JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On the Valuation of Staff Pension Funds. Part 2.—Widows' and Children's Pensions (continued). By HENRY WILLIAM MANLY, *Past-President of the Institute of Actuaries. With Tables by* WILLIAM ARTHUR WORKMAN, *of the Equitable Life Assurance Society, Fellow of the Institute of Actuaries.*

[Read before the Institute, 25 November 1907.]

AT the end of my paper on this subject, read before the Institute on the 27 April 1903 (*J.I.A.*, vol. xxxviii, p. 101), I referred to the question of fines on re-marriage, and the possibility of including the risk of second and subsequent marriages in the original contributions. I had previously explained that if the value of the risk of the first marriage only had been accurately calculated, the fine on the second and every subsequent marriage must be $a_y - a_{xy}$, or, to be more accurate, $\bar{a}_y - \bar{a}_{xy}$. This, I stated, was not likely to be tolerated in the Rules of any of these Funds.

It might be possible to substitute a fine on the death of the wife, in which case the calculations would have to be based on tables constructed similarly to those given by Dr. Sprague at the end of his paper "On the Rates of Re-marriage among Widowers" (*J.I.A.*, vol. xxii, p. 77). The fine would be much less than the value of a survivorship annuity on re-marriage, but I do not think that such a scheme would commend itself to the members of these Funds. They would be asked to pay a fine at a time when expenses were heavy, and no doubt all would declare that they had no intention of re-marrying.

There are left only two courses, either (i) to include the value of the risks of second and subsequent marriages in the original contribution, or (ii) to fix beforehand what the fine is to be on re-marriage and to calculate the contribution accordingly. Either of these methods involves the construction of a table showing the numbers who marry and re-marry at each age, as well as the numbers who die as husbands and widowers after the first, second, third, and subsequent marriages.

If we were going to estimate the exact value of the risk in respect of each person entering the Fund, irrespective of fines, we should require three sets of elaborate tables for each possible age at entry to the Fund. Thus we should start at age 15 with 10,000 bachelors, and trace them for every year of life, recording how many would die as bachelors, how many as husbands, and how many as widowers of the first, second, and subsequent marriage at every age until all were dead. As the earliest age at marriage is 20, we should next start at that age, and at every subsequent age, with 10,000 bachelors, and trace them to the end of life. For bachelors we should want 45 select tables. For the husbands we should have to start at age 20, and at every subsequent age, with 10,000 husbands, and trace each set separately to the end of life, recording at each age in each table the deaths of husbands and widowers after the first, second, and subsequent marriages. For these 44 select tables would be required. Then for widowers we should have to start at age 21, and at every subsequent age, with 10,000 widowers, tracing them to the end of life, and recording at each age in each table the number who die as widowers of the first marriage, the numbers who die as husbands and widowers of the second marriage, and so on. But if extreme accuracy be required, the tables for widowers at each age at entry should be divided into separate tables for 1st year of widowerhood, 2nd year of widowerhood, 3rd year of widowerhood, and so on, because the probability of re-marriage of a widower varies with the length of widowerhood, so that for widowers about 400 select tables would be necessary. Then if the element of withdrawals be introduced, it will be seen that the work would assume gigantic proportions. One set of tables could be made to help the others: thus, by calculating the tables for widowers first, the tables for husbands would be easier of construction; and when the tables for husbands were completed, the tables for bachelors would be easier to calculate.

Such a series of tables would be extremely useful for making a valuation, but the labour of calculating them would be out of all proportion to their value. These Funds are of the nature of Co-operative Mutual Benefit Associations, and not Societies for the insurance of individual risks. Each person contributes either a fixed annual sum or a percentage of his salary towards providing for all the widows and orphans in the whole body; and, if the bachelor and widower do happen to pay more than the true mathematical value of their risks, I think there are some members of the opposite sex who would be inclined to say "And quite right, too."

It would seem, therefore, that the table best suited for our purpose is an aggregate one, showing the number living and remaining on the staff at each age, and the numbers who die at each age as bachelors, husbands and widowers. If all those on the staff entered at about the same age, then the calculation, by such a table, of the contributions required would be fairly accurate, and the reserve by such a table would probably be greater than a reserve by Select Tables, because the contributions to be valued are not net premiums.

In the first paper in this section, "Widows' and Children's Pensions" (*J.I.A.*, vol. xxxviii, p. 105), I described how my Table 45 was constructed. My Table No. 3 (*J.I.A.*, vol. xxxvi, p. 261), showing the numbers remaining, withdrawing and dying each year out of a certain number entering at a given age, was extended to the end of life, and taken as a basis. The numbers dying were then divided into "bachelors" and "married" by means of one of Mr. A. Hewat's tables, and the married were subdivided up to age 65 into husbands and widowers by means of Mr. G. King's Table C, in his paper "On Family Annuities." The numbers of husbands and widowers dying after 65 were obtained by ascertaining how many couples out of husbands and wives surviving at age 65 would survive every year to the end of life. This was facilitated by assuming that no marriage after 65 was recognized by these Funds. A table constructed with such various materials cannot be considered as very satisfactory, and it would be better, if possible, to build up one on scientific principles.

I shall now describe how Table No. 60, on pp. 30–1, was constructed scientifically. The rates of mortality and withdrawal were to be the same as used in my Table No. 3, but, as it was necessary to have a larger radix, the table starts with 200,000 living at age 15 instead of 20,000, so that the number living at each age is 10 times greater than in Table 3.

We started with the construction of a table for husbands alone, showing, out of 100,000 husbands at the age of 20, the number of husbands which would be left at each age; the number of husbands becoming widowers and the total widowers at each age; the husbands who die as husbands and the widowers who die at each age; and the numbers of husbands and widowers who withdraw at each age. A check table based on my ${}^{w}q_x$ and modified q_x was formed showing the total numbers dying and withdrawing at each age. For the death-rate amongst wives, I used Mr. Hewat's "Probability of dying in a year" amongst Scottish bankers' wives. (Page 44 of his work). For the age of the wife, I used the same age for y, corresponding to the age x as in Table K. The table therefore will not represent the probable number who will become widowers throughout the remainder of life, out of the husbands living at any given age, by the deaths of their own wives; but, at any given age x, will give the number of husbands becoming widowers between the ages of x and $x+1$, out of the husbands living at age x married at all ages up to age x.

The Table required most careful and delicate construction.

Column 2 contains the number of husbands existing as husbands at each age.

Column 3 contains the number of husbands of the age x becoming widowers between the ages x and $x+1$. The number who become widowers and survive to age $x+1$ is evidently ${}^{H}l_x \times p_x q_y$; but then there are a certain number of cases where both husband and wife die in the year, and in half those cases we may assume that the wife dies first, so that $\frac{1}{2}{}^{H}l_x \times q_x q_y$ will be husbands who become widowers in the year and die in the year; consequently, ${}^{H}kl_x = {}^{H}l_x(p_x q_y + \frac{1}{2}q_x q_y)$.

Column 5 contains the number of husbands who die as husbands in the year. This will be ${}^{H}l_x q_x$, less those cases where both husband and wife die in the year and the wife dies first $= \frac{1}{2}{}^{H}l_x \times q_x q_y$; consequently, ${}^{H}d_x = {}^{H}l_x(q_x - \frac{1}{2}q_x q_y)$.

Column 6 contains the husbands becoming widowers in the year and dying before the end of the year. These are the cases where both husband and wife die in the year and the wife dies first $= \frac{1}{2} \mathsf{H} l_x \times q_x q_y$.

Column 8 contains the number of husbands withdrawing as husbands during the year. The number of husbands living at age x is $\mathsf{H} l_x$, and the number who withdraw during the year will be $\mathsf{H} l_x \times {}^w q_x$; but then $\mathsf{H} k l_x$ will become widowers during the year; and on the principle that they will be exposed to risk on the average, half a year, $\frac{1}{2} \mathsf{H} k l_x \times {}^w q_x$ will become widowers before they withdraw, hence those who withdraw as husbands will be $(\mathsf{H} l_x - \frac{1}{2} \mathsf{H} k l_x)\, {}^w q_x$.

Column 9 contains the number of husbands who will become widowers and withdraw as widowers during the year $= \frac{1}{2} \mathsf{H} k l_x \times {}^w q_x$.

Column 4 contains the number of widowers at age x where

$$\begin{aligned} \mathsf{K} l_{x+1} &= \mathsf{K} l_x + \mathsf{H} k l_x - \mathsf{K} d_x - \mathsf{H} k d_x - \mathsf{H} k w_x - \mathsf{K} w_x \\ &= \mathsf{K} l_x - \mathsf{K} d_x - \mathsf{K} w_x + \mathsf{H} k l_x - \mathsf{H} k d_x - \mathsf{H} k w_x, \end{aligned}$$

that is to say the widowers living at age $x+1$ will be the widowers living at age x less those who die and withdraw, and the husbands who become widowers between the ages of x and $x+1$ less those who die and withdraw before the end of the year.

Column 7 contains the number of widowers who die between x and $x+1$ out of those who enter as widowers at age $x = \mathsf{K} l_x \times q_x$.

Column 10 contains the number of widowers who withdraw between x and $x+1$ out of those who enter as widowers at age $x = \mathsf{K} l_x \times {}^w q_x$.

There should be another column showing the number of widowers living at age $x+1$ out of those who become widowers between ages x and $x+1$; that is $\mathsf{H} k l_x - \mathsf{H} k d_x - \mathsf{H} k w_x$, which for distinction I would call Hl_{x+1} or $\mathsf{H}_x l_{x+1}$.

Column 2 contains the numbers surviving as husbands at age x, so that $\mathsf{H} l_{x+1} = \mathsf{H} l_x - \mathsf{H} k l_x - \mathsf{H} d_x - \mathsf{H} w_x$, that is to say, the number of husbands existing at age $x+1$ will be the number living at age x less those who become widowers during the year less the numbers who die and withdraw as husbands during the year.

The total deaths during the year, namely, $\mathsf{H}d_x + \mathsf{H}kd_x + \mathsf{K}d_x$ must equal the d_x in the check table, and the total withdrawals, namely, $\mathsf{H}w_x + \mathsf{H}kw_x + \mathsf{K}w_x$ must equal the w_x in the check table.

Column 5 shows the number of widows to be provided for, and the sum of columns 6 and 7 the number of widowers dying.

Next it was necessary to have a table showing the number of widowers who re-marry at each age and the number who die as widowers without re-marrying; but in order to do this we must know the rate of re-marriage of widowers at each age amongst the class with which we are supposed to be dealing.

After examining all the various data and tables of re-marriage, I came to the conclusion that Mr. Hewat's table of the "Probability of a Bachelor marrying in a Year", on page 21 of his work on "An Investigation of the Marriage and Mortality Experience of a Scottish Ministers' Widows' and Orphans' Fund", would represent fairly well the probability of a widower marrying in a year, amongst the staff in a commercial or banking institution. The following is the table of rates adopted.

TABLE P.

Probability of a Widower marrying in a year $= m_x^2$.

Age (x)	Probability of Re-marrying in a Year m_x^2	Age (x)	Probability of Re-marrying in a Year m_x^2	Age (x)	Probability of Re-marrying in a Year m_x^2
21	·10	36	·12	51	·04
22	·10	37	·11	52	·04
23	·12	38	·10	53	·04
24	·15	39	·09	54	·04
25	·18	40	·08	55	·04
26	·20	41	·07	56	·04
27	·21	42	·06	57	·04
28	·20	43	·05	58	·04
29	·19	44	·05	59	·04
30	·18	45	·05	60	·04
31	·17	46	·04	61	·03
32	·16	47	·04	62	·03
33	·15	48	·04	63	·03
34	·14	49	·04	64	·03
35	·13	50	·04	65	·00

In making a table for widowers, we trace them in a similar way as we did the husbands for one complete year, but I have omitted the probability of a widower becoming a husband and dying as a widower before the end of the year, as the figures would be too small to be of any account.

