COMPOSITION OF THE F.T.-ACTUARIES
SHARE INDICES

by

ERIC SHORT and JOHN C. H. BRUMWELL

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1. INTRODUCTION

1.1. The FT-Actuaries share indices have now been published for about 11 years and calculated for a rather longer period. They have become accepted as one of the standard measures of equity share price movements. They represent a valuable contribution by actuaries in the field of investment analysis, indeed Heywood (1) states that he considered them to be the best public relations exercise of the Institute. (We assume here that he meant profession).

1.2. Yet, there are a considerable number of actuaries and actuarial students who have no more than a vague knowledge of its working, while most laymen are completely baffled by the mathematics involved. Haycocks and Plymen (2), in an extremely comprehensive and often very elegant paper, described the basic construction of the indices. But because they were actuaries addressing other actuaries they naturally used algebraic formulae. This, in itself, automatically ensures that most laymen in the investment field do not read beyond page 1.

1.3. Thus, one object of this paper is to describe, in rather simpler language, the basic construction and to give much more attention to the methods of collecting the basic data used in the calculations. This point was not covered by Haycocks and Plymen (2) and there is a steady stream of enquiries on the basic construction. We give the mathematical formulae in the appendices.

1.4. Since that famous paper, all that appears within the covers of J.I.A. and T.F.A. is the annual article (in recent years by J. C. H. Brumwell) describing the various changes that have taken place over the previous year. The joint Institute and Faculty index constituents and classification committee meets once a quarter to consider
any necessary changes to the design and construction. But, no further articles have been written and presented to the profession for discussion. The authors feel that this paper will fill a gap in the literature on the F.T.-Actuaries share indices.

1.5. We describe very briefly certain uses of the indices with which we are somewhat familiar. There should be many other uses that can and are being made, but so far nothing has been published. Finally, we conclude by advancing certain ideas for future development in this and allied fields.

2. THE FORM OF THE EQUITY INDICES

2.1. The formulae used and the reasons for their eventual choice were described in detail by Haycocks and Plymen (2) and are reproduced in Appendix I. Quite simply it is the total of the current market capitalization of the constituents in each index divided by the adjusted base capitalization for that index. Just how suitable this formula is will be shown in describing the uses of the indices.

2.2. The market capitalization of a constituent is the product of the number of shares and the market price of each share. The following sections of the paper describe how each of these elements is obtained.

2.3. The denominator in the formula is the market capitalization at the base date (for most indices 10 April 1962—the date of introduction of the original ‘short-term’ capital gains tax), adjusted for all capital changes since the base date that affect that particular index. Since rarely a day passes without at least one capital change, current base values bear little resemblance to the initial base values.

2.4. For calculating the adjustment, Haycocks and Plymen (2) gave a very elegant mathematical formula which frightens off most laymen and not a few actuaries. But it is quite simple really. The adjustment is such that the index is unaltered when recalculated with the capital change(s) incorporated. This is the whole meaning of a ‘chain-linked’ index. The formula, for those who like mathematical symbols, is given in Appendix II.

2.5. When a new Group or Subsection is brought into the series, the initial value of the index is that of its immediate superior. Thus when the Insurance Brokers Subsection was introduced at the very beginning of 1968, it took as its initial value that of the Financial Group on 29 December 1967, namely 96·67. The purpose of taking
this starting value rather than, as might be expected, 100 is that it does give a more realistic starting value, when related to the other indices, with a 10 April 1962 value of 100. It might look odd, especially to the layman, to see an index at 100 when all other indices in the group stand above 200.

2.6. Similarly, when an existing Subsection is split, the starting value for the offshoots is the value of the Subsection just before the subdivision. Thus both Engineering Subsections have a base value of 153.84, the value of the (combined) Subsection index at the end of 1971 before the split. Thus for both a new index and for the indices arising from a split, the base value is automatically determined. The only exception to this rule was made when the Industrial Group (see paragraph 3.5 below) was introduced at the beginning of 1971. Here one of the authors, after a long slog, estimated what it would have been had it started at 100 on 10 April 1962, at the end of each quarter over the intervening 8¾ years. It was later estimated by Marks & Stuart (3), that the error involved in these calculations, which are also explained in their paper, was less than ±1 point in the value on 31 December 1970.

3. CLASSIFICATION OF THE CONSTITUENTS

3.1. When the indices started in 1962, the index committee carried out the first comprehensive, and logical, classification of companies by industry that the investment world had seen. Haycocks and Plymen (2) described the methods used to determine such a classification. This was kept under review by a subcommittee as part of the running of the index. But, strange to say, there was no formal definition of what constituted each Subsection. This was a definite weakness, especially when explaining to non-actuaries, since different people had varying ideas of how to classify. Machine Tools have always been very troublesome.

3.2. Then the ‘4000 share subcommittee’ of the Institute and Faculty undertook the mammoth task of classifying all quoted companies on what was then the London Stock Exchange, publishing their findings at the end of 1969. This classification was modelled on the pattern of the existing FT-Actuaries classification, but went much further and in much greater detail. In addition, they produced a precise definition of each industrial and financial classification, which has been accepted by the investment world. A description of the initial work was given by Plymen (4).
3.3. In turn, the classification of the FT-Actuaries index constituents was brought into line with the classification of all quoted securities. The updating of these classifications, in all aspects, is now part of the work of the Index constituents and industrial classification committee, which meets quarterly as described in the Year Book. Changes made to the classification of quoted securities, which are constituents of the index, are immediately implemented in the FT-Actuaries classification when they are applicable.

3.4. The classification is shown in the Stock Exchange Daily Official List by means of a code number against the stock. Changes in classification are published in the *Stock Exchange Weekly Intelligence*. Every 12 or 15 months, which in our opinion is rather too infrequently, the Stock Exchange publishes a complete list of companies under each heading.

3.5. The method of grouping needs no comment, except on one point. The acceptance of it by the investment world is an adequate testimonial. The one point of difference is the placing of the Oil Group. This aroused comment right at the start when it was placed as part of the 500 Share Index representing U.K. industrial companies. While actuaries were unanimous on this point, others in the investment field were not. Their views were summed up in a paper by Conlong (5). To accommodate this point of view, the indices have been quoted 'cum' and 'ex' oils since the beginning of 1971, in the form of the 500-Share and Industrial indices. The compromise seems to be working well.

4. CHOICE OF CONSTITUENTS

4.1. Haycocks and Plymen (2) gave a description of the method used in determining the original constituents. From a review of all the equities with a market capitalization exceeding £1 million, quoted on what was then the London Stock Exchange, some 650 companies with a capitalization greater than £4 million accounted for 90% of the total market capitalization of these companies. The choice was based on these 650 companies.

4.2. To fulfil the objects of the indices, certain categories were and still are excluded. They were:

(a) Companies whose trading activities were almost completely outside the United Kingdom such as Inchcape (despite the recent take over of Mann, Egerton). The reason for this was
that the indices represented an attempt to reproduce the
performance of U.K. industry as reflected in the share price
of U.K. equities. With the expansion of overseas interests
within many U.K. companies, it is becoming more difficult
to apply this distinction.

(b) Companies which are controlled by other companies which
are themselves in the indices (e.g. Imperial Metal Industries
which is controlled by Imperial Chemical Industries). This
is to follow the principle of avoiding, except for investment
trusts, double counting.

(c) Companies, other than bona-fide investment trusts, where
a major part of the assets consist of shares already in the
index. Such a company is Burmah Oil, which has large holdings
in British Petroleum and Shell Transport and Trading.

4.3. Originally, industrial holding companies were excluded on the
grounds that it was impossible to allocate such companies to specific
industrial groups. Subsequently, however, seven were included in
the 500 Share and All-Share indices under Miscellaneous (Un-
classified), when the Steel Subsection disappeared on re-nationaliza-
tion in 1967, and others have been added since.

