

**REINSURANCE TREATY PRICING**

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## REINSURANCE TREATY PRICING.

### Two case studies from practice.

- cat X.L.:
  - profit loadings at treaty level
- risk X.L.:
  - aggregate deductibles
  - reinstatement premiums

## CAT X.L. TREATY : PROFIT LOADING.

A question from the underwriter.

Q : how should I load a cat X.L. treaty for the cost of capital.

A : in theory :

- stochastic A.L.M.
- marginal capital required to support writing that new treaty

A : in practice :

- take some short cuts

## CAT X.L. TREATY : PROFIT LOADING.

Short cut : the route.

- **Profit margin required at portfolio level**
  - **arising from marginal capital requirements calculated at company level**
- **allocate by zone / peril**
  - **reflecting marginal contribution to risk**
- **allocate within territory proportional to treaty standard deviation**

## CAT X.L. TREATY : PROFIT LOADING.

Sample cat X.L. portfolio.

- 300 treaties
- 40 zones / perils
- £50m premium income
- maximum gross exposure per zone / peril £35m
- reinsured down to first loss deductible £5m, retro premium £10m
- profit margin required for total cat X.L. portfolio £10m

**CAT X.L. TREATY : PROFIT LOADING.**

**In principle : stochastic A.L.M. at individual treaty level.**

- 300 treaties :

• Japan EQ	_____	
	_____	
	_____	
• Japan WS	_____	
	_____	
	_____	
• U.K. WS	_____	
	_____	
	_____	
	_____	
	_____	
	_____	
• Caribbean	_____	
	_____	
	_____	
	_____	
• etc	_____	

<u>Contract</u>	<u>Cover</u>	<u>Dedbl</u>	<u>EPI</u>	<u>alpha</u>	<u>Freqy</u>
XYZ Ins	10,000	10,000	133.5	1.1	18
XYZ Ins	30,000	20,000	133.5	1.1	10
XYZ Ins	50,000	50,000	133.5	1:1	7
ABC Ins	7,500	7,500	48.6	1.1	22
ABC Ins	15,000	15,000	48.6	1.1	12
ABC Ins	30,000	30,000	48.6	1.1	7
ABC Ins	1,500	500	2.6	0.9	27

- volume of calculations daunting
- cannot see wood for trees

## CAT X.L. TREATY : PROFIT LOADING.

### Allocate profit margin required by zone / peril.

• cost of gross claims to portfolio :

<u>Zone / peril</u>	<u>Return period</u>				
	<u>5 vrs</u>	<u>25 vrs</u>	<u>50 vrs</u>	<u>100vrs</u>	<u>250 vrs</u>
• Japan EQ	£0m	£15m	£25m	£35m	£35m
• Japan WS	£1.5m	£5m	£7.5m	£7.5m	£7.5m
• U.K. WS	£10m	£30m	£30m	£30m	£35m
• Caribbean	£6m	£8m	£10m	£10m	£10m
• etc.					

• simulate in @RISK

- gross claims to total portfolio
- recoveries from retro programme
- variability of gross results, net result
- marginal contribution to risk of each zone / peril

## CAT X.L. TREATY : PROFIT LOADING.

### Allocate profit margin required within zone / peril.

- typically treaties all subject to same peril
- variability related to “distance from event “
  - volume of underlying exposure
  - height of layer
  - encapsulated in R.O.L.
- rule of thumb : standard deviation varies as square root of R.O.L.
- now you can allocate territory profit margin to individual treaties



## RISK X.L. PRICING EXAMPLES.

### Prefer frequency / severity approach.

- intuitively appealing to underwriter
- extrapolate to higher deductibles
- can estimate variance of loss cost
- easy to value special terms and conditions
- aggregate deductibles
- reinstatement premiums

## RISK X.L. PRICING EXAMPLES.

### Frequency / severity approach.

- frequency projected by triangulation
- severity modelled by curve fitting
  - less distorted by outliers
- model claims F.G.U., then apply reinsurance terms

