Report of Working Party on Fluctuation Reserves

There are two separate papers by Mike Oakes and Henry Karsten which should be read in conjunction with this report.

Owing to the lack of time most members of the working party have not seen the final report. No doubt those members who disagree with any of the views put forward will make it known at the Cambridge Seminar.

M Trayhorn
26.8.80

GENERAL INSURANCE STUDIES GROUP

Working Party on Fluctuation Reserves

1. Members of Working Party

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    P Cooper and M Moliver also took part in the early discussions but were unable to continue owing to other commitments.

2. Subject Matter

    The incidence and size of claims varies over time and part of this variation will be of the form of statistical fluctuations which do not affect the long-term outcome of the business but do have significant short-term effects on the claims experience. A fluctuation reserve is a reserve set up in the profitable years to meet the excess claims in the worse years. Obviously, one must also consider the question of solvency because claim fluctuations are part of the variation supposed to be covered by the solvency margin. The general approach used in this report is

    (a) Consideration of risk theory and its application in practice.

    (b) Reference to studies preceding the setting up of the EEC solvency margin to determine the allowance made for claim fluctuations. A number of European papers updating and suggesting alternative methods of calculating the solvency margin have been considered.

    (c) A detailed examination of the systems in those countries, principally Germany and Finland, which already have legislation regarding fluctuation reserves.

    (d) Mention of some of the practical aspects such as taxation and reinsurance and consideration of further work required on this subject.

3. We will start by considering how far simple risk theory will take us.

3.1 Definition of solvency

    The usual concept of a solvency margin is an amount sufficient to meet the deficit (ie claim amount less premiums) in one year, with a given probability. Although, for supervisory purposes, this ties

in well with the annual reporting of results and the fact that the majority of non-life contracts are for one year, the company will consider itself as a longer term venture rather than just existing from year to year. It is theoretically possible to calculate the reserve required to remain solvent for any number of years, k(say), or even for infinite time. The calculation of such a reserve is difficult involving convolutions etc.but bearing in mind that the data will only be very approximate anyway we can gauge the effect of this by examining the total claim amount over k years. If the variance of the claim amount in one year is σ^2 then the variance of the total claim amount over k years is $k\sigma^2$. The formulae for the required reserves are (using the normal approximation) of the form:

$$U_1 = Y\sigma - \lambda P$$

$$U_k = \sigma Y\sqrt{k} - k\lambda P$$

Where λ is the safety loading and y an appropriate value for the level of security chosen. If λ is around zero $U_k \doteq \sqrt{k}U$ and the reserve required is going to be 3-4 times as large as the one year reserve if some reasonable period of 10-20 years is chosen. A positive safety loading will reduce the rate of increase in Uk,the extent depending on the relationship σ/p.

In the two countries, Germany and Finland whose fluctuation reserves are already established practice this idea of longer term solvency forms the basis of the calculation of the maximum amount of such reserves. It is difficult to see why any reserve should have a maximum limit as they are affording a higher level of protection,but it is imposed because of the tax concessions granted to these reserves in the two countries concerned.

3.2 Probability of Ruin.

One recurring problem associated with the application of risk theory is the need to decide upon,and state,the probability of ruin. Whilst it is understood in the insurance industry that a company could be adversely affected by a rare combination of events and become insolvent it is difficult to imagine a Government admitting to any stated probability of ruin in legislation. Even if such a probability were stated the danger is that for political reasons this probability would be ludicrously low. When the current EEC solvency margin was being considered the probability of ruin used was 3 in 10,000. Given the standard of data available and the approximations necessary this degree of accuracy seems dubious.

3.3 Sources of Variation

The variation in the total claim amount can be attributed to three sources. Firstly, there are purely random fluctuations around the expected number of claims and in the amount of the individual claims in any one year. It is these fluctuations that simple risk theory deals with,but in practice these are probably the least important because of the size of most portfolios and the limiting of individual risks by reinsurance. A recent paper produced by actuaries in Belgium calculated the movements of the distribution of the individual claim amount and used the Normal Power (NP) approximation to derive the reserves necessary to maintain solvency . The resulting reserves would be considered very low giving figures of the order of 13% of risk premiums or 8% of total premiums.

Rough calculations carried out on the portfolio of one UK company

indicated even lower margins ranging from 3 per cent for household to around 7 per cent for motor as a proportion of risk premiums.
However, it is clear that these random elements are not the main source of variation in practice. Introducing even a fairly small amount of variation into the expected number of claims can have a considerable influence on the size of reserves required. For example, the motor figure mentioned above increases to 25% if one allows the expected claim frequency to fluctuate slightly (up to ± 10%). Even more startling increases may be obtained by allowing the possibility of a catastrophe type accumulation of claims.

The third type of fluctuation is a trend in the number, average amount or shape of the distribution. These may be linear trends due to economic changes, sudden changes due perhaps to legislative changes or cycles in economic conditions. Whatever the cause unless the next value in the sequence can be predicted trends of any kind can only increase the variation.

3.4 Expected amount of claims.

Although risk theory refers a great deal to 'premiums' the whole basis of risk theory is the expected total amount of claims and the first few moments of the total amount. When considering solvency or fluctuation reserves the claims of interest are those occuring in the next year (or years). In practice, of course, this expected amount of claims is unknown and we need some other measure to judge whether a particular result is above or below the expected amount.

The first variable will be the amount of business written and so this suggests that the measure should be relative to the earned premiums. The earned premiums will reflect changes in the expected amount of claims if:-

(a) Expense and profit loadings remain the same from year to year.

(b) Relative underwriting rates are correct ie The earned premiums will change in line with the expected amount of claims when there are changes in the portfolio mixture.

(c) The premium rates have correctly anticipated future trends and in particular the rate of claims escalation.

The loss ratio (incurred claims/earned premiums) is then a measure of actual claims to expected claims for the past year and, therefore, an expected loss ratio together with projected earned premiums could be used to estimate the future expected claims. The variance of the loss ratio will indicate the variance of the estimate.

An alternative is to analyse the distribution of individual claims for each class and then use convolutions of these to obtain a distribution of the total claim amount. Apart from being complicated this still involves a number of problems. Account has to be taken of large claims which could occur, but did not during the period of data collection. Inflation and future trends still need to be estimated as does the possible variation in the claim frequencies. Changes in portfolio mixture will also need to be anticipated. Given the likely quality of these estimates one wonders whether the extra complexity is worthwhile (See also Sec. 5). The one advantage that this method would have is that explicit account could be taken of the company's present reinsurance arrangements whereas using the loss ratios one must assume that these arrangements have been reflected in the variance of past loss ratios.

4. The basic idea underlying fluctuation reserves is very simple in that we wish to smooth a company's results.

4.1 The total claim amount in a given year will fluctuate and so in 'good' (ie low claims) years the company should set aside reserves to make up the shortfall in 'bad' years. However, this immediately gives rise to problems such as the definition of 'good' and 'bad'. Ideally we would use the expected claim amount as the rod against which to measure the particular year but, as we have seen above, this is not practical. Instead we shall have to use an average ratio of some kind. Although claims provide the major source of variation, does it necessarily follow that the loss ratio is the measurement that needs to be smoothed?

4.2 Expenses do not fluctuate in quite the same way as claims but the expense ratio can still vary over time as the portfolio mixture changes or the rates of claims and expense escalation differ. Therefore a loss ratio that is profitable at one point of time might not be at another. A recent Italian paper by Buoro, Pavesi and Zuchiatti, described in part 3 of appendix A , has calculated solvency margins based on operating ratios. The implication is that the standard deviation is lower because there tends to be an inverse relationship between the loss ratio and the expense ratio.

4.3 In times of high inflation interest rates,and hence a company's investment earnings,are high. In these circumstances one would expect the loss ratios (and operating ratios) to be higher than in times of low inflation,although the extent will depend upon the class of business and the length of its tail.

4.4 It would seem more logical for a company to try to smooth out fluctuations in its final result (ie its trading profit) than some constituent part of it because it is this published figure out of which it has to pay dividends. Even then it is not easy to decide what constitutes 'smooth' results because a company's view of its trading result or return on capital will depend upon the rate of inflation and the rates of return available elsewhere.

4.5 Whatever measure is chosen for the smoothing process some care is needed in the definition. For example, the published loss ratio will include profit/loss on run-off of prior year claims and using published UPR figures may lead to a building up of equity in these reserves. A year that might initially appear profitable may, later turn out worse if the outstanding claims are underestimated.

5. Deriving a distribution from past data.

5.1 It is going to be necessary to derive at least the mean and standard deviation of the chosen measurement from past data, in order to be able to decide whether any given year is 'good' or 'bad'. Obviously using results over a long period of time is likely to give better statistical estimates of these parameters,but the longer the observation period used the more economic conditions etc are going to change. As mentioned above a loss ratio (say) which appears 'good' at one time may not be so if achieved in a different economic climate five years later. There is obviously a need to introduce some kind of relative measure against which to judge any ratio.

5.2 Special consideration needs to be given to companies which are expanding rapidly or changing the mix of business. The variance of any measure will be influenced by the size of the company and the type of business written,so over too long an observation period one will not really be looking at the same company. Problems will also arise with new companies and finding some method of applying market data, perhaps,to an individual company.

5.3 Provided a reasonably long observation period can be used the mean of the chosen measure can be found with some degree of accuracy but finding an estimate of the standard deviation is more difficult. This is also quite crucial because the required solvency/fluctuation reserve is going to be directly proportional to the standard deviation so any error in its estimation will be multipled up into the reserve. Appendix B show the results of some simulations designed to measure the variance of the estimate of the standard deviation of a loss ratio over different observation periods. The results show that even for fairly long observation periods the estimate of the standard deviation is still subject to significant errors of estimation.

6.1 One of the main practical complications in examining the size of fluctuation reserves required is the existing EEC solvency margin requirements. This solvency margin is generally accepted to cover all kinds of risks including investment risk, bad management and claim fluctuation although the proportion notionally allocated to claims seems to vary from Belgium where this is considered to be the major factor to Germany where the solvency margin is seen to be entirely for risks other than claim fluctuation.

6.2 It is interesting to return to the origins of the current EEC solvency margin to discover what factors were considered and look at the statistical methods and data used. The earliest work was produced by Prof. Campagne for the OECD in which he collected data on ten companies in each of seven European countries for the period 1952-57. The distribution of loss ratios for each country was fitted by a beta distribution and then combined with the average expense ratio for the country concerned to calculate the margin necessary to provide the required level of security.

Details of this calculation for the Netherlands is shown in appendix A

It should be noted that this study only considered fluctuations in loss ratios, no other factors, and used a ruin probability of 3 in 10,000. The resulting solvency margins considered necessary varied from 3% of premiums in Germany to 35% in France with most of the others around 25%.

6.3 At about the same time (1961) a committee of actuaries under de Mori collected data over a longer period (1951-60) for just four countries. The data was fitted by a normal distribution and the safety margin was taken as three times the standard deviation with the intention of allowing slightly for a non-normal distribution. The results were similar to the Campagne research with the margin for all non-life business varying from about 3% (Germany) to 35% (Belgium). They then took a weighted average of the four countries to arrive at a 'European' figure of 24% and, afterwards,'taking into account the true position of insurance companies in the six (EEC) countries' reduced this to 18%, 12% and 8% on three bands of premium income.

6.4 The data used in both the above studies is now fairly ancient and appendix A shows updated figures. A recent Dutch paper has recalculated the margin required in the Netherlands on the same method used by Campagne using data for the years 1976-78 from a much larger number of companies. Whereas the original research gave a margin of 31% for Holland this update arrives at 61% using the ruin probability of 3 in 10,000. The following changes between the two periods are notable:-

(a) The average expense ratio has dropped from 53% to 30% of premiums so that the claims (which is the only part allowed to fluctuate)

form a far higher proportion of the total.

(b) The overall underwriting profitability has declined. The overall operating ratio has increased from about 96% to 102%.

(c) The number of companies included in the survey has been increased from 10 to 71. If the original ten tended to be the larger companies, then one would expect the standard deviation of the loss ratio to increase as the smaller undertakings are included.

6.5 The reasoning used in the above studies seems a little suspect in one area, that of the use of the average expense ratio. Especially when one considers small companies there may well be a relationship (probably inverse) between the claims ratio and the expense ratio. As no account is taken of size of venture or type of business written it is not possible to guage the contribution of these factors to the overall variance. Looking at the Dutch study of recent years one must conclude that a large proportion of these companies must have had operating ratios exceeding 110% and yet, presumably, the vast majority of them survived. Between the dates of the two Dutch calculations the whole economic climate, in particular the importance of investment income, has changed completely and one wonders whether the above methods of analysis are suitable in today's conditions.

6.6 A similar calculation has been carried out using data from the DoT returns of 10 large UK companies resulting in a margin of 19% of premiums compared with the 61% produced above for the Dutch companies. It can be seen that the standard deviation of the expense ratio is almost as large as that of the claims, and similar remarks to above apply.

7.1　The existing ECC solvency margin requirements are enshrined in the 1973 non-life establishment directive. The operation of those requirements are currently being reviewed by the supervisors and separately by industry organisations. These reviews are complicated by the differing views on objectives of the margin as described in 6.3; the wording of the English version of the directive in fact refers to covering 'business fluctuations' but this is not too helpful. The directive requires the solvency margin to be covered by an excess of assets over liabilities but makes no rulings on how either assets or liabilities are to be valued, neither does it say what should be included as a liability. A non-life services directive is currently being drafted and will cover, to a limited extent, the harmonisation of technical reserves. As a precursor to this draft directive, the supervisors of the member states formed a working group headed by the German supervisor, Dr Angerer, to report on current practices and to make proposals on harmonisation.

7.2　The report eventually produced by Angerer contained a large number of reservations or minority views on different aspects. The proposals contained in the report were that the technical reserves were to consist of unearned premiums, unexpired risks, outstanding claims, claim fluctuations and atomic risks. The proposals left some room for differing practices on unexpired risks and outstanding claims. For claim fluctuations, it left devising an acceptable formula up to the supervisors of each state and did not suggest an EEC - wide standard formula. However the danger would exist of some future harmonisation movement trying to introduce a standard rule.

7.3　The proposal for a non-life Services Directive is published in the official journal of the European Communities Number C32 of 12 Feb. 1976. Article 3 covers the principles to be observed in calculating technical reserves and the important provision that such reserves should be set up under suspension of tax. The final draft of this Directive will take into account the views of the Angerer working party and possibly the proposals which have been made separately in respect of insurance company annual accounts.

7.4　A separate submission has been made to the European Communities in the review of the non-life solvency margin seeking tax-relief on the increase in that margin. This, and the tax point in 7.3 above, should be borne in mind when the subject of taxation is discussed in Sec. 13 of this paper.

7.5　The British Insurance Association has been monitoring these developments and, where possible, influencing them. Recognising that some type of claim fluctuation reserve may be forced onto the UK industry, it has set up its own working group to research current practices and company attitudes in the remainder of Europe on fluctuation reserves and to produce recommendations for an industry view. At the time of writing this working group has not produced its report, although this may well be available by the time of the GIRO seminar in Cambridge.

8. A number of European countries already have provision for some form of fluctuation reserves and this section sets out the main characteristics of these reserves for each country, excluding Germany and Finland which will be dealt with later.

8.1 <u>France</u>

Legislation:- Setting up of reserve is not compulsory but is 'expected' for certain classes of business (hail, storm, flood) subject to extreme variation. Tax relief is given on the transfer to the reserve although there is an upper limit on the reserve for tax purposes.
Amount:- Each year up to 75% of the underwriting profit may be transferred to the reserve but all losses must be met from it. The reserve may accumulate for ten years after which unused amounts must be transferred back to profits in turn.

Full details are shown in appendix C

8.2 <u>Holland</u>

Legislation:- Reserves are not compulsory, but may be set for the main classes of business. Tax relief is given but the funds are shown as part of the shareholders fund, not technical reserves.
Amount:- The total amount of the reserve is limited to 50% of the earned premiums in the latest year. The transfer each year cannot be more than 6% of the limit above but it is also restricted by various rules related to the year's profit. The position is slightly complicated by the taking account of investment losses in the calculation of the profit.

Full details are shown in appendix C

8.3 <u>Denmark</u>

Legislation:- Companies specialising in storm and hail classes are expected to set up equalisation reserves such that 'the additional premiums required are not out of proportion with the expectations of policyholders'. (The companies affected are small mutuals). Tax relief is given.
Amount:- Not specified - method of calculation is determined by company subject to approval by supervisory authority. The overall practical effect is insignificant as such companies are quite small.

8.4 <u>Italy</u>

No statutory provision except for hail insurance which is dealt with on a pooled basis. Some companies do set up fluctuation reserves but there is no tax relief.

9. Germany has legislation for fluctuation reserves covering all classes of business which has been in force for a number of years. The rules of the system are laid down in the legislation and there is very little flexibility for an individual company. For this reason, and also because much of the pressure for statutory fluctuation reserves within the EEC originate in Germany, we have examined the German system in some depth. Attached note. * gives a full description of the system and so we will limit our consideration here to commenting on the main points in the light of the earlier

* Paper 1 by Mike Oakes

theoretical discussion.

9.1 The German system aims in theory to calculate a reserve such
that the probability of 'ruin' (ie the reserve running out) is
less than 5% over a number of years known as the equalisation
period. This equalisation period is chosen such that it is
unlikely (prob. 0 .05) that one years excess loss will exceed
5% of the total risk premiums in the period. The length of
this equalisation period will obviously vary depending on the
variability of the class of business,but it is maintained that
when the distributions are examined,and the reserve discounted that
a constant figure of 4.5 standard deviations emerges for the reserve
regardless of class of business. The probability of 'ruin' of 5%
would seem high at first sight but a separate reserve is being
calculated for each class of business so the ruin probability of the
Company as a whole will depend on the number of classes written and
the mixture.

9.2 An adjustment is made to the above amount to allow for the average
profitability of the business. A 'border line' loss ratio is
calculated based on the average expense ratio over the past three
years, such that an operating ratio of 100% is achieved. The
difference between the borderline loss ratio and the average loss
ratio gives the average profitability. The multiplier of 3 used in
this adjustment is difficult to justify as one would have expected
this to be the total profit over the equalisation period suitably
discounted,but this does not seem to fit in with the figures quoted
in the literature. Perhaps this figure also 'emerged' from the tests
conducted on various classes!

The calculation of the profitability does not take account of the
fact that some of the loss ratios included in the average loss
ratio may have occurred when the expense ratio was very different,
ie A low loss ratio of ten years ago may look good when considered
with todays expense ratio but may have produced a loss combined
with the expense ratio of ten years ago.

9.3 The average and standard deviation of the loss ratio are derived
from the loss ratios of the past fifteen years. Mention has
already been made of possible errors in estimation of the standard
deviation and this could be significant when combined with the
rule that no reserve is necessary if the standard deviation is less
than 5%. For a true standard deviation near 5% the measured standard
deviation could jump around from above to below and vice versa every
few years with the reserve jumping from 0 to 22.5% of premiums (less
profit adjustment). It seems rather unnecessary to introduce a
discontinuity. If a reserve is required at 5.1% why not for 4.9%?