4.4. Investment Trusts are included, since they represent a
recognizable part of the market and of the average portfolio, despite
offending against the 'double counting' rule. They are still under-
represented proportionately in the index, although since 1971 they
have been a separate Group, rather than a Subsection of the Financial
Group, and their representation has been doubled: this could not be
done earlier without their weighting swamping the Financial Group.

4.5. A constituent is removed under the following circumstances:

(a) It is taken over by another company and its quotation ceases
on the Stock Exchange. The constituent is removed as soon
as the takeover or merger becomes effective, even though its
quotation will linger on in the Stock Exchange Daily Official
List for some weeks or even months.

(b) The quotation of a company has been suspended and it has
become apparent that the suspension is something more than
temporary. The timing of the removal of the constituent is a
matter of judgment.

(c) The company has announced that it has appointed a receiver.

(d) The market capitalization of a constituent has fallen below
£2 million and has remained below this level for a period of more than 12 months. It is removed at the end of the period. There are exceptions in the case of some of the constituents in the commodity indices, where smaller companies have to be included to maintain a reasonable number of constituents in the index.

4.6. The decision on the replacement for a constituent removed is much more of a personal choice, i.e. that of the authors. The main guide is that it is sufficiently large, at present with a market capitalization greater than £6 million (except for replacements in the commodity indices). It is usual, but not invariable, to find a replacement with the same classification as the constituent which has been removed. The main purpose is purely convenience, although it is important that the number of constituents in a section does not get too small.

5. NUMBER OF SHARES

5.1. This is the first of the two elements used in the calculation. Basically, the number of ordinary shares in issue which have a stock market quotation are used. For instance the Granada Group ordinary, which are not quoted, are excluded, leaving only the ‘A’ shares as the representation in the index.

5.2. There are other rules:

(a) Capital shares (i.e. those which receive dividends in the form of scrip issues rather than in cash) are excluded. The reason is that the portfolio performance to be measured includes income performance and that requires cash dividends. The Investment Trust Group is the one most affected, but the practice of issuing these shares is slowly growing among industrial companies.

(b) Where more than one class of equity capital, such as ordinary and ‘A’ ordinary shares, has been issued and is quoted for a company the total number of shares in issue is taken, but the price used is that of the larger or largest issue. If the nominal value is different (e.g. Hambros), then the number of shares in issue is adjusted to the equivalent number of shares of the selected nominal value. The reasons for these practices are simply to keep one price and one share number for each company.
(c) The whole of the quoted issued capital is taken. Although a portion of the capital may be firmly held by other companies or institutions, this is ignored. For instance, the whole equity capital of £386,070,116 of British Petroleum is included, even though 48·2% is held tightly by the Government, 21·2% by Burmah Oil, and 2·2% less tightly by Distillers. This line of action is purely administrative. It would be more realistic to use the number of shares that are freely marketable, but the clerical work involved in setting up and updating such a system would be considerable, far beyond the manpower at present available, even if the information required was freely available.

5.3. Share increases due to scrip and rights issues and share splits are appropriately brought into account on the day the shares are quoted ex-scrip, ex-rights or under their new nominal value. Other equity capital increases are brought into account as soon as is practicable. The Stock Exchange issues a list daily showing, inter alia, the number of shares for which that day various companies first had permission to deal on the Stock Exchange.

5.4. Some companies have not changed their equity capital for decades. Others are continually issuing small amounts of fresh equity capital to pay for acquisitions or under share option or incentive schemes. To keep the administrative work to reasonable proportions, shares are added when the market capitalization concerned is £100,000 or more. Otherwise details are recorded and accrued until it is appropriate to add them on.

6. SHARE PRICE

6.1. The second element in the calculation of the indices is the share price of the constituent. This is taken as the middle market price and it is not adjusted for any dividends declared prior to the stock going xd or for any other items. It is traditional to use middle-market prices in almost all share price indices. It is the most realistic on the Stock Exchange since, in theory at least, the jobber will vary the spread of his turn around the middle-market price according to the size of the deal. One could use the buying or selling price quoted in the Stock Exchange Daily Official List, but so far no use has been made of this source of share prices, and they are seldom altered
to reflect minor price variations, and would become available too late for use in the next morning’s daily papers.

6.2. It is axiomatic that share prices rarely stand still during the course of the day, certainly it is so with the leaders. The problem is which share price to take in the calculation of the indices. The purpose of the indices can be important. If the purpose is to measure the movement of share prices during the day, then ideally we want the latest possible share price. This is what is done for the closing index of the *Financial Times* Industrial Ordinary Share Index. If the purpose is to measure portfolio performance, then getting the latest price of the day is not so important. What is vital is that all the prices should be obtained at approximately the same time.

6.3. On the other side of the coin there is the time factor involved in the collection of the prices, transmitting them onto the computer and verifying the data. This takes a considerable amount of time and an early start has to be made in order to have the indices ready for the first edition of the *Financial Times*. Thus, the usual British compromise has to be made.

6.4. The present system is that each day a series of sheets listing certain stocks are given to the jobbers who deal in those stocks. During the afternoon, the jobber’s clerks fill in the sheets marking the latest price against those stocks where there has been a change in price. These are collected between 2.30 and 3.00 and brought back to the *Financial Times*. In due course they are transmitted to the computer. Certain specialized sectors, such as gilt-edged and oils have the closing price punched in later, where there has been a further change.

6.5. The accuracy of the calculations is almost entirely a reflection of the accuracy of the information fed in. But, because of the time limits, it is not practicable to check every price punched. A system has been devised whereby every price change exceeding certain percentage limits is queried and has to be cleared before the calculations can be done.

6.6. At the moment, virtually two price collections per day are made by the *Financial Times*. Those prices collected between 2.30 and 3.00 are used for both the Exchange Telegraph ‘Focus’ service and to calculate the FT-Actuaries indices. There is not a complete one-for-one correspondence between these prices and those which appear in the newspaper’s Share Information Service, the latter being able to incorporate some later prices. Now that the *Financial
Times have their own computer and use it to set up the Share Information Service, in due course it will be used to calculate the indices.

6.7. We are indebted to the Financial Times Stock Market Editor, F. H. Fisher, for his assistance with this section of our paper.

7. DIVIDEND YIELD

7.1. The formula for the dividend yield per cent for a particular Group or Subsection is obtained by dividing the total dividends payable for the constituents in the index by the total market capitalization of those constituents. The dividend for a constituent is the number of shares multiplied by the dividend per share. As the formula in Appendix III shows, it represents a weighted average of the dividend yields of the individual constituents, the weights being the individual market capitalizations.

7.2. From the very beginning of the indices the dividend yields calculated have been gross yields, although there is a lot to be said for calculating them on a 'net' basis. With the introduction of the so-called 'imputation' tax system, it was decided to continue to show yields on a gross basis by taking the 'actual' dividend paid grossed up by the Advance Corporation Tax (ACT) rate. It is perhaps open to question whether the change should have been made to the 'actual' dividend declared and the decision not to do so was based on the continuity of the series. A study of the two authoritative documents on the 'imputation' system—that by the Institute of Chartered Accountants (6) and that by the Society of Investment Analysts (7) shows that these august bodies were so concerned with earnings that they virtually ignored the question of dividends.

7.3. The dividend rate used is the most up-to-date annual rate for the company. This is in general the last declared annual rate (grossed up for the current rate of ACT), adjusted for any interim changes and updated for any firm (and precise) dividend forecasts made by companies for the current year or even for the coming year. However, in these days of dividend limitation, dividend forecasts are rare.

