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YEARS OF LIFE LOST AND OTHER MORTALITY INDICES

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IN 1953 together with a colleague (Benjamin and Logan) the author called attention to a paper by Haenzel (1950) describing a new index of mortality years of life lost.

The argument was that many people were living for more than the three score and ten years and that every earlier death represented a loss of potential further years of life; that adding up the total years of life lost might be a significant measure of the toll of largely preventable disease; that changes in this total year by year would maximize the improvement gained by curative and especially preventative medicine.

The aim, therefore, was to consider the years of life lost by each death rather than simply to count the number of persons whose lives were terminated; the underlying concept being that a man dying at the age of say 30, might but for the 'accident' of death have lived to the remainder of his normal span, and that it might be a greater achievement to prevent his death than to save the life of a man aged 90, who cannot have much longer to live.

There was the problem of the choice of the 'normal span of life' to be used in measuring years lost on death. There was no precise or absolute measure, since a current life table was necessarily based upon the mortality of the lives then dying and was never exactly reproduced. It was then thought any projected life table would be entirely arbitrary and speculative. Refuge was taken in the fact that the assessment of mortality improvement requires relative indices rather than absolute measures, and it was proposed therefore arbitrarily to adopt as the limit of 'normal' life that age in the life table at which the number of lives surviving was less that 10% of the original entrants, viz. the maximum span within which 90% of persons die and is survived only by an abnormally longeval 10%. For males this was 85 and for females 88 years of age in round numbers and at the then current levels of mortality.

As an example of the use of the index Table 1 is taken from the original paper of Benjamin & Logan.

The Registrar General in calculating the index for England and Wales, which is published in the Quarterly Return for the second quarter of each year (now Population Trends) has used 85 for both sexes. The years of life lost are distributed between the working age period, 15 to 64, and the remainder. For a man dying at the age of 20 it would therefore be assumed that a total potential loss of years of life (for index purposes only) of 65 years is incurred, and 45 of those years would be in the working age range.

More recently, an excellent discussion of the concept and its application to the assessment of progress in reducing mortality from particular causes has been

Table 1. *Years of life lost per 1,000 population—England and Wales*

	Persons		Males		Females	
	15–64	Total*	15–64	Total	15–64	Total
1848–72	497	1,004	542	1,047	455	964
1952	76	238	92	266	61	211
1952, per cent of 1848–72	15	24	17	25	13	22

* Total to age 85 (males) and 88 (females). Standardized on the 1952 population.

provided by Romeder & McWhinnie (1977). (They prefer to describe the index as PYLL—potential years of life lost. It does make a good acronym if one is needed.)

Professor F. D. K. Liddell in a private discussion has raised the question of the appropriateness of the 10% survival age as the marker for potential years of life. Any national life table is based on a mixture of successive generations. Those currently surviving their 85th birthday and largely determining the shape of the life table around that age were born 60 years before persons now dying at age 25 whose loss of potential years of life is probably (assuming a declining trend in mortality) more than 60 years. So should the goalposts be moved, and if so, at regular intervals or continuously? To update the life table, from which the 10% is obtained sporadically would produce discontinuities albeit not very large. The suggestion might therefore be made that at each age of death the lost years of life should be calculated by reference to a projected expectation of life for the birth cohort to which the deceased belongs. So much for theory. There are two practical difficulties. First, it will be a complicated process to refer to a different table for each age of death in a particular calendar year (i.e. each different generation) even with modern computer software. Second, the construction of generation life tables of middle and younger ages is not practicable because the age range of death rates as a basis for projection is not only short at younger ages but the curvature of age progression of death rates is different from that of later ages. The extension of this curvature to later ages would be unrealistic. Figure 1 illustrates this difference of gradient. Clearly this extension of the trend of the generation born 1951–55 would give a widely different death rate at age 70 from that given by the extension of the generation born 1911–15.

A practicable compromise would be to use at all ages a life table projected 40 years ahead on the secular trend of age death rates. This would be a compromise, being too short for young ages and too long for old ages. The Government Actuary constructs such a table each year for the United Kingdom population projection purposes and it would be sensible to use it. If this suggestion were to be adopted, it would mean substituting e_x for 85 in the present calculation where e_x is the expectation of life at age x (the age of death) and the value of e_x is taken from the projected life table.

