Dear Members

Welcome to the August edition of the VA MIG newsletter, with the Editor's apologies for the delay in publication (how standards slip when the Chairman takes his well-earned break!). This month the content follows the usual format of:

- **Chairman's Comments,** *editorial freedom to the Chairman to provide commentary and views on the market and the MIG*
- **Recent Activity,** *an update on the activities undertaken within the MIG over the last month*
- **Recent Market Activity,** *an update on what has been making the news in the prior month*
- **Resource Centre,** *an update on website tools and content*
- **Vacancies,** *requests for assistance from the membership*
- **Upcoming Events,** *outline of relevant events*
- **Recent Publications***, outline of recent publications not included in the Resource Centre*
- **Practice Area Bulletin**

If you have any comments on the newsletter, or contributions you would like to see circulated more widely, please contact our Editor-in-Chief, Jeremy Nurse, at jeremy.nurse@watsonwyatt.com

James Maher
Chairman VA Members Interest Group

Chairman's Comments

August is here and the holiday season is upon us, so as we pack our bags and head for the beach or the mountains it may be worth remembering that some important deadlines are looming early in September in respect of consultation by CEIOPS on the second set of advice on Solvency II Level 2 implementing measures. In particular, the deadline for responses is set at 4pm on the 11th September, so firms' analysis needs to be well underway by now to ensure that responses can be prepared and submitted on time.

From an initial review of the CP's there are a number of proposals that are directly pertinent to the VA space and without being too definitive or identifying the limits of relevance, I would like to bring attention to the following for consideration by members:

- **CP 56 Advice on Tests and Standards for Internal models** – this is a comprehensive document that outlines the requirements to seek/obtain regulatory approval to use an Internal Model for the purpose of the Standard Capital Requirement as an alternative to the Standard Formula. My impression is that VA guarantees fall into a bit of a no-mans-land in so far as the Standard Formula is possibly insufficiently rich in the scope of the tests to adequately capture the distribution of change in Own Funds, and for this reason there will be a significant reliance on the Own Risk Solvency Assessment within the firm, thus the case for either seeking Approval or Approval being withheld may be somewhat moot. A further item of potential interest will be to observe the development in thinking in respect of Risk Mitigation, and a clarification of where Financial Risk Mitigation ends and Future Management Actions commence is particularly pertinent for Dynamic Hedging. Considering the level and nature of the feedback to CEIOPS on CP 31 (Financial Risk Mitigation) and CP 32 (Future Management Actions) which campaigned for clarity and consistency of treatment and credit for Dynamic Hedging in the Standard Formula, it will be interesting to observe how this is taken up in the responses to this consultation.
- **CP 41 Technical Provisions** – This CP is more of a tidy up document and in particular provides guidance on when Technical Provisions can be calculated and presented as a whole, rather than being separated into a Best Estimate and a Risk Margin. In particular the requirements of this CP set stringent standards as to the observability of the reference assets, which in most case will lead to a basis which will not allow for whole presentation due to any of duration of liability, direct observability due to the indices not being traded and the impact of hybrid risks where the financial risk is crossed with a biometric risk. This is worth some consideration, in particular as to how the liability valuation may need to be reverse-engineered to separate out the respective components. In this regard it may then be worthwhile engaging and feeding back on the CP 42 (Advice on Risk Margins).
- **CP 51 Further Advice on Counterparty Default Risk** - This CP extends CP28, and appears still to be under construction, not least give the substantial adjustments it has already undergone from the preceding QIS's in respect of the

basis of calculation. The aspect of this that is perhaps most pertinent to the VA space is the calibration of the default risk calculations, rather than the process. In particular, the "exposure" for VA guarantees, as for other undertakings, comprises both the reserve recovery amounts and additionally the reduction in the Capital Requirement. So far so good. However, CEIOPS has sought to introduce distinctions in the recovery risks associated with Derivatives or other Financial Mitigation tools as compared to Reinsurance in the form of a reduced recovery amount given default. One would expect that, for a given expectation of loss on a default, adjustments to recovery risks need to be reflected in probabilities of default, and it may well be expected that such feedback finds its way into the responses. There are further areas to be built out in the guidance here, for example consistent treatment for unrated but regulated financial institutions within Europe, recognition of the qualitative impacts of collateral regimes, etc and we would expect there to be material feedback on these aspects to this particular element of the consultations.

- **CP 52 Reinsurance Mitigation** – notwithstanding the expectation that the Standard Formula may not necessarily be fit for purpose in respect of VA guarantees, it is worth noting that the message on basis risk between liability and hedge or reinsurance are consistent between Financial Mitigation and Reinsurance Mitigation. There are a number of challenges of interest to VA writers, in respect of the overriding message that no credit will be given for a mitigation technique unless the firm can demonstrate that the basis risk is not material compared to the mitigation effect (in which case the capital for the basis risk is to be set at a 1 in 200 level). The requirement for immaterial basis risk would appear to usurp the requirement for a calculation of a basis risk capital charge, which appears to be overly restrictive, and it may be more appropriate to contemplate the requirement to evaluate the basis risk as being the primary task and then perhaps to establish the materiality thereafter. This is of importance to the VA community and in particular may add further pressure to push the product in the direction of hedge-ability and away from more asset- or customer-centric aims such as freedom of fund selection.

So plenty to think about and consider as the Solvency II agenda rolls forward, and we would look to encourage you all to gain familiarity at least, if not to seek active engagement on the relevant topics. To aid in this discussion it would be great to see the membership bring these discussion to life through the Wiki.

Lest you think I have forgotten that in last months newsletter I promised to provide some commentary on the topic of realised volatility and reserve convexity, I am delighted to include a bumper holiday Practice Area Bulletin in the form of a paper drafted by me entitled "The Vexed Question of Convexity". In this paper, I look to elaborate on the topic of reserve convexity both in terms of its relationship to the other greeks, but also in respect of aspects of product and reserve design that impact on reserve convexity. I hope you will find time to review the note, and I would be delighted to receive feedback either directly or publicly through the Wiki.

The Vexed Question of Convexity

James Maher
Executive General Manager
Nexgen Reinsurance Limited
Ormonde House
12 Lower Leason Street
Dublin 2

Telephone: 353 1 439 4937
Email jmaher@nexgenfs.com

July 2009

Acknowledgements

I would like to acknowledge the support and forbearance of my employer Nexgen Financial Holdings in allowing me the time and resource to research and assemble this note and for the support of my colleagues Emmanuel Avril and Charles Monneron for their support and feedback as I develop my own understanding of the topics to hand.

I would also like to acknowledge and thank the members of the UK Actuarial Profession's Variable Annuity Members Interest Group and in particular the encouragement and support of Josh Corrigan and Mike Claffey who have provided feedback and commentary on the early drafts of this note.

Disclaimer

This note is not and does not purport to be a piece of academic literature rather it aims to translate the academic and abstract literature into credible worked examples and illustrations to aid in building the intuition of nascent risk managers or actuaries embarking on a career in Variable Annuity risk management.

The document and the underlying analysis are my own work and do not necessarily represent the thoughts or views of my employer or any professional body or other commercial organisation.

Any and all mistakes, omissions or oversights within this note are my own.

Contents

	Page Numbers
Introduction	**4-5**
Part 1	**6-8**
Part 2	**9-18**
Part 3	**19-26**
Part 4	**27**
Appendix	**28**

Introduction

The purpose of this note is to develop the readers understanding and appreciation of the second order risks that present themselves in the context of the embedded optionality in Variable Annuity Guarantees. In particular we will focus on the sub topic of reserve convexity which refers to the non linear change in the benefit reserve for changes in the underlying Unit Price.

The key learning and takeaways we hope to leave readers with are an increased understanding and intuition as to:-

- The non linear/convex relationship that exists between reserve liabilities and the level of Unit Prices.
- The interaction of liability Greeks, Gamma and Theta (time decay) of a Delta hedged portfolio, in particular an appreciation that convexity can lead to profits as well as losses thus it is not always an exposure to be eliminated and in some circumstances is a sought after exposure.
- That where a portfolio has been Delta hedged that the result realised over a period of time is no longer directly sensitive to the level of markets but is determined by the variability of markets and in particular that the aggregate result is the sum of results between rebalancing periods.
- The effect of reserve protocol or design, in particular lapse policy has on the convexity of the reserves.
- The effect of benefit design, including ratchets and pricing convention on the convexity of reserve.

The motivation for the scope and timing of the note has been prompted by a number of factors, in particular :-

- Solvency II will expose residual reserve convexity by the application of instantaneous and severe market stress tests where the ability to instantaneously rebalance first order Delta hedges is specifically disregarded. In this case the residual capital requirement from the test will be driven by the convexity of the reserves.
- In-force business issued prior to the market falls of 2008 will have moved more “in the money” and as we will see the likelihood is that those reserves have an increased convexity as compared to new business/out of the money exposures as such there is an increased sensitivity to these second order parameters
- The availability of hedge solutions and cost of the same has increased over the recent past as such companies choosing not to hedge out second order risks will have an increased interest in understanding the residual exposure and opportunity by not hedging these higher order risks.
- Industry and membership understanding and benefit of first order, Delta/Rho hedging is reasonably well understood and appears to be firmly embedded in companies current operations as such it is time to move on to the higher order risks to further develop our understanding and capabilities

The note is constructed in three sections together with a summary chapter and an appendix.

In Part 1we introduce the base product and the data set for analysis. In particular we take the case of a Guaranteed Minimum Withdrawal Benefit product as the basis of the analysis. We believe this represents an evolution from the existing literature and presentations which have focused primarily on stylised analysis of liability Greeks as apply to European Puts or Calls or in some cases presentations utilising closed form modelling of Guaranteed Minimum Accumulation Benefit. Utilising the product example allows us to better understand the implications of benefit design and reserve protocol on reserve convexity.

