The Actuarial Profession
making financial sense of the future

Mortality and Longevity Seminar
Joseph Lu & Ashley Kanter
2010

An uncertain baseline:

Credibility of mortality experience

Agenda

- The need to quantify uncertainty of mortality experience
- Solution 1: Monte Carlo Simulation
- Solution 2: Formula Approach
- Solution 3: Simplified Formula Approach
- Discussion

Background

Valuation and pricing of annuities or pensions liabilities need Mortality Assumptions

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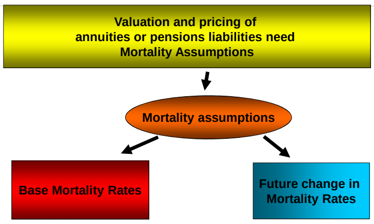
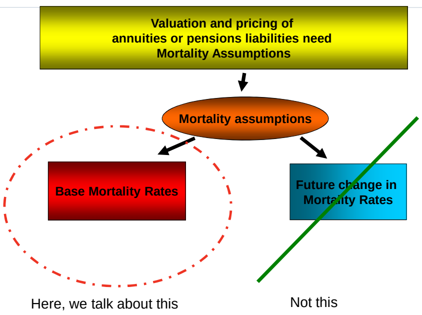
Mortality assumptions

Background



Background



Quote

"DEATH comes to everyone.
The timing is much LESS CERTAIN."

The Economist (2 Feb 2010) when commenting on longevity swaps

We must quantify uncertainty around mortality experience

- Understand Uncertainty
- Manage Risk

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The base mortality assumption
•Expressed as Actual/Expected (A/E)
•Of a life table e.g. PCMA00
•E.g. 92% PCMA00

•But rarely says 92% Plus or Minus What?

We must quantify uncertainty around mortality experience

- Understand Uncertainty
- Manage Risk
- Regulatory Requirements
- Measure Credibility of Experience

Problems

- Problem 1:
  - How do we measure uncertainty due to small number of people?

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- Problem 2
  - How do we measure increased volatility due to concentration of pension amount on a small number of people?
- Problem 3
  - How to use the solutions in practice?

We want to quantify uncertainty

	Number of 85 year-olds at the start of the year	Number of deaths *expected* in the year according to a life table which says that mortality rates of age 85 is 10%	Number of deaths *actually* observed in the year	Ratio of Actual over Expected (A/E)
Big 10K	10,000	1,000	1,000	100%
Small 1K	1,000	100	100	100%

**Intuitively, the result of Big 10K should be more certain than Small 1K.
But, how do we measure the uncertainty?**

We want to quantify uncertainty

	Number of 85 year-olds at the start of the year	Number of deaths *expected* in the year according to a life table which says that mortality rates of age 85 is 10%	Number of deaths *actually* observed in the year	Ratio of Actual over Expected (A/E)	95% confidence intervals of A/E ? This is what we try to find out
Big 10K	10,000	1,000	1000	100%	Missing?? 90-110%??
Small 1K	1,000	100	100	100%	Missing??

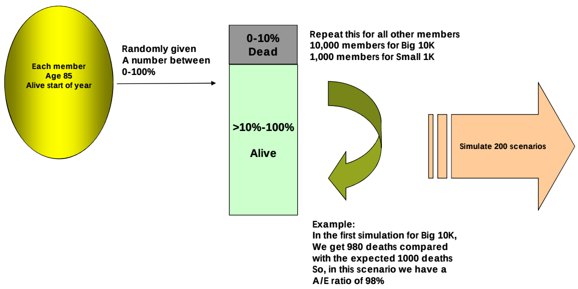
Want to work this out

Solution 1: Monte Carlo Simulation

Monte Carlo Casino (picture from Wikipedia)

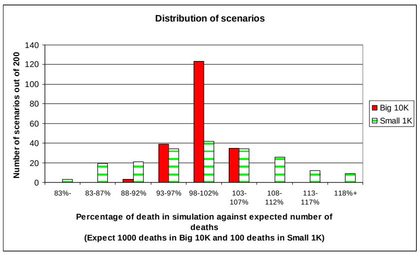
- Name coined in the 1940s after the Monte Carlo Casino
- Method involves computers to solve questions relating to random events
- Used in
  - Physics
  - Finance
  - Epidemiology/Medicine