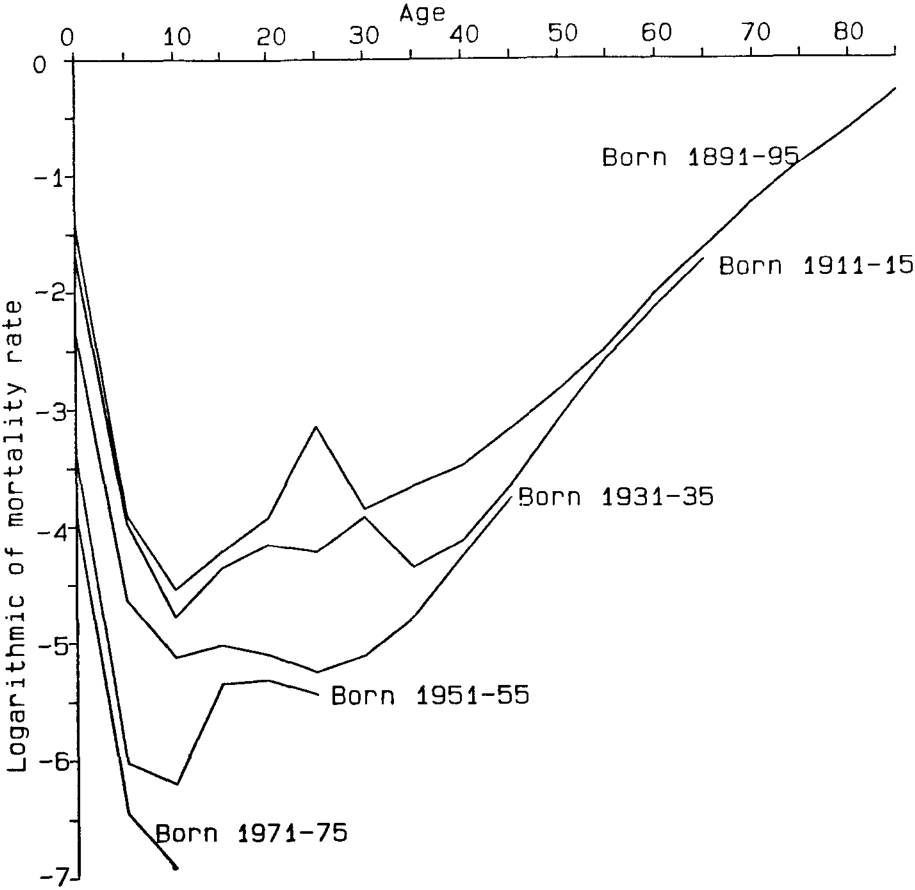

Figure 1. Cohort Mortality, males, selected cohorts.

Table 2 provides a comparison of values of the loss of expected years index calculated on the original basis and on the suggested new basis.

Since e_x is much less than $(85-x)$ at young ages the new index does not give so much relative weight to mortality at younger ages as did the old index. It is a matter of opinion but it would seem that the new basis would be preferable as not giving quite so much weight to mortality improvement at very young ages which has been a perhaps too dominant factor of the mortality trend of this century.

Table 2. *Loss of expected years of life, all ages (per 1,000 population) England and Wales*

Year	Males				Females			
	Original basis		New basis		Original basis		New basis	
1931	451	100	309	100	359	100	270	100
1951	284	63	202	65	198	55	176	65
1961	254	56	186	60	163	45	158	59
1971	239	53	164	53	148	41	154	57
1981	205	45	165	53	126	41	145	54
1982	202	45	180	58	122	34	164	61
1983	197	44	180	58	121	34	163	60
1984	194	43	162	52	120	33	164	61

A new index—The proportion of anticipated deaths

Consideration of the loss of expected years of life prompts thought about the deaths that occur 'before their time'. One way of looking at the ages at which people die is to consider the d_x column of the life table—i.e. the number out of an original birth cohort (normally 100,000) who die after attaining age x exact and before their $(x+1)$ -th birthday. The d_x column of the life table which adds to 100,000 (the radix of this table which is separately constructed for males and females) is a discontinuous distribution. But it is usual to draw a curve (known as the 'curve of deaths') through the ordinates and to assume that it is continuous (the continuous form would be $\mu_x l_x$ where μ_x is the instantaneous rate of mortality, referred to as the 'force of mortality', and l_x is the number of lives surviving at age x, where x is continuous.

