INSTITUTE AND FACULTY OF ACTUARIES

Curriculum 2019

SPECIMEN SOLUTIONS

Subject SP6 – Financial Derivatives Specialist Principles

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1 (i) From a real world viewpoint, a tradable asset is an asset where there exists a market in which it can be traded either directly or indirectly. [1]

From a theoretical viewpoint, within a complete market of tradable securities a process represents a tradable asset if its discounted price is a martingale under the risk-neutral measure. [1] [Max 1]

(ii) The market price of risk, $\gamma$, is defined as: $\gamma = \frac{\mu - r}{\sigma}$. [1]

Where $\mu$ is the expected growth rate of the stock with the dividends reinvested… [1]

…and $\sigma$ is the volatility process, and $r$ is the risk-free rate of interest. [1]

(Note to markers: students may have defined the market price of risk as:

$$\gamma = \frac{m + q - r}{\sigma}.$$ [Note to markers: If a candidate has not written down an expression but instead has defined the market price of risk in words correctly then award 1 mark] [Max 2]

(iii) For $S(t)$, the market price of risk, $\gamma(S(t))$, is:

$$\gamma(S(t)) = \frac{\mu - 0.05}{0.1}. [0.5]$$

Solve for $R(t)$ using Ito’s lemma. Let $Y(t) = \ln R(t)$. [0.5]

Differentiating $R(t) = \exp(Y(t))$ gives:

$$dR(t) = 0.25\exp(Y(t))dW(t) + (0.04\exp(Y(t)) + 0.5 \times 0.25^2 \times \exp(Y(t)))dt$$ [2]

(Note to markers: Remove 0.5 marks for each error a candidate makes up to 2 marks, in the formula above.)

$$dR(t) = 0.07125 \times R(t)dt + 0.25R(t)dW(t)$$ [0.5]
The process for a stock price, \( Z(t) \), with a constant dividend yield is of the form:

\[
dZ(t) = (\text{return} - q)Z(t)dt + S(t)\sigma(t)dW(t), \quad \text{where} \ q \ \text{is the constant dividend yield and the return is the constant total expected return of the stock.} \quad [1]
\]

Therefore, for \( R(t) \), the total expected return of this stock is

\[
0.07125 + 0.00125 = 0.0725. \quad [0.5]
\]

[Note to markers: give the above half mark if the student has used \( m + q - r \) in the numerator in the calculation of the market price of risk below, and has therefore included the 0.00125 in that step.]

Assuming that there is no arbitrage…

…implies that the market price of risk is the same for \( S(t) \) and \( R(t) \). \quad [0.5]

Equating the market price of risk for each asset gives:

\[
\frac{\mu - 0.05}{0.1} = \frac{0.0725 - 0.05}{0.25} = 0.09. \quad [1]
\]

So \( \mu = 0.059. \quad [0.5]

[Max 6]

[Total 9]

2 (i) An index-linked bond is a bond under which coupon … \quad [0.5]

… and redemption payments are increased in proportion to increases in a specified inflation index. \quad [0.5]

There may be a lag between the time used to determine the increases and when the increase is actually paid. \quad [0.5]

LPI bonds are index-linked bonds that increase coupon and redemption payments in line with an index subject to some maximum and typically a minimum (e.g. zero). \quad [1]

LPI swaps are OTC contracts typically between banks and institutions. \quad [0.5]

The institution will typically receive payments in line with an inflation index subject to a cap and floor and pay, for example, fixed cash flows. \quad [1]

They are typically traded on a zero coupon basis. \quad [0.5]

[Max 3]
(ii)

[Suggested 0.5 mark for general increase in value as inflation increase, 0.5 mark for "convex" shape.]

[Suggested 0.5 mark for floor below 0%, 0.5 mark for slope in between 0%>5%, and 0.5 mark for capped above 5%.]
(iii)  
(a) Vega risk in this context would be the change in value of the scheme’s surplus arising from changes in inflation volatility.  

A deterministic projection will not allow for the stochastic nature of inflation over time…  
… so will not allow for causing the 0% liability floor (or the 5% cap) to be breached in future….

