# Variable Capital Loads in Pricing <br> Working Party Report 

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## 1. Introduction and Overview

1.1.1.1 This working party was born out of the recommendation of the GRIP working paper which suggested that variable capital loads was an area of research that would benefit the Profession and the industry. (See Section 9.7 of the General Insurance Premium Rating Issues Working Party Report Jan 2007.)
1.1.1.2 The terms of reference are set out in full in Appendix A but the high level aims were to:
(1) Consider the purposes of variable capital loads
(2) Different methods of allowing for variable risk profiles
(3) Analyse the impact of the different methods
(4) Identify any common pitfalls
(5) Investigate the issues surrounding aggregate class level loadings
1.1.1.3 With these aims in mind we decided to split the working party into five separate work streams.

| Purposes | Considering the purposes of allocating capital, <br> the reasons for technical pricing and the <br> implications for varying capital by contract. <br> Setting the criteria by which a method should be <br> measured. |
| :--- | :--- |
| Methods | Identifying and summarising all the available <br> methods that could be used for varying capital. |
| Impacts | A selection of the methods were applied to two <br> data sets to compare and contrast the impact of <br> the methods on the premium. |


| Other Considerations | Highlighting some other issues that may need <br> consideration before selecting a method of <br> allocation. |
| :--- | :--- |
| Pitfalls | Highlighting some of the pitfalls that may be <br> encountered in trying to implement the methods. |

1.1.1.4 Generally our approach was to discuss these issues internally. The working party had a number of contributors working across the fields of pricing and capital analysis. We supplemented these discussions by some analytical research investigating the practical application of these methods. The results of this research have been set out in section 4.
1.1.1.5 This report forms the key feedback from the working party. This was supplemented by presentations at the Institute of Actuaries Pricing conference on $13^{\text {th }}$ June 2008, and at the Institute of Actuaries GIRO conference on $23^{\text {rd }}-26^{\text {th }}$ September 2008.
1.1.1.6 It is by no means clear that a well-defined concept of a technical price (allowing for variable capital loads) actually exists. In addition to the complexity of calculating such a price, and the uncertainty in selecting the most appropriate method (which is the focus of this report), the technical price also depends on the stakeholder, as it should depend on the objectives and concerns of the stakeholder.
1.1.1.7 As well as meaning that the technical price may differ between insurance firms, this also means that in any particular transaction between cedant and one insurance firm, there may well be a different technical price for each stakeholder involved. This means that there could be a different technical price for each of; the cedant, the management of the insurance firm, the debt-holders of the insurance firm, and the share-holders of the insurance firm.
1.1.1.8 We believe that a well implemented variable capital load process could offer material benefits to an Insurance firm. Our rational for this belief is set out in section 2 . The practical benefits which could be delivered include:
(1) Embedding risk considerations within the underwriting process,
(2) Charging a fair premium,
(3) Stabilising prices (e.g. due to changes in data or in company circumstances),
(4) Providing management information,
(5) Linking underwriting and business planning,
(6) Satisfying regulators,
(7) Providing confidence to rating agencies,
(8) Providing confidence to capital providers,
(9) Being easy to use,
(10) Being easy to communicate,
(11) Producing results which are easily justified,
(12) Stabilising the premium to required capital ratio,
(13) Reducing the premium to required capital ratio,
(14) Stabilising the return on capital,
(15) Increasing the return on capital,
1.1.1.9 There are many methods available for implementing a variable capital load process. This remains an active area of research and new methods continue to be developed and refined, hence the list that we have considered in section 3 is not exhaustive. We believe it does cover the methods commonly implemented in the UK at the current time, plus a representative sample of different alternatives. In
section 3 we have qualitatively discussed the strengths and weaknesses of the different approaches.
1.1.1.10 No individual method is likely to satisfy all objectives of the variable capital load process. While there is generally good agreement between the results derived from each of the methods, there are also material differences. The most appropriate method depends on the particular objectives and features of the firm introducing the policy. However we have attempted to indicate what some of the quantitative impacts to the firm of implementing these methods may be in section 4.
1.1.1.11 In our analysis, we have made a number of simplifying assumptions (for example, we have not considered reinsurance or tax; nor have we considered the impact of different lengths of tail for different business and the implied longer commitment of capital). These would, of course, need to be carefully considered in practice. In section 5 we have discussed some of these issues, with an aim to highlight areas for consideration, plus providing some potential solutions.
1.1.1.12 Implementing a variable capital load process will inevitably require detailed management to ensure that the process is embedded to the advantage of all. We discuss some of the communication challenges, and potential political pitfalls to be avoided in section 6 .
1.1.1.13 We recognise that this report has the potential to be (and it is our hope that it will be) read by people of different skill sets and experiences. Such differences inevitably mean that different parts of the report will be of interest to different readers. With this in mind the following areas may be of interest to readers who do not wish to study the entire report:

| Senior non-technical reader; for | Such a reader may find the |
| :--- | :--- | :--- | :--- | :--- | :--- |
| example a CEO considering whether | following sections particularly |
| to adopt a variable capital load | interesting; 2.5, 3.4, 5, and |
| process | possibly 6. |

Non-technical reader; for instance an Such a reader may find the
underwriter who had been informed following sections particularly
that such an approach will be interesting; 2, 3.3, 3.4, and 4.5
implemented, and wishes to
understand the potential pros and cons
of different methodologies
Senior technical reader; for example a Such a reader may find the senior actuary who has been asked to look into how to implement a variable capital load process following sections particularly interesting; 3.3, 3.4, 4.3, 4.5, 5, and 6

Junior technical reader; for example a There will be discussions of student actuary who wishes to interest to such a reader understand a variable capital load throughout the paper, but perhaps process the most interesting sections may be $2,3,4.5$, and 6
1.1.1.14 All material included in this paper is provided for information and discussion purposes only and does not constitute professional advice. No reliance should therefore be placed on such material. As such it is not the intention that this report constitutes a formal report as defined in the Profession's Guidance Note GN12 as adopted by the Board for Actuarial Standards with effect from 12 June 2006.

## 2. Purposes

### 2.1 Purposes Introduction

2.1.1.1 In this section we discuss whether an insurance firm should wish to implement a policy differentiating prices by risk.
2.1.1.2 Variable capital loads exist both as part of, and as the bridge between, the two (potentially silo) pillars of capital management and technical pricing within an insurance firm. As such the purpose of varying the capital loading applied in pricing is intertwined with the general purposes of using technical pricing and also the influences on the level of capital held by a company. It offers the potential (if successfully and appropriately embedded) to improve the technical pricing process.
2.1.1.3 In this section we aim to draw out the similarities between these pillars, and to understand the context within which a variable capital loads process may work. As such we have included discussions on the Capital pillar and the Technical pricing pillar in sections 2.2 and 2.3 respectively. In particular, we discuss the purpose of each of capital and technical pricing, and what features of the business underwritten influence each.
2.1.1.4 We then move on to discuss variable capital loads in section 2.4, discussing how these can act as the link between these pillars, allowing the influences of capital to be better reflected in the technical pricing process.
2.1.1.5 Finally we discuss how the value of the variable capital loads process could be measured in section 2.5; what benefits we would expect to see both from theoretical, and from practical perspectives.

### 2.2 Capital

### 2.2.1 The purposes of capital

2.2.1.1 The term "capital" is essentially used in two contexts in an insurance firm. In its simplest sense capital is just the assets in the business minus its liabilities. Another way of looking at this "held capital" is that it is the money that investors leave in the business (as opposed to using elsewhere).
2.2.1.2 From an investors viewpoint (with no constraints) it may make sense to run the business under a pay as you go basis, i.e. premiums in today are used to pay claims out today, which would result in negative capital; however regulatory constraints would not allow such a position. Additionally, sophisticated policyholders require a higher level of assurance that their future claims will be paid than would be available under pay as you go, hence the need for 'buffer' or 'required capital'. In this case, capital is being provided to back up promises to pay, and can be thought of as alternatives to collateral or guarantees.
2.2.1.3 There have been some suggestions that such buffers could be provided by Guarantee funds; however in practice it is difficult to ensure equality between members in such funds and this approach does not recognise the potentially material frictional costs which can occur during stressed conditions.
2.2.1.4 The other context is the required (or modelled) capital. That is, the capital that would be required as a minimum to support the business activities of the firm. For a non-stressed firm the held capital would exceed the required capital.
2.2.1.5 The required capital is driven by two main considerations.

Regulatory - i.e. the minimum level of capital that would be acceptable to regulators (in the UK this is currently based on the ICAS regime, however this level of capital will vary both between regions and over time, for example with the introduction of Solvency 2 currently planned for 2012).

Economic, e.g. capital that:
(a) protects the firm from insolvency;
(b) improves market perception/acts as a discriminator;
(c) improves the attractiveness of the firm to cedants (and hence increases demand, hence prices, and hence profits)
(d) improves the firm's rating;
(e) improves management information (e.g. the benefits of various reinsurance strategies, comparing strategic plans);
(f) improves investor information.
2.2.1.6 In theory economic capital relates to the individual firm's risk appetite, however in practice it is usually strongly related to the capital required to achieve (and be reasonably confident of maintaining) a particular rating.
2.2.1.7 The firm would normally be expected to hold sufficient capital to satisfy both the regulatory and economic constraints. However economic theory suggests that excess capital needs to be minimised, and the business should be grown in a way that optimally employs this capital.
2.2.1.8 The held capital may also deviate from the required capital as a function of the following drivers:
(1) short-term deviations due to the speed with which capital can be raised or distributed;
(2) alternative investment opportunities available (or lack thereof);
(3) stockpiling of capital either with the aim of making an acquisition or to enable organic growth;
(4) defending against hostile takeover bids;
(5) impacts derived from being a part of a wider group corporate structure;
(6) conscious or unconscious determination by management that capital should deviate from that calculated based on economic capital and risk appetite;
2.2.1.9 The potential for actual capital and 'required capital' being different raises the question as to which items should be reflected in pricing. For example, it would seem harsh to load an existing policy for capital being held within the business to fund a future takeover bid, as this capital relates to the potential to access further value in future through policies that another company is currently writing.

### 2.2.2 Influences on the capital level

2.2.2.1 Before considering the variation of capital within rating it is worth nothing some of the factors that are considered in setting the firm's overall capital.
2.2.2.2 For regulatory capital in the UK, this is defined by the FSA in INSPRU: "A firm must at all times maintain overall financial resources, including capital and liquidity resources, which are adequate, both as to amount and quality, to ensure that there is no significant risk that its liabilities cannot be met as they fall due". That is, capital should be reflective of all risks which would affect the firm's ability to meet their liabilities as they fall due.
2.2.2.3 For economic capital the risks it should be reflective of should be related to those purposes of economic capital, i.e.
(1) risk of insolvency;
(2) risk of a reduction in surplus and hence secondary effects relating to this;
(a) risk of reduction in rating;
(b) risk of a deterioration in attractiveness of the firm to cedants;
(c) risk of a deterioration in market perceptions;
(d) related to the above, the key risk is the inability to continue writing (profitable) business;
(e) of course, if the entity is confident that it can re-capitalise in the event of an adverse event then a reduction in surplus isn't necessarily a key concern;
(3) quantitative risks (which were outside of the scope of the working party);
(a) risk of a reduction in rating (over and above that related to the reduction in surplus);
(b) risk of deterioration in attractiveness of business to cedants (over and above that related to the reduction in surplus);
(c) risk of deterioration in market perception (over and above that related to the reduction in surplus);
(d) risk of misleading information adversely affecting business decisions.
2.2.2.4 Although all of those risks relate to the capital, it can be strongly argued that capital is not the most appropriate mitigating factor for some the quantitative risks. In particular, we would expect capital to reflect the financial risks; risk of insolvency, and risk of a reduction in surplus. These should potentially be net of any mitigating factors (for example security, collateral, guarantees, reinsurance), although this issue is discussed further in section 5 .

### 2.3 Technical pricing

### 2.3.1 Defining the technical price

2.3.1.1 At this stage it is worth defining what we mean by technical price for the purposes of this working party.
2.3.1.2 Underlying the technical price is the best estimate as the mean of a distribution this has an allowance for the risk in the contract, but that allowance is balanced by the potential upside. Two contracts could have the same mean costs but very different technical prices. This is the distinction which this working party is attempting to understand.
2.3.1.3 This leads us to the technical price as the actuarial recommendation of price. The actuarial recommendation of price should contain the above plus an actuarial assessment of the required profit from the contract, such that the profit expected to emerge is sufficient, given the range of outcomes, for the underwriting of the policy to be economic. This is the price considered by the working party.
2.3.1.4 Other prices we refer to in the paper are explained below:
2.3.1.5 Company price - the company price contains all of the above considerations and more besides (e.g. current market rates, allowable cross-subsidies, market power etc.).
2.3.1.6 Market price - the market price for the policy may be a known quantity. While it might contain an allowance for varying capital loads, and might be used in calibrating the allowance internally, it may not be consistent and provides little information.
2.3.1.7 Market consistent price - capital charges could be assessed on a market consistent basis, for example using the spreads on cat-bonds to help define the capital charges. Where market instruments are not available it may be possible to consider substitute investments with similar expected payoffs/risks, which could be used to determine the capital charges.

### 2.3.2 The purposes of technical pricing

2.3.2.1 The traditional control cycle for insurance business can be represented simplistically in the following manner:

2.3.2.2 Technical pricing aims to formalise the feedback at the point of underwriting to give an accurate reflection of current market pricing compared to a benchmark technical price; where the technical price is determined using a standardised definition of the price consistent with the firm's aims and objectives.
2.3.2.3 General insurance firms are selling a product which to a major degree consists of uncertain future costs (we use the term costs to cover collectively the main outflows from an insurance company, claims and expenses). Technical pricing is a method to estimate these costs in a consistent and structured manner.
2.3.2.4 The estimation of these costs is a key factor in both the supply and demand sides of the equation of setting a price for the insurance.
2.3.2.5 From the point of view of the seller, the transferred costs are usually funded by the policyholders through premiums; however in the event that the costs are above the premiums charges and any investment income earned on this premium,
these excess costs are funded by company's capital. The capital ultimately belonging to other parties (predominantly shareholders \& debt-holders).
2.3.2.6 In exchange for putting their monies at risk in this manner, capital providers will expect a long-run return above that of less risky investments. To achieve this, the insurance firm must gain an appropriate amount more in premiums and investment income than will be spent in claims and expenses.
2.3.2.7 Whilst some firms may focus on investment returns as a means of making money, from an underwriting perspective the primary focus is on the premium charged. Hence the premiums should be set in order to generate sufficient profits to satisfy investor's return requirements.
2.3.2.8 Technical prices aim to achieve this in two ways:
(1) Determining a level of premium which is sufficient to generate the required profit in respect of a portfolio of risks.
(2) Allowing the firm to understand how the underwriting fits into the objective of making the return to investors and hence take appropriate management decisions such as whether to participate, what volume to write (book level decisions), or what line to write (individual policy level decisions). In other words, ensuring price adequacy, however defined.
2.3.2.9 The buyers also require an estimate of the costs, in this case the cedant is (in general) attempting to understand the impact on their own investors (or themselves as individuals) of paying out a premium with certainty, as opposed to paying the uncertain costs themselves.
2.3.2.10 From a buyer's perspective any calculation of technical price would reflect:
(1) the value cedants gain from purchasing the insurance
(2) a fair price for the insurance
2.3.2.11 Pricing technically can also provide benefits to other stakeholders:

Management Provide enhanced management information regarding the portfolio and in particular rating adequacy.

Regulator A technical pricing framework is an important element of a firm's embedded risk management that is central to both ICA and Solvency II

Rating Agency Provide confidence in the stability of the firm's profits; as per the above it is part of risk management.

Market Improve the market's confidence that the business being written is consistent with the business aims and risk appetite.

Underwriter Provides a structured way of thinking about the elements that make up the price and aids the understanding of the levels of discount or cross-subsidy provided.

Reinsurer Provide confidence that there exists a standard rating approach that allows evaluation of risks consistent with the firm's stated objectives and can be used to demonstrate adequacy of rates.

### 2.3.3 Influences on the technical price

2.3.3.1 It is certainly worth considering the drivers of the technical price if we accept that pricing technically is of benefit to the firm. Why should the technical prices for two policies be different? In other words, what features of the business affect the technical price?
2.3.3.2 Of course, the technical prices should be different if the costs are expected to be different.
2.3.3.3 Additionally, for the seller, if Policy A adversely affects the objective of making a return to the providers of capital more than Policy B, then it should have a higher premium.
2.3.3.4 In considering the effect that a policy has on the objective of making a return on capital it is necessary to take account of several features.

Expected Costs Generally expected costs will be directly taken care of by premium and thus higher expected costs lead to higher premium.

Impact of Adverse
Outcomes

Impact of Positive
Outcomes

Secondary Effects
The higher the impact of adverse outcomes the greater the reward that will be required by the risk averse investor.

The lower the impact of positive outcomes the less compensation there is for adverse outcomes and thus the greater the reward that will be required by the risk averse investor.

Participation generates/protects other profitable opportunities.

Premium charged will act as an anchor.

Participation has secondary benefits with value (e.g. gain information, maintains minimum size for economy of scale benefits, enhances reputation).
2.3.3.5 Although the secondary effects are important for the street price they would normally not be considered when setting a technical price. Therefore these are outside the scope of this working party
2.3.3.6 For the buyer and other stakeholders the considerations are similar; although there may be differences in how the adverse outcomes affect the firm, the cedant and other stakeholders. There may also be differences in the secondary effects which will affect the street price.

### 2.3.4 The objectives of technical pricing

2.3.4.1 The criteria to be satisfied by the technical price vary by firm. A (necessarily generic) overview of the criteria would be to enable the firm to calculate the price for the policy that would be the minimum sufficient such that, if all policies were written at the technical price, the firm would be expected to meet it's risk and
return objectives. At the same time the technical price should enhance risk selection and should not allow anti-selection against the firm.
2.3.4.2 This is an investor view only. Technical prices exist from other stakeholder perspectives, for example the technical price from a policyholder perspective is the highest price at which the contract is still valuable to the cedant (valuing the contract according to the interests of the cedant, and consistent with the desires of the cedant).
2.3.4.3 Within a firm the technical price may be used in a manner which directly relates to the above, however more practically it may be used in order to produce quantifiable benefits which are related to the firm's objectives, either directly or indirectly). Some examples are:
(1) setting a hurdle rate;
(2) as a tool for underwriters;
(3) anchoring actual prices received;
(4) providing management information;
(5) charging a fair premium;
(6) linking underwriting and business planning;
(7) to satisfy regulators;
(8) to provide confidence to rating agencies;
(9) to provide confidence to shareholders.

### 2.4 Variable capital loads

### 2.4.1 Linking technical pricing and capital

2.4.1.1 The primary seller-side objective of varying the capital load by contract is to vary the technical price in a risk consistent manner that is optimal for the firm. Equivalently the primary buyer-side objective would be to vary the technical price in a risk consistent manner that is optimal for the cedant.
2.4.1.2 In particular, the capital load could vary if:
(1) The contract was expected to have higher costs;
(2) The contract's adverse/positive outcomes were expected to impact the firm's solvency more/less;
(3) The contract's adverse/positive outcomes were expected to impact the firm's level of surplus more/less;
(4) The contract's adverse/positive outcomes were expected to impact any other objectives of the firm's stakeholders more/less;
(5) The contract's adverse/positive outcomes, if the policy was not written, were expected to impact the cedant more/less;
(6) The contract's adverse outcomes could not be easily mitigated;
(7) The policy's adverse outcomes would have a large impact on either the firm's rating, or market perception of the firm (above and above any effect caused by a reduction in surplus).
2.4.1.3 This would reflect in varying the price by contract. If writing one policy leads to a different exposures to risk (even for the same expected costs) than writing another policy, then this should be reflected in the price.

### 2.4.2 Influences on the variable capital load

2.4.2.1 There are a number of features of the business being capitalised that need to be considered in ascertaining an appropriate capital. In general, the capital should vary under the same circumstances which cause the technical prices to vary, in particular;
(1) its adverse outcomes impact the investors more
(2) its positive outcomes impact the investors less
2.4.2.2 Thus the direction and degree of the impact on investors of an individual risk need to be assessed and reflected. Thus factors to be reflected in capital assessment include;
(1) The volatility of its costs
(2) The potential to call on the capital
(3) The riskiness of the business (under appropriate measures)
2.4.2.3 It may not be the loss of capital or profits per se which should be reflected; rather it may be their impact on the firm and its investors. For example, under the CAPM model it is only whether the firm's profits fall at the same time as profits from the investor's residual portfolio which matters; the capital may only require loading for systemic risks, and not those that can be diversified.
2.4.2.4 Hence it may be more appropriate to consider reasons, other than a loss of capital or profits, for an adverse income on investors.
2.4.2.5 If we focus on the financially quantifiable risk to the firm (the risk of insolvency, or the risk of a reduction in surplus) then this question can be rephrased to, under what circumstances can a policy's adverse outcomes be more likely to lead to insolvency or a reduction in surplus?
2.4.2.6 This would happen if the adverse outcomes happened when the firm was more at risk of insolvency or a reduction in surplus. That is,
(1) they happened at the same time as the adverse outcomes of other risks (increasing the vulnerability to insolvency);
(2) the occurrence of the loss would put the firm more at risk of either of these events (e.g. the risk in itself is large enough to reduce financial flexibility, either by eroding profits, or exhausting other mitigants such as reinsurance).
2.4.2.7 Thus we would conclude the variation of the capital load will depend on the correlation of that risk with the rest of the portfolio as well as the absolute size of the downside risk.