Solution 1: Monte Carlo simulation



Result 1: The more the better

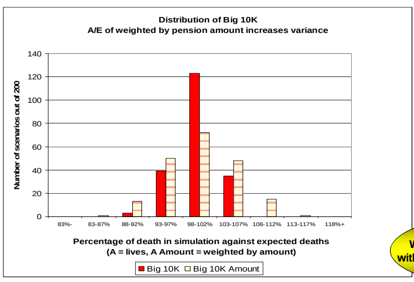
We get a distribution of A/E generated in the 200 scenarios for Big 10K and Small 1K



- Note that the distribution of Big 10K is narrower than Small 1K
- So confidence intervals for Big 10K is narrower than Small 1K,
- which is understandable because Big 10K has a larger dataset

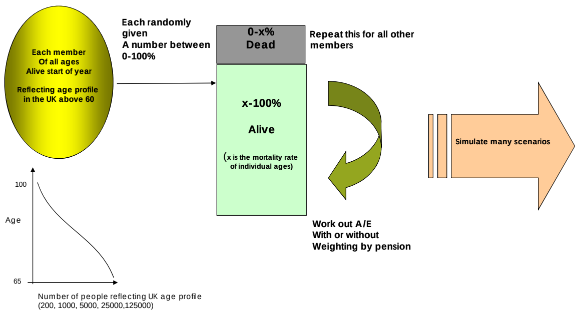
Result 2: Pension amount increase uncertainty

We can also get a distribution of A/E generated in the 200 scenarios for Big 10K and do a weighting by pension amount



Implication:
We get a wider distribution with amount weighting method,
So uncertainty increases

Solution 1: Monte Carlo simulation for all ages



Solution 1: Monte Carlo simulation

Strength

- This method has the advantage of using the full information in the data including age profile, exact pension amount of each pensioner directly
- So, it reflects uncertainty around the population's own mortality experience
- Would satisfy regulatory requirements

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Limitation

- Doesn't reflect weather – hot summer, cold winter, flu
- Need further adjustment for late reported deaths and mortality improvement
- Relatively intensive on computing power

Solution 1: Monte Carlo simulation

Discussion: Is there any less demanding method?

Solution 2: Formula to reflect results of Monte Carlo simulation

- Further details to be released in
  - 'Estimation of Confidence Intervals for Mortality Table Adjustments'
  - By A Kanter and J Lu (2010) Longevity Science on the Web Issue 2
  - www.legalandgeneral.com/lsweb

Solution 2: Formula to reflect results of Monte Carlo simulation

- var(A/E lives) = $(\Sigma\, n_x\, p_x\, q_x) / (\Sigma\, n_x\, q_x)^2$
- var(A/E amounts) = $(\Sigma\Sigma S_{ix}^2 p_x q_x) / (\Sigma\Sigma S_{ix} q_x)^2$

- x = age exact
- n = the number of population
- q_x = probability of a person age x dying over the year (standard notation)
- $p_x = 1 - q_x$
- S_i = Pension amount of person i

Compare 95% confidence intervals of Monte Carlo Simulation with Formula (3-year mortality investigation period)

Monte Carlo Simulation

Males	Three year investigation period			
	A/E (lives)	A/E (amounts) for different inequality levels		
Sample size		Low	Med	High
200	64% - 147%	55% - 161%	34% - 203%	23% - 242%
1000	83% - 120%	78% - 127%	63% - 143%	39% - 167%
5000	92% - 109%	89% - 111%	82% - 119%	69% - 133%
25000	97% - 103%	96% - 105%	92% - 108%	85% - 115%
125000	99% - 102%	98% - 102%	97% - 103%	90% - 108%

Formula (bold represents within 1% difference from above)

Males	Three year investigation period			
	A/E (lives)	A/E (amounts) for different inequality levels		
Sample size		Low	Med	High
200	60% - 143%	49% - 156%	21% - 192%	-4% - 227%
1000	**82% - 119%**	76% - 125%	61% - 142%	31% - 178%
5000	**92% - 108%**	**89% - 111%**	**82% - 119%**	**69% - 133%**
25000	**96% - 104%**	**95% - 105%**	**92% - 108%**	**85% - 116%**
125000	**98% - 102%**	**98% - 102%**	**96% - 104%**	**91% - 109%**

Compare 95% confidence intervals of Monte Carlo Simulation with Formula (1-year mortality investigation period)

