

The value of intergenerational transfers within funded pension schemes*

Jiajia Cui[†] Frank de Jong[‡] Eduard Ponds[§]

May 23, 2005

Abstract

In this paper we model the transfers of value between the various generations in a funded pension scheme (pension fund). Value-based generational accounting is used as framework of analysis. We define a setting like in the Netherlands, where pension benefits and contributions may depend on the funding ratio (the ratio of assets to liabilities of the fund). Furthermore, we define explicit risk allocation rules, specifying which stakeholders, when, and to what extent bear the risk. A pension deal may lead to substantial transfers of wealth between young and old generations, depending on the asset returns and the exact risk allocation scheme. Using contingent claims valuation methods, we calculate these transfers and the distribution of value across generations. A pension deal is recognized as a zero-sum game in value-terms, however it is potentially a positive-sum game in welfare-terms. Pension schemes that provide safer and smoother consumption streams are ranked higher in utility terms. A smoother consumption stream can be achieved by allowing risk shifting over time, using benefit indexation instruments, contribution instruments or a combination of these two type of instruments. Our results show that intergenerational risk sharing is welfare-enhancing compared with pure individual pension schemes. Even initially under-funded collective funds may still provide higher utilities than the individual benchmark.

Keywords: fair value, value-based generational accounting, intergenerational risk sharing, defined benefit plan, defined contribution plan.

*A previous version has been presented at the 4th RTN Workshop on "Financing Retirement in Europe: Public Sector Reform and Financial Market Development", Louvain Belgium, the AFIR colloquium 2004 in Boston, the Netspar workshop in Tilburg and the AFB seminar at ABP Pension fund. We would like to thank Keith Ambachtsheer, Paula Lopes and Theo Nijman for their comments on earlier versions of the paper.

[†]University of Amsterdam, ABP Pension Fund and Tinbergen Institute, Roetersstraat 31, 1018 WB Amsterdam, The Netherlands. E-mail: cui@tinbergen.nl

[‡]University of Amsterdam, Finance Group, Faculty of Economics and Econometrics, Roetersstraat 11, 1018 WB Amsterdam, The Netherlands. E-mail: fdejong@uva.nl

[§]ABP Pension Fund and Netspar, Tilburg University. E-mail: e.ponds@abp.nl

1 Introduction

Funding decisions in pension funds in the past have been dominated by the traditional actuarial approach. The main goal of this approach is to arrive at stability in the contribution rate and the funding ratio over time. The approach is typically grounded on rules of thumb as to valuation and accounting issues. The actuarial approach recently has been heavily criticized (see for example Exley *et al.* 1997, Bader and Gold 2002, Chapman *et al.* 2001). The approach contrasts sharply with the worldwide trend in accounting standards towards more transparency through market-based reporting based on fair value. The fair value approach is based on economic principles. The application of economic principles makes possible to restate funding issues of pension funds in terms of ‘economic value’. Economic value implies risk-adjusted valuation of future outcomes. This paper applies the value-based approach for pension fund issues, in particular regarding the issue of intergenerational transfers of value.

Chapman *et al.* (2001) employ the value-based approach to strategic decision making within a *company pension fund*. They model the fund not as a self-contained entity but simultaneously with the company. The stakes of the various parties aggregate to 100% of the assets of the company, including the assets of the pension scheme. Strategic decisions as to the investments, contribution rate and indexation have no impact on the total economic value of the combined stakes of the stakeholders. However, these decisions may well lead to transfers of value between the stakeholders. How and to what extent will depend on rules concerning the allocation of a funding shortage or funding surplus. Chapman *et al.* (2001) investigate different allocation rules and make clear that the pension fund must be seen as a zero-sum game in value terms. The value-based approach clarifies who gains and who loses from changes in strategic policy variables.

We combine the value-based approach with the method of generational accounting (Ponds 2003). Public finance has developed generational accounting for investigating intergenerational distributional effects of fiscal policy (Auerbach *et al.* 1999). Generational accounting is based on the intertemporal budget constraint, which requires that either the current or the future generations pay for the government spending by taxes. Generational accounting reveals the zero-sum feature of the intertemporal budget constraint of government finance: what some generations receive as an increase in net-life-time income will have to be paid by some other generations who will experience a decrease in net-life-time income. Planned increase or decrease can be used for tax smoothing in time in order to realize a sustainable fiscal policy.

Equally, the method of generational accounting may be of use to evaluate the policy of pension funds covering current and future participants. Two similarities with public finance can be discerned. Pension funds also face an intertemporal budget constraint as the

development of the net liabilities over time has to be matched by the development of assets over time being the sum of contributions, investment results and paid-out benefits. Secondly, as the government uses the tax instrument to close the budget over time, adjustments in contribution and indexation rates are the instruments of pension funds to close the balance of assets and liabilities (funding ratio) over time.

We employ the value-based approach for a pension fund based on intergenerational risk-sharing. This kind of risk-sharing can be found in *public sector pension funds* and *industry pension funds* in countries like the UK, USA, Canada and the Netherlands. We present a framework to model the transfers of value between the various generations in a pension fund. This model builds on previous contributions in this field, such as Bader & Gold (1992), Blake (1998), Chapman et al. (2001), Ponds (2003), and De Jong (2003). Members of the fund pay contributions during their working life and receive defined pension benefits after retirement. Pension benefits and contributions may depend on the funding ratio (the ratio of assets to liabilities of the fund). We define explicit risk allocation rules, specifying who of the stakeholders, when, and to what extent is taking part in risk-bearing. Such a pension deal may lead to substantial transfers of wealth between young and old generations, depending on the asset returns and the exact risk allocation scheme. The model for the returns on assets and liabilities implies a deflator that can be used to value future cash flows such as contributions, benefits and net asset transfers. Using contingent claims valuation methods, we calculate the transfers and the distribution of value across generations for alternative sets of risk-allocation. In total, we distinguish four stylized, distinct variants in funded plans based on collective risk-bearing. The 'real life' pension deals as performed by pension funds can be composed of components of these stylized variants. The alternative pension deals have no impact on the total economic value of the combined stakes of the stakeholders, however they will lead to different outcomes with respect to the distribution of total value amongst the stakeholders.

The value-based approach is applied also to individual plans. First of all we model a standard defined contribution plan (DC) with a fixed contribution rate. At retirement the capital is converted into a stream of fixed real or nominal annuities. Results may be improved by allowing flexibility in the contribution rate and the stream of benefits (variable annuity). We study three variants of individual plans with flexibility.

We also perform welfare analysis. Pension deals with safer and smoother consumption streams are ranked higher in utility terms. The smoother consumption stream can be achieved by allowing risk shifting over time via flexibility in contributions and benefits. The results show that in utility terms a pension fund as a risk-sharing arrangement is generally more useful than an individual pension saving program without risk-sharing.

The results indicate that intergenerational transfers are potentially useful and valuable in welfare terms. The private market fails to provide insurance products based on inter-

generational risk-sharing, although its usefulness has been clarified by Gordon and Varian (1988), Merton (1983), and Shiller (1999). Indeed, a funded defined benefit scheme with mandatory participation may be seen as an appropriate instrument for organizing intergenerational risk-sharing.

The recognition of the welfare aspects of risk-sharing within pension funds is important. An analysis in terms of only economic value may easily lead to the spurious conclusion that a pension fund is not relevant (Exley 2004). The pitfall in the argument is that a pension fund being a zero-sum game in value terms only imply transfers between the stakeholders where these transfers do not have any role. However, adding a welfare analysis clarifies that intergenerational transfers between stakeholders is potentially welfare-enhancing. Indeed, a pension fund is always a zero-sum game in value-terms, however it is potentially a positive-sum game in welfare terms.

Under the mandatory participation and no labor mobility assumption, our analyses take the labor supply (hours worked, retirement decision) and the wage profile as exogenously given. This simplification helps to distinguish the effect of intergenerational risk sharing, but it ignores the interplay of risk sharing arrangements and the labor / leisure choice. The welfare gain or loss due to the labor / leisure choice will be a subject of future research.

This paper is organized as follows: Section 2 introduces the basic setup of the model using individual pension arrangements. Section 3 then introduces collective pension arrangements (pension funds) that allow for intergenerational risk sharing. Section 4 presents the results of the valuations, and Section 5 concludes. The economic environment is spelled out in the Appendix A. Appendix B sets out the two methods of valuation of pension deals, this is first the analysis in economic value terms (deflator method) and secondly the analysis in welfare terms (utility function).

