Pricing in the London Market

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1. Introduction

1.1 The role of actuaries in pricing products in the London Market is an area where there is considerable interest, both from the view of the actuary and from the view of management. In the past underwriters have used their skills to identify risks that are acceptable and quote rates that, in broad terms, have provided the shareholders or names at Lloyd's with acceptable profits. The number of cases of imprudent underwriting is limited, although their consequences have often been disastrous. What new features and ideas can an actuary bring to the pricing table to ensure that his skills are valued and seen essential to all parties in the prudent management of the business?

1.2 One of the main problems in writing this paper was trying to find the audience to focus our comments. Clearly actuaries themselves would be in that audience, as would Underwriters, Chief Executives, Finance Directors, Claims Managers, and even trainees. The main conclusion was that the paper, although addressed for an Actuarial Conference, should focus on the Management Team of a London Market operation. The reasons for this will become apparent. Actuaries learning about this subject will benefit from the additional reading and sophisticated software identified in Appendices. What the Management Team needs to address is to view the actuarial role beyond the annual reserving and fully involve him in the day to day management of the operation. The role of the Actuary in Life Insurance is defined as the Financial Management of the Insurance Operation. The President of the Institute, in his presidential address endorsed the expansion of this role into General Insurance. This paper helps form a basis of how this objective may be achieved.

1.3 We need to identify the reasons why an actuary should be involved in pricing. We need to identify what skills set he can bring to the process that can be of benefit to both the underwriter and his managers. The role needs both clarification and professional guidance to avoid such problems.

The reasons identified were as follows:-

1. Actuaries are trained in Financial Management and see the broader picture. An actuary can identify assumptions in the underwriting process in an explicit form, for example, claims inflation, investment income and so on. These may often be identified only implicitly in an underwriter's rate.

2. Actuaries can interpret the numbers in ways that may bring new insight into the risk.

3. Actuaries can act as a peer review to the individual risks, an aggregate pool of risks and ultimately the whole book. The actuary can back up what may appear to be difficult decision for the underwriter to take, and maybe form a focus for discussion of particular problems as the relationship develops.

4. There is a clear relationship between pricing and reserving. An actuary involved in pricing will be able to assist the reserving actuary in identifying particular features of the account which are relevant in assessing the reserves.

5. The actuary gives a degree of comfort to the Board and shareholders. The actuary is professional in his approach, and this should outweigh any vested interest that may arise.

1 4 Given these attributes, there are clear problems which we have identified which will lead to potential problems.

1. Actuarial resources are scarce, and there are a limited number of individuals with the practical experience to undertake such a responsible task.

2. There is a need for clear communication and possible avoidance of jargon in discussions between actuary and underwriter. Lack of clear communication could lead to potential problems due to the lack of real understanding by the underwriter of the consequences of not moving the rates .

3. The involvement of actuaries in pricing needs full backing from the Chief Executive downwards. Without this backing the task becomes impossible as other issues take over. An example is the renewal of a risk not on price but because the underwriter has led the risk for 10 years.

4. The actuary is often seen as giving post event justification to the underwriter. There is often a short time between viewing the risk and acceptance, and the actuary may find the assessment very difficult in the very limited time he has to analyse the risk.

1.5 In practice the role of the actuary in pricing in the London Market depends much on the quality and quantity of information involved. There are clearly those risks were the volume of information available will enable a reasonable price to be assessed by both actuary and underwriter. It is the underwriters responsibility to write that risk, and in doing so he may adjust the actuarial rate to take account of special features which have been identified but which are difficult to price with any degree of certainty, for example good risk management or a potential health hazard. The risks of lower information quality may be pooled in some form as assessment, and eventually the whole book may be reviewed as a valuation process. In a simplistic world, an actuary may identify, for a given corporate profit objective, that the underwriter needs to write a specific book of business to achieve a specific income. Let us say the actuarial income is £10 million. If the underwriter writes those risks for £9.5 million, then the Board is aware, based on the estimates, there is likely to be a deficit of £0.5 million on the results. Actual experience may differ, but there is an early warning. Conversely if the underwriter achieves £10.5 million in premium, then the Board can rest a little easier. To achieve this level of understanding by all parties often needs input of an actuary. The comfort it may give to management is potentially enormous.

1.6 The London Market itself is a complex animal with conflicts between various risk takers. Although the insurance product is for one year, the market in general views continuity as important. This is indeed recognised in the long term planning and objectives of both insurance companies and Lloyd's. How often has the argument "We have held the risk for twenty years" been used in the past to justify accepting a risk, despite the fact the current rates may not support the risk. There is therefore a conflict between short term pricing and long term pricing. This conflict helps generate the variations we see in the rates, and managers clearly need to understand the potential costs to the balance sheet, be it a syndicate or a company, and be able to maximise profits within the risk constraints.

2. Summary of Paper

2.1 The paper is split into 2 primary sections. The purpose of these sections is to give a good but broad view of the techniques. Examples are included in each of the sections and are augmented by further examples in Appendix 3. A summary bibliography us included together with references to additional reading. It should be noted that this reference list is not comprehensive.

