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THE TREATMENT OF APPRECIATION OR DEPRECIATION IN THE ASSETS OF A LIFE ASSURANCE FUND

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I HAVE limited this paper to the consideration of appreciation or depreciation in fixed interest-bearing securities resulting from a fall or rise in the general level of interest rates and not from any alteration in the intrinsic value of the securities, i.e. there is assumed to be no change in the probabilities that the terms of the contracts will be fulfilled. My object is to discuss how this appreciation or depreciation should be treated in order to obtain the maximum practicable degree of equity between the different generations of policyholders in an office which distributes its surplus by means of a uniform reversionary bonus.

The problem can be most easily discussed by showing the effect on a model office of adopting various methods of treating this appreciation or depreciation. I propose to limit the discussion in the first place to the question of depreciation in an office employing a bonus reserve valuation.

MODEL OFFICE

In choosing the model office to be used it must be remembered that the effects of depreciation on a life office will depend principally upon the relative distributions of the maturity dates of the liabilities and the assets. It is not, therefore, of primary importance that the model office should correspond with any office which is likely to be found in practice, provided that it is tested with several funds with widely differing distributions of assets, so that all possible *relations* between the distributions of the liabilities and assets are considered. I have, therefore, chosen the very simple case of an office in a stationary position which issues on 31 December in each year l_{25} policies of 1, each policy being a 40-year endowment assurance, with profits, effected at age 25, since this reduces the work involved in making the necessary valuations and very greatly facilitates the calculation of the claims occurring in each future year (see Appendix).

I have assumed that

(1) expenses and miscellaneous sources of profit can be ignored,

(2) the premiums have always been calculated by the A 1924–29 ultimate table at $3\frac{1}{2}\%$, with a loading for a simple reversionary bonus of £2%, i.e. the annual premium for a sum assured of 1 is ·02376,

(3) the mortality experienced has always followed exactly the A 1924–29 ultimate table and will continue to do so,

(4) the interest earned on the fund has always been $3\frac{1}{2}\%$ net,

(5) quinquennial valuations have always been made by the bonus reserve method, using the A 1924–29 ultimate table at $3\frac{1}{2}\%$, valuing a future rate of bonus of K = £2%, a bonus at rate k = £2% having always been declared and an interim bonus at the same rate having been allowed on all claims,

(6) claims by death are paid at the end of the year of death,

(7) immediately before the valuation under consideration was due to be

made there was a rise of $\frac{1}{2}\%$ in the general level of interest rates, taking into consideration the redemption terms, with consequent depreciation of the fund and increase in the rate of interest from $3\frac{1}{2}\%$ to 4%, making allowance for redemption. (It is assumed that the same rate of interest is being obtained on all the investments, irrespective of their dates of maturity. This would not, of course, be the case, but the point does not materially affect the present discussion, provided that the increase in the rate of interest is the same for all terms. This seems to be a not unreasonable assumption.)

The assumed office is in a stationary condition, while most offices are expanding (although this expansion has been checked during the war years), and further, all offices issue some whole-life assurances at young ages and children's deferred assurances (the high cash option under the latter type of business meaning, however, that when considering the distribution of the maturity dates of the liabilities the policies must be treated during the deferred period as maturing after a fairly short term). On the other hand, a large proportion of the business and a very large proportion of the premium income will be derived from 10- to 25-year endowment assurances and from whole-life assurances at more advanced ages. On balance, therefore, the assumed office would appear to have liabilities maturing after at least as long a period as those of a normal office.

In view of the references in previous discussions of this subject to the special case in which the assets exactly match the liabilities, so that the claims by death and maturity in each year are met by the maturing assets, it is interesting to consider the distribution of the assets in the assumed fund which would produce this result and the distribution of the maturity dates of the new investments made each year which would be required to maintain this position. If the assets are to match the liabilities exactly, then at any given moment the assets maturing in any particular future year must be equal to the present value of the sums assured and bonuses under policies which will become payable in that year, whether by death or maturity, less the present value of the premiums under these policies, both valued by interest factors only. For an ordinary office the calculation of the present value of the emerging cost would be a laborious process, especially as future bonuses must be allowed for, but for the assumed office it is relatively simple (see Appendix) and the percentage distribution of the assets necessary under the assumed fund is shown below.

No. of years to maturity of investment	Percentage of total investments	Percentage of new investments each year
1/5	31	23
6/10	24	19
11/15	17	16
16/20	13	13
21/25	8	11
26/30	5	9
31/35	2	9
36/40	—	—

It will be noticed that although the liabilities consist of 40-year endowment assurances, no assets maturing in 36–40 years are required. This is, of course, explained by the fact that the present value of the sums assured and bonuses which will be payable on policies becoming claims at the end of 38, 39 and 40 years is less than the present value of the premiums under such policies, the resulting negative values being more than sufficient to offset the reserves required for policies becoming claims at the end of 36 and 37 years. It is, therefore, only possible to assume that no assets maturing at the end of 36 or 37 years will be held, since the total assets calculated as above must equal the total valuation reserve.

It will be seen from the figures in the third column that if a reasonable rate of interest is to be obtained, then in the assumed office or in any probable office (except a new or very rapidly expanding office) it would be impossible to make new investments each year the maturity dates of which were so distributed as to maintain an exact matching of the liabilities and assets, since only a very low yield could be obtained on the large proportion of the investments which it would be necessary to make in short-term securities. It follows that in the special case where the assets exactly correspond with the liabilities, the assets have maturity dates which are earlier relatively to the maturity dates of the liabilities than in any case which is likely to occur in practice, and this distribution is called case I in the examples which follow. On the other hand, in a fund which consists only of 30-year investments, the maturity dates of the assets would be later relatively to the maturity dates of the liabilities than in any case which is likely to occur, and this distribution is called case IV. The intermediate cases, where the fund consists only of 20-year investments and where it is equally divided between 10-, 20- and 30-year investments, are described as cases II and III respectively.

METHODS OF TREATING DEPRECIATION

Four methods (or a compromise between them) are available to the assumed office to meet the depreciation which has occurred (see assumption (7) above). Since a bonus reserve valuation is employed, the assets must be written down to their new market values and the valuation rate of interest increased from $3\frac{1}{2}\%$ to 4%. The value of the liabilities must then be equated to the value of the assets, this equality being obtained by varying the rates chosen for K, the future rate of bonus valued as a liability, and for k, the rate of bonus to be declared for the quinquennium just completed.

Method A. Take K as the rate of bonus which the existing premium scale will support under the new conditions (in the assumed office £2. 10*s*. 7*d*.%). This will ensure that policies effected after the depreciation has occurred will receive absolutely equitable treatment without any alteration having to be made in the general level of premiums charged (there will, of course, have to be minor changes in the premiums for new policies to ensure equity between the various types of assurances, since the effect of a change of the rate of interest varies with the term), and hence that there will be no material difference in the treatment of policies effected immediately before the depreciation compared with the treatment of policies effected after the depreciation.

Method B. Take k at the rate which could have been declared if the depreciation had not occurred (in the assumed office £2%). This can only be justified if it were agreed that, since all the investments are redeemable and there has been no change in the probability that the terms of the contract will be fulfilled, there has been a bookkeeping loss only and the profit-earning power of the office has not been affected.

Method C. Take K at the rate which has been used in previous valuations (in the assumed office £2%). If it were agreed that the profit-earning power of the office has not been affected, then it would be natural to assume that there would be no alteration in the future rate of bonus.

Method D. Take $k = K$. This is probably the most natural method to adopt under a bonus reserve valuation.

TABLES—WHOLE OFFICE

The fund before the depreciation occurred was equal to the reserves on the $3\frac{1}{2}\%$ basis, valuing a future bonus $K = £2\%$ as a liability, together with a surplus exactly sufficient to provide a bonus $k = £2\%$ for the quinquennium just completed. It was assumed that the fund was invested in assets yielding $3\frac{1}{2}\%$ and standing at par before the depreciation, and hence in each of the four cases the amount of the fund after the depreciation was calculated on a basis to yield 4% to redemption, using the distribution of the maturity dates of the assets described above, namely

Case I. Maturity dates of assets exactly matching liabilities.

Case II. All assets maturing in 20 years' time.

Case III. Maturity dates of assets equally divided between 10, 20 and 30 years' time.

Case IV. All assets maturing in 30 years' time.

The liabilities were then valued on a 4% basis. Under methods A and C the rate K was known and the rate k was found by equating the value of the liabilities to the amount of the fund in each of the four cases. Under method B the rate k was known and the rate K was found similarly. Under method D the rates k and K were both unknown, but were equal and thus easily found.

The rates were as follows:

Table 1

	Method A $K=£2.\ 10s.\ 7d.$	Method B $k=£2$	Method C $K=£2$	Method D $k=K$
Case I	$k=£1.\ 11s.\ 2d.$	$K=£2.\ 7s.\ 6d.$	$k=£3.\ 1s.\ 3d.$	$k=K=£2.\ 5s.\ 6d.$
Case II	$k=$ 16s. 5d.	$K=£2.\ 2s.\ 3d.$	$k=£2.\ 6s.\ 6d.$	$k=K=£2.\ 1s.\ 8d.$
Case III	$k=$ 18s. 0d.	$K=£2.\ 2s.\ 10d.$	$k=£2.\ 8s.\ 1d.$	$k=K=£2.\ 2s.\ 1d.$
Case IV	$k=$ 6s. 4d.	$K=£1.\ 18s.\ 9d.$	$k=£1.\ 16s.\ 6d.$	$k=K=£1.\ 19s.\ 1d.$

Table 2 shows the depreciation which occurred in each of the four cases and, for each case, the reduction in the surplus available by each of the four methods to provide a bonus k in respect of the quinquennium just completed, this reduction in the surplus being the net result of the decrease in the liability owing to the increase in the valuation rate of interest and the decrease in the value of the assets owing to the depreciation. All the figures have been divided by 1000 for convenience. Negative figures, i.e. increases in the surplus available, are shown in italics.