2 Generational accounts

For simplicity, we assume each cohort consists of identical individuals. The assumptions on the life cycle pattern of income and mortality are quite simple. Working life starts at 25 and lasts until 65, when there is mandatory retirement. To simplify the calculations, we assume the agents die at 80.¹ The individual has a flat salary path and earns 30000 per year during his career (from age 25 to 65). Everything is in real euro's, where it is assumed that wage inflation is identical to price inflation. There is an annual state pension of 10000 (euro, in real terms) available after retirement. The notional pension buildup percentage is assumed to be 2.25% per year over the pensionable salary, this is salary minus the state

¹This assumption is not restrictive as long as the pension fund is big enough to diversify longevity risk internally, or is able to annuitize the wealth for the cohort in the market using a full menu of fairly priced indexed or equity linked annuities.

pension income. The target annual real pension income b is determined by

$$b = \int_0^R (sal_t - statepension_t) dt \cdot 2.25\% \quad (1)$$

where $t = 0$ denotes age 25, and $t = R$ denotes age 65, when the individual retires. This gives $b = 18,000$ euro. The annual real cost price contribution rate p is solved from the following present value budget constraint:

$$p \int_0^R \exp(-rt) dt = b \int_R^T \exp(-rt) dt \quad (2)$$

This gives $p = 3,807$ euro per year (or 12.87% of pensionable salary), assuming $r = 2\%$. Throughout the paper, we assume that the only savings of the consumer are in the pension plan. All the variables are in real terms, i.e. compensated for inflation.

An important tool of analysis in our paper is the value of the assets (or surplus) that the individual generation leaves behind at the end of its lifetime. These remaining surpluses summarize the intergenerational transfers. To see this, write the dynamics of the individual account's surplus as

$$dS_t = (P_t - p)dt - (B_t - b)dt + (R_t - r)A_t dt \quad (3)$$

where P_t and B_t are the actual premiums and benefits, and p and b the target premiums and benefits; A_t denotes the value of the assets and $R_t - r$ the excess return on these assets. Working out this equation gives the remaining surplus at the end of the individual's lifetime

$$S_T = S_0 + \int_0^R (P_t - p)dt - \int_R^T (B_t - b)dt + \int_0^T (R_t - r)A_t dt \quad (4)$$

Assuming each generation starts out with a zero surplus, $S_0 = 0$, the present value of the final surplus equals²

$$PV(S_T) = PV\left(\int_0^R (P_t - p)dt\right) - PV\left(\int_R^T (B_t - b)dt\right) \quad (5)$$

When $PV(S_T)$ is positive, the generation will pay more contribution than they receive benefits. A negative outcome for $PV(S_T)$ will result when benefits received are in excess of contributions paid.

We analyze a variety of pension arrangements. One popular individual pension arrangement found in market is the conventional *Defined Contribution (DC) plan*. The individual contribute a fixed premium $P_t = p$ every year and invests this in a portfolio of assets with real return R_t (the return generating process is specified in Appendix A). At retirement, the accumulated asset is converted into a conventional (real or nominal) annuity, using

²Here we used that in an efficient market, the investment in any asset is a fair deal, i.e. has zero net present value.

the market prevailing annuity factor a_{RT} ³. The pension benefit level is determined by the asset value A_R and the annuity factor a_{RT} at the retirement date, i.e. $B_t = A_R a_{RT}$ for $R < t < T$.

The DC plan may be seen as the modeling of the standard product available in the annuity market.⁴ A disadvantage of this strategy is that in between adjustments in the contribution rate and the stream of benefits are not possible. This flexibility may contribute to an improvement of the risk management process.

One variant of the conventional DC policy is the *Individual Drawdown (IDD)* pension plan. In that plan, the individual contributes a fixed premium $P_t = p$ every year and invests this in a portfolio of assets with real return R_t . It is assumed that the asset mix remains unchanged over the life cycle. After retirement, the individual draws down the flexible annuity value $a_{tT}A_t$ of the assets, where

$$a_{tT} = \left\{ \int_t^T \exp(-r(s-t))ds \right\}^{-1} \quad (6)$$

is the annuity factor. The asset dynamics before and after retirement then are

$$\begin{aligned} dA_t &= (R_t A_t + p)dt, & 0 < t < R \\ dA_t &= (R_t - a_{tT})A_t dt, & R < t < T \end{aligned}$$

The main difference with the DC plan is that in the IDD plan, the original asset mix is maintained, whereas in the DC plan, the full capital at retirement is invested in either real (i.e. inflation indexed) bonds or nominal bonds.

Another pension scheme is the *Target Benefit scheme*. Here, the aim is to provide a fixed benefit b after retirement. The premium to be paid before retirement is flexible: depending on the returns on the investment portfolio, a higher or lower premium is paid every year. The premium policy is such that at age R (retirement), the capital is sufficient to cover the fixed benefits. The capital is converted into portfolio of index-linked bonds at retirement date. To find a reasonable premium policy, we need to define the pension liability

³The real annuity factor at the retirement date R is:

$$a_{RT} = \left\{ \int_R^T \exp(-r(s-R))ds \right\}^{-1}$$

The nominal annuity factor at the retirement date R is:

$$a_{RT} = \left\{ \int_R^T P(R, s)ds \right\}^{-1}$$

where $P(R, s)$ is the zero coupon bond prices at time R .

⁴Actually the modeled DC plan is a very good deal compared with what individuals in real life can get from the annuity market. In many countries, this market is plagued by adverse selection problems, high costs (administration and investment fees) and incompleteness (equity linked and index linked annuities are not available in most countries).

ABO_t , which is the present value of future benefits, minus the yet to be paid premiums:

$$ABO_t^{ind} = b \int_R^T e^{-r(s-t)} ds - p \int_t^R e^{-r(s-t)} ds, \quad 0 < t < R \quad (7)$$

$$= b \int_t^T e^{-r(s-t)} ds, \quad R < t < T \quad (8)$$

The dynamics of ABO_t^{ind} can be described by

$$\begin{aligned} dABO_t^{ind} &= r \cdot ABO_t^{ind} + p dt, & 0 < t < R \\ &= r \cdot ABO_t^{ind} - b dt, & R < t < T \end{aligned}$$

The surplus in the individual's account is then the difference between assets and liabilities

$$S_t = A_t - ABO_t \quad (9)$$

A premium policy that guarantees that at age R the surplus is zero can be formulated as follows

$$P_t = p - a_{tR} S_t, \quad 0 < t < R \quad (10)$$

The drawback of such a scheme is that it leads to potentially large fluctuations in premiums just before retirement, when a_{tR} increases fast. A more sensible policy is to smooth the surplus over the full remaining lifetime. We call this policy the *Life Cycle* (LC) policy. Of

course, the benefit after retirement can then be different from the target benefit b . The premium and benefit policy for this scheme is

$$P_t = p - a_{tT} S_t, \quad 0 < t < R$$

$$B_t = b + a_{tT} S_t, \quad R < t < T$$

All these policies can be seen as particular versions of more general linear premium and benefit strategies of the form

$$P_t = p - \alpha S_t \quad (11)$$

$$B_t = b + \beta S_t \quad (12)$$

where P_t and B_t are the actual premiums and benefits, and p and b are the target values as defined before. The policy parameters α and β characterize the type of pension deal. Substituting the expressions for S_t , A_t and ABO_t , the dynamics of the surplus can be written simply as

$$dS_t = (R_t - r) A_t dt + (r - \alpha) S_t dt, \quad 0 < t < R \quad (13)$$

$$= (R_t - r) A_t dt + (r - \beta) S_t dt, \quad R < t < T \quad (14)$$

where R_t is the real return on the asset portfolio and r is the real interest rate.⁵

Figure 1 displays the relationship between the relevant variables with the actual surplus at the end of each period. A variable defined contribution scheme can be modeled as fixed premium ($\alpha = 0$) and a flexible benefit ($\beta > 0$); the case $\beta = a_{tT}$ corresponds to the individual cohort IDD policy, where $B_t = a_{tT}A_t$. A defined benefit scheme, on the other hand, is characterized by fixed benefits ($\alpha > 0$) and flexible premiums ($\beta = 0$). The Target Benefit policy can be interpreted as the individual counterpart of a typical DB scheme. The TB plan has adjustment parameter $\alpha = a_{tR}$, and the cohort account at the retirement date R will always be exactly sufficient to pay out the riskless benefit b after retirement ($\beta = 0$).