Part 2 focuses on an analysis of reserve convexity and in particular looks at how the results of a hedged portfolio are impacted both by the evolution of the reserves over time and by the movement in the Unit Price. This integrated approach to Gamma and Theta better enables us to understand both the profit opportunity and the residual risk associated with the retained results. Notwithstanding the presentation is both pictorial and numeric we do digress into the maths of this relationship to support and aid in embedding the intuitive understanding as to why it is necessary to look at both Gamma and Theta together as a pairing rather than in isolation.

In Part 3 three we look to observe the impact of differing benefit and reserve design on the shape and level of reserves with the primary interest being on the convexity of the reserves. Through this analysis we make some high level commentary as to implications for product and hedge design without being drawn too deeply on these issues.

Part 4 we look to review the analysis and draw out the key findings in the analysis.

Additionally, throughout the note certain explanatory of supplemental information points are provided as an aside. These items are not necessarily core to the text or thesis and have been drafted in *italics* so as to distinguish them from the main document.

Part 1 - Introducing the Model & Data

Model Contract

For this exercise we look at the example of a term drawdown product sold in the UK. The benefit level is set at 5% per annum of the base benefit level where the base benefit level has a start value equal to the initial premium paid by the policyholder and has an annual ratchet of up to 10% with a maximum base benefit level of 150%. The annual fee charged to the policyholder for the guarantee is 90bps per annum of the account value and the administrative charges add a further 85bps per annum.

Model Fund

The underlying funds in the example are balanced 60% Equity and 40% Bonds where the fund selection is a balanced portfolio of domestic and international assets with regular rebalancing and an annual asset management charge of 75bps.

Model Reserve Basis

For the purpose of the analysis we are looking at the fair value of the guaranteed component of the contract (ie the policyholder claims in excess of the Account Value minus the Guarantee Charges), on a market consistent basis, where the lapse propensity is adjusted so that the deeper the benefits are in the money the lower the propensity to lapse.

The reserves have been generated stochastically using 2000 simulations with the reserve being the expectation of discounted future claims minus discounted future premiums.

In particular the reserves have been evaluated under the following formula :

$$\sum_i\{\sum_t\{ Max(5\%*E(t)*BB(t)/4-AV(t),0)*{}_tP_x*V^t + \sum_t Max(OS\_BB(t)- AV(t),0)*{}_tP_x*Q_{x+t}*V^t - \sum_t 1\%* AV(t)*{}_tP_x*V^t /4\}/N$$

Where:

- $\sum_I$ denotes the summation over the stochastically generated paths from the Economic Scenario Generator calibrated to the implied volatility of the fund and the risk neutral drift
- $\sum_t$ denotes the summation over the time steps for each path of the stochastic process
- E(t) = Election Factor which determines the probability of the policyholder effecting a drawdown at a future time, in this case E(t)= 0, for t<=5ys, 1 > 5yrs.
- BB(t) = Bases Benefit level at time t which can increase for AV(t) > BB(t) at an anniversary in accordance with the ratchet rules
- AV(t) = Account Value at time t = AV(t-1) + Growth – Charges - Drawdown
- ${}_tP_x$ = Survivor function allowing for decrements of death and lapse, where the lapse process is a function of AV & OS_BB(t)
- V^t = Discount factor at the risk neutral drift, in this case the discount curve for STG swaps
- OS_BB(t) = Outstanding Benefits at time (t) = BB(t) – ∑ Actual Drawdown to date
- Q_{x+t} = dependent rate of mortality for policyholder aged x+t
- N is the number of scenarios generated and evaluated

Base Presentation

For any given level of Unit Price and armed with our Economic Scenario Generator, product structure, reserving policy and discounting structure we can evaluate the level of reserves. In this example we will assume that the exposure under discussion is £100mn (i.e. the initial single premium = initial Base Benefit level). These amounts can be scaled up or down if required.

In addition to calculating the reserve levels for given levels of Unit Price we will also be interested in looking at sensitivities such as

Delta = Δ Reserve / Δ Unit Price and
Gamma = Δ Delta / Δ Unit Price/ Δ Unit Price.

The following table summarises this information (scaled to the exposure) for sample Unit Prices which will be utilised later in the analysis :

Table 1

Spot	Reserve (£mn)	Delta (£mn)	Gamma
120	-£0.25	-£0.15	0.001
100	£3.13	-£0.19	0.003
80	£7.72	-£0.27	0.006
60	£14.67	-£0.43	0.010

As one of the primary purpose of the exercise is to add to the intuitive understanding of the processes at work we will utilise graphs where possible to pictorially represent the key issues as such the following chart builds out the above table for the range of reserve levels and Unit Prices underlying this analysis.

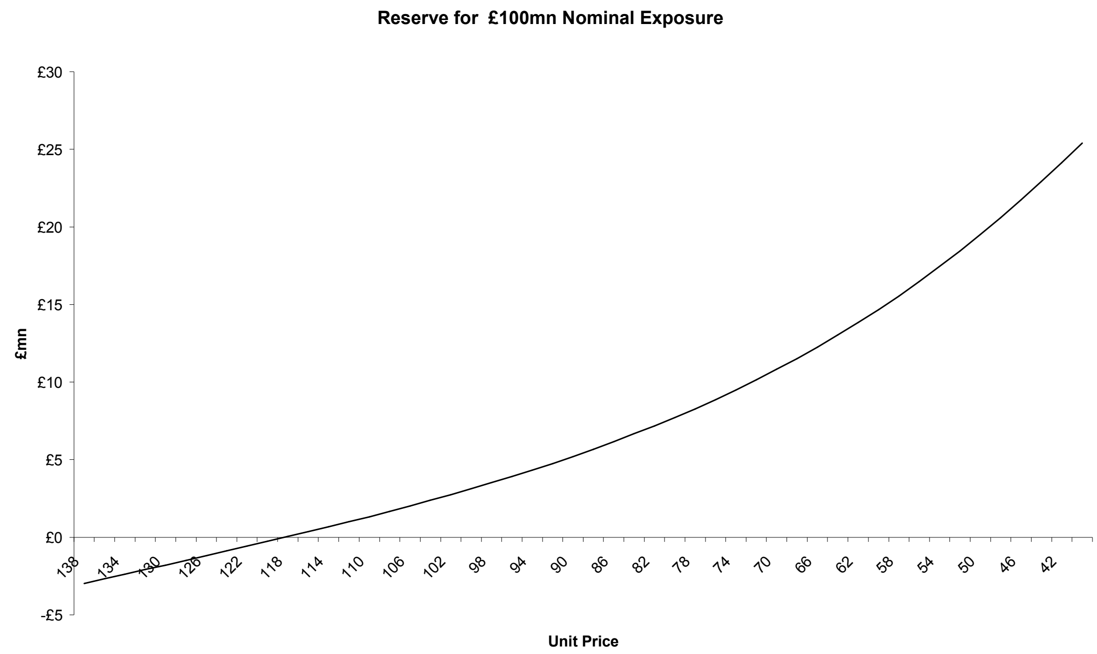

Chart 1

We will use the data in Table 1 and Chart 1 throughout the following analysis.

Aside – Observations on Contract Profitability
The above chart outlines the fair-value of reserve for the liability for different levels of Unit Price. From above it would appear that the 90bps guarantee charge is insufficient to cover the fair-value of the liability for the opening Unit Price of 100 and the reserve only moves negative for a high Unit Price. This can be argued in a number of ways, for example

- *the contract is expected to be profitable in a real world assessment. This is a challenging proposal not least as any attempt to dynamically hedge a real world proposition will realise the embedded risk neutral value of the liability. As such if a real world approach is being adopted the presentation above is a representation of the post hedging expected outcome.*
- *this is an older product that was arguably designed in an earlier time however pricing parameters such as interest rates and volatility have moved against the provider thus the current reserve is underwater.*
- *The contract has been priced for aggregate profitability as such there are margins elsewhere, for example on the asset management fees, that support the guarantee*

Irrespective of the merits of these points or alternative objections that could be raised we do not believe that current contract profitability will alter the findings as we are primarily concerned with future movements in reserve with respect to spot price and time thus we are focusing on liability Greeks, Delta, Gamma and Theta.

Sensitivity to shifts in Implied Volatility and Interest Rates
A complete analysis would bring into account movements in implied volatility (vega) and interest rates (rho) and for completeness would look at the co movements in these state parameters to enable a complete analysis.

By excluding these elements we are implicitly assuming that Interest Rates and Implied Volatility levels either do not move or that they are hedged as part of the overall risk management of the guarantees. In practice the latter assumption is more appropriate (ie that these greeks are hedged elsewhere) as it would be inappropriate to anticipate that either interest rate levels or implied volatility would be immune to the shocks to the Unit Price that we are testing in this exercise.

Part 2 Reserve Convexity & Hedge Management

Delta of a Reserve

The topic under analysis in this paper is the rate of change in the reserve as compared to the change in the level of the fund. Focussing solely on the embedded derivative within the reserve obligation we can look to the BlackScholes equation to allow us create an instantaneously riskless position through the utilisation of Delta hedging. Furthermore if we continuously rebalance these positions (without cost) over the life of the transaction then our embedded derivative can be replicated both cost and risk free. This all seems perfectly idyllic, however given that we do live in a world with frictional costs and time delays for developing portfolio hedge information and we neither are able to nor likely wish to continuously rebalance our liability hedges we need to consider the issues introduced by hedging in discrete time intervals. In particular we need to develop a sense of the departure of our liability changes from the linear assumption in continuous time to allow for the convexity or curvature of our liability profile over discrete intervals as well as the impact of the passage of time on the embedded derivative.

To aid in this let us take Chart 1 above where we present the reserve level versus Unit Price and start to look at it from a risk manager's perspective. To aid in this the chart is overlaid with a series of tangent lines at particular Unit Prices/spot levels which amplify the illustration of reserve sensitivity at various points. These tangent lines have been drawn at the relevant points on the curve using the Delta calculations outline in Table 1.