The surplus which would have been available on the $3\frac{1}{2}\%$ basis to provide the bonus of £2% which could have been declared if the depreciation had not occurred was (also divided by 1000) 18,923.

Table 2

	Depreciation in fund	Reduction in surplus available to provide k			
		Method A	Method B	Method C	Method D
Case I	9,703	5,185	1,283	*−8,091*	*−1,142*
Case II	16,211	11,693	1,283	*−1,583*	548
Case III	15,508	10,990	1,283	*−2,286*	364
Case IV	20,631	16,113	1,283	2,837	1,687

If the reserve is defined as the value of the sum assured and vested bonuses, plus the value of the future bonuses K valued as a liability, minus the value of future premiums, no allowance being made for the bonus k in respect of the quinquennium just completed, then:

Under method A the reserve is the same in each of the four cases. The increase in the valuation rate of interest and the increase in the future bonus K valued as a liability tend to offset one another and the net result is a reduction of 4518 in the reserve, the balance of the depreciation being met out of the surplus available to declare the bonus k for the quinquennium just completed.

Under method B there is a constant reduction of 1283 in the surplus absorbed in declaring a £2% bonus for the quinquennium just completed (the increase in the rate of interest from $3\frac{1}{2}\%$ to 4% reducing the cost of the bonus by this amount), and the balance of the depreciation is thrown on to the reserve and is met by varying the rate of future bonus K valued as a liability.

Under method C the increase in the valuation rate of interest results in a constant reduction of 17,794 in the reserve. In three of the four cases this reduction in the reserve exceeds the depreciation, and the surplus available to provide the bonus k for the quinquennium just completed is increased.

Under method D a constant *proportion* of the depreciation less the decrease in the reserve *excluding the value of future bonuses* K is borne by the surplus available to provide k. In case I the reduction in the reserve excluding the value of future bonuses K is greater than the depreciation, and the surplus available to provide k is increased.

TABLES—INDIVIDUAL AGE GROUPS

Three individual age groups 60, 45 and 30 were taken and treated as separate offices. On the assumptions made in choosing the model office the fund for each age group before the depreciation occurred was equal to the valuation reserve of the group on the $3\frac{1}{2}\%$ basis, valuing as a liability a future bonus K = £2% and allowing for the declaration of a bonus k = £2% for the quinquennium just completed. It was assumed that in case I each of these funds was invested in assets with maturity dates exactly matching the maturing liabilities, whether by death or maturity, of the age group. In cases II and IV it was assumed that the funds were invested in assets maturing in 20 and 30 years' time respectively. In case III it was assumed that the fund of age group 60 was invested in assets maturing in 10 years' time, of age group 45 in assets maturing in 20 years' time, and of age group 30 in assets maturing in 30 years' time. It will be seen that the assumptions made above are consistent with those made for the whole office. The fund available for each age group after the depreciation was then calculated in each case on the basis of a yield of 4% to redemption.

For each age group the value of the sum assured and vested bonus less the value of future premiums (i.e. the reserve as defined previously, but excluding the value of future bonuses K) was calculated and this reserve excluding the liability for future bonuses was deducted in each case from the fund (calculated as described in the previous paragraph) for the age group. The balance of the fund was then available to provide bonuses k and K.

The cost of providing the rates of bonus k and K already found for the whole office and shown in Table 1 was then calculated on the 4% basis for each age group.

Table 3 shows for each age group the result (divided by 1000) of deducting this cost from the balance of the fund available.

Table 3

	Age group	Method A	Method B	Method C	Method D
Case I	60	38	*−68*	*−321*	*−133*
	45	*−52*	*−36*	4	*−24*
	30	41	104	258	146
Case II	60	*−250*	*−530*	*−608*	*−550*
	45	72	118	128	121
	30	149	321	366	332
Case III	60	34	*−229*	*−325*	*−253*
	45	54	96	111	100
	30	113	272	329	287
Case IV	60	*−295*	*−696*	*−655*	*−686*
	45	77	140	133	138
	30	195	437	411	430

Table 4 shows for each age group the ratio of the balance of the fund available to this cost.

Table 4

	Age group	Method A	Method B	Method C	Method D
Case I	60	1·03	·94	·78	·90
	45	·98	·98	1·00	·99
	30	1·02	1·05	1·12	1·07
Case II	60	·72	·54	·51	·54
	45	1·03	1·06	1·06	1·06
	30	1·07	1·16	1·19	1·17
Case III	60	1·04	·80	·74	·79
	45	1·03	1·05	1·05	1·05
	30	1·05	1·13	1·17	1·14
Case IV	60	·59	·38	·39	·38
	45	1·04	1·07	1·07	1·07
	30	1·09	1·24	1·22	1·23

DEDUCTIONS FROM MODEL OFFICE

A negative figure in Table 3 or a ratio of less than 1 in Table 4 shows that the cost of the bonuses k and K allotted to that age group is greater than the surplus available in that age group to provide these bonuses, i.e. that the age group is receiving too favourable treatment. Conversely, a positive figure in Table 3 or a ratio greater than 1 in Table 4 shows that an age group is being unfavourably treated.

If the method adopted were completely equitable as between the different age groups, the fund available for each age group would be exactly equal to the value of the liabilities, including the values of future bonuses and of the bonus for the quinquennium just completed, using the rates of K and *k* found for the whole office, and the figures in Table 3 would therefore all be zero and the ratios in Table 4 all 1. Since none of the methods achieves this, the most equitable will be that which gives the results closest to zero and unity and in each case this is method A. It will be observed also that in all four cases the errors under methods B, C and D are in the same direction, i.e. they are always too favourable to the older age groups at the expense of the younger age groups, and this will apply to any conceivable distribution of the maturity dates of the assets and liabilities. Under method A, however, this is not the position in case I, so that for some distributions even greater equity might be obtained than in any of the cases chosen as examples.

The results of using method A in case I are interesting and at first sight surprising, since age groups 60 and 30 are unfavourably treated, but the intermediate age group 45 is favourably treated. In the case of age group 60, assets are held which will exactly match the claims if a bonus of £2% is declared throughout the term of the policy. As a result of the depreciation a bonus of £1. 11*s*. 2*d*.% has been declared for the quinquennium just completed and a bonus of £2. 10*s*. 7*d*.% will be allowed for the next quinquennium. Hence policies becoming claims by death will on the average require rather less than the amount of the assets maturing concurrently and policies becoming claims by maturity slightly more. There will, therefore, be little or no loss from this source and there will be a profit from the higher rate of interest obtained on the premiums and interest invested during the five years. In the case of age group 45 a fairly large reserve will be held, of which a considerable proportion will be the reserve for future bonuses, as a result, of course, of the fact that a constant profit loading is charged for a benefit of increasing value. This reserve for future bonuses has been calculated on the assumptions that the future bonuses will be at the rate of £2% and that the reserve has been invested at $3\frac{1}{2}$%. The loss from the increase in this future bonus is greater than the profit from the reduction in the bonus declared for the quinquennium just completed. In the case of age group 30, the reserve accumulated during the five years in respect of future bonuses at £2% is relatively small and the loss from this source is more than offset by the profit from the reduction in the bonus for the past quinquennium. It will be seen, therefore, that if the figures in the table had been calculated for all age groups, they would have started at a maximum for age group 64 and would have decreased with age until, having passed through zero, they would have become negative and reached a maximum numerical value; they would then have gradually increased through zero to positive values at the young age groups. In these circumstances it would appear that under a uniform reversionary bonus system no method could give absolutely equitable results and that method A is as equitable as any practicable method could be.

It is instructive to consider why methods B, C and D are unsatisfactory. One weakness is common to all the methods with the exception of method A, viz. that if the rates of premium charged for policies effected after the depreciation are not revised, these new policies will receive bonuses at a rate different from the rate which the premiums charged can support under the new conditions, while if the premiums for new policies are revised, inequity is introduced by the different treatment given to policies effected immediately prior to the depreciation compared with that given to policies effected after the depreciation.

The objections which apply more particularly to the various methods are as follows:

Method B. The justification of this method depends upon the acceptance of the argument that, if the fund is invested in redeemable securities, depreciation in the assets results only in a book loss and not in a real loss. This argument is not, however, sound unless the assets exactly match the liabilities so that the claims in each year, whether by death or maturity, are met by the maturing assets (i.e. as in case I, which explains the relatively satisfactory results obtained by method B in this case). If the assets do not exactly match the liabilities (and it has been argued previously that it is very unlikely in practice that they will) then either a real loss must be accepted immediately by writing down the book values to the market values or a real loss will emerge in the future if this is not done. In the latter event, either investments will have to be realized to meet the claims, which will involve a loss owing to the book values being higher than the market values, or the claims will have to be met partly out of current premium and interest income, which will mean in effect that some of the existing investments will be transferred from the policies becoming claims to the continuing policies at prices above the market values. In either case, therefore, there is a real loss, although in the second case (which would almost invariably occur in practice) it would be masked by the investment of new money on terms less favourable than those available in the open market.

