seems logical, so far as the tax position is concerned, to have regard to a fund closed as from the date of the new investment.

For a fund of Type C, interest income must be taken net during the taxed period of n years. If the security is redeemable within n years the net interest must be valued but the profit or loss on redemption will not be taxable; it will only modify the balance in the notional loss fund. Thus the net yield to redemption should be computed on the basis of the net interest and the gross capital profit or loss. If the security is redeemable after n years all interest must be taken net for n years and gross for the remainder of the term.

The actual redemption yield corresponding to the market price, valuing gross or net interest, whichever it happens to be, can be readily calculated. But it is little help in comparing investments with different redemption dates because the respective yields brought out would simply reflect the incidence of tax. An alternative solution amounts to a valuation of the security in two steps (a) on a gross basis after the initial n years and (b) on a net basis during the n years. Thus we have to solve for i, where

$$\text{Purchase price} = (1-t)\,Cga^{\prime(2)}_{\overline{n}|} + v'^{n}\,[Cga^{(2)}_{\overline{r-n}|} + Cv^{r-n}],$$

where accented symbols are calculated at rate i' and

C = redemption value,

Cg = annual interest or dividend payable half-yearly,

r = redemption term,

$i' = (1-t)\,i$.

i might reasonably be described as the 'equivalent gross redemption yield'.

It is possible to extend these theoretical considerations to the more complicated cases.

* The subject is further developed in a written contribution from the author on p. 251.

CONCLUSION

The major issue raised in this paper is whether 'blocks' of business should be fully self-supporting and non-subsidizing.

Are deferred annuity withdrawals morally entitled to the return of premiums (less a suitable allowance for expenses) with compound interest at more than a net rate? A withdrawal does not result in any annuity payable out of the fund and to that extent deprives the 'eligibility' of the fund to gross earnings on its investments. The arguments in this paper *theoretically* support something more than a net rate of interest in most cases. It is, I think, contrary to public policy to allow anything savouring of untaxed interest on premium accumulations withdrawn in cash.

Are immediate annuities purchased from the extreme examples of a Type C fund entitled to take full credit for the tax on annuity payments retained by the fund? The arguments in this paper *theoretically* support the title but they depend upon the assumption that the existing business will stand the existing tax position. It also seems contrary to public policy to charge only the cost of net annuity payments valued at a net rate of interest. Nevertheless it seems to be the logical outcome of taxing the capital content of annuity payments.

Whatever views may be held on these and other debatable questions, I submit that the actuary must first fix a standard which accommodates the facts. How far he departs from this standard, in the terms offered for new business and so on, is another question and for which there is no mathematical solution, but the departure should be measured quantitatively.

From the point of view of the general public it is reasonable that the terms offered by different offices should reflect such niceties as a distinctive investment policy or a low expense ratio. It does not seem reasonable that they should reflect so severely the incidence of taxation upon the different types of fund. It is hoped that the public will draw the right conclusions.

It is beyond the scope of this paper to consider alternative and more equitable systems of taxation. I have attempted merely to analyse the present situation and to point to a method of solution of the complex problems it provokes.

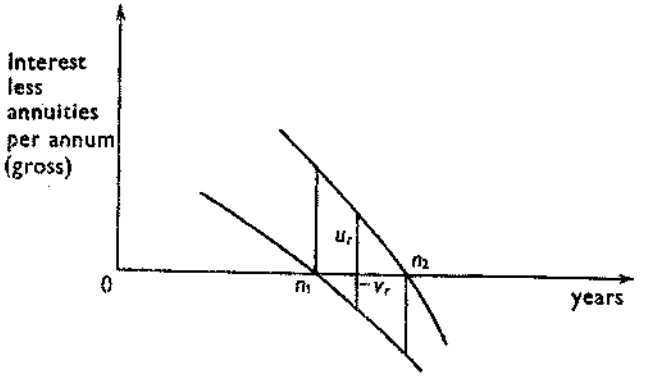
In conclusion I should like to thank M. E. Ogborn for his encouragement and valuable criticism during the preparation of this paper. His experience has been indispensable whilst exploring the general problem.