The shape of the curve of deaths can be seen from the overall outline of Figure 2; it rises at first slowly than moves sharply to a peak in the mid seventies and then declines sharply and finally levels out, becoming asymptotic to the x-axis.

Actuaries have long been interested in the curve of deaths. In particular Clarke (1950) made it the subject of a special study.

At that time he argued that mortality improvements had not extended the natural lifespan but had only allowed more to achieve it. He distinguished between 'anticipated' and 'senescent' deaths; the ages at death in the latter group were measures of natural lifespans and had a frequency distribution like other animal characteristics. Clarke's division of deaths into 'anticipated' and 'senescent' was further developed by Barnett (1955, 1958) but applied to the force of mortality, not the curve of deaths. On the basis of cause of death grouping and the actual shape of the curve of observed age rates of mortality Barnett distinguished several different groups of anticipated deaths. Clarke originally intended to define 'senescent' deaths by choosing certain degenerative diseases (e.g. cerebral vascular lesions, myocardial diseases, angina pectoris, arterio-sclerosis, other diseases of the circulatory system, bronchitis, nephritis), but he naturally found it difficult to select disease groups with sufficiently specific reference to degeneration. Ultimately therefore he arbitrarily assumed that the

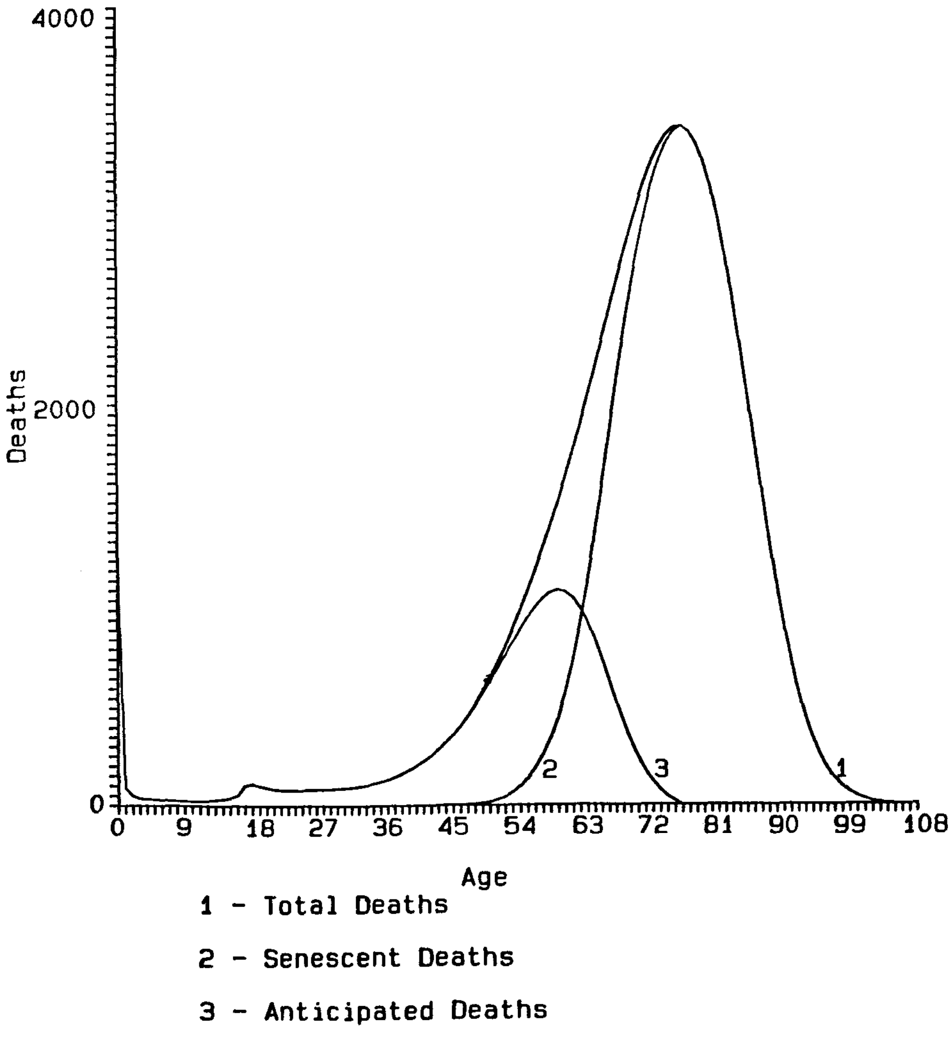

Figure 2. Curve of Deaths, Males. English Life Table no. 14

proportion of deaths that were senescent rose from ·05 at age 20 to ·10 at age 40, ·20 at age 50, ·70 at age 70, 1·00 at ages 80 and above. His limiting curve of deaths was not symmetrical. There was a sharp peak at age 80 with a tailing off rapidly on one side to age 100 or so and on the other side a rapid decline to about age 60 and then a much slower tailing off to age 20.