3 Steady state pension fund and hybrid pension schemes

In the previous section, the assets and surplus were defined for a (notional) account per cohort. In practice, policy rules of pension funds are not based on the surplus or deficits in the individual cohort's accounts, but rather in the aggregate surplus of the fund. In this section, we will model the steady state behavior of an aggregate pension fund.

For the scheme where benefits are fixed but premiums adjustable, the situation is simple. The surplus in the fund is defined as

$$S_t = A_t - ABO_t \quad (15)$$

where ABO_t denotes collective the accrued benefit obligations, which equal the present value of accrued benefits summed over all cohorts (ages) u . In the stationary case⁶ with fixed target premium and benefits, ABO_t is constant over time and equals

$$ABO^* = \int_0^T ABO_t^{ind} dt \quad (16)$$

where ABO_t^{ind} is the individual cohort liability given in equation (7).

3.1 Linear policies

The collective counterpart of the Target Benefit plan is the traditional Defined Benefit plan, where the benefits are fixed at $B_t = b$, whereas the premium policy is flexible and can be formulated as

$$P_t = p - \alpha S_t \quad (17)$$

⁵To get stability over time (i.e., non-explosive surplus values), we need to assume that either $\alpha > r$ or $\beta > r$.

⁶In a stationary scheme, the inflow and outflow of member guarantee a stable composition of members as to age and sex. In case of an aging scheme, one could take the (expected) future ageing into account when setting the cost-price contribution.

with $r < \alpha < 1$. A natural choice for the parameter α is the inverse of the average "age" of the fund, i.e. the inverse of the average duration of the liabilities, $1/D$, ($\approx \frac{1}{20}$). This makes the results most comparable to the individual Target Benefit case.

The collective counterpart of the Defined Contribution scheme ($\alpha = 0$) is slightly more involved. The fund starts out at $t = 0$ in a stationary situation with surplus

$$S_0 = A_0 - ABO_0 \quad (18)$$

with $ABO_0 = ABO^*$. The pension fund has the possibility to adjust the liabilities (and hence future benefits) if the financial situation of the funds calls for that. This is modeled in the form of an additional indexation i_t . Then every period, the (real) ABO is adjusted with the additional indexation

$$dABO_t = i_t ABO_t dt \quad (19)$$

The risk in the funding process is borne by all members with accrued benefits by a suitable adjustment in the offered indexation. The additional indexation rate is needed to close (partly) the balance of the fund at the end of the period. The additional indexation rate i_t can be solved from the expression below:

$$i_t = \beta \left(\frac{S_t}{ABO_t} \right) \quad (20)$$

Note that the additional indexation rate applies only to the accrued benefits.

The dynamics of the surplus in the collective schemes can be described by:

$$dS_t = [(P_t - p) - (B_t - b) + R_t A_t - (r + i_t) ABO_t] dt \quad (21)$$

This equation is very intuitive: the change in the fund's surplus is determined by the difference between expected and actual cash flows, the asset returns, minus the increase in the accrued benefits due to the additional indexation and the cost of capital (the real interest rate).

3.2 Hybrid schemes

Now we turn to two hybrid variants as these variants are composed of components of the collective DB scheme and the collective DC scheme.

The first variant offers a nominal guarantee on the benefits. Of course, this nominal guarantee is costly and is financed by limiting the indexation when surpluses are high. Figure 2 shows the main characteristics of such a plan being named a Collective Hybrid Defined Benefit (CHDB). It is assumed here that the maximum indexation is equal to two times the actual inflation rate. When the funding residual falls below the lower bound or exceeds the upper bound, then current workers absorb this part of the residual.

The risk allocation in the CHDB plan is according to the set of rules below. The indexation of the benefits form the initial buffer for shocks in the surplus. In that sense, this deal is comparable to the collective DC system. However, the actual indexation is bounded between zero and twice the actual inflation π_t , hence the additional indexation is between $-\pi_t$ and π_t . The premiums absorb the more extreme shocks (both positive and negative).

$$\begin{aligned}
[1] \quad & \text{when } \frac{S_t}{ABO_t} < -\pi_t \text{ then} \\
& i_t = -\pi_t \\
& P_t = p - S_t - \pi_t ABO_t \\
[2] \quad & \text{when } -\pi_t \leq \frac{S_t}{ABO_t} \leq \pi_t \text{ then} \\
& i_t = \frac{S_t}{ABO_t} \\
& P_t = p \\
[3] \quad & \text{when } \frac{S_t}{ABO_t} > \pi_t \text{ then} \\
& i_t = \pi_t \\
& P_t = p - S_t + \pi_t ABO_t
\end{aligned}$$

Alternatively, the premiums can be the initial buffer, within limits, and the extreme risks can be allocated to current and future benefits, providing a more stable stream of contributions. Suppose the workers accept a maximum contribution equal to $2p$. They buy this protection by giving up the possibility that in good times the contribution rate may fall below zero. When the surplus falls below the lower bound or exceeds the upper bound, then current retirees absorb this part of the surplus by accepting indexation risk. The risk allocation is then according to the following set of rules:

$$\begin{aligned}
[1] \quad & \text{when } S_t < -p \text{ then} \\
& i_t = \frac{S_t + p}{ABO_t} \\
& P_t = 2p \\
[2] \quad & \text{when } -p \leq S_t \leq p \text{ then} \\
& i_t = 0 \\
& P_t = p - S_t \\
[3] \quad & \text{when } S_t > p \text{ then} \\
& i_t = \frac{S_t - p}{ABO_t} \\
& P_t = 0
\end{aligned}$$

Notice that in these hybrid schemes there is no smoothing of either premiums or additional indexation, i.e. the parameters α and β are implicitly put equal to 1. A more general structure where the premiums absorb the extreme shocks

$$\begin{aligned}
[1] \quad & \text{when } \beta \frac{S_t}{ABO_t} < -\pi_t \text{ then} \\
& i_t = -\pi_t \\
& P_t = p - \alpha(S_t + \pi_t ABO_t / \beta)
\end{aligned}$$

$$\begin{aligned}
[2] \quad & \text{when } -\pi_t \leq \beta \frac{S_t}{ABO_t} \leq \pi_t \text{ then} \\
& i_t = \beta \frac{S_t}{ABO_t} \\
& P_t = p
\end{aligned}$$

$$\begin{aligned}
[3] \quad & \text{when } \beta \frac{S_t}{ABO_t} > \pi_t \text{ then} \\
& i_t = \pi_t \\
& P_t = p - \alpha(S_t - \pi_t ABO_t / \beta)
\end{aligned}$$

A similar adjustment can be made to the second structure:

$$\begin{aligned}
[1] \quad & \text{when } \alpha S_t < -p \text{ then} \\
& i_t = \beta \frac{S_t + p/\alpha}{ABO_t} \\
& P_t = 2p
\end{aligned}$$

$$\begin{aligned}
[2] \quad & \text{when } -p \leq \alpha S_t \leq p \text{ then} \\
& i_t = 0 \\
& P_t = p - \alpha S_t
\end{aligned}$$

$$\begin{aligned}
[3] \quad & \text{when } \alpha S_t > p \text{ then} \\
& i_t = \beta \frac{S_t - p/\alpha}{ABO_t} \\
& P_t = 0
\end{aligned}$$

Again, a natural choice for the smoothing parameters α and β seems the inverse of the average duration of the liabilities ($1/D$).

Table 1 summarizes the individual and collective pension deals that we analyze in the remainder of the paper.

4 Results

In this section, we make cross-deal comparisons in terms of present values and expected utilities. We distinguish two financial market settings to which the pension fund have access. In section 4.1, the pension fund invests the assets into stocks and Index Linked Bonds (which realizes the risk-free return in the portfolio). In section 4.2, we will replace the ILB with nominal bonds.

Recall that in the collective pension deals, the conditional contribution and indexation rules are based on the surplus of the collective account. Within the collective setting, we also run a generational accounting, which tracks the asset development of an individual or cohort within the collective account. The individual accounts are only artificial accounts however, because the contribution and indexation rules are defined by the collective effects.