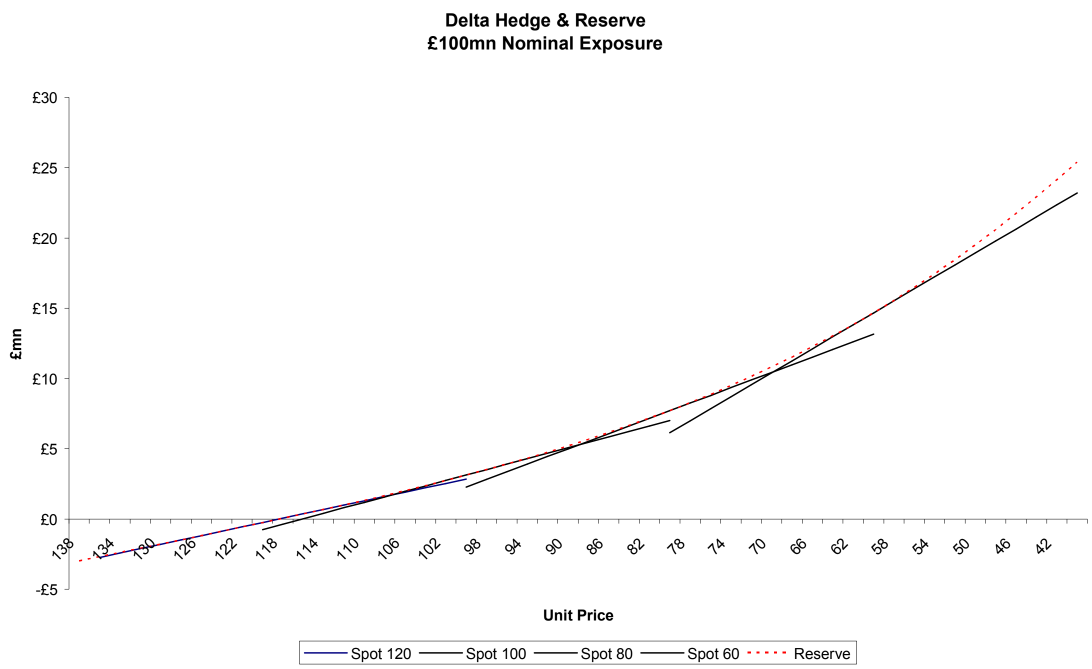

Chart 2

We can readily see that the slope of the tangent lines increase significantly as we move from high Unit Prices to low Unit Prices, i.e. from "out of the money" to "in the money".

This pictorial representation can be directly observed from table 1 above where we observe the Delta of the reserve as becoming increasingly negative as the Unit Price falls (note the sign of the £Delta is negative which results in the Reserve value increasing for decreasing Unit Price.).

This reflects the observed increase in first order sensitivity of portfolios to fund levels over the course of 2008 and into 2009 as guarantees have moved into the money and increased the amount of Delta to be hedged to maintain a neutral position.

From Delta to Gamma
The rate of change of the slope in these tangent lines (the Delta) is a function of the curvature of the liability and is described in terms of the second derivative of the Delta and is referred to as the Gamma or Convexity of the liability.

In expanding the BlackScholes formula the curvature is allowed for by including an allowance of $.5*\Gamma*(\Delta\text{Unit Price})^{\wedge 2}$, where Γ is the Gamma or convexity of the liability.

Presentation of either the mathematical curvature or the dollar exposure to sensitivity is somewhat abstract (given it is applied to the square of the change in Unit Price) however an appreciation of the relative sensitivity can be useful. The following table summarizes the convexity of the reserve curve at the Unit Prices identified in Chart 1 and presented in Table 1 and rescales the value to the convexity as applicable at Unit Price 100 (for example Gamma(120) = .0013 / Gamma(100) = .0033 = 30%).

Table 2

Unit Price	Gamma	as % of Gamma at Spot 100
120	0.0013	30%
100	0.0033	100%
80	0.0063	178%
60	0.0091	289%

From Table 2 we can immediately identify that as the Unit Price increases we are less sensitive to curvature in particular we are only 30% as sensitive as we were at Unit Price 100 and similarly if our Unit Price drops to 80 we are almost 1.8 times as sensitive to curvature as we were back at 100. Thus as markets have fallen not only have we become more sensitive to Delta but the convexity of the liabilities of in-force business has increased and thus may warrant more attention than heretofore.

Aside - Gamma for Low Unit Prices
General theory of Greeks identifies that as the contract goes deeper into the money we should expect to see the liability curvature fall off. This is due to the contract either becoming more certain as a fixed payout in the case of a limited payout contract (Delta goes to nil) or more linear in the unit price in the case of an unbounded contract (Delta goes to 1). Our above table does not illustrate this curvature; however this does not mean that the same effect does not occur under this analysis.

In particular, given the length of the liability and the effect of discounting we would need to shift the Unit Price considerably further towards zero to achieve these results. To illustrate the point we map the Gamma for the entire curve of Unit Prices from 100 down to Zero as follows :

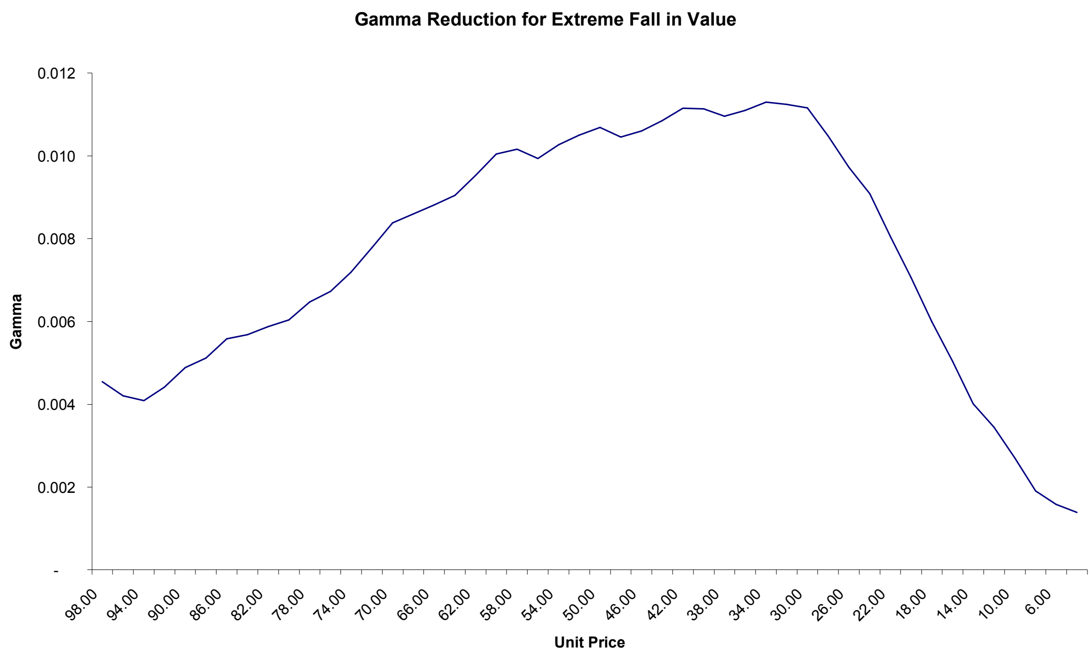

Chart 3

From above we can see that the rate of increase in curvature flattens out over the range 60 to 30 and then the curvature drops off precipitously as the Unit Price falls to zero. So the laws of physics still apply in this case however we are focusing on elements of the Unit Price curve where the observed curvature is upwards while noting that there is a point where this inverts outside our range.

Direct Calculation & Stochastic Simulation

The calculation of greeks in this exercise have been taken directly from the output of our simulations. Whereas the progression of the reserves appears smooth to the eye from Chart 1 and Chart 2 the calculation of our sensitivities at each point in the curve expose the residual noise in our calculations as is apparent from Chart 3. In fact the presentation in Chart 3 and in the ensuing charts already benefits from smoothing whereby the smoothed gamma at each Unit Price is the average of the directly calculated gamma from Unit Prices in the range +/- 5. We could look to eliminate the need for smoothing and the residual noise by increasing the number of simulations however the increased computation time would not necessarily be well rewarded for the purpose of this exercise as we are not so concerned with the precise level of the sensitivities at each point but with the overall shape and level. With this in mind as we proceed through the exercise some anomalies may present themselves in the tails of the curves where the smoothing can not be so easily applied.

From Gamma to Theta (Time Decay)
So far we have managed to describe yet again the oft described evolution of Greeks with the added texture of our obligation being closer to actual insurance company exposures than a stylized European Put. Understanding these sensitivities is important and underpins the hedge policy however many hedging examples tend to skip over the impact of theta or the advance through time in fully describing the implications for reserve evolution and hedging. In these next few lines we hope to pull this aspect out of the shadows and put it front and centre in our discussions. The main aim of this section is to identify that

- Gamma on its own does not provide the full picture of how a companies (Delta hedged) earnings are exposed to market movement in discrete time and
- the lifetime result from a Delta hedged portfolio is a summation of discrete time periods results where in some periods you can be ahead and some periods behind

The following chart takes our sample reserve line and looks to isolate the impact of advancing through time on the "time value" of the embedded options in the reserve liability.