A simple illustration

Let us first take a simplified illustration of this type of analysis of the curve of deaths. In a particular life-table the values of d_x (deaths between age x and $x+1$) have been plotted for every value of x in the table (Figure 2), thus producing an approximation to the curve of deaths (d_x is as we have stated discontinuous while the 'curve of deaths' is continuous). It has been assumed that $d_x = \mu_{x+1/2}\, l_{x+1/2}$. The curve has then been treated from its later mode (e.g. the peak at age 76 in Figure 2) to the upper limit of age as the right-hand side of the distribution of 'senescent' deaths, i.e. of normal life spans and the left-hand side of this distribution has been drawn in to mirror exactly the right-hand side. It is thus assumed for simplicity that the biometric distribution of uncurtailed life spans is symmetrical. When the deaths of this left-hand side of the distribution are subtracted from the main curve of deaths the residual (of 'anticipated' deaths) tails off to zero at the peak of the senescent deaths. In effect it is assumed in Figure 2 that no deaths before age 50 and all deaths after age 76 are senescent. It is of course a matter of doubt whether any death after age 76, even allegedly due to some accident, is other than of senile origin, but certainly some deaths before age 50 are of degenerative origin (at age 50 Clarke assumed that 20% of them were 'senescent'). Whether this degeneration is senescent in the sense of the completion of a genetically endowed life span or whether it is the cutting short of the span by departure from optimum environmental conditions and behaviour is at least arguable and we may adopt this latter hypothesis for a moment for the purpose of our simple illustration.

Table 3 illustrates the results obtained by applying the analysis already described to national life tables (for both males and females, though Figure 2 relates to males only).

There are several difficulties. In the first place the assumption of normality for the tail of the curve of deaths is an approximation although the error involved is relatively small. (The sum of squared deviation is less than ·001.) Secondly, the assumption that the distribution of 'senescent' deaths is symmetrical about the peak age of the curve of deaths has a theoretical appeal in the sense that most

Table 3. *Senescent deaths*

		Males			Females		
English Life table	Period of deaths	Peak age	Standard deviation of distribution	Senescent proportions of total deaths (%)	Peak age	Standard deviation of distribution	Senescent proportions of total deaths (%)
1	1841	72·0	9·38	39·9	73·5	9·19	41·0
8	1910–12	73·5	8·70	51·5	76·0	8·51	55·3
10	1930–32	74·3	8·51	63·3	77·5	8·19	65·0
11	1950–52	74·8	8·41	75·0	79·7	7·15	74·4
12	1960–62	74·4	8·81	81·8	80·9	7·39	76·9
13	1970–72	74·5	9·74	84·18	82·0	7·63	76·7
14	1980–82	76·7	8·52	80·6	83·5	6·99	75·5

natural events are so distributed but it does mean that in a period when a higher proportion of the under-70's survive to their 80's and the tail of the curve deaths is swollen there will be some exaggeration of the volume of 'senescent' deaths. This happened for males in 1970–72 and to a lesser extent in the opposite direction for females in 1980–82 when the tail of the curve of deaths was shortened. Such difficulties do not occur in earlier periods when mortality was heavier, when a high proportion of deaths were truly 'anticipated' and the peak of the curve of death was perhaps closer to the true peak of the distribution of 'senescent' deaths. At the present stage of mortality progression however, changes in the breadth and slope of the tail of the curve of deaths, without necessarily reflecting any persistent mortality trend, can seriously affect the proportion of 'anticipated' deaths as we have defined them, i.e. as a remainder item. This is probably the explanation of the rise in the proportion of anticipated deaths in 1981 (Table 4).

There is moreover the ever present possible disturbing effect of the mix of generations in the population life table as calculated and used in practice.