… even if current inflation expectations always sit within the range of 0% to 5%.

(b) The surplus is likely to be overstated when x is lower than the expectation of 2.5%…  
… as an increase in inflation volatility would likely cause a greater chance of the liabilities being floored at 0% rather than capped at 5%.

The vega sensitivity will be highest when an increase in volatility will cause the 0% liability floor to be breached most significantly….

… so the overstatement will be largest around \( x = 0\% \).

The exact values of \( x \) for which the surplus is overstated would be dependent on the stochastic model assumed for inflation.

(iv) **Sustained Negative Inflation**

The hedge is likely to be very effective as the payoff from the swap will not be reduced during the negative inflation due to the \( \max(\ldots, 1) \) condition…  

… which will match the liabilities which will have been floored at 0%.
Sustained Inflation Above 5%

The hedge is likely to be very effective at immunising the surplus from inflation changes as the payoff from the swap will be capped during the high inflation due to the min(…..,1.05^T) condition… [1] … which will match the liabilities which will have been capped at 5%. [0.5]

Volatile periods of high and negative inflation

The liability payment increases are determined in each calendar year, whereas the payoff on the LPI is based on the cumulative increase in inflation over the period T. [0.5]

For T = 1, the LPI will be very effective as the cumulative increase will equal the annual increase by construction. [0.5]

As T increases however, the effectiveness will reduce… [0.5] … as there is a significant chance that there is an annual instance of negative inflation or inflation in excess of 5% without causing the cumulative inflation rate to exceed the 0% floor or 5% cap…. [1] … which would mean the flooring or capping of the liabilities is not matched by any flooring or capping on the LPI swap. [0.5]

This could be beneficial if there were annual instances of very high inflation without significant cumulative inflation…. [0.5] … but overall the hedge is likely to perform better than if only RPI instruments were held but poorly compared to holding LPI instruments that apply caps and floors annually rather than cumulatively. [1]

Max 4

[Total 15]

3 (i) (a) Consider the following strategy:

Borrow an amount of cash equal to: \( S_0 e^{uT} \) and use it to purchase one unit of the consumption asset and to pay storage costs. [0.5]

Short a futures contract on one unit of the underlying consumption asset. [0.5]

\( F_0 > S_0 \exp[(r+u)T] \) can therefore be ruled out as a tradeable arbitrage opportunity. [0.5]

If \( F_0 < S_0 \exp[(r+u)T] \), then to exploit any potential arbitrage opportunity someone would need to go long on futures contract and short the consumption asset…. [0.5]
… and shorting a consumption asset is typically deemed neither possible nor desirable. [1]

(b) \[ F_0 = S_0 \exp[(r + u - y)T] \] where \( y \) is the convenience yield. [1]

[Note to markers: half mark for equals rather than less than or equals, half mark for correct \(-y\).] [Max 3]

(ii) There may be significant basis risk… [0.5]
… as the price of jet fuel will not be perfectly correlated with the price of oil… [0.5]
… or a forward may not be available on the particular grade of oil that is used to make fuel. [0.5]

The convenience yield of oil is often large and volatile. [0.5]

As a result shorter dated forward prices and spot prices may move significantly even if longer terms forward rates do not change. This results in an even lower correlation with jet fuel over the long term. [1]

And the airline will not know exactly how much fuel will be used. [0.5]

Rolling the hedge once a year would likely involve closing the contract out prior to maturity… [0.5]
… and so the company will incur basis risk on each roll. [0.5]
There will also be costs incurred each year in the governance of the forward rolling of the hedge. [0.5]

There may be insufficient liquidity on the 1-year forward… [0.5]
… so the hedge may become expensive. [0.5]
Using a forward gives away the upside if oil prices fall. [0.5]
The company may have to post collateral if the oil price falls… [0.5]
… which could cause cash flow problems. [0.5]

The company may also face counterparty risk. [0.5]

[iii] The airline may have to report the details of all open positions… [1]
… resulting in extra reporting burdens. [0.5]

