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THE THEORY OF SIMPLE AND COMPOUND INTEREST

AN EIGHTEENTH-CENTURY MANUSCRIPT

BY M. E. OGBORN, F.I.A.

Joint Actuary, The Equitable Life Assurance Society

ONE of the exhibits in the Exhibition at the Centenary Assembly of the Institute of Actuaries was a manuscript book, *The Practice of Interest*, by William Jones, which had just been bought for the Library, as it appeared to be of historical interest. Subsequent investigation has confirmed that opinion, and this note gives some account of it.

The author was William Jones, F.R.S. (1675–1749). He published an introduction to mathematics in 1706 which attracted the attention of Halley and of Newton, whose friend he became. He was elected Fellow of the Royal Society in 1712 and later became Vice-President of that Society.

The Institute Library contains a collection of Newton's papers published in 1711 under the title *Analysis per quantitatum series, fluxiones, ac differentias: cum enumeratione linearum tertii ordinis*, which was collected and edited by William Jones and which, incidentally, is wrongly ascribed to him as author in the 1935 catalogue. The collection may have had a connexion with the Liebnitz controversy.

The manuscript work is clearly referred to by John Robertson, F.R.S., in a letter which was read before the Royal Society on 13 December 1770, reprinted by F. Maseres in *Scriptores Logarithmici*, Vol. v. Robertson refers to the original mathematical work of William Jones on compound interest, and states that he 'did, many years ago, cause to be engraved on a copper plate more cases than had been exhibited before that time'. The work appears to be written, not engraved, in a copper-plate handwriting, and was presumably made by the engraver or by a professional writer. The handwriting is not that of Jones, Robertson or Dodson. A reference No. 3 appears on the end page.

The author deals with the relationships between the five quantities, the amount of an annuity, the period and the rate of interest, the present value and the accumulated sum of the payments. The problem is determined when any three of these five quantities are known. There are ten possible selections of three out of the five quantities and, in twenty cases, the author gives the formulae for the fourth and fifth quantities for all the ten possible combinations of the three given quantities. The formulae for the cases when the rate of interest has to be determined depend upon an approximation which works remarkably well provided that the period is not too long. We may illustrate the method by Case 20, for the rate of interest in an annuity-certain.

Let $G^{(n+1)/2} = n/a_{\overline{n}|}$

and $H = 6/(n-1)$

and $K = \sqrt{\{H(H-2.\overline{G-1})\}}$.

Then the rate of interest is given by

$$i = H - K.$$

It is interesting to notice that the author uses the symbol r for i and not for $(1+i)$, as was the custom later.

The method may be compared with the similar, but more powerful, method of Prof. Dr J. F. Steffensen, in *A formula for the rate of interest in an annuity-certain*, *J.I.A.* Vol. L, pp. 49–54. Dr Steffensen points out that Jones's formula comes simply to this. If $[n/a_{\overline{n}|}]^{2/(n+1)}$ be expanded in terms of i we find that the term i^3 is missing and that

$$G=[n/a_{\overline{n}|}]^{2/(n+1)}=1+i-\frac{n-1}{12}i^2$$

plus a remainder term involving fourth and higher powers of i. Jones's formula gives the solution of this quadratic when the remainder term is neglected.

The following table computed by G. V. Bayley, F.I.A., compares the numerical results (*a*) by the above formula and (*b*) by Steffensen's formula.

n	Value of approximation to $100i$		
	$i=\cdot02$	$i=\cdot04$	$i=\cdot06$
20	(*a*) 2·000	(*a*) 4·002	(*a*) 6·011
	(*b*) 1·999	(*b*) 3·998	(*b*) 5·999
40	(*a*) 2·001	(*a*) 4·016	(*a*) 6·091
	(*b*) 2·000	(*b*) 4·005	(*b*) 6·016
60	(*a*) 2·003	(*a*) 4·060	
	(*b*) 2·001	(*b*) 4·012	
80	(*a*) 2·008		
	(*b*) 2·003		
100	(*a*) 2·016		
	(*b*) 2·004		

The manuscript also gives the corresponding formulae for simple interest.

The author seems to have given general permission for his work to be published in logarithmic tables, and his twenty cases were, with acknowledgment to him, included in the prefaces to Gardiner's *Tables of logarithms for all numbers from* 1 *to* 102100 and to Dodson's *Anti-logarithmic Canon*, both published in 1742. Jones's work would have been written some years earlier. He does not seem to have published it himself but only to have circulated it among his friends. A page is reproduced to show the quality of the manuscript and the table of cases is also reproduced.

Rules for ye Annuities

Let a = ye Annuity, r = Rate of 1l per Ann., n = Number of equal Times, m = ye Amount, s = Present worth. L = Logarithm. l = its ar: com:

Let $x = n-1$, $z = n+1$, $b = r \times \overline{2s+a} - 2a$, $d = 2a - ar$. $E = \frac{3}{\frac{1}{2}n+1}$, $H = \frac{3}{\frac{1}{2}n-1}$.

$L.m + l.s$ or $nL.\overline{r+1} = L.A$ & $n\,L.A = L.B$. $L.\overline{A-1} + L.a + l.m = L.C$.

$\frac{L.m + l.a + l.n}{\frac{1}{2}n-1} = L.D$, $\frac{L.a + L.n + l.s}{\frac{1}{2}n+1} = L.G$.