The Table for widowers will be found on pages 28–9.

Column (2) shows the number of widowers existing at each age $x = \kappa l_x$.

Column (3) shows the number of widowers who marry between the ages x and $x+1 = \kappa h l_x$.

Column (6) shows the number of widowers who die as widowers after first marriage only $= (\kappa l_x - \frac{1}{2}\kappa h l_x) q_x$.

Column (7) shows the number of widowers for the first time who withdraw $= (\kappa l_x - \frac{1}{2}\kappa h l_x)^w q_x$.

Column (8) shows the number of widowers who marry and die as husbands between the ages x and $x+1$ $= \frac{1}{2}\kappa h l_x \times {}^w q_x$.

and Column (9) shows the number who marry and withdraw between the ages x and $x+1 = \frac{1}{2}\kappa h l_x \times {}^w q_x$.

Column 10 shows the number of husbands for the second time existing at the next higher age—

$$H^2 l_{x+1} = \kappa h l_x - (\kappa h d_x + \kappa h w_x).$$

A check table was made from the formula

$$\kappa l_{x+1} = \kappa l_x \{1 - [m_x^2 + (1 - \tfrac{1}{2} m_x^2)(q_x + {}^w q_x)]\}$$

We next started with the table for Bachelors, which was constructed in a similar way to the table for Widowers, as the probability of a bachelor marrying and dying a widower in the same year was omitted, the figures being insignificant. The probability of a bachelor marrying in a year was that given by Mr. A. Hewat for "Scottish Bankers" in his Table C. We started with 200,000 bachelors at the age of 15, in order to compare easily the results with Table No. 45.

The Table is printed on pages 24–5, and it will be seen that the headings are the same as in the table for Widowers, with the substitution of B for K.

Column (10) shows the number of husbands existing at age $x+1$ out of those who married between the ages x and $x+1$.

By means of our table for Husbands, it is possible to trace the fate of these husbands to their first widowerhood.

I will ask you first to look at the Table for husbands. There you will find that out of 91,418 husbands living at age 21, so many will become widowers, so many die as husbands for the first time, so many die as widowers for the first time, and so many survive as widowers, at each age to the end of the table, as set out in the respective columns. Now, if you look at the table for Bachelors, you will see that there are 306 husbands starting at the age of 21, who can be traced to their first widowerhood by multiplying the columns in the Husbands' table, commencing at age 21, by $\frac{306}{91,418}$. Similarly, the 952 husbands in the Bachelors' table, starting at age 22, can be traced to their first widowerhood by multiplying the columns in the Husbands' table, commencing at age 22, by $\frac{952}{84,190}$, and so on.

We have, therefore, to find the ratio of Hl_x in the Bachelors' table to the Hl_x in the Husbands' table for each age, which, for the moment, we will call r_x. Now if we set out the details of one of the columns in the Husbands' table, say column Hd_x, and place the ratios underneath, thus—

$$\begin{array}{ccccc}
Hd_x & Hd_{x+1} & Hd_{x+2} & Hd_{x+3} & Hd_{x+4}\ldots \\
r_x & r_x & r_x & r_x & r_x\ldots\ldots \\
 & r_{x+1} & r_{x+1} & r_{x+1} & r_{x+1}\ldots \\
 & & r_{x+2} & r_{x+2} & r_{x+2}\ldots \\
 & & & r_{x+3} & r_{x+3}\ldots \\
 & & & & r_{x+4}\ldots
\end{array}$$

it will be seen that, by making a constant addition of the ratios, we shall obtain a series of multipliers which, when multiplied into the figures in the column in the Husbands' table opposite the same age, will give us the fate of all the husbands existing at each age $x+1$ in the Bachelors' table up to their first widowerhood. We thus obtain the columns Hd_x, Hkd_x, and Kl_{x+1}.

By applying the Widowers' Table in the same way to the newly obtained Kl_{x+1} column, we shall obtain the columns Kd_x, Khd_x, and H^2l_{x+1}. Then following the same process as before with the figures in the column H^2l_{x+1} as we did with the Hl_{x+1} column, we obtain H^2d_x, H^2kd_x, and K^2l_{x+1}; and the process can be repeated until there are no widowers under the age of 65 to get married.

It might be advisable to explain that the numbers recorded as dying as husbands and widowers at age x are not the deaths among those who contracted marriages at age x alone, but also among the members who had been previously married and were existing at age x.

Considering the conglomerate way in which my Table No. 45 was constructed from materials obtained from various sources, it would be extremely interesting to compare the two tables, and, incidentally, to ascertain how far second and third marriages were included in the first table. This can be easily done by dividing all the figures in Table 60 by 10. I have prepared a diagram showing the number who die as husbands in Table 45, the number who die as husbands of the first marriage, and the total who die as husbands after any marriage in Table 60; also the number who die as widowers in Table 45, the number who die as widowers of the first marriage, and the number who die as widowers after any marriage in Table 60.

It would appear from this diagram that more than first marriages were included in Table 45 up to age 70, but that after that age the widowers increased rapidly. The actuarial calculation after age 65, based on the English Life Table No. 3, evidently had the result of increasing the mortality among wives.

This diagram is not open to the objection to diagrams showing the number of deaths at each age in two different Life Tables, because the number living at each age and the number of deaths and withdrawals at each age are the same in both tables.

Table No. 60 will only afford the means of ascertaining the values of the liabilities arising from the death of a husband or widower. I have not thought it necessary to make a table for ascertaining the value of a fine on remarriage, but, if any Member of the Institute would like to undertake the task, I am sure we should all be delighted. He would start with the Bachelors' Table, recording ${}^{\mathrm{B}}l_x$ and Hl_{x+1} for each age. Then by tracing the number of Hl_x at each age separately to their first widowerhood by means of the Husbands' Table and in the way explained on p. 8, he would record ${}^{\mathrm{H}}l_x$, Kl_{x+1}. By use of the Widowers' Table he would obtain ${}^{\mathrm{K}}l_x$, ${}^{\mathrm{K}}hl_x$, and H^2l_{x+1}, and using the Husbands' Table again for H^2l_x for each age separately, he would obtain ${}^{\mathrm{H}^2}l_x$ and K^2l_{x+1}, and so on. The total of all the Bachelors, Husbands, and Widowers living at each age, that is ${}^{\mathrm{B}}l_x + {}^{\mathrm{H}}l_x + {}^{\mathrm{K}}l_x + {}^{\mathrm{H}^2}l_x + {}^{\mathrm{K}^2}l_x +$ &c., would be equal to l_x in Table 60, and the fines would be levied on ${}^{\mathrm{K}}hl_x$, ${}^{\mathrm{K}^2}hl_x + {}^{\mathrm{K}^3}hl_x +$ &c.

A great number of Monetary Tables have been calculated on the Bachelors', Husbands' and Widowers' Tables, which might prove useful in certain cases, but I shall not publish them unless it is the wish of the Council that they should appear in the *Journal.*

The numbers of Husbands and Widowers dying and the values of

$$\frac{{}^{wa}\mathrm{M}_x}{\mathrm{D}_x},\ \frac{{}^{\mathrm{R}(16)}\mathrm{M}_x}{\mathrm{D}_x},\ \frac{{}^{\mathrm{K.YC}a}\mathrm{M}_x}{\mathrm{D}_x} \text{ and } \frac{{}^{\mathrm{O}a(16)}\mathrm{M}_x}{\mathrm{D}_x}$$

for first marriage only and for all marriages at 4 per-cent

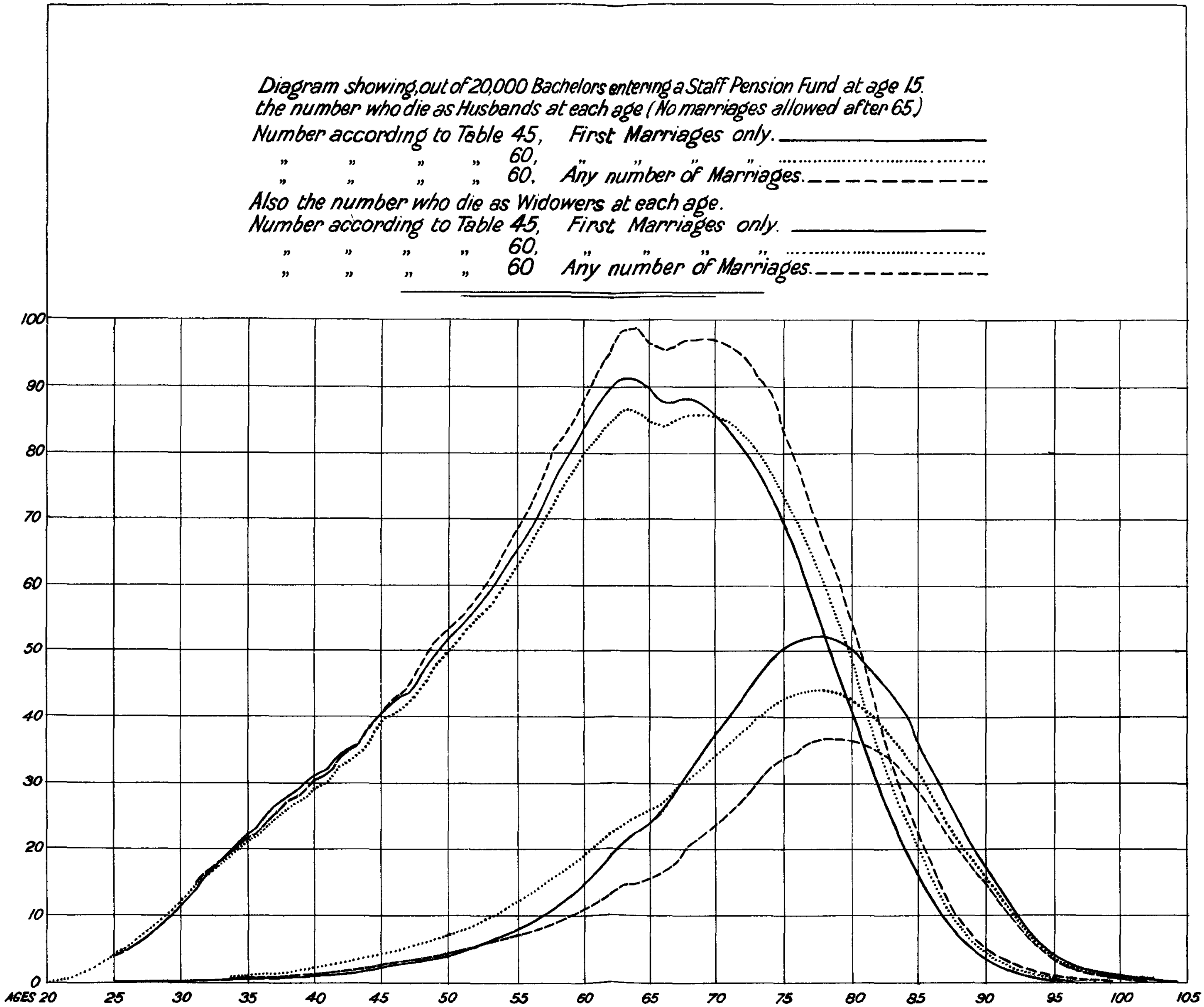

Diagram showing, out of 20,000 Bachelors entering a Staff Pension Fund at age 15.
the number who die as Husbands at each age (No marriages allowed after 65.)
Number according to Table 45, First Marriages only.
" " " " 60, " " " "
" " " " 60. Any number of Marriages.
Also the number who die as Widowers at each age.
Number according to Table 45, First Marriages only.
" " " " 60, " " " "
" " " " 60 Any number of Marriages.
100
90
80
70
60
50
40
30
20
10
0
AGES 20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
105

interest are given on pages 32–5. The number of *married* men dying at each age being the same whether first marriages only are taken, or all marriages, the value of $\frac{{}^{oa(16)}M_x}{D_x}$ will be the same for both. By these tables comparison can be made of the value of the benefits when only one marriage is allowed for, and when all marriages are included in the contribution. The difference will of course be the value, at entry, of the benefit of including all marriages after the first.