9.4 The derivation of the theoretical amount of the reserve includes
the discounting of this amount for interest. Therefore, when the
transfer to the reserve each year is being calculated one ever
present element (regardless of loss or profit) is the interest
on the theoretical amount. This amount goes to build up the
reserves and means that the profit after transfer is less than the
profit before transfer until the actual loss ratio for the year rises
several points above the average loss ratio (see section 5 of Paper 1).

9.5 Appendix E shows the results of some simulations of the German System
on a company whose loss ratios are assumed to be independant from
year and log normally distributed (some results are also shown on
the basis of a normal distribution). A measure of the 'success'

of the smoothing process is required and the obvious comparison to make is that of the standard deviation of the loss ratios after the transfer against the original standard deviation. The smoothing process is most effective when the average loss ratio is high which is what one would expect,because when there is no safety margin in the premiums all the profits and losses are transferred directly to or from the reserve. When the average loss ratio is low the profitability deduction comes into effect and it can be seen that for an average loss ratio of 50% and standard deviation below 10% no reserve at all is required throughout the 100 year period. The attitude seems to be that it is quite all right for results to fluctuate provided they are profitable results suggesting the main concern is solvency rather than smoothing. However, an interesting position arises if one considers a company writing two classes of business one of which is profitable and the other not so. The high loss ratio would tend to be smoothed to the average whilst the low one would fluctuate unaltered. If there were any negative correlation between the two classes the overall 'smoothed' result would be more variable than the original.

9.6 The only justification mentioned anywhere in the German literature for calculating reserves for each class of business separately is that they wished to avoid cross-subsidies between classes. However it can be seen that this system does not prevent this happening in any way. This subject will be returned to later.

9.7 Appendix E also show figures for the situation in which premiums are increasing at 10% p.a. It can be seen that the averages of the smoothed loss ratios are higher than the averages of the original loss ratios reflecting the cost of building up the fluctuation reserve as the premiums increase. Also,the number of years in which the reserve is entirely wiped out increases significantly as a current loss is larger,in monetary terms, that the profits stored up from previous years.

10. We now turn our attention to Finland where fluctuation reserves have been included in legislation since 1953. The reserve is held to cover all fluctuations in claims and there is no additional solvency margin requirement although there are other regulations aimed at other causes of insolvency (presumably, on assets etc). As there is no additional solvency margin the reserve has a minimum value greater than zero required before business can be written. Details of the Finnish system are shown in Paper 2* and here we will just mention the main points and compare and contrast these with the German system.
*by Henry Karsten

10.1 The formulae used in the calculations are based on risk theory. A company uses its own data to compute the distribution function of each class of business and then combines these using the expected number of claims in each class to produce the distribution function of the total claim amount. The supervisory authority publishes market data for each class,especially for the tail of the distributions,and gives factors for the effect of different levels of net retention. The supervisory authority also lays down certain adjustments to be made to allow for fluctuations in the basic claim probabilities.

In practice there are various approximations which may be used and the full calculations are carried out only if a company is near one of the limits.

10.2 The lower limit is set such that the total reserves of the company (incl. shareholders capital) are sufficient to ensure solvency over

one year with a probability of 0.99. The extra reserve (in addition to shareholders funds) cannot be negative and there is the further restriction that the total reserves must be larger than the greatest realistically possible size of a single claim (net of reinsurance).

The upper limit is such that the reserve alone (without taking account of free assets) is sufficient to ensure solvency for five-years with a probability of 0.99. The upper limit is at least twice the largest possible single claim.

Although the Finnish system takes account of the mix of business by class, the reserve is clearly calculated on the basis of the company's total business in contrast to the individual class reserves held in Germany. The system is also clearly based on the concept of solvency for a number of years rather than 'equalising' claims over some artificially designated period.

10.3 The calculation of the reserve includes discounting for interest and the transfer to the reserve each year includes interest on the initial reserve at a specified rate (5%).

10.4 As in Germany the reserves are considered part of the technical reserves and qualify for tax relief. The Finnish reserve is included in the outstanding claim reserve.

10.5 The transfer to, or from, the reserve is computed by comparing the current loss ratio with the average for the past five years and transferring the difference. This is done individually for each class of business, although there is only one reserve subject to the overall limits above. In addition the company, may agree with the supervisor to transfer a fixed percentage of premiums (between 0 and 15%) each year presumably to build up the reserve. There is no profitability adjustment.

10.6 Quite recently a special research group, headed by Pentikainen, has been set up in Finland to review the whole system. We wrote to Pentikainen, who sent us a most useful letter and the preliminary report of the above group. Most interesting, at this juncture, are his replies to our questions regarding their experience of the system in practice and why a review is necessary.

(a) The data available in 1953 was inadequate and the review will include more extensive collection of recent data.

(b) The original method was conceived in the pre-inflationary era. and may not be flexible enough in current conditions.

(c) The total fluctuation reserves held by Finnish companies have risen from 30% of premiums in 1970 to over 90% in 1979. This is upsetting both the fiscal authorities and the newly formed consumer groups.

(d) At the same time free assets now only amount to 15% of premiums.

(e) Although no insurance company has actually become insolvent nearly half the companies operating 30 years ago have disappeared through mergers, many enforced.

As the free assets of the companies have fallen so low Finnish insurers have had to give details of fluctuation reserves to foreign supervisors to show solvency, and the increasing disclosure combined with increasing size is bringing pressure for change and hence the hasty formation of the research group.

11   A basic assumption of risk theory which is built into both the
      German and Finnish systems is the assumption that the results of
      one year are totally independent of any other year's results.

11.1 There is a great deal of economic literature devoted to the subject
      of business cycles and if a company's insured are being affected
      by these cycles then they must work through to the results of the
      company itself. In addition insurance business creates its own
      cycles because of the delays involved in,firstly,assessing that
      premium rates are inadequate and then taking corrective action.
      Even if premium rates are increased immediately following a year's
      results (assuming underestimation of claims etc. is not hiding the
      facts) it will be a full year before these are fully reflected in
      the earned premiums. On top of this there is a tendency for more
      companies to enter the market when profits are high forcing rates
      down and then pulling out when profits fall.

11.2 The independence assumption means that the probability of ruin
      over the next year depends only on the level of the free assets
      at the beginning of the year and does not take account of how this
      position was reached.ie whether the reserves have been building
      up from good profits or depleted by losses to reach their current
      level. The relevant question is really whether the probability of
      a given result is the same regardless of the results of prior years.
      This is very similar to the problems involved in maturity guarantees
      on unit-linked policies where there are cycles in the equity market.

11.3 Appendix F shows the effects on the simulations of the German
      system using a four year moving average instead of the random
      values. These new results are shown in square brackets. These few
      results suggest that the German method may be vulnerable to cyclical
      results on two counts.

      1. The maximum reserve of 4.5 times the standard deviation is
          inadequate to withstand runs of high losses as one might
          expect because the independence was assumed in calculating the equal-
          isation period and standard deviation. During a prolonged period
          without   fluctuation reserve there is obviously no smoothing
          of the results.

      2. In a cyclical situation the average of the last 15 years is a
          less efficient estimator of the true underlying mean. Therefore,
          for example, where the previous 15 years includes more high
          cycles than low cycles the German method misinterprets an
          average loss as below average and makes a contribution to the
          reserve that is not justified. Comment has already been made on
          the estimation of the standard deviation and this problem is
          obviously compounded.

          It is hoped that some further results on the use of different
          cyclical models will be available at the Cambridge conference.

11.4 The research group in Finland, mentioned above, have published
      some preliminary comments on the methods they intend to employ. One
      of the suggested lines of investigation is a study of cycles,both
      those caused by exogenous economic factors and 'the market
      mechanism of the insurance industry itself'.

12.1 One of the main differences between the German and Finnish systems
      was that the German method creates a separate reserve for each class
      of business instead of the whole portfolio. In fact although the
      rules state that separate reserves must be held for a minimum number
      of classes the insurers have the option to further subdivide the
      classes. Obviously, this subdivision will usually create a greater

overall reserve. Can this practice be justified theoretically?

12.2   As we have seen the calculations for the fluctuation reserves have
as their theoretical basis a definition of solvency and surely it is
only meaningful to talk about the solvency of a company not of an
individual class of business (or some optional sub-group of a class).
To do otherwise would destroy many of the advantages of size and
diversification. The only justification put forward for this
sub-division is that each class should be self financing,but
fluctuation reserves of themselves do not ensure this because
fluctuations are merely smoothed out to the high average loss ratio.
The Finnish system adopts the opposite view in that the reserve is
calculated over the whole business and each class contributes to the
reserves according to its own individual profitability.

12.3   Whilst the objective of making each class pay for itself may be
desirable,both from the point of view of the company and the
supervisory authority,there must be easier ways of achieving it
than by accumulating unnecessary reserves. There is the suspicion
that this subdivision has more to do with the German insurers
wishing to maintain the size of the tax-free reserve than with
any other objective.

13.1   Readers will have noticed that taxation or the lack of it has been
refered to a number of times in the last few pages because in practice
it is the crucial point. At the present time increases in the EEC
solvency margin have to be financed from post-tax profits or by
raising more capital. In the UK most large companies have actual
solvency margins in excess of 40% and do not try to operate with a mar-
gin close to the statutory 16% (approx.). In effect they hold an
extra reserve to cover fluctuations, not necessarily in claims,which
could make them technically insolvent (i.e. below 16% rather than
actually bankrupt). In Germany the insurers hold large fluctuation
reserves which mean that they are able to operate on solvency
margins much nearer 16%. Obviously the tax-relief given to the
fluctuation reserves makes it far more efficient to boost the
fluctuation reserve and only increase the shareholders funds when
absolutely necessary.

13.2   Before determining whether tax-relief on either the solvency
margin or fluctuation reserve is justified one needs to decide
who these reserves are designed to benefit. The solvency margin
is quite clearly there to protect the policyholder,but the fluctuation
reserve is not so clear cut. Whilst a fluctuation reserve may smooth
results and ensure the longer term survival of the company does
the policyholder really care? If his potential liability is virtually
100% safe anyway is he concerned? The only possible advantage to
the policyholder is that companies might use the reserves to ride
out bad patches and premium rates might progress more evenly. The
management of the company itself would be happier at the extra
cushion (especially actuaries - would almost be like a life company!).
The government might be happy to see a stable industry but someone
has got to provide the money to build up these reserves and they would
not like that someone to be the policyholder (through premiums) or
themselves (through tax-relief).

13.3   If a reserve is imposed by statute to protect policyholders then
a reasonable case can be made for tax-relief and its treatment as a

technical reserve. However, as things stand at present, the increase in the solvency margin would seem to have prior claim over fluctuation reserves as presently defined.

14 Fluctuation Reserves and Reinsurance

14.1 When considering the level of fluctuation reserves which is required for a portfolio no investigation can be complete without looking at the nature and levels of reinsurance operating on that portfolio.

14.2 Initially, to optimise the requirements for reinsurance, the account should be investigated on a gross basis and the overall fluctuations within the account have to be considered in depth. The investigation has to consider all aspects of reinsurance requirements but the major consideration must be to protect the account so that if any undue fluctuation occurs on the gross account, the net account will be totally protected and solvency must be maintained.

14.3 The account first of all has to be divided into the major classes of business and the element of fluctuation must be considered within each of these classes of business, bearing in mind the expected incidence of claim and also the distribution of the size of claims expected. After detailed consideration the overall account has to be looked at to ensure that any one event will result in accumulations of liabilities is totally catered for.

14.4 The decision then has to be made on the effects of reinsurance on reducing fluctuations and then on what reserves it may be felt appropriate to hold internally for the net account to cater for situations which will not be covered by the reinsurance programme.

14.5 The level of fluctuations and the reserves required obviously varies from class of business to class of business and whether the account being looked at is mainly a direct account or a reinsurance account. Different considerations may be required for a reinsurance account where larger fluctuations and accumulations are likely to occur.

14.6 The balance between the level of internal fluctuations to be held and the balance between reinsurance required is difficult to assess at the correct level. The major criteria at the end of the day is as already stated, to protect the account for solvency purposes but also an even flow of dividend to shareholders must be considered as an equally high priority. At the end of the day the major considerations must be on the overall financial costs to the Company and although much detailed technical work can be carried out on the account overall financial considerations will determine the final policy to be followed.

15 Where Next?

Owing to the time available and our limited knowledge of fluctuation reserves at the outset this has, of necessity, been a report on what is already happening elsewhere rather than a research into the need for fluctuation reserves in the UK. The problem can really be considered in two ways, one theoretical, the other practical.

15.1 Let us first forget all existing legislation etc. and concentrate on the theorectical problem of designing a system which will fulfil two objectives.

(a) To maintain solvency with a given probability, and

(b) To smooth a company's results over a long period of time.

The relationship between the two objectives will depend on whether one accepts the basic risk theory assumption of independence between years. If independence is assumed then the amount required to maintain solvency remains the same irrespective of the earlier results unless those results alter one's opinion of the underlying model. In this case the flutuation reserve will be separate and will be designed to reach zero only occassionally at which point the excess loss will have to be met from outside sources (i.e. free assets other than the solvency margin). On the other hand if one takes the view that there is an underlying model to the results the amount required for solvency purposes will vary from year to year depending on the expectation of the future which is no longer the same regardless of past years. The fluctuation reserve would also depend on the position in the cycle and would move in the same direction as the amount required for solvency. In this case it would seem logical to have just one reserve with a more stringent restriction on the probability of ruin.

15.2 If one is going to undertake research into fluctuations in claims which is only one of the possible causes of insolvency then there should, at the same time, be some research into bases for covering the other causes.

15.3 Any solvency/fluctuation reserve requirement based solely on claims is going to produce very different answers for different classes of business and size of company. If, instead, one considered the total result of a company's business (eg. trading profit) a more uniform method is likely to emerge as the effects of expenses and interest counteract the variation in claim amounts.

15.4 The practical approach is to assume that the EEC legislation on solvency margins remains more or less the same as at present, which seems the most probable outcome of the current review, but that the non-life services directive mentioned in 7.1 will contain an opportunity for each supervisory authority to devise its own formulae for fluctuation reserves in addition. What then would be the profession's (or industry's) position in dealing with the DOT etc.?

15.5 Essential topics that need to be covered are:-

(a) Classes of business - should these be restricted to the more variable classes? If one takes the view that solvency margin makes some allowance for claim fluctuations it could be argued that an extra reserve is only required when the more volatile classes form a major part of the business.

(b) Reserve for each class? As stated earlier we feel that a fluctuation reserve can only be based on the business as

a whole.

(c) All fluctuations or just some of them? Following the
line of (a) one could argue that only extreme fluctuations
need be covered as 'normal' fluctuations could be considered
covered by the solvency margin, e.g. instead of aiming for
a constant loss ratio as the Germans do could aim to restrict
it within certain limits within plus or minus 10% of the
estimated average, for example.

(d) Basis of measurement. Loss ratios or trading profit?

(e) Theoretical models. Even for practical purposes we need a model
of the insurance market on which to base calculations.

(f) Taxation. Logically harmonisation of technical reserves in the
EEC would lead to harmonisation of taxation principles but would
it in practice?

(g) Small companies. Need to build up market data etc for use
on small or new companies with insufficient past experience
for calculation purposes.

SOLVENCY MARGIN IN NON-LIFE INSURANCE COMPANIES

In any discussion concerning fluctuation reserves the solvency margin must be taken into account. The solvency margin, though necessary in maintaining the security of a company, generates much discussion on what size of margin is needed for that security. This section mentions the ways that have been used to arrive at the size of margin required for that security. The following papers have been written on this subject, but no account has been taken of the risks in the investment field.

1.   The Solvency Margin in Non-Life Companies by De Wit and Kastelijn.

This paper reconstructs the original work done by an O.E.C.D. working party paper dated 11.3.61 by Professor Compagne which surveyed ten companies in each of Denmark, France, Germany, Gt. Britain, Italy, Holland and Sweden and updates it with reference to Holland with more recent information.

a)   The Original Work from O.E.C.D. Report.

The research covered the period of years 1952-1957. The estimated solvency margin is based on an analysis of the claims ratios defined as the claims paid for own account, expressed as a percentage of the net received premiums with the expense ratio taken into account in arriving at the solvency margin needed. The expense ratio is defined as the expenses and commission after deduction of commission received from reinsurers expressed as a percentage of the net received premium.

The information from the ten Dutch companies produced 53 figures. The average claims ratio was 43 and the average expense ratio 53.

The distribution used to represent these claims ratios was a Beta distribution.

$$f(x; p,q) = \frac{x^{p-1}(1-x)^{q-1}}{B(p,q)} \qquad \text{for} \quad 0 < x < 1$$

$$= 0 \qquad\qquad \text{for} \quad x \leq 0 \quad \text{or} \quad x \geq 1$$

$$\text{with } B(p,q) = \int_0^1 t^{p-1}(1-t)^{q-1}dt = \frac{\Gamma(p)\,\Gamma(q)}{\Gamma(p+q)} = \frac{(p-1)!(q-1)!}{(p+q-1)!}$$

where p and q are the parameters of the distribution, and x the claims ratio.

The mean of the distribution is $\mu = \dfrac{p}{p+q}$

and the variance $\sigma^2 = \dfrac{pq}{(p+q)^2(p+q+1)}$

The mean was .43 and standard deviation 0.089 with p and q becoming p = 12.9, q = 16.9

Making use of the distribution laid down above, the claims ratio, which has a probability of ruin of 0.3‰, comes out at 78. This means that if one can finance a claims ratio of 78 with the total security, the chance of bankruptcy is only 3 in 10,000.

The calculation of the solvency margin is then:

Net retained premium	100
Expense ratio	53
For claims payments remains	47
Maximum claims ratio	78
Solvency margin	31

b)   Updating the O.E.C.D. Report.

The period covered was the three years 1976-78 with information from 71 Dutch companies, giving 213 figures. The definitions were:

Expense ratio - the expenses and commission before deduction of the commission received from reinsurers, expressed as a percentage of the gross earned premium.

Claims ratio - the gross incurred claims expressed as a percentage of the gross earned premium.

The claims ratios were much higher than the O.E.C.D. report and claims ratios greater than 100 occurred frequently, and a distribution of claims ratios between 0 to 150 were chosen. The data was transformed to fit the beta distribution by dividing by 1.5 so that the range of claims ratios was 0 to 100.

The average claims ratio was 71.7 and standard deviation 19.4 with the transformed values of 47.8 and 12.9 respectively. The values of p and q were p = 6.68 and q = 7.30.

The average expense ratio was found to be 30%.

The calculation of the solvency margin is then:

With probability of ruin	1%	1‰	0.3‰
Earned premium	100	100	100
Expense ratio	30	30	30
For Claims Payments remains	70	70	70
Maximum claims ratio	116	126	131
Solvency margin	46	56	61

2. Application of the above to 10 large U.K. companies.

The data was obtained from the D.O.T. schedules I and II and covered the period 1971-78 which provided 75 figures. Net claims ratios were calculated expressed as a percentage of the net earned premium.

The mean was 66.29 and standard deviation 4.73 with $q = 33.33$ and $p = 65.55$.

The average expense ratio was 33.59 with a standard deviation of 3.16 expressed as a percentage of the net earned premiums.