	Given	Req.	Solutions: ye Simple Int.	Solutions: Compound Interest.
1	a, n, r	$m=$	$\overline{\frac{1}{2}xr+1} \times na$	$L.m = L.\overline{A-1} + L.a + l.r$
2	s, n, r		$\overline{rn+1} \times s$	$L.m = nL.\overline{r+1} + L.s$
3	a, s, r		$\frac{s}{2a} \times \sqrt{\overline{b^2+8ars}} + r \times \overline{2s+a}$	$L.m = l.\overline{a-sr} + L.s + L.a$
4	a, s, n		$\frac{saz}{2s-ax}$	$L.m = nL.\overline{H-K+1} + L.s$
5	a, n, r	$s=$	$\frac{\overline{\frac{1}{2}xr+1} \times an}{nr+1}$	$L.s = L.\overline{A-1} + L.a + l.A + l.r$
6	m, n, r		$\frac{m}{nr+1}$	$L.s = L.m - nL.\overline{r+1}$
7	a, m, n		$\frac{max}{2m-az}$	$L.s = L.m - nL.\overline{F-E+1}$
8	a, m, r		$\frac{2ma}{\sqrt{\overline{2a-ra}^2+8amr}+ra}$	$L.s = l.\overline{mr+a} + L.a + L.m$
9	s, n, r	$a=$	$\frac{\overline{nr+1} \times s}{\overline{\frac{1}{2}xr+1} \times n}$	$L.a = l.\overline{A-1} + L.A + L.s + L.r$
10	s, m, n		$\frac{2ms}{mx+sz}$	$L.a = l.\overline{A-1} + L.B\ 1 + L.m$
11	m, n, r		$\frac{m}{\overline{\frac{1}{2}xr+1} \times n}$	$L.a = l.\overline{A-1} + L.m + L.r$
12	s, m, r		$\frac{2sm \times rs}{\overline{m+s-rs} \times ms}$	$L.a = l.\overline{A-1} + L.m + L.r$
13	s, m, r	$n=$	$\frac{m-s}{rs}$	$n = \frac{L.m + l.s}{L.\overline{r+1}}$
14	a, s, r		$\frac{\sqrt{b^2+8ars}-b}{2ar}$	$n = \frac{l.\overline{a-sr} + L.a}{L.\overline{r+1}}$
15	a, s, m		$\frac{\overline{m-s} \times a + 2sm}{\overline{m+s} \times a}$	$n = \frac{L.m + l.s}{L.\overline{C+1}}$
16	a, m, r		$\frac{\sqrt{d^2+8amr}-d}{2ar}$	$n = \frac{L.\overline{mr+a} + l.a}{L.\overline{r+1}}$
17	a, m, s	$r=$	$\frac{\overline{m+s} \times \overline{m-s} \times a}{\overline{2m-a} \times s + ma \times s}$	$L.r = L.\overline{A-1} + L.a + l.m$
18	s, m, n		$\frac{m-s}{sn}$	$L.\overline{r+1} = \frac{L.m + l.s}{n}$
19	a, m, n		$\frac{m-na}{\frac{1}{2}xna}$	$\frac{L.2\overline{D-1+E} + L.E}{2} = L.F$ & $F - E = r$
20	a, s, n		$\frac{na-s}{\overline{s-\frac{1}{2}xa} \times n}$	$\frac{L.\overline{H-2\overline{G-1}} + L.H}{2} = L.K$ & $H - K = r$

Compound Interest
Case 19.

Given a, m, n: Requir'd r

Rule $\quad \dfrac{L,2\overline{D-1}+E + L,E}{2} = L,F$ & $F-E = r$

Example: If an Annuity of 357£ pr An forborn 7 Years at Compound Interest Amounts to 29966£ What Rate pr Cent An is Reckon'd?

$$
\begin{array}{rll}
L,m = & 3,4766285 & 29966 = m \\
L,a = & 7,4473318 & 357 = a \\
L,n = & 9,1549020 & 7 = n \\
\overline{n-1} = 3) & 0,0788623 \,(\, 0,0262874 = L,D &
\end{array}
$$

$$
\begin{array}{rl}
D-1 = & ,0624 \\
2D-1 = & ,1248 \\
E = & ,75 \\
2D-1+E = & ,8748
\end{array}
\qquad
\begin{array}{rl}
 & 1,8750613 = L,E \\
 & 1,9419088 = L,2\overline{D-1}+E \\
2) & 1,8169701 \,(\, 1,9084850 = L,F \\
F = & 0,81 \\
E = & 0,75 \\
F-E = & 0,06 = r
\end{array}
$$

Case 20.

Given a, s, n: Requir'd r

Rule $\quad \dfrac{L,H-2\overline{G-1} + L,H}{2} = L,K$ & $H-K = r$

Example: If an Annuity of 357£ pr An to Continue 7 years be sold for 1992,907£ Ready Mony what Rate of Comp: Int: pr Cent. pr Anm is Allow'd?

$$
\begin{array}{rll}
a = 357 & & 2,5526682 = L,a \\
n = 7 & & 0,8450980 = L,n \\
s = 1992,907 & & 6,7005126 = L,s \\
 & \overline{n+1} = 4) & 1,0982788 \,(\, 0,0245679 = L,G \\
 & G-1 = ,0582 & \\
 & 2\overline{G-1} = ,1164 & 1,9462557 = L,\overline{H-2\overline{G-1}} \\
 & H-2\overline{G-1} = ,8836 & 0,0000000 = L,H \\
 & 2) & 1,9462557 \,(\, 1,9731278 = L,K
\end{array}
$$

$$
\begin{array}{rl}
H = & 1 \\
K = & 0,94 \\
H-K = & 0,06 = r
\end{array}
$$