Monte Carlo Simulation

Males	One year investigation period			
	A/E (lives)	A/E (amounts) for different inequality levels		
Sample size		Low	Med	High
200	43% - 184%	31% - 221%	12% - 281%	9% - 373%
1000	71% - 136%	62% - 148%	41% - 177%	29% - 208%
5000	86% - 115%	82% - 120%	69% - 133%	50% - 153%
25000	94% - 107%	92% - 109%	86% - 114%	73% - 125%
125000	97% - 103%	96% - 104%	94% - 106%	85% - 113%

Formula (bold represents within 1% difference from above)

Males	One year investigation period			
	A/E (lives)	A/E (amounts) for different inequality levels		
Sample size		Low	Med	High
200	28% - 172%	5% - 195%	-51% - 251%	-106% - 306%
1000	68% - 132%	57% - 143%	29% - 171%	-20% - 220%
5000	**86% - 114%**	**81% - 119%**	**68% - 132%**	46% - 154%
25000	**94% - 106%**	**91% - 109%**	**86% - 114%**	**74% - 126%**
125000	**97% - 103%**	**96% - 104%**	**94% - 106%**	**85% - 115%**

How to use the Formula approach?
What if we want a simpler approach?

How to use the Formula approach?
What if we want a simpler approach?

- Just need a few inputs:
  - Number of annuitants
  - Average Age
  - Concentration of pension, e.g. 10% of people own 45% of pension
  - Number of years of data
  - Base table

Solution 3: A 'Simplified Formula' Approach

Variance = 1/(number of people x Observed A/E) x
Base Variances x
Inequality multiplier x
Adj factor (number of years) x
Base table adjustment

Base Variances		
Avg age	Male	Female
65	50.35	67.97
66	45.68	60.30
67	41.18	53.24
68	36.94	46.87
69	32.98	41.13
70	29.33	36.08
71	26.00	31.63
72	23.00	27.75
73	20.30	24.36
74	17.91	21.40
75	15.78	18.83

Pension Inequality			
Top 10% own	Coeff. of var	Gini	Inequality multiplier
30%	0.88	0.41	1.78
35%	1.12	0.48	2.25
40%	1.37	0.53	2.89
45%	1.69	0.59	3.84
50%	2.04	0.63	5.15
55%	2.51	0.68	7.30
60%	3.06	0.72	10.39
65%	3.91	0.76	16.26
70%	5.05	0.80	26.47

Multiyr investigation	
m	Adj factor
1	1.000
2	0.478
3	0.304
4	0.218
5	0.166
6	0.131
7	0.107
8	0.088
9	0.074
10	0.063

Base table		
Table	m adj	f adj
PCXA00	1.00	1.00
PCXL00	0.75	0.83
PEXA00	0.94	0.83
PEXL00	0.66	0.73
PXA92	0.95	1.09
PNXA00	1.05	1.04
PNXL00	0.81	0.87
PPXV00	1.44	1.55
RXV00	0.93	1.28
SPXA03	0.81	0.80
SPXL03	0.58	0.68
a55X	0.36	0.47

Compare 95% confidence intervals of Solutions 2 and 3
(3-year mortality investigation period)

Formula (Solution 2) (bold represent consistency with Solution 1)

Males	Three year investigation period			
	A/E (lives)	A/E (amounts) for different inequality levels		
Sample size		Low	Med	High
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Simplified formula close enough for key figures?

Simplified Formula (Solution 3)

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Compare 95% confidence intervals of Solutions 2 and 3
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Simplified formula close enough for key figures?

Comparison of the 3 methods

Monte Carlo

Formula

Simplified Formula

Computing complexity

Time

Level of

Skills

Conclusion

- Problem 1: How do we measure uncertainty due to small number of people?
- Problem 2: How do we measure increased volatility due to concentration of pension amount on a small number of people?
- Problem 3: How to use the solutions in practice?

Conclusion

- **Answer:**
  - Use one of the 3 methods
  - But we welcome suggestions of any other methods

Acknowledgment

- We thank the Longevity Science Advisory Panel (http://www.longevitypanel.co.uk/index.html) for their comments on the methodology
  - Sir Derek Wanless
  - Sir John Pattison
  - Professor Colin Blakemore
  - Professor Klim McPherson
  - Professor Steven Haberman

Note:

The Longevity Science Advisory Panel is independent and is not responsible for the views and methods discussed in the presentation.

Questions for Discussion

- In your capacity, what are the benefits of understanding the uncertainty around Base Mortality?
- How do you quantify uncertainty around Base Mortality for your Stakeholders?
- What would be your preferred method in practice?