### 2.5 Embedding variable capital loads

### 2.5.1 The advantages of variable capital loads

2.5.1.1 Varying the technical price by policy puts a firm ahead of competitors that do not vary at all or vary at a lesser level of sophistication. Naturally this would apply to varying capital loads as well.
2.5.1.2 This should help the firm attract business that has a positive impact on the capital structure. That is, business which;
(1) Optimises the capital to premium ratio (under the condition of achieving growth targets);
(2) Minimises the amount of excess capital.
2.5.1.3 There are also a number of secondary benefits. Varying capital by contract should help to embed the firm's risk appetite into the individual underwriting. In this way it should help to:
(1) Improve management information;
(2) Improve understanding of the firm's risk appetite throughout the firm;
(3) Relate the price charged to the firm's appetite for that risk;
(4) Encourage consideration of all outcomes and their effect on the firm;
(5) Produce a portfolio consistent with the objectives, strategy and risk appetite of the firm;
(6) Reduce the uncertainty in the estimate of the capital, by better understanding the changes in the risk profile of the firm over time (and potentially reduce the capital amount even with no change to the risk profile);
(7) Encourage appropriate risk mitigation strategies, such as reinsurance purchases;
(8) Encourage appropriate diversification across risks in the investor's portfolio, optimising risk-returns.

### 2.5.2 Measuring the value of the process

2.5.2.1 The measurement of the benefits should be related to the specific objectives each firm has. Given the generic purpose of helping to meet the firm's objective, and the generic objective of meeting investors' demand for return on their investment, it is possible to argue that a successful implementation should increase the risk adjusted return to the shareholders, i.e. either increase the mean return to shareholders, or decrease the volatility of the return. However, if all firms are implementing similar policies, and given that the level of return demanded by investors has not changed, competitive pressures are likely to lead to the enhanced return being eroded by lower prices.
2.5.2.2 Notwithstanding that, should the capital to premium ratio improve (at the same level of profitability) and the amount of excess capital decrease, then it may well indicate a benefit of varying capital loads.
2.5.2.3 However similar effects could also be caused by other issues such as the position of the market cycle, absence of catastrophes, capital availability and so on. Indeed it is likely that the changes in the capital to premium ratio or the level of excess capital caused by these factors may swamp the effect of variable capital loads to the extent that it is difficult to measure.
2.5.2.4 Of equal importance should be the reduction of operational risk due to the closer assessment of the impact of individual risks on the firm and the consequent effect of pricing those risks consistent with firm's strategy.
2.5.2.5 It is impossible to properly strip out the effects of variable capital loads from other effects, especially for other operational effects within the firm such as deviation from technical price, underwriting quality, and marketing.
2.5.2.6 It also needs to be taken into account that the assumptions behind calculation of capital might change. This would then have an impact on the capital to premium
ratio and the amount of excess capital. To neutralise the effect it might be possible to calculate an as-if capital.
2.5.2.7 The industry-wide impact should (ultimately) be to remove arbitrage prices caused by different companies' views on appropriate profit loads for a given policy being unduly skewed by the characteristics of the other policies they write.
2.5.2.8 The risk-adjusted profit for a firm should then be stabilised, as it less sensitive to risk profit cross-subsidies. The absolute profitability may still appear volatile if the company's risk profile is sensitive to changes in the book however.

### 2.5.3 Practical objectives

2.5.3.1 To provide a framework for the consideration of alternative methodologies we have defined some specific objectives for the practice of varying the capital loads. A methodology which optimised all of these objectives would, in our opinion, offer clear value to a firm.

### 2.5.4 Embedding risk considerations at underwriting

2.5.4.1 Members of the working party have, in the past, been involved in discussions with underwriters on very low frequency (but very high severity) contracts where the expected costs on the policy have been argued to be zero ("we don’t expect a loss"). We are also aware of underwriters pricing the working layer, and the very high excess layer of a layered programme at the same loss ratio.
2.5.4.2 The method employed to represent variable capital loads within the technical rate should help to encourage consideration of the downside risks at the point of underwriting. This should encourage consideration of the chance of large losses, how severe the losses could be, and whether these cause more risk to the firm or diversify away.
2.5.4.3 This should encourage a rating and risk selection culture within the firm that leaves it less exposed to "shock" losses which could otherwise severely impact solvency, or the perception of the firm to the investment community. While the firm may still wish to underwrite high-risk policies, this should be an informed underwriting decision, rather than as the result of the technical methods disguising this risk.

### 2.5.5 Providing management information

2.5.5.1 The technical premiums have the potential to contain significant information clearly up to senior management.
2.5.5.2 To do this, the technical premium would need to reflect the risk assumed. A technical premium which represented just the expected costs offers some information content; however to the extent that the technical premium also
represents more shock outcomes the information is likely to be correspondingly more valuable to senior management in understanding the potential dynamics of the business, and hence making appropriate decisions.
2.5.5.3 The risk information captured could include not only potential adverse outcomes, but also how the policy interacts with and changes the firm's overall risk profile.

### 2.5.6 Charging a fair premium

2.5.6.1 Setting the premium according to not only the expected costs of the contract, but also with regard to the variability of these costs should reduce cross subsidies between policies. For example, stable contracts within a volatile book may be carrying a cost relating to the aggregate book's volatility (e.g. by setting the book’s target loss ratio in aggregate).
2.5.6.2 A higher degree of granularity in this respect would allow the stable policies to carry a lower variable capital cost, relating more to the risk they are exposing the companies to.
2.5.6.3 By requiring each policy to pay its own way (and only its own way) the technical prices may be argued to be fairer. For example the premium charged for a policy would not change because the firm decided to change the aggregation of its business into lines for planning purposes.
2.5.6.4 One definition of fairness in this context would be that if the policy was replaced by another risk with a similar range of outcomes, then the replacement would be priced similarly. Fairness to the cedant could also imply that a policy is priced similarly by different firms; however this is likely to be overridden by the impact of each firm's circumstances and attitude to risk on the technical price.

### 2.5.7 Linking underwriting and business planning

2.5.7.1 Within the business planning process assumptions are made on the likely performance, and risk features, of the business. The acceptable price to the firm is a reflection of risk appetite, and other risk features of the firm, and of the current market environment and other strategic objectives of the firm. Overall we
assume that the objectives in the business planning process are likely to be to maximise the risk returns to shareholders (as we are not considering the street price).
2.5.7.2 The rating process should reflect the same objectives considered within the business planning process. In order to achieve this, the technical pricing mechanism should reflect how the individual policy interacts with the issues considered in the business planning process.
2.5.7.3 In practice the pricing methods considered will be, to some extent, formulaic. As such it is to be hoped that the methods used will adjust the premium charged in order to reflect the range of potential impacts to the company caused by accepting the risk.

### 2.5.8 Satisfying regulators

2.5.8.1 In the UK, the FSA states the following broad categories for it's aims;
(1) promoting efficient orderly and fair markets;
(2) helping retail consumers achieve a fair deal; and
(3) improving our business capability and effectiveness
2.5.8.2 We would expect other regulators to have broadly similar aims, although the specific details would, of course, vary from market to market, and regulator to regulator.
2.5.8.3 Elsewhere in this section we argue that widespread use of variable capital loads could help to stabilise the insurance market (in terms of both the prices offered, and the return on capital results for individual firms), and to improve fairness in the prices offered in the market (although this is more likely to affect the wholesale market given the homogeneity of the retail market). These expected macro effects could have knock-on benefits to the individual firm of helping to satisfy regulators.
2.5.8.4 Additionally any increase in the quality of understanding of the business arising to either underwriters or senior management is likely to assist in satisfying regulators.
2.5.8.5 Finally under the current ICA regime in the UK, there is a "Use test" of capital models. This requires firms to embed their capital models within the running of the business. Variable capital loads (if based in a consistent way on the risks considered and quantified in the ICA model) could help to demonstrate this embedding, and thus improve the regulators confidence in the ICA.
2.5.8.6 Another potential area of regulatory interest is in the technical adequacy of the pricing process. For example in market bulletin Y3844 in July 2006, Lloyd's of London set out Underwriting Management Standards. In the discussion on Pricing and Rate Monitoring this set a minimum standard: "In considering pricing methodology, all of the elements of the pricing calculation are expected to be included." The working party considers that allowing for the volatility of potential outcomes as one element of the pricing calculation would be best practice, and help to satisfy this criterion. However to our knowledge, this has not been an area of active focus of the Franchise Board at Lloyd's.

### 2.5.9 Providing confidence to rating agencies

2.5.9.1 Within the analytical framework of rating agencies, various factors can be positively impacted by the use of varying capital loads in pricing. These factors are already included within the other objectives (e.g. stabilising the capital to premium ratio) and should be measured directly. The rating process, however, also consists of qualitative judgement. Management can demonstrate how the use of variable capital loads contributes to underwriting quality and competitive advantage.
2.5.9.2 It is not possible to identify the contribution of the use of variable capital loadings in pricing to a company's final rating itself. But the individual feedback from analysts should indicate to management if their pricing strategy is seen as a positive contributor.

### 2.5.10 Providing confidence to capital providers

2.5.10.1 Apart from having an interest in stable and/or maximised returns, capital providers seek confidence that a company is able to cope with different situations such as a soft market. Management can present towards shareholders (e.g. in shareholder or analysts meetings) how the use of variable capital loadings helps to achieve this.
2.5.10.2 Variable capital loads provide a higher degree of granularity in technical price and subsequently in the walk away price. Within a line of business, risks that positively impact the capital (because they are far less volatile or increase diversity over proportionally) would have a justifiably lower technical price compared to a flat loading structure and therefore might still be written. This is especially important if a firm has growth targets in a soft market that it intends to achieve without compromising on result. This can be linked with excellent underwriter risk selection.
2.5.10.3 It is obviously not possible to split shareholders' confidence into possible sources, so this element can only be a broad indicator.

### 2.5.11 Ease of use

2.5.11.1 Any measure is likely to be used by a number of different parties with different skill sets and modelling tools available to them. This means that the method for calculating will therefore either have to be simple or that an easy to use tool will have to be provided to users.
2.5.11.2 Also as most quotes are provided in real-time the method will have to provide results in real-time.
2.5.11.3 If one method is to be used for all business then that method should be easily applicable to all types of business and still provide reasonable results.

### 2.5.12 Ease of communication

2.5.12.1 When assessing the framework for calculating a technical price a critical element is the extent to which it enables non-actuaries (e.g. management and
underwriters) to form their own views on both the key assumptions and the resulting technical price. In particular, if a firm wishes to demonstrate that Technical Pricing is truly embedded within the business then the end user has to understand and use the method.
2.5.12.2 Of all the stages involved in setting a technical price (e.g. developing, trending, adjusting for changes in exposure, loading for expenses and investment income) the variable capital load is likely to be hardest to communicate to non-actuaries. In practice this can be separated into ease of communication of the method use, and ease of justification of the results obtained. For example allowing nonactuaries to understand the general drivers of the variable capital load relates to ease of communication of the method, while explaining precisely what features of an individual contract have led to it's high or low variable capital load relates to the ease of justification of the results.
2.5.12.3 A primary factor in making the method easy to communicate will be the intuitiveness of the results. Results that are counter-intuitive will not find acceptance with underwriters.
2.5.12.4 Another factor will be the ability to communicate the link from the method of capital loading to other capital assumptions made within the business and to show that the method is consistent with the performance targets being set by the firm.

### 2.5.13 Stabilising prices (e.g. due to changes in data, changes in company circumstances)

2.5.13.1 By basing the required profitability for each contract on its own volatility (rather than the aggregate volatility arising from the other policies written by the firm) the prices for that policy should be less exposed to changes in the company's circumstances.
2.5.13.2 For instance, should the firm fail to renew a section of business which was known to be comparatively stable, this may suggest that the book's (and possibly the firm's) volatility will increase. If the difference in volatility has not been taken
into account historically, this may require an increase in the technical premium for the policies which remain.
2.5.13.3 Additionally the uncertainty arising from poor, or incomplete data, or only a short history could be included within the variable risk load. If this was assessed, then the price charged for each policy should become more stable over time (since in this framework, a poorly performing year, which was indicative that the previous data had not been predictive, and may otherwise have materially affected the best estimate mean of the policy's costs, may have already been allowed for within a larger loading).
2.5.13.4 If the loadings are based on the policy's effect on the firm's risk profile, then the inclusion or exclusion of other policies may still affect whether this individual policy unduly dominates the stressed scenarios for the firm, and as such have a large impact on the technical price. However this is only likely to increase the volatility of the price for large contracts, or under large changes to the firm's exposures.
2.5.13.5 Historically the appropriateness of risk loadings in aggregate may well have been performed in a backwards looking analysis, which may not have adequately captured the changes in risk exposure. As such, prices may have been more stable than theoretically justified in the past (which may lead to fewer but more extreme corrections).
2.5.13.6 However, this enhanced stability may only be identifiable in the long term, and may not be measurable as the prices should still change from renewal to renewal, due to changes in the underlying risk such as inflation, environmental factors, risk management practices, and so on. If these risk factors are changing, the effect on the premium may be so large that it becomes difficult to record this increased stability.

### 2.5.14 Stabilising the required capital to premium ratio

2.5.14.1 In many ways, the required capital to premium ratio reflects the volatility of the firm's business in that this ratio is higher when the business is more likely per unit of premium to cause company insolvency. A more stable ratio might be
achieved with a better understanding of how much additional capital is required given the additional premium of any particular contract. Variable capital loads would provide such information at the time of underwriting. It is logical that a stable ratio will follow from more information about how it could change.

### 2.5.15 Reducing the required capital to premium ratio

2.5.15.1 A reduction in the capital to premium ratio is more obviously a benefit, as it suggests that underwriters have been able to select risks that are less capital intensive per unit of premium. This may be a demonstrable outcome if it was proven that underwriters were aware of the marginal ratio for their new contract. Where underwriters are encouraged to write contracts that have a lower marginal ratio, the total company ratio is likely to reduce as a result. This reduction could be due to one of two reasons. The selected new business diversifies more than the average piece of business or the new business is less capital intensive (less volatile). Both are good results as long as the profitability of the business is not decreasing.
2.5.15.2 In reality this last assumption is unlikely to hold and a firm will want to optimise its capital to premium ratio to find good diversifying marginal business whilst discouraging low volatility business if at a lower profitability.

### 2.5.16 Stabilising return on capital

2.5.16.1 Return on capital is defined as the ratio of profits to capital employed in the firm. Return on capital is a key metric measuring the success of the firm in delivering returns to its shareholders. It is generally accepted that most shareholders would like to minimise the volatility of the returns on their investment in the firm.
2.5.16.2 The return on capital could become more stable through two separate effects. First, the use of capital loads in pricing should result in the premium charged being consistent with the level of risk. This should reduce the impact on the return due to the premium being inadequate for the risks being run.
2.5.16.3 Second, the use of variable capital loads may give management greater confidence that the firm's capital requirement will remain stable over time (due to
the capital incentives inherent in the pricing/underwriting process). This confidence could lead to a greater willingness to repatriate surplus capital to shareholders as it arises, stabilising the denominator of the return on capital ratio.
2.5.16.4 In practice it may be difficult to separate out the effects of capital loads in stabilising returns on capital from other impacts (such as the underwriting cycle) on this metric. Success may not be apparent for many years, and even then it may be difficult to attribute any improvements in stabilising results to the use of variable capital loads.

### 2.5.17 Increasing expected return on capital

2.5.17.1 Clearly, investors want to maximise their returns on the capital tied up in firms they invest in.
2.5.17.2 The use of variable capital load should ensure that greater profit is required from those policies that place a large capital strain on the firm. In theory this should enable underwriters to better select risks that maximise profit for a given level of capital strain at a micro level, leading to an overall improved return on capital at the macro level.
2.5.17.3 If management has greater confidence that the firm's capital requirement will remain stable, due to pricing disincentives to underwriting capital inefficient risks, then they may be more willing to payback surplus capital to shareholders. This will increase the return on capital achieved.
2.5.17.4 Once again it may be difficult to attribute increases in the return on capital to the variable capital loads process. However proxy measures, such as return on capital based on an estimated prospective profitability measure, could provide a stable basis for assessing whether improvements have been made in long-term return on capital. It is likely that the results would need to be de-trended for the impact of the underwriting cycle to use this type of measure.

## 3. Methods

### 3.1 Available Methods

### 3.1.1 Identification of methods

3.1.1.1 The working party considered a number of methods which could be used to implement variable capital loads. These methods ranged from those with no theoretical justification but which are currently used in practice (such as subjective methods), to those with strong theoretical justification which are rarely used in practice. The list considered was by no means exhaustive of all possible methods; indeed this remains an active area of research and new methods continue to be developed. We hope however that for the majority of methods in which the reader is interested there will at least be a similar method considered.
3.1.1.2 In this section it is assumed that the distribution of the costs arising on an individual policy is known (and as such all statistics based on this distribution such as the mean or the standard deviation are also known); as is the distribution of costs for the book in aggregate, and the joint distribution between the individual policy and the book.
3.1.1.3 In practice this is beyond the current modelling capability of most insurance firms, and as a consequence many of the methods considered do not require such detailed knowledge; however we have made this assumption to enable us to also consider the case where such data is available (as it is in some special cases currently, and which may become more prevalent in the future). We considered the level of data and of detailed modelling required as a factor in the ease of use for each of these methods.

### 3.2 Nesting of methods

3.2.1 Some of the methods considered are not necessarily easy to aggregate from policy results into book results, and then further into business unit or firm results. This is possible in some instances (for example mean proportional spread), but totally impractical in other cases (for example game theory).
3.2.2 The theoretical methods discussed in this section often require consideration of both the individual and firm risk at the same time. Risk models used within firms at the moment are unable to perform such detailed calculations, although catastrophe risk models may allow this possibility for certain risks.
3.2.3 In practice, such challenges usually result in a nested methods approach which operates "top-down". In such an approach the firm's liabilities would be aggregated into a cascaded structure, for example by business unit, then line of business, and finally at a policy level. Variable capital loads would then be applied separately at each stage.
3.2.4 The first study would be to assess the variable capital loads at business unit level. In the second stage variable capital loads would be assessed for each line of business within the business unit, treating each business unit separately with different aggregate return requirements set according to the results at the first stage. This process could then be carried out for as many levels as were used in managing the firm.
3.2.5 This method is not theoretically justified; it ignores potential interactions of risks between different business units. As such the diversification between business units is allocated to each policy on some form of pro-rata basis, rather than appropriately allowing for those sections of the business unit which caused the unit in aggregate to diversify against other units.
3.2.6 Nevertheless this method is very commonly implicitly used at present. Although many firm's currently set variable capital loads at line of business level directly (rather than passing through the business unit nested level), it is extremely rare to use a holistic process all the way down to individual policies.
3.2.7 Nesting methods is a solution to the practical problems of implementing variable capital loads, but it is important to realise that the lowest level does not require a very simple allocation; advanced methods could be used at this granular level, even if these methods are not consistent with the higher levels of the variable capital loads process.
3.2.8 There is a danger in producing results which are distorted by the use of inconsistent methods, though the working party does not consider this a material risk for a well-run insurance firm. We would, however, welcome further research on bridging this gap between theory and practice.
3.2.9 The use of such a process would require a strong, controlled process to ensure that the risk models are always run on a consistent basis.

### 3.3 Discussion of Methods

3.3.1.1 It is not the intention of this report to provide a detailed description of each of the methods considered. Rather we have aimed to provide a high-level summary; so that the method we refer to is clear to all readers (as some of the terminology is not necessarily standardised), and as an introduction to the method for those who have not seen such an approach before. Where possible we have included a reference to further information on each method.

### 3.3.2 Implicit allowance in loss curves

3.3.2.1 Loss curves are used in a number of areas of pricing, and all of them can be adjusted to allow for variable capital loads in some fashion.
3.3.2.2 Loss curves in commercial lines tend to be ILF (Increased Limit Factor) curves, which show the ratio of the price for a higher limit compared to a base limit.
3.3.2.3 Personal lines business tends to be priced using an assumed underlying frequency (e.g. negative binomial) and severity (e.g. gamma) distribution. The loss curves in this case are these two distributions.
3.3.2.4 Further details may be found in Anderson et al.
3.3.2.5 Sometimes the curves, particularly the ILFs used in commercial pricing, include an implicit loading within the calculation. In these cases, the ILF no longer refers to an expected loss cost, but rather a technical cost of the protection. The mechanism for incorporating the implicit allowance is not always clear, and is often based around historic experience of the firm rather than detailed statistical evidence. If these implicit loadings, either within the ILF or within the severity
model, are included in the pricing calculation, then it could be argued that they are allowing for additional risk, and could therefore be considered as capital allocated against the business.
3.3.2.6 This automatically loads the book premium for layers covering more extreme events which are likely to require greater capital backing relative to premium than lower and less volatile layers would.
3.3.2.7 It also frequently provides a floor to the rate on line charged, so that even high layers unlikely to ever generate a claim are still charged enough to cover capital costs for holding that potential exposure.

### 3.3.3 Loadings in different parts of loss curves

3.3.3.1 Rather than loading the whole of the curve, it is often better to only load certain parts of the curve. For most pricing exercises, there is likely to be much better quality and quantity of information on the attritional (smaller) losses, whereas there tends to be very limited information on the large losses. Since capital is, in part, a mitigant against poor models and is often used when large claims occur, it would be sensible to allocate more capital against the larger losses than the smaller losses; this could be done by loading only the tail of the distribution.