The calculation of the solvency margin is then:

With probability of ruin	1%	1‰	0.3‰
Earned premium	100	100	100
Expense ratio	34	34	34
For claims payments remains	66	66	66
Maximum claims ratio	82	84	85
Solvency margin	16	18	19

3. Observations on the Method of Calculation of the Solvency Margin by Buoro, Pavesi and Zuchiatti.

The above paper comments on the O.E.C.D. working group's findings with the object of updating the research and putting forward a new method of calculating the solvency margin.

The authors concentrated on the operating ratio rather than the claims ratio.

The data covered the period of five years from 1973-1977. The balance sheets of five Italian, French, German and British companies provided the information.

Definitions:

(a) G(Insurers Gain) = P(Premiums) - E(Expenses) - S(Claims)

(b) W(Operating Ratio) = $(E + S)$ / P

The intention is to find the value of the company's capital, Q, such that:

$$\text{Prob } (\frac{G}{P} = 1 - W < -\frac{Q}{P}) \leq P_0$$

where the P_0 is the ruin probability.

Unlike the Dutch paper, the normal distribution has been used from which the equations follow with $G/P = Z$

$$\frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{-Q/P} e^{-\frac{(z-m)^2}{2\sigma^2}} \, dz \leq P_0$$

gives $\frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{-(\frac{Q/P+m}{\sigma})} e^{-u^2/2} \, du \leq P_0$ where $m = 1 - E(W)$

$\sigma^2 = \text{Var } (W)$

Another difference from the Dutch paper is that the ruin probability used is .003, rather than .0003 in the Dutch analysis. Both analyses claim to compare with the original O.E.C.D. paper which we have been unable to obtain.

The calculation is based on the formula:

Solvency Margin (SM) required

= Normal Distribution Deviate corresponding to ruin probability x standard deviation + operating ratio minus 1.

E.g. For Italy where the mean operating ratio is 106.8% and the standard deviation is 5.23%, the calculation is:

a) With prob. 0.003

SM% = 2.75 x 5.23 + 106.8 - 100 = 21.2%

b) With prob. 0.0003

SM% = 3.43 x 5.23 + 106.8 - 100 = 24.7%

The corresponding figures are for Germany 10.3%, France 11.8% and Britain 11.4% for prob. 0.003.

The investment income was also considered so that once this has been deducted from the above figures at a rate of 6.3% of Premium,

which corresponded with the lowest rate obtained from any of
the countries analysed, the figures become Italy 14.9%,
Germany 4.0%, France 5.5% and Britain 5.1% for prob. 0.003.

Following the above, the authors put forward the following
formula for calculating the solvency margin for each company.

Solvency Margin $= P \left[(\bar{W} - 1) + 2.5\sigma \right]$

$\bar{W}$ = Average operating ratio during the last five years.

P = Annual premium volume.

= Standard deviation of the distribution of the ratios
  W during the last five years.

Application of (3) to the same ten U.K. companies analysed in (2).
The operating ratio has a mean of 99.88% with a standard deviation
of 3.59%.
a)   With prob. 0.003
        SM% = 2.75 x 3.59 + (99.88 - 100) = 9.75%

b)   With prob. 0.0003
        SM% = 3.43 x 3.59 + (99.88 - 100) = 12.19%

The above SM's required would be much lower if interest income is
taken into account.

J. Ryan
16 July 1980

This note shows the results of simulations designed to demonstrate the variance of estimates, derived from past data, of the standard deviation of the loss ratio.

The loss ratio is assumed to be log-normally distributed with mean 60 and variance s^2. Loss ratios are simulated for R years (the observation period) and the standard deviation calculated. This was then repeated 100 times for each pair (S,R) and the standard deviation of the estimate of S calculated.

		Observation period (n)		
		5	10	15
True standard deviation (s)	5	1.67(32%)	1.15(23%)	1.04(21%)
	10	3.96(40%)	2.85(29%)	2.16(22%)
	15	5.84(39%)	4.29(29%)	3.69(25%)
	20	--	6.09(30%)	5.38(27%)

Figures in brackets show standard deviation of estimate as percentage of true figure (s).

Even for fairly long observation period errors of 25% + would be common (with one standard deviation) and 50% + would occur occasionally (2 standard deviations).

<u>EQUALISATION RESERVES</u>

Report by the Europe Unit on (i) the French System of Equalisation
Reserves, and (ii) Article 3.3 of the EEC Non-Life Services Directive

1.   <u>DESCRIPTION OF THE FRENCH SCHEME</u>

  <u>Scope</u>

The scheme applies to the following classes of insurance and
reinsurance:-

  1   Hail damage
  2. Storm damage, hurricanes, cyclones
  3. Other natural elements (eg frost, floods, earthquakes etc)
  4. Atomic energy
  5. Public liability arising out of pollution

The reserve created for each class is kept distinct from any
reserve for any of the other classes.

  <u>Annual Transfer To Reserve</u>

This is limited to 75% of the "technical profit" of the class of
business concerned.   "Technical profit" is defined in Article 2
of the decree.

  <u>Legislative Background</u>

There was no fiscal "statutory" provision until 1974
when equalisation reserves, although not compulsory,
could be subject to a claim for tax exemption according
to the risk groups specified. The statistics required and
risk groups specified do not coincide with those in
the annual returns to the supervisor.

Following the change in fiscal law, the supervisory
law was extended to accommodate a claims equalisation
reserve.

<u>10-Year Limit</u>

Any annual transfer to the reserve which it has not been possible
to use by covering losses is required to be added back to the
taxable profits for the 11th year following that in which it
was made.

<u>Option To Operate The Scheme</u>

The wording of the scheme implies in several places that the
scheme is optional. The precise nature of the option is not
spelt out, except that (as noted above) the annual transfer is
limited to 75% of the "technical profit"; the implication is
that it could be any smaller amount, down to nil.

<u>Territorial Limit</u>

The scheme is confined to French business, so that the premiums,
profits and losses taken into account are confined to those
arising from French business.

<u>Entry Into Force</u>

The years for which transfers to the reserves can be debited for
tax purposes are those closing after 1st January 1975.

2.   <u>COMMENTS ON THE FRENCH SCHEME</u>

<u>Option</u>

As noted above, the scheme is optional.

<u>Not A Catastrophe Reserve</u>

From the fact that the reserve must be used whenever there is a
loss, regardless of its cause, it can be seen that this is not a
catastrophe reserve.

<u>Possible Benefit Of The Scheme</u>

Smoothing out taxable profits in a particular country - a result to
be expected from a scheme of this kind - can help mitigate a tax
similar to the UK's advance corporation tax. This point is not
relevant to France itself, but should be borne in mind in considering
whether to seek adoption of the French scheme more widely

CHARACTERISTICS OF THE PHENOMENON - KNOWN IN THE NETHERLANDS AS "EGALISATIERESERVE" (literally "EQUALISATION RESERVE").

1.The name

The term "reserve"brings to mind the usual reserves found in insurance - mathematical reserves,unexpired risks reserves,increasing age reserves etc. The equalisation reserve cannot be classed with these reserves as some of them are in fact debts.Consequently the term "equalisation reserve"is not a good choice.It is confusing and all in all not strictly correct.In French we would like to replace it by the term "fonds d'égalisation"(equalisation fund)and this will be used throughout.

2.Distinctive nature

Why was the equalisation fund set up in the first place?
When insurers calculate the appropriate premium for insuring a particular risk they take as a basis the frequency with which losses occur in this particular field.Even if they can accurately forecast the frequency they do not know how this frequency will occur over a given period of time.For example an event insured against may occur very infrequently or not at all during one particular year whilst another year the event insured against may occur frequently and be very sizeable thus giving rise to a serious situation. Seen in this light it is logical for the tax authorities to exempt insurers from declaring all the profit made in years when losses were

below average as taxable income.In return in years when results are poor they will not be completely shouldered by the tax authorities.The next step is to set up an equalisation fund so as to accomplish the scheme outlined above,or at least go some way towards doing this.

3.Accounting

The equalisation fund is not really a reserve and so it does not appear on the balance sheet.In short a company which sets up an equalisation fund has paid too little in the way of contributions in the past thus strengthening its reserves.On the other hand the tax authorities are the creditor of the company in question.In the event of investment losses, negative technical results or loss of profits the company has to add part of its equalisation fund to its tax profit.It is for this reason that Dutch insurers have made provisions vis-à-vis this debt.No accounts are kept for the equalisation fund.

EQUALIZATION RESERVE

Order in Council no. 414 of 18 th July 1972, laying down the assessment of the Insurers' Reserves Order.

(this translation is not authorized; it is only to your information).

Article 1. In this Order:

(a) "life assurer" means a taxpayer transacting the business of life assurance;

(b) "non-life insurer" means a taxpayer transacting the business of non-life insurance;

(c) "premium reserve" means actuarial reserve, net of reassurance;

(d) "premiums" means net retained premiums;

(e) "commissions" means commissions for own account;

(f) "claims" means claims incurred, net of reinsurance;

(g) "sum insured" means sum insured for own account.

Article 2. Life assurers and non-life insurers may form an equalization reserve.

Article 3.

1. In the case of a life assurer the equalization reserve shall not exceed five per cent. of the premium reserve as at the end of the year.

2. In the case of a non-life insurer the equalization reserve shall not exceed fifty per cent. of the premiums for the year. Premiums in respect of nuclear insurances shall not be taken into account.

Article 4.

1. The amount which may be transferred to the equalization reserve out of profits shall not in any one year exceed:
   (a) for:
   (1) a life assurer: four per cent. of the maximum laid down in Article 3, section 1
   (2) a non-life insurer: six per cent. of the maximum laid down in Article 3, section 2;

(b) fifty per cent. of the profit for the year available for transfer to the reserve, computed without applying Article 5;

(c) the taxable income or the taxable income from domestic operations, computed before any addition and the extra addition to the reserve as referred to in Article 5;

2. The fifty per cent. referred to in section 1, subsection (b), shall in the case of a non-life insurer be increased to three-fourths if and so far as upon such addition the reserve does not exceed ten per cent. of the premiums for the year.

Article 5.

1. An extra addition may be made to the reserve up to the amount by which the equalization reserve has been reduced in accordance with Article 6, section 1 subsection (b). Such addition shall not in any one year exceed:
   (a) the sum of:
      (1) the amount by which the maximum computed on the basis of Article 4, section 1, subsection (b) exceeds the maximum computed on the basis of the said Article, section 1, subsection (a); and
      (2) the positive balance of profits and losses for the year in relation to the value of the investments up to an amount not exceeding half of the maximum computed on the basis of Article 4, section 1, subsection (b);
   (b) the taxable income or the taxable income from domestic operations, computed without any extra addition to the reserve.

2. In the case of a non-life insurer, Article 4, section 2, shall apply in respect of the first-mentioned maximum referred to in section 1, subsection (a), sub (1).

Article 6.

1. From the equalization reserve shall be added to profits so far as possible and in the following order:
   (a) for:
      (1) a life assurer: an amount equal to that by which the premium reserve for the year increases as a result of a revision of the bases and methods adopted in computing the premium reserve;

(2) a non-life insurer: an amount equal to the underwriting loss in respect of a class of business, not exceeding the amount which on the basis of the premiums from the lines of business constituting the said class of business may be included in the maximum of the reserve to be computed on the basis of Article 3, section 2;

(b) an amount equal to the negative balance of profits and losses for the year in relation to the value of the investments; when calculating the said balance, profits and losses arising from a substantial reduction in business operations shall not be taken into account.

(c) an amount equal to the negative outcome of the computation of the taxable income or of the income from domestic operations applying subsections (a) and (b).

2. Any addition to profits in pursuance of section 1, subsections (a) and (b), shall be restricted to the amount of the reserve before applying Articles 4 and 5.

3. If as at the end of any one year the reserve exceeds the shareholders' equity less the paid-up capital and less the other allowable reserves, the excess shall be added to the profit for the year. When determining the shareholders' equity, distributions not deductable when ascertaining the profit and similar payments made after the end of the year but relating to that particular year or previous years shall also be considered as liabilities.

4. The underwriting result in respect of a class of business of a non-life insurer means the balance of the premiums for the year - net of commissions due thereon - and the claims for the year, with the proviso that premiums, commissions and claims so far as they apply to nuclear insurances shall not be taken into account.

5. The insurances transacted by a non-life insurer shall be divided into the following four classes of business:
(a) fire, including windstorm;
(b) accident and sickness;
(c) miscellaneous;
(d) marine and aviation.

Article 7.

Non-life insurers may form a catastrophe reserve in respect of nuclear risks.

Article 8.

The catastrophe reserve shall not exceed the sum insured in respect of nuclear risks as of the end of any one year.

Article 9.

1. The amount which may be transferred to the catastrophe reserve out of profits shall not in any one year exceed:
   (a) fifty per cent. of the underwriting profit in respect of nuclear insurances;
   (b) the profit for the year available for transfer to the reserve, computed without applying Articles 4 and 5;
   (c) the taxable income or the taxable income from domestic operations, computed before any addition to the reserve and without the additions to the equalization reserve as referred to in Articles 4 and 5.

2. The underwriting result in respect of nuclear insurances means the balance of the premiums for the year - net of commissions due thereon - and the claims for the year.

Article 10.

1. From the catastrophe reserve shall be transferred to profits so far as possible and in the following order:
   (a) an amount equal to the underwriting loss in respect of nuclear insurances;
   (b) an amount equal to the negative outcome of the computation of the taxable income or of the income from domestic operations applying subsection (a) and after addition of the equalization reserve.

2. If as at the end of any one year the reserve exceeds the shareholders' equity less the paid-up capital and less the other allowable reserves - as regards the equalization reserve after applying Article 6, section 3 - the excess shall be added to the profit for the year.
   When determining the shareholders' equity, Article 6, section 3, last full sentence, shall apply similarly.

Article 11.

So far as the underwriting loss in respect of nuclear insurances exceeds the catastrophe reserve the excess shall be deemed to be part of the underwriting result in respect of the class of business Miscellaneous.

Article 12.

For the computation of the profit for the year available for transfer to the equalization reserve and the catastrophe reserve the tax - except for a discretionary addition from the untaxed reserve as referred to in Article 3 of the Tax Amendment Act 1950 (Government Gazette K 423) - shall be forty per cent. of the taxable income or the taxable income from domestic operations.

Article 13.

1. In the case of a non-life insurer transacting exclusively or almost exclusively the business of windstorm insurance or hail insurance the following amendments shall apply:

   (a) Article 3, section 2: the maximum of the equalization reserve shall be two hunderd per cent. of the premiums for the year;

   (b) Article 4, section 1, subsection (a), sub (2): the maximum referred to in respect of the annual addition to the equalization reserve shall be twenty-five per cent. of the maximum of the reserve as laid down in subsection (a);

   (c) Article 4, section 1, subsection (b): the maximum referred to in respect of the annual addition to the equalization reserve - except for the application of Article 5, section 1, subsection (a), sub (2) - shall be three-fourths of the profit for the year available for transfer to the reserve computed without applying Article 5.

2. In the case of a non-life insurer transacting exclusively or almost exclusively the business of war risk insurance, Article 4, section 1, subsection (a), sub (2), shall not apply, and furthermore the following amendments shall apply:

   (a) Article 3, section 2: the maximum of the equalization reserve shall be the sum insured as at the end of the year;

(b) Article 6, section 1, subsection (a), sub (2): the addition to profits referred to shall be the underwriting loss in respect of war risk insurance, being the excess of the claims for the year over the premiums for the year net of commissions due thereon.

Article 14.

If Article 15 of the Corporation Tax Act 1969 applies in respect of two or more companies which are not all either life assurers or non-life insurers, the provisions of this Order - except for Article 4, section 2, Article 5, section 2, and Article 13 - so far as they do not already apply, shall apply similarly with the proviso that the following amendments shall apply:

(a) Article 3: the maximum of the equalization reserve shall be the sum of the maxima laid down in the said Article for a life assurer and a non-life insurer respectively;

(b) Article 4, section 1, subsection (a): the maximum referred to in respect of the annual addition to the equalization reserve shall be the sum of the maxima laid down in the said subsection for a life assurer and a non-life insurer respectively.

Article 15.

1. The amount of the equalization reserve within the meaning of the Sixth Supplementary Regulation Corporation Tax 1942 (Government Gazette 1945, 101) as at the end of the year to which the said Regulation last applies, shall be the opening balance of the equalization reserve within the meaning of the present Order. If the first-mentioned reserve has been created before the beginning of the year in which the 1st January 1950 falls, the opening balance being an adjusted transitional reserve, the amount referred to in the preceding full sentence shall be decreased by two-fifths of the said transitional reserve, if and to the extent that this reserve is still part of the first-mentioned reserve.

2. If and so long as the opening balance as referred to in section 1 causes the equalization reserve as at the end of any one year to exceed the maximum computed on the basis of Article 3, any addition to profits in pursuance of the said Article shall be allowed only in the event of the said maximum being lower than the maximum as

at the end of the preceding year computed in like manner. In that case an amount shall be added to profits bearing the same relationship to the opening balance as does the difference between the maxima referred to in the preceding full sentence to the maximum similarly computed as at the end of the year preceding that to which this Order first applies.

3. With regard to the taxpayers referred to in Article 13, sections 1 and 2, and Article 14, the provisions contained in Article 13, section 1, subsection (a), Article 13, section 2, subsection (a) and Article 14, subsection (a) respectively shall govern the application of Article 3 as referred to in section 2.

4. The Inspector of Taxes shall determine by regulation:
   (a) the amount of the opening balance referred to in section 1;
   (b) the amount of the maximum referred to in section 2 as at the end of the year preceding that to which the present Order first applies.

5. In the event that any fact gives rise to the assumption that any amount referred to in section 4 has been set too low the Inspector of Taxes is empowered to vary the said regulation. A fact of which the Inspector of Taxes was aware or of which he could reasonably have been aware shall constitute no cause for any such variation. The relevant authority shall lapse on expiry of five years as from the date of establishment of the regulation.

Article 16.

1. This Order shall become effective as from the second day after the date of issue of the Government Gazette in which it has been published.

2. This Order may be cited as "Insurance Companies Reserves Order".

THE FUNCTION AND CHARACTERISTICS OF A FLUCTUATION RESERVE

R.J. Hunter

Suppose that the underwriting experience of a hypothetical company could be represented by Fig. I below.

fig I
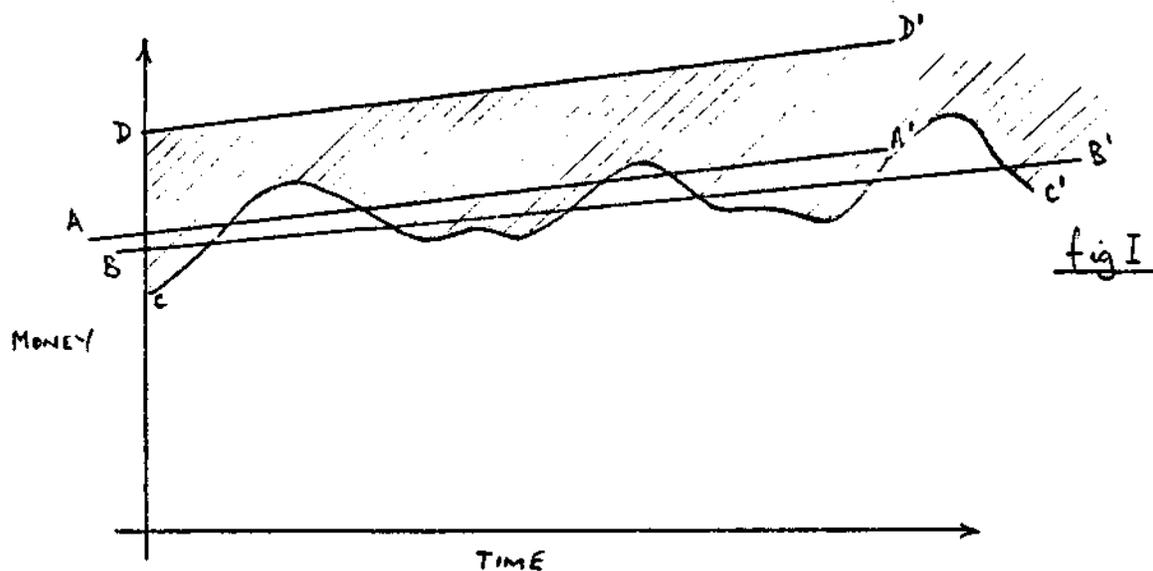
The line AA' represents the trend of premiums (net of expenses and reinsurance costs) during an inflationary period.