I should like here to acknowledge my great indebtedness to Mr. W. A. Workman, F.I.A., for his careful construction of these tables, over which he has expended a great amount of labour, and exercised the most exemplary patience. If it had not been for his perseverance, sometimes under the most trying circumstances, when the work seemed to go wrong, I do not think these tables would have seen the light.

THE IMPORTANCE OF A CORRECT SCALE OF SALARIES.

In the first part of this work I stated that the amount of the salary was not very important so long as we had a table representing the relative yearly increase; and I showed on page 232 (vol. xxxvi) that the formulas were independent of the amount of salary. The reason was that all the benefits there investigated were functions of the salary, and could only vary in relation to the salary according to the ratio of increase in such salary. The case is very different with the benefits we are now investigating, for they are in no sense a function of the salary, yet the contributions are dependent on the amount of salary and its probable increase; consequently, it is very important that the scale of salaries used in the calculations should represent the actual salaries and their average increase as accurately as possible. In the illustrations to this second part of the work, I have used the scale adopted in the first part, because the tables were already calculated; but I have another scale representing the average salary in an old but progressive Bank, and I will place it side by side with the other scale representing the salaries in a Railway Company, together with the quinquennial ratio of increase in each. When the ratios of increase alone form the basis of the calculations, the difference in the results would probably not be very great; but when the amounts of the salaries as well as the ratio of increase have to be taken into consideration the difference would be large.

TABLE S.

Showing two different scales of Average Annual Salaries, representing those of a great Railway Company, and a large Bank, and the ratio of increase in every 5 years.

Age (x)	Railway Company	Quinquennial Ratio of Increase	Bank	Quinquennial Ratio of Increase	Age (x)
15	20	...	20	...	15
16	25	...	25	...	16
17	30	...	30	...	17
18	35	...	40	...	18
19	40	...	50	. .	19
20	45	2·250	60	3·000	20
21	50	...	70	...	21
22	55	...	80	...	22
23	60	...	90	...	23
24	65	...	100	...	24
25	70	1·555	110	1·833	25
26	74	...	120	...	26
27	78	...	130	...	27
28	82	...	140	...	28
29	86	...	150	...	29
30	90	1·286	160	1·455	30
31	94	...	170	...	31
32	98	...	180	...	32
33	102	...	190	...	33
34	106	...	200	...	34
35	110	1·222	210	1·313	35
36	114	...	220	...	36
37	118	...	230	..	37
38	122	...	240	...	38
39	126	...	250	...	39
40	130	1·182	260	1·238	40
41	134	...	270	...	41
42	138	...	280	...	42
43	142	...	290	...	43
44	146	...	300	...	44
45	150	1·154	310	1·192	45
46	154	...	320	...	46
47	158	...	330	...	47
48	162	...	340	...	48
49	166	...	350	...	49
50	170	1·133	360	1·161	50
51	174	...	370	...	51
52	178	...	380	...	52
53	182	...	390	...	53
54	186	...	400	...	54
55	190	1·118	405	1·125	55
56	194	...	410	...	56
57	198	...	415	...	57
58	202	...	420	...	58
59	206	...	425	...	59
60	210	1·105	430	1·062	60
61	214	...	435	...	61
62	218	...	440	...	62
63	222	...	445	.	63
64	226	...	450	...	64

CONCLUSION.

In taking farewell of this work, permit me to say that I greatly appreciate the good opinions which have been expressed upon it by all members of the profession. I have been encouraged to go much further than I ever intended; but the work has had a fascination which I could not resist.

The formulas I have deduced are universal in their application, but the material on which we have to base our calculations is often of very inferior quality. It would be impossible to make a standard Table of Experience to apply to all Funds alike, for rates of mortality, marriage, withdrawal, retirement and salary will vary, not only in different trades and institutions, but in different districts.

When we have to value a Staff Pension Fund, it is often possible to obtain enough material to deduce a fairly good experience; but for the valuation of a Widows' and Orphans' Fund, there is no institution large enough to afford all the material necessary to make such a table as we require. Our only hope is to induce every Fund to keep its records on cards of one common form, and at some future time for the Institute or some Committee to undertake the task of collecting these cards and extracting therefrom the experience for each class of risk. The following is the form of card which I would suggest. Strict injunctions should be given that no card is ever to be destroyed.

Front of Card.

Name of Fund ....

No. ....

Name ....

Occupation or Rank ....

DATE OF			Age of			
			Member		Wife	
	Y.	M.	Y.	M.	Y.	M.
Birth						
Entry on Fund ...						
1st Marriage						
Death, 1st Wife ...						
2nd Marriage						
Death, 2nd Wife ...						
Withdrawal						
Retirement on Pension						
Death						

Christian Name of

1st Wife ....

2nd Wife ....

N.B.—This for identification.

CHILDREN							CHILDREN						
	Date of				Age at D'th of Memb'r			Date of				Age at D'th of Memb'r	
	Birth		Death					Birth		Death			
	Y.	M.	Y.	M.	Y.	M.		Y.	M.	Y.	M.	Y.	M.
1							7						
2							8						
3							9						
4							10						
5							11						
6							12						

REMARKS:—

N.B.—This for 3rd and subsequent marriages.

Back of Card.

No. ....

Name ....

Occupation or Rank ....

Year	Salary	Year	Salary	Year	Salary
19		19		19	
19		19		19	
19		19		19	
19		19		19	
19		19		19	
19		19		19	
19		19		19	
19		19		19	
19		19		19	
19		19		19	
19		19		19	
19		19		19	
19		19		19	
19		19		19	
19		19		19	

Annuity of £.......... commenced ....

Widow died .... Aged ....

Continued to Children till ....

£.......... paid to ....
on death of Member as a (Bachelor) (Widower without children under 16).

REMARKS :—

N.B.—This for fines on re-marriage, &c.

POSTSCRIPT.

In his paper "On Staff Pension Funds" (*J.I.A.*, xxxix, p. 129), Mr. George King says in a postscript (p. 179) that he fortunately showed a proof of his Addendum to Mr. E. C. Thomas, who brought to his notice a shorter and more elegant way, due to Mr. Manly, of finding the value of ${}^{f}_{y}\mathrm{F}^{ra}_{x}$.

Let $${}^{s-1}_{y}\mathrm{M}^{ra}_{x} = s_{x-1} \times {}_{y}\mathrm{M}^{ra}_{x}$$

and $${}^{s-1}_{y}\mathrm{R}^{ra}_{x} = \Sigma^{s-1}_{y}\mathrm{M}^{ra}_{x}$$

Then $${}^{f}_{y}\mathrm{F}^{ra}_{x} = \frac{{}^{s-1}_{y}\mathrm{R}^{ra}_{x+t}}{{}^{s}\mathrm{D}_{x}}$$

This puzzled me terribly. I naturally felt flattered by Mr. King's description of my work, but could not recognize my own child in the garb in which he had dressed it. I asked Mr. Thomas what it was supposed to represent, and he said it was what we called the ψ formula which we used in a heavy piece of work in which he ably assisted me in 1903–4. Whenever I have had occasion to refer to Mr. King's paper, I have always been attracted to that page, but I could never understand the demonstration, and could never recognize in it anything which I had done. This is so unlike Mr. King, because as a rule his demonstrations are very clear.

Recently I had occasion to open out the pile of papers connected with that piece of work, and I found the formula. My demonstration of it took more than three lines, it, in fact, took ten. I did not think anything particular of it at the time, but Mr. King has made the problem famous. There will be, therefore, no harm in my giving a full demonstration of the Problem in my own way.

Problem XVIB. To find the value of a pension based on a varying proportion of average salary according to the number of years of service, no pension allowed if retirement takes place within t years.

This is a variation of my Problem XB (*J.I.A.*, xxxvi, p. 238); and as the simplest way of dealing with problems in average salary is to alter the form and deal with total salary, it will be desirable to change "proportion of average salary" into "proportion of total salary."

Let us call the completed years of membership t, and the proportion of total salary corresponding to t, ψ_t.

To make the problem clearer, I give two Specimen Pension Scales. The first is taken from Mr. King's paper (*J.I.A.*, xxxix, p. 162), and the other is one-fiftieth of average salary for each completed year, up to a maximum proportion of two-thirds of average salary.

Specimen Pension Scale. No. 1.

Completed Years of Membership t	Pension in percentage of average Salary	Equivalent proportion of total Salary (ψ_t)	Completed Years of Membership t	Pension in percentage of average Salary	Equivalent proportion of total Salary (ψ_t)
10	25	·02500	30	48	·01600
11	26	·02364	31	50	·01613
12	27	·02250	32	52	·01625
⋮	⋮	⋮	⋮	⋮	⋮
19	36	·01895	39	66	·01692
20	37	·01850	40	$66\frac{2}{3}$	·01667
21	38	·01810	41	$66\frac{2}{3}$	·01626
22	39	·01773	42	$66\frac{2}{3}$	·01587
⋮	⋮	⋮	⋮	⋮	⋮

Specimen Pension Scale. No. 2.

Completed Years of Membership (t)	Pension in percentage of average Salary	Equivalent proportion of total Salary (ψ_t)	Completed Years of Membership (t)	Pension in percentage of average Salary	Equivalent proportion of total Salary (ψ_t)
10	20	·02	33	66	·02
11	22	·02	34	$66\frac{2}{3}$	·01961
12	24	·02	35	$66\frac{2}{3}$	·01905
			36	$66\frac{2}{3}$	·01852
⋮	⋮	⋮	37	$66\frac{2}{3}$	·01802
			38	$66\frac{2}{3}$	·01754
20	40	·02	39	$66\frac{2}{3}$	·01709
21	42	·02	40	$66\frac{2}{3}$	·01667
22	44	·02	41	$66\frac{2}{3}$	·01626
⋮	⋮	⋮	⋮	⋮	⋮

As the scale varies with years of membership, and not according to age, it will be necessary to make a table for each probable age at entry.

I proceed to demonstrate the solution in the same way as I have done in all my problems.

Out of l_x persons entering at age x, r_{x+t} will retire between the years t and $t+1$, and the value of the pensions will be (assuming that s is paid for the whole year),

$$v^{x+t+\frac{1}{2}}r_{x+t}\bar{a}_{x+t+\frac{1}{2}}\psi_t s_x + v^{x+t+\frac{1}{2}}r_{x+t}\bar{a}_{x+t+\frac{1}{2}}\psi_t s_{x+1} \ldots$$
$$+ v^{x+t+\frac{1}{2}}r_{x+t}\bar{a}_{x+t+\frac{1}{2}}\psi_t s_{x+t-1}$$

The value of the pensions to those who retire in the year $t+1$ to $t+2$ will be

$$v^{x+t+1\frac{1}{2}}r_{x+t+1}\bar{a}_{x+t+1\frac{1}{2}}\psi_{t+1}s_x + v^{x+t+1\frac{1}{2}}r_{x+t+1}\bar{a}_{x+t+1\frac{1}{2}}\psi_{t+1}s_{x+1} \ldots$$
$$+ v^{x+t+1\frac{1}{2}}r_{x+t+1}\bar{a}_{x+t+1\frac{1}{2}}\psi_{t+1}s_{x+t-1} + v^{x+t+1\frac{1}{2}}r_{x+t+1}\bar{a}_{x+t+1\frac{1}{2}}\psi_{t+1}s_{x+t}$$

Now it is quite evident that $v^{x+\frac{1}{2}}r_x\bar{a}_{x+\frac{1}{2}}$ will be common to all the tables. We shall therefore construct that table and call it ${}^{ra}\overline{C}_x$. Then making $x=16, 17, 18, 19 \ldots 40$ successively,* we multiply ${}^{ra}\overline{C}_{x+t}$ by ψ_t, ${}^{ra}\overline{C}_{x+t+1}$ by ψ_{t+1}, and so on. These values we will call ${}^{\psi.ra}\overline{C}_x$. Summing these values like the M column, we get ${}^{\psi.ra}\overline{M}_x$. The summation of ${}^{\psi.ra}\overline{C}_x$ must be continued to age x as in my Table 9 for ${}^{r}M_x$ (*J.I.A.*, xxxvi, 267).