### 3.3.4 Different loss curves used for different segments

3.3.4.1 Where classes of business have different risk profiles, they should attract different capital loadings. These could be reflected by having a different loss curve for each segment of the business which is representative of their different risk characteristics.
3.3.4.2 This method is similar to the other loss curve methods but recognises that different risk types have differing loss profiles and capital requirements that change differently with increasing excess points.
3.3.4.3 However, as the information is segmented more and more, the quantity of data backing differential loss curves will be lower, and therefore the statistical quality of the pricing model will be poorer. Therefore, there is a natural tension between
having loss curves at a sufficiently homogenous risk level and quality of the output.
3.3.4.4 This provides an extra level of sophistication to the attempt to make the price charged reflective of the capital costs of writing a particular risk, but otherwise is subject to broadly the same advantages and disadvantages.

### 3.3.5 Pricing according to management led plan

3.3.5.1 Planning processes are intended to generate the most up to date assessment from underwriters of the potential profit margins and business volumes that they can achieve over the coming years. As such they may inform management's decisions in the strategic direction of the firm.
3.3.5.2 However following strategic overlay they may also serve as target loss ratios; taking into account the required level of profitability given the capital loads of the business. These should then inform the underwriters decisions at case pricing level, either in terms of participation (if a price taker), or quote (if a price setter).
3.3.5.3 The variance in these targets between different segments gives an indication of the variance in rate strength possible given differing capital requirements. As loss ratios are a familiar measure for underwriters and management, this is an easy way of communicating varying capital requirements, and can help unit heads in targeting segments where they can achieve a higher return on capital, while also providing a high degree of consistency between pricing, capital modelling and business planning.
3.3.5.4 The measure is at best fairly crude however, with the segmentation unlikely to be detailed enough to truly reflect varying capital requirements between different risks.
3.3.5.5 In practice the planning process is also unlikely to be used so explicitly and specifically for capital loads. The strategic decisions for the required return on capital for a segment will tend to vary dramatically, for example management could accept a low return on capital for a segment due to its importance for
market presence or brand image, or could ask underwriters to target a high return on capital because current market conditions would make that possible.

### 3.3.6 Judgemental pricing at case level

3.3.6.1 Adding a risk load based on a judgemental consideration of the outcomes may result in premium levels which are appropriate. However any such approach is only as good as the practioner.
3.3.6.2 It could be argued that in many cases this is the system which is in current practice, with the underwriter setting the price based on many factors, including consideration of the uncertainty.
3.3.6.3 However to apply these methods well would require a detailed understanding of the book and of the range of results which is in practice, beyond any individual. The system is open to both drift over time, and deliberate abuse. As such it seems unlikely that it would provide meaningful management information, sufficient confidence to regulators, rating agencies, or shareholders.
3.3.6.4 It is not the intention of the working party to remove expert underwriter consideration from the pricing process as this is vital to the correct business decisions being taken. Rather the methods discussed are designed to provide enhanced information to the underwriter and senior management, both helping in, and reducing the risks of, the application of subjective judgement.

### 3.3.7 Capital markets

3.3.7.1 Some portfolios of policies may be highly correlated to capital market instruments e.g. a book of US property risks to futures contracts based on the US Property Claims Service Index.
3.3.7.2 The price of the instruments should give an indication of the risk profile of the underlying book of business. Such information can be fed into pricing formula.
3.3.7.3 This approach will have increasing importance as capital markets offer more insurance related products. Currently these are limited however.

### 3.3.8 Incremental marginal capital value

3.3.8.1 A policy can be viewed as adding value if it generates a return greater then the opportunity cost of the capital supporting it. The contribution of the policy to the insurer is thus expected profit minus capital usage costs.
3.3.8.2 In theory the value of the firm could be assessed both just before, and just after the writing of the policy. It is assumed that the value of the firm would be affected by the premium received and the extra risk assumed. Most obviously this extra risk would affect the capital requirement of the firm. Other risk metrics would also be affected and may in turn affect the firm's value (such as the probability of a loss for the firm, rather than the (ICA) value of the size of a loss at a $99.5 \%$ confidence level); however for practicalities sake we have considered only the marginal change in the ICA capital.

### 3.3.9 Mean of transformed loss

3.3.9.1 Transformed loss measures, in their generic form, apply a transformation to the loss distribution to give more weight to the key focus areas of the loss distribution for the decision maker. To put it more simply, a transformed loss measure would set the technical premium according to a weighted average of all possible outcomes, with more weight being applied to those scenarios which impact the objectives of the firm.
3.3.9.2 This translates to a capital allocation in one of two ways. Either the difference between the mean of the transformed distribution and the original distribution is assessed for each policy and used as the basis for a pro-rata allocation of capital, or the expectation of the transformed loss distribution is used itself as the technical premium. It is the later approach that we have considered here.
3.3.9.3 The tail risk measures such as Value-at-Risk and Tail-Value-at-Risk give a linear weighting to extreme outcomes, often this can constitute underweighting in the eyes of the risk adverse insurer. This can be overcome through the use of different transformation measures, which increase the weighting on extreme outcomes.
3.3.9.4 The transform function applied is of critical importance to the suitability of the resultant risk measures and technical price. We have focussed on the most common transform functions applied to assist in capital allocation considerations:
(1) Proportional hazards transform
(2) Wang transform
(3) Outcome specific charges
3.3.9.5 The generalisation of all transform methods is the approach of applying outcome specific charges to the parts of the loss distribution depending on how much influence those outcomes should have on the risk assessment.
3.3.9.6 It is possible to be relatively crude or extremely sophisticated when deriving the charges to apply. It is important to ensure that the weightings rule can be easily understandable and accessible to the ultimate decision-maker.

### 3.3.10 Mean of transformed loss - Proportional hazard

3.3.10.1 The proportional hazards transform is of the form $S_{X}^{*}=\left(S_{X}\right)^{1-\lambda}$ with $0 \leq \lambda<1$; where $S_{X}=\left(1-F_{X}\right)$ and $F_{X}$ is the cumulative distribution function for the random loss variable $X$.
3.3.10.2 This transform results in more weight being given in the risk assessment to the severe loss outcomes hence it is appropriate where the decision maker is genuinely more concerned by the tail end of the loss distribution. The redistribution of the weighting given by the proportional hazards transform is shown below.


### 3.3.11 Mean of transformed loss - Wang transform

3.3.11.1 While the proportional hazard transform shifts the loss distribution exponentially, the Wang transform essentially shifts the percentile of the distribution looked at. That is the $90^{\text {th }}$ percentile (say) of the transformed distribution is determined as the $93^{\text {rd }}$ percentile of the untransformed distribution. The degree of the shift follows the transform of a standard Normal distribution. More precisely, it is calculated as $S_{X}^{*}=\Phi\left(\Phi^{-1}\left(S_{X}\right)+\lambda\right)$ or equivalently $F_{X}^{*}=\Phi\left(\Phi^{-1}\left(F_{X}\right)+\lambda\right)$
3.3.11.2 This transform extends the theory behind the Sharpe ratio and applies it to fat tailed distributions; the Sharpe ratio is the foundation to CAPM and financial market pricing more generally.
3.3.11.3 As with the proportional hazard transform, the Wang transform inflates the probability of adverse outcomes whilst correspondingly deflating the probability of favourable outcomes (for pricing analysis purposes). However the degree of adjustment is different to the proportional hazard transform. The redistribution of the weighting in this case is shown below.

3.3.11.4 It has been shown that risk pricing of catastrophe bonds based on expectations formed after the Wang transform has been applied give risk premiums that are consistent with financial models used in catastrophe bond pricing.

### 3.3.12 Capital Asset Pricing Model (CAPM)

3.3.12.1 CAPM is a well-known investment model which assesses the required return from an asset (and hence, given an estimate of the income from the asset, its price) as the risk-free rate plus a factor, known as the beta of the asset, times the
difference between the average market return and the risk-free rate. The beta represents the correlation between the asset and the market. Details of CAPM are readily available, and are covered by the current Actuarial Profession examinations syllabus.
3.3.12.2 Considering individual policies, or investments in books of insurance business as investible assets, this approach could be extended to produce pricing for general insurance applications; calculating the technical premium based on an assessment of the expected costs and the required return. However it would be impractical to assess the beta between all market assets and each individual policy, so this method would in practice reduce to a mean proportional spread.
3.3.12.3 Furthermore, failure of CAPM has been identified (for a brief summary with references to further papers, see Cummins). However the method has been considered here due to its familiarity.

### 3.3.13 Adjusted Capital Asset Pricing Model (Adjusted CAPM)

3.3.13.1 If the firm did not hold capital, the same economic effect could be obtained by purchasing a put option on the firm's overall profitability at zero. There would be no further cost to the firm (above the option premium) if the firm was profitable, and losses would be paid for exactly from the payout of the option. As each line expands or contracts, this affects the risk of payouts on the option and hence the price of the put. The Myers-Read approach (discussed below) essentially allocates capital such that (at the margin) any change in exposure to the line would require capital which exactly pays for the increase in cost of the put option.
3.3.13.2 Under certain simplifying assumptions it was shown in Butsic that this capital allocation method simplifies to a simple function of the correlation of the line with the overall liability risk profile (Butsic showed this under the assumption of zero correlation between the assets and the liabilities, and for the cases of normally, and lognormally distributed individual liability profiles). This method requires an assessment of; the expected loss to the line, the correlation between the line and the total liability profile, the total capital to be allocated, and a scaling factor (denoted Z) which essentially represents how sensitive the put
option price is to changes in the liability risk profile and for which a formula is given in the paper.
3.3.13.3 Essentially this generates a CAPM-like formula for the allocation of capital to a line.
3.3.13.4 Underlying this method are assumptions, in particular surrounding the availability of such options, the underlying liability profiles, and the method for valuing the put option (which is assumed to follow a Black-Scholes model). The availability of such an option in practice is not a necessity of the methodology however; so long as a consistent value for that option could be calculated (the availability of such an option may be necessary to ensure a market-consistent valuation and hence market-consistent pricing). The assumptions on the underlying liability profiles and method for valuing the put option could be replaced. For example since a put option on the firm's profitability is analogous to a stop loss on the firm's results at $100 \%$, the firm may wish to consider common general insurance individual case pricing methods applied to this stop loss. Under such an approach the correlation assumption may need to be replaced by consideration of the firm's risk profile both before and after perturbation by a slight increase in the line.
3.3.13.5 For the remainder of this paper, the CAPM method refers to the allocation method as presented in Butsic's paper, however calculating the correlations between each line and the total liability portfolio explicitly.

### 3.3.14 Capital allocation methods

3.3.14.1 One approach to planning is to directly allocate capital to different business segments. This method encourages direct and explicit consideration of capital requirements for particular types of business.
3.3.14.2 This may often involve linking the business planning inputs to the capital modelling information (e.g. ICA), and can consider the interaction between different segments and the marginal capital costs of growing or contracting each segment. The capital costs appropriate for a particular segment (in conjunction with other assumptions such as expenses, reinsurance costs etc) and the required
return on capital for that segment then suggest a maximum target loss ratio that the unit cannot exceed without failing to meet ROC targets.
3.3.14.3 On submission of an expected business plan, the capital that would be required to write those premium volumes for that particular type of business can be estimated, and if the underwriter is comfortable with that requirement and their ability to meet the plan they can book the capital for that underwriting year.
3.3.14.4 Such an explicit measure of capital naturally encourages underwriters to consider the viability of different types of business in terms of the efficiency of capital use, and is clear and easy to communicate.
3.3.14.5 In addition, tying plans to a specific capital measure encourages greater caution and realism in planning, as profit on lower than expected premium volumes would still be compared to the initial capital allocation. In practice there is scope for an internal capital market to mitigate the impact of this, with units able to surpass income targets borrowing capital from units unable to meet theirs.
3.3.14.6 Depending on the sophistication of the allocation process, this method can provide some detailed feedback on varying capital requirements. In addition to preparing an overall capital allocation for a segment, the mix of risks within that segment can be considered in the allocation. For example, an estimate of capital allocation could be prepared for each of several different limit profiles, giving a clear guide as to how the unit could structure their account to use capital more efficiently.
3.3.14.7 Relativities between target loss ratios in plan will often bear little similarity to relativities between the capital requirements for lines of business. This is more of a process issue however, and although the final headline targets in plan may be distorted by other factors, the underlying maximum target loss ratios based on capital requirements can be circulated separately.
3.3.14.8 There are many methods of allocating capital down to individual constituent business; these are considered separately below.

### 3.3.15 Risk measure

3.3.15.1 In order to apply many of these capital allocation methods discussed later in this section a risk measure is required. This is usually consistent with, but may not be the same as, the overall capital setting risk measure. Common examples are;
(1) Value-at-Risk
(2) Expected Policyholder Default
(3) Tail-Value-at-Risk
(4) Excess-Tail-Value-at-Risk
(5) Standard Deviation
(6) Variance
(7) Semi-Variance
3.3.15.2 Further details may be found in Venter.
3.3.15.3 A common combination used in practice is to use Tail-Value-at-Risk, or/and Excess-Tail-Value-at-Risk, for the risk measure in the proportional spread or apply co-measure methods (discussed below); even when using an overall Value-at-Risk risk measure. In practice using a Value-at-Risk measure for these methods may result in highly unstable results; and zero allocations if the probability of a claim is low.
3.3.15.4 Historically the Expected Policyholder Default measure was used in capital setting at Lloyd's of London under the Equalise relative risk method (discussed below), by setting capital such that the expected loss to the Central Fund as a proportion of premium income was the same for all members.
3.3.15.5 The Game Theory methodology (discussed below) would generally be assumed to use the same risk measure as the overall risk capital setting; however the computational complexity of the calculation suggests that the simplest credible measure may be used in practice.

### 3.3.16 Proportional spread

3.3.16.1 At their base level, proportional spread methods allocate capital by considering a risk measurement calculated separately for each section of business. Capital is allocated to the piece of business in accordance with its proportion of the total of the measurements. As such this method is naturally complementary with the nesting of variable capital loads.
3.3.16.2 As a general comment, the method does not allow for the impacts of correlation and diversification, and as such may potentially under- or over price individual sections of the business. However due to its practicality and reasonable level of success, this method is commonly used in practice.
3.3.16.3 While this method in theory requires all of the policies to be considered at the same time, in practice simple proxies allow this method to reduce to the level of individual case pricing as a stand-alone entity. This is the approach that is often carried out in practice.
3.3.16.4 The success of the method is highly dependent on the risk measurement used. We have therefore considered several variants of this method.

### 3.3.17 Mean proportional spread

3.3.17.1 This method allocates capital in accordance to the expected costs. As such it is equivalent to pricing to achieve the same target loss ratio on each piece of business.

### 3.3.18 Standard deviation proportional spread

3.3.18.1 This method allocates capital in accordance to the standard deviation. In theory this requires calculation of the standard deviation of all policies, before any can be priced, however in practice historical data and benchmarks can be used to derive rules of thumb, such as setting the price as the expected costs plus $35 \%$ of the standard deviation, which approximate the overall capital allocation multiplied by the required return on capital.

### 3.3.19 Value-at-Risk proportional spread

3.3.19.1 This method allocates capital in accordance to a certain Value-at-Risk statistic. This may be achieved either with, or without, deduction of premium, and this is sometimes approximated by deduction of expected costs to avoid recursive formula. For the purposes of this paper we have assumed deduction of premium.
3.3.19.2 This method could be carried out at any selected percentile; however the results are sensitive to the choice of percentile. Too low a percentile and the business may not produce losses (and if carrying this out at an individual policy level, this can for some books mean $95 \%$ or even higher), and hence the statistic is meaningless. Too high a percentile and the method may over-allocate capital to policies with extreme, but highly unlikely, losses.
3.3.19.3 For the purposes of this paper we have assumed that the percentile will be set such that the aggregate capital on each piece of business adds up to the ICA capital at aggregate level. This would normally imply a percentile lower than $99.5 \%$ for allocation purposes. Again a proxy would normally be used which would approximate this addition.

### 3.3.20 Tail-Value-at-Risk proportional spread

3.3.20.1 This method allocates capital in accordance to a certain Tail-Value-at-Risk statistic. Again this may be achieved either with, or without, deduction of premium; and again for the purposes of this paper we have assumed deduction of premium.
3.3.20.2 Similar considerations on the selection of the percentile apply as with Value-atRisk, (although the risk of too low a percentile producing a meaningless statistic is lessened). Again we have assumed for the purposes of this paper that the percentile will be set such that the aggregate capital on each piece of business adds up to the ICA capital at aggregate level.

### 3.3.21 Game theory

3.3.21.1 Game theory is a relatively young branch of mathematics that provides a framework for decision making and identifying equilibrium in competitive situations where different parties interact and conflict. The application of this branch of mathematics to capital allocation problems falls out quite naturally when it is recognised that adding a sub-portfolio to a business brings additional diversification benefits to the whole but creates conflict as all sub-portfolios vie for the largest share of the benefit. This theory provides a solution to some of the limitations of the marginal and proportional spread methods.
3.3.21.2 The application of Game Theory in this case allows for the formation of coalitions between sub-portfolios and considers the marginal capital cost associated with adding a further sub-portfolio to the coalition. Individual subportfolios have an incentive to be part of the largest coalition as they enjoy greater diversification benefits yet each member will also seek to minimise their allocation of the coalition's capital burden. In particular, no individual subportfolio will tolerate an allocation of capital that exceeds its standalone capital requirement, i.e. as if it were to be viewed as a separate business.
3.3.21.3 The 'equilibrium' allocation of capital, for a given sub-portfolio, represents the average marginal impact of adding the sub-portfolio when it is the first, mth and last portfolio to be added to the business. In taking this average all possible combinations of the (m-1) sub-portfolios comprising the starting 'coalition' are considered. Put simply the marginal capital for a unit is calculated for every group of units it could be a part of, and these are averaged.
3.3.21.4 The Game Theory approach is computationally intensive as it averages across a total of $2^{n}$ marginal scenarios and hence has obvious practical limitations. However if implemented Game Theory variable capital loads do exhibit certain useful properties, for example natural additivity.
3.3.21.5 A further limitation of the Game Theory method is that it gives rise to different allocations when sub-portfolios are broken down into further sub units (although this also applies to the majority of measures considered).

### 3.3.22 Myers-Read

3.3.22.1 Myers-Read refers to a family of methods based around financial options. We assume that all insurance firms contribute to a guarantee fund which will pay all claims in the event of insurer insolvency i.e. it will pay liabilities less any remaining assets if the liabilities exceed the assets. This can be viewed as an exchange option.
3.3.22.2 Assuming the insurer holds no capital then valuing this option gives both the economic capital and expected profit as dictated by the market place.
3.3.22.3 Merton and Perold first proposed this methodology. They allocated capital between lines of business by considering the changes in option prices when different lines are added. This allocation approach is non-additive.
3.3.22.4 The methods assumed that the value of such an option is observable in the market place - an assumption which may not be realistic in the insurance world.
3.3.22.5 Myers and Read use the above definition of capital (value of an exchange option) within their general capital allocation formula. This approach is additive.
3.3.22.6 Mildenhall investigates the assumptions underlying Myers and Read’s results and find them to be unrealistic for typical general insurance portfolios.
3.3.22.7 Whilst the above methods show interesting mathematical results underlying assumptions can often fail, for example the lack of a complete market, nonexistence of arbitrage prices, and unrealistic loss distributions. In addition the method's complexity can make communication difficult.

### 3.3.23 Equalise relative risk

3.3.23.1 Given a method of quantifying risk, that method could be applied to each policy (or book) and the price adjusted until the risk per unit of exposure (generally premium) is equal for all policies. Usually the target risk level would be set down in advance by senior management (following a firm-wide analysis) and then utilised in the pricing.
3.3.23.2 For example, setting the risk quantification as the probability of a loss, and setting the target risk level as $5 \%$ (say) would effectively set a pricing mechanism which required all policies to be written so that they were expected to be profitable $95 \%$ of the time. At the other extreme, setting the risk quantification as the expected costs, the exposure as the premium and setting the target risk level as $85 \%$, would involve varying the premium until the expected costs were $85 \%$ of the premium essentially setting the premium for all policies so that the expected combined ratio on the policy is $85 \%$. The idea was originally developed in Butsic and further developed for pricing applications in Zhang.
3.3.23.3 This method has historically been used in capital setting at Lloyd's of London, with a risk quantification of the expected cost to the central fund (i.e. the expected value of the losses less the Syndicate's capital, if losses exceed capital, and zero otherwise), an exposure of the Syndicate's capacity, and a set loss cost. This set capital levels for Lloyd's Syndicates such that the expected loss to the Central Fund per unit of capacity was the same for all Syndicates.
3.3.23.4 A similar approach could be used as a methodology for variable capital loads by setting the risk quantification as the expected cost to the firm's net assets (downside risk only), exposure of the technical premium, and a target risk level set so that in aggregate, the firm met its planned objectives. This would involve varying the premium level such that the each policy had the same expected exposure to draw down on the firm's net assets.
3.3.23.5 The method replaces the cost of capital calculations, and sets premium by a trial and error approach. As such it is slightly more difficult to follow the calculations, although these are not difficult with modern computing performance, and the validity of the solution is easy to check.