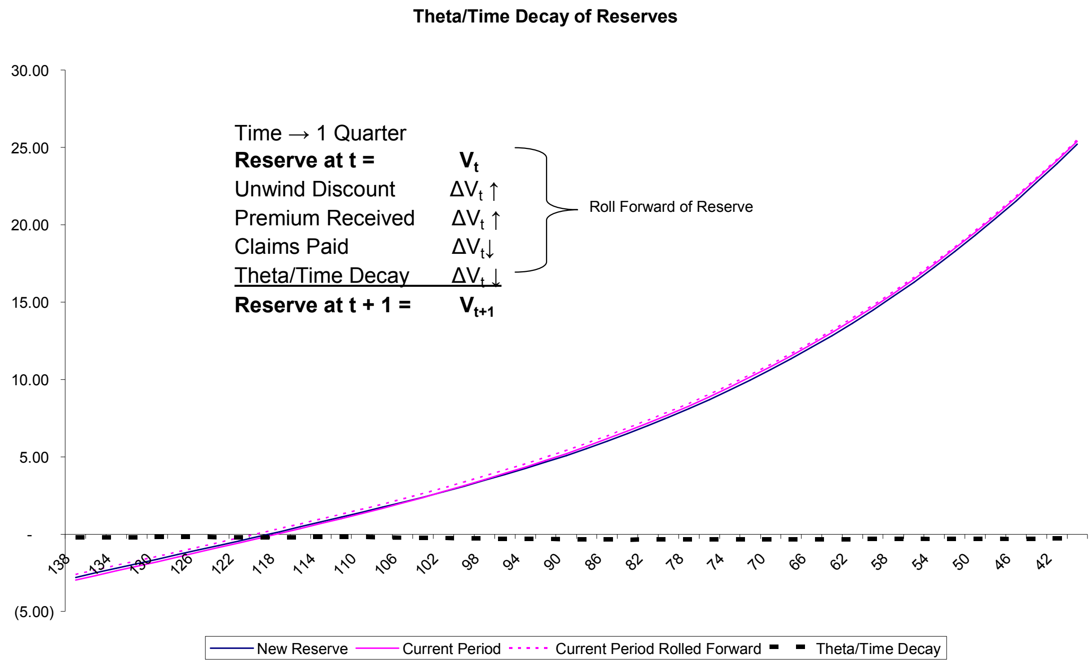

Chart 3

Chart 3 above is unfortunately quite a busy chart as we are trying to separate out a number of moving parts. In particular the chart and the commentary overlaid on it seek to identify that two processes are at work as we move through time.

- There is the effect of the linear movements in the reserve due to cash-flow in the period that is familiar to reserving actuaries thus we unwind our discount rate and allow for premiums and claims in the period.
- Additionally we have an impact from a reduction in the "time value" of the option, ie the portion of the premium that relates to uncertainty in the next period.

For the purpose of this note we are interested in the change in this "time value" of the reserve, which we will variously refer to as Time Decay or Theta. In particular we are interested to see how it interacts with the effect of movements in the Delta hedged liability vis a vis the Unit Price as we move forward in time.

In order to illustrate the impacts we can look to isolate the Time Decay of the liability where the Unit Price is 80.

From Table 1 we know the Start reserve is £7.72mn. From modeling we can calculate the End reserve being the current provision evaluated one period hence assuming growth in the Unit Price at the risk neutral drift and no volatility in the period, which from calculation is £7.55mn. We know that there will be flows in the period for a) unwind of discount rate, b) receipt of premium and c) expected claims. The following table outlines the analysis undertaken to isolate the Time Decay:

Table 3

Development of Reserve for Unit Price = 80

		£mn
Start Reserve		**7.72**
Unwind Discount		0.04
Add Back premium		0.18
Takeaway Claims	-	0.04
Rolled Forward Reserve		**7.89**
End Reserve		**7.55**
Theta/Time Decay	**-**	**0.34**

Thus from our analysis we identify that there has been a release of £340,000 from the reserve in respect of expected volatility during the period.

Delta Hedged Reserves

Understanding the Time Decay component is an important component to fully understanding the residual exposure under the Delta hedged portfolio as we are looking at the combined result of the movement in the Unit Price and the advance of the portfolio through time.

Numerically we can describe the situation as follows :-

From Table 1 the Delta of the reserve at Unit Price 80 is -£270,000, thus for every drop of Unit Price by 1 the hedge payoff is £270,000.

The payoff for the hedge has the formula (Unit Price – 80)* -£270,000

Additionally the reserve will release £340,000 for Time Decay, as thus we have this additional income source to include in our period earnings.

The payoff for the hedge together with release of provision for volatility has the formula £340,000 + (Unit Price – 80)* -£270,000

Turning now to the development of our reserve liability for the unknown price evolution we can develop the formula for this as being Reserve (Unit Price = x) – Reserve(Unit Price = 80). For completeness we can further refine this movement for the known cash flows however the effects of this are somewhat modest.

We now have the components of our balance sheet and can calculate the combined impact of change in Unit Price and movement through time as presented in the following table :

Table 4

	1	2	3	4
Unit Price	Delta Hedge	Delta Hedge & Time Decay	Change in Reserve	Net Result 2 - 3
100	-£5.44	-£5.10	-£4.52	-£0.58
98	-£4.90	-£4.56	-£4.14	-£0.42
96	-£4.35	-£4.01	-£3.74	-£0.27
94	-£3.81	-£3.47	-£3.34	-£0.13
92	-£3.26	-£2.93	-£2.92	£0.00
90	-£2.72	-£2.38	-£2.49	£0.10
88	-£2.18	-£1.84	-£2.02	£0.18
86	-£1.63	-£1.29	-£1.54	£0.24
84	-£1.09	-£0.75	-£1.03	£0.29
82	-£0.54	-£0.21	-£0.52	£0.32
80	**£0.00**	**£0.34**	**£0.00**	**£0.34**
78	£0.54	£0.88	£0.55	£0.33
76	£1.09	£1.43	£1.14	£0.29
74	£1.63	£1.97	£1.76	£0.21
72	£2.18	£2.51	£2.41	£0.11
70	£2.72	£3.06	£3.08	-£0.02
68	£3.26	£3.60	£3.78	-£0.17
66	£3.81	£4.15	£4.51	-£0.36
64	£4.35	£4.69	£5.28	-£0.59
62	£4.90	£5.23	£6.08	-£0.84
60	£5.44	£5.78	£6.90	-£1.13

From the above we can identify that the combined result is at its highest for no change in Unit Price and decreases as the price moves away from 80 (either up or down), thus we are no longer sensitive to the level of the Unit Price but to its variability over the period. We can present the above table as a chart to get a pictorial sense of the factors at play :

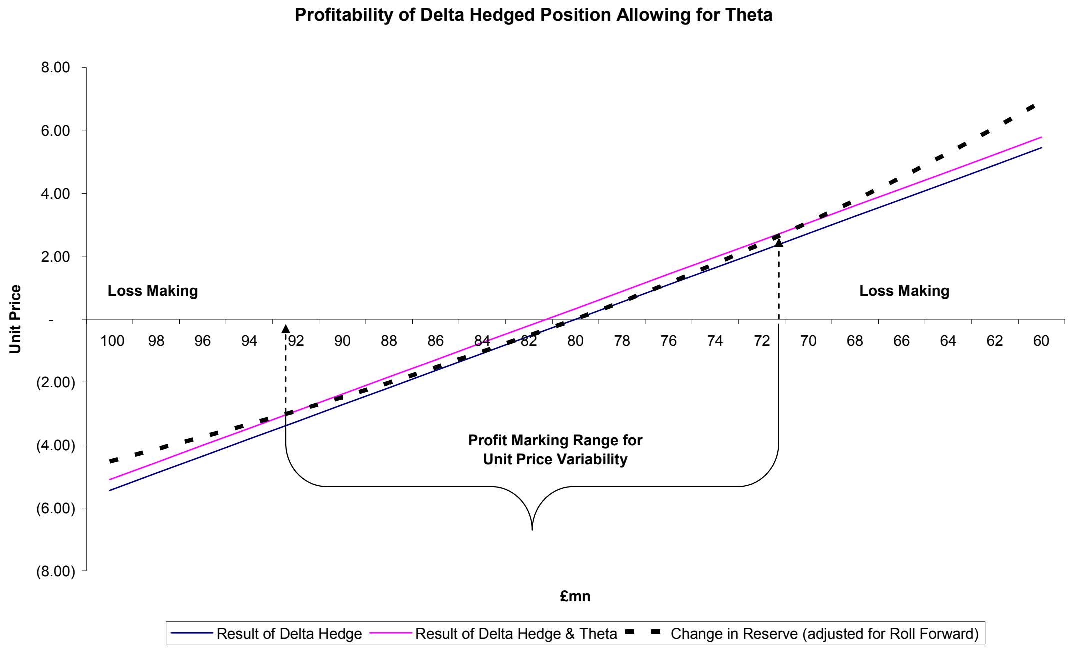

Chart 4

From Theta to Vega

The actual amount of the reduction in reserve or time decay is proportionate to the reserving assumption for volatility in the reserves and an alternative view of the above graph is the release of the premium the company has charged for volatility over the period. This is a useful perspective as we can then readily interpret the act of increasing our volatility assumption (a Vega charge) as an increase in reserves for future volatility which will be released as we advance through time.

Digression from Pictures to Formulae

The release of the premium for volatility is expected to pay for something and that something is realized volatility, or more correctly realized variance. In order to provide some mathematical framework to embed the observations we will briefly revert to a stylized presentation of Greeks in the context of a European option before proceeding with our numeric and pictorial example.

In particular we will use standard notation and the case of a short (sold) European Put option where:

- Γ_t represents the convexity of the liability at time t,
- S_t is the Unit Price at time t and
- σ_r^2 is the realized variance (calculated as Annualisation Factor*$\sum \ln(S_t / S_{t-1})^{\wedge 2}$/n where the Annualisation Factor adjusts the realized variance over periods of length n days to the appropriate measurement of one year),

Then the realized gamma to the put seller for the period t to t+Δt is $\mathbf{\{.5*\Gamma_t*S^2_t * \sigma_r^2\}*\Delta t}$

Coincident with the realized experience there is a release of Theta which follows the same formula as above with the important substitution of σ_r^2 by σ_i^2, where

- σ_i^2 is the Implied Variance being the parameterization of volatility for the reserve (and Delta hedging) for the realized variance parameter.