An oil forward may be considered a “standardised OTC derivative”… [0.5]
… so the airline may have to clear the forwards through a central counterparty (CCP) [0.5]
… which would require the provision of margin… [0.5]
… specifically both initial and variation margin [0.5]

If the forward is not considered “standardised”, then there are additional operational risk management requirements… [0.5]
… and as the airline would likely be classed as a “non-financial” counterparty and is using the derivatives for risk management, collateralisation would only be required if the positions are large. [1]

As fuel is a major component of operating costs for an airline the position is likely to be large. [0.5]

The benefits of EMIR include reducing the risk, including counterparty risk, of using this hedging approach. [1]

[Other reasonable points could be made, for 0.5 marks each.]

[Max 3]

[Total 11]

4 (i) The payoff to the option holder at time $T$ is:
$$\max\{0, S(T) - K\}.$$ [1]

[Subtotal 1]

(ii) For an expectation to be meaningful a probability measure $\mathbb{P}$ is assumed to exist. [0.5]

Further, it is assumed that a distribution of the random variable $S(T)$, the terminal stock price, exists with respect to $\mathbb{P}$. [0.5]

The expected present value is therefore:
$$\mathbb{E}_\mathbb{P}[e^{-rT} \max\{0, S(T) - K\}].$$ [1]

[Note to markers: if candidates state that a distribution is required for $S(t)$ for all $t$ then 0.5 marks can also be awarded instead of the 0.5 marks for the distribution of $S(T)$.] [Subtotal 2]

(iii) Let $n$ be the integer number of loops in the algorithm ($n > 1$). [0.5]

Assume that independent samples of the price of the underlying at expiry, $S(T)$, can be generated. [1]

Let $S_i$ be a sample drawn from the distribution for $S(T)$. [0.5]

Algorithm:

For each $i = 1, \ldots, n$ [0.5]

- generate a sample $S_i$, [0.5]
- set $V_i = e^{-rT} \max\{0, S_i - K\}$. [0.5]
Set $V_n = (V_1 + \ldots + V_n)/n$. \hfill [0.5]

Using the hint given in the question: as $n \to \infty$, $V_n \to V = \mathbb{E}_p[e^{-rT}\max\{0, S(T) - K\}]$ with probability 1. \hfill [1]

[Note to markers: half marks can be awarded if an algorithm is sketched out as above but in words.]

(iv) $p(S)$ has been defined as the probability density function of $S(T)$. Writing $S$ for $S(T)$ and noting that $p(S)$ and $S$ are continuous, then using the definition of the expectation of a continuous function:

$$
\mathbb{E}_p[e^{-rT}\max\{0, S - K\}] = \int_{-\infty}^{\infty} e^{-rT}\max\{0, S - K\}p(S)dS,
$$

$$
e^{-rT}\int_{-\infty}^{\infty} \max\{0, S - K\}p(S)dS \quad \text{(as $e^{-rT}$ does not depend on $S$)},
$$

$$
e^{-rT}\int_{K}^{\infty} (S - K)p(S)dS \quad \text{(as $\max\{0, S - K\}$ is 0 for $S \leq K$, so the lower limit can be changed)}.
$$

[Total 1]

(v) Note that: $f(x) = \frac{f(x)}{g(x)}g(x)$. \hfill [0.5]

We need to ensure this is well defined for all $x \in \mathbb{R}$. \hfill [0.5]

One approach is to assume that $f(x)$ and $g(x)$ are equivalent measures. \hfill [0.5]

A more detailed argument from first principles is to assume that $g$ is such that:

$$
f(x) > 0 \Rightarrow g(x) > 0 \text{ for all } x \in \mathbb{R}.
$$

This is required for the integral to be well defined. \hfill [0.5]

Then, $h(x)f(x) = h(x)\frac{f(x)}{g(x)}g(x)$ and $h(x)\frac{f(x)}{g(x)}$ is a function from $\mathbb{R}$ to $\mathbb{R}$. \hfill [0.5]

Hence $\int_{\mathbb{R}} h(x)f(x)dx = \int_{\mathbb{R}} h(x)\frac{f(x)}{g(x)}g(x)dx$. \hfill [0.5]

The right-hand side can be interpreted as $\mathbb{E}_Q[h(X)f(X)/g(X)]$ and therefore $\mathbb{E}_p[h(X)] = \mathbb{E}_Q[h(X)f(X)/g(X)] \ldots$ \hfill [0.5]
…where \( \mathbb{E}_Q[.]. \) is an expectation with respect to the probability density function \( g(x) \) and \( \mathbb{E}_P[.]. \) is an expectation with respect to the probability density function \( f(x) \).