4.1 Valuation of the pension deals with ILB and equity

Table 3 and 4 show the valuation of pension deals outlined before, including four collective plans and four individual plans. The top, middle and bottom panels of Table 3 display the results of the collective plans when the fund is initially fully funded, over-funded and under-funded respectively. The results of the individual plans are listed in Table 4. The results are expressed in real terms, as multiples of annual salary, which is easier to read and to interpret. For each deal, we assume three different asset mixes: 100% ILB; 50% ILB and 50% Equity; and 100% Equity⁷.

Collective deals

Table 3 shows a number of interesting characteristics of the pension deals. First, the table provides an estimate of the present value of the actual contributions made to the fund and the actual benefits received from the fund over the lifetime. In several of the collective deals the value of the actual contributions may deviate from the cost-price contribution, and the actual benefits may differ from the targeted level of real benefits, which corresponds to the nominal case with the unconditional indexation. The table further provides in column (7) the present value of the net transfer to the individual member, that is the present value of actual benefits minus the present value of actual contributions. This number can also

⁷This paper pays no attention to an age-dependent asset mix for the individual plans. This strategy, known in the literature as life-cycle investment strategy, prescribes a risky investment mix when the young individual starts saving for retirement and the risk is decreased gradually when the individual is getting older. The risk-exposure is at a minimum during the retirement period. For a fair comparison of individual and collective plans, we strive for a 'level playing field' which implies the exclusion of the shrinking horizon aspect of individual plans. However, the individual DC plan, deal 1, actually will differ with respect to the mix before and after retirement as the built-up capital during the working period is converted at retirement in a fixed annuity offered by the market. The implied rate of return then is the rate of interest the insurance company is using when converting pension capital into a stream of fixed annuities.

be seen as the net present value (for the individual) of joining the pension deal. To gain more insight in the potential size of the transfer, we split the net transfer into positive and negative parts, that is, $\max(\text{transfer}, 0)$ and $\min(\text{transfer}, 0)$, which are the value of a call and a put option respectively. In most of the cases, the value of the option is very large, indicating a substantial amount of transfers. The call and put option can be regarded as contracts written between current generation and all other generations. The current generation holds a call option from the other generations, and writes a put option to the other generations. When the value of call equals the value of put, the pension deal is a fair deal in value terms *ex ante*.

Let's first focus on the fully funded situation (i.e. funding rate is 100%). For the collective deals CDB and CDC, the present values of the net transfers (and also the call and put value) are exactly zero for the 100% Index Linked Bond investment case. This is no surprise, as with this investment strategy the cost price contribution is exactly the right price to guarantee the benefits. Also, since the growth of the assets exactly tracks the liabilities (inflation), there is no real risk. With 50% or 100% equity, the picture changes. First, the call and put values indicate the size of potential intergenerational transfers. These transfers will be higher the more risk is taken in the investment policy. Second, the call and put values are higher in the CDB case than in the CDC case. So fixing the benefit during retirement calls for large redistribution of value between the generations. Third, let's take a look at the size and distribution of the actual contributions. Figure 3 (b) plots the quantiles of the normalized actual contributions of three collective plans (CDB, CHDB and CHDC) with adjustable contributions. The distribution of CDB actual contribution is skewed, however, the present values of the actual contribution in CDB equals the cost-price contribution. This is due to the equity premium: due to the high average return on equity, contribution reductions are more frequent (and also larger) than contribution increases. However, this does not add value to the pension deal: the lower contributions occur in scenarios where the equity returns are high, but the deflators in such scenarios are low.

The analysis of the two hybrid deals (CHDB and CHDC) is more complex. Recall that in CHDB, the additional benefits are adjusted up to limits, and the additional contributions pick up the extreme (high and low) returns. In CHDC it is the other way around: contributions can be adjusted up to limits, and the benefits pick up the extremes. Perhaps not surprisingly, the dispersion of actual contributions in CHDB is still large (Figure 3(b)). One special observation is that the 80% and 50% quantiles of the actual contributions in CHDB coincide with each other. It means that most of the funding deficits can be absorbed by first reducing indexations (to zero), and therefore additional contributions are not required in many cases.

In present value terms, the limits on indexation and contributions now have a substantial impact. For CHDB, having larger portion of the assets in equity leads to lower present

value of the contributions. However, the value of the actual benefits (that pick up the bad returns!) is also lower. The net transfer is relatively small, however. In many ways, CHDC is the mirror image of CHDB: actual contributions are higher with more equity, but the value of the benefits is correspondingly higher too. Again, the net present value of the deal is relatively small. The sizes of the potential transfers of the two hybrid plans (indicated by calls and puts) lie in between that of the cases CDB and CDC.

The cases with initial over- and underfunded deserve some attention. We assume that the fund has a surplus at the collective level. However, it is hard to attribute this surplus to the individual generations and therefore we start off the account for an individual generation at zero. With initial overfunding, then, the average premiums will typically be below the target premiums (in the DB scheme), and the benefits will be above the target benefits (in the collective DC scheme), or both (in the hybrid schemes). This is translated into larger positive transfers and smaller negative transfers to this generation. Hence, the value of the "call" is larger than in the fully funded case and the value of the "put" is smaller. For the initially underfunded case the result is exactly the opposite. Notice that this calculation ignores the transfer of the initial surplus, on which we assume the generation has no direct claim. The impact of the initial fund surplus is only indirect via lower premiums and higher benefits.

Individual deals

For the four individual deals (DC, TB, IDD and LC), in Table 4, the actual contributions and benefits are always equal. That is, what an individual can draw out of the pot is what he has saved into the pot. Furthermore, there is no remaining asset in the individual account, hence no intergenerational transfer taking place. Figure 3 (a) compares the level and distribution of the actual contributions of TB, LC and the collective counterpart CDB, where in the contributions are adjustable. One distinct feature of TB is that the actual contributions are extremely dispersed before the retirement, because an individual has limited time period to smooth out funding shocks. One year before the retirement, in our discrete model, the annuity factor increases quickly up to one. The full amount of the funding surplus must be absorbed at once. The LC plan provides much less dispersed actual contributions, because the individual can smooth out funding surplus in the whole life time.

Figure 4 compares the level and distribution of the actual benefits of IDD (LC) and their collective counterpart CDC (CHDC), where in benefits vary depending on asset levels. Three observations are clear: The conventional DC benefit fully depends on the asset value at retirement date, while the IDD and LC benefits have the potential of growth. IDD and CDC provide very similar benefit profiles, while LC and CHDC provide very similar benefit profiles; The probability mass of the benefit distributions are higher than b , but the present value of the actual benefits all equals the targeted value. This is because the deflators

weight the low-return states much higher than the states of high returns.

4.2 Valuation of the pension deals with nominal bonds and equity

In a market setting where ILB is not available, investing in nominal bonds is an obvious alternative. Inflation risks are not perfectly hedged by this portfolio. But replacing ILB with nominal bonds in the asset mix does not change the overall picture of the above stories. To approximate the returns on an actively managed bond portfolio, which may include various coupon-bonds with different term-to-maturity and different coupon rates, we use the return process of a zero-coupon bond, with term-to-maturity equals the (modified) duration of the bond portfolio. Here we assume that the duration of the bond portfolio is 7 years. The results⁸ are very comparable with the results shown in Table 3. Since there is no risk-free asset in this case, the calls and puts are nonzero for 100% nominal bonds investment in the collective deals, although they are relatively small comparing with the more risky asset of equities. The intuitions from section 4.1 hold.

4.3 Welfare analysis

This section compares the pension deals in utility terms, with special attention to the order of preferences regarding the different deals and different asset mix from a newly-entry-cohort's point of view. The specification of the utility functions are given in Appendix B. The results presented in the paper are calculated based on CARA utility function⁹. The bar charts of the certainty equivalent consumption levels are shown in Figure 5 (left: ILB/Equity; right: Nominal Bond/Equity) for different value of risk aversion. For a given value of risk aversion parameter, the deals that are able to provide smoother and higher level of consumption streams are ranked higher in the bar chart.

First we look at the individual pension deals. As we have noted before, the DC plan may be seen as the standard product available in the annuity market: saving up till retirement and then converting the capital into a stream of fixed real or nominal annuities. We use this conventional DC plan as a benchmark for the cross-deal comparison. The three other individual plans allow for flexibility in contributions and benefits with the aim to improve the results. Two observations are clear. First, allowing flexibility indeed may improve the outcome. The IDD and the LC plans provides higher utility than the standard DC plan. The best result is achieved with the LC plan. This is due to the fact that LC plan smooths funding shocks out over a long period. The TB plan ranks the lowest, primarily because it leads to potentially large fluctuations in contributions just before retirement.