### 3.3.24 Apply co-measure

3.3.24.1 If the pricing actuary has an understanding of the risk profile of the policy under consideration, the aggregate book risk profile, and how these interact, he or she can apply a co-measure methodology. This method allocates risk to constituent
drivers by examining the impact of those drivers on the risk for the aggregate book.
3.3.24.2 For example aggregate book capital may have been set by a 99.5\% Value-at-Risk methodology. If using the same risk measure for the co-measure in question, this method would allocate the book capital to sub-units according to their profit or loss in the particular simulation which was at the 99.5th percentile. In practice it would be far more common to allocate capital according to a more robust comeasure such as TVaR or XTVaR, parameterised consistently with the aggregate capital setting, for example allocating capital according to the expected costs or losses on each policy in the scenarios which constituted the worst $5 \%$ for the firm in aggregate (rather than the worst $5 \%$ for that specific risk). Such measures consider the impact of the policies on the firm in stressed scenarios.
3.3.24.3 A modification to TVaR co-measure capital allocation was proposed in Mumford, Nielsen, and Poulson which suggested conditioning on the firm's aggregate capital being sufficient. They argued that this method was more stable, less affected by the higher parameter uncertainty in tail losses, and had a sound common sense interpretation, as allocation of capital in events in which the total capital was exhausted was a moot point.

### 3.3.25 Insurance capital as a shared asset

3.3.25.1 The Capital Consumption approach of Mango (2003) makes no attempt to allocate capital to each policy; rather, each policy has a right to call upon the firm's capital pool. It must pay a charge for this right - a rental fee. It must also pay a charge for any contingent capital calls, i.e. when it has to use some of the capital pool.
3.3.25.2 One by-product of the consumption method is that it is automatically included within the premium rating formula.
3.3.25.3 Mango acknowledges the consumption method shares many of the weaknesses of allocation methods. Whilst the method cannot be used for return on capital type calculations it is simple, intuitive, and flexible in stochastic modelling.
3.3.25.4 Mango (2005) expands his idea further, splitting assets into a consumptive and non-consumptive element.

### 3.4 Selection of methods

### 3.4.1 Selection criteria

3.4.1.1 Each of the methods was assessed against the practical objectives discussed in section 2.5. Each of the methods was assessed against these criteria and graded Low/Medium/High. These assessments are shown in the table below, and the rationale for the assessment is subsequently discussed.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Implicit allowance methods <br> Allowance in loss curves |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Implicit allowance in loss curves | Low | Medium | High | Low | Low | Low | Low | Low | High | High | Medium | Low | Medium | Low | Medium |
| Loadings in certain parts of the loss curves | Low | Medium | High | Low | Low | Medium | Medium | Medium | Low | High | Medium | Low | Medium | Low | Medium |
| Different loss curves used for different segments | Medium | High | High | Medium | Low | Medium | Medium | Medium | Medium | High | High | Low | Medium | Low | Medium |
| Judgemental price setting |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Price according to management led plan | Medium | Low | Medium | Low | High | Low | Low | Medium | High | High | Low | Medium | Low | Medium | Medium |
| Judgemental pricing at case level | Low | Medium | Low | Low | Low | Low | Low | Low | Low | Low | Medium | Medium | Medium | Medium | Medium |
| Portfolio value pricing <br> Aggregate book approaches |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Capital markets | Medium | Medium | Medium | Medium | Low | Medium | High | High | Medium | High | High | Medium | Medium | Medium | Medium |
| Incremental marginal capital value add | Medium | Low | Low | Medium | Low | High | High | High | Medium | High | Low | High | High | High | High |
| Policy value |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Mean of transformed loss |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Proportional hazard | Medium | Medium | Medium | Medium | Low | High | High | High | Medium | Medium | Medium | Medium | Medium | Medium | Medium |
| Wang | Medium | Medium | Medium | Medium | Low | High | High | High | Medium | Low | Low | Medium | Medium | Medium | Medium |
| CAPM | Medium | Medium | Low | Medium | Low | Medium | Medium | Medium | Medium | Low | Medium | Low | Medium | Low | Medium |
| Capital allocation methods |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Proportional spread |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Mean | Low | Low | Medium | Low | Low | Low | Low | Low | High | High | High | Low | Low | Low | Low |
| Standard Deviation | Medium | Low | Medium | Low | Low | Low | Low | Low | High | High | High | Low | Low | Low | Low |
| VaR | Medium | Medium | Medium | Medium | Medium | Medium | Medium | Medium | Low | High | Medium | Low | Medium | Low | Medium |
| TVaR | Medium | High | Medium | Medium | Medium | High | High | Medium | High | Medium | Medium | Medium | Medium | Medium | Medium |
| Game theory / Shapely | High | Medium | Low | High | Low | High | High | High | Low | High | Low | Medium | Medium | Medium | Medium |
| Myers-Read | High | Medium | Low | High | Low | High | High | High | Medium | Low | Low | Medium | Medium | Medium | Medium |
| Equalise relative risk | High | Medium | Medium | High | Low | High | High | High | Medium | High | Medium | Medium | Medium | Medium | Medium |
| Apply co-measure | High | Medium | Low | High | Low | High | High | High | Low | Medium | Low | High | High | High | High |
| Insurance capital as a shared asset | High | Medium | Medium | High | Low | High | High | High | Medium | Medium | Medium | Medium | Medium | Medium | Medium |

### 3.4.2 Allowance in loss curves

3.4.2.1 Allowance in loss curves in general do not support the embedding of risk considerations at underwriting, as the allowances are often not fully understood or even known about. Explicitly using different loss curves for different segments makes this consideration clearer however. It can be argued that these methods
generate a fair premium, in that the price reflects the risk assumed and is not distorted to the buyer by other aspects of the firm's risks - especially so the more granular the loss curves. As the loss curves change irregularly, these methods should produce relatively stable prices over time.
3.4.2.2 The technical prices derived by these methods provide little information to management (for example, it is not easy to identify the policies most increasing the firm's risk), as the information they provide is, in effect, smoothed and insufficiently granular. The use of different loss curves in different segments may help, but is unlikely to provide sufficient granularity. The methods are not related to business planning, or necessarily the firm's objectives (for example, if management decides to aggressively pursue a particular line as it is a diversifier for the aggregate book, implicit loadings within the loss curves may make it difficult to underwrite to this guideline).
3.4.2.3 In general these methods may produce some distortion, especially due to potential inappropriate selection of a loss curve, and lack of understanding. This is likely to reduce the confidence these methods provide to regulators, rating agencies, and capital providers.
3.4.2.4 The implicit allowance method is easy to use, however the selection of different loss curves is less easy, and the lack of certainty over the loadings in different parts of the loss curve makes the appropriate use of this method difficult. The methods are well known in the market and hence relatively easy to communicate and justify. The increased (and obvious) granularity of the different loss curves method makes this especially easy to justify.
3.4.2.5 However these methods are not particularly good volatility discriminators, and as such may produce anomalies which distort the capital to premium ratio and hence the return on capital. These methods do not provide sufficient information to identify opportunities to reduce the capital to premium ratio, and hence increase the return on capital.

### 3.4.3 Judgemental price setting

3.4.3.1 Judgemental price setting methods do not aid the embedding of risk considerations at underwriting (although if applied at the case level it is possible that they are reflected in the prices). Such judgemental methods have little guarantee or transparency of fairness, and are subject to alteration, for example if a different underwriter prices the risk.
3.4.3.2 In general these methods are not informative (beyond the expected costs), and judgemental pricing at a case level is not necessarily aligned to the planning process; although as the pricing mechanism in the management led plan approach is derived as part of the planning process, this method is perfectly aligned.
3.4.3.3 We would not expect these methods to produce much confidence to other stakeholders, although as the plan is set by shareholders' representatives, senior management, they may be more satisfied with this method.
3.4.3.4 Individual pricing is formulaic in the management led plan approach, which leads to an easy to use method, which is easy to communicate. However it may be difficult to justify the resulting prices. By contrast the judgemental pricing at case level approach requires an expert practioner overseeing each risk; however it may be clearer how specific considerations have been reflected in the price.
3.4.3.5 These methods are only as good as the management plan, or the individual underwriter. As such it is not clear if the prices will be appropriate to stabilising, or provide sufficient information to reduce the capital to premium ratio, and hence have knock-on effects for the return on capital.

### 3.4.4 Aggregate book approaches

3.4.4.1 The aggregate book approaches include a consideration of the risk inherent in the deal (either implicitly as this is expected to be reflected in the capital market prices, or explicitly based on the incremental effect on marginal capital). However there is potential in both methods for the information content to become disguised in the overall noise of the results. Recent sub-prime issues have highlighted that the capital market price for risks can be volatile; as such it may
be, or at least be perceived by cedants to be, unfair under certain circumstances. This is more of a problem for the incremental marginal capital, as it is potentially exposed to a price bias based on the order of the policies.
3.4.4.2 These methods contain information about the current market price, or the effect of the policy on the firm, but hold little individual risk information and as such do not provide as much detailed information to management as possible. They are unlikely to generate technical prices directly linked to management strategic plans.
3.4.4.3 Both of these approaches consider the effect of the policy on the value and risk to the firm as a whole. As such we would expect these methods to provide significant confidence to third parties. The capital markets approach may not be as satisfactory to regulators, as it may lead to pricing and acceptance of risk at levels which are market-efficient, but place the firm at higher risk of defaulting on policy-holder liabilities (unless that risk is also adequately captured and mitigated).
3.4.4.4 The capital markets approach requires finding an equivalent capital market instrument. This method would be a departure from current market techniques, and finding such an instrument may not be clear, however similar methods have been used successfully in investment funds. The method is both easy to communicate and justify, having no subjective assumptions or detailed calculations (once an appropriate instrument has been found). The incremental marginal capital method requires a consideration of the firm's risk profile both before and after the underwriting which would require a detailed, but easy to use, model. The purpose of the method is easy to communicate; however the results are difficult to understand, discuss, and debate in practice.
3.4.4.5 Practically the marginal capital method is intensive at a policy level but may be most appropriate for catastrophe policies where aggregate monitoring is routine.
3.4.4.6 If the capital instruments are appropriately priced (taking into account the risk profiles) and not excessively driven by liquidity and market supply and demand issues, they should produce appropriate risk prices and hence assist in stabilising
and potentially reducing the required premium to capital ratio. The incremental marginal capital method considers the capital (and hence premium to capital ratio) explicitly so should prove most effective at these objectives.

### 3.4.5 Policy value

3.4.5.1 By consideration of the risks in the policy, the mean of transformed loss methods explicitly embed risk considerations in the underwriting, however they do not consider any correlation or diversification effects to the firm. This results in a price which is fair in terms of the risk presented, but not necessarily fair in terms of the firm. As the price is only dependent on the individual risk profile it may be more stable over time, but is exposed to changes in estimation of extreme results arising from the policy. The CAPM model explicitly considers correlation with the firm's risk profile and hence embeds only that aspect of the risk considerations in underwriting. As such the considerations for this method are almost opposite those of the mean of transformed losses.
3.4.5.2 These methods do produce risk adjusted prices, and as such provide information to management; however as before each considers only one dimension of risk. The technical prices derived are unlikely to be directly linked to management strategic plans.
3.4.5.3 The recent investigations into the CAPM methodology may lead to some discomfort from third-parties surrounding over-use of the method. However the link to financial economics and a detailed technical price should provide some comfort. We expect that the mean of transformed loss method would provide a high degree of comfort due to its explicit consideration of downside risk and information content.
3.4.5.4 None of these methods are particularly easy to use, though IT solutions can overcome these difficulties. We do not consider the methods particularly easy to communicate (we were not able to conceive of appropriate graphics and analogies to replace the formulae, although we did not spend much time on this), or to justify if the results are out of line with expectations.
3.4.5.5 The mean of transformed loss methods are reasonable volatility discriminators, and as such should limit the anomalies which distort the capital to premium ratio and hence return on capital. The CAPM method is essentially a standard deviation method and as such may produce some capital anomalies. Both methods should provide reasonable information to identify opportunities to reduce the capital to premium ratio.

### 3.4.6 Proportional spread

3.4.6.1 Proportional spread methods vary considerably in terms of meeting these objectives based on the underlying risk measurement that they are based on.
3.4.6.2 Mean loss measures do not embed risk considerations at underwriting; the other methods considered do contain a risk analysis at the individual policy level, but do not consider how well these link to the firm's risk. The mean and standard deviation methods are not good discriminators of risk to either the cedant or the firm (unless the risk is small in relation to the cedant or firm) and as such are unlikely to provide "fair" prices. The Value-at-Risk, and Tail-Value-at-Risk methods are better discriminators of risk (especially to the cedant for whom the risk may be relatively larger) and hence may well produce fairer prices. All of the methods are exposed to instability predominantly through changes in the opinions of potential outcomes (especially the extreme outcomes in the case of the Value-at-Risk and Tail-Value-at-Risk methods), but also through high level decisions such as which percentile to use.
3.4.6.3 The technical prices derived by these methods may provide good stand-alone risk information to management (the mean method is unlikely to produce sensible information, as is the standard deviation for heavy-tailed classes) but will not provide clear firm level risk information. Assuming that the method has been set at an appropriate risk measurement, they should produce results according to the management plan.
3.4.6.4 The confidence provided to third parties by these methods depends heavily on the underlying risk measurement. A mean or standard deviation basis would probably not deliver confidence, especially if the capital being allocated has only
been assessed at a high level. The Value-at-Risk method would provide more confidence, but its sensitivity to unreasonable results (such as zero allocations) means that significant practical alterations would be needed which might reduce the confidence in the method. The Tail-Value-at-Risk method would provide the most confidence, and would probably satisfy rating agencies and regulators, but its potential misalignment with the firm's overall objectives may reduce the confidence this method provides to capital providers.
3.4.6.5 The methods are generally easy to use (with the exception of the Value-at-Risk method which would require significant practical adjustments to avoid unreasonable results). The methods are easy to explain, with the Tail-Value-atRisk method being slightly more difficult in practice; however the members have found that, while it is not difficult to discuss the results, the importance and consideration of extreme results (which by their nature have usually not been experienced) can often provide a communicative challenge.
3.4.6.6 The methods are not particularly aligned with the firm's overall capital (the Tail-Value-at-Risk method is arguably more aligned as it considers a wider range of stressed scenarios) and this reduces the effectiveness of the methods at stabilising the capital to premium ratio (and hence the return on capital). The Value-at-Risk and Tail-Value-at-Risk methods may provide information about the volatility and hence proxies for the impact of individual policies on the firm's overall capital to premium ratio, and so help to reduce this ratio (and hence improve expected return on capital).

### 3.4.7 Other allocation methods

3.4.7.1 The other allocation methods (Game Theory, Myers-Read, Equalise Relative Risk, Apply Co-measure, and Insurance Capital as a Shared Asset) generally consider the risk from the individual policy as it relates to the firm's risk (with the exception of the Equalise Relative Risk method which contains a detailed consideration of the risk that the individual policy presents to the firm's assets, but no consideration of diversification) and as such should help to embed risk considerations in underwriting. They are dependent on the other policies written by the firm and hence volatile, which could be seen as unfair to the cedant.

Equalise Relative Risk and Insurance Capital as a Shared Asset contain a consideration of the stand-alone risk and are hence less volatile.
3.4.7.2 The information provided in the technical premium from these methods is indicative of the risk assumed and as such these prices form useful management information. However it is unlikely that the prices generated directly relate to the firm's objectives.
3.4.7.3 We would expect any of these methods would provide significant confidence to third parties, as they should leave the firm with a better understanding (and hence management) of their risk and return profile.
3.4.7.4 The methods are not easy to use. In particular the Game Theory method is highly computationally intensive, and the Co-measure approach has high information processing requirements. The Game Theory and Equalise Relative Risk methods are easy to explain in concept, while Myers-Read was considered difficult (especially as it is particularly unfamiliar in concept compared to common general insurance methods). In general the results from these methods are potentially difficult to interrogate, understand, and discuss. The Equalise Relative Risk and Insurance Capital as a Shared Asset methods were felt slightly better in practice in this regard.
3.4.7.5 All of these methods are reasonably good risk differentiators and as such will help to stabilise the capital to premium ratio, and help identify actions to reduce this ratio (with knock-on effects for stabilising and enhancing return on capital). The Apply Co-measure method, being specifically related to the firm's overall risk profile is very closely linked with these objectives.

### 3.5 Selected methods

3.5.1.1 Based on the above grid, and the discussions of the working party, we identified the following methods as worthy of impact analysis.
(1) Incremental marginal capital
(2) Mean of transformed loss - proportional hazard transform
(3) Mean of transformed loss - Wang transform
(4) Proportional spread - Mean
(5) Proportional spread - Tail-Value-at-Risk
(6) Game theory
(7) Myers-Read
(8) Equalise relative risk
(9) Apply co-measure (Tail-Value-at-Risk)
(10) Insurance capital as a shared asset

## 4. Impact Analysis

### 4.1 Introduction

4.1.1.1 In this section we compare the performance of different capital allocation methods on two sets of theoretical policy data. The sets correspond to credit risk and property direct and facultative (D\&F) business. For each policy there are 10,000 simulated outcomes. All outcomes are from the same 10,000 events so we can model the effects of correlations.
4.1.1.2 The methods considered are those identified in Section 3.5.
4.1.1.3 The intention of this section is not to identify the "best" method. Rather to help the reader understand the relative merits of each.

### 4.2 Aggregate Capital and Premium

4.2.1.1 For each set of policies the overall capital (and hence premium) is calculated. This investigation considers different ways of allocating this aggregate capital.
4.2.1.2 The aggregate capital is set using 99.5 ${ }^{\text {th }}$ percentile of the overall costs distribution (the $99.5^{\text {th }}$ Value-at-Risk (VaR)). The aggregate premium is then calculated using the formula:

$$
\text { Premium }=\text { Expected Claims }+\left(99.5^{\text {th }} \text { VaR }- \text { Premium }\right) * \text { Return on Capital }
$$

4.2.1.3 Return on capital is taken as $20 \%$.

### 4.3 Practical Issues in Programming the methods

### 4.3.1 Mean Proportional spread.

4.3.1.1 For the mean proportional spread, the methodology applied was simply to take the expected loss for the policy being priced and divide this by the target loss ratio. This was the aggregate book target loss ratio derived as discussed in section 4.2
4.3.1.2 As such the methodology had no parameterisation or calibration process required, had low data requirements for each policy's risk profile (only needing the mean losses on the policy), and was fast to apply. It was also reliable, in the sense that it did not derive any obviously flawed premiums which necessitated complicating the methodology to deal with special cases.

### 4.3.2 Tail-Value-at-Risk (TVaR) proportional spread

4.3.2.1 For the TVaR proportional spread, the methodology applied was to take the TVaR of the costs distribution for each policy (that is, without deduction of premium or expected claims), scale this according to a factor (the same factor for each policy within the dataset, but a different factor was used for each dataset and for each different percentile at which this methodology was applied), and then apply the pricing formula in section 4.2 , using this scaled TVaR as the stressed liability (i.e. instead of the $99.5 \% \mathrm{VaR}$ ), but using the same Return on Capital as the aggregate book.
4.3.2.2 The methodology required calibration so that the aggregate book premium reconciled to the target derived as discussed in section 4.2. This was the point of the scaling factor. This factor was calculated as the aggregate book 99.5\% VaR divided by the sum of the individual policy TVaRs. As such this parameter could only be derived with knowledge of all of the policies to be written in the period under consideration. In practice a proxy would be likely to be derived, and section 4.5 shows the sensitivity of the aggregate book's results to uncertainty in this parameter.
4.3.2.3 With this implementation the minimum stressed liability requirement for any policy is zero, although this could only happen if the policy did not have any probability of producing a loss (as some of our policies did); in this case the technical premium derived would be zero, as intuitively seems reasonable. The stressed liability may be very close to zero even where there is some potential of losses in the tail of the distribution however, especially for the methodology based on the $80^{\text {th }}$ percentile TVaR. This translates to a technical premium which gives an expected loss ratio for the policy of above $100 \%$ (up to a maximum of $100 \%$ plus the required rate of return).
4.3.2.4 In our analysis we generated a number of policies with an expected loss ratio above $100 \%$. These are valid results given the method, as the writing of these policies actually decreases the capital requirement (they are not producing costs, but still producing premiums in stressed scenarios for the aggregate book), and hence the same target return on capital can be obtained from lower profitability. In practice we do not believe that current capital modelling methodologies would necessarily recognise and allow for this decrease in capital requirement, and arguments could certainly be made over whether this was a fair premium for the risk ceded. We would expect a practical pricing implementation to restrict such results, and this would add an additional level of complexity to the calibration of this pricing formula to generate an aggregate premium for the book at the target level.
4.3.2.5 Once the scaling factor had been derived, the methodology was relatively simple to apply, requiring only knowledge of the risk profile of the policy under consideration; however knowledge of the full distribution for the individual policy was required to calculate the TVaR value itself. The method was simple to apply in standard spreadsheet packages (although more advanced methods may be necessary if the number of simulations increases).
4.3.2.6 For the Targeted TVaR methodology, the percentile was selected so that a scaling factor was not required. This removes the sensitivity of the method to the selection of the percentile (demonstrated by the differing results for these methods), but does so by making an implicit selection which may cause the pricing to exhibit certain biases in relative pricing, specific to the percentile chosen. Again, this selection is likely to only be approximate due to lack of certainty in knowledge of the business that will be underwritten.
4.3.2.7 Due to the effect of diversification, this method would price an aggregate policy (for example an aggregate policy covering both Employers Liability and Public Liability risks) differently than the sum of the prices for each component if they were priced separately.

