INSTITUTE AND FACULTY OF ACTUARIES

SPECIMEN EXAMINATION

Subject CM1B
Actuarial Mathematics

Time allowed: One hour and 45 minutes

INSTRUCTIONS TO THE CANDIDATE

1. You are given this question paper and two Excel files.

2. Mark allocations are shown in brackets. This exam has a total of 100 marks.

3. There are 2 questions. Attempt all questions. Each question is to be answered in a separate Excel file. A separate Excel file has been provided for each question.

4. The working of each part of the question should be on a separate sheet (tab). For example, question 1i should be worked out within the sheet (tab) i of the spreadsheet file named CM1 Q1.

5. Where possible, summarise your answers for each question in the sheet (tab) named ‘answers’.

4. Upload the two Excel files with your solutions.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.
1 A property developer has bought some land for £25,000,000 on which a block of apartments will be built. These apartments will be built over a period of five years starting immediately. Each apartment will take one month to build and will cost £160,000 to build. It is assumed that these building costs will be incurred at the beginning of each month.

The number of apartments built each month are given in the spreadsheet CM1B Q1. It is further assumed that the apartments will each be sold for £250,000 three months after they have been built so that e.g. the first apartments are sold after four months.

The developer assumes a risk discount rate of 10% per annum effective.

(i) Calculate
   (a) the present value of the land and building costs of the project
   (b) the present value of the sale proceeds

(ii) Calculate
   (a) the accumulated value of the project at the time that the final apartments are sold.
   (b) the discounted payback period of the project
   (c) the internal rate of return for the project.

It is now assumed that the developer will fund the project by borrowing from a bank at an interest rate of 12% per annum effective. The developer can repay the borrowing at any time from the income from the sale of the apartments and can invest any surplus income in a bank account paying 6% per annum effective.

(iii) Calculate the revised accumulated value of the project at the time that the final apartments are sold.

The developer is concerned that the original assumptions are too optimistic and now wishes to consider the effects of changing the following assumptions:

- the building costs are £160,000 for each apartment in the first month but then increase monthly in line with an assumed inflation rate of 3% per annum effective
- a delay in selling the apartments such that 60% of the apartments are sold three months after they have been built with the other 40% being sold nine months after being built.

(iv) Recalculate the answers to part (ii) assuming a risk discount rate of 10% per annum effective assuming both of the above changes to the assumptions are made.

[Total 44]
A life insurance company issues a 20-year endowment assurance to a life aged 35 exact. Premiums of £3,000 per annum are paid annually in advance. The sum assured of £100,000 is payable at the end of the year of death or on survival to the end of the term.

The life insurance company carries out a profit test on this policy assuming no surrenders occur. This profit test is entitled Base Scenario in the spreadsheet (CM1B Q2) provided. For each year, \( t \), the end of year cashflows per policy in force at the start of year \( t \) are shown.

(i) Calculate the expected present value of profit and the profit margin for this contract using a risk discount rate of 7% per annum. The other elements of the basis used to carry out the profit test are given within the formulae of the Base Scenario spreadsheet provided. [12]

The life insurance company is considering introducing a benefit payable on surrender for this contract. The benefit on surrender is to be 109% of total premiums paid to date and will be paid at the end of the year of surrender. It is assumed that surrenders do not generate any claims expenses.

The assumed independent rates of surrender are 10% in year 1, 5% in year 2 and 1% thereafter until year 20. There are no surrenders in year 20. All other assumptions remain unchanged.

(ii) Calculate the expected present value of profits and the profit margin for this contract after the introduction of the surrender benefit. [18]

(iii) Explain your results. [4]

The life insurance company introduces this surrender benefit and its assumption on surrender rates are unchanged. The company now investigates the impact of holding prospective gross premium reserves for this contract. The gross premium reserves are set up at policy duration \( t \) (for duration \( t = 1, 2, 3, \ldots 19 \)) immediately before the premium then due.

Gross Premium Reserve Basis
Mortality 100% Base \( q_x \) shown in the Base Scenario sheet
Interest 4% per annum
Surrenders None
Expenses None

(iv) Calculate the expected present value of profit and the profit margin for this contract after the introduction of gross premium reserves. [18]

(v) Comment briefly on your results. [4]

[Total 56]

END OF PAPER