8. EARNINGS

8.1. The history of the FT-Actuaries indices earnings is largely a history of the changes in corporate taxation. Since 1962 there have been three different systems operating.
8.2. The first was the system of income tax and profits tax, with the dividends being paid net. Earnings yields were shown for all Groups and Subsections except certain Financial Subsections, such as Banks, Insurance and Discount Houses.

8.3. In 1965, corporation tax was introduced. This taxed company profits and dividends were payable gross. After the changeover the price earnings ratio was quoted, a concept familiar to foreign analysts but until then little used in the U.K. Under this system the price earnings ratio was the inverse of the earnings yield per cent. This may seem superfluous, but at that time, the investment world quickly accepted the concept of price earnings ratios. Earnings yields did not appear to be much used outside actuarial circles, and number of times dividend cover was the statistic which had been most frequently quoted.

8.4. 1973 has seen the introduction of the 'Imputation' tax system. Briefly, the tax on the dividends could be regarded as an advance payment of corporation tax and the recipient of the dividend has a corresponding tax credit 'imputed'. How to calculate earnings under this system has caused a storm of controversy in investment circles. The authors and other members of the Index committee attended many meetings between accountants and investment analysts and listened to innumerable arguments as to which method to use to calculate a price earnings ratio. A summary of the methods is given in Appendix IV.

8.5. The final result was an impasse. The Institute of Chartered Accounts issued a circular (6) recommending the 'net' method. The Society of Investment Analysts (7) issued their report advocating the 'nil' method. The Financial Times opted in February 1973 for the 'net' method in the Share Information Service despite certain pressures; the Investors Chronicle opted for 'nil', while other daily papers initially sidestepped the issue but have now come into line, The Times adopting the 'net' method on 11 October 1973.

8.6. There has been no official discussion by the actuarial profession on this subject, which in view of their contribution to investment thought is perhaps a disappointment. The concensus amongst the committee was for the 'net' method, although there was a strong minority backing the 'nil' advocates. The arguments for and against are summed up in two articles by Short (8) which appeared in the Financial Times.

8.7. There are valid arguments for both methods, so with the
agreement of the Financial Times, it was decided to show price earnings ratios in the FT-Actuaries Index Display on both the 'net' and 'nil' basis and the earnings yield on a maximum distribution basis. By this means the price earnings ratio most appropriate for a task can be chosen and used. The full use of these price earnings ratios on an 'imputation' basis will come out as the investment world gets to grips with the system. At present, it is being notoriously slow to accept that 'imputation' is with us.

8.8. The formula used for the earnings yield is similar to that for dividend yields, replacing dividend per share by earnings per share. Again it represents a weighted average of individual earnings yields. (See Appendix V).

8.9. Similarly, the price earnings ratio for a Group or Subsection is the total market capitalization divided by the sum of the earnings (on the appropriate basis) of the constituents. This is a weighted average of the individual ratios, the weights being the individual earnings (see Appendix VI). Under 'imputation' the earnings yield is no longer the inverse of the price earnings ratio.

8.10. The object of calculating the earnings is to obtain the annual rate of earnings. Such a concept, hypothetical though it is, comes naturally to actuaries, but many laymen just cannot visualize it. Thus from the presented accounts all items of a capital, exceptional or non-recurring nature are excluded from the calculations. This procedure is taught to actuarial students in ascertaining priority percentages.

8.11. The basic material is the annual report and accounts of a company. At present, earnings are updated for quantitative pre-tax profits forecasts relating to the current year, and use is made of the preliminary announcements of certain of the leading companies. But no updating is done for half-yearly figures, a bad omission if the objective is to ascertain the latest annual rate of earnings. We can only plead a present lack of staff at the Financial Times although the position is under review. Negative earnings are included if they arise.

8.12. When a merger or takeover occurs, the earnings are consolidated as far as possible. Similarly, on the conversion of a large amount of loan stock, a re-calculation of the diluted earnings is made. For a rights issue, it is assumed, perhaps harshly, that the new money will not earn anything in the current year.

8.13. There is a time lag of a few days between the issue of a company's report and the updating of the earnings whilst the figures
are calculated and checked. A discontinuity in the price earnings ratio arises here since the reaction of the share price to the change in earnings takes place when the preliminary figures are announced, some weeks before the accounts appear and the earnings are updated.

8.14. With a change in the rate of corporation tax, we endeavour to estimate the earnings on the new rate and change the file as soon as possible after announcement of the change. After a considerable amount of trial and error, we had devised a system which enabled us to make the alteration on the same day as the announcement, with the computer doing the work. On the new ‘imputation’ basis we had to rearrange the layout of each record and we were back to square one regarding effecting a change in earnings with a tax rate change.

8.15. Possibly, we have not given this aspect of the indices enough thought or consideration. It certainly requires considerable clerical time and effort and, as far as the authors’ experience goes, is the aspect which most people enquire about.

9. FIXED-INTEREST INDICES

9.1. The space devoted to these indices has been deliberately kept short for the following reasons:

(a) To do it justice would require a complete paper on its own.
(b) Since the series began in 1962, the techniques used in the fixed interest market have changed and improved out of all recognition. They have become very sophisticated, actuaries having played a dominant role in this development.
(c) Considerable changes are being considered by the committee, so this is not the best time to make a detailed analysis of the fixed interest indices quoted.

9.2. The formula used is given in Appendix VII. The aim was to show yields (i) on long-term and medium-to-long-term gilt-edged, (ii) on redeemable debentures and unsecured loan stocks of comparable term and (iii) on preference shares.

9.3. All the yields, except that on Consols, are converted into indices. To an actuary this may seem to be a rather doubtful and unnecessary step. Actuaries are taught from infancy to think in terms of yields. In contrast, the layman thinks almost exclusively
in terms of stock prices and possibly running yields. It is primarily for his benefit that indices are shown, since they can be related to price.

10. FIXED-INTEREST PRICES

10.1. Gilt-edged stocks are the most readily dealt in of stocks and consequently the prices obtained from the stock jobbers are precise and reflect fractional changes.

10.2. The prices of the preference shares, redeemable debentures and loans are the very opposite, the least reliable. The reason is that there is a very thin market in all these stocks. Often the jobber will not deal for weeks in a particular stock. Movements, when they occur, tend to be quite large as the jobber brings the prices into line with the movement of interest rates. This factor has to be carefully considered in any design of fixed-interest indices.

11. CHECKING THE INDICES

11.1. In addition to the precautions taken by the Financial Times organization, a number of other checks are applied to the indices aimed at the avoidance of errors which would permanently distort them. This means effectively a continual check on the number of shares of each constituent.

11.2. No check outside the Financial Times organization is applied to dividend yields or earnings; the reasoning behind this, apart from practicality, is that if a wrong dividend or earnings figure is fed into the computer, then the error should be corrected when the new figure is fed in a year later. If the newspaper does arrive at the wrong figure this will also appear in the Share Information Service, and some discerning reader is likely to spot the error; then the wrong figure in the index will also be corrected.

11.3. The checks which are applied outside the Financial Times organization take two forms. There is a daily check that capital changes (of which there are nearly 500 a year) are made correctly, or made at all, and there is a further check made every six months, on the shares held in the computer records. These checks are made by the co-author who is not on the Financial Times staff.

11.4. The daily check takes the form of a telephone call from the Financial Times listing the capital changes to be made that day,
adding $x$ shares at a price $p$ to company $A$, a scrip issue for company $B$, or a 1-for-$k$ rights issue at price $q$ for company $C$, or (perhaps forty times a year) that company $D$ has replaced company $E$. From this information the (net) increase or decrease in the capitalization of each index Group and Subsection is calculated. Consequently, after division by the previous day's indices (see Appendix II), the change in each denominator is telephoned back to the *Financial Times*.