### 4.3.3 Mean of transformed loss

4.3.3.1 In practice, the mean of transformed loss methods set the premium as the weighted average of all the simulation outputs, with more weight being given to the more adverse outcomes. The exact weighting functions varied for the different methods, but were easy to code into standard spreadsheet packages.
4.3.3.2 Both mean of transformed loss methodologies have one parameter. For the Wang method this relates to the shift in the exceedance probability applied, for the PH method it applies to the exponential increase in weight to more severe outcomes. For each of these methods we used these as free parameters to alter the aggregate book premium, so that this reconciled with the target derived as discussed in section 4.2. The derivation of this parameter required knowledge of the distributions of all policies to be written, at the time of writing the first policy, but once this parameter (or a proxy) was derived the pricing methodology can be applied at the point of underwriting with no more information requirements than the individual policies distribution.
4.3.3.3 The full distribution of costs on the policy was required for the processing of this method, and needed to be sorted (independently for each policy) before being utilised (to identify the more adverse outcomes), although this didn’t provide a material practical barrier, and in general pricing would be applied to only one policy at a time.
4.3.3.4 Due to the effect of diversification (and their impact on the ordering of the adverse outcomes), this method would price an aggregate policy (for example an aggregate policy covering both Employers Liability and Public Liability risks) differently than the sum of the prices for each component priced separately.

### 4.3.4 Incremental marginal capital

4.3.4.1 In theory, an incremental marginal capital approach would price each policy as it adds to the capital need of the on-risk book at the time of underwriting. Potentially (depending on the exact implementation) this could penalise the first policies to be written. While we felt that this could be a useful risk metric for an
insurance firm to consider and record (as it measures the actual utilisation of capital over time), for the purposes of the working party we felt that applying a specific ordering would be likely to distort the results and make comparisons difficult, as a different ordering could produce materially different results. Instead, the approach we tested was to allocate the stressed liabilities (i.e. the term replacing the $99.5 \% \mathrm{VaR}$ in the pricing formula in section 4.2) according to the average of two methods; setting stressed liabilities for each policy on a first-in basis, and setting stressed liabilities for each policy on a last-in basis. We felt this approach would reflect both the risk being assumed from each policy, but also the diversification effects.
4.3.4.2 There were no parameters to derive for this method, however the stressed liabilities derived for each policy (being the average of the value derived under each method) did not sum to the aggregate book 99.5\% Value-at-Risk, and hence required rescaling to ensure that the method priced the aggregate book consistently with other methods.
4.3.4.3 With this implementation the minimum stressed liability requirement for any policy is zero. This translates to a technical premium which gives an expected loss ratio for the policy of $100 \%$ plus the required rate of return (from the formula in section 4.2).
4.3.4.4 In our analysis we generated a number of policies with an expected loss ratio above $100 \%$. The method allows this as the writing of these policies actually decreases the capital requirement (as they are not producing losses in stressed scenarios for the aggregate book, but still produce premiums), and hence the same target return on capital can be obtained from lower profitability. In practice we do not believe that current capital modelling methodologies would necessarily recognise and allow for this decrease in capital requirement, and arguments could certainly be made over whether this was a fair premium for the risk ceded. We would expect a practical pricing implementation to restrict such results, and this would add an additional level of complexity to the calibration of this pricing formula to generate an aggregate premium for the book at the target level.
4.3.4.5 The application of the method required knowledge of the aggregate book at the point of pricing of any policy. As we are including a first-in element, diversification would mean that an aggregate policy would be priced differently than the sum of each of its components; however this would not be a problem with a true theoretical implementation, as the later processed individual sections would have reduced profit requirements due to their diversification with the earlier segments, and ultimately the incremental capital for the aggregate policy would be the same whether that policy was added in one piece or in separate sections.

### 4.3.5 Game theory

4.3.5.1 An implementation of the full theoretical Game Theory model is impractical - for the Credit Risk class, if one aggregate portfolio could be priced per second this would take approximately $4 \times 10^{51}$ times the current age of the universe to complete! Instead we tried to approximate this method using the following algorithm:
(1) Define an ordering for the policies at random,
(2) Calculate the stressed liability requirement for each policy using a true theoretical marginal capital approach (i.e. assess the change in the cumulative aggregate book $99.5 \% \mathrm{VaR}$ from adding each policy in turn),
(3) Perturb the ordering such that first policy becomes the last, the second becomes the first, the third becomes the second and so on,
(4) Repeat steps (2) and (3) until the ordering returns to the initial ordering (i.e. repeat the process once with each policy in the first-in position),
(5) Set the stressed liability requirement for a policy as the average requirement for that policy over all of the perturbed orderings,
(6) Choose another random ordering and repeat steps (2)-(5), repeat until three random orderings have been fully processed, and average over the results from each.
(7) Set premium according to the formula in 4.2.
4.3.5.2 As such, for the Credit Risk dataset, this method required 690 full implementations of an incremental marginal capital approach, and 158,700 individual policy marginal capital calculations. Even at this level the results were not perfectly stable, with deviations between the final allocated stressed liability, and the stressed liability allocated from just one full processing of one random ordering of up to $50 \%$ of the average allocation. As such, given the importance of the pricing process, if such a method were implemented in practice, more than three random orderings should be processed. However even at this level the method was slow and unwieldy to implement, although a purpose built approach may improve speed and ease of application.
4.3.5.3 With this implementation the minimum stressed liability requirement for any policy is zero. This translates to a technical premium which gives an expected loss ratio for the policy of $100 \%$ plus the required rate of return (from the formula in section 4.2).
4.3.5.4 In our analysis we generated one policy from each dataset with an expected loss ratio above $100 \%$. The method allows this as the writing of these policies actually decreases the capital requirement (as they are not producing losses in stressed scenarios for the aggregate book, but are still producing premium), and hence the same target return on capital can be obtained from lower profitability. In practice we do not believe that current capital modelling methodologies would necessarily recognise and allow for this decrease in capital requirement, and arguments could certainly be made over whether this was a fair premium for the risk ceded. We would expect a practical pricing implementation to restrict such results, and this would add an additional level of complexity to the calibration of this pricing formula to generate an aggregate premium for the book at the target level.
4.3.5.5 There were no parameters of this method, and it automatically generated the target aggregate book premium.
4.3.5.6 The application of the method required the knowledge of the full distribution of all policies (and their joint distributions) at the time of underwriting any policy. We were not able to think of a credible proxy for this data requirement.
4.3.5.7 The method would price aggregate policies equal to the sum of each of their components.

### 4.3.6 Myers-Read

4.3.6.1 The methodology applied for the Myers-Read calculation was the simplified method described in Butsic.
4.3.6.2 The method required additional assumptions for asset volatility, and the correlation between assets and liabilities. Essentially the Myers-Read method can recognise that, for classes with a material asset requirement (being the sum of premium and capital) the firm's profitability is highly dependent on market risk, and this risk should be represented in the pricing formula. This risk is (very roughly) allocated in accordance with the expected claims. For consistency with the other methods we considered that this risk was already included within the target aggregate book premium, and hence set the asset volatility to zero. It should be noted that this means we are not making full use of the power of this method.
4.3.6.3 We found some of the calculations to be complex and difficult to assess for reasonableness meaning that if the method is not carefully implemented it could contain errors; however the calculations can be easily embedded in a pricing spreadsheet. Once this spreadsheet has been created the method is easy to implement.
4.3.6.4 It is possible for the method to produce technical premiums below the expected losses (i.e. an expected loss ratio above $100 \%$ ); this happens if the policy is relatively uncorrelated with the aggregate book (it does not require negative correlation; low positive correlation may be sufficient). Indeed, it is possible for
this method to produce negative technical premiums, which is clearly an unreasonable result. This happened for twelve of our Property D\&F policies. This is less likely to be a problem for more stable books.
4.3.6.5 The method does not require any parameterisation or calibration, but at the point of underwriting requires the mean and coefficient of variation of the losses from the policy being prices, as well as correlation of that policy's losses with the total book. As such knowledge of the full distribution of losses from that policy and the aggregate book, and the full joint distribution is needed at the point of underwriting any policy, although it may be possible to derive sensible proxies.
4.3.6.6 Due to the impact of diversification aggregate policies would be priced lower than the sum of the prices for their component sections, however we would expect this to be less of an issue than for, say, the TVaR method, due to inclusion of the correlation parameter.

### 4.3.7 Equalise relative risk.

4.3.7.1 For the Equalise Relative Risk method we need to allocate the overall book premium to each policy such that the expected policy deficit divided by premium (risk measure per exposure) is equal for all policies.
4.3.7.2 We used a double optimisation (bisection algorithm) in excel The overall method is given below:
(1) Initial guess at risk measure per exposure;
(2) Calculate premium for each policy to meet risk measure / exposure. Using a bisection algorithm;
(3) Sum up all the premiums;
(4) Go back to (1) using a refined guess at risk measure per exposure given from the bisection algorithm.
4.3.7.3 The above algorithm when run will (theoretically) guarantee that the individual policy premiums sum to the required target aggregate book premium. However
we do require the two initial estimates of risk measure per exposure to be either side of the actual value to guarantee convergence. Of course setting these values arbitrarily large will ensure this but will also make the algorithm prohibitively slow. This will especially be the case if a large number of policies are used and/or the policies have vastly different distributions. Of course there may be better ways of programming the algorithm which deals with this problem.
4.3.7.4 Once the risk measure per exposure has been set by this process, the method is quick and easy to apply. The method will always produce technical premiums above expected costs, and produces additive premiums (that is the premium of a book is equal to the sum of the premiums of each policy).

### 4.3.8 Co-measure

4.3.8.1 For the Co-measure methodology we applied a TVaR Co-measure, with the TVaR percentile calibrated so that the aggregate book premium was equal to the target. We allocated the stressed liabilities (the tail of the losses distribution) and then set the premium using the formula in section 4.2.
4.3.8.2 The main parameter estimate in the method was the percentile of the TVaR measure, and this was calibrated so that the target aggregate premium was achieved. This required some iteration, but only required the aggregate book distribution as data in this process so was not difficult.
4.3.8.3 With this implementation the minimum stressed liability requirement for any policy is zero. This translates to a technical premium which gives an expected loss ratio for the policy of $100 \%$ plus the required rate of return (from the formula in section 4.2).
4.3.8.4 In our analysis we generated a number of policies with an expected loss ratio above $100 \%$. The method allows this as the writing of these policies actually decreases the capital requirement (as they are not producing losses in stressed scenarios for the aggregate book, but are still generating premium), and hence the same target return on capital can be obtained from lower profitability. In practice we do not believe that current capital modelling methodologies would necessarily recognise and allow for this decrease in capital requirement, and arguments could
certainly be made over whether this was a fair premium for the risk ceded. We would expect a practical pricing implementation to restrict such results, and this would add an additional level of complexity to the calibration of this pricing formula to generate an aggregate premium for the book at the target level.
4.3.8.5 To apply the method, the full joint distribution of the policy being underwritten and the aggregate book was required. As such the loss distribution of all policies was needed at the time of underwriting the first policy.
4.3.8.6 The method derives additive premiums, i.e. an aggregate policy would be priced equal to the sum of the premiums on each of its sections.

### 4.3.9 Insurance Capital as a Shared Asset

4.3.9.1 For the Insurance Capital as a Shared Asset method, we attempted to implement two versions of the method i.e. top down evaluation by applying the proposed approach on an entire portfolio mix and bottom up evaluation by simply applying the proposed approach on an individual contract. Both versions of the method can be implemented on an Excel spreadsheet. The methods can be found in Mango (2005).
4.3.9.2 However we found that the bottom up approach performed less well at differentiating policies, perhaps because this method was more dominated by the rental capital (which is allocated based on expected cost), and does not reflect the correlation between policies and the aggregate book risk profile. We have hence only presented the top-down approach in these results.
4.3.9.3 Again we have set a target loss ratio for both versions/formats in order to achieve the capital required at 99.5th.
4.3.9.4 We then "goal-seek" both versions in order to arrive at zero value on the Economic Value Added (EVA) metric. In this context, the EVA is the supernormal profit expected to be made by the policy or book, over and above the profit required to satisfy the return on capital; setting this at zero implies that the policy has been "correctly" priced to achieve the return on capital target. For bottom up evaluation, this is done by initially setting a nominal value for the
opportunity cost. A macro is written to iteratively seek the appropriate value for the opportunity cost in order to achieve the overall target premium. For the top down evaluation, the percentage of the consumption charge (i.e. capital call cost) at portfolio level is evaluated in a similar fashion.
4.3.9.5 Broadly, the top-down evaluation follows these steps;
(1) Determine the aggregate profit required;
(2) Split this profit into a rental requirement and a consumption requirement. In our results we have assumed that this required profit is based on a risk-free rate applied to the aggregate capital. We understand that in practice, this split is normally interpreted as a key risk management process, and could have material implications on the kinds of business written.
(3) Allocate this rental profit requirement to each policy. Again there would appear to be flexibility in this allocation, and, given the nature of the method, it may be more appropriate to use an allocation which reflects the likely change in the aggregate rental capital requirement (for instance Myers-Read or a Co-measure approach). However we have followed the approach set out in the original paper and allocated this back based on the expected costs.
(4) Calculate the aggregate book capital consumption for each simulation. The consumption charge follows a utility cost of this capital utilisation. We have used a simple utility function applying:
(a) Zero charge if the simulated losses are less than the expected losses;
(b) A percentage, p , multiplied by the excess of the simulated costs over the expected costs, up to a level of simulated costs being twice the expected costs;
(c) Four times p times the excess of the simulated costs over twice the expected costs (if greater).
(d) The percentage, p , was solved so that the expected consumption cost charge was equal to the aggregate profit requirement less the expected profit requirement.
(5) In this way the consumption charge is designed to reflect a higher risk-aversion to more serious outcomes, similarly to the weighted expectation approaches. In practice, our distributions are very skew, and if they produce costs, often produce more than twice the expected costs, so it may well be that the performance of this method could be improved by varying this utility function; however we did not have time to investigate this further.
(6) This consumption charge defines capital as the excess of costs over expected costs, rather than premium. This is different to some of the other methods we investigated (although either set of methods could be adjusted to a consistent basis). This approach does avoid recursion in the consumption calculation however, which simplifies this method, as the utility percentage p also required solving.
(7) The aggregate consumption charge was then allocated to policies. This allocation was performed on a simulation by simulation basis; allocating by each policy's individual capital consumption on that simulation. In this way this element of the pricing method is similar to a Co-measure approach, albeit based on a utility based riskmeasure.
(8) The premium for each policy is then set as the expected costs, plus the rental charge allocated to that policy, plus the expected consumption charge allocated to that policy.
4.3.9.6 With the approach we followed the minimum premium charged for any policy would produce a loss ratio below $100 \%$ (as it would cover the expected costs plus the rental capital allocated; no negative consumption capital was allocated).
4.3.9.7 To apply the method, the full joint distribution of the policy being underwritten and the aggregate book was required. As such the loss distribution of all policies was needed at the time of underwriting the first policy.
4.3.9.8 The method derives additive premiums, i.e. an aggregate policy would be priced equal to the sum of the premiums on each of its sections.

### 4.4 Data

4.4.1.1 We used two sets of policy data, corresponding to credit risk and property direct and facultative. For each policy there are 10,000 simulated outcomes. All outcomes are from the same 10,000 events so correlations may be modelled. Each set of policies were split into a number of groups. Each group having policies with similar characteristics (mean \& standard deviation of losses). The data sets are summarised in the tables and graphs below. The graphs show the mean, standard deviation, and probability of a claim on each policy. The standard deviation should be interpreted as the green line only. That is the top of the green line indicates the mean plus one standard deviation. The policies are in group order.

| Data Summary - Credit Risk |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | :---: | :---: | :---: |
| Group | No. of <br> Policies |  |  |  | Average Mean <br> Loss | Average St Dev <br> of Loss |
|  | 1 | 11 | 11.3 |  |  |  |
|  | 25 | 0.0 | 173.1 |  |  |  |
|  | 2 | 0.9 | 1.0 |  |  |  |
|  | 25 | 60.3 | 41.1 |  |  |  |
|  | 20 | 298.6 | 246.8 |  |  |  |
|  | 5 | 50 | 152.4 |  |  |  |
|  | 48 | 205.9 | 375.5 |  |  |  |
|  | 7 | 51 | $\mathbf{1 4 8 . 3}$ |  |  |  |
| Total | $\mathbf{2 3 0}$ |  | 404.7 |  |  |  |


| Data Summary - D\&F Property |  |  |  |
| :---: | :---: | :---: | :---: |
| Group | No. of Policies | Average Mean Loss | Average St Dev of Loss |
| 1 | 21 | 0.1 | 5.6 |
| 2 | 12 | 0.5 | 14.8 |
| 3 | 30 | 1.8 | 27.8 |
| 4 | 15 | 5.0 | 46.7 |
| 5 | 9 | 15.0 | 82.8 |
| Total | 87 | 3.1 | 29.6 |

## Credit Risk Dataset



Property D\&F Dataset


### 4.5 Analysis

### 4.5.1 Comparison of results

4.5.1.1 As each dataset has been grouped into sections with similar attributes, we considered the aggregate premium for each group, as a practical analysis of the impact of the methods. We reiterate however that the technical premiums were derived at individual policy level, and then aggregated into these groups for review, rather than having been derived at this level.
4.5.1.2 In particular, we considered the expected profit for each group (that is the aggregate premium from each policy in that group, which varied according to the pricing method, less the sum of the expected losses from each policy in the group, which is fixed irrespective of the pricing method), and the expected loss ratio from each group (that is the sum of the expected losses from each policy in that group, divided by the aggregate premium from each policy in the group).

### 4.5.2 Credit Risk Dataset Results

4.5.2.1 The table below shows the technical premium for each group of the Credit Risk dataset under each of the pricing methods.

| Technical Premium by Pricing Method Credit Risk Data Set |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Group | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Total |
| Method |  |  |  |  |  |  |  |  |
| Mean of transormed loss - PH | 839 | 5 | 301 | 3,607 | 25,976 | 16,635 | 20,633 | 67,996 |
| Mean of transformed loss - Wang | 513 | 2 | 128 | 3,187 | 27,071 | 16,259 | 20,837 | 67,996 |
| Mean proportional | 249 | 0 | 47 | 2,406 | 29,770 | 14,584 | 20,940 | 67,996 |
| 80\% TVaR propotional | 275 | 1 | 52 | 2,657 | 28,073 | 16,015 | 20,924 | 67,996 |
| 95\% TVaR propotional | 447 | 1 | 85 | 3,701 | 25,920 | 17,015 | 20,827 | 67,996 |
| 99\% TVaR propotional | 1,151 | 2 | 218 | 4,177 | 24,697 | 17,277 | 20,474 | 67,996 |
| Targeted TVaR propotional | 422 | 1 | 80 | 3,746 | 25,451 | 17,313 | 20,983 | 67,996 |
| Incremental marginal capital | 1,092 | 3 | 62 | 4,300 | 23,542 | 18,784 | 20,213 | 67,996 |
| Game theory | 1,442 | 0 | 152 | 3,621 | 23,184 | 19,358 | 20,238 | 67,996 |
| Myers-Read | 2,428 | 5 | 563 | 4,563 | 17,871 | 22,401 | 20,165 | 67,996 |
| Equalise relative risk | 519 | 1 | 101 | 3,736 | 26,076 | 17,501 | 20,062 | 67,996 |
| Apply co-measure (TVaR) | 1,621 | 4 | 336 | 3,575 | 23,355 | 18,818 | 20,287 | 67,996 |
| Insurance capital as a shared asset | 721 | 1 | 142 | 3,046 | 25,911 | 17,328 | 20,847 | 67,997 |

4.5.2.2 The table below shows the expected profits for each group of the Credit Risk dataset under each of the pricing methods.

| Expected Profits by Pricing Method Credit Risk Data Set |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Group | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Total |
| Method |  |  |  |  |  |  |  |  |
| Mean of transormed loss - PH | 714 | 5 | 277 | 2,401 | 11,047 | 9,321 | 10,132 | 33,896 |
| Mean of transformed loss - Wang | 388 | 1 | 104 | 1,980 | 12,141 | 8,945 | 10,336 | 33,896 |
| Mean proportional | 124 | 0 | 23 | 1,199 | 14,841 | 7,270 | 10,439 | 33,896 |
| 80\% TVaR propotional | 150 | 0 | 28 | 1,451 | 13,144 | 8,701 | 10,423 | 33,896 |
| 95\% TVaR propotional | 323 | 1 | 61 | 2,494 | 10,991 | 9,702 | 10,326 | 33,896 |
| 99\% TVaR propotional | 1,026 | 2 | 194 | 2,970 | 9,767 | 9,963 | 9,973 | 33,896 |
| Targeted TVaR propotional | 297 | 1 | 56 | 2,539 | 10,522 | 10,000 | 10,482 | 33,896 |
| Incremental marginal capital | 967 | 3 | 39 | 3,093 | 8,612 | 11,470 | 9,712 | 33,896 |
| Game theory | 1,317 | 0 | 128 | 2,415 | 8,255 | 12,044 | 9,737 | 33,896 |
| Myers-Read | 2,303 | 5 | 539 | 3,357 | 2,941 | 15,088 | 9,664 | 33,896 |
| Equalise relative risk | 394 | 1 | 77 | 2,530 | 11,146 | 10,187 | 9,560 | 33,896 |
| Apply co-measure (TVaR) | 1,497 | 3 | 313 | 2,369 | 8,426 | 11,504 | 9,785 | 33,896 |
| Insurance capital as a shared asset | 596 | 1 | 118 | 1,840 | 10,982 | 10,015 | 10,345 | 33,897 |

4.5.2.3 The chart on the following page shows the expected loss ratio for each group of the Credit Risk dataset under each of the pricing methods.