The value of the pension, at age $x+n$, in respect of past contributions will be

$$(\text{total past salary}) \times \frac{{}^{\psi.ra}\overline{M}_{x+n}}{D_{x+n}}.$$

If the contribution is $2\frac{1}{2}$ per-cent of salary, the total past salary will be total contributions multiplied by 40.

We next proceed to find the value in respect of future salary.

It will be seen from the detailed demonstration of the values that s_x is common to the first column, s_{x+1} common to the second column and so on. If now we multiply ${}^{\psi.ra}\overline{M}_x$ by s_x, we obtain the total of the first column, and if we multiply ${}^{\psi.ra}\overline{M}_{x+1}$ by s_{x+1}, we get the total of the second column, &c. But the value of ${}^{\psi.ra}\overline{M}_x.s_x$ is too large by the value in respect of half the salary for the first year, that is, by $\frac{1}{2}s_x.{}^{\psi.ra}\overline{C}_x$, so that we must make a column in each of the tables, of ${}^{\psi.ra}\overline{M}_x.s_x - \frac{1}{2}s_x.{}^{\psi.ra}\overline{C}_x$, which we will call ${}^{\psi.ra}\overline{M}{}^s_x$.

We then proceed to sum this column like the R column, obtaining thereby a column which we will call ${}^{\psi.ra}\overline{R}{}^s_x$; and the

* Probably 20 tables would be sufficient for the First Pension Scale, and 16 for the Second.

value of the pension, at age $x+n$, in respect of future salary will be

$$(\text{salary at age } x+n) \times \frac{{}^{\psi.ra}\overline{\mathrm{R}}^{s}_{x+n}}{\mathrm{D}^{s}_{x+n}}.$$

It does not matter whether s_x or $\frac{s_x}{100}$ is used in the calculations. By the latter the decimal point is set two places back in both numerator and denominator.

It will, no doubt, be noticed that I have adopted Mr. King's method of making the correction for the excess of half-year's salary in ascertaining the second formula. It is better than mine as it only involves one deduction and one summation, whereas mine involves two summations and one deduction. My formula for future salary was $\frac{{}^{\psi.ra}\mathrm{R}^{s}_{x+n} - \frac{1}{2}{}^{\psi.ra}\mathrm{M}^{ls}_{x+n}}{\mathrm{D}^{s}_{x+n}}$.

The agreement of the two formulas will be seen at once by the following explanation. Mr. King's formula is obtained from

$$\Sigma({}^{\psi.ra}\overline{\mathrm{M}}_x . s_x - \tfrac{1}{2}s_x {}^{\psi.ra}\mathrm{C}_x) = {}^{\psi.ra}\overline{\mathrm{R}}^{s}_{x},$$

while mine is obtained from

$$\Sigma{}^{\psi.ra}\mathrm{M}_x . s_x - \Sigma\tfrac{1}{2}s_x {}^{\psi.ra}\mathrm{C}_x = {}^{\psi.ra}\mathrm{R}^{s}_{x} - \tfrac{1}{2}{}^{\psi.ra}\mathrm{M}^{ls}_{x}.$$

I should like here to be allowed to express in the strongest terms my protest against representing these formulas by a central symbol F with a lot of little letters round it. The beauty, power, character and identity of the formulas are entirely lost in an attempt to represent them by one insignificant and unrepresentative symbol.

Is it more trouble to write

$$\frac{{}^{\psi.ra}\mathrm{M}_x}{\mathrm{D}_x} \text{ or } \frac{{}^{\psi.ra}\mathrm{M}_x}{} \text{ than } {}^{p}_{y}\mathrm{F}^{ra}_{x},$$

or

$$\frac{{}^{\psi.ra}\mathrm{R}^{s}_x}{\mathrm{D}^{s}_x} \text{ or } \frac{{}^{\psi.ra}\mathrm{R}^{s}_x}{} \text{ than } {}^{f}_{y}\mathrm{F}^{ra}_{x}?$$

When you have learnt my notation (*J.I.A.*, xxxvii, 236) you can see at a glance what the formulas represent. $\frac{{}^{\psi.ra}\mathrm{M}_x}{\mathrm{D}_x}$ is the assurance of an annuity at retirement of ψ at the date of retirement, and when multiplied into past salary will give the present value of pension, at date of retirement, of ψ times the past contributions. $\frac{{}^{\psi.ra}\mathrm{R}^{s}_x}{\mathrm{D}^{s}_x}$ is the increasing assurance of an annuity, at retirement, of future salary multiplied by ψ at the date of retirement equated to 1 of salary at x, so that $\frac{{}^{\psi.ra}\mathrm{R}^s_x}{\mathrm{D}^s_x}$ multiplied by salary at x is the present value of the pension at retirement of ψ times the future salary.

With the knowledge of the characteristic significance of these formulas it is often possible to build up a solution of a difficult problem. Let us take the special form of return on death in Mr. King's paper (*J.I.A.*, xxxix, p. 144), which gave him so much trouble.

Problem XVII B. Find the present value, at age at entrance x, of the return on death of the whole of the member's and the company's contributions if death occurs within ten years of entry, and the return, if death occurs after ten years, of one half of the average salary from date of entry.

Mr. Thomas has pointed out (*J.I.A.*, xxxix, p. 206) that the best way to solve this problem is to follow my invariable practice, when average salary is involved, of altering the form of the question from proportion of average salary to proportion of total salary. The problem will thus present itself in the following form, if we assume that the member's contribution is $2\frac{1}{2}$ per-cent of salary, and the company's contribution the same.

Special Return on Death.

Completed years of Membership (t)	Return in percentage of average salary	Equivalent proportion of total salary ψ_t
0 to 9	5 per-cent of total salary	·05
10	5	·05
11	5	·04545
12	5	·04167
13	5	·03846
14	5	·03571
⋮	⋮	⋮

The value of the benefit in respect of past salary will evidently be the assurance at death of ψ times the total past salary, that is to say (Total past salary) $\times \frac{{}^{\psi.d}\overline{\mathrm{M}}_{x+n}}{\mathrm{D}_{x+n}}$; and in respect of future salary it will be an increasing assurance of future salary multiplied by ψ at the date of death equated to 1 of salary at present age, that is to say (Present salary) $\times \dfrac{\psi . d\overline{\mathrm{R}}^{s}_{x+n}}{\mathrm{D}^{s}_{x+n}}$. The same results could be obtained by substituting d for ra in the demonstration in the previous Problem.

Members may be pleased to have the investigation of another formula which I used on the same occasion.

Problem XVIIIB. To find the value of the return, in the event of death after retirement, and before the payments of pension amount to the total contributions without interest, of the difference between the total contributions and the payments of pension.

Let c be the total contributions to date of retirement, and let ψ_t be the proportion of total salary payable as pension on retirement at age $x+t+\frac{1}{2}$.

Then the number of years during which there will be a risk of paying something in excess of the pension will be

$$\frac{c}{\psi_t}=\beta, \text{ and the number of months}=12\beta.$$

All pensions are payable monthly, so that the risk for the first month will be c, for the second month $c-\frac{1}{12}\psi_t$, for the third month $c-\frac{2}{12}\psi_t$, and so on.

Now, $c=12\beta\times\frac{1}{12}\psi_t$, so that the value of the risk will be

$$\frac{\psi_t}{12\mathrm{D}_{x+t+\frac{1}{2}}}\Big\{12\beta(\mathrm{M}_{x+t+\frac{6}{12}}-\mathrm{M}_{x+t+\frac{7}{12}})+(12\beta-1)(\mathrm{M}_{x+t+\frac{7}{12}}-\mathrm{M}_{x+t+\frac{8}{12}})+\dots$$
$$+1.(\mathrm{M}_{x+t+\beta+\frac{5}{12}}-\mathrm{M}_{n+t+\beta+\frac{6}{12}})\Big\}$$

The values of M and D would have to be calculated by the Table used in valuing the pension annuities.

Let us call the value of the temporary decreasing assurance, $\gamma_{x+t+\frac{1}{2}}$, then the value of the total risk on retirement at age $x+t+\frac{1}{2}$ will be $\psi_t.\bar{a}_{x+t+\frac{1}{2}}+\gamma_{x+t+\frac{1}{2}}$. For our purpose, however, it would be better to express the value in terms of $\psi_t.\bar{a}_{x+t+\frac{1}{2}}$, so that we can write it as $\psi_t.\bar{a}_{x+t+\frac{1}{2}}(1+\kappa_{x+t+\frac{1}{2}})$ where $\kappa_{x+t+\frac{1}{2}}=\dfrac{\gamma_{x+t+\frac{1}{2}}}{\psi_t.\bar{a}_{x+t+\frac{1}{2}}}$. We can then subtsitute for $\psi_t\bar{a}_{x+t+\frac{1}{2}}$ in the demonstration in Problem XVIB above, the term $\psi_t\bar{a}_{x+t+\frac{1}{2}}$

$(1+\kappa_{x+t+\frac{1}{2}})$. The value of the pension, together with the return of the excess of total contributions over pension payments in event of death before β years, will be, at age $x+n$,

$$(\text{total past salary}) \times \frac{^{\psi.ra(1+\kappa)}\overline{\mathrm{M}}_{x+n}}{\mathrm{D}_{x+n}} + (\text{salary at age } x+n) \times \frac{^{\psi.ra(1+\kappa)}\overline{\mathrm{R}}^s_{x+n}}{\mathrm{D}^s_{x+n}}.$$

It would be extremely laborious and troublesome to calculate these values if ψ_t varied according to t, as in the specimen pension scale in Problem XVIB.

If ψ were constant, κ would still vary slightly; but if ψ and κ were both constant, there would be no difficulty, for the value of the risk for both pension and assurance would be

$$\left\{(\text{total past salary}) \times \frac{^{ra}\overline{\mathrm{M}}_{x+n}}{\mathrm{D}_{x+n}} + (\text{salary at age } x+n) \times \frac{^{ra}\overline{\mathrm{R}}^s_{x+n}}{\mathrm{D}^s_n}\right\} \times \psi(1+\kappa).$$

This presents a possible solution of the Problem by approximation.

If we go a step further and suppose $\bar{a}_{x+\frac{1}{2}}$ to be constant, then we shall have, as the value of both benefits,

$$\left\{(\text{total past salary}) \times \frac{^{r}\overline{\mathrm{M}}_{x+n}}{\mathrm{D}_{x+n}} + (\text{salary at age } x+n)\frac{^{r}\overline{\mathrm{R}}^s_{x+n}}{\mathrm{D}^s_{x+n}}\right\} \times \bar{a}_{x+\frac{1}{2}} \times \psi(1+\kappa)$$

The proposal to treat $\bar{a}$ as a constant is not so very outrageous, for if you will look at my Table 8 (*J.I.A.*, xxxvi, 266), you will see that the extreme limits of a' are 6·99 at age 20, and 9·01 at age 55. But if we take the ages from 50 to 64, when the majority of the retirements take place, it will be found that the average of a' is 8·75, which is the value for age 60, and that the extreme divergence from that is $+$·26 at age 55, and $-$·78 at age 64.