(vi) The change in representation can be identified as a change in measure from \( P \) to \( Q \).

The ratio \( \frac{f(x)}{g(x)} \) is the Radon-Nikodym derivative at \( x \).

(vii) \[
\mathbb{E}_Q \left[ \left( h(X) \frac{f(X)}{g(X)} \right)^2 \right] = \int_{\mathbb{R}} h^2(x) \frac{f^2(x)}{g^2(x)} g(x) dx,
\]

\[
= \int_{\mathbb{R}} h^2(x) \frac{f(x)}{g(x)} f(x) dx,
\]

\[
= \mathbb{E}_P \left[ h^2(X) \frac{f(X)}{g(X)} \right].
\]

(viii) Changing the measure is often used in financial mathematics, for example in
the pricing of options…

… and changing the numeraire.

A change in the measure can result in obtaining a more convenient
representation of the expectation for use in calculations or numerical
estimation.

\( Q \) is typically a risk-neutral probability measure…

…which is useful for pricing calculations.

As the actual expected values are the same under each measure …

… an important consideration is the second moments.

The previous part shows that the second moments are different: \( \mathbb{E}_P \left[ h^2(X) \right] \)

and \( \mathbb{E}_P \left[ h^2(X) \frac{f(X)}{g(X)} \right] \).

A suitable choice of \( g(X) \) may result in \( \mathbb{E}_P \left[ h^2(X) \frac{f(X)}{g(X)} \right] \) being smaller than
\( \mathbb{E}_P \left[ h^2(X) \right] \).
As a result this may be useful in the numerical estimation of integrals. For example, in a Monte Carlo simulation this may result in more efficient estimates, as …

…fewer simulations will be needed to produce an estimate with lower variance compared to one in not changing measures. [0.5]

This could be done by selecting an appropriate distribution which emphasises “important” values, for example in the tails of distributions. [1]

5 (i) A panel of banks is appointed to set the LIBOR rate… [1]
…for each of the ten major international currencies, [0.5]
…and for several funding periods. [0.5]

The panel of banks reflects the balance of the market, by country and by type of institution [1]
The constituents of this panel are reviewed annually. [0.5]

The LIBOR administrator assembles the interbank borrowing rates from the contributor panel banks… [0.5]
… in the morning of each business day and…. [0.5]
… discards the top and bottom quartile of the rates…. [0.5]
… and calculates the average of the middle two quartiles… [0.5]

This process is repeated for all maturities and currencies. [0.5]

LIBOR is not a compounded rate, but is calculated on the basis of actual days in funding period… [0.5]
… divided by 360 (or 365 for Sterling). [0.5]

Rates for periods for which LIBOR is not set are obtained by linear interpolation. [0.5]

[Note to markers: the number of currencies and other details of how LIBOR has been calculated has changed in recent years, e.g. it is now only calculated for 5 currencies. The core reading students were revising from was the one dated for 2016 exams, and the answer above is consistent with the factual details in that. However, do not deduct marks for factual comments that differ from the above but which are correct now.] [Max 4]

(ii) It is unlikely that an individual bank, acting alone, would be able to materially influence the LIBOR rate…. [1]
… as if the bank wanted a significantly different LIBOR rate from other banks, it would get discarded due to the use of the middle quartiles. [0.5]
Even if the rate disclosed provided was within the middle two quartiles, the impact would be reduced due to the averaging process. [0.5]
However, a small change in the LIBOR rate could have a large absolute impact, depending on the type and size of position. [0.5]