It might be argued that although there is a real loss it does not occur until a policy becomes a claim and that it should be allowed to emerge at that time. If this is done, however, the whole of the loss is thrown on to the continuing policies and none is borne by the policy causing the loss.

The weakness of method B is shown in Table 2 and the remarks following the table, where it is pointed out that only a small, constant reduction is made in the surplus absorbed by the bonus k declared for the quinquennium just completed whatever the size of the depreciation, i.e. no distinction is made between a case where the investments all mature in 10 years' time and one where they all mature in 50 years' time. This is reflected in the fact that in case IV a lower rate of bonus will be declared in the future than in respect of the quinquennium just completed (see Table 1) in spite of the higher rate of interest which will be obtained on new investments in the future.

Method C is based on the argument that depreciation does not affect the profit-earning power of the office, but this is obviously not true. There is a real immediate loss from the depreciation, while in the future the profit-earning power of the office will be improved by the higher rate of interest obtainable.

It is obvious from Table 2 that the method is unsatisfactory, since under it the result of the depreciation in three of the four cases is an increase in the surplus.

Method D. By giving the same value to k and K this method accepts only a small proportion of the depreciation as an immediate loss, and spreads forward the balance of the loss over the future of all the existing policies, thus anticipating some of the profit which will be earned in the future as a result of the increase in the rate of interest obtainable.

The spreading forward of the loss is inequitable since it places the largest share of the loss on the policies with the longest unexpired term to maturity, although on the average these would be the policies with the smallest reserves at the date of the depreciation and consequently responsible for the smallest part of the loss.

It is unjustifiable to anticipate interest profit which will be earned in the future in order to increase the bonus k to be declared in respect of the quinquennium when in fact no excess interest has been received.

COMPARISON OF RESULTS UNDER BONUS RESERVE AND NET PREMIUM VALUATIONS

It is clear that of the methods considered under a bonus reserve valuation the most equitable is that which writes down the assets to their new market values, increases the valuation rate of interest to the new experience rate, and values as a liability a future bonus K, where K is the rate which the existing premium scale will support under the new conditions. It is now desirable to consider whether there is any method available to an office employing a net premium valuation which will give results at least equally equitable. In the discussion which follows it is assumed that a net premium valuation has been made in the past on a basis which has given results similar to, and has set up reserves equal to those under, the $3\frac{1}{2}\%$ bonus reserve valuation.

Three courses (or a compromise between them) are open to such an office:

(1) The office can write down its assets to their new market values without altering the basis on which its liabilities are valued. This will throw the whole depreciation on to the surplus available to declare the bonus k for the quinquennium just completed. It will be seen from Table 2 and the remarks following the table that under method A a considerable amount of the depreciation is absorbed by the decrease in the reserve. It follows, therefore, that if the method under discussion is adopted, a higher future rate of bonus will emerge than under method A, i.e. higher than the rate which the existing premium scale will support under the new conditions (in the model office the rates are £2. 14*s*. 2*d*.% and £2. 10*s*. 7*d*.% respectively). Policies entering after the depreciation will, therefore, be too favourably treated unless the premiums for new policies are increased. It is unlikely that an office would increase its premium scale as a result of an increase in the rate of interest obtainable, but even if it were to do so, then although new policies might not receive higher bonuses than their premiums would support, the policies which entered shortly before the depreciation would benefit at the expense of the older policies.

(2) The office can continue to value its liabilities on the same basis as before and ignore the reduction in the market values of its assets in so far as these are redeemable securities, a certificate being given in its balance sheet that all terminable securities have been valued on a basis which provides for the equalization of the book and redemption values at the date of redemption, i.e. the amortization method. The results of this method will be similar to those of method B, i.e. a bonus k will be declared in respect of the quinquennium just completed at the same rate as would have been declared if the depreciation had not occurred; and the justification of the method depends

upon the same argument, viz. that the depreciation causes only a book and not a real loss. As argued in discussing method B, it is, therefore, unjustifiable in all except the special case where the assets exactly match the liabilities. In this special case, however, the results will be more equitable than those obtained in case I by method B, for under the latter a bonus of £2% having been declared for the quinquennium just completed a level future bonus of £2. 7*s*. 6*d*.% will emerge (assuming the fund is closed or the premiums for new entrants suitably reduced), while under the amortization method an increasing rate of bonus will emerge at future valuations as the proportion of the fund invested at the higher rate of interest increases. This will benefit the younger age groups at the expense of the older age groups which were too favourably treated under method B. In this special case, therefore, the amortization method will attain a high degree of equity as between existing policyholders, but a difficult problem will arise in deciding upon the premiums to be charged to new entrants (since an increasing rate of bonus will emerge at future valuations) unless the fund is closed after the depreciation and a new fund opened. Such a course would hardly be practicable if depreciation or appreciation occurred frequently.

(3) The office can write down its assets to their new market values and increase the valuation rate of interest by the amount of the increase in the experience rate. This will maintain the margin between the valuation and experience rates of interest and at first sight, therefore, it would appear to make provision for the same rate of future bonus as was allowed for in the past and thus to be similar to method C. This is not, however, quite true, for under a net premium valuation an increase in the valuation rate of interest involves a decrease in the net premiums valued and the decrease in the reserve under a net premium valuation will, therefore, be less than the decrease in the reserve under method C.

Whatever the margin between the valuation and experience rate of interest, a net premium valuation will always obtain the correct reserve at duration 0, viz. zero. If, however, the margin between the valuation and experience rate of interest is inadequate to maintain the rate of bonus required, the reserves for policies at all other durations will be too low, this deficiency increasing as the duration increases until a maximum is reached and then decreasing steadily until the correct reserve is obtained for policies at maturity. Under method C, however, the deficiency in the reserve is greatest for policies at duration 0 (large negative values will be introduced by method C at the early durations) the deficiency decreasing steadily as the unexpired term decreases. The exact effect of using this net premium method will, therefore, depend upon the composition of the particular office, but as regards the amount of the depreciation thrown on to the surplus for the quinquennium just completed it will be intermediate between methods A and C (i.e. it will normally be less equitable than method A but more equitable than method C). The reserve set up should be sufficient to maintain a future bonus at a rate intermediate between the rates of K under methods A and C, but since the margin between the valuation and experience rate of interest is inadequate to maintain this rate, it is difficult to forecast the actual results of future valuations. It would appear probable that too high a rate of bonus would emerge from the next valuation, the rate at subsequent valuations decreasing if the fund were closed.

CONCLUSIONS

To sum up, it appears that the most equitable method of dealing with depreciation under a uniform reversionary bonus system is to write down the assets to their new market values and to increase the valuation rate of interest to that obtainable under the new conditions, valuing as a liability a future bonus at the rate which the existing premium scale will support under the new conditions. This method will not obtain absolute equity, but the degree of equity will probably be as high as can be obtained by any practicable system of bonus distribution, since the position will in practice always be obscured by the profit or loss from mortality, loading, lapses and other miscellaneous sources. There is no convenient method of obtaining the same results under a net premium valuation. Similar results could be obtained under a net premium valuation by increasing the valuation rate of interest by an amount less than the increase in the experience rate of interest, but the increase in the valuation rate required would vary with the composition of the office and could only be ascertained in any particular case by trial, having first made a bonus reserve valuation, so that the work involved would be very great. Moreover, it would be difficult to predict the results which would actually emerge from future valuations.

It is obvious that exactly the reverse must apply if appreciation occurs, i.e. the most equitable method will be to write up the assets to their new market values, decreasing the valuation rate of interest to the new experience rate and the future bonus to the bonus supported by the premium scale under the new conditions.

If these arguments are accepted, depreciation will involve a decrease and appreciation an increase in the rate of bonus declared in respect of the valuation period during which the change in values occurs, the magnitude of the decrease or increase depending upon the relative distributions of the maturity dates of the assets and liabilities, while if conditions again become stable, future bonuses will be at a higher rate than in the past following depreciation and at a lower rate following appreciation. Such abrupt changes in the level of bonus rates are contrary to the usual practice of offices and it would undoubtedly be difficult for one office to follow such a policy. If, on the other hand, the suggested policy were to be adopted by all offices, it might cause difficulties until it was understood, or at least accepted, by the policyholders. Eventually, however, it should produce a more intelligent attitude among them towards the office and towards the bonus; they would come to understand that the latter really was dependent on profits, and that it was not a guaranteed addition to the sum assured, a reduction of which—even in war-time—would be almost a breach of faith. If valuations by this method were made by all offices and the full basis and results published, it would be possible for a prospective proposer to make an intelligent choice between competing offices, which is hardly possible at present when the only guide is the past bonus record—obviously an unsatisfactory guide, since a low bonus in the past *may* be a source of present strength and vice versa.

The comparatively violent fluctuations in the rate of bonus would not cause hardship in the great majority of cases, for a with-profit policy is usually intended to provide an income from the date of maturity. If severe depreciation occurred during a valuation period so that no bonus could be declared, then a policyholder whose policy matured shortly after the valuation would still be able to obtain a higher income by investing the maturity money at the higher rates then obtainable than he could have obtained if the depreciation had not occurred. Similarly, in the event of appreciation, the high bonus would tend to compensate for the lower yield obtainable.