2.2 Section 3 of this paper deals with the individual risk methods which are used currently in the day to day assessment of risks. Methods in this section are of value to those actively assisting the underwriter.

Section 4 deals with the other issues which are normally implicit in the current pricing but need to be expressed explicitly. These include risk/reward approaches and detailed analysis of the cash flows and investment income (profit testing). These more general approaches have been used, and possibly of more substantial benefit to management than the "premium checking" approach in section 3. The concept of the insurance cycle is introduced.

In Section 5 the challenging issue of actuaries being actively used in underwriting is broached. There are many views on this subject, and some of the issues are addressed in that section.

In Appendix 1 we try to identify other methods which have been used by one or two individuals to assess rates, but are generally not well known or adequately tested. These include approaches from Financial Theory and from Neural Networks . These methods are rapidly becoming part of today's actuary's toolbox and is an area of continual interest. These are both new directions and esoteric methods. Such methods, if they pass the test, will become the standard pricing techniques of tomorrow.

3. PRICING TECHNIQUES USED IN PRACTICE

This section represents a simple "tool box" of techniques used in practice. It deals with the loss cost only, i.e. does not consider expense and profit loadings. The structure of the section is as follows:-

A list of the various pricing techniques available is given.

For each technique we try to provide the following:

Introductory remarks describe how it is used .

The basic recipe is described.

Fuller details together with an illustrative example is given

References to further reading which contain more detail are provided.

Comments are made on application of the technique in practice,

The list of headings is as follows:

In addition to these see chapter 6 of The Foundations of Casualty Actuarial Science which has many excellent worked examples.

3.1. <u>Risk excess of loss treaties</u> (Risk X/L)

3.1.1 Experience rating

(i) Introductory remarks

Experience rating (or burning cost) is an intuitively appealing approach to pricing, taking into account as it does the cedant's past loss history in trying to estimate future losses to a layer. In theory it can be used for any class of business where there is an adequate loss history, but it is perhaps most often used to price commercial fire, the non-liability element of motor and liability business. It is also frequently used to price medical malpractice business.

(ii) Basic recipe

The following is an example of how a layer might be priced using experience rating. The FGU (full ground up) development of the cedant's large claims for a number of past years of account is obtained. "Large" claims would typically mean claims which do, or are likely to, impact the layer to be priced; the reinsurer's reporting requirement might typically be for all losses with an incurred equal to 50-75% of the deductible after allowing for any indexation clause. **(Note: in the US, development may not be available by individual claims, only on an aggregate basis).** The development is adjusted for an appropriate measure of claims inflation to the period to be covered and for any changes to, for example, policy wordings, business profile, claim size and frequency; in market jargon it is reworked on an "as if" basis.

The ultimate for each loss in each year is then estimated using appropriate methods, for example, a traditional triangulation method might be used, with tail factors and market statistics applied where the development available is inadequate. Having adjusted the loss development for IBNR, it can be put to the layer to be priced to produce an estimate of the ultimate losses to the layer for each year under consideration.
Next, the premium in each year of account is [projected to ultimate and] inflated to the period of cover allowing for any changes in coverage, past rate inadequacy, appropriate sum insured inflation, etc. The estimated ultimate loss ratio, or perhaps more frequently the burning cost (equal to the loss cost to the layer, the burn, divided by the original cedant's premium) to the layer for each year can then be calculated, a

suitable average selected, and loadings added for the reinsurer's expenses, to get to the indicated rate, per unit of direct premium written.

(iii) Illustrative example

Risk Excess of Loss

Casualty Burning Cost Rating Example

250,000 XS 250,000

Allocated Loss Adjustment Expenses Covered Pro Rata pro Loss

1. For each accident year, adjust each loss on as "as if" basis, for both indemnity and expenses, for an appropriate measure of claims inflation, to the period to be covered. For example consider 1991 accident year, 4 years after the start of the accident year

Loss	Reported Indemnity	Reported Expenses	Inflation Factor	Adjusted Indemnity	Adjusted Expenses	Indemnity Expenses
	(1)	(2)	(3)	(4)	(5)	(6)
1	500,000	50,000	1.464	732,050	73,205	805,255
2	450,000	45,000	1.464	658,845	65,885	724,730
3	325,000	24,000	1.464	475,833	35,138	510,971
4	300,000	7,000	1.464	439,230	10,249	449,479
5	240,000	11,000	1.464	351,384	16,105	367,489
Total	1,815,000	137,000		2,657,342	200,582	2,857,923

Use an appropriate method to estimate loss development factors to ultimate, hence to estimate the ultimate value of each loss from 1. above, for each accident year under consideration. For example by using a triangulation of the adjusted losses from 1:

Accident	Development Year				
Year	3	4	5	6	7
1988	2,277,920	2,961,295	3,858,565	4,784,620	5,980,775
1989	1,793,420	2,510,790	3,422,050	3,969,755	
1990	1,568,855	2,353,285	3,136,930		
1991	1,754,203	2,857,923			
1992	3,855,907				