11.5. These answers are then compared with a computer print out of the capital changes and new denominators. Usually any discrepancy proves that the Extel computer's arithmetic is better than that of the checker, but sometimes an error in the data given to the computer is discovered, and either a further computer run is made with correct figures, or it is agreed (in minor cases) to make a further adjustment the next day.

11.6. Further to the daily check, the capital changes actually made the previous week are compared with the new shares first quoted during that week as set out in the *Stock Exchange Weekly Official Intelligence*. This results in a few further capital changes, but since these figures are not always satisfactory (for instance ‘permission to deal’ may be given for the full conversion of a convertible issue, at the first opportunity, when in fact all holders probably do not then convert), an additional check is carried out from the annual report of each constituent comparing shares in issue, with those ‘in the index’.

11.7. Every six months (at the end of March and September—so as not to coincide with ‘end of the year’ pressures) a further check on each constituent is carried out. These checks were more comprehensive and important in the early years of the index since then the daily checking system was not so well developed.

11.8. The first stage of the check is to compare the ‘sedol’ code number of each constituent in the computer with that in the Stock Exchange Daily Official List (initials = sedol). It is not unusual for a name change or perhaps change in denomination of a share to result in a change of ‘sedol’ number, and if it is not picked up then (as has happened) price changes under the new number are not accepted by the computer which ‘sticks’ at the value fed in under the old number.

11.9. The number of shares held in the computer for each constituent is then checked against the manuscript records (against which, of course, changes are considered). In earlier years, the multiplication
of this number of shares by the latest share price was checked, since the figure was then built up by feeding the computer with the day-to-day changes in price—and any mis-punch would lead to permanent error. The procedure is now to punch in the new price, rather than the change in price, so that since any mis-punch would 'come right' at the next price change for that stock (assuming it is correctly punched), the multiplication by the computer is no longer checked. However, the capitalizations recorded for each constituent are recorded, and when a constituent fails to reach a qualifying level (currently £2 million, but probably to be increased) for three such successive six-monthly valuations (i.e. for at least twelve months), the opportunity is taken of replacing it with a more 'weighty' company.

12. USES OF THE INDICES

12.1. Haycocks and Plymen (2) listed at the beginning of their paper five principal uses to which they considered the FT-Actuaries share indices would be suitable. They were: (a) Investment Policy, (b) Portfolio Performance, (c) Historical Studies, (d) Economic Surveys and (e) Other day-to-day investment problems.

12.2. It is, in our opinion, somewhat disappointing that over the ten years since that paper nothing has been presented in actuarial circles describing in detail how the indices can be used in practice and giving the opportunity for discussion on these ideas. From time-to-time enquiries are made to the Financial Times on the use of the FT-Actuaries share indices and other than the original paper there is nothing to which they can be referred.

12.3. In sections 13 and 14 we attempt to describe, in bare outline, some of the uses which can be made of the FT-Actuaries indices and of which the authors have some little experience. There are other uses where we are not really qualified to discuss and we can only hope that somebody, somewhere, will be prompted to present something on this particular aspect to the profession. Portfolio Performance, the most important use of the indices, merits the more detailed treatment given in sections 15 to 17.

13. MEASURING SHARE PRICE MOVEMENTS

13.1. In using the FT-Actuaries indices in this respect, there are two separate areas to be considered—short-term movements and
those over much longer periods. There are rather different emphases to be placed on the requirements of an index for these two purposes.

13.2. For short-term movements the prime consideration is the sensitivity of the market and price changes of shares as a whole. In reviewing much longer periods, what is required is an overall picture of the market which incorporates the changing importance of the various sectors and stocks within those sectors.

13.3. Haycocks and Plymen (2) did not discuss short-term movements, but described in detail the use of the indices for historical studies, which must include the past movements of the stock market. In this field, the FT-Actuaries share indices are eminently suitable, the weighting in effect adjusting for the varying conditions of the stock market. Records covering a period of 11½ years have been built up so the indices should now be used more and more for this purpose. The disadvantages of a geometrically constructed index, such as the FT 30 share index, have often been quoted, *inter alia* in the *Financial Times*—see Short (9).

13.4. For short-term movements, there are certain considerations, admittedly more theoretical than practical in many cases. Firstly, the weighting used in the index is most important. Haycocks and Plymen (2) stated that ‘If a purely price index is considered then theoretically the weights should be fixed because changes in the index should reflect changes in price only’. The strong advocacy for the use of the FT-Actuaries to replace the FT 30-Share as summarized by Heywood (1) thus dates after this paper.

13.5. For a price index a change of \( x \) per cent in the price of a constituent today should have the same effect on the index as a change of \( x \) per cent yesterday. This does not necessarily happen with the FT-Actuaries share index. The formula in Appendix I shows the effect of the weights on the averaging of the share price changes. This was very noticeable when in 1969 the two oil companies BP and Shell accounted for 15 per cent of the market capitalization of the All-Share index.

13.6. Secondly, an index of 651 constituents is comparatively sluggish in movement compared with one based on many fewer stocks. A sensitive index is essential for measuring short-term movements. The stock market is very complex: it is usual for the leading stocks to move first with the reaction of the second liners being somewhat delayed. Thus the movement of the All-Share index
is not so clear-cut as that of an index based on a smaller number of leading shares.

13.7. Thirdly, the large number of constituents makes it extremely difficult to make frequent calculations of the All-Share index. The stock market can move very rapidly during the course of the day and an almost continuous calculation of an index would be desired. One of the noticeable features of the stock market is the changes that take place after 3.30 p.m., the official close of the Stock Exchange. At present, late changes are not picked up by the FT-Actuaries indices.

13.8. After pointing out the disadvantages of the FT-Actuaries index, we consider its big advantage in measuring short-term price movements. The number of different subsections for which indices are calculated means that a picture of sector movements is given daily. No other index series provides such an overall picture. A word of caution is needed here. Certain Subsections are dominated by one or two constituents, such as Distillers accounting for 84% of Wines and Spirits, and Rank Organization, 74% of Office Equipment. In these cases, the change in the index largely reflects changes in the share price of the dominant constituent.

13.9. Nevertheless, the FT-Actuaries indices have a useful part to play in the measuring of short-term price movements and are ideal for showing the historic picture of past movements of the stock market as a whole and the relative movement of the Groups and Subsections.

14. A YARDSTICK TO COMPARE INDIVIDUAL SHARE CHARACTERISTICS

14.1. When the financial statistics of an individual share, such as dividend yield, dividend cover or price earnings ratio, are being assessed, absolute values can be considered in forming a judgment, but it is very useful to go much further and look at the values relative to those of other shares.

14.2. To do this, the choices available are either to pick out the statistics of other shares and do a direct comparison, or to make a comparison with some sort of average value.

14.3. The former choice involves first the time-consuming task of choosing the shares for comparison, and then if one has not access to a computer service, tabulating all the statistics of the companies
probably occupying different places in the newspaper Share Information Service.

14.4. The second choice enables a very quick comparison to be made with the relevant Groups or Subsections of the FT-Actuaries share indices. Financial commentators and journalists often talk about an average yield or price earnings ratio, by which they usually mean those given by the FT-Actuaries. It is fairly certain that more use could be made of this information.

14.5. The use of forecast figures in the index averages means that the yardstick is constantly being updated thus ensuring that any comparisons being made will be, as far as possible, between like and like. However, this use of forecasts can be a drawback in portfolio comparisons where it is historical figures that are required. This objection is probably more theoretical than practical.

14.6. Because of the weighting process used, care must be taken when using section yields and price earnings ratios. For some sections dominated by one or two constituents, the section values will be little different from those of the dominant constituent. If for some reason these are distorted, then the section values will be distorted and of less value. Perhaps we should consider giving supplementary information in such cases, although it will mean much more work and a much closer watch on the data than is now given.