COMPARISON WITH PAST PRACTICE

It is interesting to compare the suggestions made in the preceding pages with the general practice between 1918 and 1939. During the war of 1914–18 and the years immediately following, there was a very substantial rise in the level of interest rates and consequent depreciation of investments. It would appear that in many cases life offices wrote down their assets to their new market values and continued to make their valuations on the same net premium basis as had been used in the past. If the argument developed in this paper is correct, this procedure benefited the more recent policies at the expense of the older. Further, if the level of premiums charged for new policies remained unaltered, it benefited policies entering after the depreciation occurred, while if the premiums were reduced on account of the increase in the rates of interest obtainable, new policies were even more favourably treated.

In 1932, as a result of the conversion of the 5% War Loan, there was a reduction in the level of interest rates and consequent appreciation in investments, but it would appear that the majority of offices did not write up the assets to their new market values so that the difference between the book values and the market values constituted a hidden reserve which was not made available for distribution in bonuses. This again benefited the more recent policies at the expense of the older policies, and greatly benefited the policies entering after the appreciation unless the premiums for new policies were substantially increased.

It may be, of course, that in some cases the policies which benefited on the earlier occasion suffered from the method adopted on the second occasion, but such a balance is hardly scientific. It would seem that in many cases equity was sacrificed in order to obtain greater strength and safety in the fund. Whether this is justifiable is a matter of opinion, but it seems to me that the bonus loading gives such a margin that it should never be necessary to sacrifice equity to secure additional safety.

I should like to take this opportunity of recording my gratitude to Mr T. W. Haynes, F.I.A., not only for his work in checking the calculations required, but also for his encouragement and constructive criticism.

APPENDIX

From the assumptions made on pages 203 and 204 when choosing the model office to be used, the following valuation data can be easily deduced for 31 December at which the valuation under consideration is being made:

Sum assured in force at age $x = l_x$.

Vested bonus in force at age x:

Each policy will have received a bonus of ·02 for each premium paid since entry, with the exception of the premiums paid in the current quinquennium. Policies at age 29 or less will not have received any bonuses. Policies over that age will have paid $(x-24)$ premiums and will, therefore, have received bonuses for $(x-29)$ years. The vested bonus in force at age x is, therefore, $\cdot 02\,(x-29)\,l_x$.

Premiums in force at age $x = \cdot 02376 l_x$.

Value of 1% *future bonus at age* $x =$

$$l_x \times \frac{1}{100} \times \frac{1}{D_x}[R_{x+1} - R_{65} + (65 - \overline{x+1})(D_{65} - M_{65})]$$
$$= \frac{(1+i)^x}{100}[R_{x+1} - R_{65} + (64 - x)(D_{65} - M_{65})].$$

In order to ascertain the distribution of the maturity dates of the assets which would be required to obtain an exact matching of the assets and liabilities, it is necessary to calculate the sums assured and bonuses payable in any future year in respect of claims by death and maturity on the assumption that there will be no further entrants, that a bonus of £2% will be maintained and that there will be no withdrawals other than claims.

The bonus (including interim bonus) payable when a claim by death arises between age $(25+n-1)$ and age $(25+n)$ is $\cdot 02n$ and the number of such claims is d_{25+n-1}.

The total sums assured and bonuses payable at the end of t years from the date of valuation in respect of claims will be (remembering that it is assumed that there will be no new entrants after the date of the valuation):

$$(1+\cdot 02t)\,d_{25+t-1} + (1+\cdot 02\overline{t+1})\,d_{25+t} + \dots + (1+\cdot 02\times 40)\,d_{64} + (1+\cdot 02\times 40)\,l_{65}$$
$$= \sum_{n=t}^{40}(1+\cdot 02n)\,d_{25+n-1} + 1\cdot 8 l_{65}.$$

The number of policies which will become claims in the tth year from the date of valuation is

$$\sum_{n=t}^{40} d_{25+n-1} + l_{65} = l_{25+t-1}.$$

Hence the total premiums payable under such policies are $\cdot 02376 l_{25+t-1}$.

In order, therefore, that the assets may exactly match the liabilities, it is necessary to hold assets maturing at the end of t years of amount:

$$v^t\left[\sum_{n=t}^{40}(1+\cdot 02n)\,d_{25+n-1} + 1\cdot 8 l_{65}\right] - a_{\overline{t-1}|} \times \cdot 02376 l_{25+t-1}.$$

ABSTRACT OF THE DISCUSSION

Mr T. R. Suttie, in introducing the paper, said that it had been suggested to him that $3\frac{1}{2}\%$ and 4% were rather high rates of interest to use for a valuation at the present time. Their use was explained by the fact that he had commenced the calculations for the paper in the summer of 1940, when they had seemed not unreasonable rates to use. As it was only a change in the valuation rate of interest which was of importance, he had not thought it necessary to undertake the very considerable amount of work involved in making the calculations at a lower rate of interest.

Mr L. Brown, in opening the discussion, said that in order to confine consideration to the effect of an increase in the rate of interest and to the best method of securing equity between policyholders, the author had assumed that the other variables remained unchanged. Comment might be made upon the data on the grounds that his model office gave an average unexpired term for the business which was too long for a normal office, even allowing for whole-life business and for the fact that most offices had a growing fund and not a stationary one. Such points, however, were not vital to the problem, as it was the relation of the average term of the liabilities to the average term of the assets which was material, and he did not intend, therefore, to criticize the basis of the data.

The author had taken as his first case a distribution of assets exactly corresponding with the distribution of liabilities of the existing business. Such a distribution of the assets was the basis from which an investment policy should start, and variations from that basis had to be supported by the belief that such variations were justified. One cause of variation was mentioned in the paper, namely, that the theoretical distribution required substantial investments in short-term securities giving very low yields. Other variations arose from practical difficulties in finding securities with appropriate dates, from the present necessity for investment in Government 'tap' loans, and from a deliberate choice of investment policy.

The author had deduced a distribution of liabilities on a basis which called for comment. He had taken the expected claims in year t by death and maturity and suggested that, in a theoretical distribution, assets should be held maturing in year t which, with the whole of the future premiums on those policies similarly invested, would accumulate to the total claims in that year. If the policies issued in one year only were considered, the reserve value at the date of issue was, of course, nil. The theoretical distribution threw up positive reserves required to meet the claims by death in the early years and negative reserves for the claims of the later years. By aggregating the reserves for all the outstanding policies the author had set off those negative reserves against the positive reserves required for other policies which would become claims in the same years, but at the end of the table the balance of reserve was negative, and those negative values had been arbitrarily deducted from the reserves of the latest years for which positive values occurred. That appeared to mis-state the average term. The point was incidental to the real subject matter of the paper but he thought it was of some general importance.

The theory of a level premium for an increasing risk implied that the excess of a year's premiums over the year's claims was invested to provide for the years in which the outgo would exceed the current premiums and interest income. It would, therefore, be right to assume that, for each valuation group, future claims would be met out of premiums and interest of the year and that existing reserves were required to mature when there was a net outgo for the group. For endowment assurances, with the exception of long-term policies maturing at advanced ages, that meant that assets would be required maturing only in the year in which the policies matured, and the valuation reserves would, he thought, be a proper measure of those assets. For whole-life assurances there was a net outgo to be provided over a series of years which would be met out of existing reserves and the investment of future premiums and interest after meeting current outgo each year. There was, therefore, some option as to the term required for the existing reserves, and probably a spread over the period of net outgo would be most suitable. From a practical point of view it was only necessary to provide for assets to mature in each year to the extent that the outgo in respect of all the existing business would exceed the total premiums and interest, i.e. the majority of the earlier policy maturities could be met out of premiums and interest. The adoption of such a course would clearly not provide the theoretical distribution described; in fact, if future business were also taken into account it would lead, with a stationary or expanding fund, to the position that no redeemable assets would be required and the investments could all be perpetual. The distribution of liabilities according to the valuation reserves clearly gave an average term somewhat longer than the distribution given on p. 204, where the average term of a little over ten years seemed surprisingly short. In practice the position of different offices was likely to vary considerably, but it was probable that the average unexpired term of the model office was rather long compared with most offices, except in cases of recent expansion.

As a matter of interest he had recently examined the redemption terms of the Stock Exchange securities held by a large fund. By generous approximation it appeared that the assets were distributed as to just over 70% in securities, the redemption of which could be reasonably relied upon, with an average term of about fifteen years. Under present conditions of investing all new money in Government war issues, it appeared that the average term was likely to increase. The remaining 30% was in securities of a perpetual type, including preference and ordinary shares and freehold property, together with a number of redeemable securities where the present credit-standing of the debtors threw doubt on their ability to repay on the due dates. Perpetual securities, of course, showed greater depreciation on a rise in the interest rate, and with an assumed rise from $3\frac{1}{2}\%$ to 4% on all securities, the resulting depreciation was equivalent to that which would have applied to an average term of the order of 24 years for the whole fund. He had ignored a difficult point which arose in practice. Many securities had rather wide optional redemption clauses, and a change in the market rate of interest would, in a number of cases, cause an alteration in the assumed redemption date.

The other cases taken in the paper assumed assets with terms definitely longer than those of the liabilities. That appeared to cover the relationship between liabilities and assets which was likely to occur in practice. The current and future bonuses were calculated by four different methods, and the results were analysed by examining three representative age groups or durations in Tables 3 and 4. Such an analysis was quite sound for case I, and possibly also for case III, but for cases II and IV the assumption of the same term for all assets had very different effects when dealing with the fund as a whole and when dealing with policies of different durations. That seemed to invalidate the comparisons shown in the Tables for those cases. He appreciated the many practical difficulties, but suggested that more reliable results could have been obtained by taking distributions of assets which had proportionately similar effects for each duration, e.g. by taking the distribution of assets provided by case I and doubling or trebling the term of each asset, so that a series of assets $m_1, m_2, \ldots, m_t$, with terms of $1, 2, \ldots, t$, years would become the same series with terms of $2, 4, \ldots, 2t$, years, etc.