⁸Results are available upon request.

⁹CARA utility function can accommodate the possible negative consumptions, which happens in TB plan. For other deals, the certainty equivalent consumption calculated using CARA and CRRA utility function, are very much the same. The results are available upon request.

The second observation is that holding 100% nominal bonds is preferred over holding only ILB. This result is due to two effects: notice that when nominal bonds are in the portfolio, the fluctuations in the consumption stream is increased, hence the utility will be lower; on the other hand, the average level of the consumption stream is increased due to the risk premium in the nominal bonds, hence the utility will be increased. The second effect dominates, given our current set of model specifications.

Let's then look at the collective deals. Several interesting things are observed. In many cases the size of potential intergenerational transfers is large, especially for portfolio with 100% equities. However what seems interesting is that, the potential significant transfer does not necessarily decrease the expected utility level. In most cases (except for CDC with $A=8$), the consumer prefers a high equity stake (how much depends on his risk aversion). This is the effect of the high equity premium: although the equity premium doesn't add anything in market value terms, in utility terms, equity investments seem to be a good deal.

From the comparison between CDB and CDC, it is clear that CDB is preferred over CDC for most asset mixes and risk aversion profiles. Remember that pension fund can shift risk over time and absorb funding shocks gradually, hence it smooths consumptions greatly. In CDB, contributions are greatly smoothed and pension benefits are fully indexed. Furthermore, the contributions are more often reduced rather than increased due to the risk premiums. In CDC, real contributions are fixed, benefits are gradually adjusted. Therefore, the (relatively long) working period consumptions are higher in CDB on average; the (relatively short) retirement period consumptions are higher in CDC. Due to the discounting effect in the utility calculation, where utilities from the near future are weighted more than the utilities from the far future, CDB gives higher utility than CDC.

Next, the two collective hybrid plans (CHDB and CHDC) show additional interesting points. First, in most cases, the hybrid collective deals provides higher utilities over the benchmark (DC), how much depending on degrees of risk aversion and asset allocation. Other DC variants like IDD and LC come close to the collective schemes, however they are hard to implement. This result shows the advantage of collective risk sharing of a pension fund over the individual schemes. Second, the hybrid DB is preferred over the pure CDB; the hybrid DC is preferred over the pure CDC, which shows the advantage of using both risk absorbing instruments. Third, CHDB provides the highest utility in all cross-deal comparisons. The reason can be found from Figure 3 (b): Both hybrid deals utilize two risk absorbing instruments. When the indexation rule is first exercised as in CHDB, a large part of funding deficits are absorbed by cutting indexations, leaving contributions unchanged for most of the times (especially during bad years with low returns!). Therefore the 80% and 50% quantiles of the actual contributions of CHDB overlap. The unabsorbed funding surpluses leads to lower contribution and benefit than CHDC (Figure 3(c)). In CHDC, the contribution rule is exercised first. The unabsorbed funding surpluses push

the benefits upwards (higher than CHDB benefits), and hence require higher contributions. CHDB leads to higher working period consumption, and a relatively lower retirement period consumption. Due to the discounting effect in the utility calculations, CHDB gives higher utilities level.

Finally, let's look at the initially over-funded and under-funded situations for the collective deals. The present values of contributions, benefits and transfers are summarized in Table 3. Clearly, a larger scale of intergenerational transfers take place in these situations. A new-entry cohort joining an over-funded pension fund will definitely benefit from the initial funding surplus. The older generations transfers wealth to this current young. A new-entry cohort joining an under-funded fund will obviously share the funding deficits, by transfer wealth to older generations. Figure 6 (right panel) shows that a new cohort joining an under-funded collective scheme is not necessarily worse-off compared with the alternatives of individual DC and IDD plans, especially for risk averse investors. Within certain range of the funding ratio, it is possible for well-structured collective pension schemes to absorb funding deficits by intergenerational risk sharing and meanwhile enhance the welfare for her participants.

How does the risk premium affect the utility outcome? Figure 7 show some additional results with risk premiums lowered to half of the default values ($[\lambda_E = 0.1, \lambda_\pi = -0.1, \lambda_u = -0.1]$). In present value terms, the results are exactly the same as in Table 3, since the level of risk premiums has no impact on the present values. From Figure 7 we see that the levels of CEC are lowered due to the lower risk premiums, but the order is not changed comparing to the default setup.

Another remark is on the smoothing parameters (α, β) implemented by the collective pension deals. We also observed the results when using the maximum smoothing parameters $(\alpha = r, \beta = r)$, the CEC outcomes are lower than the cases with $(\alpha = 1/D, \beta = 1/D)$. It means that shifting risk into infinite future is not optimal.

5 Conclusion

In this paper, we have employed the method of value-based generational accounting as a formal framework to model the transfers of value between generations in a pension fund based on intergenerational risk-sharing. The institutional setting of public sector pension funds or industry pension funds based on intergenerational risk-sharing is used, where pension benefits and contributions may depend on the funding ratio (the ratio of assets to liabilities of the fund). Explicit risk allocation rules have been defined, specifying who of the stakeholders, when, and to what extent is taking part in risk-bearing. Using contingent claims valuation methods (deflators), we have calculated these transfers and the distribution of value across generations for alternative sets of risk-allocation. The alternative pension

deals have no impact on the total economic value of the combined stakes of the stakeholders, however they will imply different distribution of the value amongst the stakeholders. Apart from risk allocation rules, choices with respect to the asset mix also will have impact on the value distribution. An investment strategy with 100% index linked bond guarantee a full match between the assets and the value of indexed liabilities. No transfers of value between generations will occur. Any other asset allocation will lead to substantial transfers, where the risk allocation rules determine who, when and to what extent is bearing the risk.

Utility analysis is added to the valuation framework with the aim to highlight the welfare aspect of intergenerational risk-sharing within the construct of a pension fund. We have evaluated generational risk-sharing for alternative settings with respect to risk-allocation and to the asset mix. Deals which rely on risk absorption by a combination of adjustments in both indexation and contribution instruments are preferred over deals which rely on risk absorption by only one of these two instruments.

Furthermore we make clear intergenerational risk sharing within collective plans is welfare-enhancing compared with a standard defined contribution plan (DC). This standard DC plan is designed as saving a fixed contribution rate up till retirement and then converting the acquired capital into a stream of fixed benefits (real or nominal annuities). The collective schemes are superior to this standard DC product offered by the market. The results of the standard DC plan may be improved by allowing for flexibility in contributions and benefits. Ideal DC variants like the Life Cycle variant will come close to collective schemes, however they are hard to implement because of problems in the annuity market.

Even initially under-funded collective funds may still provide higher utilities than the individual benchmark. It means that young participants joining an under-funded collective scheme are not necessarily worse-off. Within certain range, it is possible for well-structured collective pension schemes to absorb funding risks by intergenerational risk sharing and meanwhile enhance the welfare for her participants. Hence the results indicate that inter-generational transfers are potentially useful and valuable in welfare terms. Indeed, a pension fund may be a zero-sum game in value-terms, however it is potentially a positive-sum game in welfare-terms.

The individual makes a trade-off between the level and the degree of riskiness of the consumption stream over the life time. The results suggest that individuals generally prefer risk-taking above a riskless position in index-linked bonds. It turns out that the increase in utility because of higher consumption level over the life cycle more than outweigh the decrease in utility because of extra risk. The consumer prefers a positive equity stake (how much depends on his risk aversion): although the equity premium doesn't add anything in market value terms, in utility terms, equity investments seem to be a good deal.