If, then, we had in the valuation schedule the totals of the values of

$$\text{(i)}\quad (\text{total past salary}) \times \frac{^{r}\overline{\mathrm{M}}^s_x}{\mathrm{D}_x} + (\text{present salary}) \times \frac{^{r}\overline{\mathrm{R}}^s_x}{\mathrm{D}^s_x},$$

$$\text{(ii)}\quad (\text{total past salary}) \times \frac{^{ra}\overline{\mathrm{M}}_x}{\mathrm{D}_x} + (\text{present salary}) \times \frac{^{ra}\overline{\mathrm{R}}^s_x}{\mathrm{D}^s_x},$$

$$\text{(iii)}\quad (\text{total past salary}) \times \frac{^{\psi.ra}\overline{\mathrm{M}}_x}{\mathrm{D}_x} + (\text{present salary}) \times \frac{^{\psi.ra}\overline{\mathrm{R}}^s_x}{\mathrm{D}^s_x},$$

we could find the average $\bar{a}_{x+\frac{1}{2}}$ by dividing (ii) by (i), and the average of ψ by dividing (iii) by (ii), and the average age at retirement could be found by reference to $\bar{a}_{x+\frac{1}{2}}$.

These values are not, as a rule, separately calculated, so we must exercise our best judgment in selecting the average age at retirement and the average ψ.

Suppose we take a concrete example. Say total ordinary contributions 5 per-cent of salary; average $\psi = 1\frac{2}{3}$ per-cent of total salary $= \cdot 01667$; average age at retirement 63. It would not be wise to select a much younger age. Rate of interest 4 per-cent guaranteed, and mortality after age 63 the same as O^M Table.

To avoid very small values, let us say $c=5$ and $\psi = 1\frac{2}{3}$, then $\beta = \dfrac{c}{\psi} = 3$, and the number of months will be 36. $\dfrac{\psi}{12} = \dfrac{5}{36}$.

The value, at the moment of retirement, of the risk of having to make a return of excess of contributions over pension payments will be

$$\frac{5}{36} \cdot \frac{1}{D_{63}} \left\{ 36(M_{63} - M_{63\frac{1}{12}}) + 35(M_{63\frac{1}{12}} - M_{63\frac{2}{12}}) + \ldots + 1(M_{65\frac{11}{12}} - M_{66}) \right\}.$$

Now, if we assume

$$M_{63} - M_{63\frac{1}{12}}) = (M_{63\frac{1}{12}} - M_{63\frac{2}{12}}) = \ .\ .\ .\ = (M_{65\frac{11}{12}} - M_{66}) = \frac{1}{36}(M_{63} - M_{66}),$$

which is most likely to be the case, seeing that for the first two years after retirement the mortality is above the normal, we have

$$\frac{1}{36}(M_{63} - M_{66}) = \frac{1}{36}(3004\cdot 80 - 2509\cdot 36) = 13\cdot 762.$$

This has to be multiplied by 666, the sum of 36 terms of an arithmetical series of which the first term is 36 and the last term 1.

$D_{63} = 4771\cdot 0$, and the value of the assurance will be

$$\frac{5}{36} \times \frac{1}{4771} \times 666 \times 13\cdot 762 = \cdot 2668.$$

The value of $\bar{a}_{63}$ is $9\cdot 119$, so that the value of the pension and assurance, that is $\psi \bar{a}_x + \gamma_x$, will be

$$(9\cdot 119 \times 1\tfrac{2}{3}) + \cdot 2668 = (15\cdot 1983 + \cdot 2668) = 15\cdot 1983(1 + \cdot 0176)$$

$$\kappa = \cdot 0176,$$

and the value of the risk is about $1\frac{3}{4}$ per-cent of the value of the pensions.

There now remains the question of the value of the risk in respect of those who have been pensioners for less than three years. On the payment of the first monthly pension the value will be

$$\frac{5}{36}\cdot\frac{1}{D_{63\frac{1}{12}}}\left\{35(M_{63\frac{1}{12}}-M_{63\frac{2}{12}})+34(M_{63\frac{2}{12}}-M_{63\frac{3}{12}})+\ldots+1(M_{65\frac{11}{12}}-M_{66})\right\}$$

and on the second monthly payment, it will be

$$\frac{5}{36}\cdot\frac{1}{D_{63\frac{2}{12}}}\left\{34(M_{63\frac{2}{12}}-M_{63\frac{3}{12}})+\ldots\right\}.$$

The best way to do this is to make a double summation like the M and R columns, of 13·762, so that $\frac{5}{36}\cdot\frac{1}{D_{63}}\cdot S.S(13{\cdot}762)_1$ will be the value of the risk at the moment of retirement. $\frac{5}{36}\cdot\frac{1}{D_{63\frac{1}{12}}}\cdot S.S(13{\cdot}762)_2$ will be the value immediately on the payment of the first monthly pension, $\frac{5}{36}\cdot\frac{1}{D_{63\frac{2}{12}}}\cdot S.S(13{\cdot}762)_3$ the value on the second monthly payment, and so on. The values in the column of double summation must be multiplied by $\frac{5}{36}$ and divided by the appropriate $D_x \times \psi\bar{a}_x$ (*i.e.*, 15·1983). The result will be the ratio of the value of the pension to be set aside for the risk. It has been my practice to make additional calculations for those who have retired after age 63.

Hypothetical Experience of Staff Pension Fund for Widows and Orphans.

TABLE 57.—THE BACHELORS' TABLE.

Showing out of 200,000 Bachelors at age 15, the numbers living and dying at each age as Bachelors, the number who marry before the next age, and the number who pass out of observation at the next age as Husbands, after allowing for withdrawals.

Age x	Bl_x	$Bhl_x = Bl_x \times m_x$	$\frac{1}{2}Bhl_x$	$Bl_x - \frac{1}{2}Bhl_x$	$Bd_x = (Bl_x - \frac{1}{2}Bhl_x)q_x$	$Bw_x = (Bl_x - \frac{1}{2}Bhl_x){}^wq_x$	$Bhd_x = \frac{1}{2}Bhl_x \times q_x$	$Bhw_x = \frac{1}{2}Bhl_x \times {}_wq_x$	$Hl_{x+1} = Bhl_x - Bhd_x - Bhw_x$	Age $x+1$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
15	200,000	...	...	...	720	15,000	...	...	...	16
16	184,280	...	...	...	682	16,438	...	...	...	17
17	167,160	...	...	...	635	15,212	...	...	...	18
18	151,313	...	...	...	590	12,680	...	...	...	19
19	138,043	...	...	...	552	10,547	...	...	...	20
20	126,944	317	159	126,785	520	8,850	...	11	306	21
21	117,257	985	493	116,764	490	7,356	2	31	952	22
22	108,426	1,605	802	107,624	462	6,145	4	46	1,555	23
23	100,214	2,175	1,088	99,126	436	5,134	5	56	2,114	24
24	92,469	2,728	1,364	91,105	410	4,299	6	64	2,658	25
25	85,032	3,240	1,620	83,412	384	3,595	8	70	3,162	26
26	77,813	3,564	1,782	76,031	365	3,011	9	70	3,485	27
27	70,873	3,714	1,857	69,016	338	2,512	9	68	3,637	28
28	64,309	3,730	1,865	62,444	312	2,092	9	63	3,658	29
29	58,175	3,648	1,824	56,351	293	1,740	10	56	3,582	30
30	52,494	3,491	1,745	50,749	274	1,447	9	50	3,432	31
31	47,282	3,277	1,638	45,644	260	1,205	9	43	3,225	32
32	42,540	3,025	1,512	41,028	242	997	9	37	2,979	33
33	38,276	2,775	1,388	36,888	225	833	9	31	2,735	34
34	34,443	2,401	1,200	33,243	213	695	8	25	2,368	35
35	31,134	2,080	1,040	30,094	201	587	7	20	2,053	36
36	28,266	1,795	897	27,369	189	493	6	16	1,773	37
37	25,789	1,553	776	25,013	185	415	6	13	1,534	38
38	23,636	1,344	672	22,964	179	354	5	10	1,329	39
39	21,759	1,168	584	21,175	172	303	5	8	1,155	40
40	20,116	1,016	508	19,608	170	259	4	7	1,005	41
41	18,671	887	444	18,227	164	221	4	5	878	42
42	17,399	774	387	17,012	167	190	4	4	766	43
43	16,268	678	339	15,929	163	161	4	3	671	44
44	15,266	594	297	14,969	165	137	3	3	588	45
45	14,370	520	260	14,110	167	115	3	2	515	46
46	13,568	456	228	13,340	168	96	3	2	451	47
47	12,848	401	200	12,648	168	78	3	1	397	48
48	12,201	350	175	12,026	173	64	3	1	346	49
49	11,614	305	153	11,461	179	49	2	1	302	50
50	11,081	267	134	10,947	181	36	2	...	265	51
51	10,597	233	117	10,480	184	25	2	...	231	52
52	10,155	202	101	10,054	188	14	2	...	200	53
53	9,751	176	88	9,663	193	5	2	...	174	54
54	9,377	152	76	9,301	201	...	2	...	150	55
55	9,024	132	66	8,958	208	...	2	...	130	56
56	8,684	116	58	8,626	219	...	2	...	114	57
57	8,349	101	51	8,298	226	...	1	...	100	58
58	8,022	90	45	7,977	236	...	1	...	89	59
59	7,696	82	41	7,655	243	...	1	...	81	60

Hypothetical Experience of Staff Pension Fund for Widows and Orphans.

TABLE 57.—THE BACHELORS' TABLE—*continued.*

Showing out of 200,000 *Bachelors at age* 15, *the numbers living and dying at each age as Bachelors, the number who marry before the next age, and the number who pass out of observation at the next age as husbands, after allowing for withdrawals.*

Age x	$\mathrm{B}l_x$	$\mathrm{B}hl_x = \mathrm{B}l_x \times m_x$	$\frac{1}{2}\mathrm{B}hl_x$	$\mathrm{B}l_x - \frac{1}{2}\mathrm{B}hl_x$	$\mathrm{B}d_x = (\mathrm{B}l_x - \frac{1}{2}\mathrm{B}hl_x)q_x$	$\mathrm{B}w_x = (\mathrm{B}l_x - \frac{1}{2}\mathrm{B}hl_x)^w q_x$	$\mathrm{B}hd_x = \frac{1}{2}\mathrm{B}hl_x \times q_x$	$\mathrm{B}hw_x = \frac{1}{2}\mathrm{B}hl_x \times {}_wq_x$	$Hl_{x+1} = \mathrm{B}hl_x - \mathrm{B}hd_x - \mathrm{B}hw_x$	Age $x+1$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
60	7,371	75	38	7,333	254	...	1	...	74	61
61	7,042	69	35	7,007	260	...	1	...	68	62
62	6,713	63	32	6,681	268	...	1	...	62	63
63	6,382	57	29	6,353	274	...	1	...	56	64
64	6,051	52	26	6,025	273	...	1	...	51	65
65	5,726	...	...	...	270	...	...	...	...	66
66	5,456	...	...	...	270	...	...	...	...	67
67	5,186	...	...	...	277	...	...	...	...	68
68	4,909	...	...	...	282	...	...	...	...	69
69	4,627	...	...	...	289	...	...	...	...	70
70	4,338	...	...	...	292	...	...	...	...	71
71	4,046	...	...	...	294	...	...	...	...	72
72	3,752	...	...	...	295	...	...	...	...	73
73	3,457	...	...	...	293	...	...	...	...	74
74	3,164	...	...	...	290	...	...	..	...	75
75	2,874	...	...	...	284	...	...	...	...	76
76	2,590	...	...	...	275	...	...	...	...	77
77	2,315	...	...	...	265	...	...	...	...	78
78	2,050	...	...	...	253	...	...	...	...	79
79	1,797	...	...	...	237	...	...	...	...	80
80	1,560	...	...	...	222	...	...	...	...	81
81	1,338	...	...	...	203	...	...	...	...	82
82	1,135	...	...	...	184	...	...	...	...	83
83	951	...	...	...	165	...	...	...	...	84
84	786	...	...	...	146	...	...	...	...	85
85	640	...	...	...	126	...	...	...	...	86
86	514	...	...	...	108	...	...	...	...	87
87	406	...	...	...	90	...	...	...	...	88
88	316	...	...	...	75	...	...	...	...	89
89	241	...	...	...	60	...	...	...	...	90
90	181	...	...	...	48	...	...	...	...	91
91	133	...	...	...	37	...	...	...	...	92
92	96	...	...	...	28	...	...	...	...	93
93	68	...	...	...	21	...	...	...	...	94
94	47	...	...	...	15	...	...	...	...	95
95	32	...	...	...	11	...	...	...	...	96
96	21	...	...	...	8	...	...	...	...	97
97	13	...	...	...	5	...	...	...	...	98
98	8	...	...	...	3	...	...	..	...	99
99	5	...	...	...	2	...	...	...	...	100
100	3	...	...	...	1	...	...	...	...	101
101	2	...	...	...	1	...	...	...	...	102
102	1	...	...	...	1	...	...	...	...	103
103	...	...	...	...	...	...	...	...	...	104
104	...	...	...	...	...	...	...	...	...	105

Hypothetical Experience of Staff Pension Fund for Widows and Orphans.