Thus the time decay of the reserve held by the put seller for the period t to t+Δt is $\mathbf{\{.5^* \Gamma_t{}^*S^2_t{}^* \sigma_i^2 \}^*\Delta t}$

Putting these together the realized net earnings in the period are:

$$\{.5^* \Gamma_t{}^*S^2_t{}^* \sigma_i^2 \}^*\Delta t - \{.5^*\Gamma_t{}^*S^2_t{}^* \sigma_r^2 \}^*\Delta t ,$$

This simplifies to:

$$.5^*\Gamma_t{}^*S^2_t \{\sigma_i^2 - \sigma_r^2 \}^*\Delta t,$$

Aside – Long or Short
Depending on whether we are long or short the exposure we can multiply the above formula by -1,(i.e. change the order of " $\sigma_i^2 - \sigma_r^2$" above *) to realize the appropriate result. Thus where we are a protection seller we are concerned that realized variance may exceed implied variance and conversely the holder of the option would be much happier to see realized volatility exceed implied volatility.*

The product of the sensitivity and the square of the Unit Price is commonly referred to as the "Dollar Gamma" and is a measure of the financial sensitivity of the position to the difference between realized and implied volatility

When we look to the aggregate result over a period of time we get the following generalized result

$$\sum_t .5^*\Gamma_t{}^*S^2_t \{\sigma_i^2 - \sigma_r^2 \}^*\Delta t,$$

Thus we can directly observe that the results of our Delta hedged reserves over time become independent of the level of market and become a path dependent summation of results realized over each rebalancing period.

Where the "Dollar Gamma" is deemed to be static or stable then this can be brought outside the summation as follows $.5^*\Gamma_t{}^*S^2_t{}^*\sum \{\sigma_i^2 - \sigma_r^2 \}^*\Delta t$, to derive the simplified but intuitive understanding that the residual exposure is in effect a position in realized variance minus implied variance, where the sensitivity is a function of convexity:

$$\text{Convexity}^* \{\sigma_i^2 - \sigma_r^2 \}^*n/N$$

Where n is the number of days over which the result is determined and thus over which the realized variance is averaged and N is the period for which the measures of Variance are calculated. In practice the valuations for hedge purposes may be limited to trading days so a measure of 252 is in practice the denominator for annual measurement.

Aside - Variable Annuity Guarantees and Variance Swaps

The above formula for the retained exposure is the basic structure of a variance swap. In the case of a Variance Swap the contract replaces Convexity with some Nominal amount which will determine the scale of the payout and the implied volatility parameter σ_i^2 *is replaced with an agreed Strike level,* σ_{strike}^2 *(which may be set at the implied volatility sought for hedging purposes).*

*In the case of the contract having the format Nominal**$(\sigma_{strike}^2 - \sigma_r^2) * n/N$*, one party to the agreement will in effect be selling Nominal**$\sigma_{strike}^2 * n/N$ *and receiving Nominal**$\sigma_r^2 * n/N$ *from their counterparty thus the description of a "swap". The party selling the fixed leg of the contract is in effect paying a known premium in return for the uncertain payoff and thus is in effect purchasing insurance against realized volatility.*

Armed with this understanding of the basic structure of variance swaps it is of interest to reflect on a number of items of practical interest.

Firstly, if one of the primary motivations of an insurance company to sell a variable annuity guarantee is to access exposure to realized volatility as an asset class in its own right then it is worth noting that there are likely much more direct and cheaper ways of access to these exposures through the wholesale markets for example through the sale of Variance Swaps without going through the complexity of selling the guarantees and isolating the volatility exposures.

Secondly, and a related point, if retail policyholders will not pay the market consistent rate for volatility demanded by the wholesale markets does it not make sense for the insurance company to look to sell volatility in the wholesale markets ?

Finally, and getting back on track, the assumption of a constant "dollar gamma" in our formulae above, which allowed us to bring the convexity measurement outside of the brackets for summation over time, underpins the weakness or limitation of Variance Swaps in completely matching the residual exposure to movement in the reserve. In particular the hedged position will not adequately allow for movement or variation in Dollar Gamma over the period as the protection acts on the relationship between realized and implied volatility rather than directly eliminating the convexity of the exposure.

Gamma Hedged Reserves

Should the company look to transfer this exposure to realized volatility, the company in effect looks to emulate its reserve convexity with a replicating convex hedge asset such as an option or a variance swap and in effect sells the theta that it expects to accrue from decay in reserve liability in return for protection against convexity the movement in the hedge program evolves.

Chart 5 below illustrates the coverage or sensitivity of our Delta & Gamma hedge asset as compared to the reserve liability where the greeks are set for neutrality at Unit Price 80 as per prior examples.

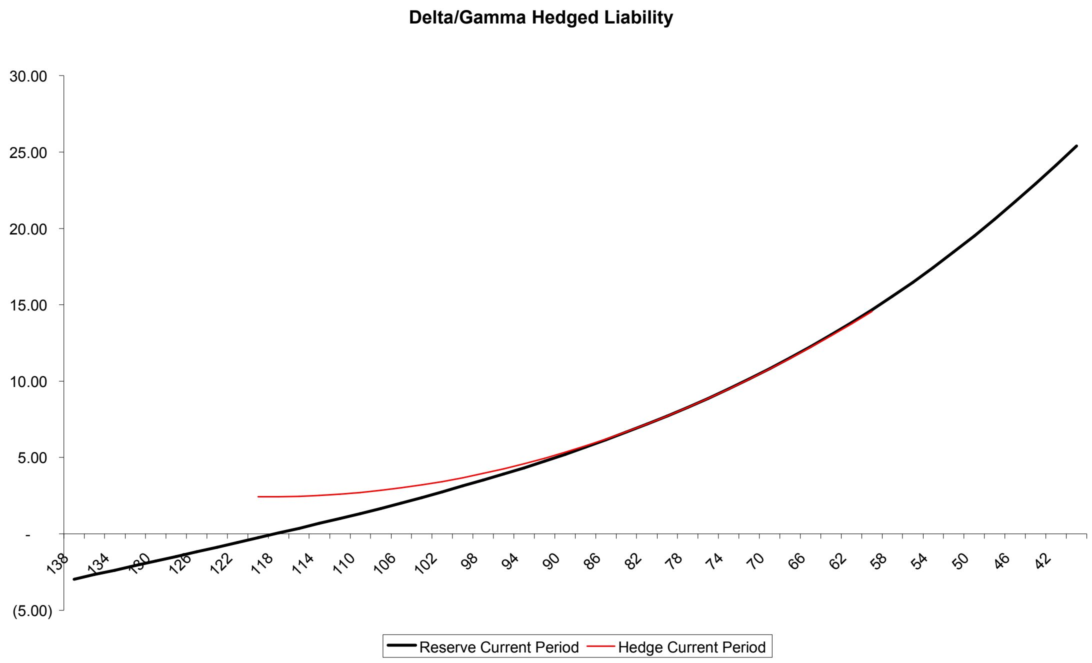

Chart 5

Neutrality is again instantaneous however the range of coverage that is afforded by the additional curvature of the liability hedge provide protection over an extend range of Unit Price movements.

As we move through time we will retain our neutral position in so far as the Theta of the liability hedge will decay concurrently with that of the underlying reserves and thus offset each other.

From a practical perspective it is worth identifying that where options are utilized to hedge the liability convexity the hedge process will need to eliminate the Delta component of the option as we are only seeking the convexity. This to some extent identifies part of the attraction of Variance Swaps in so far as the hedge asset has no Delta.

Part 2 - Factors impacting on Realized Volatility & Reserve Convexity

The discussion and analysis in Part 1 sought to illustrate the impact of convexity on the residual exposure of a Delta hedged reserve liability. The exposure and sensitivity to convexity needs to take into account both the sensitivity of liabilities to large movements but also the propensity of the underlying Unit Price to exhibit large movements not anticipated by the risk neutral framework (i.e. returns that are inconsistent with log normal). In the next number of sections we look at some of these components to better understand their contribution to our exposure and consider what responses can be put in place to aid in the management of these.

Unit Price Movement and Realized Volatility

In the case of Variable Annuities where the underlying Unit Price is in effect a basket of indices the performance of the Unit Price will start to move away from the log normal assumption underpinning the risk neutral framework. In particular the limitations of the risk neutral framework for single stocks is readily acknowledged in so far as real world price evolution does not follow a random walk from a log normal distribution and in particular is prone to greater spikes in volatility than is expected or anticipated. These shortcomings are well enough known in pricing and hedging and are compensated for either in pricing through the use of Implied Volatility surfaces or modeling using stochastic or jump based process.

Furthermore as the underlying unit in a VA contract is in most cases a basket of indices there are confounding effects of realized correlation to contend with. Thus the pricing and risk management needs to have some awareness of the skew introduced in assessing underlying index volatility for the component indices together with the next layer of skew and tail extenuation from integrating the sub indices of the basket into the underlying Unit Price.

For this reason there is a significant gap risk associated with derivative protection on unit fund prices that needs to be addressed through a combination of fund design, pricing and hedge design,

Lapse Process and Reserve Convexity

Looking briefly at the reserving policy a factor that significantly impacts on the convexity of the liability is the lapse model applied in the setting of reserves. In the following charts we look at three lapse protocols, variously being a) Dynamic Lapse, b) Nil Lapse and c) Static Lapse.

In this analysis the Dynamic Lapse model acts on the Static Lapse basis by the application of a factor such that it reduces the level of lapsing as account values fall, however it does not reduce the lapse propensity for rising markets as such it is not a model of fully rational behavior. The effect of this model is that the Dynamic Lapse propensity will tend to Nil lapse for low Unit Prices and tend towards the Static Lapse basis for high Unit Prices.