## Expected Loss Ratio if policies priced at Technical Premium <br> Credit Risk Dataset


4.5.2.4 These results show consistency between the methods, in that different methods tend to deviate from the aggregate book target loss ratio in the same direction for a particular group of policies. That is, the methods agree that the expected loss ratio for groups one to four, and group six should be priced at a lower loss ratio than the aggregate book; group five should be priced at a higher expected loss ratio, and group seven should be priced at around the aggregate book target loss ratio.
4.5.2.5 In general there is also a high degree of agreement between the relative materiality of the deviations, for example groups one to three are generally priced at a lower loss ratio than group four.
4.5.2.6 Where the methods disagree is in the extent of the deviation appropriate for each group. This is most obvious for groups one to three, which show a range of expected loss ratios from $5 \%$ to $50 \%$, and hence a tenfold difference in the average premium in these groups! Although less material, this is also a problem for groups four to six. Group seven does not seem to be exposed to this volatility, perhaps as it represents a large bulk of attritional claims, although it is unclear why it is less exposed than group five.
4.5.2.7 The pattern of rates differs for the Incremental marginal capital method. The results for group two are largely spurious with only two policies in that group producing any probability of a loss, and those only producing a claim on one simulation each, but the results for group three are credible. It is not clear why the Incremental method tends to price this group at lower rates (relative to the comparison between methods on group one, say), although this group is slightly less correlated than the other groups.
4.5.2.8 Of the remaining methods, the Myers-Read method tends to differentiate between the groups the most, followed by the Co-measure, $99 \%$ TVaR and Transformed loss methods. The Shared Asset, $95 \%$ and targeted TVaR, and the Equalise relative risk methods come next, followed by the $80 \%$ TVaR and the Mean proportional (which does not differentiate between groups at all, of course).

### 4.5.3 Property D\&F Dataset Results

4.5.3.1 The table below shows the technical premium for each group of the Property D\&F dataset under each of the pricing methods.

| Technical Premium by Pricing Method Property D\&F Data Set |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Group | 1 | 2 | 3 | 4 | 5 | Total |
| Method |  |  |  |  |  |  |
| Mean of transormed loss - PH | 44 | 78 | 428 | 406 | 513 | 1,469 |
| Mean of transformed loss - Wang | 28 | 57 | 384 | 415 | 586 | 1,469 |
| Mean proportional | 14 | 32 | 293 | 401 | 729 | 1,469 |
| 80\% TVaR propotional | 14 | 32 | 293 | 401 | 729 | 1,469 |
| 95\% TVaR propotional | 14 | 34 | 309 | 403 | 709 | 1,469 |
| 99\% TVaR propotional | 22 | 51 | 436 | 483 | 478 | 1,469 |
| Targeted TVaR propotional | 15 | 35 | 321 | 412 | 685 | 1,469 |
| Incremental marginal capital | 7 | 17 | 283 | 515 | 648 | 1,469 |
| Game theory | 15 | 37 | 299 | 422 | 697 | 1,469 |
| Myers-Read | 14 | 42 | 362 | 416 | 635 | 1,469 |
| Equalise relative risk | 17 | 39 | 337 | 417 | 659 | 1,469 |
| Apply co-measure (TVaR) | 13 | 35 | 327 | 410 | 685 | 1,469 |
| Insurance capital as a shared asset | 13 | 32 | 296 | 398 | 730 | 1,469 |

4.5.3.2 The table below shows the expected profits for each group of the Property D\&F dataset under each of the pricing methods.

| Expected Profits by Pricing Method <br> Property D\&F Data Set |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Group | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | Total |
| Method |  |  |  |  |  |  |
| Mean of transormed loss - PH | 42 | 72 | 373 | 331 | 378 | $\mathbf{1 , 1 9 7}$ |
| Mean of transformed loss - Wang | 25 | 51 | 330 | 340 | 451 | $\mathbf{1 , 1 9 7}$ |
| Mean proportional | 11 | 26 | 239 | 327 | 594 | $\mathbf{1 , 1 9 7}$ |
| 80\% TVaR propotional | 11 | 26 | 239 | 326 | 594 | $\mathbf{1 , 1 9 7}$ |
| 95\% TVaR propotional | 12 | 28 | 255 | 328 | 574 | $\mathbf{1 , 1 9 7}$ |
| 99\% TVaR propotional | 19 | 45 | 382 | 408 | 343 | $\mathbf{1 , 1 9 7}$ |
| Targeted TVaR propotional | 13 | 29 | 267 | 338 | 550 | $\mathbf{1 , 1 9 7}$ |
| Incremental marginal capital | 5 | 11 | 228 | 440 | 512 | $\mathbf{1}, \mathbf{1 9 7}$ |
| Game theory | 12 | 31 | 245 | 347 | 562 | $\mathbf{1 , 1 9 7}$ |
| Myers-Read | 11 | 36 | 308 | 342 | 500 | $\mathbf{1 , 1 9 7}$ |
| Equalise relative risk | 14 | 33 | 282 | 343 | 524 | $\mathbf{1 , 1 9 6}$ |
| Apply co-measure (TVaR) | 10 | 29 | 272 | 335 | 549 | $\mathbf{1 , 1 9 7}$ |
| Insurance capital as a shared asset | 10 | 27 | 241 | 323 | 595 | $\mathbf{1 , 1 9 7}$ |

4.5.3.3 The chart on the following page shows the expected loss ratio for each group of the Property D\&F dataset under each of the pricing methods.

4.5.3.4 Similarly to the Credit Risk dataset, the methods generally agree on the pattern of pricing, requiring higher rates for groups one and two, and lower rates for group five in particular.
4.5.3.5 While virtually all of the methods price group five at an expected loss ratio above the aggregate book target of $18.5 \%$, there is a great deal of spread in the expected loss ratios for groups one and two in particular.
4.5.3.6 The Myers-Read and Co-measure approaches, actually price group one at or above the aggregate book loss ratio, and above the expected loss ratio for group two on the same method. These methods take account of correlations between policies, and this may be explained as group one is generally less correlated to the other groups. These methods have been able to identify that feature and adjust premium rates accordingly.
4.5.3.7 The Game Theory method reacts in a similar way, but in a much more muted fashion, whereas the Incremental Marginal Capital method is highly volatile to those correlations. It was not clear to the working party why there should be such a disparity.
4.5.3.8 For the remainder of the methods, those that price group one at the highest rates tend to price group five at the lowest rates and vice versa. The transformed loss methods tend to differentiate between the groups the most, followed by the $99 \%$ TVaR method. All of these methods are highly geared to the tail risk of the individual policies which explains the discrimination seen in these results. Next come the Equalise Relative Risk, targeted and 95\% TVaR and Shared Asset methods, with the $80 \%$ TVaR and Mean proportional differentiating between the groups the least.

### 4.5.4 Summary

4.5.4.1 The results are generally encouraging, as they suggest that any of the methods would help to satisfy a few of the key objectives; to provide enhanced management information, and through the identification of policies which are producing surplus profits, or deficits, to allow the book to be managed to improve short and long term profitability.
4.5.4.2 In general the Myers-Read, Co-measure, $99 \%$ TVaR and Transformed Loss method appear to differentiate between policies the most, followed by the Equalise relative risk, 95\% and targeted TVaR, Game Theory and Shared Asset methods, with the $80 \%$ TVaR and Mean Proportional methods differentiating the least. Of these the first group are heavily dominated by extreme tail events, the second group are affected by a wider range of downside outcomes, and the last group take account of a large proportion of the distribution. Of the methods tested, the Myers-Read, Co-measure and Game Theory results are also affected by the correlation between the policies.
4.5.4.3 There does seem to be a tendency for the Incremental Marginal Capital method to be much more reactive to correlations than any other method, with this feature dominating its pattern of results. This may be a result of the method's heavy weighting to the "last-in" results, and could be considered unfair.
4.5.4.4 Given the volatility in these technical premiums, it would appear extremely hazardous to rely entirely upon one technical method. Rather the contrast between the different levels for different methods provides additional information which could be of benefit in the underwriting process.
4.5.4.5 Consideration of these high level results suggest there may be material benefits in the inclusion of detailed technical calculation of the risk premium, however there are implicit dangers. Many of these methods rely on the same data requirements, and are easy to embed in quick calculation tools and hence the calculation of a few of these methods would seem practical, and may be a valuable input into the underwriting process, but the use of any of these methods as a black box mechanical pricing process themselves may be highly unfavourable.

### 4.5.5 Aggressive Competitor

4.5.5.1 In order to understand the potential impact on a firm of using one of these pricing methods, we considered the impact of a specialist competitor aggressively targeted a subset of the business. In separate tests we assumed that each group unexpectedly failed to renew and investigated the impact on the results of the residual aggregate book. We did not adjust the premiums charged for the
remaining policies; in particular these were still dependent on parameters designed to achieve aggregate book performance if all policies were written.
4.5.5.2 The following tables show, separately for each of the two datasets, the impact on the Return on Capital of such a situation assuming that the business had been priced according to each method.

| Change in Expected Return on Capital following non-renewal of a Group Credit Risk Dataset |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Group | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Average movement |
| Method |  |  |  |  |  |  |  |  |
| Mean of transormed loss - PH | +0.4\% | -0.0\% | -0.1\% | -0.1\% | -2.3\% | +3.0\% | -0.1\% | +0.8\% |
| Mean of transformed loss - Wang | +0.6\% | -0.0\% | -0.0\% | +0.2\% | -3.3\% | +3.4\% | -0.3\% | +1.1\% |
| Mean proportional | +0.8\% | -0.0\% | +0.1\% | +0.8\% | -5.7\% | +5.4\% | -0.4\% | +1.9\% |
| 80\% TVaR propotional | +0.8\% | -0.0\% | +0.1\% | +0.6\% | -4.2\% | +3.7\% | -0.4\% | +1.4\% |
| 95\% TVaR propotional | +0.7\% | -0.0\% | +0.0\% | -0.2\% | -2.2\% | +2.5\% | -0.3\% | +0.8\% |
| 99\% TVaR propotional | +0.1\% | -0.0\% | -0.1\% | -0.5\% | -1.1\% | +2.2\% | +0.1\% | +0.6\% |
| Targeted TVaR propotional | +0.7\% | -0.0\% | +0.0\% | -0.2\% | -1.8\% | +2.2\% | -0.4\% | +0.8\% |
| Incremental marginal capital | +0.2\% | -0.0\% | +0.0\% | -0.6\% | -0.0\% | +0.5\% | +0.3\% | +0.2\% |
| Game theory | -0.1\% | -0.0\% | -0.0\% | -0.1\% | +0.3\% | -0.1\% | +0.3\% | +0.1\% |
| Myers-Read | -0.8\% | -0.0\% | -0.3\% | -0.8\% | +5.6\% | -3.3\% | +0.4\% | +1.6\% |
| Equalise relative risk | +0.6\% | -0.0\% | +0.0\% | -0.2\% | -2.4\% | +2.0\% | +0.5\% | +0.8\% |
| Apply co-measure (TVaR) | -0.2\% | -0.0\% | -0.1\% | -0.1\% | +0.1\% | +0.5\% | +0.3\% | +0.2\% |
| Insurance capital as a shared asset | +0.5\% | -0.0\% | -0.0\% | +0.3\% | -2.2\% | +2.2\% | -0.3\% | +0.8\% |

Change in Expected Return on Capital following non-renewal of a Group Property D\&F Dataset

| Group | 1 | 2 | 3 | 4 | 5 | Average movement |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Method |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| Mean of transformed loss - Wang | -0.2\% | +0.0\% | -2.1\% | -1.1\% | +2.6\% | +1.2\% |
| Mean proportional | +0.0\% | +0.6\% | +0.1\% | -0.7\% | -2.5\% | +0.8\% |
| 80\% TVaR propotional | +0.0\% | +0.6\% | +0.1\% | -0.7\% | -2.5\% | +0.8\% |
| 95\% TVaR propotional | +0.0\% | +0.5\% | -0.3\% | -0.8\% | -1.8\% | +0.7\% |
| 99\% TVaR propotional | -0.1\% | +0.2\% | -3.4\% | -2.8\% | +6.7\% | +2.6\% |
| Targeted TVaR propotional | +0.0\% | +0.5\% | -0.6\% | -1.0\% | -1.0\% | +0.6\% |
| Incremental marginal capital | +0.2\% | +0.9\% | +0.4\% | -3.7\% | +0.3\% | +1.1\% |
| Game theory | +0.0\% | +0.5\% | -0.0\% | -1.3\% | -1.4\% | +0.6\% |
| Myers-Read | +0.0\% | +0.3\% | -1.6\% | -1.1\% | +0.8\% | +0.8\% |
| Equalise relative risk | -0.0\% | +0.4\% | -1.0\% | -1.2\% | -0.1\% | +0.5\% |
| Apply co-measure (TVaR) | +0.1\% | +0.5\% | -0.7\% | -1.0\% | -1.0\% | +0.6\% |
| Insurance capital as a shared asset | +0.1\% | +0.5\% | +0.1\% | -0.6\% | -2.5\% | +0.8\% |

4.5.5.3 The cells in red show the largest change (either positively or negatively) for the Return on Capital over all pricing methods following the loss of the particular group. The cells in green show the pricing method which exhibited the most stability in the return on capital following the loss of that group. The final
column shows the average of the (absolute value of the) changes in the Return on Capital for that method over the loss of each of the groups.
4.5.5.4 For the Credit Risk dataset, the Game Theory, Incremental marginal capital, and Co-measure methods show the greatest stability. Next are the $95 \%, 99 \%$ and targeted TVaR, Proportional hazard, and the Equalise Relative Risk methods. The Wang, Shared Asset, and $80 \%$ TVaR measures follow, and finally the Myers-Read and Mean proportional methods exhibit significant volatility.
4.5.5.5 The results show significant volatility. Admittedly the assumption that the entire group will be lost is extreme, however there is a potential loss of up to $6 \%$ in the expected Return on Capital (which is expected to be $20 \%$ under each of the method if all business is written), without any market cycle, mis-pricing, or even adverse claims events occurring. Further, the Return on Capital would actually improve by over $5 \%$ on the Mean proportional method if the firm simply stopped writing any policies in group six (and recognised the impact of this on the aggregate capital requirement).
4.5.5.6 In general the result is most sensitive to the loss of either group five or six. These are amongst the largest groups (by exposure), and were priced by most methods at noticeable deviations from the aggregate book loss ratio. The results are also reasonable sensitive to group one; these were the high risk, and highly priced policies.
4.5.5.7 For the Property D\&F dataset, the Equalise Relative Risk, Co-measure, and Game theory methods are most stable, followed by the Shared Asset, 80\% and 95\% TVaR, Myers-Read, and, perhaps surprisingly, the Mean Proportional methods. For this dataset the Incremental marginal capital, Transformed loss, and in particular the $99 \%$ TVaR methods appear to have over-reacted to the more volatile policies, so that too much of the return comes from these policies, leaving firms which price by these methods susceptible to price competition.
4.5.5.8 The range of results is similar to the Credit Risk dataset, with up to 4\% Return on Capital at risk to such competition events. Again, for one method, in this case the

99\% TVaR method, the firm could materially improve their Return on Capital by ceasing to underwrite one group, in this case group five.
4.5.5.9 These methods, in general, require calibrating to generate the aggregate book premium, and such a calibration requires an assumption as to the loss dynamics of the (rest of the) future book. Further, some methods require knowledge of, or at least a proxy to the aggregate book loss distribution, or even each policy. Since these results are uncertain at the time of underwriting, the methods are exposed to sub-optimal performance due to mis-estimation of these proxies.
4.5.5.10 In this context, the above results highlight which methods are most subject to volatility in the overall performance as a result of this uncertainty. Another way of interpreting these results is that they indicate the results if a different book from that planned, and assuming in the pricing formulae is eventually written. This emphasises that stability in the above results is a highly desirable feature of any practical pricing method.

### 4.5.6 Comparison (price sensitive)

4.5.6.1 One issue which should be considered in the selection of a pricing framework is whether that framework will influence the type of business underwritten by the firm. That is, if the method is, relatively, cheaper for certain kinds of risks (for example the low-risk attritional type policies) then it is likely to lead to an increased proportion of the book becoming dominated by those risks.
4.5.6.2 To investigate this we assumed total price elasticity in the market, and compared the business attracted by two firms, both pricing with a different method selected from the set considered above.
4.5.6.3 Both firms had calibrated their pricing formulae to give the same return assuming that they wrote all policies in the market. In practice, we would expect the firms to base their calibration on their own data, and take into account the impact of a smaller book reducing the return on capital compared to an aggregate market study. However so that we did not bias the attraction of business we used the results shown above, with both firms targeting the same aggregate loss ratio (50\%
for the Credit Risk dataset, $18.5 \%$ for the Property D\&F dataset). The tests were carried out for each dataset separately.
4.5.6.4 For any pair of pricing methods we assumed that each policy would be written by the firm with the pricing method which generated the lowest premium for that policy. All of the policies in the dataset were written by one of the two firms.
4.5.6.5 This resulted in a set of business for each firm and hence both a total loss distribution and an aggregate written premium for each firm. The market premium (being the sum of the premium for each firm) was generally lower than the target used in calibrating each method, since each policy was written by the method which was cheapest. These results may be seen in appendix C.
4.5.6.6 We investigated the expected Return on Capital that would be achieved by each firm. In the following table, a green cell indicates that the method in the row title achieved a higher Return on Capital than the method in the column title, when the two methods were directly compared.

## Credit Risk

|  |  |  |  |  |  |  |  |  |  |  |  |  | $\stackrel{\rightharpoonup}{0}$ $\omega$ 0 0 0 0 $\omega$ $\omega$ 0 $\pi$ $\pi$ 0 0 0 0 0 0 0 0 0 0 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mean of transormed loss - PH |  | TRUE | TRUE | TRUE | FALSE | FALSE | FALSE | FALSE | FALSE | TRUE | FALSE | FALSE | FALSE |
| Mean of transformed loss - Wang | FALSE | A | TRUE | TRUE | FALSE | FALSE | FALSE | FALSE | FALSE | TRUE | FALSE | FALSE | FALSE |
| Mean proportional | FALSE | FALSE | JA | FALSE | FALSE | FALSE | FALSE | FALSE | FALSE | FALSE | FALSE | FALSE | FALSE |
| 80\% TVaR propotional | FALSE | FALSE | TRUE | A | FALSE | FALSE | FALSE | FALSE | FALSE | TRUE | FALSE | FALSE | FALSE |
| 95\% TVaR propotional | TRUE | TRUE | TRUE | TRUE | A | FALSE | FALSE | TRUE | FALSE | TRUE | FALSE | FALSE | FALSE |
| 99\% TVaR propotional | TRUE | TRUE | TRUE | TRUE | TRUE | IA | TRUE | TRUE | FALSE | TRUE | TRUE | FALSE | TRUE |
| Targeted TVaR propotional | TRUE | TRUE | TRUE | TRUE | TRUE | FALSE | VA | TRUE | FALSE | TRUE | FALSE | FALSE | FALSE |
| Incremental marginal capital value add | TRUE | TRUE | TRUE | TRUE | FALSE | FALSE | FALSE | A | FALSE | TRUE | TRUE | FALSE | FALSE |
| Game theory | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | IA | TRUE | TRUE | FALSE | TRUE |
| Myers-Read | FALSE | FALSE | TRUE | FALSE | FALSE | FALSE | FALSE | FALSE | FALSE | IA | FALSE | FALSE | FALSE |
| Equalise relative risk | TRUE | TRUE | TRUE | TRUE | TRUE | FALSE | TRUE | FALSE | FALSE | TRUE | A | FALSE | FALSE |
| Apply co-measure (TVaR) | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | A | TRUE |
| Insurance capital as a shared asset | TRUE | TRUE | TRUE | TRUE | TRUE | FALSE | TRUE | TRUE | FALSE | TRUE | TRUE | FALSE | IA |

## Property D\&F

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mean of transormed loss - PH |  | FALSE | FALSE | FALSE | FALSE | TRUE | FALSE | TRUE | FALSE | TRUE | FALSE | FALSE | FALSE |
| Mean of transformed loss - Wang | TRUE | IA | TRUE | TRUE | FALSE | TRUE | FALSE | TRUE | FALSE | TRUE | FALSE | FALSE | FALSE |
| Mean proportional | TRUE | FALSE | IA | TRUE | FALSE | TRUE | FALSE | TRUE | FALSE | FALSE | FALSE | FALSE | FALSE |
| 80\% TVaR propotional | TRUE | FALSE | FALSE | VA | FALSE | TRUE | FALSE | TRUE | FALSE | FALSE | FALSE | FALSE | FALSE |
| $95 \%$ TVaR propotional | TRUE | TRUE | TRUE | TRUE | VA | TRUE | FALSE | FALSE | FALSE | TRUE | FALSE | FALSE | FALSE |
| 99\% TVaR propotional | FALSE | FALSE | FALSE | FALSE | FALSE | JA | FALSE | FALSE | FALSE | TRUE | FALSE | FALSE | FALSE |
| Targeted TVaR propotional | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | IA | FALSE | FALSE | TRUE | FALSE | FALSE | FALSE |
| Incremental marginal capital value add | FALSE | FALSE | FALSE | FALSE | TRUE | TRUE | TRUE | IA | FALSE | TRUE | FALSE | FALSE | TRUE |
| Game theory | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | A | TRUE | TRUE | TRUE | TRUE |
| Myers-Read | FALSE | FALSE | TRUE | TRUE | FALSE | FALSE | FALSE | FALSE | FALSE | VA | FALSE | FALSE | FALSE |
| Equalise relative risk | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | FALSE | TRUE | IA | FALSE | FALSE |
| Apply co-measure (TVaR) | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | FALSE | TRUE | TRUE | IA | TRUE |
| Insurance capital as a shared asset | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | FALSE | FALSE | TRUE | TRUE | FALSE | A |

4.5.6.7 Firms using the Co-measure, Game theory, and Equalise relative risk methods generally managed to achieve a higher Return on Capital than a single competitor firm using a different methodology; while firms using the Mean proportional, 80\% TVaR, and Myers-Read methods tended to underperform compared to their competition. The firm using the $99 \%$ TVaR method performed well on the Credit Risk dataset, but poorly on the Property D\&F dataset.
4.5.6.8 However a more detailed analysis of the results shows that a high Return on Capital was often achieved at the expense of business volumes, through aggressively pricing the more stable policies, and pricing the firm out of the more volatile, capital exposed policies. This may be an efficient strategy if there is limited capital; however if a firm has excess capital they may be pricing themselves out of business which would still produce an adequate risk-adjusted return.
4.5.6.9 The following table shows a green cell if the firm using the pricing method shown in the row achieved a higher expected profit cash amount than the firm using the method shown in the column heading.