TABLE 58.—THE HUSBANDS' TABLE.

Showing, out of 100,000 *Husbands at the age of* 20, *the numbers living at each age as Husbands and Widowers, and the numbers dying at each age as Husbands and Widowers. Also, out of a certain number of Husbands at each age, the number who become Widowers within a year, after allowing for withdrawals.*

Age x (1)	$\mathrm{H}\,l_x$ (2)	$\mathrm{H}kl_x = \mathrm{H}\,l_x \times (p_x q_y + \frac{1}{2} q_x q_y)$ (3)	$\mathrm{K}\,l_x$ (4)	$\mathrm{H}\,d_x = \mathrm{H}\,l_x(q_x - \frac{1}{2} q_x q_y)$ (5)	$\mathrm{H}kd_x = \mathrm{H}\,l_x \times \frac{1}{2} q_x q_y$ (6)	$\mathrm{K}\,d_x = \mathrm{K}\,l_x \times q_x$ (7)	$\mathrm{H}w_x = (\mathrm{H}\,l_x - \frac{1}{2}\mathrm{H}kl_x) . {}^w q_x$ (8)	$\mathrm{H}kw_x = \frac{1}{2}\mathrm{H}kl_x \times {}^w q_x$ (9)	$\mathrm{K}w_x = \mathrm{K}l_x \times {}^w q_x$ (10)	Age x (11)
20	100,000	1,237	...	408	2	...	6,937	43	...	20
21	91,418	1,123	1,192	381	3	5	5,724	35	75	21
22	84,190	1,017	2,197	360	2	9	4,778	29	126	22
23	78,035	927	3,048	342	2	13	4,018	24	158	23
24	72,748	857	3,778	325	2	17	3,414	20	178	24
25	68,152	782	4,418	312	2	20	2,921	17	190	25
26	64,137	710	4,971	306	2	24	2,525	14	197	26
27	60,596	647	5,444	295	2	26	2,194	12	198	27
28	57,460	590	5,853	286	2	29	1,915	10	196	28
29	54,669	545	6,206	283	2	32	1,681	8	192	29
30	52,160	505	6,517	280	1	35	1,480	7	186	30
31	49,895	473	6,793	283	1	39	1,312	6	179	31
32	47,827	444	7,041	281	1	42	1,157	5	171	32
33	45,945	413	7,266	279	1	45	1,034	5	164	33
34	44,219	392	7,464	282	1	48	920	4	156	34
35	42,625	370	7,647	284	1	51	828	4	149	35
36	41,143	349	7,812	283	1	54	737	3	141	36
37	39,774	333	7,962	293	1	59	657	3	132	37
38	38,491	318	8,100	299	1	63	590	3	125	38
39	37,284	305	8,226	301	1	67	531	2	117	39
40	36,147	295	8,344	313	1	73	475	2	111	40
41	35,064	283	8,452	314	1	76	423	2	102	41
42	34,044	271	8,554	332	1	84	380	2	96	42
43	33,061	263	8,642	336	1	88	333	1	88	43
44	32,129	252	8,727	352	1	96	295	1	80	44
45	31,230	245	8,801	367	1	104	255	1	72	45
46	30,363	238	8,868	381	2	112	217	1	63	46
47	29,527	229	8,928	391	2	119	182	1	55	47
48	28,725	222	8,980	412	2	129	152	1	47	48
49	27,939	216	9,023	434	2	140	120	1	38	49
50	27,169	208	9,058	446	2	150	89	...	30	50
51	26,426	202	9,084	461	2	159	63	...	22	51
52	25,700	196	9,103	479	2	170	35	...	13	52
53	24,990	193	9,114	498	2	182	13	...	5	53
54	24,286	187	9,118	522	2	197	...	...	...	54
55	23,577	184	9,106	547	2	213	...	...	...	55
56	22,846	183	9,075	578	2	230	...	...	...	56
57	22,085	179	9,026	601	3	246	...	...	...	57
58	21,305	176	8,956	627	3	265	...	...	...	58
59	20,502	174	8,864	649	3	282	...	...	...	59

Hypothetical Experience of Staff Pension Fund for Widows and Orphans.

TABLE 58—(*continued*).—THE HUSBANDS' TABLE.

Showing, out of 100,000 Husbands at the age of 20, the numbers living at each age as Husbands and Widowers, and the numbers dying at each age as Husbands and Widowers. Also, out of a certain number of Husbands at each age, the number who become Widowers within a year, after allowing for withdrawals.

Age x (1)	$\mathrm{H}l_x$ (2)	$\mathrm{H}kl_x = \mathrm{H}l_x \times (p_xq_y + \frac{1}{2}q_xq_y)$ (3)	$\mathrm{K}l_x$ (4)	$\mathrm{H}d_x = \mathrm{H}l_x(q_x - \frac{1}{2}q_xq_y)$ (5)	$\mathrm{H}kd_x = \mathrm{H}l_x \times \frac{1}{2}q_xq_y$ (6)	$\mathrm{K}d_x = \mathrm{K}l_x \times q_x$ (7)	$\mathrm{H}w_x = (\mathrm{H}l_x - \frac{1}{2}\mathrm{H}kl_x).{}^wq_x$ (8)	$\mathrm{H}kw_x = \frac{1}{2}\mathrm{H}kl_x \times {}^wq_x$ (9)	$\mathrm{K}w_x = \mathrm{K}l_x \times {}^wq_x$ (10)	Age x (11)
60	19,679	174	8,753	677	3	303	...	...	...	60
61	18,828	174	8,621	697	3	321	...	...	...	61
62	17,957	176	8,471	717	3	340	...	...	...	62
63	17,064	177	8,304	731	4	358	...	...	...	63
64	16,156	178	8,119	728	4	367	...	...	...	64
65	15,250	180	7,926	714	4	374	...	...	...	65
66	14,356	183	7,728	705	5	382	...	...	...	66
67	13,468	186	7,524	714	5	402	...	...	...	67
68	12,568	188	7,303	718	6	421	...	...	...	68
69	11,662	189	7,064	720	6	440	...	...	...	69
70	10,753	187	6,807	717	6	458	...	...	...	70
71	9,849	185	6,530	709	7	475	...	...	...	71
72	8,955	182	6,233	696	7	490	...	...	...	72
73	8,077	176	5,918	677	8	502	...	...	...	73
74	7,224	172	5,584	654	8	511	...	...	...	74
75	6,398	165	5,237	624	8	517	...	...	...	75
76	5,607	159	4,877	588	9	519	...	...	...	76
77	4,862	152	4,508	548	9	516	...	..	..	77
78	4,162	143	4,135	504	9	509	...	...	...	78
79	3,515	134	3,760	455	10	497	...	...	...	79
80	2,926	123	3,387	406	10	480	...	...	...	80
81	2,397	112	3,020	355	9	459	...	...	...	81
82	1,930	100	2,664	305	9	432	...	...	...	82
83	1,525	87	2,323	257	8	403	...	...	...	83
84	1,181	75	1,999	211	8	370	...	...	..	84
85	895	63	1,696	169	7	334	...	...	..	85
86	663	51	1,418	133	6	298	...	...	...	86
87	479	41	1,165	102	5	259	...	...	...	87
88	336	31	942	75	4	222	...	...	...	88
89	230	23	747	54	3	187	...	...	...	89
90	153	17	580	38	3	153	...	...	..	90
91	98	12	441	25	2	123	...	...	...	91
92	61	8	328	17	1	97	...	...	...	92
93	36	5	238	10	1	73	...	...	...	93
94	21	3	169	6	1	55	..	...	...	94
95	12	2	116	4	...	40	...	...	...	95
96	6	1	78	2	...	28	...	...	...	96
97	3	...	51	1	...	19	...	...	...	97
98	2	...	32	1	...	12	...	...	...	98
99	1	...	20	1	...	8	...	...	...	99
100	...	...	12	...	...	5	...	...	...	100
101	...	...	7	...	...	3	...	...	...	101
102	...	...	4	...	...	2	...	...	...	102
103	...	...	2	...	...	1	...	...	...	103
104	...	...	1	...	...	1	...	...	...	104
105	...	...	...	...	...	...	...	...	...	105

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TABLE 59.—THE WIDOWERS' TABLE.

Showing, out of 100,000 *Widowers at age* 20, *the numbers living and dying at each age as Widowers, the number who re-marry before the next age, and the number who pass out of observation at the next age as Second Husbands, after allowing for withdrawals.*

Age x (1)	Kl_x (2)	$Khl_x = Kl_x \times m_x^2$ (3)	$\frac{1}{2}Khl_x$ (4)	$Kl_x - \frac{1}{2}Khl_x$ (5)	$Kd_x = (Kl_x - \frac{1}{2}Khl_x)q_x$ (6)	$Kw_x = (Kl_x - \frac{1}{2}Khl_x)^wq_x$ (7)	$Khd_x = \frac{1}{2}Khl_x \times q_x$ (8)	$Khw_x = \frac{1}{2}Khl_x \times {}^wq_x$ (9)	H^2l_{x+1} (10)	Age $x+1$ (11)
20	100,000	...	...	100,000	410	6,980	...	...	...	21
21	92,610	9,261	4,631	87,979	370	5,542	20	292	8,949	22
22	77,437	7,744	3,872	73,565	316	4,201	17	221	7,506	23
23	65,176	7,821	3,911	61,265	270	3,174	17	203	7,601	24
24	53,911	8,087	4,044	49,867	224	2,354	18	191	7,878	25
25	43,246	7,784	3,892	39,354	181	1,696	18	168	7,598	26
26	33,585	6,717	3,359	30,226	145	1,197	16	133	6,568	27
27	25,526	5,361	2,680	22,846	112	831	13	98	5,250	28
28	19,222	3,844	1,922	17,300	87	579	10	64	3,770	29
29	14,712	2,796	1,398	13,314	69	411	7	43	2,746	30
30	11,436	2,059	1,029	10,407	56	296	6	29	2,024	31
31	9,025	1,535	767	8,258	47	218	4	20	1,511	32
32	7,225	1,156	578	6,647	39	161	3	14	1,139	33
33	5,869	880	440	5,429	33	123	3	10	867	34
34	4,833	677	338	4,495	29	94	2	7	668	35
35	4,033	524	262	3,771	25	74	2	5	517	36
36	3,410	409	205	3,205	22	58	1	4	404	37
37	2,921	321	161	2,760	20	47	1	3	317	38
38	2,533	253	127	2,406	19	37	1	2	250	39
39	2,224	200	100	2,124	17	30	1	1	198	40
40	1,977	158	79	1,898	17	25	1	1	156	41
41	1,777	124	62	1,715	16	21	1	1	122	42
42	1,616	97	49	1,567	15	18	1	1	95	43
43	1,486	74	37	1,449	15	14	1	1	72	44
44	1,383	69	35	1,348	15	13	1	...	68	45
45	1,286	64	32	1,254	15	10	...	...	64	46
46	1,197	48	24	1,173	15	8	...	...	48	47
47	1,126	45	23	1,103	15	7	...	...	45	48
48	1,059	42	21	1,038	15	6	...	...	42	49
49	996	40	20	976	15	4	...	...	40	50
50	937	37	19	918	15	3	...	...	37	51
51	882	36	18	864	15	2	...	...	36	52
52	829	33	17	812	15	1	...	...	33	53
53	780	31	16	764	15	1	...	...	31	54
54	733	29	15	718	16	...	...	...	29	55
55	688	27	14	674	16	...	...	...	27	56
56	645	26	13	632	16	...	...	...	26	57
57	603	24	12	591	16	...	...	...	24	58
58	563	23	12	551	16	...	...	...	23	59
59	524	21	10	514	16	...	...	...	21	60

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TABLE 59.—THE WIDOWERS' TABLE—(*continued*).