The line BB' represents the underlying mean claims experience, after reinsurance recoveries, consistent say with an underwriting target of 97½% of net premium.

The wavy line CC' represents the actual claims experience over the period.

The difference between DD' and AA' represents the interest, net of expenses, earned on the funds.

The difference between DD' and CC' (shaded) is therefore the profit emerging over the period. The emerging profit, as illustrated, is extremely variable.

In practice, companies hold free reserves far in excess of the statutory solvency requirement. These free reserves are used to smooth distributed profits and to finance future growth.

The existence of a calculated fluctuation reserve puts the fixing of part of these excess free reserves on a more scientific basis.

Suppose the above average profits (below line BB') are reserved and used to fully relieve the above average losses (above BB'). The emerging profit would then look like Fig. II

fig II
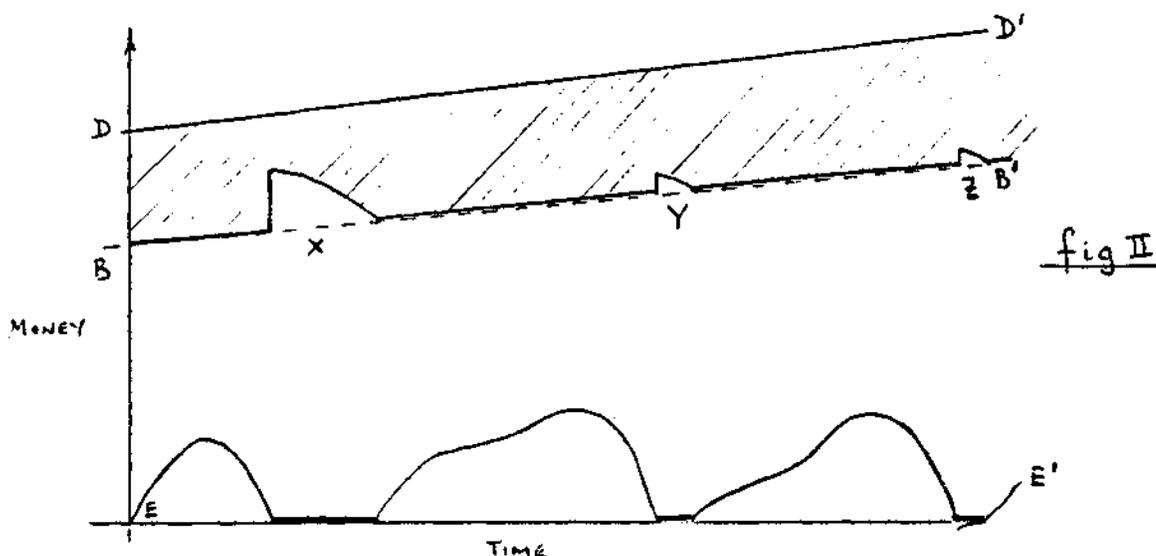
The EE' bumpy line shows the movement in the fluctuation reserve over the period.

It can be seen that even this very simplest of fluctuation reserves can be very effective in smoothing the results. Of course, this method assumes that the underlying mean is known. However, the result would not be much different if a moving average were used to estimate the underlying mean.

The discontinuities at X,Y and Z would be typical in the early years of most fluctuation reserving systems but in this simple system would be typical throughout.

From the shareholders point of view, this system has the advantage that the fluctuation reserve often reduces to zero so there is a minimal interference with the average level of emerging profit.

By redefining BB' at a level slightly higher than the underlying average incurred claims, we would build up a reserve (at the expense of emerging profit equal to the difference between the two lines) which would eliminate the discontinuities after the early years.

Unrestricted, such a reserve would, apart from temporary fluctuations, continue to grow.

Restricting the reserve to a maximum has the effect that the long term average level of emerging profit falls back more in line with the average unsmoothed level and there would be the occasional discontinuity in the emerging profit when a run of bad years extinguished the reserve. The fund would then of course be again vunerable to normal fluctuations, until the fluctuation reserve built up again.

It is debatable whether, with an effective fluctuation reserve, companies would need to hold further free reserves to protect their solvency margin during times when the reserve is low. One could argue that the E.E.C. dual solvency system accepts temporary insolvencies (providing that the lower level has not been breached) and this facility should be utilised.

SIMULATION OF GERMAN METHOD FOR CALCULATING FLUCTUATION RESERVES

Each analysis was based on 100 loss ratios (plus 15 more to provide the start values) which were lognormally distributed with input mean and standard deviation. For selected means and standard deviations the normal distribution was used.

A different set of random variables was used for each run.

Other assumptions incorporated into the simulation model were:-

1. The premium income increases at a given compound rate per annum (0% and 10% were used).

2. The cost ratio was assumed to be 30% constant.

3. The Borderline Loss Ratio was assumed to be 100%.

4. When the calculated, maximum reserve was less than the reserve carried forward from the previous year, the maximum was reduced linearly over 5 years from the carried forward amount to the new required level.

Summaries of the results of the simulation are attached. The figures shown in brackets are the results from normally distributed loss ratios.

The smoothed loss ratios are the observed loss ratios plus the movement in the fluctuation reserve.

The maximum fluctuation reserves and the numbers of zero reserves are volatile and should be interpreted with care, particularly for the 5% standard deviation results.

Where the underlying mean loss plus cost ratio is below the borderline (i.e. in our case where the mean loss ratio is less than 70%), the deduction from the maximum allowable reserve of three times the difference between 70% and the mean is very effective in reducing the size of the allowable reserve and consequently reduces the effectiveness of the system in smoothing fluctuations.

For the higher means and standard deviations the German system produces an effective smoothing of the results with only a small deterioration in the average level of emerging profit.

There appears to be no significant difference between the results derived from the lognormal distribution and those derived from the normal distribution.

It is notable that in none of the 48 simulations did the German method simply build up and maintain a reserve. In every case, there were considerable fluctuations in the size of the reserve between zero and the maximum shown. This suggests that the German method should not be considered to be merely a means of accumulating tax free funds.

R.J. Hunter
25.6.80

RESULTS OF SIMULATION OF GERMAN METHOD
FOR CALCULATING FLUCTUATION RESERVES

Standard Deviation of Loss Ratios = 5%

Premiums increasing at 0% p.a.

Mean L.R.%	Mean Smoothed L.R.%	S.D. Smoothed L.R.%	Max. Fluct. Res % of Prem.	No. of Yrs. with Zero res.
50	50	5	0	100
60	60.0	5.0	3.3	90
65	65.0 (65.2)	4.7 (4.5)	15.2 (15.6)	52 (52)
70	70.3 (70.0)	3.6 (3.9)	30.2 (25.4)	25 (45)
80	80.0 (80.0)	4.6 (4.2)	25.4 (28.0)	61 (36)

Standard Deviation of Loss Ratios = 5%

Premiums increasing at 10% p.a.

Mean L.R.%	Mean Smoothed L.R.%	S.D. Smoothed L.R.%	Max. Fluct. Res % of Prem.	No. of Yrs. with Zero res.
50	50	5	0	100
60	60.1	4.8	10.3	64
65	65.1	4.5	6.5	49
70	70.5	4.0	33.1	43
80	80.2	4.4	17.7	74

Standard Deviation of Loss Ratios = 10%

Premiums increasing at 0% p.a.

Mean L.R.%	Mean Smoothed L.R.%	S.D. Smoothed L.R.%	Max. Fluct. Res % of Prem.	No. of Yrs. with Zero res.
50	50	10	0	100
60	60.3	7.7	29.2	14
65	65.2(65.1)	6.7 (6.7)	50.5 (44.7)	4.(8)
70	70.5	5.7	51.3	4
80	80.3 (80.3)	5.3 (5.3)	67.5 (56.5)	4 (6)

Standard Deviation of Loss Ratios = 10%

Premium increasing at 10% p.a.

Mean L.R.%	Mean Smoothed L.R.%	S.D. Smoothed L.R.%	Max.Fluct. Res.% of Prem.	No. of Yrs. with Zero Res.
50	50	10	0	100
60	60.8	8.1	23.2	18
65	66.7	6.3	45.5	11
70	72.1	4.6	45.2	10
80	82.0	4.6	42.3	10

Standard Deviation of Loss Ratios = 15%

Premiums increasing at 0% p.a.

Mean L.R.%	Mean Smoothed L.R.%	S.D.Smoothed L.R.%	Max. Fluct. Res.% of Prem.	No. of Yrs. with Zero Res.
50	50	13.4	49.2	38
60	60	10.7	49.2	13
65	65.3	10.3	59.9	4
70	70.4	9.3	87.7	4
80	80.4	7.6	82.8	2

Standard Deviation of Loss Ratios = 15%

Premium increasing at 10% p.a.

Mean L.R.%	Mean Smoothed L.R.%	S.D. Smoothed L.R.%	Max. Fluct. Res.% of Prem.	No. of Yrs. with Zero res.
50	50.1	14.8	10.1	57
60	62.6	10.1	70.2	4
65	67.8	8.3	80.3	11
70	72.7	7.0	71.4	16
80	82.6	4.7	67.1	14

Standard Deviation of Loss Ratios = 20%
Premiums increasing at 0% p.a.

Mean L.R.%	Mean Smoothed L.R.%	S.D. Smoothed L.R.%	Max.Fluct. Res % of Prem.	no. of years with zero Res.
50	50.1 (50.1)	16.3 (16.5)	98.1 (71.2)	12 (11)
60	60.3	13.1	136.3	7
65	65.3 (65.4)	10.7 (14.6)	90.3 (87.2)	4 (5)
70	70.2	10.5	124.6	5
80	80.5 (80.1)	11.6 (10.0)	117.1 (129.8)	5 (4)

Standard Deviation of Loss Ratios = 20%
Premiums increasing at 10% p.a.

Mean L.R.%	Mean Smoothed L.R.%	S.D. Smoothed L.R.%	Max. Fluct. Res % of Prem.	No. of years with zero Res.
50	52.2	14.0	69.8	12
60	62.9	12.6	86.0	8
65	69.7	9.7	90.0	8
70	74.3	11.4	101.1	10
80	83.8	9.0	106.3	12

RESULTS OF SIMULATION OF GERMAN METHOD
FOR CALCULATING FLUCTUATION RESERVES

Standard Deviation of Loss Ratios = 5%

Premiums increasing at 0% p.a.

Mean L.R.%	Mean Smoothed L.R.%	S.D.Smoothed L.R.%	Max. Fluct. Res % of Prem.	No. of Yrs. with Zero res.
50	50	5	0	100
60	60.0	5.0	3.3	90
65	65.0 (65.2)	4.7 (4.5)	15.2 (15.6)	52 (52)
70	70.3 (70.0)	3.6 (3.9)	30.2 (25.4)	25 (45)
80	80.0 (80.0)	4.6 (4.2)	25.4 (28.0)	61 (36)

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Mean L.R.%	Mean Smoothed L.R.%	S.D. Smoothed L.R.%	Max. Fluct. Res % of Prem.	No. of Yrs. with Zero res.
50	50	5	0	100
60	60.1	4.8	10.3	64
65	65.1 [65.2]	4.5 [5.3]	6.5 [22.0]	49 [85]
70	70.5 [70.4]	4.0 [4.6]	33.1 [34.7]	43 [71]
80	80.2 [80.1]	4.4 [5.1]	17.7 [22.3]	74 [83]

Standard Deviation of Loss Ratios = 10%

Premiums increasing at 0% p.a.

Mean L.R.%	Mean Smoothed L.R.%	S.D. Smoothed L.R.%	Max. Fluct. Res % of Prem.	No. of Yrs. with Zero res.
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70	70.5	5.7	51.3	4
80	80.3 (80.3)	5.3 (5.3)	67.5 (56.5)	4 (6)

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60	60.8	8.1	23.2	18
65	66.7 [66.1]	6.3 [10.1]	45.5 [54.2]	11 [24]
70	72.1	4.6	45.2	10
80	82.0 [81.7]	4.6 [7.8]	42.3 [47.6]	10 [25]

Standard Deviation of Loss Ratios = 15%

Premiums increasing at 0% p.a.

Mean L.R.%	Mean Smoothed L.R.%	S.D.Smoothed L.R.%	Max. Fluct. Res.% of Prem.	No. of Yrs. with Zero Res.
50	50	13.4	49.2	38
60	60	10.7	49.2	13
65	65.3	10.3	59.9	4
70	70.4	9.3	87.7	4
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60	62.6	10.1	70.2	4
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70	72.7	7.0	71.4	16
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Premiums increasing at 0% p.a.

Mean L.R.%	Mean Smoothed L.R.%	S.D. Smoothed L.R.%	Max.Fluct. Res % of Prem.	no. of years with zero Res.
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60	60.3	13.1	136.3	7
65	65.3 (65.4)	10.7 (14.6)	90.3 (87.2)	4 (5)
70	70.2	10.5	124.6	5
80	80.5 (80.1)	11.6 (10.0)	117.1 (129.8)	5 (4)

Standard Deviation of Loss Ratios = 20%
Premiums increasing at 10% p.a.

Mean L.R.%	Mean Smoothed L.R.%	S.D. Smoothed L.R.%	Max. Fluct. Res % of Prem.	No. of years with zero Res.
50	52.2 [51.3]	14.0 [20.5]	69.8 [123.0]	12 [44]
60	62.9	12.6	86.0	8
65	69.7 [69.1]	9.7 [18.8]	90.0 [159.5]	8 [24]
70	74.3	11.4	101.1	10
80	83.8 [83.4]	9.0 [16.9]	106.3 [124.2]	12 [15]

FLUCTUATION RESERVES

THE GERMAN SYSTEM

M.W. Oakes
Royal Insurance Co. Ltd.
JULY 1980.

FLUCTUATION RESERVES. THE GERMAN SYSTEM

1. Introduction

This note deals specifically with the German System for establishing Fluctuation Reserves (FR). Such reserves have existed officially in Germany for the last 50 years and their estimation has been via several sets of Regulations. The latest of these is dated 1978 and it is with this that we will be primarily concerned, it being based on actuarial/ statistical principles.

It is not the intention of this note to justify the need for Fluctuation Reserves since a justification is the outcome - or otherwise - of very fundamental considerations. Accepting that in Germany a Fluctuation Reserve, in addition to a Solvency Margin, is considered to be justified, the 1978 (i.e. current) method for its calculation will be outlined, together with some of the relevant reasoning.

Unfortunately, literature on the System and its rationale is limited, is not always easy to translate and appears to contain inconsistencies. Nevertheless, it has been possible to establish, with some reliability, the broad basis of the method.

2. Basic Philosophy

By definition, general insurance is such that for any insured risk there is only a small probability that a loss will occur, and no guarantee as to how large it will be. Consequently, regardless of the number of policies in a homogenous grouping, an insurer cannot rely on his loss expenditure remaining constant from one year to the next. The loss expenditure fluctuates to some degree around a certain average amount which can be determined, together with a measure of its fluctuation, from several individual years' figures.

Insurance premiums are based upon this average loss expenditure. They are therefore calculated on the basis that fluctuations in losses will balance out over the number of years used in determining the average. This number of years is referred to as the Equalisation Period.

Since the Premiums are based upon an average loss, determined as above, they effectively remain constant from year to year (inflation effects and other features necessitating premium adjustments are separate considerations) and do not vary according to the fluctuations in annual loss. Fixed premium rates on the one hand and the obligation to pay the entire losses on the other form the basis for setting up Fluctuation Reserves.

These Reserves are intended to allow for the fact that above-average and below-average losses are caused by chance factors affecting the loss experience which cannot be avoided even by the collective balancing of risks. They therefore deal with a different kind of situation from, say, special provisions for large risks or provisions for expected losses.

In order to fulfil their function of equalising the fluctuations in annual loss requirements, the Fluctuation Reserves (or Equalisation Provisions) should be capable of adjusting the actual amount of loss expenditure in any year to an average value. It is therefore necessary to withdraw above-average loss amounts (i.e. the excess above the average requirement) from the Fluctuation Reserves, and to allocate to them below-average loss amounts.

As technical provisions of the first degree, Fluctuation Reserves function according to the laws of large numbers and therefore on portfolios of risks rather than individual policies. Since the Reserves are set up to cover liabilities of uncertain amount, their evaluation should be based on actuarial principles, it having been recognised that "the application of actuarial principles in the calculation of equalisation provisions has the advantage that better allowance can be made then by any other provisions method for the fortuitous nature of the fluctuations to annual requirements".

As stated above, the fixed insurance premium is calculated on the basis that fluctuations in claims will balance out over the Equalisation Period. This is the expected effect. However, as with all statistical samples, variations will occur round this expected value. The maximum amount of excess loss to be expected with 95% certainty during the Equalisation Period is the maximum amount which can be carried in the Fluctuation Reserve. This maximum can be regarded as a notional amount (although it must not be exceeded), the actual amount in the Fluctuation Reserves being determined by the accumulation of below-average loss allocations or above-average loss withdrawals over successive years.

3. Basic Definitions

Def.1 Observation Period

For most classes of business, the observation period (which should not be confused with the Equalisation Period) consists of the 15 business years preceding the current year. This period supplies the random sample from which data for evaluating the current Fluctuation Reserves requirements can be obtained. Whilst it is accepted that it would be better for the length of the Observation Period to reflect the size of annual loss fluctuations, experience has shown that a period of 15 years is generally long enough to minimise the effect of random errors and short enough to avoid the risk of systematic errors. There are exceptions to the 15 year rule, viz Hail and Credit insurance, where the Observation Period is 30 years.

Def.2 Equalisation Period

This is the period (no. of years) over which above-average and below-average losses will, with 95% certainty, cancel each other out. This has to be determined using statistical methods (see Section 6).

Def.3 Loss Ratio

The loss ratio is the ratio between loss expenditure, (which include claims payments, claims handling expenses, return of premium, surrenders and refunds) and the net earned premium. It is evaluated for each individual class of business, and separately for reinsurance and direct business.

Def.4 Average Loss Ratio

The average loss ratio for a particular class is the arithmetic average of all the loss ratios for that class occurring during the observation period.

Def.5 Cost Ratio

The cost ratio is the ratio between the costs of providing the cover (including costs of loss prevention) and the gross earned premium.