The problem was to achieve equity between different classes of policyholders. The author had offered no definition of equity, but he had implied that each group of like policies should receive the benefit of the interest and of the appreciation or depreciation arising from the investments representing the valuation reserves of that group—no more and no less. The author's general arguments and conclusions were evidently the product of long and careful thought. Considering them and examining the figures for case I in Tables 3 and 4, he thought the author had made out his case that, with a uniform reversionary bonus and a bonus reserve valuation, method A would give results which were, on the whole, most nearly equitable. With a net premium valuation a similar result should be aimed at, but it was difficult to say how it would work out in the future. Actuaries should try to avoid holding back present profits and giving to future policyholders more profits than those to which they were entitled. The conclusions, however, ought not, without due consideration, to be taken beyond the limits of the model office from which they were deduced.

In practice the circumstances of the office concerned had to be examined, and every office could be expected to differ in some respects from the model office in the paper. There were also broader conceptions of equity which had to be taken into account. It should be remembered that a large company partook of the nature of a national institution which had been built up from small beginnings by a carefully balanced consideration of the just claims of the current policyholders for security first and profit second, and of the equally proper desire and need to provide a progressive improvement in the security to new generations of policyholders. Looking back, it might be argued that that course had failed to provide equity to the early policyholders. The prestige of life offices had come to be taken for granted to such an extent that there was a tendency to forget that, for a long-term contract such as life assurance, security was an essential without which the business could not have grown to the dimensions it had attained. Obviously, if the returns offered did not compare favourably with those obtainable from other companies and by other methods of providing for the future, the company or the industry would not prosper; but if the security offered were not adequate, the question of such a comparison would not arise. Those who entered into assurance contracts immediately enjoyed the security which was offered by the reserves which had been built up in the past. He therefore suggested that broad equity allowed or required a contribution from each generation of policyholders towards the maintenance of the fundamental strength of the company, and that any course which, straining at a precision of equity between new and old policyholders, was in fact disadvantageous to the progress of the company as an entity, was inequitable and not likely to lead to the advantage of the recent or new policyholder any more than to that of the old policyholder.

It was suggested in the paper that policyholders should be educated to appreciate that, following a rise in interest rates, it was quite logical to reduce the current rate of bonus and, at the same time, to look for even higher rates of bonus in the future if conditions should again become stable. He felt that, without a basic actuarial training for all, the more probable result would be a loss of public confidence in the solidity of life assurance and in the soundness and sagacity of those in control of life offices. With most offices, part of the profits of the past had been used to create contingency reserves which should be drawn upon to deal with the depreciation of the assets, and possibly some alteration in the valuation basis should be made in the light of the new conditions. Assuming that a net premium valuation was published and that there was general confidence that the higher interest rates were likely to continue indefinitely, the aim should be to smooth out the effect of the depreciation by declaring a current bonus at a reasonable level and by rebuilding the contingency reserves gradually in the future, using for that purpose some part of the expected increase in profits from higher interest rates. Incidentally, the position would be much more difficult where an annual valuation was made, although the depreciation was likely, in practice, to occur gradually and so to be spread over several years. If, on the other hand, there was some doubt whether the higher interest rates would continue for long, consideration would doubtless be given to the adoption of an amortization method of valuing redeemable securities in order to achieve the same end. That method was not in favour, but the extent to which they had been forced into long-term, low interest-bearing securities, by subscription to war issues and by the conversion of other securities to lower rates of interest, would be an argument in favour of such an alteration if depreciation should occur.

The author was concerned that both the depreciation of the last war and the appreciation of more recent times should have been used in a way which favoured the future policyholder. He felt that a different position now obtained, in that the large volume of investments made at low interest rates during the present phase would affect the yield obtained on the fund for a very long time and that there seemed to be little possibility of bonus rates returning to the high levels previously attained.

Mr R. W. A. Fowler said that to test the equity, as between various policyholders, of a method of allocating surplus derived from a particular source, the standard method was to assume a model fund in which everything had been and was proceeding according to plan and to examine for separate groups of policyholders the effect of introducing the particular form of distribution of surplus under investigation. If the method of allocation was such that each group of policies was treated fairly, the method was deemed to be equitable.

The author had dealt with depreciation in that way and had reached the conclusion that the most equitable method, as between various age groups, of dealing with the particular type of depreciation under discussion was to declare a smaller immediate bonus than would normally be distributed and to reserve for larger bonuses in the future. In effect, the position was that, as the depreciation was of a temporary nature, the policies nearing maturity suffered a loss of bonus because the money held on their behalf would not be retained by the office for its ultimate appreciation. Taking that argument to its logical conclusion, it seemed that, in the event of there not being any securities of a very short term which could be matched with the maturities, those policies just about to mature should receive no bonus at all, or even a negative bonus if that were possible, in order that absolute equity should be satisfied. As was usual, however, in most arguments on equity, the eventual conclusion was reached that practical needs outweighed absolute equity, and he felt that most offices would tend to adopt method D of the paper rather than method A, despite its greater inequity.

It seemed that actuaries were always trying to achieve an ideal in bonus apportionment which they were never able to reach and which, if they could attain it by practicable means, they would not consider desirable. He asked whether in those circumstances it would not be better to set a different standard by which to judge equity, one which, whilst being possible of attainment, would also be intelligible to policyholders. In his view, a with-profit contract was really two contracts: first, the without-profit portion which was exactly similar to ordinary without-profit business, and secondly, the profit-sharing portion which involved the payment of an annual bonus loading for a fixed term of years in consideration of the right to a share in the surplus earned by the office. The amount of loading was in effect the market value of the bonus rights. There was thus a certain analogy between the latter portion of the contract and any marketable security carrying the right to profits, such as an ordinary share, and that analogy could be extended to provide the standard required. A market price was paid for an ordinary share which was the market's assessment of the value of the future dividends. A company would determine from time to time how much money was to be allocated to its ordinary shares, after meeting all prior charges and setting up contingency reserves which quite possibly included what might be called 'dividend maintenance reserves'. Once, however, the total sum available was determined, its allocation among shareholders was fixed on the basis of the nominal amount of capital held. That was considered to be an equitable arrangement, even though the actual sums paid by individual shareholders for their rights might have varied considerably.

By analogy, a life assurance company could make a valuation of its without-profit business, including the without-profit portion of its with-profit business, on experience bases; it might in effect include a contingency reserve by using a slightly stronger basis, and it might attempt to even out bonuses over the future by adopting either a net premium valuation at an arbitrarily low rate of interest or a bonus reserve valuation bringing in the value of future bonuses. Having decided on the amount of surplus available, it should then distribute it among policyholders according to a certain formula. It did not matter whether that formula apportioned the bonus according to the sum assured, the sum assured plus bonuses, or the premiums paid, provided that the basis was consistently used and did not change from declaration to declaration. That was what he felt to be practical equity. Where inequity would arise would be if the life assurance company were to change its formula, just as it would be inequitable for a trading company suddenly to decide to distribute its profits according to the prices paid for the shares by their owners. He admitted that the analogy between an ordinary share and the bonus portion of a with-profit contract was not complete, but he did not think that that vitiated the argument.

In effect, what a life assurance company said to a prospective policyholder was: 'If you will pay us an additional £x per annum on this policy, you will be entitled to share in profits over the term of the policy. It is our responsibility to determine the surplus available, but what there is will be divided among all with-profit policies according to a definite formula (such as the simple reversionary bonus plan).' If that argument were accepted, it meant that in the particular case under discussion the life assurance company could deal with the depreciation in whatever way it chose, without reference to equity. If, however, its practice in the past had been to value on a bonus reserve basis with $k=K$, it had in equity to continue that basis at the present valuation and use method D. If its customary basis had been a net premium valuation with simple reversionary bonuses, the choice of the new rate of interest to be used could be decided entirely with reference to its own views on strength of valuation and continuity of bonus scales, but the resulting surplus would still have to be divided on the simple reversionary bonus plan.

Mr M. E. Ogborn said that if reference were made to old books on economics, it would often be found that the author transported his readers to a desert island where he constructed a financial and economic system to suit his own ideas and drew conclusions which sometimes were not valid in the everyday world. He thought that the author had tended to fall into that error, for the inferences he drew might not hold in the actual conditions with which actuaries had to deal. For example, he had assumed a sudden change—overnight—in the rate of interest, from $3\frac{1}{2}\%$ to 4%, and had excluded all the other considerations affecting the rate of interest. He had had to do that, of course, for the purposes of the paper; but in assessing the value of his conclusions it was necessary to remember the assumption. In practice there were many factors which affected the rate of interest, such as the possibility of default and of variations in the rate of income tax, uncertainty as to the permanence of economic conditions, and so on. Actuaries would feel much easier if they knew that the rate of interest would be, say, i this year and next year, without any variation, or that, if a change occurred, the change would be permanent and that no other factor would affect it.

Turning to market values, the author had suggested that the book values of the securities should be written up to the full market values. He thought that most people would have some sympathy with that view, as bringing them into touch with reality, but when the attempt was made to give effect to it there were difficulties in the way. It was not known to what extent current market values of stocks were likely to be permanent. Further, a redeemable security bearing a high nominal rate of interest would have to be carried for the time being at above its redemption price. Not many people, he thought, would wish that course to be adopted.