## FREQUENCY / SEVERITY APPROACH.

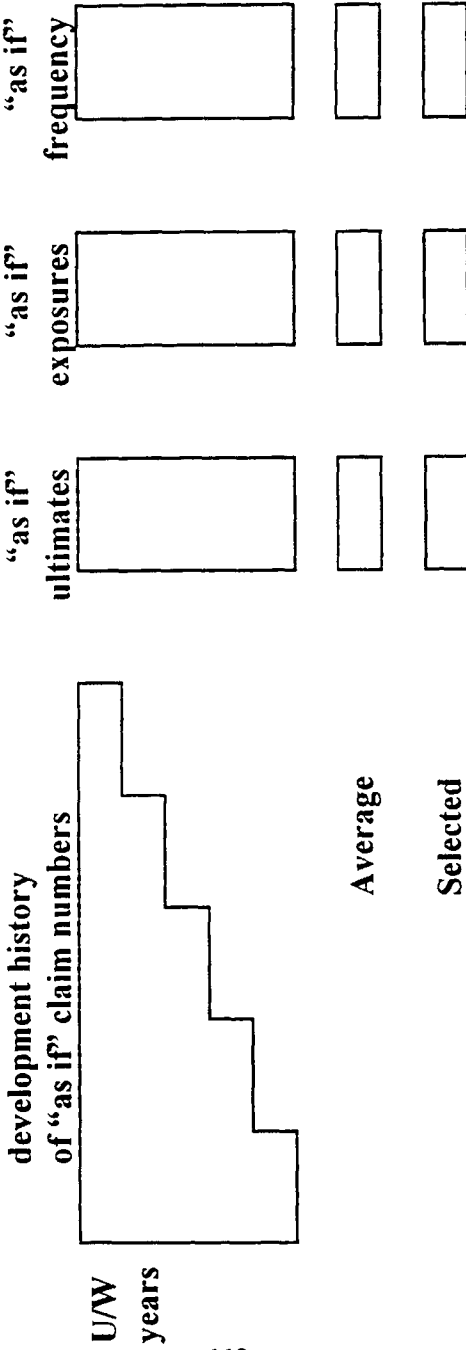
### Frequency projected by triangulation.

- individual F.G.U. data triangulated on an “as if” basis
- revalued claims set against revalued exposures
- slice at different deductibles
  - get frequency IBNR at different deductibles
  - get ultimate frequency at different deductibles
- fit curve to these ultimate frequencies

# FREQUENCY / SEVERITY APPROACH.

## Frequency projected by triangulation.

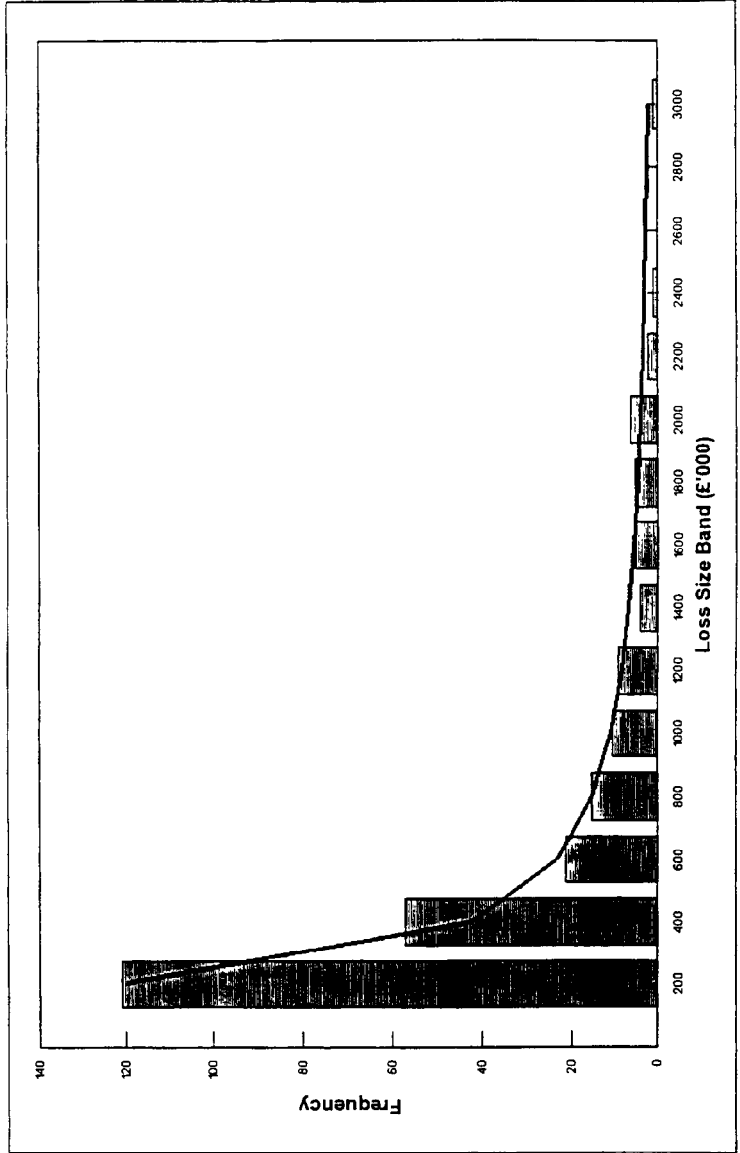
- example : claims numbers above £750,000