Showing, out of 100,000 *Widowers at age* 20, *the numbers living and dying at each age as Widowers, the number who re-marry before the next age, and the number who pass out of observation at the next age as Second Husbands, after allowing for withdrawals.*

Age x	$\mathsf{K}l_x$	$\mathsf{K}hl_x = \mathsf{K}l_x \times m_x^2$	$\frac{1}{2}\mathsf{K}hl_x$	$\mathsf{K}l_x - \frac{1}{2}\mathsf{K}hl_x$	$\mathsf{K}d_x = (\mathsf{K}l_x - \frac{1}{2}\mathsf{K}hl_x)q_x$	$\mathsf{K}w_x = (\mathsf{K}l_x - \frac{1}{2}\mathsf{K}hl_x)^w q_x$	$\mathsf{K}hd_x = \frac{1}{2}\mathsf{K}hl_x \times q_x$	$\mathsf{K}hw_x = \frac{1}{2}\mathsf{K}hl_x \times {}^w q_x$	$H^2 l_{x+1}$	Age $x+1$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
60	487	19	10	477	17	...	...	...	19	61
61	451	13	7	444	17	...	...	...	13	62
62	421	13	7	414	17	...	...	...	13	63
63	391	11	6	385	17	...	...	...	11	64
64	363	11	5	358	16	...	...	...	11	65
65	336	...	...	...	16	...	...	...	...	66
66	320	...	...	...	16	...	...	...	...	67
67	304	...	...	...	16	...	...	...	...	68
68	288	...	...	...	17	...	...	...	...	69
69	271	...	...	...	17	...	...	...	...	70
70	254	...	...	...	17	...	...	...	...	71
71	237	...	...	...	17	...	...	...	...	72
72	220	...	...	...	17	...	...	...	...	73
73	203	...	...	...	17	...	...	...	...	74
74	186	...	...	...	17	...	...	...	...	75
75	169	...	...	...	17	...	...	...	...	76
76	152	...	...	...	16	...	...	...	...	77
77	136	...	...	...	16	...	...	...	...	78
78	120	...	...	...	15	...	...	...	...	79
79	105	...	...	...	14	...	...	...	...	80
80	91	...	...	...	13	...	...	...	...	81
81	78	...	...	...	12	...	...	...	...	82
82	66	...	...	...	11	...	...	...	...	83
83	55	...	...	...	10	...	...	...	...	84
84	45	...	...	...	8	...	...	...	...	85
85	37	...	...	...	7	...	...	...	...	86
86	30	...	...	...	6	...	...	...	...	87
87	24	...	...	...	5	...	...	...	...	88
88	19	...	...	...	5	...	...	...	...	89
89	14	...	...	...	4	...	...	...	...	90
90	10	...	...	...	3	...	...	...	...	91
91	7	...	...	...	2	...	...	...	...	92
92	5	...	...	...	2	...	...	...	...	93
93	3	...	...	...	1	...	...	...	...	94
94	2	...	...	...	1	...	...	...	...	95
95	1	...	...	...	1	...	...	...	...	96
96	...	...	...	...	...	...	...	...	...	97
97	...	...	...	...	...	...	...	...	...	98
98	...	...	...	...	...	...	...	...	...	99
99	...	...	...	...	...	...	...	...	...	100
100	...	...	...	...	...	...	...	...	...	101

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TABLE 60.

Showing, out of 200,000 persons of the age of 15, the number living and remaining on the Staff at each age; and the numbers who die at each age as Bachelors and as Husbands and Widowers of first marriages only, of second marriages only, of third marriages only, and of fourth marriages only, after allowing for withdrawals.

Age (x)	l_x	Bd_x	Hd_x	Kd_x	${}^2d_x$	K^2d_x	H^3d_x	K^3d_x	H^4d_x	Age (x)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
15	200,000	720	...	...	...	...	...	...	...	15
16	184,280	682	...	...	...	...	...	...	...	16
17	167,160	635	...	...	...	...	...	...	...	17
18	151,313	590	...	...	...	...	...	...	...	18
19	138,043	552	...	...	...	...	...	...	...	19
20	126,944	520	1	...	...	...	...	...	...	20
21	117,566	490	4	...	...	...	...	...	...	21
22	109,665	462	10	...	...	...	...	...	...	22
23	102,932	436	17	...	...	...	...	...	...	23
24	97,147	410	27	...	...	...	...	...	...	24
25	92,126	384	40	...	...	...	...	...	...	25
26	87,730	365	55	1	...	...	...	...	...	26
27	83,836	338	71	2	...	...	...	...	...	27
28	80,373	312	87	2	1	...	...	...	...	28
29	77,279	293	106	2	1	...	...	...	...	29
30	74,489	274	124	3	1	...	...	...	...	30
31	71,963	260	144	4	2	...	...	...	...	31
32	69,653	242	162	4	3	...	...	...	...	32
33	67,549	225	179	5	3	...	...	...	...	33
34	65,610	213	197	6	4	...	...	...	...	34
35	63,819	201	214	8	5	...	...	...	...	35
36	62,147	189	226	8	6	...	...	...	...	36
37	60,600	185	247	9	7	...	...	...	...	37
38	59,145	179	263	11	8	...	...	...	...	38
39	57,774	172	275	12	9	...	...	...	...	39
40	56,479	170	295	14	11	1	...	...	...	40
41	55,242	164	304	16	12	1	...	...	...	41
42	54,077	167	330	18	14	1	...	...	...	42
43	52,941	163	342	20	15	...	...	...	...	43
44	51,866	165	364	23	18	1	...	...	...	44
45	50,818	167	387	26	18	1	1	...	...	45
46	49,802	168	408	30	20	1	1	...	...	46
47	48,817	168	424	33	22	1	1	...	...	47
48	47,865	173	453	37	24	1	1	...	...	48
49	46,922	179	482	41	27	2	1	...	...	49
50	45,989	181	500	45	30	2	1	...	...	50
51	45,079	184	521	49	32	2	1	...	...	51
52	44,181	188	546	53	36	2	1	...	...	52
53	43,293	193	571	57	40	3	1	...	...	53
54	42,405	201	602	65	43	3	1	...	...	54
55	41,490	208	635	70	48	4	2	...	...	55
56	40,523	219	674	76	54	4	2	...	...	56
57	39,494	226	704	82	59	5	2	...	...	57
58	38,416	236	738	89	67	5	2	...	...	58
59	37,279	243	766	95	72	6	3	...	...	59

Hypothetical Experience of Staff Pension Fund for Widows and Orphans.

TABLE 60—(*continued*).

Showing, out of 200,000 persons of the age of 15, the number living and remaining on the Staff at each age; and the numbers who die at each age as Bachelors, and as Husbands and Widowers of first marriages only, of second marriages only, of third marriages only, and of fourth marriages only, after allowing for withdrawals.

Age (x)	l_x	$\mathrm{B}d_x$	$\mathrm{H}d_x$	$\mathrm{K}d_x$	H^2d_x	K^2d_x	H^3d_x	K^3d_x	H^4d_x	Age (x)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
60	36,094	254	800	107	78	6	3	1	...	60
61	34,845	260	826	114	85	7	3	1	...	61
62	33,549	268	853	122	91	8	3	...	...	62
63	32,204	274	872	133	96	9	3	1	...	63
64	30,816	273	872	136	100	10	4	1	...	64
65	29,420	270	855	144	102	10	4	...	...	65
66	28,035	270	844	156	101	12	4	...	...	66
67	26,648	277	856	168	103	13	4	1	...	67
68	25,226	282	860	191	102	15	4	...	...	68
69	23,772	289	862	205	103	17	4	...	...	69
70	22,292	292	859	220	103	20	4	1	1	70
71	20,792	294	850	237	102	22	4	1	1	71
72	19,281	295	835	254	100	24	4	2	1	72
73	17,766	293	812	272	97	26	4	2	1	73
74	16,259	290	783	290	93	28	3	2	...	74
75	14,770	284	746	309	88	29	3	...	...	75
76	13,311	275	705	315	84	31	3	2	1	76
77	11,895	265	653	333	77	32	3	...	...	77
78	10,532	253	603	335	71	33	2	...	...	78
79	9,235	237	545	337	65	34	2	1	...	79
80	8,014	222	486	334	58	35	2	...	...	80
81	6,877	203	425	329	51	34	2	1	...	81
82	5,832	184	364	322	43	33	1	...	...	82
83	4,885	165	305	310	35	32	1	...	...	83
84	4,037	146	253	288	29	30	1	...	...	84
85	3,290	126	203	266	24	27	1	1	1	85
86	2,641	108	160	239	19	25	1	1	1	86
87	2,087	90	122	212	15	23	1	1	...	87
88	1,623	75	90	185	11	21	1	...	...	88
89	1,240	60	65	157	8	20	...	...	...	89
90	930	48	46	129	6	17	...	...	...	90
91	684	37	30	106	4	13	...	1	...	91
92	493	28	20	84	2	11	...	...	...	92
93	348	21	12	65	1	8	...	...	...	93
94	241	15	7	50	1	6	...	...	...	94
95	162	11	5	33	1	4	...	1	...	95
96	107	8	3	24	...	2	...	1	...	96
97	69	5	2	18	...	1	...	...	...	97
98	43	3	1	13	...	...	...	...	...	98
99	26	2	...	8	...	...	...	...	...	99
100	16	1	...	3	1	...	...	1	1	100
101	9	1	...	2	...	...	1	...	...	101
102	5	1	...	1	...	...	...	...	...	102
103	3	...	...	1	1	...	...	...	...	103
104	1	...	...	...	1	...	...	..	..	104

Hypothetical Experience of Staff Pension Fund for Widows and Orphans.

TABLE 61.