In dealing with the distribution of surplus, the author had assumed that appreciation and depreciation could be allocated in proportion to the reserves of the policies. He did not know what would be said by the holder of a policy nearing maturity if, just before maturity, there were a fall in prices and, as a consequence, a reduction of the sum payable at maturity. He felt that such a type of distribution would be bound to cause dissatisfaction. Conversely, if there were appreciation and if, in consequence thereof, companies declared a higher rate of bonus than could subsequently be maintained, those companies were likely to find themselves in difficulties if in fact the appreciation proved to be merely transient. He thought that actuaries were now more suspicious than ever of market appreciation.

It seemed impossible to compute exactly what bonus the existing fund would support or what the new business fund would support, so that most actuaries would feel that, while they had to bear in mind the bonus-earning power of the new business fund, in fact method D of distribution was the only method which should be adopted, unless there was some default or loss in the current quinquennium which had to be set against the current bonus.

He had looked up what had been said on the subject of distribution of surplus more than a hundred years ago by William Morgan. He commended Morgan's rules for distributing surplus 'as the best means of securing and improving the interests of the present members and in due time the interests also of those that shall succeed them'. That, he suggested, was as far as it was possible to go towards defining an equitable distribution of surplus.

Mr H. G. Jones thought that, where previous speakers had disagreed with the author, it was partly because the results were unpalatable; in fact, that had been his own first reaction. Notwithstanding that the reasoning indicated that the correct course to adopt in certain circumstances was to declare a high bonus now and a lower bonus in the future, such a course seemed illogical, and he had therefore looked for some argument against its adoption. Various reasons had been found for disagreeing with the author's conclusions, but there was one angle of attack which, while largely parallel to what had been already said, put the matter in a slightly different way. Was not the author looking at the position after the event had occurred, instead of remembering that the actuary dealt with probabilities?

It was not always possible to divide the business into portions and to say that one portion had made a profit and that another had made a loss. It was necessary to see that for each new policyholder the premiums which he expected to pay balanced the benefits which he expected to receive. A person might be told: 'If you come into our fund, you will pay premiums at such and such a rate and, if there are any profits, they will be distributed in proportion to the sum assured (or, perhaps, in proportion to the premiums paid).' It would not matter what the scale of distribution was, so long as any additional profit or any reduction in profit was allocated according to the scale, and the probability of the one would, he thought, balance the probability of the other. If actuaries were to arrive at an equitable bonus distribution, they should think in that way instead of imagining that certain changes had taken place and then considering what the result was in the case of each individual policy. If the latter principle were taken to its extreme, would it not mean that policies becoming claims by death would receive no bonus because they involved a loss to the office, whereas the claims by survival implied a profit?

Mr R. I. MacIntosh said that the results shown in the paper were very interesting, but it seemed to him that the assumptions governing the operation of the model office rendered the results too artificial for practical application. They might help an office making a bonus reserve valuation, where miscellaneous profits were not large. If real profits as such did not arise to any extent and surplus in the past had been distributed in the form of a level reversionary bonus, it might be possible and reasonable to consider equity between different classes of policyholders.

It was not possible to define equity in two or three words. The contribution method was perhaps the only way of distributing surplus in direct proportion to the earning power of a policy, but if policyholders were led to expect their bonus to be calculated in a different manner—such as in the form of a level reversionary addition—it was correct to say that equity was achieved when that method was used. In the paper, however, the author had considered the case where a level addition had been declared in the past but where, as soon as some unusual item of profit or loss presented itself, it was dealt with by the contribution method. He did not think it true to say that the results of Tables 3 and 4 proved method A to be the most equitable. In practice, of course, miscellaneous sources of profit or loss would always be present, thus further complicating any possible application of the results of the fairly simple example in the paper.

It was, he thought, fair to say that equity would be achieved by the maintenance of a level reversionary bonus, if that were possible, but it would depend on the results of the bonus reserve valuation. Consideration of the nature of the profit or loss was irrelevant. Where, however—as in the majority of cases—a net premium valuation was published, even if a bonus reserve valuation was made for internal guidance, the cost of the new bonus had to bear some relationship to the surplus disclosed. It was broadly true to say that a net premium valuation, like a bonus reserve valuation, held up surplus in the early years and released it later on, but it was very difficult to judge whether surplus would be thrown up in the right amounts and at the right time to enable a level reversionary bonus to be declared, even apart from the question of miscellaneous profits.

Mr F. M. Redington thought that the author's standard of equity was mathematical and individual—a penny for a penny or a pound for a pound. He himself did not believe that that was the only form of equity, or even the best one. The basic principle of assurance was the replacing of individual equity by another—and he believed a higher—form of equity, which might for convenience be called 'group equity'. He saw no reason why that higher principle should not be imported into bonus distribution, just as it was into the basic claim distribution; indeed, it was his conviction that that ought to be done. In his work he had come across the question of equity on many occasions and he thought he could safely state that it had never been possible to say exactly what equity was; often enough, a purely mathematical analysis produced a result which was seemingly unreasonable and inequitable.

The same difficulties arose with regard to the author's solution of the problem of depreciation. Was it really fair to place the burden of depreciation on the policyholders at the time the depreciation occurred? If it was, why were contingency funds and reserve funds built up? It was presumably a corollary of the author's conclusions that he would not set up investment reserve funds, or that, if he did, he would not use them. Why did not offices 'zillmerise' reserves and refrain from granting bonuses for the first five or ten years? He supposed it was, at least partly, because they were obeying—perhaps not too consciously—the higher principle of group equity. Moreover, even if it were admitted that the author had correctly expounded a precise mathematical criterion, was it really desirable to attempt to educate the public as he suggested? It was certain that the public would regard his solution as the reverse of equitable. To the public, equity was as equity seemed, and he was not sure whether there was not a deeper instinctive truth in that feeling than there was in precise mathematical equations.

He had expressed opinions which were directly contrary to those of the author, but he recognised that in practice it was as difficult to apply the principle of group equity as it was to apply rigid principles of individual equity. It was necessary to compromise, and to do that with justice and discretion the factors affecting the decision had to be clearly perceived.

Mr E. J. W. Dyson regretted that the author had made no reference to non-profit policies and the treatment of appreciation or depreciation in relation to that class of business. It seemed to him that appreciation or depreciation was bound to lead to some profit or loss arising from non-profit business, which would be passed on to the with-profit policyholders. How could that be distributed on the basis of mathematical equity? Surely it could only be distributed on very broad and general lines. Similarly, miscellaneous sources of profit could be used to level up a distribution in a way which it would obviously be hard to justify on the ground of strict equity.

Another point concerned the treatment of policies effected after the change in conditions had taken place. It seemed necessary for the rates of premium to be adjusted to ensure that broad equity would be maintained, but it had not usually been considered necessary in the past to alter premium rates frequently.

Mr R. J. W. Crabbe agreed that the author's results were artificial to some extent, but that obviously followed from the fact that without artificiality there could not have been a paper. It was necessary to examine the particular feature of depreciation in isolation if its operation was to be seen. The conclusions which were set out in the paper having been arrived at, they could be combined with other considerations regarding the questions of bonus distribution and of profits available for that purpose, in order to arrive at results which perhaps would be satisfactory both to actuaries and to the public.

He agreed that method D was satisfactory from the point of view of the public, but, like the last speaker, he wished to consider the effect on new business. So far as group or general equity was concerned, the method might produce a reasonable result for existing policies, which would in the future be a group shrinking in size and earning a bonus at rate K. It was possible that at the same time there might be a steadily increasing group of new policies earning a rate of bonus substantially higher than K. He was assuming, of course, that the new conditions—the 4% conditions—continued indefinitely without change, and that there was no other disturbing factor, and furthermore that premium rates for new policies were not altered. In such circumstances it seemed to him that method A was the only one which could in equity be adopted, even though it might be unsatisfactory for practical purposes. Alternatively, if the rates of premium for future policies were altered so as to provide for a rate of bonus of K, or, at any rate, the rate of bonus assumed in the valuation, method D could be adopted. But if the premium tables were not altered, it seemed that method D would result in a worse position than method A, because the fund would produce profits which would provide constantly increasing rates of bonus. In those circumstances it was to be hoped that the laboratory conditions would not be realised, and that the changes would be gradual so that their effects could be masked by merging them with those of other factors. He thought that that was really what had happened in the past; companies had been able to match, one against another, factors which by themselves would have produced results which, however equitable they were in theory, would have been unsatisfactory to the public.

Mr A. T. Haynes wished to draw attention to a slightly different aspect of the problem, one which had been developed by the opener, namely, that a matched investment policy had a possible conception other than that which the author had developed. As he understood the author's assumptions underlying case I, the investments of the fund were arranged to mature to meet claims as they arose in future years even although in those same years there would be an income from premiums and from interest on investments which would have to be invested. Under the opener's conception of a matched investment policy, it was expected that those two items, premium income and investment income, could be used to meet claims, and investments would only be required to mature in years where they were needed to meet a net cash outgo. A suitable distribution of investments could, he thought, lead to a position where, on a depreciation arising purely as a result of a rise in the interest rate, there would be exactly the same change in the capital value of the investments as in the reserves. For example, in Table 1, method C (where the reserve allowed for exactly the same rate of bonus in the future as had been declared in the past), it appeared that a distribution of assets somewhere between those of cases III and IV would permit the declaration of a bonus of exactly £2 %.