## Credit Risk

|  |  |  |  |  |  |  |  |  | $\begin{aligned} & \text { Z } \\ & 0 \\ & \stackrel{0}{ \pm} \\ & \pm \\ & \stackrel{1}{E} \\ & \vdots \\ & 0 \\ & \hline \end{aligned}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mean of transormed loss - PH |  | TRUE | TRUE | TRUE | TRUE | FALSE | TRUE | TRUE | TRUE | TRUE | TRUE | FALSE | TRUE |
| Mean of transformed loss - Wang | FALSE | VA | TRUE | TRUE | FALSE | FALSE | TRUE | TRUE | TRUE | TRUE | FALSE | FALSE | FALSE |
| Mean proportional | FALSE | FALSE | JA | FALSE | FALSE | FALSE | FALSE | FALSE | FALSE | FALSE | FALSE | FALSE | FALSE |
| 80\% TVaR propotional | FALSE | FALSE | TRUE | A | FALSE | FALSE | FALSE | FALSE | FALSE | TRUE | FALSE | FALSE | FALSE |
| $95 \%$ TVaR propotional | FALSE | TRUE | TRUE | TRUE | IA | FALSE | TRUE | TRUE | TRUE | TRUE | TRUE | FALSE | TRUE |
| 99\% TVaR propotional | TRUE | TRUE | TRUE | TRUE | TRUE | NA | TRUE | FALSE | TRUE | TRUE | TRUE | TRUE | TRUE |
| Targeted TVaR propotional | FALSE | FALSE | TRUE | TRUE | FALSE | FALSE | VA | TRUE | TRUE | TRUE | TRUE | FALSE | TRUE |
| Incremental marginal capital value add | FALSE | FALSE | TRUE | TRUE | FALSE | TRUE | FALSE | IA | TRUE | TRUE | FALSE | FALSE | FALSE |
| Game theory | FALSE | FALSE | TRUE | TRUE | FALSE | FALSE | FALSE | FALSE | IA | TRUE | FALSE | FALSE | FALSE |
| Myers-Read | FALSE | FALSE | TRUE | FALSE | FALSE | FALSE | FALSE | FALSE | FALSE | A | FALSE | FALSE | FALSE |
| Equalise relative risk | FALSE | TRUE | TRUE | TRUE | FALSE | FALSE | FALSE | TRUE | TRUE | TRUE | IA | TRUE | TRUE |
| Apply co-measure (TVaR) | TRUE | TRUE | TRUE | TRUE | TRUE | FALSE | TRUE | TRUE | TRUE | TRUE | FALSE | VA | TRUE |
| Insurance capital as a shared asset | FALSE | TRUE | TRUE | TRUE | FALSE | FALSE | FALSE | TRUE | TRUE | TRUE | FALSE | FALSE | A |

## Property D\&F


4.5.6.10 The PH and Wang methods now perform the best overall; while the Game Theory and Myers-Read methods perform the worst. The Mean proportional and $80 \%$ TVaR perform poorly on the Credit Risk dataset, but very well on the Property D\&F dataset.
4.5.6.11 The situation is even more confusing if more than two firms exist in the market (as in practice, of course). If there was one firm in the market using each pricing method, again using the same calibration of the pricing formulae, the results would be as overleaf.


|  | $\begin{gathered} \text { Mean of } \\ \text { transormed } \\ \text { loss }-\mathrm{PH} \end{gathered}$ | $\begin{array}{\|c\|} \hline \text { Mean of } \\ \hline \text { transformed } \\ \text { loss - Wang } \end{array}$ | Mean proportional | $80 \%$ TVaR propotional | $95 \%$ TVaR propotional | 99\% TVaR propotional | Targeted TVaR propotional | Incremental marginal capital value | Game theory | Myers-Read | Equalise relative risk | $\begin{gathered} \text { Apply co- } \\ \text { measure } \\ \text { (TVaR) } \end{gathered}$ | Insurance capital as a shared asset |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Premium Income Expected Loss Ratio Expected Profits | 0\% | 0\% | $\begin{array}{r} 17,489 \\ 500 \\ 8,718 \\ \hline \end{array}$ | 0\% | 0\% | 0\% | \% | $\begin{array}{r} 6,933 \\ \hline 576 \\ 2,966 \\ \hline, 966 \end{array}$ | $\begin{array}{r} 3,935 \\ 386 \\ 1,651 \\ \hline \end{array}$ | $\begin{array}{r} 21,639 \\ 7996 \\ 4,605 \\ \hline \end{array}$ | $\begin{aligned} & 3,512 \\ & 536 \\ & 1,660 \end{aligned}$ | 369 $52 \%$ 176 | \%\% |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Capital Requio | 0\% | 0\% | 485\% | 0\% | 0\% | 0\% | \% | 309\% | 267\% | ${ }^{573 \%}$ | 288\% | ${ }_{485 \%}^{1,787}$ | 0\% |
| Expected Return on Capital | 0\% | 0\% | 10\% | 0\% | 0\% | 0\% | \% | 14\% | 16\% | 8\% | 16\% | 10\% | 0\% |













This business is the most volatile and capital intensive though, so the Mean proportional method is exceeded by other methods on Return on Capital. The Myers-Read method is also worth discussing. This method attracts volumes (it writes most of the policies in group four in fact), but at poor rates, and makes little profit. The business is capital intensive and hence the method achieves a low return on capital. This business was diversifying against the market book, but as the Myers-Read method failed to win any more business it was left with nothing to diversify against, leading to its poor performance.
4.5.6.14 The Incremental marginal capital method attracts most of the policies from section three, but at unprofitable rates. These policies diversify away against the market loss distribution, but the methodology would be exposed in the same way as the Myers-Read method, however it also attracts profitable business from section seven.
4.5.6.15 The policies in section seven are generally shared between Equalise relative risk, Game theory, Myers-Read and Incremental marginal capital.
4.5.6.16 One way to interpret these results is to look at the relative competitiveness of each method, segregated by the method which proved most competitive. The following graphs show the level of competitiveness of each pricing method, where a value of $20 \%$ (say) indicates that the method in question sets a technical premium five times the lowest technical premium over all methods. The data has been segregated into groups according to which technical premium was lowest. That is, for a given line in the graph below all polices will be written by the same firm, using the pricing method indicated in the legend.

## Comparative Competitiveness

Credit Risk Data


## Comparative Competitiveness

Property D\&F Data

$\square$ Mean of transormed loss - PH
■Mean of transformed loss - Wang
$\square$ Mean proportional
$\square 80 \%$ TVaR propotional
-95\% TVaR propotional

- 99\% TVaR propotional
$\square$ Targeted TVaR proportional
-Incremental marginal capital value add
■ Game theory
$\square$ Myers-Read
- Equalise relative risk
$\square$ Apply co-measure (TVaR)
■Insurance capital as a shared asset
4.5.6.17 The results show that certain methods, most particularly the Incremental Marginal Capital, Game Theory, higher percentile TVaR methods, and, to a lesser extent, Myers-Read methods are clearly the most competitive for the business they win. That is, they only charge the cheapest premium when they are much lower than alternative methods; they rarely win in what could be called a competitive situation. In contrast the Mean proportional, and Transformed Mean methods tend to win business in exactly these situations.
4.5.6.18 These results do tend to agree that certain methods are competing for the same types of business; broadly splitting into: low volatility, medium volatility, high volatility, and diversifying business.
4.5.6.19 The results suggest that the most appropriate method can only be selected in light of a detailed understanding of the objectives of the insurance firm. This may mean that different methods are appropriate in different classes; and at different points in the insurance cycle. Given the need to cover fixed expenses (which we excluded from our analysis) there may well be a need to deviate from technical models in order to attract profitable (if less desirable) business.


## 5. Other Considerations

### 5.1 Other Considerations

5.1.1.1 In this section, we highlight other issues that will require thought when discussing and explaining variable capital loads within the business. The purpose of this section is not to discuss these additional issues in detail, but only to raise them for potential future discussions and possible research.
5.1.1.2 These issues have been grouped under the headings:
(1) The Market Cycle
(2) Marginal Capital
(3) Updating and Communicating Methods and Parameters
(4) Risk Profiles
(5) Reinsurance
(6) Risk Appetite
(7) Tax
(8) Length of Tail
(9) Mergers and Acquisitions, Group and Market considerations

### 5.1.2 The Market Cycle

5.1.2.1 How should capital loads reflect overall over/under supply of capital in the market as a whole?
5.1.2.2 As a tool for the underwriter a method which does not give a price that reflects the optimal use of capital in the current market conditions may be seen as inadequate.
5.1.2.3 On the other hand from a technical point of view it is certainly simpler if the method does not reflect the market capital position. In this case it is likely to be the target e.g. the target ratio of the Actual to the Technical price may change through the cycle but the underlying technical price remains unaffected.
5.1.2.4 If the objective of a variable capital load methodology is help the company meet its objectives, then the methodology should change only to reflect those objectives. It is therefore unlikely that there should be a direct link between the market cycle and the capital load. However there may be an indirect link - a known, or an expected, deterioration in the market cycle may affect investor expectations and reduce their required return. This would then lead to a modification of the capital load methodology.

### 5.1.3 Marginal Capital

5.1.3.1 Marginal capital methods have been considered in more detail above from a technical perspective but for the firm the top-line is often as important as important as the bottom line. Thus although the selected capital loading may be different there is always likely to be interest in the marginal capital required by a given contract.

### 5.1.4 Updating and Communicating Methods and Parameters

5.1.4.1 For underwriters to implement the chosen methodology they will need to be ready willing and able to implement the chosen method. Thus they not only need to have explained which method to use and when but also what parameters to use in any given situation.
5.1.4.2 As both business planning and capital modelling may be an annual (probably at best quarterly) activity the firm will need to decide on an approach to updating and communicating macro parameters related to the calculation of technical price to the business.
5.1.4.3 To enable the calculated technical price to be understood and analysed the firm will have to ensure that all technical prices (including capital loads) can be
reconciled to the parameters/method in force at that time to ensure that appropriate conclusions will be drawn from the resulting MI.

### 5.1.5 Risk profiles

5.1.5.1 Clearly there should be a relationship between the risk profiles within a portfolio of business and the capital requirement modelled within the business. The capital model is dependent on the risk profile of the underlying business. In turn the risk profile of the business, and the price charged for that risk, should reflect risk and the cost of capital identified through the capital modelling process.
5.1.5.2 When considering attributing a capital load to an individual contract the magnitude of the loading could be reflective of some or all of the following factors
(1) Gross limit written
(2) Net limit written
(3) Attachment point
(4) The impact on non-insurance risks, such as reinsurer credit risk or operational risks
(5) Rating level
(6) The ability to mitigate the risk e.g. current reinsurance pricing and the volatility of future reinsurance pricing
5.1.5.3 There are various aggregation issues that require consideration:
(1) Exposure to a particular risk on different layers. The weighting in this case could be non-linear, reflecting the non-diversifiable nature of the risk.
(2) Sideways risk on a particular risk.
(3) Aggregation of different risks.
(4) Claims from a single event (such as WTC).
(5) Knock on impacts (i.e. large claim event leading to an increase in reinsurance pricing)
(6) Impact on investment portfolio
5.1.5.4 Another factor that should theoretically impact the capital loading is parameter uncertainty. Although conceptually underwriters are likely to understand the implications, technically it is a difficult area. There are various ways to approach parameter uncertainty but a simplistic approach where the parameters of the capital loading method are changed based on a simple classification of the data as high or low quality may be most practical.
5.1.5.5 There will be a number of pragmatic issues to consider when identifying which of the factors identified above are used. The communication of any capital loading process needs to be considered, bearing in mind that the implementation of any process would require the training and education of underwriters and others within the business.

### 5.1.6 Reinsurance

5.1.6.1 It is logical that any capital loading applied will be net of any risk mitigation that can be applied but we would anticipate a separate loading for the cost of this mitigation.
5.1.6.2 Thus the capital loading may be defined to take account of the type and extent of any reinsurance coverage purchased.
5.1.6.3 Depending on how the loadings are constructed, the existence of capital loadings may provide incentives to optimise reinsurance purchasing, through providing a reduction in loading equal to the economic benefit derived from the reinsurance purchase. Purchase of facultative reinsurance would, in particular, lend itself to scenario.
5.1.6.4 Additionally, capital loadings could be used to provide incentives to:
(1) Select reinsurers in an economically beneficial way (i.e. diversify by reinsurer)
(2) Encourage efficient purchasing of reinsurance as rates increase and decrease through the cycle by providing greater transparency and visibility around the economic value of reinsurance (in comparison to the price).

### 5.1.7 Risk Appetite

5.1.7.1 The conversion of the firm's risk appetite into the variable capital load will require consideration. Although in general at group level it may be relatively straightforward to express the firm's risk appetite in terms of risk measure and level.
5.1.7.2 At an individual risk level this may be harder; an aversion to Florida windstorm exposure or Taxi drivers may not be easy to convert into a risk measure and level.
5.1.7.3 The approach here will again depend on whether the intention of the technical price is to directly help the underwriter select risk given the firm's risk appetite or to provide a more general indication of a benchmark price.

### 5.1.8 Tax

5.1.8.1 Corporate taxation usually has a limited impact on the capital requirement of a company. However, to the extent that taxation on losses can be carried forward to future years it may have an impact on capital requirements and therefore on the capital loadings required. In these circumstances, taxation will usually reduce the capital required, with the effect being proportionately larger in higher taxation environments.
5.1.8.2 It is likely that there will be limitations to the amount that losses can be carried forward (if they can be carried forward at all) for the purposes of capital requirements. Thus, higher risk companies, with potential for larger losses, will gain a proportionately lower benefit in their capital requirements. The extent of
the benefit derived may be linked to the regulators' view of the ongoing financial viability of the company.
5.1.8.3 Higher tax environments are also likely to be matched with higher pre-tax return on equity targets in order to compensate the investor for the taxation payable on profits generated.
5.1.8.4 Capital loadings could be made to be reflective of the issues outlined above.

### 5.1.9 Length of tail

5.1.9.1 At the most basic level, longer tailed business has an inherent higher uncertainty, and as a result will require more capital. In reality, this relationship between "length of the tail" and "required capital" is much more complex.
5.1.9.2 The late reporting of claims in longer tailed business, accompanied by the higher likelihood for latent claims, results in increased insurance risk, both underwriting and reserving risk. As time passes, and underwriting risk transfers into reserving risk, the overall risk profile of the business has not really changed for long tailed business, where, for short tailed business, the uncertainty would have reduced fast with time. This indicates a high correlation between underwriting and reserving risk for long tailed business, which may be underestimated in many of today's models.
5.1.9.3 Insurance companies attempt to match assets to its liabilities. The duration of assets for longer tailed business should be longer than for shorter tailed business. Because different asset classes will be used for long-tailed business, we expect the asset risk to perform differently for long-tailed classes than for short-tailed classes. Although these differences may disappear at an aggregate firm level, they are important in the allocation of capital to individual classes of business or risks. In everyday business, the target combined ratios in long-tailed business will hold an allowance for investment returns. This allowance should include an appropriate capital load not just for the insurance risk but also for the investment risks.
5.1.9.4 Traditionally, catastrophic risks are linked to short-tailed classes. The occurrence of one or more catastrophes may result in higher liquidity risk, and the release of surplus funds in the reserves. This indicates that shorter tailed classes may carry higher liquidity risk, with a potential knock-on effect on the held reserves. Again, we believe these differences to be important at individual class of business or risk level.
5.1.9.5 The situation with regards to credit risk appears to be more complex. Long-tailed business carries higher credit risk as liabilities span many future years, but history has shown that catastrophic risks, mainly apparent in short-tailed classes, have a direct negative impact on the credit worthiness of (re)insurers, which may in turn impact both the long and short tailed classes.
5.1.9.6 It is clear that model risk, process risk and parameter risk will be higher on longtailed classes. It takes years for claims to develop, time in which risk profiles, the legal and regulatory environment, claims and underwriting practices and the economic environment may have changed.
5.1.9.7 On balance, it appears that longer tailed classes carry more insurance risk, asset risk and possibly lower liquidity risk. The balance seems to disfavour long-tailed business. This is not accounted for in many of today's capital allocation models, with the exception of insurance risk. However, this is likely to be captured to some extent or in full in the aggregate capital model. Correct capital allocation between long and short tailed business, considering the issues mentioned above, may result in a lower correlation between long and short tailed business.

### 5.1.10 Mergers and Acquisitions, Group and Market considerations

5.1.10.1 The risk profile of a company may change dramatically after a merger or acquisition, when the market suddenly changes or when considered within the context of a group. This section will address a few of the issues with regards to variable capital loads.
5.1.10.2 Not only the liability or asset mix may change, but also the matching between assets and liabilities may change as a result of a merger or acquisition. This shows that, any major change will result in significantly different key assumptions in the underlying models (e.g. asset classes may be allocated to different lines of business after the merger to better reflect the duration of the liabilities). In practice, this would be difficult to evaluate due to e.g. integration issues between different companies (different data systems, different capital calculation methods, different allocation methods, etc.).
5.1.10.3 Acquisition activity may put strains on the capital where capital has been used for financing the deal. This may be especially an issue in stressed markets like the current environment ("credit crunch", "sub-prime crisis"). This indicates that, after an acquisition, many of the relationships between the different risk elements (insurance, credit, liquidity, etc.) may suddenly look very different, and that exposures to catastrophes around the acquisition activity increases risks in a nonlinear way.
5.1.10.4 A more fundamental question is if companies can account for "potential mergers with other companies", i.e. for the probability that a merger of two or more companies may result in capital efficiencies. This is especially important for those groups where capital can be easily moved between entities. As the market changes daily, capital loadings should change daily as well.
5.1.10.5 Much more complicated issues relate to intellectual capital. If we would assume that competent staff prefers to work for highly rated insurance providers, there is a link between capital and intellectual capital. This is especially important in the insurance industry where intellectual capital is one of the key drivers of the business. Would high intellectual capital result in lower capital requirements? These effects are likely to be included in current models implicitly (using historic performance).
5.1.10.6 A similar point can be made around brand name and reputation. Customers prefer to deal with solid insurance companies, resulting in a more stable environment, so lower capital requirements. Again, these effects are likely to be included in current models implicitly (using historic performance).
5.1.10.7 When the market is very competitive (customers shopping around, more new business and less renewals, more M\&A activity, etc.), there will be more strains on the capital. This leads again towards the need for a regular review of capital loadings. It is generally understood that new business carries more uncertainty than renewal business, although we haven't seen this reflected in many capital allocation models. The addition of tenure to capital models in customer value models - to spread capital costs over time - are in general not explicitly considered in capital allocation models.

### 5.1.11 Negative capital loads

5.1.11.1 Are there circumstances where negative capital loads can be justified? We believe this to be unlikely for the more traditional insurance products. However, where more exotic products are considered, this may be very well the case.

### 5.1.12 Other Factors

5.1.12.1 Another consideration is the duration of capital. In stressed situations, e.g. the 9/11 terrorist attacks, it can be expected that the stressed situation is likely to be followed by an increase in rates and capacity as a consequence. These longer term impacts may or may not be considered in capital loading models.
5.1.12.2 In cases where profit-shares are defined with the broker, affinity group, supplier or other third parties, the risk profile may be altered and reflected in capital loads.
5.1.12.3 It is clear from these examples that at the individual risk level, the considerations around capital loadings are much more complex than at a portfolio level. All these considerations will require additional research which is outside of scope for this working party.