Def.6 Average Cost Ratio

The average cost ratio for an individual class is the arithmetic mean of the cost ratios of the year under review and the previous two years. It is considered that, since costs are not subject to the same random fluctuations as losses, an average based on 3 years' individual values should remove any small irregularities which might occur.

Def.7 Borderline Loss Ratio

The borderline loss ratio is the percentage difference between 0.95 in the case of business written direct (0.98 for direct Legal Expense insurance and 0.99 for reinsurance business) and the average cost ratio.

The borderline loss ratio is used to calculate the "Safety Margin" allowed for in the premium - see Def.8 below. The reason for using 0.95 in the calculation is because it is assumed that 5% of the premium is used to cover non-insurance costs incurred in running the business e.g. expenditure on pension schemes, depreciation of equipment etc.

Def.8 Premium

The premium written for any cover is assumed to consist of several component parts.

(a) A portion to cover average losses only (based on the average loss ratio) i.e. the risk premium.

(b) A portion to cover insurance-related costs (based on the average cost ratio).

(c) A portion to cover non-insurance related costs. This is taken as 5% of the premium.

(d) A safety margin, when applicable. See Definition 9.

Def.9 Safety Margin

When the average loss ratio in any one year is less than the borderline loss ratio, there will (according to the breakdown of premium given in Definition 8) be some Premium unaccounted for. This residual amount is regarded as a safety margin.

It is assumed that 60% of this safety margin (if it exists) is available to cover losses. The figure of 60% is solely empirical.

NB. The safety margin cannot be allocated to the Fluctuation Reserves, nor can it be withdrawn from same.

4. The 1978 Regulations: Formulae for Calculation

4.1 Nomenclature

X : Maximum amount permissible in Fluctuation Reserves.

P : Premium Income in year under consideration (Def.8).

q : Loss Ratio in year under consideration (Def.3).

q : Average Loss Ratio over Observation Period, (Def.4) (Normally the last 15 years).

$\bar{c}$: Average Cost Ratio (over last 3 years). (Def.6).

$\bar{q}'$: Borderline Loss Ratio = $0.95 - \bar{c}$ (Def.7).

σ_q : Standard Deviation of q, evaluated over Observation Period.

B : Below-average loss allocations to FR.

D : Above-average loss withdrawals from FR.

4.2 Conditions for creating Fluctuation Reserves

According to the 1978 Regulations, Fluctuation Reserves can be created in those classes of indemnity and accident insurance where:

(a) The earned premiums of the past 3 years exceed, on average, DM 250,000.
(b) The loss ratio standard deviation is at least 0.05.
(c) A loss (measured as the sum of loss and cost ratios) has occurred in at least one year of the observation period.

These conditions have to be applied to 6 specified classes of business, but may also be applied to any other class, and separate Reserves may be set up for Reinsurance and direct business.

The Fluctuation Reserves are to be liquidated when these conditions no longer exist. Liquidation can be distributed over 5 years.

4.3 Comments on Conditions

Condition (a) : The specified minimum portfolio size, even though it is in monetary units rather than number of policies, is an attempt to ensure that the laws of large numbers apply.

Condition (b) : This presumably is intended to indicate that loss fluctuations are large enough to justify a special provision.

Note : No justification for the choice of 0.05 has been found. Further, one source gives this particular condition as

$$\sigma_q \geqslant 0.05\, \bar{q}$$

which appears far more sensible, statistically.

Condition (c) : There is no justification for this condition other than that it appears to be a relic of previous Regulations.

4.4 Calculations for Establishing FR Amounts

In this section, the formulae used for calculating the Fluctuation Reserves accounting entries for a particular year will be given, together with what appears to be the underlying rationale.

The only statistical material required is that relating to loss experience and costs over the Observation Period. As absolute sums, the loss and cost amounts include the effect of inflation. In order to remove this and other factors having no bearing upon the calculation of Fluctuation Reserves, it is considered better to use ratios to premiums as a basis for calculation.

The actuarial definition of the Fluctuation Reserve as the cash value of the excess losses which could occur in total during the equalisation period supplies automatically a condition that all below-average loss amounts must be allocated to the Reserves and all excess losses withdrawn. This follows from the definition of the Equalisation Period as being that within which below-average losses and excess losses balance each other out. This first consideration indicates that:-

Allocation to $FR\left(= B\right)$ should contain $P\ (\bar{q} - q)$ when $q < \bar{q}$

Withdrawal from $FR\left(= D\right)$ should contain $P\ (q - \bar{q})$ when $q > \bar{q}$

If, however, $\bar{q}' > \bar{q}$ i.e. the premiums contain a Safety Margin, then that part of this margin (60%) attributable to claims cover cannot be withdrawn from the Fluctuation Reserve, just as it cannot be allocated to it. By definition, the Safety Margin component of Premium available for claims cover is:-

$A = 0.6\ P\ (\bar{q}' - \bar{q})$, provided $\bar{q}' > \bar{q}$

and so, generally, we may write

$B = P\ (\bar{q} - q)$, minimum zero

$D = P\ (q - \bar{q}) - A$, minimum zero

where

$A = 0.6\ P\ (\bar{q}' - \bar{q})$, minimum zero

In addition to the above expressions for allocation and withdrawal, there is further component of allocation which must be considered.

Since the Fluctuation Reserves are the cash value of possible total excess loss occurring during the equalisation period, interest which will be accrued must be one of the factors taken into account in determining their size. Therefore, for any year of account, even though the full amount of the Fluctuation Reserve is likely to be only notional, the interest which would accrue from this full amount should be paid into the Fluctuation Reserve.

In order to determine this interest we need to know the maximum notional requirement (Y) of the Fluctuation Reserve. The formula for this is:-

$$Y = 4.5\ P\ \sigma_q$$

and the theoretical justification for this is given in Section 6.

However, should the Premium contain a Safety Margin, Y should be reduced by the claims portion of this margin in order to obtain the maximum amount (X) permissible in the Fluctuation Reserve. The formula used is:-

$$X = Y - \max\left\{0,\ 3P(\bar{q}' - \bar{q})\right\}$$

where $3\ P\ (\bar{q}' - \bar{q})$ is the Safety Margin reduction.

Note

We have been unable to find any justification for the use of the factor 3 in this equation. It is stated that it is obtained "by means of an approximation method on the basis of the average period required to achieve equalisation, effect of interest and the empirical assumption that 60% of the Safety Margins are used to cover losses."

Having determined the notional amount X as above, the assumption of an interest rate of $3\frac{1}{2}$% means that the allocation to the Fluctuation Reserves for this particular item is:-

$$0.035X$$

Consequently, in any year the change in the actual Fluctuation Reserve is given by:-

$$\triangle FR = 0.035X + B - D \text{ subject to}$$

$$
\begin{aligned}
0 &\leqslant FR \leqslant X \\
X &= 4.5 \, P\sigma_q - \max \left\{ 0, 3 \, P \, (\bar{q}' - \bar{q} \,) \right\} \\
B &= P \, (\bar{q} - q), \text{ minimum zero} \\
D &= P \, (q - \bar{q}) - A, \text{ minimum zero}
\end{aligned}
$$

where

$$A = 0.6P \, (\bar{q}' - \bar{q}), \text{ minimum zero.}$$

4.5   Examples of Calculation

$$P = £10^6$$

Average Loss Ratio, $\bar{q} = 0.70$

Average Cost Ratio, $\bar{c} = 0.20$

Therefore, Borderline Loss Ratio, $\bar{q}' = 0.95 - 0.20 = 0.75$

Therefore, a Safety Margin component exists $(\bar{q}' > \bar{q})$, value $£0.6 \, (0.75 - 0.70) \, 10^6$

$$= £3. \times 10^4$$

Standard Deviation $\sigma_q = 0.25$

i.e. $X = (4.5)(0.25)10^6 - 15 \times 10^4 = £97.5 \times 10^4$

(i)   Loss Ratio = 0.60

Below-average losses $= £(0.70 - 0.60)10^6 = £10^5$

Withdrawals = zero.

Therefore, $\triangle FR$ = Below-average losses + Interest on X.

i.e. $\triangle FR = 10^5 + (0.035)(97.5 \times 10^4) = £13.4 \times 10^4$

(ii) <u>Loss Ratio = 0.90</u>

Excess losses $= £(0.90 - 0.70) \, 10^6 = £2 \times 10^5$

Withdrawals $=$ Excess losses $-$ Safety Margin

$$= 2 \times 10^5 - 3 \times 10^4 = £17 \times 10^4$$

Allocation $=$ Interest on X $= £3.4 \times 10^4$

Therefore $\triangle$ FR $= -£13.6 \times 10^4$

5. <u>The Effect of the Fluctuation Reserves Formulae on Profit figures</u>

The broad objective of Fluctuation Reserves is to equalise profit from year to year. It is interesting, therefore, to study the effect on profit figures of applying the formulae given in Section 4.

Whilst it is possible to carry out computer simulations to investigate the effect of the German formulae on the build-up of Fluctuation Reserves etc (as is done elsewhere in the Working Party Report) the effect of the formulae can be examined in a simpler, but less effective way.

There are two separate considerations.

1. No safety margin exists in the Premiums.) each of which contain two
2. The Premiums contain a Safety Margin.) different situations:

(A) The Fluctuation Reserves are building up and are nowhere near the notional maximum requirement i.e. $FR \ll X$.

(B) The Fluctuation Reserves have built up and are near the maximum requirement.

In order to facilitate examination of the effect of the formulae, we will write

$$q = \bar{q} + Z \sigma_q$$

and make observations concerning magnitudes of effect in terms of the size of Z i.e. the number of Standard deviations difference between the year's loss ratio and the average loss ratio.

5.1 <u>Premiums contain no Safety Margin</u>

A. <u>FR $\ll$ X</u>

The formulae of Section 4.4 indicate that:-

$$\triangle FR = 0.1575 \, P \sigma_q - PZ \sigma_q$$

Over a long enough period $\sum Z \to 0$ by definition, and so the Fluctuation Reserves will increase by an average amount per year of $0.1575 \, P\sigma q$. It will therefore eventually reach its maximum permissible value.

Taking $\triangle$ FR into account.

Equalised Profit $= P(1 - \tilde{c} - \bar{q}) - 0.1575 \, P\sigma_q$.

Therefore the effect of the application of the FR formulae in this instance is to equalise the profit but to a lower level than that obtained by straight application of the average loss ratio.

B. **FR Close to Maximum FR**

Suppose that the difference between the notional maximum FR (i.e. X) and the existing, actual FR, is a $\sigma_q P$ (a > 0). a $\sigma_q P$ is therefore the maximum allowable allocation to the Fluctuation Reserve.

It can then be shown that

(i) <u>$Z \leqslant -a + 0.1575$</u>

Equalised Profit $= P(1 - \bar{c} - \bar{q}) - P \sigma_q(Z + a)$

Since a > 0, $Z < -a$, the equalised profit will be higher than the equalised profit in 5.1A.

(ii) <u>$Z \geqslant -a + 0.1575$</u>

Equalised Profit $= P(1 - \bar{c} - \bar{q}) - 0.1575 P \sigma_q$.

Fig. 1 gives an illustration of the effect on profit in the above two cases.

5.2 <u>Premiums contain a Safety Margin</u>

Denote the size of the calculated Safety Margin by $W \sigma_q$ ($= 0.95 - \bar{c} - \bar{q}$)

A. **FR $\ll$ X**

(i) <u>$q < \bar{q}$</u>

$\triangle$FR $= (0.1575 - 0.105 W - Z) P \sigma_q$

Equalised Profit $= P(1 - \bar{c} - \bar{q}) - (0.1575 - 0.105W) P \sigma_q$

(ii) <u>$\bar{q} \leqslant q \leqslant \bar{q} + 0.6W$</u>

$\triangle$FR $= (0.1575 - 0.105W) P \sigma_q$

Equalised Profit $= P (1 - \bar{c} - \bar{q}) - (0.1575 - 0.105W + Z) P \sigma_q$

(iii) <u>$\bar{q} + 0.6W \leqslant q$</u>

$\triangle$FR $= (0.1575 + 0.495W - Z) P \sigma_q$

Equalised Profit $= P (1 - \bar{c} - \bar{q}) - (0.1575 + 0.495W) P \sigma_q$

B. **FR close to Maximum FR**

If, again it is assumed that the maximum allowable allocation to the FR is aP σq, then the above equations (5.2A) apply except for:-

(i) <u>$Z \leqslant -a + 0.1575 - 0.105W$</u>

Equalised Profit $= P (1 - \bar{c} - \bar{q}) - P \sigma_q(Z + a)$

For $Z \geqslant -a + 0.1575 - 0.105W$, the formula given in 5.2A (i), (ii) and (iii) apply.

Fig. 2 gives an illustration of the effect of cases 5.2A, 5.2B on profit.

Fig. 2.

ACTUAL PROFIT, EQUALISED PROFIT CONSIDERATIONS.
PREMIUMS CONTAIN SAFETY MARGIN $w \sigma_q P$.

A: FR << X

+ve PROFIT/PREM.

ACTUAL PROFIT.

$\bar{q}$ PROFIT.

EQUALISED PROFIT

$0.6 \sigma_q$

DIFF $= \begin{pmatrix} 0.1575 \\ -0.105w \end{pmatrix} \sigma_q$

0.6w.

z

-ve

B. FR CLOSE TO MAX FR. $(\Delta FR)_{MAX} = a \sigma_q P.$

ACTUAL PROFIT

PROFIT/PREM.

$a \sigma_q$

EQUALISED PROFIT

$\bar{q}$ PROFIT.

$-a+0.1575 -0.105w.$

0.6w.

z
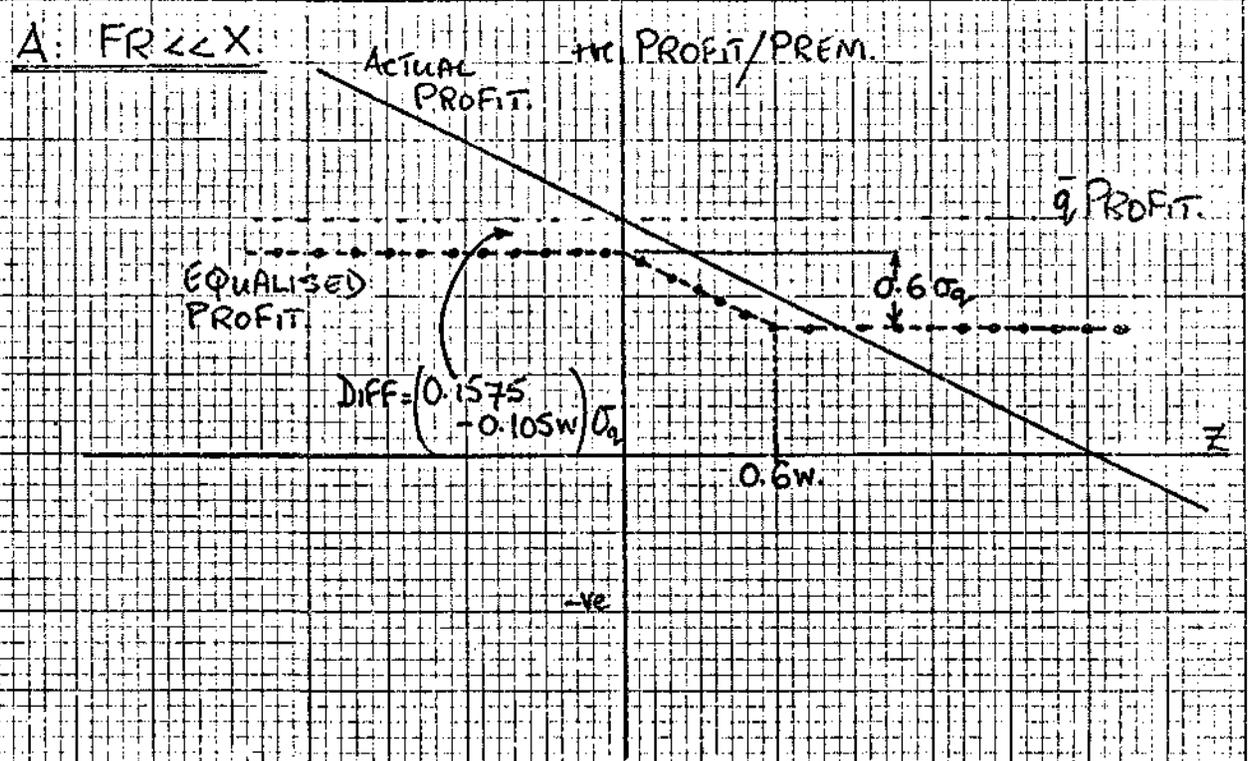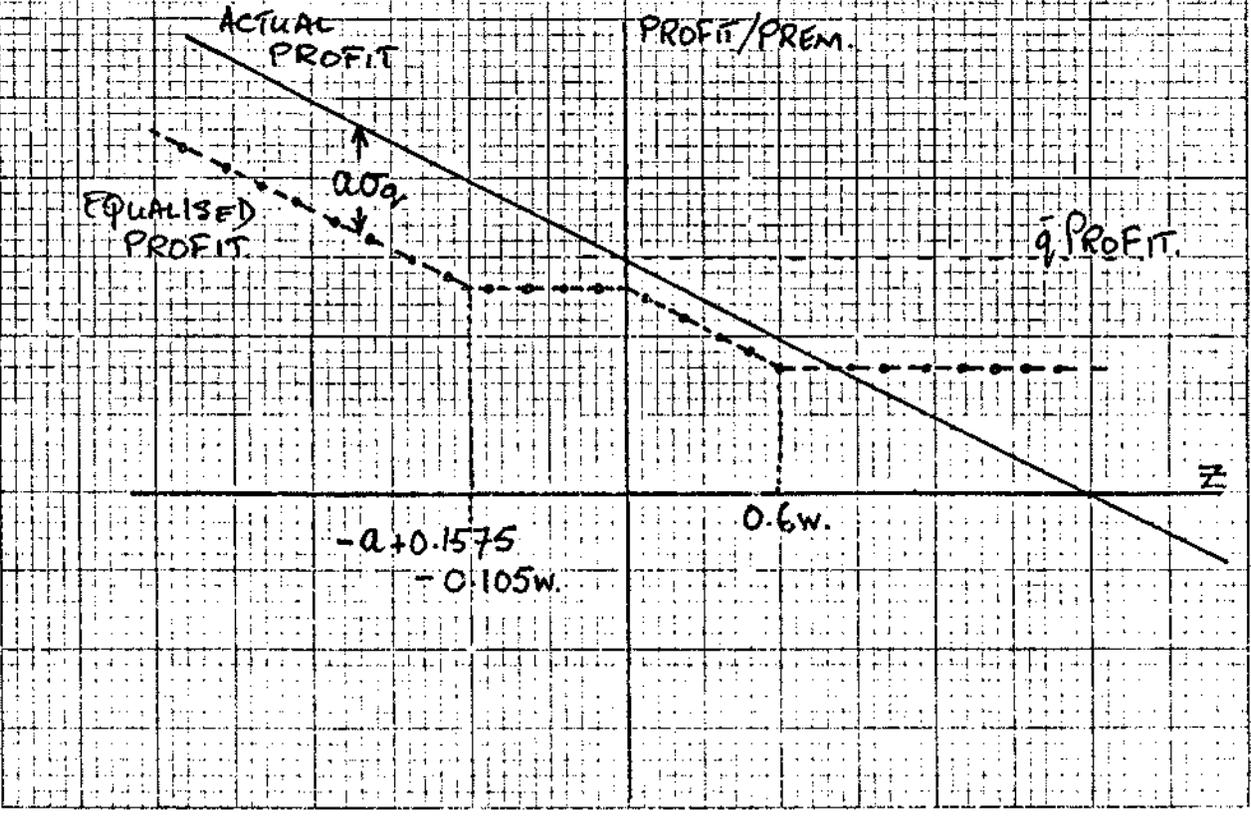
FIG. 1

ACTUAL PROFIT, EQUALISED PROFIT CONSIDERATIONS. NO SAFETY MARGIN. $q = \bar{q} + z\sigma_q$.