- repeat for other deductibles to get frequency in different loss size bands

# FREQUENCY / SEVERITY APPROACH.

Fitting a curve to severity data.



## **FREQUENCY / SEVERITY APPROACH.**

### **Pricing a reinsurance layer.**

- model produces frequency / severity distribution for F.G.U. losses above a threshold
- 14 ● “integrate” between deductible and limit
- “layering” - easy to produce alternative quotes that are consistent
- extrapolate to higher layers - with care

# RISK X.L. PRICING EXAMPLES - AGGREGATE DEDUCTIBLE (1)

Lotus 1-2-3 Release 5 - [AGGEDBL.WK4]

File Edit View Style Tools Range Window Help

AC15 @RISK>PARETO(\*\$D16,\$F\$3)

**RISK XL MODEL - AGGREGATE DEDUCTIBLE**

1	<b>RISK XL MODEL - AGGREGATE DEDUCTIBLE</b>	
2	Layer	500 xs 500
3		
4	Burning Cost	850
5	Pareto Shape Parameter	2.1
6	Implied Average Claim Size	955
7	Implied Claim Frequency	3.5
8		
9	Aggregate Deductible	500
10		
11		
12		
13		
14		
15	F.G.U. Claim Sizes	1 2 3 4 5 6
16	Cost To Layer	862.59 545.15 1573.47 712.63 675.39
17	Claims For Sample	41.73 45.15 500.00 212.63 175.39
18		41.73 0.00 0.00 0.00 0.00
19	Total Claim Cost	404
20	Average Claim Cost	202
21	Cost With Agg Ded	0
22		

Claim Frequency For Sample

Field 2 12 23/05/97 5:18 [F12] Ready

# RISK X.L. PRICING EXAMPLES - AGGREGATE DEDUCTIBLE (2).

Lotus 1-2-3 Release 5 - [AGGDEDBL.WK4]

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ACIS <<CRISKS>>PARETO(\$D16\*\$F\$)

**RISK XL MODEL - AGGREGATE DEDUCTIBLE**

Layer	600	xs	500					
Burning Cost	850							
Pareto Shape Parameter	2.1							
Implied Average Claim Size	955							
Implied Claim Frequency	3.5							
Aggregate Deductible	500							
				Claim Frequency For Sample	5			
						1	2	3
F.U. Claim Sizes	528.57	905.34	563.25	1008.87	802.16	834.06		
Cost To Layer	28.57	405.34	63.25	500.00	302.16	334.06		
Claims For Sample	28.57	405.34	63.25	500.00	302.16	0.00		
Total Claim Cost	1,299							
Average Claim Cost	260							
Cost With Agg Ded	799							

Ready



# RISK X.L. PRICING EXAMPLES - AGGREGATE DEDUCTIBLE (3)

**@RISK**

File Edit Settings Variables Execute Results Window Help

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**Simulation**

Simulation #1 of 100  
 AGGREGATE.DBL.WKS  
 for device 100  
 Simulation 1

**Results**

Cell	Minimum	Mean	Maximum
AC19 Total Claim Cost/Claim Freq.	0	845.0696	3460.13
AC20 Average Claim Cost/Claim Freq.	3.976638	241.2445	500
AC21 Cost With Agg Ded/Claim Freq.	0	421.141	2960.13