The publication of all quotes on screen would also highlight any odd quotes from a specific bank. [0.5]

In order to significantly affect the LIBOR rate, a bank would likely need to influence other banks to similarly distort their LIBOR submissions. [0.5]

(iii) (a) The quarterly coupon amount is \( \frac{(i + x)}{4} \times$100m. [0.5]

So, the present value of the FRN in $m is:

\[
100 \frac{(i + x)}{4} \sum_{t=1}^{T} v^t + 100v^{4T}
\]

where \( v = \left(1 + \frac{i + s}{4}\right)^{-1} \) [0.5]

Which simplifies to:

\[
100 \frac{(i + x)}{4} \left(1 - v^{4T} \right) \left( \frac{1}{i + s} \right) + 100v^{4T}
\]

\[
= 100 \left[ \frac{(i + x)}{(i + s)} (1 - v^{4T}) + v^{4T} \right] \text{ as required.} \quad [0.5]
\]

(b) \( v = \left(1 + \frac{0.03 + 0.01}{4}\right)^{-1} = 0.99010 \) [0.5]

\[
PV = 100 \left[ \frac{(0.03 + 0.02)}{(0.03 + 0.01)} \left(1 - 0.99010^{40}\right) + 0.99010^{40} \right] = $108.2m \quad [0.5]
\]

[Note to markers: give the full mark if correct answer reached, due to “Calculate” command verb.] [Max 3]

(iv) (a) By calculation:

\[
v = \left(1 + \frac{0.04 + 0.01}{4}\right)^{-1} = 0.98765 \quad [0.5]
\]
\[ PV = 100 \left[ \frac{(0.04 + 0.02)}{(0.04 + 0.01)} \right] (1 - 0.98765^{40}) + 0.98765^{40} = 107.8 \text{m} \] 

Hence the change in value is £0m to the nearest £1m

By reasoning:

£0m

… as the FRN will have very limited interest rate exposure given that the coupons float in line with LIBOR

… and the spread payments are of a similar magnitude to the discount spread.

(b) By calculation:

\[ PV = 100 \left[ \frac{(0.03 + 0.02)}{(0.03 + 0.02)} \right] (1 - v^{40}) + v^{40} = 100 \text{m} \]

Hence a fall in market value of £8m (to the nearest £1m)

By reasoning:

£8m fall in market value

… as the coupon rate and discount spread is now the same, so the value will be par

[Total 3]

(v) As \( x \) increases:

The interest rate sensitivity will increase materially…

… as the coupon spread will now significantly exceed the discount spread…

… so the extra coupons will be similar to interest-sensitive fixed payments.

The spread sensitivity will also increase…

… as the higher coupon payments will increase the present value of the FRN...

… although the spread sensitivity from the redemption payment at maturity is unchanged.
The spread sensitivity will still significantly exceed the interest rate sensitivity though, even for very high rates of $x$. [0.5]

[Max 3]

(vi) This feature will help mitigate credit risk… [0.5]
… as the increased coupons following a downgrade will compensate for the widening spread as the credit quality deteriorates. [0.5]

[Total 1]

(vii) The credit rating represents one simple comparator statistic… [0.5]
… that may not fully reflect the credit risk of the FRN. [0.5]

For example ratings may not fully allow for:
The recovery amount on default [0.5]
The liquidity of the FRN [0.5]

[1 maximum for two relevant examples]

Responsiveness – the credit rating may not be refreshed frequently … [0.5]
… so the spread could widen, or there could even be default, before the FRN was downgraded. [0.5]
This could also cause significant spikes in the price of the FRN when the bond is finally downgraded and the coupons increased. [0.5]

Scope – it is possible that the FRN may cease to be rated by the agency. [0.5]

Consistency – the downgrade is triggered using a single credit rating rather than gaining wider perspective from a group of credit rating agencies. [0.5]

Transparency of fee structure – this may not be sufficiently transparent… [0.5]
… and consequently the credit rating agency may have too familiar a relationship with the FRN issuer… [0.5]
…, possibly opening themselves to undue influence or being misled. [0.5]