We do not know where the true peak of the senescent distribution lies. It must be to the right of the peak of the curve of all deaths but at a decreasing distance as mortality improves and a higher proportion of total deaths are 'senescent'. This is the defect of the proposed senescent curve that it would not reflect this shift. On the grounds that some deaths at earlier ages than the peak of the total curve of deaths are 'senescent' (the persons did attain their endowed potential) and that some deaths later than the peak age are 'anticipated', we could make the arbitrary assumption that all deaths to the left of the peak age of the curve of deaths are 'anticipated'. In life table notation this means that $(l_0 - l_{x_1})/l_0$ is the proportion of anticipated deaths (PAD). The values of this arbitrary index are shown in Table 4. (Note that, because of the change of definition, the proportions in Table 4 are twice those implied by Table 3.)

If thought to be useful the index is easily calculated. Given the abridged life table produced from time to time by OPCS it is necessary to interpolate to find the peak age x_p of d_x, to further interpolate to find l_{x_p} whence $(l_0 - l_{x_p})/l_0$ is the value of PAD (if it was thought undesirable to use historic data, it would be possible to use a projected life table as suggested for 'years' of life lost).

Table 4. *Proportion of anticipated deaths* (%) *(revised definition)*

English Life table	Central year of experiences	Males	Females
8	1911	72	72
10	1931	68	68
11	1951	63	63
12	1961	59	62
13	1971	58	62
14	1981	60 (a)	64 (a)

(a) See text for a suggested explanation of the rise in the index.

Before leaving PAD it should be mentioned that Clarke in a late contribution (1962) to the consideration of senescent deaths referred to this difficulty of there being a mix of anticipated and senescent deaths at all adult ages and to the difficulty of reporting the actual senescent distribution. He suggested that the medical profession might be able to mount a study in which deaths might be labelled (as a matter of medical opinion) as 'anticipated' or 'senescent'. This would be a very useful experiment and might be commended to the Royal Colleges.

Back to Gompertz

Perhaps we should turn back to Gompertz as Redington did in 1969 and as Heligman & Pollard effectively did in 1980 and as Thatcher has done more recently (1987).

Let us briefly recapitulate. Gompertz in 1825 proposed on physiological grounds that the intensity of mortality (in his terms the average exhaustion of man's powers to avoid death) gained equal proportions in equal intervals of age, giving rise to an increasing force of mortality, i.e. μ_x the instantaneous rate of mortality may be represented by Bc^x. Later Makeham in 1867 added a constant A to allow for a level incidence of chance causes (accidents etc.) to give $\mu_x = A + Bc^x$. Later several workers developed more complicated curves to fit more closely the observed variation of mortality with age. Notably Perks (1932) produced a family of curves in the general form:

$$\mu_x = \frac{A + Bc^x}{Kc^{-x} + 1 + Dc^x}.$$

In 1969 Redington examined mortality in later life and showed that as mortality in England and Wales had declined so the B of Gompertz had decreased and c had increased. He examined the possibility that the population value of B was the average of individual genetically endowed values, while c reflected environmental influences.

In 1980 L. Heligman and J. H. Pollard obtained promising results over the entire life span with the 'law':

$$q_x/p_x = A^{(x+B)^C} + D\exp\{-E(\log x\text{-}\log F)^2\} + GH^x$$

The number of parameters at first sight appears excessive.

However, when it is recalled that the curve reproduces three distinct features—the mortality of a child adapting to its new environment, the mortality associated with the ageing of the body and the superimposed accident mortality—and the 'law' is applicable throughout the life span of more than 100 ages, the number of parameters seems very reasonable. Most of the parameters are also readily interpreted. A, for example, is almost the same as q_1. C measures the rate of decline in mortality in early life (the rate at which a child adapts to his environment). G indicates (in the author's view) the level of senescent mortality, while H measures the rate of increase of that mortality. D represents the intensity