If such a distribution could be regarded as an 'optimum' distribution of assets—providing protection against the effect of depreciation or appreciation resulting from changes in the interest rate—then it followed that in Table 1, method C, cases I, II, and III represented assets distributions shorter than the optimum and case IV longer. The result of the investment policies underlying cases I, II and III was less depreciation, owing to the shorter average term of the assets, than under the optimum distribution of assets. Therefore there was a surplus available to permit a higher rate of bonus if desired. In case IV there was a longer-dated assets distribution than the optimum, and therefore greater depreciation, so that the fund was not sufficient to allow of the declaration of a bonus of £2 %. That effect was also shown in Table 2, method C, where there was a surplus in cases I, II and III and a loss of surplus in case IV. He thought that that view of the situation would perhaps throw some light on what was the equitable and proper course to adopt. If the investment policy had been such that the investments were on the average shorter-dated than the optimum distribution, a declaration of a bonus higher than £2 % would be possible, and the decision should be made in the light of practical considerations. If, on the other hand, the investments were longer-dated than the optimum distribution, then the investment policy had led to a loss of surplus, and there should be an immediate reduction in the rate of bonus.

Mr V. W. Tyler, in closing the discussion, said that the author, by means of a model office and by the use of an ingenious set of theoretical assumptions, had come to certain practical conclusions—very briefly, that when depreciation occurred, an office using a bonus reserve valuation should write down the assets to their new market values, increase the valuation rate of interest to that obtainable in the new conditions, and value a rate of bonus that the existing scale of premiums would support. An office employing the net premium or gross premium method should use such a rate of interest as would produce approximately the same effect. If appreciation occurred, the reverse process should be followed.

Those who had taken part in the discussion had commented on the author's assumptions, on which, after all, the validity of his conclusions depended. He thought like Mr Crabbe, however, that, had it not been for the artificiality of some of the assumptions, a useful paper would not have been before the Institute at all. He drew attention to the remark made by the author in parenthesis at the end of assumption (7) on p. 204, and said that, quite apart from the fact that a very large part of the assets of a life office consisted of investments other than Stock Exchange securities—investments such as mortgages, real property and so on, which were not affected in the same way as were Stock Exchange securities—he did not think that the remark would apply even to Stock Exchange securities themselves. It was possible that small changes would be fairly uniform, but he felt sure that large changes would not be so, for in periods of high rates of interest short-term securities sometimes gave a better yield than long-term securities, and in times of low rates of interest the reverse position existed. The current yields on Government securities demonstrated that point. For instance, for periods of up to one year the gross yield was about 1 %; for periods of from seven to ten years it was $2\frac{1}{2}$ %; and for periods of from 20 to 25 years, 3 %.

Coming to the practical side of the conclusions, it was certain that many of the author's assumptions would not be borne out in practice, but that did not necessarily affect the validity of the conclusions, provided that in his assumptions the author had assumed a sufficiently wide range to cover all the possibilities. In that connexion, it was interesting to note that thirty years ago R. R. Tilt, using different methods, had arrived at similar conclusions (*J.I.A.* Vol. XLVIII, p. 261). The author, therefore, had very useful theoretical confirmation. Actuaries had endeavoured, as witness the evolution of the bonus system through the cash bonus and the contribution systems to the simple and compound reversionary bonus methods, to deal as equitably as possible with different generations of policyholders, and yet they had not, in general, adopted the author's conception of equity. Presumably they had had their reasons for not having done so, and he therefore felt that they would not now be readily persuaded to adopt in practice what might be called a theoretical counsel of perfection.

Equity was a very desirable thing, but history in general, and, he thought, that of life assurance in particular, had proved that in practice strict equity was unobtainable when a multitude of unknown factors had to be taken into account. There was, for example, the difficulty of differentiating between one form of depreciation and another, particularly in the case of an institution such as a life office where the assets were of various kinds which were not all affected in the same way as Stock Exchange securities. Moreover, a practical point to be borne in mind when striving for theoretical accuracy in connexion with depreciation and appreciation was the fact that, whereas the quoted prices of Stock Exchange securities might be a very fair guide to value in the case of securities in which there was a free market, securities in which there were only infrequent dealings were often quoted at prices very different from those at which it was possible actually to deal. It was true that a policy of writing down the value of assets without making a corresponding increase in the valuation rate of interest might at times have caused hardship to with-profit policyholders; yet, in spite of that, it was also true to say that with-profit policyholders had nearly always in the past fared better than without-profit policyholders. The hidden reserve gradually accumulated by following such a policy enabled the office to withstand setbacks without the necessity for frequent changes in the valuation rate of interest or, in normal times, in the rates of bonus or premium tables.

By the Assurance Companies Act 1909 an actuary was left with a fairly free hand in the treatment of assets and the valuation of liabilities, but with not nearly so free a hand by the actuaries who had preceded him in his office. In most cases he had been left with the legacy of an institution strong and well equipped to carry on its work for the benefit of the present and future generations, and it was his duty to hand over his trust in due course with its strength unimpaired. Except in the case of a closed fund, the actuary was dealing with an indefinite number of persons whose well-being had to be secured over an indefinite period of future time, and he must beware lest, in attempting to achieve a nearer approach to equity as between individuals, he reduced the security of the policyholders as a whole. In the case of a closed fund the position was different, because there the actuary was dealing with a fixed number of people for a more or less definite period, and it was permissible, and even desirable, in order to distribute the assets as the liabilities ran off, to take account of capital appreciation provided that it was done judiciously. A practical point that had to be borne in mind in connexion with closed funds was that, whereas the actuary was usually free to deal with the assets as he wished, his hands were frequently tied by the agreement for the transfer of the fund to a specified basis of valuation for the liabilities.

The President (Mr H. E. Melville) recalled that when he delivered his Presidential Address in the previous autumn he had ventured to ask whether the changed financial conditions, to which actuaries had become accustomed of recent years and which seemed to him likely to persist for some time, might not make it necessary for them to examine their methods of bonus distribution and to ask themselves whether the systems which were appropriate in the days of high interest rates were still equitable today. He was then thinking of the fall in interest rates which had occurred, while the author had been thinking of what some people might hope for or expect tomorrow, namely, a rise in interest rates. The author had set himself a problem, and was seeking an answer to it; he welcomed the paper and thought it served an excellent purpose.

The author had started by making some comparatively simple assumptions and had confined his examination to the case of a simple bonus. He agreed with some of the speakers that without those simple assumptions it would have been very difficult to produce the paper. It was no criticism to say that other assumptions might have been made or that in practice life was much more complicated than had been assumed in the paper. The author had sought to find 'the maximum practicable degree of equity between the different generations of policyholders'. Quite a lot had been said about equity during the course of the discussion and with a large part of it he found himself in disagreement. Very little had been said about the rates of premium which were charged. More had been said about the custom of perpetuating certain types of bonus without much regard to changed conditions. He doubted whether a certain type of bonus which was in use in years gone by for a certain scale of premiums and a certain scale of profits was necessarily right today, when there was a much lower rate of profit due to a lower rate of interest. That seemed to him to be worth investigating, but the author had not done that; he had confined himself to one entry age only, age 25. What he had done had been valuable and it was a first and very useful step towards a bigger investigation. It was to be hoped that this paper would be followed by other papers dealing with other conditions and other aspects of the problem.

Actuaries could only hope to do broad justice in their distributions, but they should at least see that what was done was not far removed from justice. They should not be afraid of having a flexible system of distribution if in fact that was more equitable than some of the rigid systems in existence which most offices were very reluctant to change.

He thought that the author should be very gratified with the discussion to which his paper had given rise.

Mr T. R. Suttie, in reply, expressed his appreciation of the reception given to the paper, even though that reception had been rather critical.

He had been criticised for looking at the position after the event had occurred instead of dealing with probabilities. He regarded that, however, as support for the paper, for his whole argument was that the premiums charged for new entrants should correspond to their expectation of bonuses. The alternative to his method of allowing bonuses to fluctuate to meet depreciation or appreciation was to allow premiums to fluctuate. If that were done, it would ensure that new policyholders received equitable treatment but it would undoubtedly introduce inequity between the policyholders who joined very shortly before the change occurred and future entrants. For that reason he did not think it was fair to say that he was concerned with individuals and not with groups. His argument was based on a consideration of all future entrants. As the President had said, actuaries could not hope to do more than broad justice but they should see that what was done was not far removed from justice, and he did not think that miscellaneous sources of profit or similar complications in any way excused them from making an attempt to attain equity.

The opener and Mr A. T. Haynes had criticised his interpretation of the matching of assets to liabilities. The former had suggested that, at any rate for endowment assurances, it should mean that claims by maturity were met by maturing assets, whilst he himself took it as meaning that claims by both maturity and death were met by maturing assets. As the opener had said, that was not vital to the problem; it did not matter which interpretation was correct, even if it were possible to say whether there was any correct interpretation. Case I was, on his argument, the only case in which the amortization method was justified; but any other definition of the matching of assets to liabilities would increase the superiority of method A over method B (i.e. the amortization method).

Mr Haynes had suggested that the optimum distribution of investments would be one which allowed the future rate of bonus, K, to remain constant at £2 %, but he could not possibly agree with that view. If conditions were such that future premiums would support a bonus of £2. 10*s*. 7*d*. % and the depreciation were such that a future bonus of only £2 % could be allowed for in the valuation, considerable difficulties would have to be faced and the position could hardly be regarded as an optimum one.