A Economic and Financial Risk factors

In this section we describe the economic environment and model the returns on investment assets and the evolution of the liabilities. We adopt the structure of the economy in Brennan and Xia (2002). Their model has four underlying risk factors: stock returns, expected inflation, unexpected inflation, and real interest rate shocks. For simplicity, we assume for now that real interest rates are non-stochastic. Real stock (equity) prices follow a geometric Brownian motion with drift:

$$dE_t/E_t = \mu_E dt + \sigma_E dZ_{E,t} \quad (22)$$

The real Sharpe ratio of the investment portfolio equals

$$\lambda_E = \frac{\mu_E - r}{\sigma_E} \quad (23)$$

Inflation is modeled as the sum of expected inflation π_t^e and an independent unexpected inflation shock. Expected inflation follows a mean reverting process

$$d\pi_t^e = a(\bar{\pi} - \pi_t^e)dt + \sigma_\pi dZ_{\pi,t} \quad (24)$$

The price level changes according to

$$d\Pi_t/\Pi_t = \pi_t^e dt + \sigma_u dZ_{u,t} \quad (25)$$

where the last term is the unexpected inflation. The corresponding Brownian motions are summarized in $\begin{bmatrix} Z_{E,t} & Z_{\pi,t} & Z_{u,t} \end{bmatrix}$ with correlation matrix

$$\rho \equiv \begin{bmatrix} 1, \rho_{E,\pi}, 0; & \rho_{E,\pi}, 1, 0; & 0, 0, 1 \end{bmatrix}. \quad (26)$$

The solution for the price level is

$$\Pi_t = \Pi_0 \exp \left\{ \int_0^t \pi_s^e ds - \frac{1}{2} \sigma_u^2 t + \int_0^t \sigma_u dZ_{u,s} \right\} \quad (27)$$

where

$$\int_0^t \pi_s^e ds = \bar{\pi}(t - B(t)) + B(t)\pi_0^e + \sigma_\pi \int_0^t B(s) dZ_{\pi,s} \quad (28)$$

with $B(t) = \frac{1}{a}[1 - e^{-at}]$.

The default investment portfolios are a mix of equity and index linked bonds, with portfolio weight w for equities. The real value dynamics of this asset portfolio are given by

$$dA_t/A_t = [r + w\sigma_E\lambda_E]dt + w\sigma_E dZ_t \quad (29)$$

In shorthand, we will sometimes denote the real asset price dynamics as $dA_t = R_t A_t$, where R_t is the right hand side of equation (29). Both the expected real return and the volatility increase linearly with the fraction w invested in equities.

As an alternative investment, we also consider portfolios of equities and nominal bonds. The time t price for a nominal bond with term-to-maturity τ is given by

$$P_t(\tau) = \exp \{A(\tau) - B(\tau)\pi_t^e\} \quad (30)$$

where $A(\tau) = B(\tau)\bar{\pi} - (\bar{\pi} + r - \sigma_u\lambda_u)\tau + \frac{1}{2}\sigma_\pi^2 \int_0^\tau B^2(s)ds$. The real return on the nominal bonds are given by

$$dP_t(\tau)/P_t(\tau) = [r - B(\tau)\sigma_\pi\lambda_\pi - \sigma_u\lambda_u] - B(\tau)\sigma_\pi dZ_\pi - \sigma_u dZ_u \quad (31)$$

The return on portfolio's of nominal bonds and equities follows the same logic as before

$$\begin{aligned} dA_t/A_t &= [r + w\sigma_E\lambda_E + (1-w)(-B(\tau)\sigma_\pi\lambda_\pi - \sigma_u\lambda_u)]dt \\ &+ w\sigma_E dZ_t - (1-w)B(\tau)\sigma_\pi dZ_\pi - (1-w)\sigma_u dZ_u \end{aligned} \quad (32)$$

Table 2 shows the default values for the model parameters to be used in the calculations. The parameters are partly based on estimates reported in Brennan and Xia (2002), but some values are superseded to numbers of our own preference. For example, we set the average inflation at 2%, in line with the informal ECB target for the euro area. We also assume the real interest rate is constant and equals 2%. Finally, the expected real return on equity is assumed to be 6%, implying an equity premium of 4%, which is in line with the long run estimates in Fama and French (2002).

B Valuation of the pension deals

Important step in the analysis of the pension deals is the valuation exercise. We make use of value analysis and welfare analysis (utility).

Value analysis

We look at all the cash flows generated by the pension deals and value these using the deflator method. This assumes all the risks are tradable. For an individual member this may be difficult once he stepped into a pension contract, but a market valuation is useful ex-ante, when deciding which pension offer to accept, or when transferring pensions from one fund to another. From the point of view of the fund member, the value of the pension deal is the value of the actual benefits minus the premiums. For a member with retirement date R and life expectancy T , this can be modeled as

$$PV = \mathbb{E} \left[- \int_0^R M_t P_t dt + \int_R^T M_t B_t dt \right] \quad (33)$$

where P_t and B_t are the actual premiums paid and benefits received. The stochastic discount factor M_t is the deflator for (real) risky cash flows, which in our model evolves according to

$$dM_t/M_t = -r dt - \kappa_E dZ_t - \kappa_\pi dZ_{\pi,t} - \kappa_u dZ_{u,t} \quad (34)$$

where $\kappa_t \equiv [\kappa_E, \kappa_\pi, \kappa_u]' = \rho^{-1}\lambda$ and λ is the vector of real market prices of risk.

Welfare analysis

For the welfare analysis, utility is defined as the expectation of the discounted sum of the utility of consumption in every period. Consumption before retirement equals labor income minus pension contributions, $c_t = Y_t - P_t$, and after retirement consumption equals benefits plus the fixed state pension, $c_t = b_{statepension} + B_t$. Since people prefer smoother consumption patterns, for a given average level of consumption, the larger the volatility of the consumption stream, the lower the utility is. Formally, the utility function takes the form:

$$U = E_0 \left[\int_0^T \exp(-rt) u(c_t) dt \right] \quad (35)$$

The utility function is either CRRA, which is the usual choice in this literature and takes the form

$$u(c_t) = c_t^{(1-\gamma)} / (1-\gamma) \quad (36)$$

or CARA with absolute risk aversion parameter A with¹⁰:

$$u(c_t) = -\exp(-Ac_t). \quad (37)$$

For ease of comparison, we translate the utility levels into certainty equivalent consumption levels (c^{CE}), designed to be constant over time. The c^{CE} can be easily backed out from the following equality

$$U = \int_0^T \exp(-rt) u(c^{CE}) dt \quad (38)$$

¹⁰When consumption streams remain positive in all the scenarios, the CRRA utility function becomes applicable. If there are scenario's with negative consumption, only CARA utility can be applied.

References

- Auerbach, A.J., Kotlikoff, L.J., and Leibfritz W. (eds) (1999), '*Generational Accounting around the World*', NBER. Chicago, Chicago University Press.
- Bader L.N. and Gold J. (2002), 'Reinventing pension actuarial science', working paper.
- Blake D. (1998), 'Pension schemes as options on pension fund assets: implications for pension fund management', *Insurance, Mathematics and Economics*, vol. 23, pp 263-286.
- Brennan, M.J. and Xia Y. (2002), 'Dynamic asset allocation under inflation', *Journal of Finance* 57, 1201-1238.
- Chapman R.J., Gordon T.J. and Speed C.A. (2001), 'Pensions, funding and risk', *British Actuarial Journal*, vol. 7, nr 4, pp. 605-663.
- De Jong, F. (2003), '*Pension Fund Investments and the Valuation of Liabilities under Conditional Indexation*', International AFIR Colloquium 2003 proceedings.
- Exley, C.J. (2004), '*Stakeholder Interests Alignment Agency Issues*', paper presented at the Centre for Pension Management Colloquium October 5-6 2004, University of Toronto.
- Exley, C.J., Mehta S.J.B. and Smith A.D. (1997), 'The financial theory of defined benefit schemes', *British Actuarial Journal*, vol. 3, nr 4, pp. 835-938.
- Gordon, R.H. and Varian H.R.(1988), 'Intergenerational risk sharing', *Journal of Public Economics*, vol 14, pp. 1-29.
- Fama, E.F., and K.R. French (2002), 'The equity premium', *Journal of Finance* 57, 637-659.
- Kotlikoff, L.J. (2001), 'Generational Policy', Auerbach A.J. and Feldstein M.S. (eds.), '*The Handbook of Public Economics*', North Holland.
- Lancaster, T. (1990), '*The Econometric Analysis of Transition Data*', Cambridge University Press, Cambridge.
- Merton, R.C. (1987), 'On the role of Social Security as a Means for Efficient Risk Sharing in an Economy where human capital is not tradeable', in Bodie Z. and Shoven J.B (eds), '*Financial aspects of the United States pension system*', Chicago.
- Ponds, E.H.M. (2003), 'Pension funds and value-based generational accounting', *Journal of Pension Economics and Finance*, vol. 2, nr. 3, pp. 295-325.
- Shiller R.J. (1999), 'Social security and institutions for intergenerational, intragenerational and international risk sharing', *Carnegie-Rochester Conference Series on Public Policy*, vol 50, pp 165-204.