Accident	Loss Development Factors				
Year	3:4	4:5	5:6	6:7	7:Ult
1988	1.300	1.303	1.240	1.250	
1989	1.400	1.363	1.160		
1990	1.500	1.333			
1991	1.629				
1992					
Average	1.457	1.333	1.200	1.250	
Selected	1.500	1.333	1.250	1.225	1.200
Cumulative	3.674	2.449	1.838	1.470	1.200

3. For each accident year under consideration, increase the adjusted losses in 1. by the appropriate factor to ultimate, and then put each loss to the layer to be priced to calculate the loss in the layer For example the 1991 accident year.

Loss	Adjusted Indemnity (7)	Adjusted Expenses (8)	Develop Factor (8)	Est Ult Indemnity (10)	Est Ult Expenses (10)	Indemnity in Layer (11)	Pro-rata Expenses (12)	Total in Layer (13)
1	732,050	73,205	2.449	1,793,074	179,307	250,000	25,000	275,000
2	658,845	65,885	2.449	1,613,767	161,377	250,000	25,000	275,000
3	475,833	35,138	2.449	1,165,498	86,068	250,000	18,462	268,462
4	439,230	10,249	2.449	1,075,844	25,103	250,000	5,833	255,833
5	351,384	16,105	2.449	860,676	39,448	250,000	11,458	261,458
	2,657,342	200,582		6,508,859	491,302	1,250,000	85,753	1,335,753

4. Adjust subject premiums to current premium levels, for example, for changes in rate adequacy and changes in exposure and calculate exposure premium.

Accident Year	Adjusted Subject Premium (14)	Adjusted Losses in Layer (15)	Indicated Loss Rate (16)
1988	10,000,000	2,657,892	0.266
1989	12,000,000	3,125,000	0.260
1990	14,500,000	4,125,036	0.284
1991	17,000,000	1,335,753	0.079
1992	19,000,000	2,501,420	0.132
	72,500,000	13,745,101	0.204

(17) Indicated loss rate	0.224
(18) Reinsurer loading (say)	100/80
(19) Exposure premium per unit of cedant's direct premium	0.255

(iv) Practical considerations

In theory experience rating requires a statistically significant historical loss development to be available for the cedant, and as such is much more suited to working layers of an X/L programme. In practice there can be other sources of loss development, for example ISO development factors in the US, so full cedant FGU data might not be needed. ISO does a lot of work on development factors within layers (see Pinto & Gogol), and when pricing US X/L it may not be necessary to work with full FGU losses, but simply to work with losses to the layer. Consideration must be given to losses below the deductible; and how ISO and similar development factors cope with the losses, which incurred below the deductible, which **might** impact the layer in the future'

Loss adjustment expenses must be allowed for appropriately depending on whether they are included with the loss to be put to the layer, or are allocated on a pro-rata basis.

3.1.2 Curve fitting[1]

(i) Introductory remarks

A method which is not dissimilar to experience rating, but instead of using the FGU data directly, a curve, often a Pareto, is fitted to the FGU data and rates are derived by reference to the fitted curve. The data required is very similar to that required for experience rating, in particular, the cedant's FGU data for large claims is needed, where "large" claims are defined as before. It is a method which can be used for both property and casualty business and is often used for hospitals' medical malpractice in the US.

Much work has been carried out on the single parameter Pareto. This simplification arises if the FGU data is normalised, by dividing each loss by the observation point, the size below which losses are not included in the FGU data used. This paper does not repeat the derivation of the various formulae quoted, but the interested reader is referred to, in particular, Stephen Philbrick's paper and the reading list included in that.

(ii) Basic recipe

Obtain FGU data from the cedant for the past few years, perhaps five or six years, for losses which satisfy the reporting requirements for the layer to be priced (i.e. losses in excess of the observation point). Project this data to ultimate to obtain an estimate of the IBNR . This can be done after fitting the curve. Although claims inflation has a more marked effect on frequency of losses in the layer, rather than the average claim size of yjr layer, it is generally better toadjust the loss data for claims inflation to a common point in time. Philbrick argues that thr Pareto described in his paper does not require the original loss data to be adjusted for claims inflation, but this only holds if the Pareto fitted is a good fit for all claims above a lower observation point in earlier years, being the current observation point reduced for claims inflation.

Next, normalise the adjusted data by dividing each loss by the observation point and then fit a one parameter Pareto by estimating the shape parameter α.

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1 For further details see "A Practical Guide to the Single Parameter Pareto Distribution, Stephen W. Philbrick, Proceedings of the Casualty Actuarial Society, 1985.