Giving the numbers of Husbands and Widowers dying at each age of the first marriage only and of any number of marriages, and the present value per member (whether Bachelor, Husband or Widower) of an annuity of 1 *to a Widow to commence at the death of a Husband.*

Age x	First Marriages only		All Marriages		$\frac{^{wa}M_x}{D_x}$		Δ	Age x
	Hd_x	Kd_x	Hd_x	Kd_x	First Marriages only	All Marriages		
20	1	...	1	...	·793	·848	·055	20
21	4	...	4	...	·891	·952	·061	21
22	10	...	10	...	·992	1·060	·068	22
23	17	...	17	...	1·098	1·173	·075	23
24	27	...	27	...	1·206	1·289	·083	24
25	40	...	40	...	1·318	1·409	·091	25
26	55	1	55	1	1·431	1·530	·099	26
27	71	2	71	2	1·545	1·653	·108	27
28	87	3	88	2	1·660	1·777	·117	28
29	106	3	107	2	1·774	1·901	·127	29
30	124	4	125	3	1·889	2·025	·136	30
31	144	6	146	4	2·002	2·149	·147	31
32	162	7	165	4	2·114	2·271	·157	32
33	179	8	182	5	2·224	2·391	·167	33
34	197	10	201	6	2·332	2·511	·179	34
35	214	13	219	8	2·439	2·629	·190	35
36	226	14	232	8	2·543	2·745	·202	36
37	247	16	254	9	2·647	2·860	·213	37
38	263	19	271	11	2·747	2·972	·225	38
39	275	21	284	12	2·845	3·082	·237	39
40	295	26	306	15	2·942	3·191	·249	40
41	304	29	316	17	3·036	3·297	·261	41
42	330	33	344	19	3·128	3·402	·274	42
43	342	35	357	20	3·216	3·503	·287	43
44	364	42	382	24	3·302	3·602	·300	44
45	387	46	406	27	3·384	3·695	·311	45
46	408	52	429	31	3·460	3·785	·325	46
47	424	57	447	34	3·532	3·869	·337	47
48	453	63	478	38	3·600	3·949	·349	48
49	482	71	510	43	3·662	4·022	·360	49
50	500	78	531	47	3·715	4·088	·373	50
51	521	84	554	51	3·762	4·148	·386	51
52	546	92	583	55	3·804	4·200	·396	52
53	571	101	612	60	3·838	4·245	·407	53
54	602	112	646	68	3·865	4·282	·417	54
55	635	124	685	74	3·884	4·311	·427	55
56	674	136	730	80	3·897	4·333	·436	56
57	704	148	765	87	3·902	4·346	·444	57
58	738	163	807	94	3·900	4·351	·451	58
59	766	176	841	101	3·890	4·347	·457	59
60	800	195	881	114	3·874	4·333	·459	60
61	826	210	914	122	3·846	4·309	·463	61
62	853	224	947	130	3·811	4·274	·463	62
63	872	242	971	143	3·765	4·227	·462	63
64	872	251	976	147	3·711	4·169	·458	64

Hypothetical Experience of Staff Pension Fund for Widows and Orphans.

TABLE 61—(*continued*).

Giving the numbers of Husbands and Widowers dying at each age of the first marriage only and of any number of marriages, and the present value per member (whether Bachelor, Husband or Widower) of an annuity of 1 to a Widow to commence at the death of a Husband.

Age x	First Marriages only		All Marriages		$\frac{^{wa}M_x}{D_x}$		Δ	Age x
	Hd_x	Kd_x	Hd_x	Kd_x	First Marriages only	All Marriages		
65	855	260	961	154	3·650	4·103	·453	65
66	844	273	949	168	3·587	4·032	·445	66
67	856	289	963	182	3·521	3·958	·437	67
68	860	312	966	206	3·447	3·875	·428	68
69	862	329	969	222	3·365	3·782	·417	69
70	859	349	967	241	3·274	3·681	·407	70
71	850	367	957	260	3·174	3·567	·393	71
72	835	385	940	280	3·066	3·445	·379	72
73	812	402	914	300	2·949	3·313	·364	73
74	783	416	879	320	2·824	3·171	·347	74
75	746	429	837	338	2·693	3·023	·330	75
76	705	436	793	348	2·556	2·872	·316	76
77	653	445	733	365	2·413	2·708	·295	77
78	603	441	676	368	2·269	2·550	·281	78
79	545	439	612	372	2·120	2·383	·263	79
80	486	429	546	369	1·970	2·215	·245	80
81	425	417	478	364	1·817	2·053	·236	81
82	364	399	408	355	1·665	1·870	·205	82
83	305	378	341	342	1·513	1·701	·188	83
84	253	348	283	318	1·365	1·536	·171	84
85	203	320	229	294	1·217	1·374	·157	85
86	160	286	181	265	1·072	1·214	·142	86
87	122	252	138	236	·932	1·053	·121	87
88	90	218	102	206	·798	·901	·103	88
89	65	185	73	177	·675	·762	·087	89
90	46	152	52	146	·560	·634	·074	90
91	30	124	34	120	·451	·510	·059	91
92	20	97	22	95	·358	·406	·048	92
93	12	74	13	73	·275	·317	·042	93
94	7	57	8	56	·217	·257	·040	94
95	5	39	6	38	·175	·203	·028	95
96	3	27	3	27	·120	·120	·000	96
97	2	19	2	19	·067	·067	·000	97
98	1	13	1	13	...	...	...	98
99	...	8	...	8	...	...	...	99
100	...	6	2	4	...	...	...	100
101	...	3	1	2	...	...	...	101
102	...	1	...	1	...	...	...	102
103	...	2	1	1	...	...	...	103
104	...	1	1	...	...	...	...	104

Hypothetical Experience of Staff Pension Fund for Widows and Orphans.

TABLE 62.

Giving the present values per member (whether Bachelor, Husband, or Widower) of an annuity of 1 *to commence at the death of a Widow and to continue until the youngest surviving child reaches the age of* 16 *; of an annuity of* 1 *to each of the children of a married man (whether Husband or Widower) until they reach the age of* 16 *; and of an annuity of* 1 *to commence at the death of a Widower and to continue until the youngest surviving child reaches the age of* 16.

Age (x)	$\frac{^{E(16)}M_x}{D_x}$ First Marriages only	$\frac{^{E(16)}M_x}{D_x}$ All Marriages	Δ	$\frac{^{Oa(16)}M_x}{D_x}$ First and all Marriages	$\frac{^{K.YCa(16)}M_x}{D_x}$ First Marriages only	$\frac{^{K.YCa(16)}M_x}{D_x}$ All Marriages	Δ	Age (x)
20	·020	·021	·001	·593	·040	·024	·016	20
21	·022	·023	·001	·667	·045	·027	·018	21
22	·025	·026	·001	·743	·050	·030	·020	22
23	·027	·029	·002	·822	·055	·033	·022	23
24	·030	·032	·002	·905	·061	·037	·024	24
25	·033	·035	·002	·989	·067	·040	·027	25
26	·036	·038	·002	1·075	·073	·044	·029	26
27	·038	·040	·002	1·162	·079	·048	·031	27
28	·041	·043	·002	1·248	·086	·051	·035	28
29	·044	·046	·002	1·333	·092	·055	·037	29
30	·046	·049	·003	1·415	·099	·059	·040	30
31	·048	·051	·003	1·494	·106	·064	·042	31
32	·051	·053	·002	1·567	·114	·068	·046	32
33	·053	·056	·003	1·634	·121	·072	·049	33
34	·055	·057	·002	1·693	·128	·077	·051	34
35	·056	·060	·004	1·743	·136	·081	·055	35
36	·058	·061	·003	1·784	·143	·085	·058	36
37	·059	·063	·004	1·818	·150	·090	·060	37
38	·060	·064	·004	1·841	·158	·094	·064	38
39	·061	·065	·004	1·856	·165	·099	·066	39
40	·062	·066	·004	1·866	·172	·103	·069	40
41	·063	·067	·004	1·868	·179	·107	·072	41
42	·063	·068	·005	1·865	·185	·111	·074	42
43	·063	·068	·005	1·854	·191	·115	·076	43
44	·063	·068	·005	1·837	·197	·118	·079	44
45	·063	·068	·005	1·813	·202	·122	·080	45
46	·062	·067	·005	1·782	·207	·125	·082	46
47	·061	·066	·005	1·742	·211	·127	·084	47
48	·060	·065	·005	1·699	·214	·129	·085	48
49	·058	·063	·005	1·647	·216	·130	·086	49
50	·056	·061	·005	1·585	·217	·131	·086	50
51	·054	·059	·005	1·519	·217	·131	·086	51
52	·052	·056	·004	1·448	·216	·130	·086	52
53	·049	·053	·004	1·370	·213	·128	·085	53
54	·046	·050	·004	1·288	·209	·126	·083	54
55	·043	·047	·004	1·201	·203	·123	·080	55
56	·039	·043	·004	1·108	·196	·118	·078	56
57	·036	·039	·003	1·012	·187	·113	·074	57
58	·032	·036	·004	·914	·177	·108	·069	58
59	·029	·032	·003	·814	·165	·101	·064	59
60	·025	·028	·003	·715	·152	·094	·058	60
61	·022	·025	·003	·617	·137	·086	·051	61
62	·020	·022	·002	·522	·122	·077	·045	62
63	·017	·019	·002	·434	·107	·070	·037	63
64	·015	·017	·002	·357	·094	·062	·032	64

Hypothetical Experience of Staff Pension Fund for Widows and Orphans.

TABLE 62—(*continued*).

Giving the present values per member (whether Bachelor, Husband, or Widower) of an annuity of 1 *to commence at the death of a Widow and to continue until the youngest surviving child reaches the age of* 16; *of an annuity of* 1 *to each of the children of a married man (whether Husband or Widower) until they reach the age of* 16; *and of an annuity of* 1 *to commence at the death of a Widower and to continue until the youngest surviving child reaches the age of* 16.

Age (x)	$\frac{^{E(16)}M_x}{D_x}$ First Marriages only	$\frac{^{E(16)}M_x}{D_x}$ All Marriages	Δ	$\frac{^{O\alpha(16)}M_x}{D_x}$ First and all Marriages	$\frac{^{K.YC\alpha(16)}M_x}{D_x}$ First Marriages only	$\frac{^{K.YC\alpha(16)}M_x}{D_x}$ All Marriages	Δ	Age (x)
65	·013	·015	·002	·297	·082	·056	·026	65
66	·011	·012	·001	·254	·074	·052	·022	66
67	·010	·011	·001	·223	·068	·048	·020	67
68	·009	·010	·001	·197	·062	·046	·016	68
69	·008	·009	·001	·175	·057	·043	·014	69
70	·007	·007	·000	·155	·053	·040	·013	70
71	·006	·006	·000	·137	·049	·038	·011	71
72	·005	·005	·000	·121	·045	·035	·010	72
73	·004	·005	·001	·106	·041	·033	·008	73
74	·003	·004	·001	·093	·037	·030	·007	74
75	·003	·003	·000	·081	·034	·028	·006	75
76	·002	·003	·001	·069	·030	·026	·004	76
77	·002	·002	·000	·059	·027	·023	·004	77
78	·002	·002	·000	·051	·024	·021	·003	78
79	·001	·001	·000	·043	·021	·018	·003	79
80	·001	·001	·000	·037	·019	·016	·003	80
81	·001	·001	·000	·030	·016	·014	·002	81
82	·001	·001	·000	·023	·013	·012	·001	82
83	...	...	...	·018	·011	·010	·001	83
84	...	...	...	·012	·007	·007	·000	84
85	...	...	...	·008	·005	·005	·000	85
86	...	...	...	·004	·003	·003	·000	86
87	...	...	...	·002	·001	·001	·000	87
88	...	...	...	...	...	...	...	88
89	...	...	...	...	...	...	...	89
90	...	...	...	...	...	...	...	90
91	...	...	...	...	...	...	...	91
92	...	...	...	...	...	...	...	92
93	...	...	...	...	...	...	...	93
94	...	...	...	...	...	...	...	94
95	...	...	...	...	...	...	...	95
96	...	...	...	...	...	...	...	96
97	...	...	...	...	...	...	...	97
98	...	...	...	...	...	...	...	98
99	...	...	...	...	...	...	...	99
100	...	...	...	...	...	...	...	100
101	...	...	...	...	...	...	...	101
102	...	...	...	...	...	...	...	102
103	...	...	...	..	...	...	...	103
104	...	...	...	...	...	...	...	104