A. $FR << X$.

+ve PROFIT/PREM.

ACTUAL PROFIT.

0.05
↓ NO SAFETY MARGIN.

$\bar{q}$ PROFIT.

0.1575 σ_q

EQUALISED PROFIT.

z

−ve

B. FR CLOSE TO MAX FR. $(\Delta FR)_{MAX} = a\sigma_q P$.

ACTUAL PROFIT

+ve PROFIT/PREM.

$a\sigma_q$

EQUALISED PROFIT.

$\bar{q}$ PROFIT.

0.1575 σ_q

$a - 0.1575$

z

−ve
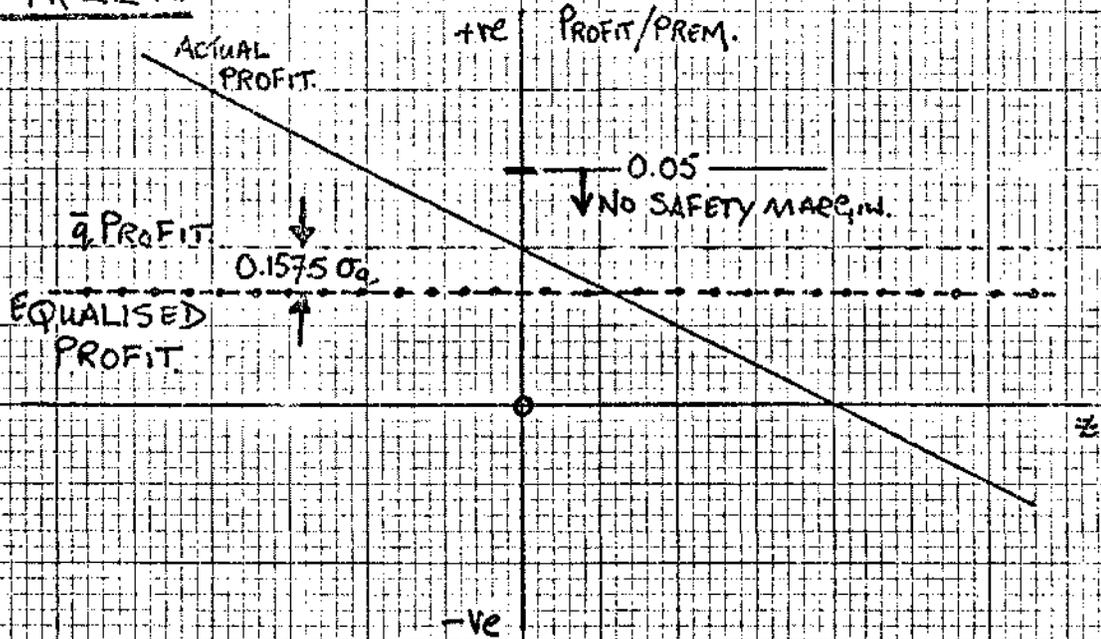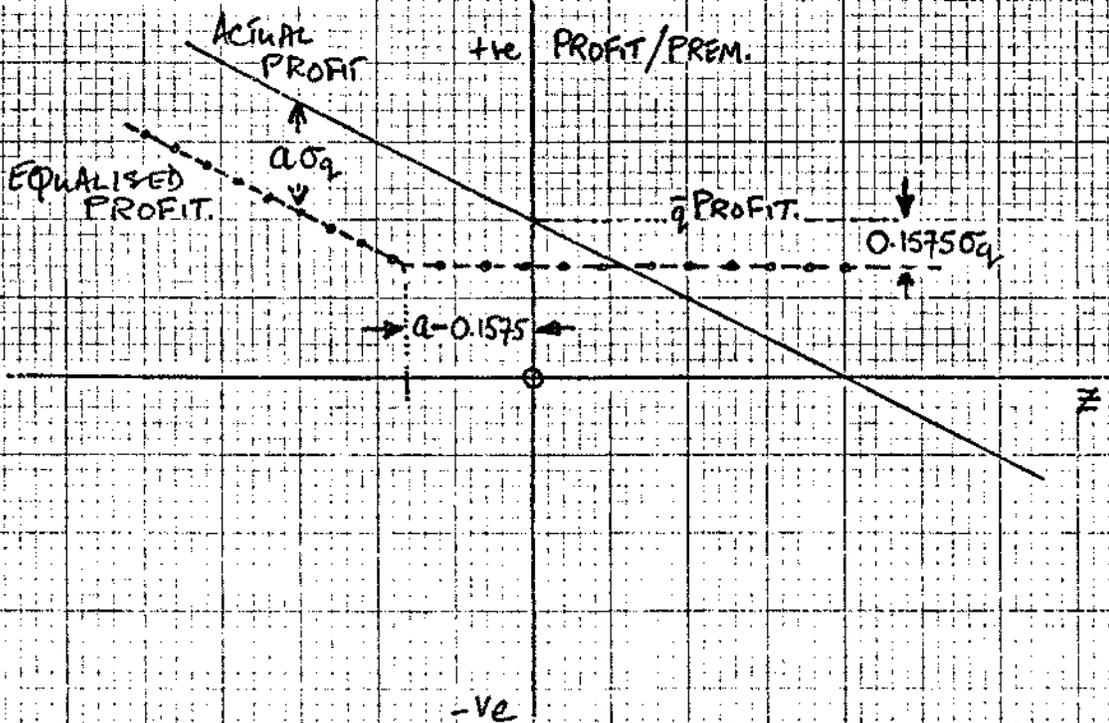
6.   Actuarial/Statistical Considerations

The German literature contains brief details of the statistical theory
associated with two components of their approach to Fluctuation Reserves.

(a) The Equalisation Period

(b) The maximum amount permissible in the Fluctuation Reserves.

In order to apply their theoretical equations, it is necessary to know, for
instance, the underlying distribution of loss ratios. It would appear that
a log-normal distribution is used in some cases, although it cannot be
discovered whether this is true generally. Indeed, it is impossible to
determine how thorough the transition from theory to practice has been e.g.
how much data has been used, how many possible loss-ratio distributions have
been examined, how well inter-class consistencies have been checked etc.,

However, whilst actual data scrutiny is an essential ingredient of the
application of any method, the first step is to develop a soundly based
theoretical, but simple, approach. The Germans, given their basic philosophy
towards Fluctuation Reserves, appear to have made good progress in this direction.

6.1   The Equalisation Period

The Equalisation Period is that within which below-average and above-average
losses can reasonably be expected to balance each other out. This statement,
as it stands, is completely open insofar as the word "reasonably" can assume
any probability criteria. The approach adopted, in order to set criteria for
actively determining a value for the length of the period, appears to be as
follows:-

Given a period of k years, the maximum excess loss occurring, with 95%
certainty, in any one of those years should not be greater than 5% of the
total risk premium collected during the k years.

Consequently, if

k = length of Equalisation Period (years)

$\tilde{q}$ = average loss ratio

q_{max} = maximum individual loss ratio expected with 95% certainty.

it follows, from the above criteria, that

$q_{max} - \tilde{q}$ = $0.05 \, k\tilde{q}$.

Given that we know the underlying loss ratio distribution, q_{max} can be
determined in the standard way. $\tilde{q}$ is known and so k can be calculated.

German data on hail insurance (where a log normal distribution is assumed
for loss ratios) gives k = 17.2 years.

6.2 <u>The Maximum Amount Permissible in the Fluctuation Reserves</u>

Given that the Equalisation Period has been calculated as above, it is now necessary to determine the <u>total</u> excess losses which can accrue over the k years.

The Germans assume that the upper limit on these totalexcess losses is such that, in 95% of cases, the actual total excess losses will be less than this upper limit.

Suppose that

k = length of Equalisation Period j

q_i = loss ratio in year

Q_j = total of loss ratios, = $\sum_{i=1}^{k} q_i$ in Equalisation Period j,

Q_M = upper limit on Q_j

$f(q)$ = distribution function of individual loss ratios

$F(Q)$ = distribution of total of loss ratios over the Equalisation Period.

σ_q = standard deviation of q_i

σ_φ = standard deviation of Q.

Since Q = $\sum_{i=1}^{k} q_i$,   it follows that, if it is assumed that the individual q_i are independent,

Var. Q = k Var.q

and so,

$$\sigma_\varphi = \sqrt{k}\ \sigma_q$$

Further, if $\bar{Q}$ is the average value of Q_j (obtained from a large number of Equalisation Periods), any value of φ (say Q_j) can be written as

$$Q_j = \bar{Q} + b_j\ \sigma_\varphi$$

i.e.

$$Q_j = \bar{Q} + b_j\ \sqrt{k}\ \sigma_q$$

If the distribution function, $F(Q)$, of Q can be determined, then λ can be found such that

$$Q_j < \bar{Q} + \lambda \sqrt{k} \, \sigma_q \text{ in 95\% of cases.}$$

This value (i.e. $\bar{Q} + \lambda \sqrt{k} \, \sigma_q$) is the upper limit of Q_j denoted by Q_M

It then follows that the required 95% upper limit on total excess losses, EL, is given by

$$EL = Q_M - \bar{Q} = \lambda \sqrt{k} \, \sigma_q$$

Any total excess losses which occur will do so, by definition, over the whole of the Equalisation Period. On average, therefore, they will occur at the mid-point of this Period and so the actual requirement is the full one with a $k/2$ year discount.

Therefore, the actual upper limit on the total excess loss, Y, (referred to in Section 4.4) is given by

$$Y = V^{k/2}. \; \lambda. \; \sqrt{k}. \; \sigma_q$$

$V^{k/2}$ and k can be determined fairly easily. The main problem lies in determining the distribution function $F(Q)$ so that a value of λ can be obtained.

Since $Q_j = \sum\limits_{i=1}^{K} q_i$, a fixed value of Qj, say Qo,

can be obtained from a very large number of Equalisation Periods (since these can contain any sample, size k, of all possible q_i). Each such sample will have an associated probability of occurrence, and the probability of occurrence of Q_0 will then be the sum of these individual sample associated probabilities.

Suppose that there are n Equalisation Periods, denoted by S_r (r = 1 $\rightarrow$ n), containing loss ratios q_{ri} (i = 1 $\rightarrow$ k) such that

$$\sum\limits_{i=1}^{K} q_{ri} = Q_0 \qquad \text{for all } r$$

Since the distribution functions of q is f(q), the probability $P(S_r)$ obtaining set S_r is given by

$$P(S_r) = \prod\limits_{i=1}^{K} f(q_{ri})$$

It then follows that the probability of obtaining the value Q_0 is given by

$$F(Q_0) = \sum\limits_{r=1}^{n} P(S_r) = \sum\limits_{r=1}^{n} \prod\limits_{i=1}^{K} f(q_{ri})$$

We are therefore concerned with "folding" (i.e. convoluting) to the kth power, the distribution of q.

This has been done empirically, using the computer, for hail insurance and a value of λ determined from the calculated F (Q). This value is $\lambda = 1.8$.

In the case of sufficiently high values of k, $F(Q)$ approximates to the normal distribution, by the central limit theorem. In this case, $\lambda = 1.64$.

Using hail insurance as an example, it has thus been found that

$\hat{\lambda} = 1.80$

$K \cong 17$

Taking an interest rate of 5.5% (should this be consistent with 3.5% in the Interest Allocation Component?)

$$v^{k/2} \simeq 0.63$$

and so
$$v^{k/2} \lambda. \sqrt{\kappa} = 4.68$$

It is maintained, in the literature, that this value has been found to be common to most classes of business. Hence, it is taken that the Maximum permissible amount in the Fluctuation Reserves (ignoring Safety Margin considerations - discussed in Section 4) is given by

$$Y = 4.5 \sigma_q$$

FLUCTUATION RESERVES

THE FINNISH SYSTEM

1. INTRODUCTION AND BACKGROUND

1.1 The Finnish Insurance Companies Act of 1953 has enabled companies to retain reserve funds in respect of fluctuation reserves. It contains the section: "The outstanding claims reserve shall include the amounts of occurred but unpaid claims plus a fluctuation provision for years with excessive losses calculated according to risk theory."

1.2 In the published accounts the amount of the fluctuation reserve is not shown separately but is amalgamated with the outstanding claims reserve.

1.3 Fluctuations in claims are due in part to random variation in the number and size of claim. Additionally certain factors affect the basic probability of loss, such as weather conditions in some branches.

1.4 Taxation of surplus may make it difficult to build up reserves to meet fluctuations.

1.5 Insurance companies may not wish to report results fluctuating grossly from year to year. Achieving smoothness by transfers from reserves other than fluctuation reserves may be undesired since analysts can interpret such transfers as signs of ill health.

1.6 If fluctuation reserving is not used, then there is a considerable amount of reinsurance. This is a factor that increases costs for companies and for the whole economy.

1.7 The fluctuation reserve is limited to the coverage of the fluctuation of the claims. Investment losses etc. are covered by separate requirements and reserve funds.

1.8 Instructions issued by the Finnish Insurance Department of the Ministry of Social Affairs and Public Health provide for an upper and lower limit to the fluctuation reserve. The upper limit represents the amount required to ensure a low probability of ruin over the next five years, the lower limit represents the amount required to ensure a low probability of ruin over the next year.

1.9 The fluctuation reserve applies only to future years of exposure and has no relation to IBNR or outstanding claims.

1.10 Transfers to and from the fluctuation reserve are not subject to taxation. They are determined by a formula agreed at the outset in advance between the company and the supervisory authority. This prevents companies from using the fluctuation reserve system to determine their taxable profit. Changes in the formula are only permitted under changing conditions or special circumstances.

1.11 A fluctuation reserve may be covered by hidden assets, for example undervaluation of assets in the balance sheet. So for some companies the minimum fluctuation reserve may be zero.

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1.12 The formulae applied may be modified by agreement between the Actuary and the supervisory authority where appropriate, for example in the presence of stop loss reinsurance.

1.13 The rate of interest assumed by the supervisory authority is 5%.

1.14 This paper is based upon the Finnish system as defined in Pentikainen(1970).

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2. THE METHOD

2.1 For the fluctuation reserve there is an upper limit and a lower limit. The upper limit is referred to as E_{max} and the lower limit as E_{min}. Formulae for E_{max} and E_{min} are given in section 3.

2.2 Transfers to and from fluctuation reserves are determined automatically by a formula. Such a formula is given in section 3.6. Changes in a formula once adopted are only allowed under special circumstances. The effect is that in years when the claims ratio is favourable the surplus is to be deposited in fluctuation reserves provided that the upper limit is not exceeded. On the other hand, the loss in a year when claims are high is to be covered by a reduction in the fluctuation reserve provided that the reserve does not thereby drop below the lower limit.

3. THE FORMULAE

3.1 Formulae are given below. These are described in more detail in Section 4. Section 4 also gives the notation used in the formulae below, and the notation is summarised in Section 8.

3.2 $\quad E_{min} = 0.976 \; \Sigma \; q_k P_k + 2.270\sigma + 0.714 . \dfrac{\mu_3}{\sigma^2} - U$

3.3 $\quad E_{min} = M \; y(\tau) - P - U$

3.4 $\quad E_{max} = 4.436 \; \Sigma \; q_k P_k + 4.626\sigma + 0.658 . \dfrac{\mu_3}{\sigma^2}$

3.5 $\quad E_{max} = 5 \; \Sigma \; q_k P_k + \dfrac{1}{\sqrt{n}} \; \Sigma \; (1 + q_k) P_k$

3.6 $\quad \Delta E_k = .05 E_k^o + 1.0247((\overline{f}_k + a_k) P_k - x_k)$

4. THE DESCRIPTION OF THE FORMULAE

The formulae below are justified within Section 7 and have been summarised in Section 3. The notation introduced below is summarised within Section 8.

4.1 The formulae used by the Finnish method are given in Section 3 and are described below.

4.2 E_{min} is the absolute lower limit of the fluctuation reserve. The formula for E_{min} is given in Section 3.2. as

$$E_{min} = 0.976 \ \Sigma \ q_k P_k + 2.270\sigma + 0.714.\frac{\mu_3}{\sigma^2} - U$$

In this expression the following notation has been used:

q_k is a constant coefficient representing excess loading owing to the risk of the fluctuation of the basic probabilities. Values of q_k are supplied by the Finnish Supervisory Service and are typically in the range .2 to .4, although in special cases it may be higher. Examples of q_k are: car insurance .25, fire .4, credit .5, forest 6.0.

P_k is the premium income (net of expense loading and reinsurance) for branch k.

$\sigma^2 = \Sigma_k \ n_k (1 + q_k)\alpha_{k2}$ where n_k is the number of claims expected in branch k and α_{k2} is the second moment of the amount of one claim.

$\mu_3 = \Sigma_k \ n_k (1+q_k)\alpha_{k3}$ where α_{k3} is the third moment of the amount of one claim.

U is the company's own reserves and free reserves (including "hidden reserves").

In Finland it has been possible to assist companies wishing to estimate α_{k2} and α_{k3} merely from the values of P_k, n_k and M_k (where M_k is the maximum retention for branch k). This has been done by means of industry-wide statistics on claim amounts which are amalgamated into tables for each branch. Examples are given in Hovinen(1969).

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4.3 An approximate formula for E_{min} is given in Section 3.3 as follows:

$$E_{min} \simeq M \, y(\tau) - P - U.$$

This approximate formula generally yields a result higher than that given by formula 3.2.

In this formula the following notation has been used:

M = Maximum retention on a single claim

$\tau = \Sigma \, (1 + q_k)P_k / M$

$y(\tau)$ = Smallest integer ≥ 2 satisfying $e^{-\tau} \sum_0^{y-1} \frac{\tau^r}{r!} \geq .99$.

4.4 E_{max} is the upper limit of the fluctuation reserve. The formula for E_{max} is given in Section 3.4 as

$$E_{max} = 4.436 \, \Sigma \, q_k P_k + 4.626\sigma + 0.658.\frac{\mu_3}{\sigma^2}.$$

In this expression the notation described in Section 4.2 above has been used.

4.5 An approximate formula for E_{max} is given in Section 3.5 as follows:

$$E_{max} = 5 \, \Sigma \, q_k P_k + \frac{1}{\sqrt{\bar{n}}} \cdot \Sigma \, (1 + q_k)P_k \, .$$

In this expression the notation q_k, P_k has been described in Section 4.2 above, while

$\bar{n} = \Sigma \, (1 + q_k)n_k$ where n_k is the expected annual number of claims

for branch k.