This can be done, for example, by Maximum Likelihood, and Philbrick's paper gives the appropriate formula:

$$\alpha = n / \sum_{i=1}^{n} \ln X_i$$

for α fitted by Maximum Likelihood, where n *is* the number of losses in the observed FGU data and X_i is the size of the (normalised) i^{th} loss . If α is the Pareto parameter, U is the upper limit (or censorship point) and R is the retention of the layer to be priced. and OP is the observation point, then the expected loss size in the layer iscalculated as:

$$R \times [(\alpha - (U/OP)^{1-\alpha})/(\alpha - 1) - 1] \quad \alpha \neq 1$$

which simplifies to $R(1 - (U/R)^{1-\alpha})/(1 - \alpha)$

$$R \times [(1 + \ln(U/OP) - (1 + \ln(R/OP))] = R\ln(U/R) \quad \alpha = 1$$

Having fitted a curve for severity, one must consider frequency. A distribution for frequency, typically Poisson, might be assumed, and the parameter(s) estimated from the data. First estimate the number of losses in excess of the observation point for each accident year under consideration, then divide by the exposure, for example the original cedant's premium adjusted to date on an "as if" basis, to estimate the frequency above the observation point for each accident year. (Note: For frequencies, an increase in claims inflation, and in claim sizes due to development of a loss, both tend to increase the frequency of losses to the layer. Therefore if the original loss data has not been trended, these frequencies must be adjusted by $(1 + i)^{n\alpha}$ where i is an appropriate measure of claims inflation, α is the Pareto parameter, and n is the number of years between each accident year and the year to which frequencies are being projected)

A "suitable" averaged or trended value is selected from the adjusted observed frequencies, and an estimated number of losses in excess of the observation point is calculated, using (an estimate of) the exposure for the year to be priced. This frequency then needs to be adjusted to get to the frequency of losses above the retention

This is done by multiplying the projected frequency of losses in excess of the observation point by:

$$(R/OP)^{-\alpha} \times (1 - (U/R)^{-\alpha})$$

Having fitted a curve, and calculated the expected claim cost to the layer and the expected claim frequency to the layer, the actuary should consider both these estimates for reasonableness, and it will often be useful to get the view of the underwriter.

The risk premium can then be calculated and loaded for the reinsurer's expenses, contingency margins, etc. as before. The variance of the aggregate losses to the layer can be calculated, and under the assumption of a Poisson distributed frequency, again using formulae demonstrated in Philbrick, the variance of the total loss to the layer is given by:

$$nR^2 \times (\alpha - 2(U/OP)^{2-\alpha})/(\alpha - 2) \qquad \alpha \neq 2$$
$$nR^2 \times (1 + 2\ln(U/OP)) \qquad \alpha = 2$$

from which it is possible to calculate a risk loading.

(iii) Illustrative example

Risk Excess Of Loss

Observation Point = 25,000 Claims Inflation = 8 %

Excess = 50,000

Upper Limit = 100,000

Acc Yr	Loss	Amount (1)	Infl Adj (2)	Infl Amount (3)=(1)*(2)	IBNR Factor (4)	Estimated Ult Amount (5)	Normalised (3)/25000 (6)	Ln(Normalised) ln((6)) (7)
1990	1	37,775	1.469	55,504	1.00	55,504	2.220	0.798
1990	2	17,365	1.469	25,515	1.00	25,515	1.021	0.020
1990	3	27,121	1.469	39,850	1.00	39,850	1.594	0.466
1990	4	58,196	1.469	85,509	1.00	85,509	3.420	1.230
1990	5	20,328	1.469	29,868	1.00	29,868	1.195	0.178
1990	6	17,564	1.469	25,807	1.00	25,807	1.032	0.032
1991	7	392,477	1.360	533,961	1.30	694,149	27.766	3.324
1991	8	23,167	1.360	31,519	1.30	40,974	1.639	0.494
1991	9	19,918	1.360	27,098	1.30	35,227	1.409	0.343
1991	10	128,396	1.360	174,682	1.30	227,087	9.083	2.206
1991	11	19,123	1.360	26,016	1.30	33,820	1.353	0.302
1991	12	21,872	1.360	29,757	1.30	38,684	1.547	0.437
1992	13	19,870	1.260	25,030	1.80	45,054	1.802	0.589
1992	14	33,324	1.260	41,979	1.80	75,563	3.022	1.106
1992	15	25,293	1.260	31,862	1.80	57,351	2.294	0.830
1992	16	75,335	1.260	94,900	1.80	170,819	6.833	1.922
1992	17	80,735	1.260	101,703	1.80	183,066	7.323	1.991
1993	18	84,648	1.166	98,733	2.30	227,087	9.083	2.206
1993	19	32,556	1.166	37,973	2.30	87,337	3.494	1.251
1993	20	30,373	1.166	35,427	2.30	81,483	3.259	1.182
1993	21	408,062	1.166	475,964	2.30	1,094,717	43.789	3.779
1993	22	327,335	1.166	43,547	2.30	100,157	4.006	1.388
1994	23	60,388	1.080	65,219	2.90	189,135	7.565	2.024
1994	24	947,030	1.080	1,022,792	2.90	2,966,096	118.644	4.776
1994	25	422,180	1.08	455,954	2.90	1,322,267	52.891	3.968
								36.842