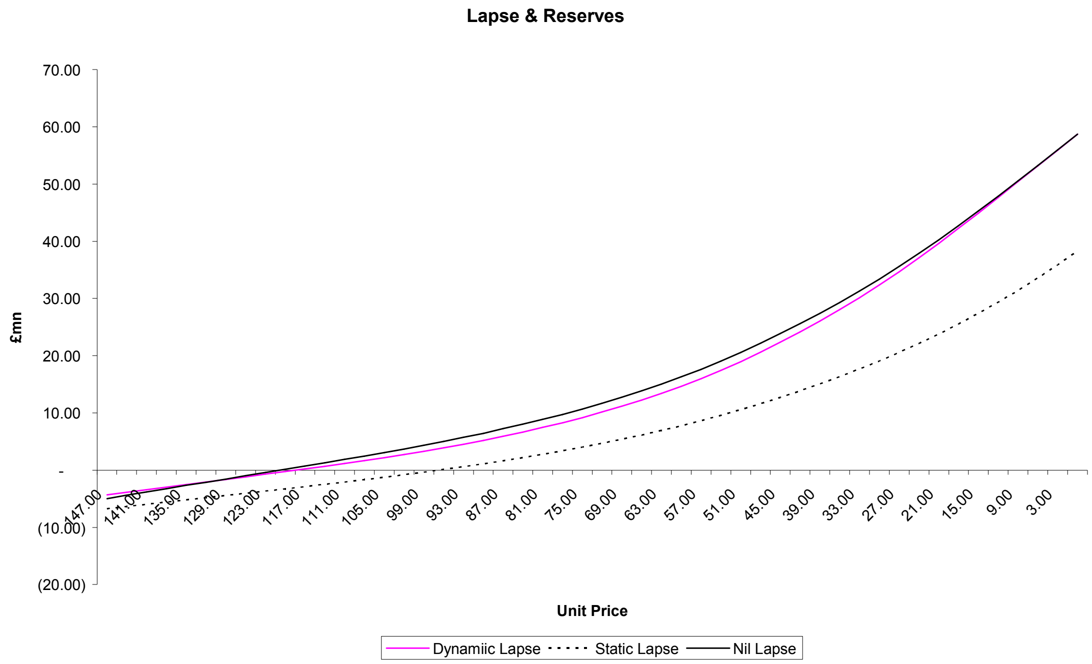

Chart 6

As can be seen from the charts the reserve levels for Nil Lapse and Dynamic are not dissimilar over most unit prices and are almost identical for very low Unit Prices. However the reserves for our Static Lapse basis are materially lower than for the alternatives, in particular for the lower Unit Prices. If we now look directly at the convexity of the reserves at each Unit Price the impacts are equally apparent.

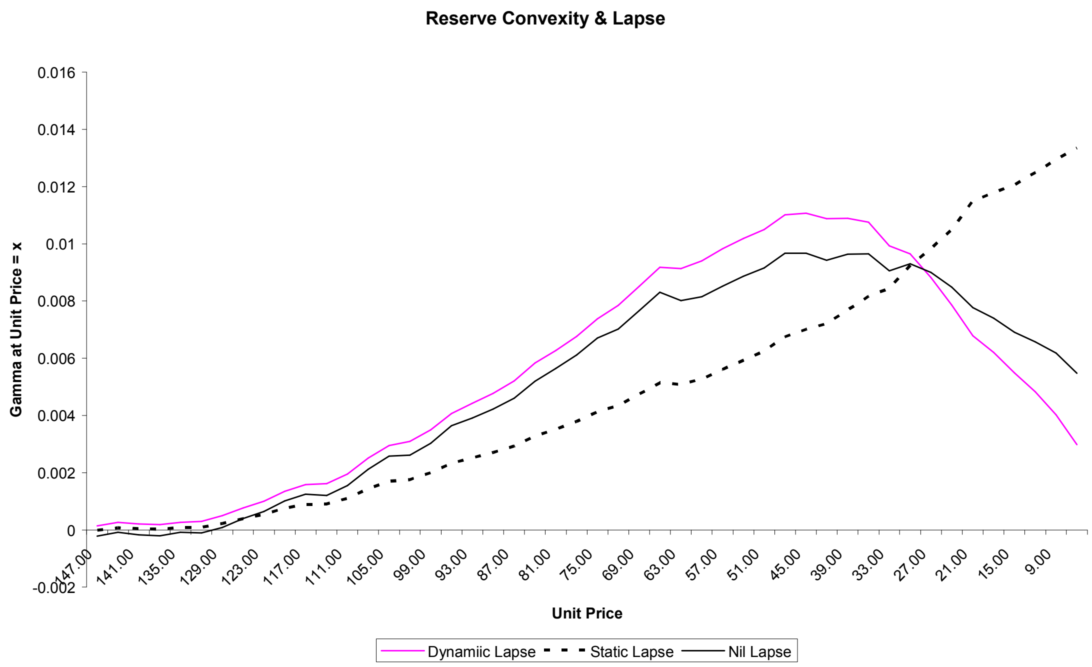

Chart 7

As anticipated given the somewhat similar shapes of the reserves for Nil Lapse and Dynamic Lapse the level of curvature is of the same level and shape. The Dynamic model does however add some additional curvature and that is somewhat intuitive given the nominal exposure of the coverage in effect increases with a fall in Unit Prices given the operation of the dynamic lapse assumption.

The level of curvature of the Static Lapse reserves is materially different in shape and level as compared to the alternative reserve protocols. A significant component of this is due to the shift in moneyness of the reserves which will be discussed in more detail further below however part is also due to the altered benefit shape.

Aside Gamma for Unit Price Nil
Without trying to insult the reader it is worth identifying that when the account value goes to nil it can neither go negative nor attain a future positive value such that the liability becomes a fixed annuity with no further sensitivity to Unit Price. In this case the liability theoretically has a Delta of nil and thus no further convexity.

Impact of Product Features :
Over the next few paragraphs we will look variously at the impact of a range of product initiatives on both the level and the shape of our reserves with respect to Unit Price. For this exercise we will not alter the contract pricing structure as such inference as to impact on convexity will need to be treated with some caution and in particular we bring your attention to the aside "Shifting Moneyness & Observed Changes in Convexity".

In particular we will make the following alterations to our current benefit and charging structure :

- Shifting the Base Benefit level from 5% down to 3.5%
- Altering the Ratchet to increase the benefit richness through allowing annual increases of up to 20% and increasing the aggregate cap to 500% and alternatively reducing the richness of the guarantee by moving it to a bi annual feature
- Altering the charging convention from charge on Account Value to charges on Base Benefit Level and a "higher of" calculation

Aside – Shifting Moneyness & Observed changes in Convexity
As we go through the exercise it will be important to keep in mind that when we observe the convexity of the liabilities we are observing the convexity of the liabilities for the given Unit Prices under consideration. The implication of this statement is that some of the variations in the benefit structure have the impact of shifting the moneyness of the guarantee. In particular if we hark back to Chart 3 where we exposed the convexity over the entire Unit Price spectrum down to zero we could identify a build up in convexity as we approach and operate at the money with a rapid descent once the contract becomes significantly in the money. Thus as we review the following charts it is worth keeping in mind and seeking to identify what elements of the convexity are attributable to the shift in moneyness of the guarantee and what represents absolute movements in the level of convexity due to the alteration in pay off structure.

Base Benefit Level

In the following chart we look at the impact of reducing the maximum withdrawal guaranteed from 5% of base benefit level to 3.5% of the Base Benefit level. It is readily identifiable that the reserve for this reduced benefit structure is materially lower that for our original benefit and it would appear that the 3.5% is a more realistically supportable level of benefit under the current parameters given that the reserve is close to zero for the Unit Price close to 100.

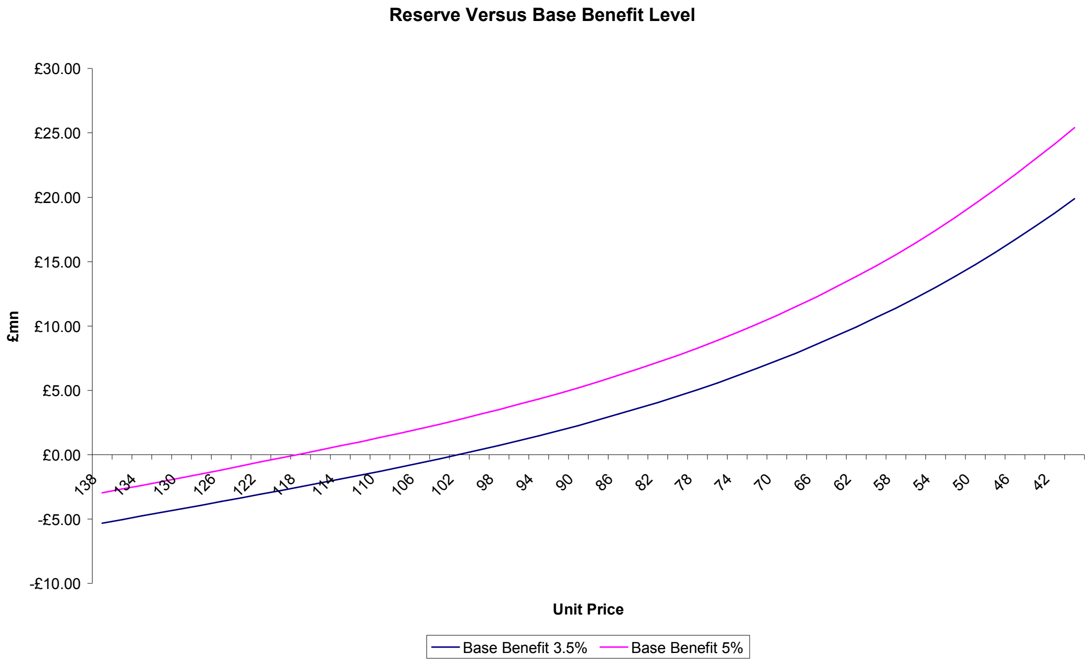

Chart 8

To the eye the reserves in Chart 8 appear to have similar shapes and levels and thus we do not anticipate a significant difference in the shape or level of the convexity however we may anticipate that the location of the peak in convexity may have shifted.

In chart 9 below we expand the range of observation down below a Unit Price of 40 (the limit of the range in Chart 8) to zero and as anticipated we can identify the peak and drop for the Gamma on the Base Benefit has shifted to the right.

Thus we can identify that the curvature is shifting for the change in moneyness and perhaps unexpectedly we can observe a slightly higher peak in the curvature under the lower base benefit structure; thus our intuitive assessment that the impact of the change to the level of withdrawal benefit would have a modest impact on convexity appears to be borne out.