APPENDIX

THE TECHNICAL ADJUSTMENT CAUSED BY NEW BUSINESS

In what follows, all quantities of income and outgo at a given moment are dealt with in terms of amounts per annum.

A closed fund of Type C is assumed to have emerging interest income (gross) I_r, and annuity payments (gross) A_r, at a point of time r years hence. After a taxed period of n_1 years the fund is assumed to change to Type A permanently, so that, when $r = n_1$, $I_r = A_r$ for the first time, and thereafter $I_r < A_r$. The lower curve in the figure below is a graph of $(I_r - A_r)$ against r.



The upper curve shows the effect of admitting a block of mixed deferred and immediate annuity new business at time 0, after which the fund is again assumed to be closed. The new business produces increases in interest income (gross) i_r, and annuities (gross) a_r, at time r. As a result, the total interest income is equal to the total annuities at some point of time, say n_2 years hence, which will differ in general from n_1. In the diagram n_2 is shown greater than n_1.

From the diagram, when $n_1 < r < n_2$,

$$u_r = (I_r + i_r) - (A_r + a_r),$$

$$-v_r = I_r - A_r \text{ which is negative at this point, and}$$

$$u_r + v_r = i_r - a_r.$$

Consider the actual net income (or outgo) arising from interest and annuities alone with allowance for income tax. As a closed fund, before the introduction of new business, the aggregate net outgo is

$$v_r, \tag{1}$$

since all items can be dealt with on a gross basis after n_1 years.

Considering the new business alone, the aggregate additional net income is the gross additional difference $(i_r - a_r)$ less tax on u_r, i.e.

$$(u_r + v_r) - tu_r. \tag{2}$$

This expression may be put into the form

$$(u_r + v_r) - t'(u_r + v_r) = (1 - t')(i_r - a_r),$$

where
$$t' = t\frac{u_r}{u_r + v_r}$$
$$= t \quad \text{when } r = n_1$$
and
$$= 0 \quad \text{when } r = n_2.$$

No strain will be inflicted on the fund provided the new business enters on terms which are self-supporting. This means that the terms must allow for the fact that, so far as new business is concerned, interest and annuities are to be taken as

(i) net for n_1 years, allowing for tax at rate t,

(ii) net during the following $(n_2 - n_1)$ years, allowing for tax at the varying rate t',

(iii) gross thereafter.

Thus, strictly, for the intervening period (ii), the two items cannot be brought into account on a net basis at the constant rate of tax t or on a gross basis. There must however be some point of time between n_1 and n_2 at which an assumed change from 'net' to 'gross' is equivalent to the actual situation. Evidently a reasonable approximation would be to take the point where I = A if only half of the supposed new business is written.

Expressions (1) and (2) may be combined to give the aggregate net income for the combined fund at time r, namely,

$$(u_r - tu_r) = (1 - t)u_r$$
$$= (1 - t)[(\mathrm{I}_r + i_r) - (\mathrm{A}_r + a_r)].$$

This result shows that for the fund as a whole interest and annuities may now be brought into account on a net basis for the slightly longer period of n_2 years. Moreover the whole of the strain upon the existing business due to the alteration of n_1 to n_2 has been borne by the new business.

This analysis of the position suggests alternative ways of regarding a fund supplemented by a block of new business.

(i) The tax position of the fund can be reviewed in the light of the changed circumstances and a fresh value of n_2 may be computed.

(ii) The fund can be split into two parts consisting of the existing and the new business. The former may be regarded as not having sustained any change to its tax position (n_1 unaltered) and the latter can be allotted a unique value of n' (between n_1 and n_2 years). Other things being equal the next block of new business can be allotted a taxed period between n_2 and n_3 years and so on.