Much of the criticism had been on practical grounds, which was, of course, perfectly legitimate, but his principal object in writing the paper had been to attempt to arrive at a theoretical standard of equity by which it would be possible to measure the inequity involved in any concession to practical needs. He agreed that security was more important than equity, but he felt that, with the profit loading contained in the rates of premium charged and the flexibility created by the bonus reserve valuation, there was no reason why there should not be both equity and security.

Mr Suttie subsequently wrote as follows:

There was considerable criticism of the suggestion in the paper that, if appreciation occurs, the assets should be written up to their new market values, but perhaps the discussion did not make it clear that there are really two questions involved, namely,

(*a*) Should the assets be written up to their new market values?

(*b*) If so, should the increase in the value of the assets result in a material increase in the rate of bonus for the current valuation period, followed by a decrease in the rate of bonus at future valuations?

As regards (*a*), there does not seem to be any object in making a bonus reserve valuation unless it is based upon the true facts, i.e. the actual premiums to be received, the actual renewal expenses, the experience rate of interest, and the market value of the assets. Hidden reserves seem entirely contrary to the principles of a bonus reserve valuation; if they exist, they can easily be absorbed by reducing the valuation rate of interest (which will have to be done, since the increase in the published fund will reduce the rate of interest earned thereon) and, if necessary, by increasing the value of K. The position is quite different under a net premium valuation. The valuation basis being more or less fixed, hidden reserves are necessary, not only to avoid violent fluctuations in the rate of bonus if appreciation or depreciation occurs, but also to enable the results of the published net premium valuation to correspond with those of the unpublished bonus reserve valuation.

The disagreement over (*b*) is much more fundamental. If the results of Tables 3 and 4 are accepted and the attainment of equity between the existing policyholders is considered to be of primary importance (subject always to the security of the office), then it appears that the question must be answered in the affirmative. A majority of the speakers, however, seemed to feel that the primary objective should be stability of the rates of bonus and to argue, explicitly or implicitly, that equity was achieved if, first, the distribution of bonus was made upon a consistent basis at each valuation, and, second, the premiums charged to new policyholders corresponded with their expectations of bonus at the date of entry. This argument involves the alteration of the premiums charged to new policyholders following any appreciable change in interest rates and the adoption of method D. It will be seen from Tables 3 and 4 that this method involves serious inequities among existing policyholders, but it may be that, within the limits set by the uniform reversionary bonus system and the attitude of policyholders to their bonuses, it is the most equitable of the practicable methods. It is at least much more equitable than the methods used in the past (and discussed on p. 214), provided that it is applied consistently to both appreciation and depreciation.

From Table 1 it will be seen that, in three of the four cases considered in the paper, the use of method D would involve a reduction in the rate of bonus for a quinquennium during which appreciation occurs. Such a result would obviously be undesirable and this leads to the question of the distribution of the maturity dates of the assets. Table 1 shows that, in the model office, assets with an average maturity date of approximately 27 years from the valuation date would give $k = \text{K} = £2\,\%$. If, therefore, stability of bonus rates is the primary objective, Mr A. T. Haynes is obviously correct in his interpretation of the distribution required in order that the assets may exactly match the liabilities. (The position of non-profit business is similar, so that it should be possible to maintain a distribution of the maturity dates of the assets such that appreciation or depreciation causes no real profit or loss under the non-profit business; this meets the point raised by Mr Dyson.) If, however, equity towards the policyholders is the objective, it is desirable to declare a bonus of $k = £2\,\%$ and to provide for a future bonus $\text{K} = £2.\ 10s.\ 7d.\,\%$ and to do this a distribution even shorter-dated than that under case I would be required.

Mr Redington asked whether it was really fair to place the burden of depreciation on the policyholders at the time the depreciation occurred. Clearly it would not be desirable in practice to declare a bonus k of less than $£2\,\%$ in circumstances such as those of Table 1, case I. No attempt was made, however, to find a practical solution for the problem under case I, since it is extremely unlikely that such a distribution would occur in practice. It was only desired to show that within the limits set (i.e. a uniform reversionary bonus) method A gave the most equitable results.

The following communications have also been received:

Mr J. L. Anderson: On p. 212 a brief reference is made to the possibility of declaring an increasing rate of bonus in certain circumstances. There is no reason, however, why this method should be confined to the particular case mentioned in the paper, where the office uses a net premium valuation and proposes to continue this method of valuation without modification after a rise in interest rates, and it would be interesting to add to the methods of distribution described earlier in the paper, a further method, E, involving increasing or decreasing rates of bonus. In case I, which for reasons which I mention later seems to me to be the most important, k would have a value of £2 % and K would increase until it reached a rate of £2. 10*s*. 7*d*. %, i.e. the rate earned on the average by the premiums under the new conditions. Some discretion would be open to the actuary in fixing the intermediate values of K and the period to elapse before the limiting rate was reached, always provided that the total value of benefits, including future bonuses, equalled the value of the assets at 4 %. Clearly, the most convenient arrangement, if practicable, would be to make the period during which the rate of bonus was increasing fairly short, say ten years. If figures for method E, case I, were worked out on the lines of Tables 3 and 4, it would probably be found that from the standpoint of equity, method E would be less favourable than method A but more favourable than method B. It would, in fact, reach a high standard of equity.

The author mentions one objection to method E, i.e. the fact that the premium scale for new policies would be out of line, assuming that they were to receive the same rates of bonus as the old policies. Strictly speaking, the premiums ought to be reduced immediately and then gradually increased again until they reverted on the average to their former level when the ultimate rate of bonus was reached. The reductions would, however, be small except in the case of short-term policies, and it is doubtful whether any serious loss of equity would be involved if the premium scales were left unaltered for most classes of policy, in which case inconsistency between policies effected just before and just after the rise in interest rates would be largely avoided.

Of the four cases used throughout the paper to exemplify the distribution of assets, case I seems to be of special importance. It is true that the average maturity date of the assets would in general be later than according to case I, but it may nevertheless be argued that case I should be the basis for comparing the bonus-earning power of policies of different classes, durations and entry ages. The argument in favour of this procedure is that, in a closed fund, the maturity of the assets should conform fairly closely to the expected falling-in of the liabilities, and the investment of the assets in longer-dated securities is only justified in the case of a continuing institution. The profit or loss arising from a change in the rate of interest may be divided into two parts, (*a*) the profit or loss which would have occurred if the assets were invested in accordance with case I, and (*b*) the additional profit or loss due to their being invested on the average in longer-dated securities. Since the profit or loss under (*b*) would not have occurred if the office were not a continuing institution, is it unreasonable that future policies should receive a share of the profit or suffer part of the loss? The basic rate of bonus earned by a particular class of existing policies would then be the appropriate rate according to case I and the basic rate for future policies would be the rate earned by their premiums in the new conditions. These basic rates would then be adjusted by the same amount for all policies on account of the profits or losses under (*b*).

The paper does not refer to the possibility of a proportion of the funds being invested in irredeemable securities, including preference and ordinary shares. In some offices, a considerable proportion of the funds, e.g. from one-quarter to one-half, might be invested in this way. In that event, method A might result in a negative value of k, especially if the rise in interest rates were greater than that assumed in the paper. Conversely, a fall in interest rates might involve an exceptionally high value for k. This provides some support for my suggestion that the profit or loss from appreciation or depreciation should be divided into two parts. In the case of appreciation, part (*b*) would not be distributed, but would provide a reserve against future depreciation, the interest on which would earn additional bonuses for all policies, old and new. In the case of depreciation, the course to be followed would depend on the strength of the office's free reserves, whether arising from previous appreciation or from other sources. If these reserves were sufficient to cover part (*b*) of the loss due to depreciation and were applied to write off this loss, new and old policies alike would in future fail to receive the additional bonus paid in the past out of interest on this reserve, but the new policies should nevertheless still receive a bonus not less than that earned by their premiums under the new conditions.

Mr A. W. Joseph: The table on p. 204, which purports to show the percentage distribution of assets necessary so that the assets shall exactly match the liabilities of the assumed model office, is quite different from what would have been expected. E. H. Lever, in his paper to the Institute in 1937, when discussing the principle of finding investments to form the counterpart of policies issued, said (*J.I.A.* Vol. LXIX, p. 25) 'its complete application would exclude the investment of more than a small proportion of the funds in short-dated securities...'. Yet the author finds it necessary for the greater part of the investments to be short-dated.

It would seem that the author and Lever have quite different ideas of what constitutes matching the assets to the liabilities of an office. Surely the author's conception is one which has little practical value. Under his plan investments maturing each year meet the emerging claims, and all the premiums and investment income have to be invested. No doubt what Lever had in mind was a distribution of investments such that the investments maturing each year together with all the premiums and investment income accruing to the fund are just sufficient to meet the emerging claims. It is not at all difficult to calculate the form of the fund to satisfy this condition for the author's simple model office, and it will be found that the necessary maturities run nearly as a uniformly increasing series starting at a very low figure and increasing each year until the fortieth. The following table, set out in the same form as the author's on p. 204, shows the percentage distribution of the fund to satisfy the conditions postulated. Naturally it would be a matter of considerable practical difficulty to obtain and maintain such a distribution of assets, but the same applies to any other distribution discussed in the paper.

No. of years to maturity of investment	Percentage of total investments
1/5	1
6/10	4
11/15	7
16/20	10
21/25	14
26/30	18
31/35	21
36/40	25