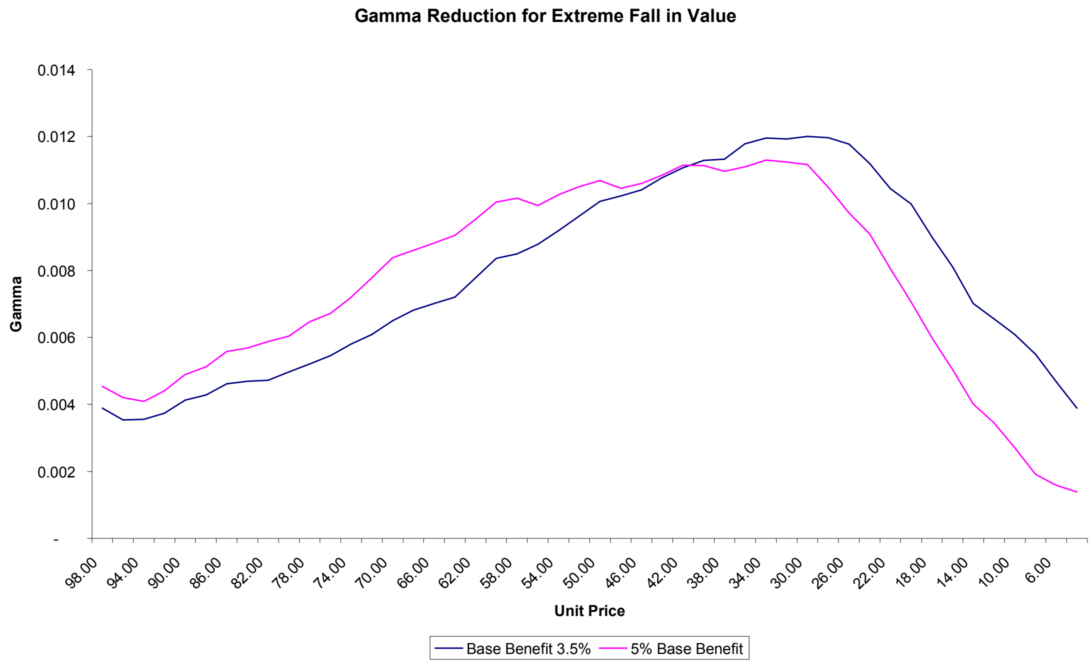

Chart 9

Benefit Ratchet/Step Up Level

In the following example we look at shifting the current ratchet/step up features both to increase the richness of the guarantee whereby we increase the maximum annual step up to 20% and increasing the aggregate cap to 500% and additionally to decrease the benefits by maintaining the current step up structure of 10% per annum, with a maximum cap of 150% but limiting the increases to once every 2 years.

Chart 10 below outlines the reserves under the various benefit structures and it is readily identifiable that the effect of altering the ratchet mechanism are most noticeable where the Unit Price rises and thus where the limiting features of the ratchets are most significant. In fact as the Unit Price falls away the impact of the different ratchet structures fall away as they are not pertinent to the ultimate turn out of the liability.

Given that the impact of the ratchet will be most observable in the tails we have expanded the range of Unit Prices over which we will observe the reserve level and curvature such that we will review Unit Prices over the range 150 down to Zero.

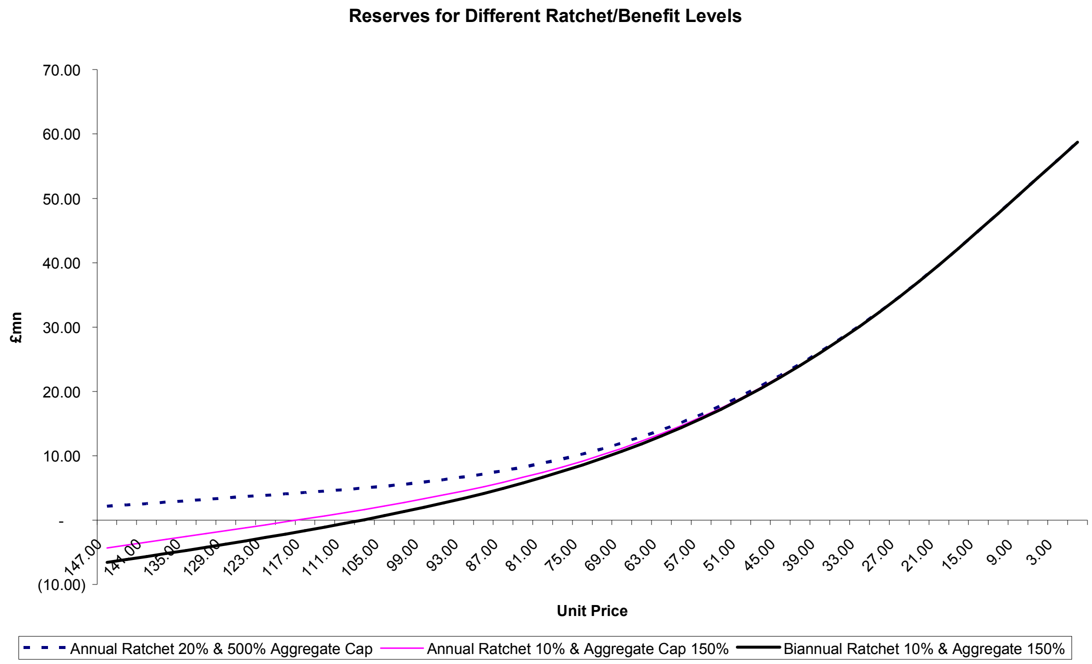

Chart 10

The chart immediately below outlines the curvature for the observed ranges. As expected, given that the reserves converge for very low prices the peak in moneyness are all located in the same range of Unit Prices. As we move out of the money however we can start to discern the disparate effects of the ratchets.

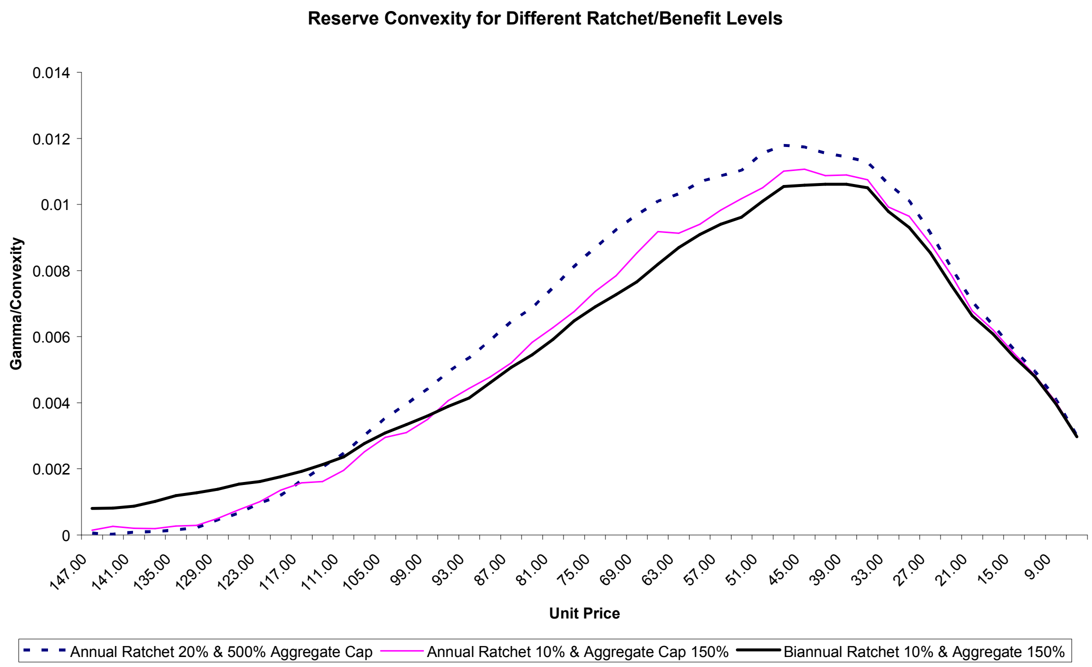

Chart 11

In particular the higher ratchet benefit maintains a persistently higher convexity over most Unit Prices lower than 100. For the higher unit prices there is a not unexpected drop off in reserve convexity as the aggregate liability caps start to bite and the liabilities move out of the money and become relatively insensitive to the Unit Price.

Benefit Charging Structure
Our final product feature for analysis is the impact of the charging structure on the reserves. In this case we look at the alternatives of applying the charge to the Base Benefit level as such it becomes immune to falls in the level of account value and only partially linked to increase in the account value through the step up of the Base Benefit level. An alternative approach that provides the insurer with an even greater participation in the upside (in earlier period particularly) will be to apply the guarantee charge as being the greater of the charge applied to base benefit and that applied to the account value.

Chart 12 below illustrates the reserve under these alternatives and it is identifiable and unsurprising that the hybrid "higher of" pricing regime produces reserves that are at or lower than the other reserves at each Unit Price.

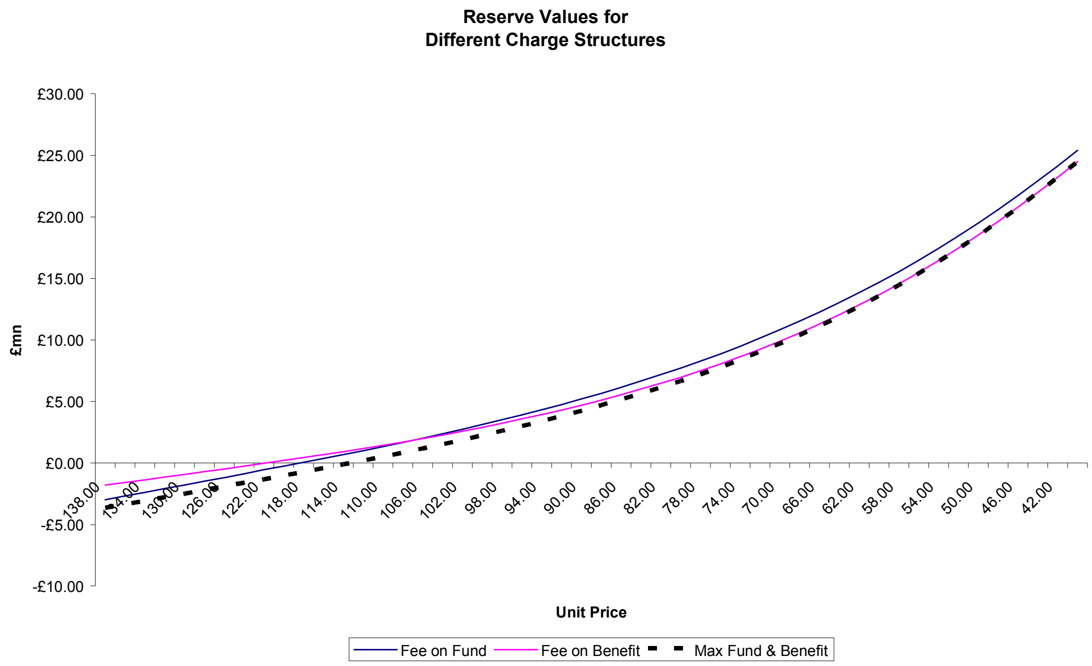

Chart 12

From the above we expect that there will be little difference between the convexity of the "higher of" and simple fee on benefit structure. Additionally we are likely more interested in observing the impact of the higher guaranteed charge under lower fund value scenarios given that it will likely accelerate the erosion of account values as funds run out. For this reason we again focus our assessment on the lower Unit Prices when wee look to compare the gamma maps for the liabilities.