## 6. Pitfalls

### 6.1 Communication as a potential pitfall

### 6.1.1 The need for communication

6.1.1.1 A common pitfall of any actuarial analysis is the inadequate communication of the work to the various stakeholders. It is necessary to overcome any potential pitfalls by effective communication.
6.1.1.2 We have separated two key dimensions in our discussion on communication. The first is our audience, 'type of person'. The second is the 'type of knowledge'.
6.1.1.3 As every organisation is different, these role descriptions may not be wholly appropriate. We would hope, however, that they should act as a guide to our thinking. Equally, the types of knowledge have been chosen to be broad, and each reader may wish to adapt our thoughts as they see fit for their organisation.
6.1.1.4 The grid below outlines how the two dimensions could interlink.

|  | Risk <br> Committee | Product <br> Heads | Line <br> Underwriters |
| :--- | :--- | :--- | :--- |
| High level understanding of capital | YES | YES | YES |
| Overview of allocation methodology |  |  |  |
| including pros and cons | YES | YES |  |
| Relevance of certain KPIs to allocated <br> portfolio capital | YES |  |  |
| Account level features such as risk load <br> credit / debit allowances | YES | YES |  |

6.1.1.5 Moreover to the dimensions outlined above, we have given communication its own life-cycle as we would expect the type and format of communication to evolve over time. This life-cycle has been split into three distinct parts:
(1) The initial introduction of the topic
(2) Further discussion and real understanding
(3) Use, fine tuning and active debate
6.1.1.6 These stages will be addressed separately also.

### 6.1.2 Type of person

6.1.2.1 An important consideration is who in the organisation needs to know what. It is important to understand the types of audience as different stakeholders will care about different things. The table above considers this issue and splits the types of person involved into three broad groups: Risk Committee; Product heads; and Line underwriters.
6.1.2.2 The Risk Committee are senior members of management who need to oversee the project. They are effectively the project sponsors and need to know the high level issues involved. Importantly they are impartial with regards to product line and so their decisions on allocation methodology are key. Their responsibilities and incentives require them to think about:
(1) The big picture - does the approach meet the current and future requirements for use tests?
(2) Choosing the allocation method early - changing the method is not something that will be welcome after it has been cascaded throughout the organisation. They will need to know pros and cons of each method.
(3) What are the key sensitivities and uncertainties of the method selected?
(4) They will want to ensure that management reporting can be created in line with the methods and establish KPIs.
6.1.2.3 Product heads are members of management who manage a particular class profit centre. e.g. Head of Marine. Product heads need to be actively involved in managing their units allocated capital through their underwriting strategies. They are not impartial and so should not set allocation methodologies. Once a method is agreed, however, they should be free to 'play' their position as they see fit to maximise their unit's return on capital. Their responsibilities and incentives require them to think about:
(1) Allocation methodology - though they will not be changing the allocation, they will need to understand what drivers affect it. For example, is it more based on individual risk characteristics or marginal diversification against a portfolio.
(2) Unit Strategies - how business volumes, geographical diversification, industry diversification, limit profiles, reinsurance programmes etc. affect their capital allocation. These should be picked up in the KPIs as agreed with the Risk Committee.
(3) Frequency of review - they will be keen to ensure that changes in the portfolio are reflected in a timely manner in return on capital reports to senior management.
(4) Senior management information - what is important to senior management. Do they care more about return on capital or the quantum of profits? Strategies may need to adapt accordingly.
6.1.2.4 Line underwriters see risks account by account and don't focus on portfolio decisions. They need to be educated as to the link between the risk load on their accounts with the overall portfolio, but as a rule will not need to be thinking about portfolio issues day to day. The consideration of Line underwriters assumes that the allocation approach is cascaded to policy level on at least some accounts. Their responsibilities and incentives require them to think about:
(1) Day to day risk loads - underwriters will think about what makes any particular account have a higher risk load. They will need to consider
the calculation of risk load transparent so as to start to understand and buy in to the overall process.
(2) Flexibility - though a risk load is likely to be automatically generated, each account is individual. The ability to flex risk load due to certain features would be reasonable e.g. attachment point.

### 6.1.3 Type of knowledge

6.1.3.1 In considering communication, it might also be useful to segment the type of knowledge. Not all information is always required for someone to effectively do their job. It is best to target specific types of knowledge, rather than attempt to teach everything in a more 'scattergun' approach. The table above considers this issue and splits the types of knowledge into four broad groups: High level understanding of capital; Overview of allocation methodology including pros and cons; Relevance of certain KPIs to allocated portfolio capital; and Account level features such as risk load credit / debit allowances. The following sections outline the components of each 'type of knowledge'.

### 6.1.3.2 High level understanding of capital

(1) What is capital and why is it a useful concept used across all industries?
(2) The history of capital modelling and what our company has done thus far.
(3) The future of capital modelling with Solvency II and increased modelling capability

### 6.1.3.3 Overview of allocation methodology including pros and cons

(1) Methods available as outlined in the previous sections of this report
(2) Pros and Cons of each method spanning technical points as well as softer issues such as transparency.
(3) Any implicit biases associated with methods
(4) How the allocation methodology for pricing may differ from ICA values.
6.1.3.4 Relevance of certain KPIs to allocated portfolio capital
(1) Introduce the concept and language used to measure the volatility for any individual account
(2) Introduce the concept of diversification and why individual accounts need to be considered against existing portfolios e.g. Property CAT can be used as an example
(3) Discuss the high level principles of the method selected
(4) Agreement of the sensitivity of the different KPIs and how best to include them in reporting and capital allocation.
(5) Explain the relationship between the risk loading calculated by the method and the risk.
6.1.3.5 Account level features such as risk load credit / debit allowances:
(1) How the risk load has been calculated.
(2) By line of business, what features need to be allowed for by way of built in flexibility tolerances.

### 6.2 Communication life cycle

6.2.1.1 The temporal dimension of communication is also important. Different companies may be at different stages of their journey, and we considered that depending on this, communication might be approached differently.

### 6.2.2 The initial introduction of the topic

6.2.2.1 This introductory phase is the kick-start. The aim is to get as many people aware of the initiative as possible, in the shortest space of time.
6.2.2.2 This step is all about awareness and education. It is likely best done by way of presentation to reach the wider audience quickly and efficiently. As it will be a new concept to the majority of participants, presentations should start from first principles and educate in line with the type of person/knowledge. Given the amount of material available, a single presentation is unlikely to be sufficient, with a number of presentations required.

### 6.2.3 Further discussion and real understanding

6.2.3.1 This section heading captures the potentially long period from introduction of a topic through to real understanding of each user. Beyond the initial presentation, follow up meetings are probably the most effective way to promote long term understanding and buy in. Buy in can be slow and is the process of real understanding.
6.2.3.2 It is vital that all stakeholders in the process, in particular product heads and line underwriters can see that their actions affect the capital attaching to their portfolio. If this link is not transparent then it is more difficult to integrate the new methods into the work flow and thinking of the underwriting teams.

### 6.2.4 Use, fine tuning and active debate

6.2.4.1 Reports need to be designed to capture all the key information affecting the process. Each report can be unique to the business and needs of management, but in general a report which provides all the key information in a transparent way is ideal. This report will enable communication and questions that open up any potential 'black box' so that assumptions and methodology can be challenged and adapted accordingly.
6.2.4.2 Reports need to be regular, say monthly, so that it can keep in tune with business forecasting / planning. Indeed depending on the style and aim of the 'capital report' it may well become the same report as the business planning schedules.

### 6.2.5 Technical Concepts

6.2.5.1 Jargon and use of technical language needs to be considered against the audience. A complaint against the actuarial profession is that use of such language inhibits effective communication to a non-technical audience. Of course, to a technical audience, such language is wholly appropriate and indeed enables effective communication.
6.2.5.2 Communication is included in the actuarial examination syllabus and this module is being adapted to make it as business oriented as possible. This and practice within the working environment are required to build up communication skills.

### 6.2.6 Subjectivity

6.2.6.1 Inevitably, the whole process relies on various actuarial assumptions. The results can be sensitive to particular assumptions and generally it can be these areas which are challenged by underwriters.
6.2.6.2 Evidence of challenge is proof of strong process understanding by stakeholders and will no doubt be a good demonstration of 'integration' to any third parties that review the models and their use. Though challenge may be frustrating at first, this is a hurdle that must be tackled for true integration. One major communication pitfall that is common in technical work is shying away from challenge, but this can often leave the undesirable consequences of work which isn't sense checked and therefore not believed by the wider business.

### 6.3 Politics as potential pitfalls

### 6.3.1 Governance

6.3.1.1 Having the proper governance structure around the capital function is essential to manage a number of the internal and external political implications of building and using an economic capital model.
6.3.1.2 Ideally a risk committee should be formed to own the capital project and outputs. Senior executives and senior underwriters need to be represented in order to
demonstrate the strategic importance of the risk committee. Potential representatives include:
(1) Chief Executive Officer
(2) Chief Risk Officer (possibly as chair)
(3) Chief Operations Officer
(4) Chief Underwriting Officer
(5) Chief Actuarial Officer (possibly as chair)
(6) Chief Finance Officer (possibly as chair)
(7) Senior underwriters
(8) Head of Capital Modelling
6.3.1.3 The Head of Capital Modelling is likely to be the main person doing the work, and will use the model within the business, offering strategic capital insight to the risk committee. They will also ensure that the model is reviewed internally by business experts, in order to ensure that there is business buy-in to the model.
6.3.1.4 The other members are there to take advantage of the strategic risk insights that the capital model gives, and to offer support to the capital modelling team within the company.
6.3.1.5 As well as steering the risk modelling work, a sub-set of the committee could be part of the overall escalation process for the model parameterisation. There may be a natural tendency for optimism by people that are charged with controlling risk (for example underwriters), and tensions can arise when the modelling team view risk differently from the risk holder. In such disputes, a formal escalation process to provide a definite view of risk for a particular aspect of the model will help remove some of the politics that could otherwise arise.

### 6.3.2 Aligning interests

6.3.2.1 Another aspect of politics arising as a result of capital modelling work is the linking of incentives to capital, for example underwriting bonuses being directly linked to return on capital for each class of business within the company.
6.3.2.2 If a capital model is being used for this purpose, then it is arguably the clearest demonstration that senior executives believe that the capital model adequately captures the risk inherent within their business; if it were not, then they would not rely on the model to affect the remuneration of their staff.
6.3.2.3 In such a scenario, underwriters have a financial incentive to "game" the model to their advantage, and try to portray an optimistic view of their accounts. Some underwriters will aggressively pursue the modelling team, and adverse responses (indeed any non-positive response) by the team can cause internal friction.
6.3.2.4 These politics are much more difficult to control, but aiming to have a transparent method to allocate capital, and a clear escalation process (through a sub-set of the risk committee) can help alleviate problems. Also, giving the Chief Executive Officer clear ownership for the capital allocation process will ensure that the business will place less pressure on the capital modelling team since they would have to justify themselves to the top level of management in order for changes to their allocation to be made.
6.3.2.5 One area of aligned interest that also gives rise to politics is the tension between rewarding top-line growth against bottom-line profitability. Most companies will focus on the latter, although this does not necessarily encourage underwriters to write less business in a declining market. If this behaviour could be incorporated within the overall risk framework, then business objectives are more likely to be met to the mutual benefit of both the company and individual underwriters.

### 6.3.3 Total capital pot

6.3.3.1 When allocating capital, there is often politics around the overall level of capital that should be allocated in the first place.
6.3.3.2 The first decision is usually whether to allocate total capital held, rating agency capital (i.e. ignoring any surplus above that required to maintain the current credit rating) or the pure regulatory capital.
6.3.3.3 Allocating total capital seems like a natural start point, since it is this that the shareholders require a return on.
6.3.3.4 Regulatory capital is rarely used since it does not reflect the economic fundamentals of the business being written (how much retro business would be written through a BBB rated company?).
6.3.3.5 However, parts of the business may not have any control over how surplus capital is spent, and therefore a more natural measure becomes the rating agency capital (with the surplus capital all being allocated to Group functions who have the responsibility to either return the capital to shareholders, or use it more effectively such as by making acquisitions).
6.3.3.6 Even going to a rating agency level of capital has a number of political ramifications. For example, a motor portfolio could argue that it does not need an AA rating in order to write business, whereas this is much more important for an inwards reinsurance writer. Therefore, it could be argued that direct business should be allocated less capital (for the same level of risk) since they do not require the enhanced credit rating. If such a decision is made, then who has to make a return on the difference between these two levels of rating agency capital?
6.3.3.7 There are also potential politics over which components of capital should be allocated.
6.3.3.8 Underwriting capital is fairly clear-cut - the business is writing business and therefore this capital should be allocated in order that shareholders get a return on the capital.
6.3.3.9 Reserving capital is slightly less obvious. The business has to capitalise business that has been written in the past, and therefore it should arguably enter in the allocation exercise at some point. However, current underwriters should not be forced to pay for poor business that was written in the past by a different set of
underwriters. If reserve capital is allocated out (which is the natural thing to do since otherwise casualty lines tend to get a small capital allocation due to contagion risk not being fully reflected), then should it be based on actual reserves, or a notional "steady-state" reserve amount. The latter is arguably better since it allows for new business lines to be allocated capital, and for shrinking lines of business not to be overly penalised by the past volumes they have written.
6.3.3.10 Reinsurance protections and brokers used are commonly controlled by the underwriters, albeit with a corporate strategy overlaid. Therefore credit risk should probably be incorporated within the allocation exercise.
6.3.3.11 Market risk and liquidity risk are usually outside the remit of most underwriters, and should therefore probably be excluded from the allocation exercise.
6.3.3.12 Operational risk is partially controlled by underwriters, but given the inherent difficulties in modelling operational risk in the first place, allocating out operational risk capital is likely to be too subjective to get proper buy-in.

### 6.3.4 International sensitivities

6.3.4.1 Further political issues can arise from international sensitivities. People across the globe have very different cultures, and these can manifest themselves within the overall capital framework. Quite often there is also a lack of understanding of business written in different territories, and again this can give rise to further politics. Finally, local legislation requirements differ and this can give rise to a few politics, such as should countries with high statutory capital requirements be allocated extra capital to reflect these regulatory constraints.
6.3.4.2 Resolving international sensitivities is difficult, and is best controlled through the risk committee. If representatives from each country are included in the authorising committee, then they are likely to be helpful in managing the crossborder politics.

### 6.3.5 Professionalism

6.3.5.1 Professionalism is a key skill in all actuaries, and capital related work is no different. To alleviate some of the political pressures, it is important to maintain a slightly detached view in order that undue pressure for the business does not damage the overall integrity of the work.
6.3.5.2 When feedback on the model is given by business experts, it needs to be assessed as to whether it is of no use, useful but not incorporated within the model, or useful and incorporated. Such decisions should be based on the merit of the information, rather than the personalities involved, in order to alleviate potential political issues.

## Appendix: A Terms of Reference

## Introduction

The General insurance premium Rating Issues working Party ("GRIP") recommended a range of research topics of value for the profession in its report of 12 January 2007. One of these was variable capital loads, and the recommendation is repeated below.
"3.32 Capital allocation and loading methods are discussed in detail in Appendix C. All of these methods, however, derive an overall capital loading that does not differentiate risk by segment (or which makes a loading in proportion to the expected cost of claims). For example, within a motor portfolio, a higher capital loading might be appropriate for segments that generate more large claims or have higher levels of inherent variability. There has been little research to date as to how this link may be best achieved."

This working party has been set up in order to investigate these issues.

## Issues to consider

The purposes of variable capital loads;
o To consider the objectives of varying the capital load, and whether these influence the most appropriate methodologies;
o To consider the level of aggregation appropriate;
Investigate different methods of allowing for variable risk profiles;
o To identify the methods used in practice within the industry at present;
o To identify the theoretical methods available;
o To discuss the appropriateness of these methods;
o To consider these methods from the point of view of various stakeholders; insured, underwriters, senior management, and shareholders;

Analyse the impact of the different methods;
o Investigate whether use of different capital allocation methods would tend to exhibit trends in the pricing of different segments;
o Consider whether such trends might be likely to affect the insurance companies overall results;

Identify any common pitfalls;
o Investigate whether there are any common pitfalls of the results of capital loading which may result in inappropriate pricing;
o Discuss how to deal with the politics inherent;
o Discuss the best methods for communicating the results;
Investigate the issues surrounding applying these methods;
o Consider the practical application of these methods, e.g.
o length of the tail;
o allowance for reinsurance;
o allowance for tax;
o Consider whether the time horizons used are appropriate for pricing;
o Consider any issues arising from acquisitions;

## Classes of business

The intention is that the working party results consider the issues arising from pricing varying risk profiles in direct insurance and reinsurance at a technical level considering the risk profiles (both independently and jointly) in isolation. It is hoped that the majority of issues which are considered will relate purely to this and will not be class of business specific. Where we believe class of business specific issues are highly relevant these will be discussed and identified as such.

## Input from outside the working party members

The working party intends to research published work in this field, however there is no intention at this stage to seek further input.

## Deliverables

o Paper to be issued by GIRO 2007
o Presentation of paper at GIRO 2007

## Membership

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## Appendix: B Comparison (price sensitive) results

The following graphs show the results that would be obtained by various firms operating in a perfectly price elastic market.

The policies available in the market are those described under the respective datasets in section 4 of the report. In each test we have hypothesised two firms in the market, each pricing with a different pricing formula as described in section 4. The technical prices used are those generated in section 4. In particular the pricing formula have been parameterised on the assumption of the acquisition of all the market policies.

Each policy has then been assumed to be written by the firm with the lower technical premium for that policy. This means that the aggregate market premium (summed over both firms) will be lower than the value that the pricing formulae have been calibrated to.

The policies written be each firm have then been aggregated to determine the total results. The tables on the following pages show for each test;
o The aggregate premium income for each firm;
o The expected loss ratio for each firm;
o The expected aggregate profits for each firm;
o The capital requirement for each firm, given the business they have written, determined as a $99.5 \%$ Value-at-Risk measure;
o The capital to premium ratio for each firm;
o And the expected return on capital of each firm.

The aggregate premium, expected loss ratio, expected aggregate profits, and expected return on capital are also displayed graphically on the following pages. Additionally, the aggregate profit distribution and return on capital distribution for each firm are displayed for each test.

Credit Risk Dataset







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Credit Risk Dataset


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Credit Risk Dataset


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Credit Risk Dataset







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Credit Risk Dataset






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Credit Risk Dataset






| Return on Capital CDF |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 100.00\% |  |  |  |  |  |  |  |  |
| 80.00\% |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| $70.00 \%$ <br> $60.00 \%$ |  |  |  |  |  |  |  |  |
| 50.00\% |  |  |  |  |  |  |  |  |
|  | 40.00\% \% $\quad$ - |  |  |  |  |  |  |  |
| ${ }_{20.0076}^{30.0076}$ |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $10.00 \%$ - |  |  |  |  |  |  |  |  |
| -250\% | -200\% | -150\% | -100\% | -50\% | 0\% |  | 50\% |  |

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Credit Risk Dataset





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## Credit Risk Dataset








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## Credit Risk Dataset






| Profit CDF |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $100 \%$ |  |  |  |  |  |  |  |
| $-90 \%-$ |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|    <br>    <br>    <br> $50 \%$   <br> $50 \%$   |  |  |  |  |  |  |  |
| 40\% - $\quad$ Myers-Read |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  | 30\% |  |  |
| $\begin{aligned} & 20 \% \\ & 100 \% \end{aligned}$ |  |  |  |  |  |  |  |
|  |  |  |  |  | $10 \%$ |  |  |
| -10,000 | -8,000 | -6,000 | -4,000 | $-2,000$ | - | 2,000 |  |



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## Property D\&F Dataset








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## Property D\&F Dataset








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## Property D\&F Dataset






| Profit CDF |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{100 \%}$ |  |  |  |  |  |  |  |  |  |
|  |  |  | 90\% |  |  |  |  |  |  |
| 80\% |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  $60 \%$   <br>  $50 \%$   <br>  $40 \%$   |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|    <br>  $30 \%$  <br> $20 \%$   |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| -12,000 | -10,000 | -8,000 | -6,000 | -4,000 | -2,000 | - | 2,000 | 4,000 |  |



## Property D\&F Dataset



## Property D\&F Dataset








## Property D\&F Dataset



## Property D\&F Dataset








## Property D\&F Dataset







## Property D\&F Dataset






| Profit CDF |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $100 \%$ |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  | -80\% |  |  |  |  |  |  |
| ${ }^{70 \%}$ |  |  |  |  |  |  |  |
| ${ }_{50 \%}^{60 \%}$ |  |  |  |  |  |  |  |
| 40\% $\quad \square$ Myers-Read |  |  |  |  |  |  |  |
|  | 30\% |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
| ${ }_{100}^{20 \%}$ |  |  |  |  |  |  |  |
| -10,000 |  |  |  |  |  |  |  |
|  | -8,000 | -6,000 | -4,000 | -2,000 | $\cdot$ | 2,000 |  |



## Property D\&F Dataset



Property D\&F Dataset






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Property D\&F Dataset







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## Property D\&F Dataset



|  | Game theory | Myers-Read |
| :---: | :---: | :---: |
| Premium Income | 579 | 690 |
| Expected Loss Ratio | 13\% | 29\% |
| Expected Profits | 505 | 492 |
| Capital Requirement | 2,524 | 3,568 |
| Capital Ratio | 436\% | 517\% |
| Expected Return on Capital | 20\% | 14\% |







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## Property D\&F Dataset



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Property D\&F Dataset






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## Property D\&F Dataset








## Property D\&F Dataset





Property D\&F Dataset







## Appendix: C Bibliography

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