Pareto parameter = 1.474 (See Note 1)

Acc Yr	Number of Losses (8)	Ultimate Number (9)	Adjusted Subj Premium (10)	Obs Frequency (9)/(10) (11)
1990	6	6	100	0.060
1991	6	7	110	0.064
1992	5	8	120	0.067
1993	5	8	130	0.062
1994	3	9	135	0.067
	25	38		

(12) Selected frequency for 1995, say	0.07
(13) 1995 subject premium (millions)	140
(14) Expected claims > 25,000	9.800
(15) Expected claims > 50,000 retention (see note 2 below)	2.830
(16) Average claim size to layer4 (see Note 3 below)	30,830
(17) Reinsurer loading	100/80
(18) Exposure premium ((15) x (16) x (17))	109,082
(19) Exposure premium per million cedant's direct premium ((18) / (13))	789

Notes:
1 - q = number of losses/sum ln(normalised est ult) ie 25/36.842
2 - (14)*((R/OP)^(-q)*(1-(R+L)/R)^(-q))
3 - R*[(q-{(R+L)/OP}^(1-q))/((q-1)-(q-(R/OP)^(1-q))/(q-1)]

(iv) Practical Considerations

Care must be taken, especially in extrapolating to layers beyond that for which data is available. As a general rule of thumb this method should not really be used to extrapolate to losses in excess of three times the deductible of the layer for which data is available to fit the Pareto in the first place. The choice of alpha is especially crucial when extrapolating to a higher layer, and only a small change in alpha can have a very marked effect on the risk premium - see the example.

Other curves may be appropriate to certain situations, and the lognormal might be used if there are unlikely to be too many aggregations of losses, so that the tail of the business is not too long.

The frequency at the observation point is used to infer the frequency at the retention, and is often not the best guide to use. In practice, it can prove useful to make judgmental changes to the frequency at the observation point, effectively to treat the Pareto as a two parameter distribution.

The Pareto has a fairly "thick" tail, and is usually censored. It does not fit well over the full range of losses, and is only typically used for the upper tail of a given loss distribution.

3.1.3 Exposure rating[2]

(i) Introductory remarks

Exposure rating is a technique for pricing X/L reinsurance which does not rely on the availability of historical loss data. Instead the ceding company's distribution of direct premium by policy limit, (its "risk profile") either current, or projected future, is used together with, typically, increased limit factors (ILFs) for casualty business or Ludwig tables for property or other published tables appropriate to a particular class and territory, or a reinsurer's own internal tables.

(ii) Basic recipe

For the class of business for which an X/L policy is to be priced, an estimate is needed of the ceding company's direct premium by policy limit over the period of cover. For a Property cover, the first step would be to express the retention for the layer to be priced as a percentage of each of the policy limits written by the cedant direct. The retention plus the limit for the layer are then expressed as a percentage of each of the direct limits in the same way. The former can be thought of as the proportion of each policy limit being retained by the cedant, and the latter less the former as the proportion of each policy limit which falls in the layer to be priced.

The next step is to choose appropriate values from Ludwig's table of cumulative loss amount distributions, depending on the class of business and type of coverage being priced, for each of the percentages already calculated. For each direct policy limit, the difference between the Ludwig values, multiplied by the direct premium for the limit gives the expected value of the exposure premium for the limit falling into the layer being priced.

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2 For further details see "An Exposure Rating Approach to Pricing Property Excess of Loss Reinsurance", Stephen J. Ludgwig, proceedings of the Casualty Actuarial Society, 1991.

This exposure premium must then be adjusted for

1. The portion of the direct premium in respect of losses,
2. The loss adjustment expenses to be allocated,
3. Any premium inadequacy,
4 Reinsurer's expenses,
5. The absolute amount of direct premium projected to be written by the cedant to get to a final exposure premium rate, per unit of direct premium written by the cedant.

For a casualty treaty, the method would be similar, but would use ILFs instead of Ludwig values.

(iii) Illustrative example

<u>*Example A*</u>

Property Exposure Rating Example,

Homeowner's (Non-Catastrophe) Wind Losses: 150,000 xs 50,000

Allocated Loss Adjustment Expense Covered Pro-Rata to Loss

(1) Policy Limit	*(2)* Subject Premium	*(3)* Retention as % of Pol Limit	*(4)* (Retention + Limit) as % of Pol Lim	*(5)* Value from Ludwig Curve for (3)	*(6)* Value from Ludwig Curve for (4)	*(7)* Exposure Premium ((6)-(5))*(2))
100,000	4,000,000	50.00 %	200.00 %	0.980	1.000	80,000
300,000	3,000,000	16.67 %	66.67 %	0.951	0.986	160,000
500,000	2,000,000	10.00 %	40.00 %	0.934	0.976	84,000
750,000	1,500,000	6.67 %	26.67 %	0.892	0.966	110,500
1,000,000	2,000,000	5.00 %	20.00 %	0.871	0.959	176,500
2,000,000	750,000	2.50 %	10.00 %	0.860	0.934	55,000
	13,250,000					612,000