Table 5. *Values of G and H of the Heligman/Pollard curve*

English Life Table	Central years of experience	Heligman/Pollard constants Males $G \times 10^5$	H	Females $G \times 10^5$	H
1	1841	12·379	1·0938	8·6592	1·0976
2	1841	19·044	1·0878	13·243	1·0913
3	1846	23·169	1·0852	13·401	1·0915
4	1876	19·391	1·0880	11·310	1·0939
5	1886	19·480	1·0889	14·179	1·0915
6	1896	23·886	1·0858	10·978	1·0947
7	1906	26·036	1·0838	8·8769	1·0963
8	1911	23·370	1·0851	7·5496	1·0981
9	1921	11·580	1·0946	5·1298	1·1030
10	1931	12·875	1·0930	4·8421	1·1036
11	1951	5·4925	1·1052	2·1927	1·1125
12	1961	4·0468	1·1090	1·2486	1·1184
13	1971	3·7853	1·1093	2·8313	1·1047
14	1981	3·200	1·113	2·400	1·106

of the accident hump, while F indicates the location of the hump and E its spread. (G and H are clearly analagous to the B and c constants of Gompertz.)

Forfar & Smith (1987) have fitted the Heligman & Pollard curve to every English Life Table and the values of the constants G and H are shown in Table 5. It would be seen that G representing the level of senescent mortality does not fall significantly until about 1921. It was about this time that economic and social conditions improved (the ending of the Poor Law and the extension of Social Insurance) and survival prospects at older ages began significantly to be enhanced.

In 1987, Thatcher looked at mortality at advanced ages and demonstrated the applicability of the Gompertz law. More recently (in publication) he has demonstrated the close agreement at advanced ages between the Heligman & Pollard curve and the Gompertz curve.

So at a time when we all are dying at later ages it may be advantageous to look at the Gompertz constant B which measures the current level of mortality and the constant c which measures the rate of deterioration in that mortality. Alternatively we might look at the Heligman & Pollard constants G and H which have the same functions. In fact the latter would be preferable since the Heligman & Pollard curve is applied to the whole span of life and the form GH^x is specifically provided for advanced ages.

This discussion leads on to the suggestion that for the present perhaps a better index of senescent mortality for countries with an advanced decline in mortality might be the G of the Heligman & Pollard mortality curve (the B of Gompertz); that is if ever the human race does stop incurring new health hazards and does allow natural senescence to rule.

Table 6. *Comparison of indices*

Sex	Year	Standardized death rate	Reciprocal of expectation at birth	Years of life lost (new basis)	Proportion anticipated deaths*	Heligman & Pollard
Males	1931	100	100	100	100	100
	1951	79	88	65	93	43
	1961	76	88	60	87	31
	1971	70	82	58	85	29
	1981	65	82	53	88	25
Females	1931	100	100	100	100	100
	1951	71	89	65	93	45
	1961	64	89	59	91	26
	1971	57	81	57	91	58
	1981	53	81	54	94	50

* Assumed to be all deaths to left of peak age of curve of deaths.

In Table 6 the various indices discussed in this paper have been compared. In each case the index is rated to 1931 = 100. In order to produce a following trend the reciprocal of the expectation of life (Brownlee called this the Life Table Death rate), and the proportion of 'anticipated' deaths (the complement of the proportion of senescent deaths) have been used.

The difference between the trends of these indices reflects their orientation to different aspects of mortality. The expectation of life is heavily influenced by mortality in infancy and childhood the major improvement in which occurred before 1931, so that it is not surprising that this index shows the least change since that year. For the same reason it is not surprising that there is little difference in trend as between the sexes since any change in the sex differential in mortality since 1931 will have been largely confined to ages 45 and over, i.e. at ages which have less weight in this index. Conversely one would expect the Heligman & Pollard parameter G to indicate a larger improvement in the mortality of males at later ages since 1931. This index is also that showing the largest fall since 1931. The reasons have been briefly referred to earlier.

But it is possible that G which covers what is left after the operation of the parameters largely but not wholly dealing with mortality of younger ages, may exaggerate the decline in senescent deaths.

The other indices are intermediate. 'Years of life lost' is based on adult ages and therefore shows more change than 'expectation of life' which tends to be influenced by experience of very young ages as is also the general (standardized) death rate which has a trend similar to that of $(e_x)^{-1}$.

This leaves us to consider the trend of the division between anticipated and senescent deaths. This trend shows the wavering to which reference has already been made, i.e. the lack of definition of the division between the two classes of deaths on the basis of the curve of death at least in a country in which there has been an advanced decline in mortality. It is probable that the truth about senescent mortality lies somewhere between PAD and G.

On the whole it seems to be a question of 'horses for courses'. One chooses the index best designed to emphasis the mortality feature in which there is particular interest.

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