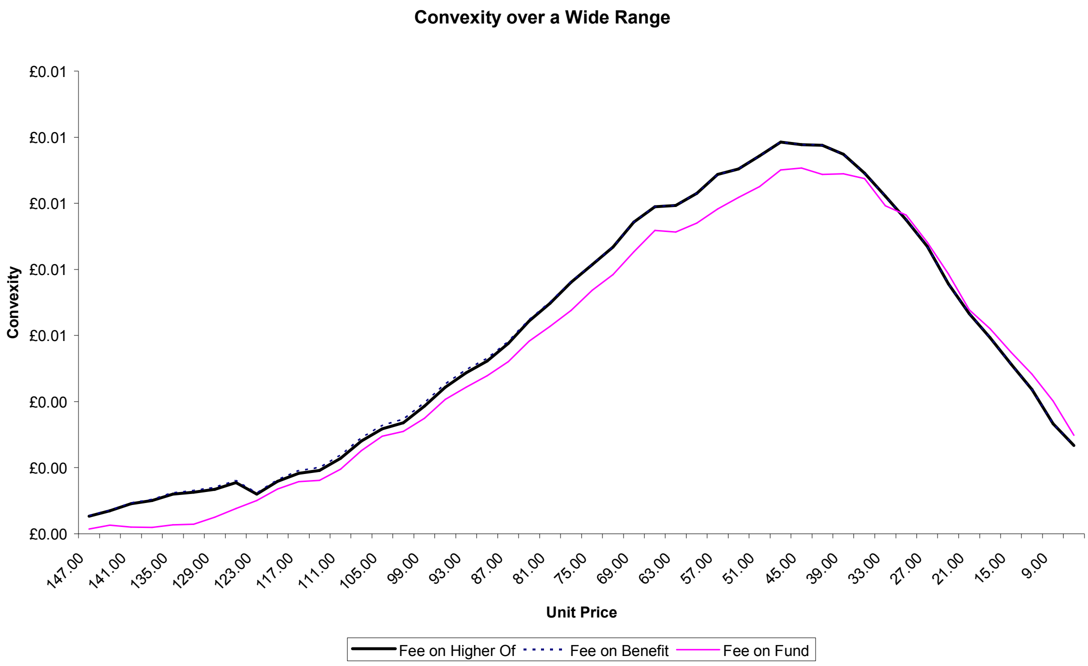

Chart 13

From the above chart we identify that the overall levels of convexity are not materially altered for the variation in charging structure. However to the extent that there is a difference the charging on Base Benefit and the hybrid "Higher of" lead to modestly higher convexities over the range of Unit Prices above 50.

Part 4 Summary & Conclusion

The above note was prepared to

- Evolve the often presented examples of Greek hedging of a European put into a more nuanced presentation of the reserve convexity of a drawdown product.
- Introduce the theoretical and practical consequence of time value on the reserve liability and to present it in the context of a balance sheet provision against realized volatility.
- Discuss the implications of product design and reserving methodology on reserve convexity.

In Part 1 we identified that the hedging strategy for a book of liabilities is dynamic and in particular noted that the sensitivities of reserve liabilities to convexity were low during the heady days of early VA development in Europe pre 2008 however as markets have fallen more guarantees have moved closer to or into the money there is a greater need for awareness of convexity management. Convexity itself can be a source of profit or loss for a company depending on the realization of actual volatility and thus each company will adopt its own position as regards to convexity management. Where a company is looking to reduce the earnings effect of convexity then utilizing a gamma management strategy will eliminate exposure to convexity and thus the result becomes less sensitivity to realized volatility, or alternatively the company could purchase variance protection in the form of a variance swap which will not eliminate the sensitivity to changes in convexity but will counter the effect of realized volatility.

In Part 2 we identified a number of non market risk items and examined their impact on the convexity of the liabilities. In assessing the alteration to the convexity of the liability we identified the need to keep in mind what benefit alterations led to a shift in moneyness and a consequent shift in the shape of the convexity and which benefit and reserving features impacted on the overall level of convexity.

From our analysis we identified that assumptions as to lapse in the reserving methodology has the most significant impact on the level of reserve convexity and in particular the utilization of a dynamic lapse formula significantly increased convexity by comparison to liability valuations which have no regard to rational behavior.

Regarding the benefit structure it was observed that in most cases the impact of varying the benefit structure led to a shift in moneyness owing to our not changing the product pricing in concert with these changes. That said the impact on the ultimate peak levels or the width of the peak levels was discernable for most benefit variations.

In terms of further research or more definitive findings there is likely a diminishing return from further exploration of analysis of convexity as the idiosyncratic shapes and levels of convexity from different benefit, parameter and reserve structures are best left to the individual practitioners as they assess their own exposures.

Appendix - Summary Policy Design and Parameterisation

Product:-
Benefit Type: Drawdown, 5% per annum, GMDB = O/S Limit
Ratchet: Annual Ratchet, Maximum 10% with Aggregate Increase Limit 150%
Term Product : ie guarantee terminates once the aggregate payments = Base Benefit
Fund Basis: Equity 60%/Bond 40%
Currency: Sterling
Taxation: - No tax in fund
Model Point: 60 Year Old Single Male

Fund Charges

Asset Management	75bps
Product charge	85bps
Guarantee Charge	90bps
Total Deduction	250bps

Additional Scenarios

- Guarantee Charge on Base Benefit Level
- Guarantee Charge on Max (Base Benefit Level, Account Value)
- Maximum Guaranteed Withdrawal 3.5%
- Annual Ratchet/Step Up capped at 20% per annum with an aggregate cap at 500%
- Ratchet/Step Up only available every 2nd year

Modeling Frequency Quarterly

Finally I would like to feel that the attached note provides encouragement and engagement to budding commentators and authors who would like to put pen to paper, but who are uncertain as to the requirements or who have a concern as to an implicit obligation to "present" the materials. We see the MIG as offering an appropriate forum within which to publish papers that are not necessarily of sufficient length, or perhaps innovation, to be considered ground-breaking or meriting a sessional meeting or CPD event in their own right, but are perhaps on topic, insightful and of interest to our MIG community. These documents can then be held in either the VA MIG space of the Actuarial Profession's website (and thus would be open to all to review) or perhaps contained in the protected space of the Wiki, according to the would-be author's preference.

So happy vacation time to all and I look forward to hearing from you all in September when we hope to be able to move away from risk management for a while and back towards the Product, Pricing and Opportunity practice area for an update.

Recent MIG and Market Activity

For some reason, July seems to have been a quiet month for formal MIG activity, and it looks like August will be as well. Maybe everyone is watching cricket, or else on holidays? Or maybe people are just too busy with the in-tray and day-to-day activities to get involved? Note to VA MIG members – your MIG needs you!

Also the VA market itself has been fairly quiet, which in a way is giving us an opportunity to catch our breath! We hope to publish a list of the VA products that have so far been launched in Europe in next month's newsletter, which we hope members will find interesting and/or useful – E&OE!! Market statistics are not readily available from most countries, but we should be able to report on the aggregate Q2 2009 data for the UK market next month as well.

Resource Centre – Discussion Forum and Wiki User Guide

As a reminder, the discussion forum/wiki is managed under the Knowledge Area Network which is a separate secure site to the Actuarial Profession's website, and can be found at:

http://kan.actuaries.org.uk/Wiki%20Pages/Home.aspx

There is a link to the site under the popular links section of the main website. When you get to the site you will not automatically see the Variable Annuities forum, as you need to sign in - the sign-in icon is at the top right of the page. Once you sign in, if you click on the "Life" area in the top banner a link to our forum will appear on the left hand side of your screen.

For those looking for a more detailed exposition, please refer to the user's guide included in the June newsletter.

Vacancies – Operations and Events Manager

Another reminder - the Management Committee would like to appoint a member to take on the role of organising and co-ordinating relevant VA meetings and events. The person will provide a pivotal link and liaison between the MIG, the profession and other host societies where we look to hold networking and sessional meetings. We are open to volunteers whether from the general membership or from the existing management committee, and the successful candidate will become a member of the Management Committee and its operating sub committee.

We are always happy to hear from others who are willing to contribute towards our ongoing efforts, so please feel free to contact Rachel Smith at rachel.smith@actuaries.org.uk if you would like to get involved with the MIG in any other way.

Upcoming Events

Nothing to add at this stage, but if you have anything you would like others in the MIG to be aware of, please contact Jeremy Nurse with details at jeremy.nurse@watsonwyatt.com

Recent Publications

Nothing to add at this stage, but if you have read anything of interest recently that may be usefully disseminated to the rest of the MIG then please feel free to contact James Maher with details at jmaher@nexgenfs.com.

Practice Area Bulletin

Nothing further to add at this stage, but watch out for next month's input from the Product, Pricing and Opportunity practice area!