15. ASSESSMENT OF PORTFOLIO PERFORMANCE

15.1. In paragraph 43 and part of paragraph 44 of their paper, submitted to the Institute about ten years ago, Haycocks and Plymen (2) wrote:

The Financial Times 30-share Index, however, with its daily quotation, has been widely used as a check on the price performance of life funds, investment trusts, unit trusts, etc. Sometimes, annual reports mention how much better the results have been than the index. Such comparisons are rarely published when the results are adverse! Now that alternative daily indices are available, the 30-share index should no longer be so used, except perhaps for short-term comparisons. Over a period of, say, more than a year the 30-share index will normally lag behind wider-based and more representative series because:

1. The geometric averaging makes for a downward bias.
2. The shares concerned, having been selected 30 years ago, are not perhaps fully representative of the new and growing industries.
3. The 30 constituents representing the industrial 'giants', may have below average scope for growth, merely because of their size.

In fact, former index techniques, designed for manual computation, simply did
not permit the construction of a daily index acceptable as a performance standard. The new Financial Times—Actuaries index does, however, represent a 'standard portfolio'. . . .

In fact we have now seen fairly general adoption of a comparison with the FT-Actuaries All-Share Index by such investment trusts as publish a comparison of their performance and by unit trusts, but it is equally noticeable that where the performance has not been very good the 'easier to beat' 30-share index is still used as the yardstick.

15.2. Again, little if any reference to investment management performance has been published by the Institute since Haycocks and Plymen's paper (2). However there have been several papers in the Investment Analyst, and attention is drawn to the full treatment given to the subject in four recent articles (10). These articles discuss the use of portfolio performance assessment, with formulae, on a basis of whether the management of a fund has come to the right decision as to the optimum time to invest new money, and whether the fund has chosen rightly to invest in equities, fixed interest securities, or property, introducing such concepts as the time-weighted rate of return. The subjects are fully treated in these articles and no useful purpose would be served by repeating the ideas developed there, except to emphasize three points which they correctly make (and which we shall studiously ignore in the concepts on which we enlarge below).

15.3. First, in considering the success or otherwise of investment decisions, comparison should be made of the overall rate of return for alternative investments: (a) net of any capital gains tax liability either payable or in respect of unrealized gains, and (b) after payment of any tax on the income, with due allowance if one investment provides franked income, and the other does not. As an example of the latter point it should be noted that the FT-Actuaries Investment Trust Group index yield is usually some 0.75% less than the yield for the All-Share index. 'Par' for the average trust ought therefore to be not to keep pace with the All-Share index, but (taking the difference in net yield) to beat it by some ½ % per annum.

15.4 Secondly, in the matter of timing, it is not unusual for the highs and lows of the indices to occur in January and December of a year (either way round) indicating that the market was falling (or rising) throughout the year. This would mean that any test of investment management would have its optimum performance for that
year if all the new money was invested at the appropriate (low) end: this would be splendid in theory, but not many fund managers are in a position to decide to invest in that way, particularly with a growing fund, when presumably the money would not be available for investment in a rising market in January.

15.5. The third point we would emphasize is that too elegant a system of analysis of overall investment expertise may easily founder because of the lack of an appropriately objective measure of the value of the property element in the portfolio, since this is not subject to any precision of valuation in the way one can specify the value of a quoted share or government security.

15.6. The assessment of portfolio performance breaks down into two major factors, which, we suggest, are best treated quite separately. These are, first the actual performance relative to the index of the purchases and sales initiated by the fund managers, and secondly the performance of the fund they inherited, for the sake of illustration at the beginning of the year under consideration, or perhaps their period of responsibility.

15.7. So far as the normal fund is concerned the inherited fund is likely to be several times as big as the notional fund built from their activities in both directions but, since their responsibility for their actual activity is greater, the consideration given to the two factors ought to close the gap. Clearly if, for example, the fund managers fail to sell shares which they inherited and which subsequently go down disproportionately they should not be able to disclaim responsibility. But it should be noted that there may have been a reason for such inactivity—for example a large capital gains tax liability.

15.8. The authors are indebted for many of the ideas developed in this section to a number of colleagues in the red building opposite; in particular, Messrs. G. S. Minto, B.A., D. Sirkett, F.I.A., P. J. Nowell, M.Sc., F.I.A., and D. A. Roberts, B.Sc. Certain similar ideas have also been developed by Messrs. Phillips and Drew (11).

16. ASSESSMENT OF THE PERFORMANCE OF THE ACTIVITY PORTFOLIO

16.1. Let us consider first the assessment of performance on actual purchases and sales. A good purchase may be defined as a purchase which appreciated (after an appropriate adjustment for expenses) by the end of the period under consideration by a greater percentage
than the All-Share index; under this definition a purchase which depreciated would still be good provided the index went down further. Similarly a good sale should depreciate more, after disposal, than the index (or appreciate less).

16.2. The way in which this is calculated is to accumulate, for all purchases, 'units' which are the quotient when the cost of a purchase is divided by the All-Share index on the day of purchase. When the performance is to be assessed, the accumulated units are multiplied by the All-Share index on the date of the analysis, and the resultant 'index value' is compared (after an expenses adjustment) with the value of the shares actually bought. It should be noted that this procedure eliminates 'timing' altogether, except in so far as a share bought when depressed relative to the market will earn credit, when it has recovered.

16.3. It might be that purchases could be shown on this basis to have appreciated 5% more than the index. What would be interesting and useful would be to know why—and this is where the structure of the FT-Actuaries Indices is critical. Such appreciation could arise in three quite different ways:

(a) by a selection of shares which performed in line with their Group indices, but with a disproportionately greater weight by purchases in Groups which did well, and less in those which did badly (for example in 1972 the Financial Group index beat the All-Share index by 10.6%, but the Consumer Durables Group trailed it by 11.93%),

(b) by a selection of shares in the same Group proportions as the All-Share, but which performed in line with the Subsection indices, and with a disproportionately greater weight in Subsections which did better than their Group and less in those that underperformed their Group (in 1972 the Electronics, Newspapers, Banks and Insurance brokers Subsection indices all beat their respective Group indices by at least 15%, and the Motor, Packaging, Discount Houses and Composite Insurance Subsections fell short by at least that margin), or

(c) by selection of good shares throughout the market which appreciated on average 5% more than their Subsection indices.

16.4. Of course, this appreciation would really have arisen partly in all three ways; it might well have arisen by positive factors in
two (or one) of them overcompensating for a disappointing performance in the remaining one (or two). Bearing in mind that it is likely that a big fund may well be managed separately by specialists in the main market Groups, it would clearly be instructive to identify the reason for good and bad performance between Groups, using the terms (a) Group selection, (b) Subsection selection, and (c) Company selection to distinguish the three factors enumerated above.

16.5. Now that investment managers can call on computers to maintain their records, this analysis can be achieved quite easily by dividing the cost of each purchase not just by the All-Share index to produce All-Share units, but also by the appropriate Group and Subsection indices for each share purchased, so as to produce additionally Group units, and Subsection units. These can also be accumulated to the end of the period, and by multiplying by the appropriate indices then, the performance of each purchase (or sale) can be assessed:

first, relative to the Subsection index, to give when accumulated a measure of Company selection (c) within each Subsection, or by further accumulation within Groups or overall,
secondly, relative to the Group index, to give when compared with the aggregate Company selection (c) within that Group, a measure of Subsection selection (b) within the Group or as a whole, and
finally, relative to the All-Share index, to give when compared with the aggregate of Subsection selection (b) and company selection (c) for each Group, a measure of Group selection (a).