(8)	Exposure Premium, prior to loadings	612,000
(9)	Times portion of subject premium for losses	0.600
(10)	Times allocated loss adjustment expense loading	1.05
(11)	Times adjustment for subject premium inadequacy	1.10
(12)	Times reinsurer loading	100/80
(13)	Indicated exposure premium	530,145
(14)	Divided by Subject Premium	13,250,000
(15)	Final indicated exposure per unit of cedant's direct premium	0.04000

Example B

Casualty Exposure Rating Example: 700,000 xs 300,000

Allocated Loss Adjustment Expense Covered Pro-Rata to Loss

(1) Policy Limit	*(2) Subject Premium*	*(3) Limited Average Severity*	*(4) Increase Limit Factors*	*(5) Excess Factors*	*(6) Exposure Prem (2)*(5)*
300,000	2,000,000	63,962	1.563	0.000	0
500,000	2,000,000	74,884	1.830	0.146	291,803
750,000	1,000,000	83,417	2.039	0.233	233,448
1,000,000	2,000,000	89,356	2.184	0.284	568,681
2,000,000	500,000	103,173	2.522	0.246	123,117
	10,500,000			0.116	1,217,049

(7)	Exposure Premium, prior to loadings	1,217,049
(8)	Times portion of subject premium for losses	0.75
(9)	Times allocated loss adjustment expense loading	1.15
(10)	Times adjustment for subject premium inadequacy	1.10
(11)	Times reinsurer loading	100/80
(12)	Indicated exposure premium	1,443,344
(13)	Divided by Subject Premium	10,500,000
(14)	Final indicated exposure per unit of cedant's direct premium	0.13750

(iv) Practical considerations

Exposure rating can be a useful tool, in particular in circumstances where little or no historical loss information is available, so an experience rating approach cannot be used. It can also be combined with experience rating into a credibility type approach to pricing a layer of X/L.

Clearly care is needed in the use of, for example, the Ludwig tables, or the derivation of ILFs to make sure they are appropriate to the coverage being priced.

In an ideal world excess of loss reinsurance rating should be a combination of both experience rating and exposure rating. The rates under the different approaches are unlikely to be the same, and the question of what weights are to attach to what methods. This issue is addressed in "Credibility for Treaty Reinsurance Excess Pricing" by Gary Patrick and Isaac Mashitz in CAS 1990 Discussion Papers on Pricing.

If both exposure and experience rates have been successfully estimated and they differ, then there is a clear question of which to use. If the book has changed dramatically over a period of time, then experience rating will be meaningless. If the future exposure is likely to have changed, then the exposure rate is in doubt. In practice the rate lies between the two.

3.2. Catastrophe excess of loss treaties

Introductory remarks

The available methods depend crucially on the information available, which varies widely from territory to territory.

Good information is available in Australia where following the Darwin loss, the ICA (Insurance Council of Australia) set up a detailed exposure tracking system.

Reasonable quality information is available from Cresta for earthquake zones for many territories.

The data available for European Windstorm is generally moderate to poor, with inconsistent monitoring of aggregates.

The UK flood is an example where historically information has not been available. This may change with the publication of the Halcrow Report commissioned by the ABI and other alternative studies being undertaken.

Information from United States is detailed.

It is of interest that the so-called developed territories are often worse at providing aggregates information on a regular basis than those territories which are less well developed.

Some markets have a Tariff rate for primary cover which may be used. The question as to its adequacy may arise. For example, the rates for earthquake cover in Turkey were set by the Government on the advice of a leading reinsurer so may be felt to be reliable.

Information can be obtained from R&D departments of consultancies on damagability and return periods of events.

3.2.1 Aggregate-based methods

(i) Basic recipe

1. Determine target perils and zones.
2. The cedant provides the aggregate exposure for the key zone or zones and perils,
3. Determine return periods and damage degrees for natural perils of different severities and for different risk types using research undertaken by meteorological experts, construction engineers and so on.
4. Calculate the pure premium for the excess layer as [damage degrees exposed] times [aggregate exposure],
5. Rate for secondary perils, e.g. fire following earthquake, Tsunami, business interruption, workers compensation and so on.

(ii) Illustrative example

Perils: Earthquake

Aggregate: $100m

Cover: $30m xs $20m

Intensity	*Return Period*	*Estimated Damage*	*Cost to Layer*	*Estimated XL Cost*
MMVII	35	16 %	0 %	0.0000 %
MM IX	160	38 %	18 %	0.1125 %
MM X	400	67 %	30 %	0.0750 %
MM XI	1,500	90 %	30 %	0.0200 %
MM XII	5,000	100 %	30 %	0.0060 %
			Total Cost	0.2135 %

The expected net loss cost is therefore 0.2135% of $100m or $213,500.

Here MM refers to the Modified Mercalli scale of assessing earthquake damage. Richter scale is not used .