Competition – the credit rating agency may have a monopoly position in the market… [0.5]
… which may reduce its incentive to continue to develop and adjust to changing market conditions. [0.5]

[Other valid points may be made]

[Max 3]
[Total 19]

6 (i) Minimum $X = 0$ [0.5]
Maximum $X = 0.20$ [0.5]

[Total 1]

(ii) Let $P(Y)$ = the probability of default of $Y$ and $P(\bar{Y})$ = the probability of survival of $Y$. 
The following probabilities of default in year 1 are as follows (where strike through indicates survival):

\[
P(A \cup B) = P(A) + (B) - P(A \cap B) = 0.45
\]
[1]
[or could calculate this as the sum of the two values below plus
\(P(A \cap B) = 0.15\) as stated in the question, i.e. 0.25 + 0.05 +0.15 = 0.45]

\[
P(A, B) = P(A) - P(A \cap B) = 0.25
\]
\[
P(\overline{A}, B) = P(B) - P(A \cap B) = 0.05
\]
[1]

There are four possibilities that would lead to payoff of the derivative at the end of the second year with the following losses:

Both default in Y1: \(P(A \cap B) \times 100 \times 0.7 = 10.5\) [0.5]

No default Y1, both default in Y2:

\[
(1 - P(A \cup B)) \times P(A \cap B) \times 100 \times 0.7 = 5.775
\]
[1]
[or \((1 - P(A \cup B)) \times P(A \cap B) = 0.0825\)]

A defaults Y1, B defaults Y2:

\[
P(A, B) \times P(B) \times 100 \times 0.7 = 3.5
\]
[1]
[or \(P(A, B) \times P(B) = 0.05\)]

B defaults Y1, A defaults Y2:

\[
P(\overline{A}, B) \times P(A) \times 100 \times 0.7 = 1.4
\]
[1]
[or \(P(\overline{A}, B) \times P(A) = 0.02\)]

Hence the total present value of the losses, is equal to:

\[
21.175 \times e^{-0.03 \times 2} = $19.9m
\]
[1]
[or \((0.15+0.02+0.05+0.0825) \times 100 \times 0.7 \times e^{-0.03 \times 2} = $19.9m\]

[Note to markers: give the full marks if correct answer reached, due to “Calculate” command verb.] [Max 5]
Appropriate x axis based on answer to (i) [0.5]
Correct plot against \( X = 0.15 \) based on answer to (ii) (even if figure there was incorrect) [0.5]
Upward sloping [0.5]
Intersects y axis at a non-zero present value [0.5]
Plausible curvature (i.e. close to linear) [0.5]

Max 2

(iv) Sample random values from a \( N(0,1) \) distribution. [0.5]

Need two sets of such random values, one for each bond. [0.5]

Need to sample from a multivariate distribution using a correlation of \( \rho_{AB} \). [0.5]

\( \rho_{AB} \) could be estimated from the correlation between equity returns for the two companies. [0.5]

This produces correlated pairs of values \( x_A, x_B \) with the correct correlation structure, but still with a \( N(0,1) \) distribution. [0.5]

Carry out a percentile-to-percentile transformation from these values… [0.5]

… to simulate the times to default of Chain A and Chain B, \( t_A, t_B \). [0.5]

To do so, first estimate the cumulative probability distributions, \( Q_A, Q_B \), for these times to default … [0.5]

… from data produced by credit rating agencies… [0.5]

… or from bond prices. [0.5]

Simulated pairs of default times \( t_A, t_B \) are then derived by comparing the \( x_A, x_B \) to the cumulative probability distributions, \( Q_A, Q_B \) using the Gaussian
copula relationship \( x_i = N^{-1}[Q(t_i)] \), where \( N^{-1} \) is the inverse of the standard cumulative normal distribution.