16.6. By grouping and accumulating these various results as desired, the overall performance of the dealing in the fund can be assessed in any way which management may desire into these components of (a) Group selection, (b) Subsection selection, and (c) Company selection for both purchases and sales in what we may call ‘voluntary’ transactions.

16.7. If such a ‘performance measurement’ system is set up, it is necessary to adopt a number of conventions as to how to deal with such items as rights issues or dealing in a share whose classification is outside the scope of the All-Share index (e.g. Mining Finance).

16.8. In assessing the performance of a fund, it is suggested that
a separate analysis should be made for the transactions made in each
calendar year. If the fund managers are buying or selling on taking
a three-year or longer view, a fair result may not be obtained if
the analysis is made at an earlier date, although it would be as well
to be aware of the situation during the intervening period.

16.9. If, at this stage, members of the stockbroking fraternity are
enjoying the prospect of their client fund-managers belabouring
each other with critical appraisals of their respective fund manage-
ment, all over the City, they should note that this concept need not
be applied to an actual fund. It could quite well be applied by a
fund-manager to the recommendations he receives from different
brokers: at its most simple he would notionally invest £1,000 in each
recommendation, or on a more sophisticated scale he would weight
his notional investment according to the degree of enthusiasm in the
recommendation.

16.10. The fund-manager would then be able to assess the value
of advice he received on, for example, the Financial Group against
that index rather than the All-Share index. This would be fairer
since the broker’s analyst would himself probably profess to be an
expert in the financial field rather than in the whole market. Similarly,
it could be argued that while several stockbroking firms produce
comprehensive reviews of, say, the brewery industry, the use of these
to the fund-manager is to advise him which brewery to buy; it should
not be to advise him to buy breweries rather than stores or electricals:
since the brewery analysts probably do not know enough about the
other two industries to give such advice. If that thesis is accepted
then any such analysis of a ‘brewery’ review should be made using the
Brewery Subsection index, not the Consumer Non-Durable Group
index or the All-Share index.

17. ASSESSMENT OF THE PERFORMANCE OF THE
INHERITED PORTFOLIO

17.1. We previously suggested that the assessment of the invest-
ment performance of a fund should be divided into two parts, that
of the Activity Portfolio, and that of the Inherited Portfolio. In
section 16 we suggested how the Activity Portfolio might be assessed,
we must now therefore consider the Inherited Portfolio, and to do
this we must isolate it from the Activity Portfolio.

17.2. For example, let us consider the whole equity portfolio at
the end of 1971 as our Inherited Portfolio, and review its progress during the calendar year 1972. We have to relate the entire portfolio at the end of 1972 to that a year earlier. Starting, therefore, with the 31.12.72 portfolio we deduct all the ‘voluntary’ purchases made in 1972, and add back the ‘voluntary’ sales made in 1972 (using end-1972 values throughout): these ‘voluntary’ deals can be obtained from the activity analysis to which reference is made above.

17.3. It is then necessary to exclude ‘involuntary’ deals: these will normally be take-overs partly or wholly for cash, or loan stock, or rights issues, conversions and any other item which affects an inherited holding, other than a purchase or sale which has been treated as ‘voluntary’. These ‘involuntary’ happenings are not a result of a management dealing decision, and should not therefore have been dealt with in the analysis of the Activity Portfolio. Since the items above suggested the introduction of cash into the involuntary element, it will be fair to adjust this cash by the index movement from the date of the ‘involuntary’ transaction to the date of analysis. The use of cash instead of later evaluation of a loan-stock is suggested, since the review is of the equity portfolio; the substitution of a convertible is sufficiently near to that of an equity for the stock price to be taken when the analysis is made.

17.4. Once these ‘involuntary’ 1972 deals have been excluded, we are left with the end-1971 portfolio, revalued at the end of 1972. The percentage change over the year is then directly comparable with the rise of 12.82% in the All-Share index over the year in the assessment of the performance of the Inherited Portfolio.

17.5. As in the case of the Activity Portfolio, a much more meaningful review of the year’s performance will be obtained if the whole review is broken down into the FT-Actuaries index Groups and Subsections, and each of these is followed from the end of 1971, excluding the ‘voluntary’ purchases, adding back the ‘voluntary’ sales, and adjusting for the ‘involuntary’ deals. It should be noted here that the simplest ‘involuntary’ take-over, an all-equity bid, would need no adjustment in the All-Share treatment first considered; but if the classification of the two companies concerned differed, would at the subdivided level have to be treated as an ‘involuntary’ purchase in one Subsection and an involuntary sale in the other.

17.6. It is therefore possible to tabulate how each Group and Subsection of the end-1971 portfolio of the fund under consideration
has performed, and from this tabulation we can analyse why the performance of our fund has differed from that of the All-Share index. The initial factor is the 'shape' of the fund—that is to say, how its weighting differs from the weighting of the All-Share index. Once again the performance can be broken down into three components: (a) Group distribution, (b) Subsection distribution and (c) Company distribution.

17.7. If the 'shape' of the fund diverges from the distribution of the All-Share index then, other things being equal, its performance must be expected to diverge appropriately. For example, at the beginning of 1972 the All-Share index constituents were distributed *inter alia* 14.61% in Capital Goods, 28.82% in Non-Durables, and 19.98% in the Financial Group, and during 1972 the Capital Goods index underperformed the All-Share index by 9.41%, whilst the Financial Group beat it by 10.60%.

17.8. If we consider a fund which at the beginning of 1972 had the same distribution as the All-Share index, except that 10% extra was held in Capital Goods, and 10% less in the Financial Group, then this factor alone should produce underperformance of this fund against the All-Share index of 2% (i.e. 10% (9.41% + 10.60%)) in 1972.

17.9. Similarly had a fund had the same shape as the index except that it held no composite insurance shares (index proportion 4.26%) and an extra 4.26% in banks (index 5.09%) then since the Bank Subsection index beat the Financial Group index by 17.84% and the Composite Insurance Subsection index underperformed it by 19.86%, these factors alone should result in the fund beating the performance of the All-Share index by 1.59% (i.e. 4.26% (17.84% + 19.86%)).

17.10. Comparison of the change in value of each Subsection portfolio with the change in its index gives a measure of the effect of Company distribution within Subsections, and if these changes are totalled for the entire portfolio, the contribution of Company distribution to the overall performance of the Inherited Portfolio is obtained.

17.11. It is necessary to adopt some convention to deal with those parts of the equity portfolio outside the scope of the FT-Actuaries All-Share index (probably largely mining finance and overseas trade): a possible solution is to treat those shares within the All-Share index as 100% of the portfolio, and then to
treat all the others as 'excess weightings' relative to a nil contribution to the All-Share index.

17.12. At this stage, we should remark that the portfolio of a fund will probably not be restricted to FT-Actuaries constituents, but by using the Stock Exchange classification of all equities, published in the Daily Official List, the appropriate FT-Actuaries classification can immediately be derived, and should be used in the analysis of both Activity and Inherited Portfolios.

17.12. In Appendix VIII the underlying formula of this analysis of an Inherited Portfolio is derived, and in Appendix IX are set out the suggested headings for a schedule on which such an analysis could be made.

18. FUTURE DEVELOPMENTS

18.1. A popular misconception of our profession is that we are extremely good at predicting the future. In this final section, we shall make no attempt to forecast the future developments likely to take place, but just indicate useful fields which could and possibly should be considered.

18.2. To make readily available the vast amount of statistics that are collected during the course of calculating the indices. At the moment the data is literally going down the drain after it has been used. A financial data bank is being developed outside the profession. We certainly missed the boat in this respect, but there are possibilities of co-operation with those who are setting up data banks based on the FT-Actuaries indices.