4.6 ΔE_k is the fluctuation reserve transfer in respect of branch k. The formula for ΔE_k is given in Section 3.6 as

$$\Delta E_k = 0.05E_k^0 + 1.0247((\bar{f}_k + a_k)P_k - X_k) \, .$$

In this expression the following notation has been used:

E_k^0 = Amount of fluctuation reserve in respect of branch k at the end of the preceding year.

4.6 (continued)

$\bar{f}_k$ = Expected claims ratio (computed by a formula supplied by the Finnish Supervisory Service on the basis of the actual claims ratio of at least five preceding years) based on net premiums.

a_k = A correction coefficient that must be chosen in advance by the company in the range 0 to 0.15.

X_k = Actual total of claims net of reinsurance for branch k.

5. EXAMPLES

5.1 Suppose for a company with just one branch (the notation used is that of Section 4) that

$q_k = 0.2$

$P_k = £1,000,000$

$\alpha_{k2} = £^2 2,000,000 \quad (= (£1400)^2)$

$\alpha_{k3} = £^3 5,000,000,000 \quad (= (£1700)^3)$

$n_k = 1,000$

$U = 0$

$M = £10,000$

$\bar{f}_k = 1.0$

$a_k = 0.10$

$X_k = £1,000,000$

$E_k^0 = £500,000$

5.2 From Sections 5.1 and 4.2 we have

$\sigma^2 = £^2 2,400,000,000 \ , \quad \sigma = £50,000$

$\mu_3 = £\ 6,000,000,000,000,000$

So $E_{min} = 0.976.£200,000 + 2.270.£50,000 + 0.714.\dfrac{£6,000,000,000,000}{2,400,000,000}$

So $E_{min} = £195,000 + £113,500 + £1785 = £310,000$

5.3 From Sections 5.1 and 4.3 we have

$\tau = £1,2000,000\ /\ £10,000 = 120$

So $y(\tau) \simeq 120 + 2.326\sqrt{120} = 145$

So $E_{min} \simeq £1,450,000 - £1,000,000 = £450,000$

5.4 From Sections 5.1 and 4.4 we have

$E_{max} = 4.436.£200,000 + 4.626.£50,000 + 0.658.\dfrac{£6,000,000,000,000}{2,400,000,000}$

So $E_{max} = £887,200 + £231,300 + £1645 = £1,120,000$

5.5 From Sections 5.1 and 4.5 we have

$E_{max} \simeq 5.£200,000 + \dfrac{1}{\sqrt{1000}}.£120,000 = £1,000,000$

5.6   From Sections 5.1 and 4.6 we have

$$\Delta E_k = .05(\pounds 500,000) + 1.0247((1.0 + 0.1)\pounds 1,000,000 - \pounds 1,000,000)$$

So $\Delta E_k \approx \pounds 25,000 + \pounds 103,000 = \pounds 128,000.$

6. THEORETICAL CONSIDERATIONS

6.1 Suppose we wish to consider just one branch of business for one year. We suppose that N claims will be made, labelled $X_1, X_2, \ldots, X_N$. We further suppose that the first three moments of X_i are known to be m, α_2 and α_3 and that the set $\{X_i\}$ form a set of independent identically distributed random variables. We suppose that N is a Poisson random variable with mean Qn where n is a constant and Q is a random variable with mean 1 and variance V_q. We use the notation

$$\xi = \sum_{i=1}^{N} X_i \quad \text{to denote the total amount claimed.}$$

6.2 The variable ξ is said to have the compound generalised Poisson distribution.

6.3 We wish to obtain the moments of ξ. However ξ is a function of N. Accordingly in Section 6.4 the moments of N will be calculated, these will then be used in Section 6.5 to calculate the moments of ξ.

6.4 The moments of N (N = number of claims) are calculated in this Section 6.4. N and Q have been defined in Section 6.1 above.

6.4.1 $E(N) = e^{-nQ} \sum_{r=0}^{\infty} \frac{r(nQ)^r}{r!} = e^{-nQ} nQ \sum_{s=0}^{\infty} \frac{(nQ)^s}{s!} = nQ$

6.4.2 $E((N(N-1))) = e^{-nQ} \sum_{r=0}^{\infty} \frac{r(r-1)(nQ)^r}{r!} = n^2Q^2$

6.4.3 $E((N(N-1)(N-2))) = e^{-nQ} \sum_{r=0}^{\infty} \frac{r(r-1)(r-2)(nQ)^r}{r!} = n^3Q^3$

6.5 The moments of ξ (ξ = total claims amount) are calculated in this Section 6.5. N, Q, ξ, are defined in Section 6.1 and the moments of N are calculated in Section 6.4.

6.5.1 The first moment of ξ is calculated in this Section 6.5.1.

6.5.1.1 $E(\xi|N) = E\left(\sum_{i=1}^{N} X_i \Big| N\right) = N E(X_i) = N m$.

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6.5.1.2 $E(\xi|Q) = E(N\,m|Q) = nmQ$

6.5.1.3 $E(\xi) = E(nmQ) = nm$

6.5.2 The second moment of ξ is calculated in this Section 6.5.2.

6.5.2.1 $E(\xi^2|N) = E((\sum\limits_{i=1}^{N} X_i)^2|N)$

$$= E((\sum X_i^2 + \sum\sum\limits_{i\neq j} X_iX_j)|N)$$

$$= N\,E(X_i^2) + N(N-1)\,E(X_iX_j\,(i\neq j))$$

$$E(\xi^2|N) = N\alpha_2 + N(N-1)\,m^2$$

6.5.2.2. $E(\xi^2|Q) = E(N\alpha_2 + N(N-1)\,m^2|Q)$

$$E(\xi^2|Q) = nQ\alpha_2 + n^2Q^2m^2$$

6.5.2.3 $E(\xi^2) = n\alpha_2 + n^2m^2\,(1 + V_q)$

6.5.2.4 $Var(\xi) = E(\xi^2) - (E(\xi))^2$

$$Var(\xi) = n\alpha_2 + n^2m^2\,(1 + V_q) - n^2m^2$$

$$Var(\xi) = n\alpha_2 + n^2m^2V_q$$

6.5.3 The third moment of ξ is calculated in this Section 6.5.3.

6.5.3.1 $E(\xi^3|N) = E((\sum\limits_{i=1}^{N} X_i)^3|N)$

$$= E(\sum X_i^3 + 3\sum\sum\limits_{i\neq j} X_i^2X_j + \sum\sum\sum\limits_{i\neq j\neq k} X_iX_jX_k|N)$$

$$E(\xi^3|N) = N \ E(\Sigma \ X_i^3) + 3N(N - 1) \ E(X_i X_j (i \neq j))$$

$$+ \ N(N - 1)(N - 2)E(X_i X_j X_k (i \neq j \neq k))$$

$$E(\xi^3|N) = N\alpha_3 + 3N(N - 1)\alpha_2 m + N(N - 1)(N - 2)m^3$$

6.5.3.2 $\quad E(\xi^3|Q) = E(N\alpha_3 + 3N(N - 1)\alpha_2 m + N(N - 1)(N - 2)m^3|Q)$

$$E(\xi^3|Q) = nQ\alpha_3 + 3n^2Q^2\alpha_2 m + n^3Q^3m^3$$

6.5.3.3 $\quad E(\xi^3) = n\alpha_3 + 3n^2 m\alpha_2(1 + V_q) + n^3 m^3 E(Q^3)$

6.5.3.4. $\quad E((\xi - E(\xi))^3) = E \ (\xi^3) - 3E(\xi^2)E(\xi) + 2(E(\xi))^3$

$$= n\alpha_3 + 3n^2 m\alpha_2(1+V_q) + n^3 m^3 E(Q^3)$$

$$-3(n\alpha_2 + n^2 m^2(1+V_q))nm + 2 \ n^3 m^3$$

$$E((\xi - E(\xi))^3) = n\alpha_3 + 3n^2 m\alpha_2 V_q + n^3 m^3 (\hat{E}((Q-1)^3))$$

6.6 A special case of the model considered in Section 6.1 occurs when $Q = 1$ (i.e. where no fluctuation in the basic probability of claim is assumed). In this case ξ is said to have the simple generalised Poisson distribution and the results of Section 6.5 become

$E(\xi) = nm$	for $Q = 1$
$E(\xi^2) = n\alpha_2 + n^2 m^2$	for $Q = 1$
$Var(\xi) = n\alpha_2$	for $Q = 1$
$E(\xi^3) = n\alpha_3 + 3n^2 m\alpha_2 + n^3 m^3$	for $Q = 1$
$E((\xi^3 - E(\xi))^3) = n\alpha_3$	for $Q = 1$

6.7 The effect of fluctuations in the basic probability of claim can be studied by comparing the moments computed in Section 6.5 with the moments computed in Section 6.6. It can be seen that allowing for

6.7 (continued)

such fluctuations will increase the second and third moments of the total amount claimed. For example the variance increases from $n\alpha_2$ to $n\alpha_2 + n^2m^2V_q$ when fluctuations in the basic probabilities are considered. If n is large then this part of the variation will come to dominate other sources of fluctuation in the total amount claimed.

6.8 Section 6.5 explained how the moments of the compound generalised Poisson distribution may be calculated. If ξ_1, ξ_2 etc. are compound generalised Poisson variates (e.g. total amounts claimed in successive years or in separate branches) then quantities of interest will be of the form $\Sigma\ c_i\xi_i = \zeta$. Under suitable assumptions the moments of ζ can be calculated, for example if the joint distribution of ξ_i are known. This is done for one special case in Section 6.9. In Section 6.10 it will be assumed that the moments of ζ are known so that $\mu_1 = E(\zeta)$, $\sigma^2 = Var(\zeta)$, $\mu_3 = E((\zeta - E(\zeta))^3)$. Section 6.10 will also hold in the special case where $\zeta = \xi$.

6.9 In this Section 6.9 a special case of the process put forward in Section 6.8 above is considered. It is supposed that there are K variates $\xi_1,\ldots,\xi_K$ (which may correspond to the claims from the K branches) and that the first three moments of ξ_k are (as in Section 6.5) such that

$$E(\xi_k|Q) = Qm_k n_k$$

$$Var(\xi_k|Q) = Qn_k\alpha_{2k}$$
$$E((\xi_k - E(\xi_k))^3) = Qn_k\alpha_{3k} \ .$$

It is supposed in this Section 6.9 that Q, the underlying claims probability variation parameter (first introduced in Section 6.1) is identical over all k. In this case we have

$$E(\Sigma\xi_k|Q) = Q\ \Sigma\ m_k n_k = Q\beta_1 \quad \text{say}$$

$$Var(\Sigma\ \xi_k|Q) = Q\ \Sigma\ n_k\alpha_{2k} = Q\beta_2 \quad \text{say}$$

$$E((\Sigma\ \xi_k - E(\Sigma\ \xi_k))^3|Q) = Q\Sigma\ n_k\alpha_{3k} = Q\beta_3 \quad \text{say}$$

So

$$E(\Sigma\xi_k|Q) = Q\beta_1$$

$$E((\Sigma\xi_k)^2|Q) = Q\beta_2 + Q^2\beta_1^2$$
$$E((\Sigma\xi_k)^3|Q) = Q\beta_3 + 3Q\beta_1(Q\beta_2 + Q^2\beta_1^2) - 2\beta_1^3Q^3 = Q\beta_3 + 3\beta_1\beta_2 Q^2 + Q^3\beta_1^3$$
So $E(\Sigma\xi_k) = \beta_1$
$$E((\Sigma\xi_k)^2) = \beta_2 + \beta_1^2(1+V_q)$$

6.9 (continued)

$$E((\Sigma\ \xi_k)^3) = \beta_3 + \beta_1\beta_2 3(1 + V_q) + \beta_1^3 E(Q^3)$$

Therefore

$$E(\Sigma\xi_k) = \beta_1 = \Sigma\ m_k n_k$$

$$Var(\Sigma\ \xi_k) = \beta_2 + \beta_1^2(1 + V_q) - \beta_1^2 = \beta_2 + \beta_1^2 V_q$$

$$Var(\Sigma\ \xi_k) = \Sigma\ n_k\alpha_{2k} + V_q(\Sigma\ n_k m_k)^2$$

$$E((\Sigma\ \xi_k - E(\Sigma\ \xi_k))^3) = \beta_3 + 3\beta_1\beta_2(1 + V_q) + \beta_1^3 E(Q^3)$$
$$- 3\beta_1(\beta_2 + \beta_1^2)(1 + V_q) + 2\beta_1^3$$

$$= \beta_3 + 3\beta_1\beta_2 V_q + \beta_1^3(E((Q - 1)^3))$$

$$E((\Sigma\ \xi_k - E(\Sigma\ \xi_k))^3) = \Sigma\ n_k\alpha_{3k} + 3(\Sigma\ n_k\alpha_{3k})(\Sigma\ n_j\alpha_{2j})V_q$$
$$+ (\Sigma\ m_k n_k)^3(E((Q - 1)^3))$$

6.10 ζ was introduced in Section 6.8. An approximate distribution for ζ, if its moments are known, will be derived below.

Suppose $\mu_1 = E(\zeta)$, $\sigma^2 = Var(\zeta)$, $\mu_3 = E((\zeta - E(\zeta))^3)$.

Let $\psi(s) = E(e^{is\zeta}) = e^{is\mu_1}E(e^{is(\zeta - \mu_1)})$

$\psi(s) = e^{is\mu_1}(1 + \frac{(is)^2}{2}\sigma^2 + \frac{(is)^3}{3!}\mu_3 + ...)$ if the moments all exist.

$\psi(s) = e^{is\mu_1}\exp \log (1 - \frac{\sigma^2 s^2}{2!} + \frac{(is)^3}{3!}\mu_3 + ...)$

$\psi(s) = e^{is\mu_1 - s^2\sigma^2/2}(1 + \frac{(is)^3}{3!}\mu_3 + ...)$ under suitable conditions on the moments within some range of s.

6.10 (continued)

Suppose ζ has density function f and distribution function F.

Then
$$f(x) = \frac{1}{2\pi} \int \psi(s)e^{-isx}\, ds$$

$$= \frac{1}{2\pi} \int e^{is\mu_1 - s^2\sigma^2/2} e^{-isx}(1 + \frac{(is)^3}{3!}\mu_3 + \ldots)\, ds$$

$$= (1 - \frac{\mu_3}{3!}\frac{d^3}{dx^3})\, \frac{1}{2\pi} \int e^{is\mu_1 - s^2\sigma^2/2}\, ds + \ldots$$

So
$$F(x) = (1 - \frac{\mu_3}{3!}\frac{d^3}{dx^3})\, \Phi(\frac{x - \mu_1}{\sigma}) + \ldots$$

$$F(x) = \Phi(\frac{x - \mu_1}{\sigma}) - \frac{\mu_3}{6\sigma^3}\Phi^{(3)}(\frac{x - \mu_1}{\sigma}) + \ldots$$

This expansion is known as the Edgeworth expansion.

6.11 Section 6.10 computed an approximate distribution for ζ given its first three moments. Suppose we denote by z_ϵ the amount such that the probability z_ϵ is exceeded by ζ is ϵ. In that case, by definition,

$$F(z_\epsilon) = 1 - \epsilon\ .$$

Similarly define y_ϵ so that

$$\Phi(y_\epsilon) = 1 - \epsilon\ .$$

Then
$$\Phi(y_\epsilon) = 1 - \epsilon$$

$$= F(z_\epsilon)$$

$$= F(\ (\mu_1 + \sigma y_\epsilon) + (z_\epsilon - (\mu_1 + \sigma y_\epsilon))\ )$$

$$= F(\mu_1 + \sigma y_\epsilon) + (z_\epsilon - (\mu_1 + \sigma y_\epsilon))F^{(1)}(\mu_1 + \sigma y_\epsilon) + \ldots$$

6.11 (continued)

$$\phi(y_\epsilon) = \phi(y_\epsilon) \;-\; \frac{\mu_3}{6\sigma^3}\, \phi^{(3)}(y_\epsilon) \;+\; \ldots$$

$$+\; (z_\epsilon - (\mu_1 + \sigma y_\epsilon))\, \frac{1}{\sigma}\, \phi^{(1)}(y_\epsilon) \;+\; \ldots$$

Therefore

$$z_\epsilon \simeq \mu_1 \;+\; \sigma y_\epsilon \;+\; \frac{\mu_3}{6\sigma^2}(y_\epsilon^2 - 1)$$

This approximation has been found to be good in practice when $M < 2.5\sigma$. This is normally the case. For very skew distributions with very high retention ratios (M/σ) the approximation may be checked for a given distribution for ζ by comparison with the $(1 - \epsilon)$ quantile obtained from the distribution.

7. JUSTIFICATION OF THE FORMULAE

The formulae that require justification are given in Section 3 and described in Section 4. The notation used is that described in Section 4 and summarised in Section 8.

7.1 E_{min} is supposed to ensure solvency over the next year. So E_{min} is defined conceptually by

$$Pr\{(1+i)(E_{min}+U) + \sqrt{1+i}((1+\lambda)P - x) \geq 0\} = 1 - \epsilon \ .$$

In this equation

i = interest assumed earned net of tax
U = the company's own capital and free reserves (including "hidden reserves")
λ = the security margin in the premiums
x = the total amount of claims during the next year net of reinsurance
P = net premium income net of expenses and reinsurance

7.2 The Finnish Supervisory Service requires the values

i = .05
λ = 0
ϵ = .01

and additionally imposes the constraint $E_{min} \geq M - U$ where M is the company's maximum net retention.

The conceptual definition of E_{min} now becomes

$$Pr\{1.05(E_{min}+U) + 1.0247(P-x) \geq 0\} = .99 \ .$$

In order to determine E_{min} from the conceptual definition it can be seen that it becomes necessary to consider the distribution of x.

7.3 Suppose that there are K branches and that the total amount of claims during the next year is X_k in respect of branch k. Then

$x = \sum\limits_{k=1} X_k$. Accordingly it becomes necessary to consider the

distribution of X_k.

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7.4 X_k has the compound generalised Poisson distribution described in Section 6. Section 6 refers to the compound generalised Poisson variate as ξ. In this section 7.4 we suppose that the model described in Section 6.1 is appropriate for X_k. Suppose n_k to be the number of expected claims in branch k and P_k the net premium income. As in Section 6.1 suppose V_q to be the variance of the underlying claim frequency parameter.