(iii) Additional examples

An alternative method can be used for Australia where original gross premium is split by zone, peril and coverage. Thus, the percentage relating to catastrophe ground up can be determined and spread using a Pareto-type loss curve.

Various studies for Caribbean windstorm have been undertaken to determine damagability and return periods.

(iv) Points in practice

Care must be exercised over the reliability of, and sensitivity to, the underlying return period and damage assumptions. The choice of key zone is also problematic, especially for windstorm exposures but also for earthquake. For instance, the impact of earthquake in Mexico is very difficult to model. This is because Mexico City is built on landfill, surrounded by mountains which amplifies the earth's movements; a quake falling in this mountainous area will therefore cause a higher degree of damage to the city than at its epicentre. The 1985 Earthquake had its epicentre in the Pacific.

Premium is independent of market and cedant rates.

The need to understand original coverage, coinsurance and deductibles make the practical application very difficult.

3.2.2 Loss based methods

(i) Basic recipe

Obtain the historic loss amount from specified events,

Revalue for inflation,

"As if" for changes in the portfolio/exposure,

Assign return periods to the catastrophic events,

Add "ghost" events and return periods outside historical experience.

(ii) Illustrative examples

"90A" for UK wind,

Typhoon 19 Mireille for Japanese wind.

(iii) Points in practice

A common problem can be rating for <u>one</u> event only. There is the need to recognise that two or more events are possible. Some lower layer catastrophes are effectively second loss policies.

3.2.3 Simulation/modelling methods

(i) Basic recipe

Obtain the cedant exposure split by postcode, prefecture, etc.,

Determine a distribution of events with data either from external consultancies or an internal R&D department. This will need frequency and severity components, together with an implied damage ratio for each severity.,

Run a 1,000 year simulation, say. It is better to use stratified than random sampling to cope with the rare events. A good example is the Latin Hypercube sampling used by "Crystal Ball". Monte Carlo simulation does not have this feature, and could lead to underpricing.

Determine the average and standard deviation (and possibly higher moments) of losses to the layer concerned.

(ii) Illustrative example

The output of a simulation on a UK property account is part of Appendix 2 to this paper. This has been achieved by using a commercially available package. As an alternative see "Storm Rating in the Nineties" [21].

(iii) Points in practice

Very complex and subjective modelling is necessary,

Packages can be bought in the marketplace.

(iv) Example

Historically in the US the following procedure was used:

Establish the total wind exposed premium. This was a percentage of total premium varying by line, e.g. 25 to 30% for homeowners or 7½ to 10% for auto property damage.

Apply a 20% PML. This gave the attachment point for a reasonable catastrophe programme for which a rate on line of 20 to 25% would be charged. Higher layers were then priced at around two-third of the previous layer and experience adjustments were made.

However, following the 1989 Hurricane Hugo loss rates increased dramatically. This led to pressure from carriers who were less exposed to refined pricing methods, The simulation programme CATMAP is now widely used. This at a standard level runs off the income split by State, but enhanced versions use a county or even zip code breakdown. The results cover wind and earthquake exposure, but an additional load needs to be made for other perils such as riot, flood and terrorism, or additional coverages such as California workers' compensation or business interruption. Other packages are available for UK and other territories.

3.2.4 Burning Cost Rating

Under Burning Cost Rating actual losses incurred are used to determine the cost. The keys to assessing these rates are:-

(a) **Loss Frequency**

A burning cost method is only suitable if there are a sufficient number of losses to obtain a suitable loss frequency.

(b) **Indexation**

Losses should be revalued into current terms. This involves both inflation and the increase in number of policies. A suitable index could be premium income adjusted for any rate changes.

(c) **Changes in Policy Conditions**

(d) **Changes in Retention**

3.2.5 Exposure Rating

Exposure rating is used to rate areas and covers with little or no loss experience. There are three stages:-

(1) Establish a Catastrophe Estimated Maximum Loss (E.M.L.).

(2) Establish a Catastrophe Premium - this is normally From The Ground Up - (F.G.U.).

(3) Establish a suitable Loss Distribution Curve. A Pareto type distribution is the norm, although this is derived from "feel" rather than parameters, due to the lack of data..

Set out below is an example of a calculation for a UK direct writer requiring a quote of £25 million excess of £50 million. Reinstatements and brokerage are ignored. This is based on a 1992 quotation.