18.3. Indices of the share prices of medium and small companies (only) could be calculated. We have been missing a good opportunity here. The indices in *The Times* have separate series for 'largest' and 'smaller' companies.

18.4. With regard to portfolio performance, the indices reproduce the behaviour of a closed fund, such as an investment trust. But most actuaries are concerned with open-ended funds and various methods are used to adapt the present indices: one was given by Ferguson (12). The methods revealed are ingenious, but the main point is that only one specific index should be used for one specific purpose. If you try to use it for several, then errors will occur and misleading results be obtained.

18.5. If the performance of an open-ended fund is to be measured, then an index should be used which reinvests income when it
becomes due. If, on the other hand, an approximation is considered sufficient, then there should be a roll-up index that is officially approved and calculated. At present there is a plethora of, what are claimed to be, FT-Actuaries 'roll-up' indices calculated and quoted in advertising material for bonds and units. In no case has the Index committee even been consulted.

18.6. The lack of a suitable fixed-interest index provoked comment in the report by a working group on the measurement of portfolio performance for pension funds. Actuaries are playing a leading role in a relatively new area as suggested by Haycocks and Plymen (2) and it would be natural if actuaries provided the necessary index.

18.7. Finally, the provision of past records of the FT-Actuaries indices has been non-existent. It is hoped that sheets giving twice-monthly figures of the indices will be available soon. But since this task has been on the stocks for some years, the whole problem of producing adequate records quickly needs a fresh approach.

19. CONCLUSION

19.1. We have endeavoured to describe the present workings of the FT-Actuaries indices and to indicate uses and possible developments. The ideas are our own, and should not necessarily be taken to be those of either the Financial Times or the joint committee of the Institute and Faculty which supervises the index. Our hope is that it will stimulate thought and discussion on the subject.

BIBLIOGRAPHY

(5) Conlong, A. 'The first eight years of the FT–Actuaries All-Share Index', The Investment Analyst, 27, 3.
(6) The Institute of Chartered Accountants in England and Wales: Accounting for Corporation Tax under the Imputation System (September 1972).
APPENDIX I

Equity index formula

The index at time \( t \) is obtained from the formula

\[
I_t = \frac{\sum_{r=1}^{m} N_{r,t} \cdot P_{r,t}}{B_t}
\]

where \( B_t \) is the value of the base at time \( t \),
\( N_{r,t} \) is the number of shares, and \( P_{r,t} \) is the price per share,
both of the \( r^{th} \) constituent at the time \( t \), and
\( m \) is the number of constituents in the index.

Now

\[
I_{t-1} = \frac{\sum_{r=1}^{m} N_{r,t-1} \cdot P_{r,t-1}}{B_{t-1}} = \frac{\sum_{r=1}^{m} N_{r,t} \cdot P_{r,t-1}}{B_t}
\]

since the adjustment to the base for capital changes is such that
\( I_{t-1} \) is left unaltered.

The percentage change in the index is given by

\[
\left( \frac{I_t - I_{t-1}}{I_{t-1}} \right) \times 100 = \frac{\sum_{r=1}^{m} N_{r,t} \cdot (P_{r,t} - P_{r,t-1}) \times 100}{\sum_{r=1}^{m} N_{r,t} \cdot P_{r,t-1}} = \frac{\sum_{r=1}^{m} N_{r,t} \cdot \frac{P_{r,t} - P_{r,t-1}}{P_{r,t-1}}}{\sum_{r=1}^{m} N_{r,t} \cdot P_{r,t-1}} \times 100
\]
Thus the percentage change in the index is the weighted average of
the percentage changes of the price relatives of the constituents,
the weights being the market capitalizations prior to the change.

APPENDIX II

Formula for new base value after capital change
The original base value is

$$B_0 = \sum_{r=1}^{n} N_{r,o} \cdot P_{r,o}$$

Between time \(t-1\) and \(t\), let \(n_{r,t-1}\) be the number of shares added to
the \(r^{th}\) constituent (\(n\) could be negative). Then the value of the
capital changes applicable to time \(t-1\) is

$$C_{t-1} = \sum_{r=1}^{m} n_{r,t-1} \cdot P_{r,t-1}$$

\((P_{r,t-1}\) is as defined in Appendix I, except that it is adjusted where
there has been a rights issue, or capital repayment).

The existing market capitalization is

$$M_{t-1} = \sum_{r=1}^{m} N_{r,t-1} \cdot P_{r,t-1}$$

and the revised market capitalization is

$$M_{t-1} + C_{t-1}.$$

Thus if the Index at time \(t-1\) remains unchanged

$$I_{t-1} = \frac{100 M_{t-1}}{B_{t-1}} = \frac{100(M_{t-1} + C_{t-1})}{B_{t}}.$$

Thus

$$B_{t} = B_{t-1} \left(1 + \frac{C_{t-1}}{M_{t-1}}\right).$$

This can be written as

$$B_{t} = B_{t-1} + \frac{B_{t-1} \cdot C_{t-1}}{M_{t-1}} = B_{t-1} + \frac{100 \cdot C_{t-1}}{I_{t-1}}.$$

This latter form is the one used both in the calculation programme
and in the checking.
APPENDIX III

Dividend yield formula

Dividend Yield % at time \( t \) = \( 100 \times \frac{\sum_{r=1}^{m} N_{r,t} \cdot D_{r,t}}{\sum_{r=1}^{m} N_{r,t} \cdot P_{r,t}} \)

where \( D_{r,t} \) is the dividend per share at time \( t \) of the \( r \)th constituent, this can be written as

\[
\frac{\sum_{r=1}^{m} (N_{r,t} \cdot P_{r,t}) \{(D_{r,t}/P_{r,t}) \times 100\}}{\sum_{r=1}^{m} (N_{r,t} \cdot P_{r,t})}
\]

(i.e. the weighted average of the individual dividend yields per cent—the weights being the market capitalizations).

APPENDIX IV

Earnings under 'imputation' tax

Example to illustrate the calculation of earnings under 'imputation' (the high overseas proportion is to emphasize the differences in the earnings on the three bases):

<table>
<thead>
<tr>
<th></th>
<th>U.K.</th>
<th>Overseas</th>
<th>Total</th>
<th>Total</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit before tax</td>
<td>30</td>
<td>70</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>less tax (50% U.K. &amp; overseas)</td>
<td>15</td>
<td>35</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Profit after tax</td>
<td>15</td>
<td>35</td>
<td>(A) 50</td>
<td>(A) 50</td>
<td>(A) 50</td>
</tr>
<tr>
<td>less 'actual' dividends</td>
<td>28</td>
<td>28</td>
<td>(B) 47</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>A.C.T. on dividends (3/7)</td>
<td></td>
<td></td>
<td>-12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum relieved (30% U.K. pft)</td>
<td></td>
<td></td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unrelieved A.C.T.</td>
<td></td>
<td></td>
<td>-3</td>
<td>-3</td>
<td>-3</td>
</tr>
<tr>
<td>Retentions</td>
<td></td>
<td></td>
<td>19</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(A) Earnings on a 'nil' distribution, which ignores unrelieved A.C.T.

(B) Earnings on a 'net' distribution, defined as retained earnings plus actual dividends paid (i.e. takes unrelieved A.C.T. into account).

(C) Maximum distribution—the maximum dividend (grossed up for A.C.T.) which could theoretically be paid from the profits for the year = Profit after tax plus maximum relieved A.C.T.
APPENDIX V

Earnings yield formula

Earnings yield % at time \( t \) = \( 100 \times \frac{\sum_{r=1}^{m} N_{r,t} \cdot E_{r,t}}{\sum_{r=1}^{m} N_{r,t} \cdot P_{r,t}} \)

where \( E_{r,t} \) is the earnings per share on a maximum distribution at time \( t \) of the \( r^{th} \) constituent.