Table 1: Pension deals

Deal	α	β	name	Code
1	0	a_{RT}	Defined Contribution	DC
2	0	a_{tT}	Individual Drawdown	IDD
3	a_{tR}	0	Target Benefit	TB
4	a_{tT}	a_{tT}	Life Cycle	LC
5	$1/D$	0	Collective Defined Benefit	CDB
6	0	$1/D$	Collective Defined Contribution	CDC
7	$1/D$	$1/D$	Collective Defined Benefit (hybrid)	CHDB
8	$1/D$	$1/D$	Collective Defined Contribution (hybrid)	CHDC

Table 2: Default parameters for the stochastic models

parameter	value
μ	0.06
r	0.02
$\bar{\pi}$	0.02
a	0.2
σ	0.20
σ_{π}	0.01
σ_u	0.01
λ_{π}	-0.2
λ_u	-0.2
$\rho_{E\pi}$	-0.05

Table 3: Valuation of intergenerational transfers in terms of gross salary of 30000 euro (the cost-price contribution is 3.53)

Collective Deals							
funding Ratio		Equity as %	act. Contrib (3)	act. Benefit (4)	pos. transfer (5)	neg. transfer (6)	net transfer (7) = (5) - (6)
100%	CDB	0%	3.53	3.53	0.00	0.00	0.00
		50%	3.53	3.53	0.65	0.65	0.00
		100%	3.53	3.53	1.38	1.38	-0.01
	CDC	0%	3.53	3.53	0.00	0.00	0.00
		50%	3.53	3.53	0.43	0.43	0.00
		100%	3.53	3.54	0.85	0.85	0.00
	CHDB	0%	3.44	3.52	0.01	0.08	-0.07
		50%	3.40	3.46	0.51	0.57	-0.06
		100%	3.19	3.21	1.13	1.16	-0.03
	CHDC	0%	3.53	3.53	0.00	0.00	0.00
		50%	3.70	3.81	0.59	0.69	-0.11
		100%	4.19	4.23	1.16	1.20	-0.03
funding Ratio							
110%	CDB	0%	3.10	3.53	0.43	0.00	0.43
		50%	3.10	3.53	0.89	0.47	0.43
		100%	3.10	3.53	1.62	1.19	0.43
	CDC	0%	3.53	3.80	0.27	0.00	0.27
		50%	3.53	3.80	0.60	0.33	0.27
		100%	3.53	3.81	1.05	0.77	0.28
	CHDB	0%	3.37	3.75	0.38	0.00	0.38
		50%	3.22	3.62	0.79	0.41	0.38
		100%	2.92	3.31	1.40	1.02	0.39
	CHDC	0%	3.10	3.53	0.43	0.00	0.43
		50%	3.37	3.91	0.95	0.52	0.43
		100%	3.98	4.40	1.43	0.99	0.44
funding Ratio							
90%	CDB	0%	3.96	3.53	0.00	0.43	-0.43
		50%	3.96	3.53	0.45	0.88	-0.43
		100%	3.97	3.53	1.13	1.59	-0.45
	CDC	0%	3.53	3.26	0.00	0.27	-0.27
		50%	3.53	3.25	0.29	0.56	-0.27
		100%	3.53	3.25	0.67	0.95	-0.29
	CHDB	0%	3.53	3.29	0.00	0.25	-0.25
		50%	3.58	3.29	0.42	0.67	-0.25
		100%	3.47	3.10	1.02	1.29	-0.27
	CHDC	0%	3.96	3.53	0.00	0.43	-0.43
		50%	4.07	3.73	0.37	0.81	-0.43
		100%	4.43	4.06	0.89	1.34	-0.45

Table 4: Valuation of individual deals in terms of gross salary

Individual Deals			
	Equity	act. Contrib	act. Benefit
DC	0%	3.53	3.53
	50%	3.53	3.53
	100%	3.53	3.53
TB	0%	3.53	3.53
	50%	3.53	3.53
	100%	3.52	3.53
IDD	0%	3.53	3.53
	50%	3.53	3.53
	100%	3.53	3.54
LC	0%	3.53	3.53
	50%	3.53	3.53
	100%	3.53	3.53

Figure 1: Linear premium policies

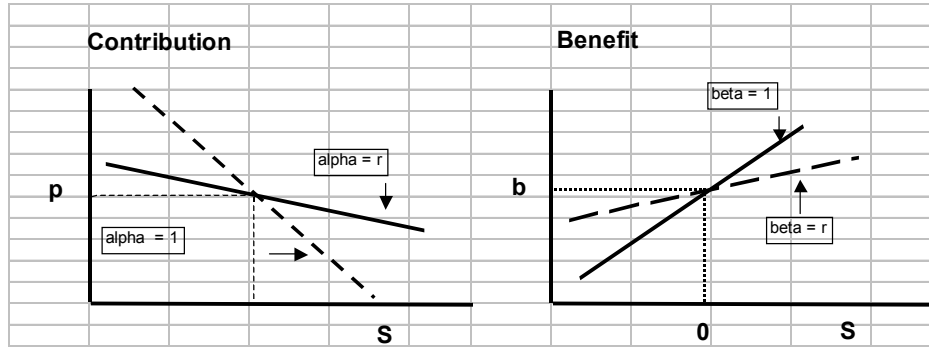


Figure 2: CHDB and CHDC contribution and indexation rules

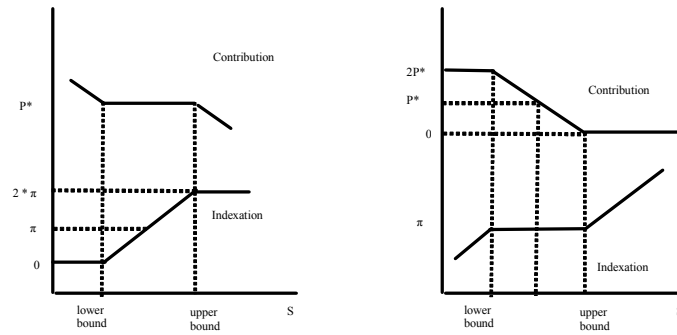


Figure 3: Quantile plot of the normalized actual contribution (ILB / Equity)

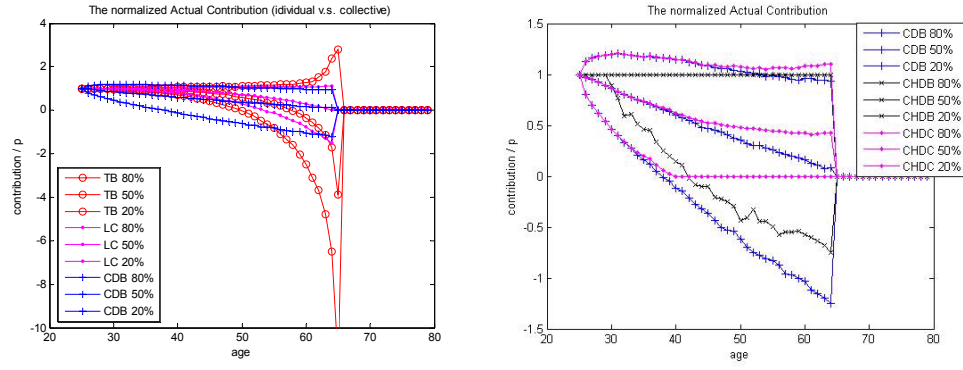


Figure 4: Quantile plot of the normalized actual benefit (ILB / Equity)