[Note to markers: the above could be described using the normal rather than inverse normal (i.e. \( N[x] = Q(t) \)), or alternatively by comparing the sampled \( x \) against \( N^{-1}(p_i) \) where \( p_i \) is the cumulative probability of default up to the end of year \( i \) and then assuming default in year \( i \) if \( x \) is less than \( N^{-1}(p_i) \) but greater than \( N^{-1}(p_{i-1}) \) etc.]

The simulated default times across all sampled pairs can then be used to determine simulated numbers and hence probabilities of default of each bond within the two years. [0.5]

Allowance would then also need to be made for the loss given default and discounting. [0.5] [Max 4]

(v) The financial institution (FI) may wish to hedge existing exposure to credits of Chain A and Chain B… [1]
… and using a Second-to-default basket would be cheaper than buying separate protection on both chains. [0.5]

Or it would allow a reduction in concentration risk to global supermarkets… [0.5]
… even if the FI had a positive view on both credits. [0.5]
The FI may speculate that the correlation implied by the banks pricing is too low… [0.5]
… and hence wants to go long on credit correlation (which buying the second-to-default basket effectively is). [0.5]
The basket swap also provides protection against default tail risk. [0.5] [Max 2]

(vi) The equity price of Chain C is volatile, and so market convention is now likely to require the swap to be over-collateralised [1]

Crucially, the equity of Chain C will be likely be negatively correlated with the price of the derivative… [1]
… given that Chain C is also a global supermarket. [0.5]
So as the likelihood of a combined default of Chain A and Chain B increases, the bank would have to purchase more and more equity of Chain C to ensure the swap remains fully (or over) collateralised… [0.5]
… which would be unattractive as this the equity may not be available in the market… [0.5]
… and it could lead to the bank being selected against by market participants. [0.5]

In the extreme scenario when Chain A and B default and the derivative is required to be settled, it is likely that the equity of Chain C will have fallen dramatically, or even be worthless. [0.5]
Even if the equity of Chain C did have some residual value, the liquidity of the equity may be limited in a stressed environment…

… and it may not be possible to liquidate the holding sufficiently quickly to settle the derivative

So on settlement the holder of the derivative would likely have to pursue a claim against the bank once the collateral has been exhausted.

It could be argued that post the default of Chain A and Chain B, Chain C may face less competition and so its shares might perform well.

This would be most likely if the default of Chain A and Chain B were due to specific factors only affecting these Chains (e.g. aggressive expansion into new markets).

Overall, using the equity of Chain C would likely be an ineffective source of collateral.

[Other valid points may be made.] [Max 4] [Total 18]

7  (i)  Without loss of generality let $t > s$.

From the definition of covariance:

$$\text{cov}[W_s, W_t] = \mathbb{E}_\mathbb{P}[\{W_s - \mathbb{E}_\mathbb{P}[W_s]\}\{W_t - \mathbb{E}_\mathbb{P}[W_t]\}] .$$

From the definition of $\mathbb{P}$-Brownian motion,

$$\mathbb{E}_\mathbb{P}[W_s] = \mathbb{E}_\mathbb{P}[W_t] = 0 .$$

Therefore, $\text{cov}[W_s, W_t] = \mathbb{E}_\mathbb{P}[W_s W_t] .$

The corresponding time intervals $[0,s]$ and $[0,t]$ are overlapping. Expressing $W_t$ as the sum of independent random variables $W_s$ and the increment $W_t - W_s$ gives:

$$\text{cov}[W_s, W_t] = \mathbb{E}_\mathbb{P}[W_s (W_s + W_t - W_s)] ,$$

$$= \mathbb{E}_\mathbb{P}[W_s^2 + W_s (W_t - W_s)] ,$$

$$= \mathbb{E}_\mathbb{P}[(W_s)^2] + \mathbb{E}_\mathbb{P}[W_s (W_t - W_s)] .$$

Due to the independence of the terms in the second expectation it follows that

$$\text{cov}[W_s, W_t] = \mathbb{E}_\mathbb{P}[(W_s)^2] + \mathbb{E}_\mathbb{P}[W_s] \mathbb{E}_\mathbb{P}[(W_t - W_s)] ,$$

$$= s + 0 ,$$

$$= s .$$


Similarly, if $s < t$ then $\text{cov}[W_s, W_t] = t$. \hfill [0.5]