In the aggregate both methods theoretically produce the same result. For valuation purposes (ii) seems cumbersome in view of the multiple subdivision which is entailed.

REFERENCES

(1) Ogborn, M. E. (1947). The taxation of annuities. *J.I.A.* Vol. LXXIV, p. 31.

(2) Rowland, S. J. and Wales, F. H. (1937). The taxation of the annuity fund and some practical points arising therefrom. *J.I.A.* Vol. LXVIII, p. 447.

(3) Shrewsbury, A. H. (1943). Income tax as affecting life assurance offices. *J.I.A.* Vol. LXXII, p. 35.

The author has sent the following remarks in amplification of his paper for publication in the *Journal*:

After further consideration I feel that the section entitled 'Deaths and Withdrawals' was inadequate and that a number of difficulties in that section were over-simplified.

The problem of establishing suitable surrender terms for deferred annuities is a complex one—the more so since they are almost invariably guaranteed at the outset. The principles applied in the appendix to new business terms, however, should also be applied to surrender values. A withdrawal from the fund affects the liabilities for all other contracts owing to the consequential adjustment to the taxed period. Theoretically therefore the terms of surrender should allow for the alteration to the total liability of the remainder of the business.

To simplify the problem, assume that the premium and valuation bases are identical and accord with actual experience; assume also that new entrants and withdrawals take place on terms which satisfy the conditions put forward in the paper. Accordingly the sum available on withdrawal is the amount of the valuation liability which is released: this is the valuation reserve calculated on the basis of the taxed period that would obtain at the date of withdrawal if half the contract only were surrendered. If surrender values were not guaranteed this liability would be a theoretical starting point in fixing terms for surrender.

A widespread practice exists of guaranteeing a surrender value on death or withdrawal amounting to the premiums paid, possibly less expenses, accumulated at a net rate of interest. It is interesting to compare, in the ideal conditions just discussed, the two quantities:

(i) the surrender value equal to a return of the premiums paid accumulated at a net rate of interest, and
(ii) the valuation reserve.

For this purpose I have prepared the following table using the basis put forward in the paper for calculating the annual premium for a deferred annuity at age 60 and have assumed that the fund was Type C ($n=10$) when the contract was effected. For simplicity, expenses have been disregarded altogether in the calculation of the annual premium, the reserve and the surrender value, as they are largely irrelevant to the matter under consideration.

Surrender value and percentage it bears to valuation reserve for a deferred annuity of £10 at age 60 effected when the taxed period of the fund was 10 years.

Years in force at surrender	Surrender value	Percentage of surrender value to valuation reserve		
		Taxed period of fund (in years at surrender)		
		0	10	20
		Female life aged 30 at entry		
		%	%	%
10	34·7	100	87	72
20	76·9	92	82	97
30	128·4	86	98	107
		Female life aged 50 at entry		
		%	%	%
5	70·6	108	113	146
10	148·5	100	113	124

There are two important conclusions. First, for a contract of short duration the conventional guaranteed surrender value and cash option may be larger than can be afforded unless special provision for withdrawals is made in the premium basis. Secondly, the amount of the surrender value that should be offered depends upon the taxed period at the date of surrender which itself depends upon the new business and withdrawals that have taken place since the contract was effected.

The valuation procedure requires special consideration. The guaranteed surrender values and cash options under existing contracts may be at a level low enough to make any necessary contribution to the fund when withdrawal takes place to allow for the consequential technical adjustments to the remainder of the business. If so, no special treatment is necessary at valuation and withdrawals would not have to be taken into account in estimating the taxed period. It is more likely that some of the guaranteed surrender values and cash options may be too high and some too low; it might be possible to set the one against the other, or rates of withdrawal may have to be associated with surrender values and cash options.

It is ironic that public pressure for guaranteed surrender values should be greatest where conditions are so manifestly unsuited to them.