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**Simulation Statistics**

Name	Description	Output	Average Claim Cost/Claim Frequency For	Cost With Agg Ded/Claim Frequency For	Std Deviation	Variance	Skewness	Kurtosis	First Cumulative Prob	Second Cumulative Prob	Third Cumulative Prob	Fourth Cumulative Prob	Fifth Cumulative Prob	Sixth Cumulative Prob	Seventh Cumulative Prob	Eighth Cumulative Prob	Ninth Cumulative Prob	Tenth Cumulative Prob	
110.048	Output	505.6447	AGGREGATE.DBL.WKS>>A:20	505.6447	1.9														
12110.57	Output	2556.765	AGGREGATE.DBL.WKS>>A:20	2556.765	3.61														
7.60125E-02	Output	1.868567	AGGREGATE.DBL.WKS>>A:21	1.868567	6123														
2.863545	Output	818025	AGGREGATE.DBL.WKS>>A:21	818025	3.467														
3	Output	0	AGGREGATE.DBL.WKS>>A:21	0	0														
313.9912	Output	0	AGGREGATE.DBL.WKS>>A:21	0	3														
29.00402	Output	0	AGGREGATE.DBL.WKS>>A:21	0	3														
71.00848	Output	0	AGGREGATE.DBL.WKS>>A:21	0	1														
122.9983	Output	0	AGGREGATE.DBL.WKS>>A:21	0	1														
140.801	Output	0	AGGREGATE.DBL.WKS>>A:21	0	2														
263.7946	Output	0	AGGREGATE.DBL.WKS>>A:21	0	2														
175.4951	Output	0	AGGREGATE.DBL.WKS>>A:21	0	2														
158.8723	Output	65.53349	AGGREGATE.DBL.WKS>>A:21	65.53349	2														
210.0872	Output	134.499	AGGREGATE.DBL.WKS>>A:21	134.499	2														
223.1118	Output	153.2367	AGGREGATE.DBL.WKS>>A:21	153.2367	3														
228.7183	Output	236.4717	AGGREGATE.DBL.WKS>>A:21	236.4717	3														
244.9149	Output		AGGREGATE.DBL.WKS>>A:21		3														

# RISK X.L. PRICING EXAMPLES - REINSTATEMENT PREMIUMS.

Lotus 1-2-3 Release 5 - [REINSTAT.WK4 [R0]]

	A	B	C	D	E	F	G	H	
1	<b>RISK XL MODEL - TWO REINSTATEMENTS</b>								
2									
3	Layer		500	xs		500			
4									
5	Burning Cost		850						
6	Pareto Shape Parameter		2.1						
7	Average Claim Size		955						
8	Implied Claim Frequency		3.5						
9									
10									
11									
12									
13	F.G.U. Claim Sizes		1575.66	1	2	3	4	5	6
14	Cost To Layer		500.00	645.21	571.56	1828.81	624.78	552.88	
15	Claims Used For This Sample		500.00	145.21	71.56	500.00	124.78	52.88	
16				145.21	71.56	0.00	0.00	0.00	
17	Reinstatement Analysis								
18	Recovery		500	145	72	0	0	0	
19	Cover Remaining		1,000	855	783	783	783	783	
20									
21	Revised Claim Cost (2 reinstatements only)			717					
22	Premiums received			2.43					
23	Revised Risk Premium (2 @100)			295					
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**RISK X.L. PRICING EXAMPLES.**

**Be careful with loadings !**

- need to preserve cash value of loadings :

	<u>Full Cover Premium</u>	<u>With Aggregate Deductible</u>	
Risk Premium	£850k	£425k	- 50%
Expense Loading	£55k	£55k	n/c
Profit Loading	£125k	£106k	- 15%
Commission	£115k	£65k	
<b>Total Premium</b>	<b>£1,145k</b>	<b>£651k</b>	<b>- 41%</b>

- note how profit loading has decreased in cash terms, but increased as % premium