Then P_k/n_k is the expected amount of one claim. Suppose α_{k2} and α_{k3} are the second and third moments of the amount of one claim. Then, from Section 6.5 we obtain the first, second and third moments of X_k as

$$E(X_k) = P_k$$

$$Var(X_k) = n_k \alpha_{k2} + \frac{P_k^2}{n_k} V_q$$

$$E((X_k - E(X_k))^3) = n_k \alpha_{k3} + 3n_k P_k \alpha_{k2} V_q + P_k^3 E((Q-1)^3)$$

7.5 The Finnish Supervisory Service have issued instructions that it is desirable to treat the fluctuations in basic probabilities approximately. Instead of assuming (as in Section 7.4) that for a branch the mean of N has a distribution that is centred on n_k with variance $n_k^2 V_q$, the companies have been asked to assume that the mean of N is equal to $n_k(1+q_k)$ where q_k is a constant. On this simplifying assumption it follows from Section 6.6 that

$$E(X_k) = P_k(1+q_k)$$

$$Var(X_k) = n_k(1+q_k)\alpha_{k2}$$

$$E((X_k - E(X_k))^3) = n_k(1+q_k)\alpha_{k3}$$

7.6 The approximation given in Section 7.5 is justifiable to some extent on the following grounds.

7.6.1 It is possible to arrange a rough equivalence between the two schemes by comparing the capital required under each scheme. Assuming the normal distribution within this Section 7.6.1 and using the moments computed in Section 7.4 then the capital required for branch k alone would be

$$P_k + y_\varepsilon (n\alpha_{k2} + \frac{P_k^2}{n_k} V_q)^{\frac{1}{2}}$$

7.6.1 (continued)

But if the approximation described in Section 7.5 is used then the corresponding capital required is approximately

$$P_k(1+q_k) + y_\epsilon(n_k(1+q_k)\alpha_{k2})^{\frac{1}{2}} \ .$$

By equating these two expressions we can obtain an estimate of the q_k required. In particular if n is large we obtain the following approximation for q_k:

$$q_k \simeq y_\epsilon V_q^{\frac{1}{2}}. \quad V_q \text{ is defined in Section 6.1. Values of } q_k$$

between .2 and .4 are commonly used, although in special cases it can be higher. Examples of q_k are given in Section 4.2.

7.6.2 Unless some approximation on the lines of Section 7.5 is made it becomes very difficult to compute the moments of $x = \Sigma \ X_k$. The alternative assumption in Section 6.9 is hard to justify on practical grounds. The possibility of correlations between Q for different branches is a very real one (e.g. a weather episode may affect several branches, likewise a financial/ inflationary episode). It is desirable therefore that the reserve in respect of the fluctuation of the basic probability should be essentially additive across the branches. The approximation of Section 7.5 permits this additivity, and furthermore removes the link between the {X_k} so that they may be treated as independent variables. By this means the approximation of Section 7.5 permits the computation of the moments of x.

7.7 Using the assumption in Section 7.5 we may now regard {X_k} as a set of independent random variables. So we may now compute the first three moments of x. The derivation below uses characteristic functions.

Let $\phi(s) = E(e^{is(x-E(x))})$

Then $\phi(s) = 1 + \dfrac{(is)^2}{2!} \text{Var}(x) + \dfrac{(is)^3}{3!} E((x-E(x))^3) + \ldots$ under

suitable conditions on the moments of x.

7.7 (continued)

Now $\phi(s) = E(\prod_{k=1}^{K} e^{is(X_k - E(X_k))})$

So $\phi(s) = \prod_{k=1}^{K} E(e^{is(X_k - E(X_k))})$ since $\{X_k\}$ are independent

So $\phi(s) = \prod_{k=1}^{K} (1 + \frac{(is)^2}{2!} Var(X_k) + \frac{(is)^3}{3!} E((X_k - E(X_k))^3) + \dots)$

So $\phi(s) = 1 + \sum_k \frac{(is)^2}{2!} Var(X_k) + \sum_k \frac{(is)^3}{3!} E((X_k - E(X_k))^3) + \dots$

Therefore

$Var(x) = \sum_k Var(X_k)$

$E((x - E(x))^3) = \sum_k E((X_k - E(X_k))^3)$.

The moments of X_k are given in Section 7.5, and can be substituted in the formulae above. So

$E(x) = \sum_{k=1}^{K} (1+q_k)P_k$

$Var(x) = \sum_{k=1}^{K} n_k(1+q_k)\alpha_{k2} = \sigma^2$ in the notation of Section 4.2

$E((x - E(x))^3) = \sum_{k=1}^{K} n_k(1+q_k)\alpha_{k3} = \mu_3$ in the notation of Section 4.3.

7.8   Section 7.7 computed the moments of the total amount of claims x. From Section 6.11 the claims for which reserves must be adequate are then

$\sum(1+q_k)P_k + \sigma y_\epsilon + \frac{\mu_3}{6\sigma^2} \cdot (y_\epsilon^2 - 1)$

where P_k, σ, μ_3 are as defined in Section 4.2 while y_ϵ is defined in Section 6.11?

7.9 From Sections 7.2 and 7.5 we have

$$1.05(E_{min}+U) + 1.0247(P - (\Sigma(1+q_k)P_k + \sigma y_\epsilon + \frac{\mu_3}{6\sigma^2}(y_\epsilon^2-1))) = 0$$

So

$$E_{min} = .976(\Sigma q_k P_k + \sigma y_\epsilon + \frac{\mu_3}{6\sigma^2}(y_\epsilon^2-1)) - U$$

$$E_{min} = .976\Sigma q_k P_k + 2.270\sigma + .718.\frac{\mu_3}{\sigma^2}$$

since $y_\epsilon = 2.326$, since ϵ is .01 from Section 7.2.

This is almost equivalent to equation 3.2 described in Section 4.2.
(The writer has been able to find no explanation for the small
discrepancy in the coefficient of μ_3/σ^2). Equation 3.2 was

$$E_{min} = .976\Sigma q_k P_k + 2.270 + 0.714.\frac{\mu_3}{\sigma^2}$$

7.10 Equation 3.2 is somewhat complicated to apply in practice because of
the necessity to estimate μ_3 and σ^2. Accordingly in situations where
the reserve is substantially greater than E_{min} it is possible to
save the labour involved in calculating E_{min} by calculating an
approximation that is in general higher than E_{min}. This is outlined
in Section 7.11.

7.11 Let $\bar{n} = \Sigma(1+q_k)n_k$

Let $S(x)$ be the distribution function of a single claim, with mean m, second moment α_2, third moment α_3.

Then, following the assumption of Section 7.5, in the next year it is expected that there will be $\bar{n}$ claims with the average claim being of amount m, where $m = \Sigma P_k(1+q_k)/\bar{n}$. This actual process will now be contrasted with an alternative imaginary process.

An alternative imaginary process would expect $\dfrac{\bar{n}m}{M}$ claims, each claim

being for an amount M (where M is the maximum retention and satisfies $M \geq m$).

It is intuitive, although difficult to prove, that the alternative imaginary process is more dangerous than the actual claims process. For example for both processes the mean is $\bar{n}m$. For the actual process the variance is $\bar{n}\alpha_2$. For the imaginary process the variance is $\bar{n}mM$. Now

$$\bar{n}\alpha_2 = \bar{n}\int_0^M x^2\, dS(x) \leq M\bar{n}\int_0^M x\,dS(x) = M\bar{n}m.$$

So the variance of the actual process is less than the variance of the imaginary process.

The approximation of E_{min} proceeds via assuming the above imaginary process. The expected number of claims of size M is

$$\dfrac{\bar{n}m}{M} = \dfrac{1}{M}\Sigma P_k(1+q_k) = \tau \quad \text{in the notation of Section 4.3. If } \tilde{N} \text{ is the}$$

number of claims in the imaginary process then $\tilde{N}$ is a Poisson variate with mean τ. The claims incurred in the imaginary process are NM. If $y(\tau)$ is the smallest integer satisfying $\Pr\{N<y(\tau)\} \geq .99$ then the probability that N is less than $y(\tau)$ is at least 99%. So $y(\tau)M$ is a satisfactory cautious estimate of the annual total claim. The constraint $y(\tau) \geq 2$ is an assurance that two large claims, however unlikely, can both simultaneously be met. So $E = My(\tau) - P - U$ is a cautious estimate of the minimum fluctuation reserve defined in Section 7.1. This completes the justification of Formula 3.3 described in Section 4.3.

7.12 E_{max} is supposed to ensure solvency over the next five years. So E_{max} is defined conceptually by

$$\Pr\{(1+i)^r E_{max} + \sum_{t=1}^{r}(1+i)^{r-t+\frac{1}{2}}((1+\lambda)P-x_t) \geq 0 : 1 \leq r \leq 5\} = 1 - \epsilon.$$

7.12 (continued)

The notation is that of Section 7.1 and as in Section 7.2 the Finnish Supervisory Service requires the values

$i = .05$
$\lambda = 0$
$\epsilon = .01$

The conceptual definition of E_{max} can now be rewritten

$$E_{max} = Max\{E_r : (Pr\{(1.05)^r E_r + \sum_{t=1} (1+i)^{r-t+\frac{1}{2}}(P-x_t) \geq 0\} = .99)\}$$

7.13 In view of the assumption of Section 7.5 E_r is of the order of $r\Sigma q_k P_k$.

So generally E_5 above equals E_{max}.

So the equation defining E_{max} can be rewritten

$$Pr \{(1.05)^5 E_{max} + \sum_{t=1}^{5} (1.05)^{5-t+\frac{1}{2}}(P-x_t) \geq 0\} = .99.$$

7.14 The moments of x_t have been computed in Section 7.7. However we now require the moments of $\sum_{t=1}^{5} x_t(1.05)^{\frac{1}{2}-t} = T$ say. $\{x_t\}$ may be assumed to be independent random variables.

Now from Section 7.7

$$E(x_t(1.05)^{\frac{1}{2}-t}) = \sum_k (1+q_k)P_k(1.05)^{\frac{1}{2}-t}$$

$$Var(x_t(1.05)^{\frac{1}{2}-t}) = \sigma^2(1.05)^{1-2t}$$

$$E((x_t(1.05)^{\frac{1}{2}-t} - E(x_t(1.05)^{\frac{1}{2}-t}))^3) = \mu_3(1.05)^{1\frac{1}{2}-3t} .$$

We may also use the results of Section 7.7 concerning the moments of sums of random variables. This yields

7.14 (continued)

$$E(T) = \sum_k \sum_t (1+q_k)(1.05)^{\frac{1}{2}-t} P_k$$

$$Var(T) = \sum_t \sigma^2 (1.05)^{1-2t}$$

$$E((T-E(T))^3) = \sum_t \mu_3 (1.05)^{1\frac{1}{2}-3t}$$

Now

$$\sum_1^5 (1.05)^{\frac{1}{2}-t} = (1.05)^{\frac{1}{2}} a_{\overline{5}|} = 1.024695 \times 4.329486 = 4.436$$

$$\sum_1^5 (1.05)^{1-2t} = (1.05) \frac{a_{\overline{10}|}}{s_{\overline{2}|}} = 1.05 \times 7.72175 \div 2.05 = 3.955042$$

$$\sum_1^5 (1.05)^{1\frac{1}{2}-3t} = (1.05)^{1\frac{1}{2}} \frac{a_{\overline{15}|}}{s_{\overline{3}|}} = \frac{1.0759297 \times 10.379678}{3.1525} = 3.542522$$

So

$$E(T) = 4.436 \sum (1+q_k) P_k$$

$$Var(T) = 3.955\sigma^2 = (1.989\sigma)^2$$

$$E((T-E(T))^3) = 3.543\mu_3$$

7.15 The definition of E_{max} can be rewritten (from Sections 7.13 and 7.14)

$$Pr\{E_{max} + 4.436P - T \geq 0\} = .99$$

From Section 6.11 we obtain the $(1-\epsilon)$ quantile for T for which reserves must be adequate as

$$4.436 \sum (1+q_k P) + 1.989\sigma y_\epsilon + \frac{3.543}{3.955} \frac{\mu_3}{\sigma^2} \frac{1}{6} (y_\epsilon^2 - 1)$$

Therefore

7.15 (continued)

$$E_{max} = 4.436 \Sigma q_k P_k + 4.626\sigma + .658 \frac{\mu_3}{\sigma^2} \quad \text{since } y_\epsilon = 2.326.$$

This completes the justification of Formula 3.4 described in Section 4.4.

7.16 If in a typical year the mean number of claims is $\bar{n}$ then the average claim is $\dfrac{\Sigma P_k(1+q_k)}{\bar{n}}$ so that α_{k2} is of the order of $\left(\dfrac{\Sigma P_k(1+q_k)}{\bar{n}}\right)^2$

From Section 6.6 this implies σ is of the order of $\Sigma P_k(1+q_k)//\sqrt{\bar{n}}$

Thus the approximate formula of Section 4.5 is of the order

$$5\Sigma q_k P_k + \sigma.$$

This is close to the formula of Section 4.4. The writer can see no further justification for the formula 3.5 described in Section 4.5.

7.17 The formula applied by the Finnish Supervisory Service for fluctuation reserve transfers is

$$\Delta E_k = .05 E_k^0 + 1.0247((\bar{f}_k + a_k)P_k - X_k) \quad \text{as described in Section 4.6.}$$

This formula is applicable without difficulty provided that the constraints $E_k \geq 0$ for all k, $E_{min} \leq \Sigma E_k \leq E_{max}$ are not breached. In this case the effect is to remove both the profit due to investment income earned by the reserve and the deviation off expected profit (or loss) on underwriting from the reported profits. The profit and loss account will merely show the outgoings as $\Sigma(\bar{f}_k + a_k)P_k$ regardless of the actual claims experience.

If the result of using the formula of Section 4.6 would be to satisfy the constraint $E_{min} \leq \Sigma E_k \leq E_{max}$ but to breach the constraint $E_j \geq 0$ for some j then E_j is zeroed by proportionate transfers from branches l satisfying $E_l > 0$.

If the result of using the formula of Section 4.6 would be to breach the constraint $E_{min} \leq \Sigma E_k \leq E_{max}$ then the amount transferred is reduced so that the constraint is satisfied. So, for example, a succession of poor years would exhaust the fluctuation reserve and would after exhaustion of the fluctuation reserve come through as outgoings in the profit and loss account.

8. NOTATION

Where symbol best defined	Where symbol used	Symbol	Description of symbol
2.1	3,4,5,7,10	E_{min}	Lower limit of the fluctuation reserve
2.1	3,4,5,7,10	E_{max}	Upper limit of the fluctuation reserve
4.2	3,4,5,7	q_k	A constant for branch k
4.2	3,4,5,7	P_k	Net annual premium income for branch k
4.2	3,4,5,7	σ^2	Adjusted variance of annual total claims
4.2	3,4,5,7	μ_3	Adjusted third central moment of annual total claims
4.2	3,4,5,7	U	Free reserves
4.2	4,5,6,7,10	α_{k2}	Second moment of single claim amount
4.2	4,5,6,7,10	α_{k3}	Third moment of single claim amount
4.3	3,4,5,7	τ	Ratio of adjusted expected annual claims to retention
4.3	3,4,5,7	M	Retention
4.3	3,4,5,7	$y(\tau)$	Inverse Poisson variate
4.5	3,4,7	$\bar{n}$	Adjusted expected annual number of claims
4.5	3,4,5,7,10	n_k	Expected annual number of claims in branch k
4.6	3,4,7	E_k	Fluctuation reserve for branch k
4.6	3,4,7	ΔE_k	Change in E_k
4.6	3,4,5,7	E_k^0	Initial value of E_k
4.6	3,4,5,7	$\bar{f}_k$	Expected claims ratio
4.6	3,4,5,7	a_k	Constant addition to fluctuation reserve transfer for branch k
4.6	3,4,5,7	X_k	Annual total claims for branch k
6.1	6	N	Number of claims
6.1	6	$X_i \ 1 \leq i \leq N$	Single claim amount
6.1	6,7	m	Expected single claim amount
6.1	6,7	α_2	Second moment of single claim amount
6.1	6	n	Mean number of claims
6.1	6	Q	Underlying probability random variable
6.1	6,7	V_q	Variance of Q
6.1	6	ξ^q	Compound generalised Poisson variate
6.8	6	c_i	Arbitrary coefficient

8. NOTATION (continued)

Where symbol best defined	Where symbol used	Symbol	Description of symbol
6.8	6	ζ	Annual total claims for company
6.8	6	μ_1	Expectation of ζ
6.8	6	σ^2	Variance of ζ
6.8	6	μ_3	Third central moment of ζ
6.9	6	ξ_k	An indexed compound generalised Poisson variate
6.9	6	m_k	Expected single claim amount for branch k
6.9	6	β_1	Summed first moment
6.9	6	β_2	Summed second moment
6.9	6	β_3	Summed third moment
6.10	6	$\psi(s)$	Characteristic function of
6.10	6	f	Density function for
6.10	6	F	Distribution function for
6.10	6	ϕ	Distribution function for standard normal distribution
6.11	6	z_ϵ	$(1-\epsilon)$ quantile for
6.11	6	y_ϵ	$(1-\epsilon)$ quantile for standard normal distribution
7.1	7	i	Rate of interest
7.1	7	λ	Security margin in the premiums
7.1	7	x	Annual total claims for company
7.1	7	P	Annual net premium for company
7.2	6,7	ϵ	Probability of ruin
7.3	7	K	Number of branches
7.7	7	$\phi(s)$	Characteristic function of x
7.8	7	$\bar{n}$	Adjusted mean number of claims
7.11	7	$S(x)$	Distribution function for a single claim amount
7.11	7	$\tilde{N}$	Ratio of annual claims to retention
7.11	7	x_t	Total claims for company during year t
7.12	7	$E_r \quad 1 \leq r \leq 5$	Fluctuation reserve for next r years
7.14	7	T	Discounted total claims for next five years

9. EXPERIENCE

9.1 When the system came into force in Finland in 1953 the insurance companies were allowed to create "initial funds" from "hidden assets" within the upper and lower limits.

9.2 Figures for the fluctuation reserves actually held are not available to me, however it is believed that they are commonly in the range 50% to 100% of premium income.

9.3 The fluctuation reserves have, according to Pentikainen(1970) increased the financial standing and action potential of the insurance companies.

9.4 The implementation of the idea of fluctuation reserves was helped by the fact that actuaries in Finland had received training in applied risk theory.

9.5 From 1953 onwards there occurred a substantial increase in net retentions and a reduction in the proportion of premiums reinsured. For example in transport the ratio Reinsurance premiums/Total premiums fell from 70% in 1952 to 60% in 1955 and it is believed this fall is due to fluctuation reserves.

10. COMMENTS AND CRITICISMS

10.1 The method is not completely automatic. Determination of the upper
and lower limits requires estimation of the quantities $n_k, \alpha_{k2}, \alpha_{k3}$
introduced in Section 4.2. This requires some judgement and
experience and also requires some study of the experienced
distribution of claim amount so that α_{k2} and α_{k3} can be estimated.

This is a disadvantage since it generates work for the company. On
the other hand the fact that the method is not completely automatic
allows the reserve to be set with due regard to the circumstances
of the company's portfolio, and this is an advantage of the method.

10.2 The method does not specifically cover reinsurance inward, although
it may be amalgamated with direct business net of reinsurance.

10.3 A fluctuation reserve does not deal with long term trends. Long term
trends are best dealt with by alteration of premium rates in a
process of adaptive control.

10.4 The formulae (especially for E_{max}) assume a stable premium income
and there is apparently no flexibility with regard to projected
premium and claims growth.

10.5 The amount E_{min} is inadequate for a company to hold as its
fluctuation reserve since it would then have no cover against
fluctuations reducing its free reserves to below E_{min}, at which
point presumably the Supervisor may prevent new business being
written. So perhaps $2E_{min}$ would be a realistic minimum fluctuation
reserve from the company's point of view.

10.6 The method does not deal with stop-loss reinsurance, which is allowed
for separately.

10.7 The method used in Finland is scientifically based, using the
principles of risk theory. It is related to theory and has
apparently worked well in practice. The effect of the method has
apparently been to increase the security of policyholders and, by
virtue of the reduction in the reinsurance requirement, reduced
costs for policyholders and throughout the economy.

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