APPENDIX VI

Price earnings ratio formula

The price earnings ratio at time \( t \) (on 'net' or 'nil' basis as appropriate)

\[
= \frac{\sum_{r=1}^{m} N_{r,t} \cdot P_{r,t}}{\sum_{r=1}^{m} N_{r,t} \cdot E'_{r,t}}
\]

where \( E'_{r,t} \) is the earnings per share ('net' or 'nil distribution' at time \( t \) of the \( r^{th} \) constituent).

This can be written as

\[
= \frac{\sum_{r=1}^{m} (N_{r,t} \cdot E'_{r,t})(P_{r,t}/E'_{r,t})}{\sum_{r=1}^{m} N_{r,t} \cdot E'_{r,t}}
\]

(i.e. the weighted average of the individual price earnings ratios, the weights being the appropriate earnings of each constituent).

APPENDIX VII

Redeemable fixed-interest yield formulae

Both 20-year Redeemable Fixed-Interest indices are based on yield averages made up for three groups (with terms each separated by about five years) and with weights varying so that the exact term for all the constituents is always precisely 20 years thus, where \( G_{n} = \) average yield of the \( n^{th} \) group of constituents, and \( t = \) the number of days since the current groups were adopted:
The average yield is converted into an average price by ascertaining in the usual way, the present value at this yield of a 20-year stock with a coupon of \( r \) %; this value is multiplied by a factor \( f \) which is determined whenever a new group is incorporated in the formula so as to ensure continuity in the price series.

The current values of the factors indicated for the two indices are:

<table>
<thead>
<tr>
<th></th>
<th>Govt. Stocks</th>
<th>Debs. &amp; Loans</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of constituents</td>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>Number in each group</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Date current groups adopted</td>
<td>20.3.1971</td>
<td>8.3.1973</td>
</tr>
<tr>
<td>( x )</td>
<td>1617</td>
<td>1943</td>
</tr>
<tr>
<td>( r ) %</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>( f )</td>
<td>1.15331</td>
<td>0.991914</td>
</tr>
</tbody>
</table>

A new group has to be chosen for each index approximately every five years, and the coupons \( r \) % have been increased from their 1962 values of 4 and 5 respectively.

APPENDIX VIII

**Inherited portfolio performance analysis formula**

Let

- \( R = \) percentage change in FT—A All-Share Index
- \( R_i = \) percentage change in \( i \)th Group index
- \( R_{ij} = \) percentage change in \( j \)th Subsection index of \( i \)th Group
- \( W_i = \) index weighting of \( i \)th Group
- \( W_{ij} = \) index weighting of \( j \)th Subsection of \( i \)th Group
- \( r = \) percentage capital appreciation of portfolio
- \( r_i = \) percentage capital appreciation of holding of shares classified in one or other Subsection of the \( i \)th Group
- \( r_{ij} = \) percentage capital appreciation of holding of shares classified in \( j \)th Subsection of the \( i \)th Group
- \( w_i = \) portfolio weighting in \( i \)th Group
- \( w_{ij} = \) portfolio weighting in \( j \)th Subsection of \( i \)th Group

\[
W_i = \sum_j W_{ij} \\
\sum_i W_i = \sum_i \sum_j W_{ij} = 1 \\
\sum_i w_i = \sum_i \sum_j w_{ij} = 1 \\
R_i = \frac{1}{W_i} \sum_j W_{ij} \cdot R_{ij} \\
R = \sum_i R_i \cdot W_i \\
r = \sum_i r_i \cdot w_i
\]
The analysis is based on the following decomposition of the difference between $r$ (appreciation of the portfolio) and $R$ (change in index):

$$r - R = \sum_i \sum_j (w_{ij} \cdot r_{ij} - W_{ij} \cdot R_{ij})$$

$$= \sum_i (w_i - W_i)(R_i - R) \quad \text{(Group distribution)}$$

$$+ \sum_i \sum_j (w_{ij} - W_{ij})(R_{ij} - R_i) \quad \text{(Subsection distribution)}$$

$$+ \sum_i \sum_j w_{ij}(r_{ij} - R_{ij}) \quad \text{(Company distribution)}$$

The components of these sums would be displayed in columns 11 (Group distribution), 12 (Subsection distribution) and 13 (Company distribution), of a schedule set out as suggested in Appendix IX (below).

**APPENDIX IX**

*Inherited portfolio performance analysis headings*

Suggested column headings for the analysis of the performance of the Inherited Portfolio for the year 1972:

**Column (1)** *Market Value of Initial Portfolio at 1.1.1972*
Shows the market value at 31.12.1971 of quoted U.K. equity holdings, subdivided according to the Groups and Subsections of the FT-Actuaries Indices.

**Column (2)** *Initial Market Value: Percentage of All-Share Classification*
Expresses the valuations in Column (1) as percentages of the valuation of that part of the portfolio which is in those industrial classifications represented in the FT-Actuaries All-Share Index.

**Column (3)** *Indices at 1.1.1972: Percentage of All-Share*
Shows the capitalizations of the Groups and Subsections of the indices expressed as percentages of the capitalization of the All-Share index at 31.12.1971.

**Column (4)** *Deviation from Index Shape: 1.1.1972*
Shows the deviation from the index 'shape', being the difference between the percentages in columns (2) and (3).
<table>
<thead>
<tr>
<th>Column (5)</th>
<th><strong>Index Movement during 1972</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Shows the movement, expressed as a percentage, of each index, at the end of 1972, related to the value on 31.12.1971.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Column (6)</th>
<th><strong>Excess Movement of Group Index over that of All-Share</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Shows the difference between the movements of the Group indices in column (5) and the 12.82% increase during 1972 in the All-Share index.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Column (7)</th>
<th><strong>Excess Movement of Subsection Index over Group Index</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Shows the difference between the movements of the Subsection indices in column (5) and those of the appropriate Group indices, also in column (5).</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Column (8)</th>
<th><strong>1.1.1972 Portfolio Revalued at 31.12.1972</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Shows the value on 31.12.1972 of the 1.1.1972 portfolio (column 1)—which is derived from a full valuation at 31.12.1972 by excluding all the 1972 ‘voluntary’ purchases and adding back the ‘voluntary’ sales and also the net ‘involuntary’ disposals (all valued at 31.12.1972).</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Column (9)</th>
<th><strong>Percentage Change in Portfolio Valuation during 1972</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Shows the movement, expressed as a percentage, of the value of each Group or Subsection of the 1.1.1972 portfolio, at the end of 1972.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Column (10)</th>
<th><strong>Excess Movement of Portfolio over that of Index</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Shows the difference between the movements in column (9) and the movements in the corresponding FT-Actuaries indices shown in column (5).</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Column (11)</th>
<th><strong>Effect of Group Distribution</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Represents the product of the two percentages in columns (4) and (6), effectively showing, as a percentage of the initial portfolio, the effect on the relative performance of the portfolio of over- and under-representation in each Group: over-representation in a Group beating the All-Share index and under-representation in one not beating it both giving a positive percentage, and the converse of each giving a negative percentage.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Column (12)</th>
<th><strong>Effect of Subsection Distribution</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Represents the product of two percentages in columns (4) and (7), showing the corresponding percentages for</td>
<td></td>
</tr>
</tbody>
</table>
THE F.T.-ACTUARIES SHARE INDICES

Subsections to those for Groups given in column (11).

Column (13) *Effect of Company Distribution*

Represents the product of the two percentages in columns (2) and (10), showing as a percentage of the entire initial portfolio, the effect of the performance of holdings in each Subsection or (not subdivided) Group relative to the appropriate index.