Therefore, for any $s$ and $t$:

$\text{cov}[W_s, W_t] = \min\{s, t\}$. \hfill [0.5] \hfill [Max 3]

(ii) Using Taylor’s theorem for $V(S_t, R_t, t)$:

$$dV = \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial S_t}dS_t + \frac{\partial V}{\partial R_t}dR_t + 0.5 \frac{\partial^2 V}{\partial t^2}(dt)^2 + 0.5 \frac{\partial^2 V}{\partial S_t^2}(dS_t)^2 + 0.5 \frac{\partial^2 V}{\partial R_t^2}(dR_t)^2 +$$

$$\frac{\partial^2 V}{\partial S_t \partial t}(dS_t)(dt) + \frac{\partial^2 V}{\partial R_t \partial t}(dR_t)(dt) + 0.5 \left( \frac{\partial^2 V}{\partial S_t \partial R_t}(dS_t)(dR_t) + \frac{\partial^2 V}{\partial R_t \partial S_t}(dR_t)(dS_t) \right) +$$

higher order terms. \hfill [2]

$(dt)^2 = 0 + \text{higher order terms.} \hfill [0.5]$

$(dt)(dW_t) = 0 + \text{higher order terms.} \hfill [1]$

$(dt)(dZ_t) = 0 + \text{higher order terms.} \hfill [1]$

$(dW_t)^2 = dt + \text{higher order terms.} \hfill [1]$

$(dZ_t)^2 = dt + \text{higher order terms, as both } W \text{ and } Z \text{ are Brownian motions.} \hfill [1]$

Using the definition for $Z$:

$$dZ_t = \rho dW_t + \sqrt{1-\rho^2} dB_t$$ and so

$$dW_t dZ_t = \rho (dW_t)^2 + \sqrt{1-\rho^2} dW_t dB_t,$$

$$= \rho dt + \sqrt{1-\rho^2} dW_t dB_t + \text{higher order terms [using the work above]},$$

$$= \rho dt + \text{higher order terms [using the result given in the question} \hfill [1]$$

$$dW_t dB_t = 0].$$

Using the results above gives:

$$(dS_t)^2 = \sigma^2 S_t^2 dt + \text{higher order terms,} \hfill [1]$$

$$(dR_t)^2 = \alpha^2 R_t^2 dt + \text{higher order terms,} \hfill [1]$$

$(dt)(dS_t) = 0 + \text{higher order terms,} \hfill [1]$$
(\text{dt})(dR_t) = 0 + \text{higher order terms},
(dS_t)(dR_t) = (dR_t)(dS_t) = \rho \sigma \alpha S_t R_t \text{dt} + \text{higher order terms.}

[2]

Putting these into the Taylor expansion gives the required result (by ignoring the higher terms as stated in the question).

[0.5]

[Max 6]

(iii) The equation is Ito’s lemma generalised to two assets…
with different Brownian motions.

[1]

[0.5]

[Max 1]

(iv) **Construction of** $Z_t$

The correlated Brownian processes can be used in the simulation of two correlated assets.

[1]

For example, equity prices generally move together in the same direction.

[1]

Such assets may also arise in the case of an asset which is dependent on another asset.
This may be useful in determining the price of derivatives based on such assets, such as a basket option.

[0.5]

[0.5]

For example by using a Monte Carlo simulation approach.

[0.5]

The method set out in the question for the construction of $Z_t$ from two independent standard Brownian motions can instead be used in reverse, to take two correlated Brownian motions and create independent Brownian motions.

[0.5]

Independent Brownian motions can be easier to work with.

[0.5]

**Equation in (ii) (Ito’s lemma for 2 assets)**

This can be used for modelling processes which involve two assets.

[0.5]

For example, it can be used as a basis for deriving a Black-Scholes equation using two assets.

[0.5]

This can lead onto pricing options whose payoff is dependent on two assets.

[0.5]

[Max 3]

[Total 13]

**END OF SOLUTIONS**