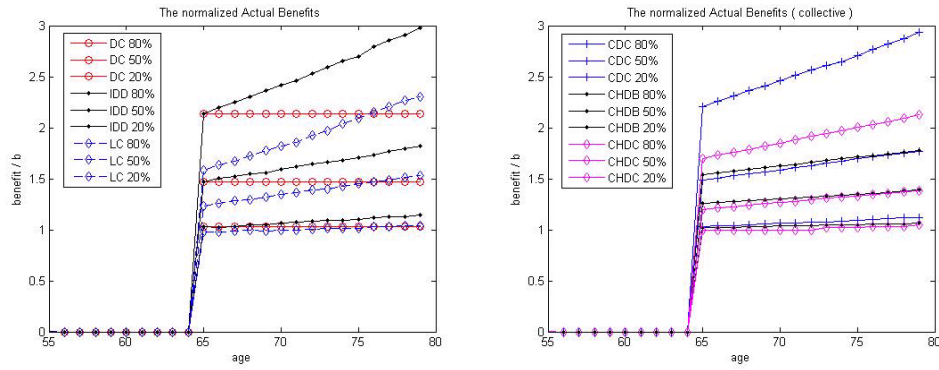


Figure 5: Certainty equivalent consumptions, when funding ratio is 100%

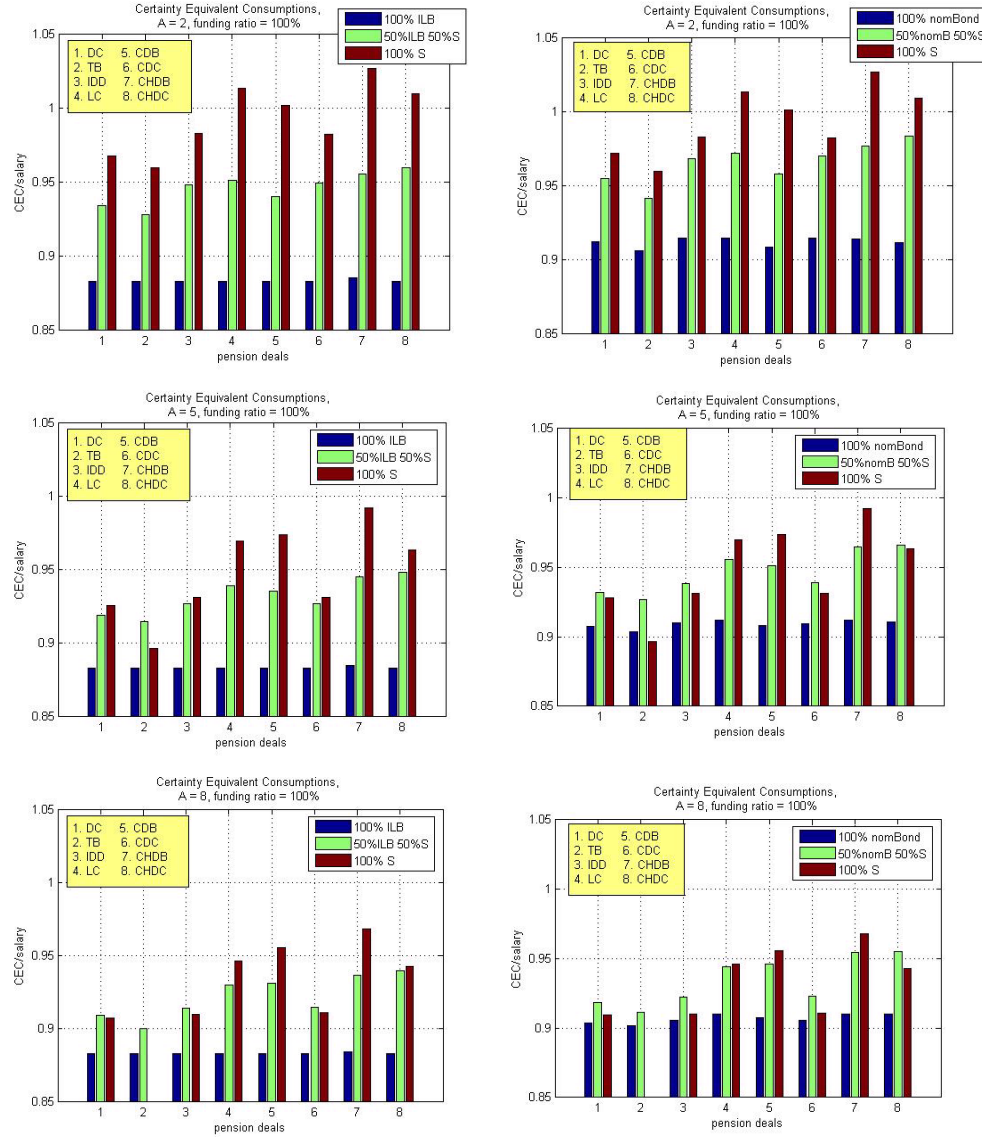


Figure 6: Certainty equivalent consumptions, when the initial funding ratio is 110% (left panel) and 90% (right panel) respectively

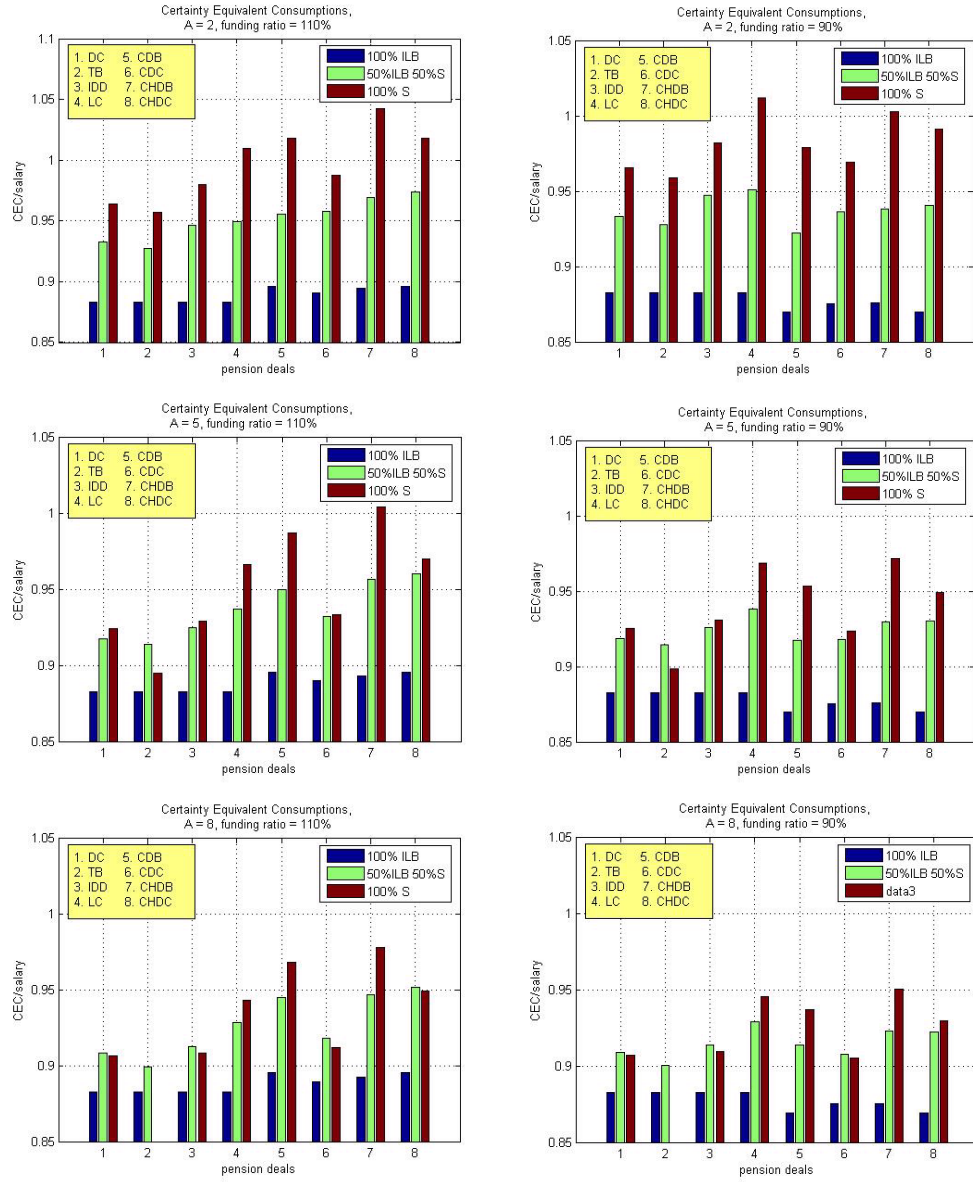


Figure 7: Certainty equivalent consumptions, in an economy with relatively lower risk premiums $[\lambda_E, \lambda_\pi, \lambda_u] = [0.1, -0.1, -0.1]$ (for funding ratio of 100%)