The estimated Gross Premium income for 1992 is £230 million and the loss data is as follows:

	Premium	**F.G.U. Losses**	**Indexed**	
1991	220,000,000	Nil	Nil	
1990	200,000,000	95,000,000	109,250,000	(90A)
		22,000,000	25,300,000	(90D)
1989	180,000,000	Nil	Nil	
1988	170,000,000	Nil	Nil	
1987	160,000,000	65,000,000	96,451,612	(87J)
1986	155,000,000	Nil	Nil	
1985	150,000,000	Nil	Nil	
1984	145,000,000	6,500,000	10,310,344	
1983	120,000,000	Nil	Nil	
1982	100,000,000	Nil	Nil	

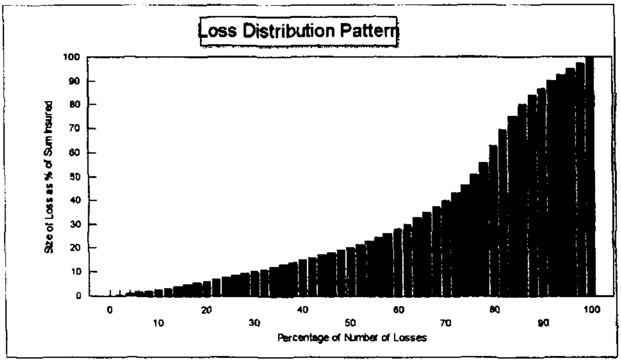
First calculate the Maximum Possible loss. This is assumed to be twice the 90A Loss Indexed i.e. £220 million (2 x 109.250). This is based on "current market practice." Flood damage is not considered, but could give rise to a substantially higher PML.

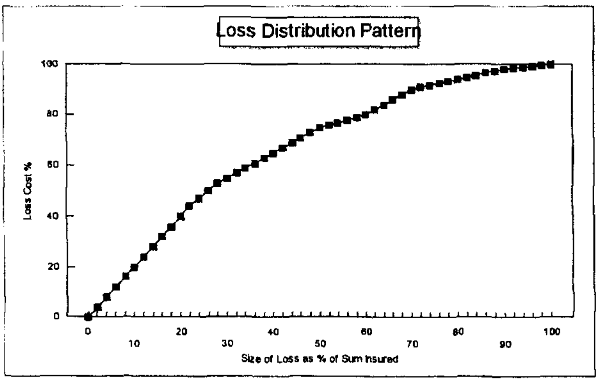
Next, we calculate a loss for a specific layer. The practice is to use 90% xs of 10% of the largest loss (109,250,000) say £90 million xs £10 million.

The losses to this treaty at the current index would be £90 million + £15.3 million + £86.451 million + £0.310 million = £192.151 million. (This is similar to the burning cost). The average cost is £19.215 million per annum.

This cost, from the Pareto curve, represents about 50% of the total cost. This is taken from the size of loss curve below looking at the size of loss of 10 (giving 20%) and 50 (giving 70%). Therefore, the total catastrophe programme should cost £38.42 million.

The £50 million point represents about 22.5% of the E.M.L. of £220,000,000 and £75 million (i.e. £25 million xs of £50 million) is about 34% of E.M.L. Using the second graph 22.5% is about 45% of loss cost, 34% is 60% of loss cost and so the premium is 15% of the total cost of £38.42 million or £5.73 million (before expense, commission and safety loading).





3.3 Proportional treaties

(i) Introductory remarks

Traditionally, the attitude of "follow the fortunes" meant that the only pricing decision was how much over-rider commission to add to the cedant's original commission. In recent years, poor underlying performance, and growing recognition of the unlimited nature of the cover and the extent of natural perils exposures, have led reinsurers to make their own assessment of the total reinsurance commission affordable, independent of the original commission. This section will use a property proportional example to illustrate the ideas, then discuss application to other lines of business.

(ii) Basic recipe

1. Split the exposure into three separate components:

   Basic loss cost (ordinary losses)

   Large individual fire losses (peak losses)

   Catastrophe/event losses (natural perils losses),

2. The basic loss cost may be priced using normal burning cost/experience rating methods as follows:

   a.) Take the 100% cost of claims to the treaty for the last 5 or 10 years, and subtract peak and natural perils losses above a certain level, to obtain the experience of ordinary losses

   b) Put the historic experience onto an "as if" basis of the current exposures, e.g. adjusting for changes in primary underwriting terms and conditions, changes in portfolio mix and any other appropriate factor.

   c) Revalue the experience to reflect claims inflation and changes in primary rates.

   d) Take a suitable average or trend of historic experience, to arrive at an expected basic loss cost for the forthcoming treaty year.

3. For peak losses, a suitable deductible is chosen. The cost of claims below this level falls into the basic loss cost. The cost of exposure to claims above this level should be priced using normal risk X/L methods, e.g.:

   a) Obtain a risk profile

   b) Use a loss curve to apportion the original premium to exposures above and below the deductible.

   c) If a risk profile is not available, you may be able to use a Pareto curve extrapolation of the recent large loss history, but this is less satisfactory

   d) upper limit equals treaty maximum sum insured.

4. Catastrophe losses should be priced using a normal X/L method of rating suitable to the risk and territory concerned:

   a) The choice of deductible will depend on the territory and peril Sometimes the cedant will only report natural perils losses above a certain amount, e.g. minor snow/freeze losses in the UK, e.g. minor typhoons in SE Asia. Sometimes the cedant will report all losses, and the peril should be rated from the ground up

   b) The upper limit can be chosen as the event limit if the treaty is capped, otherwise treat as an unlimited treaty.

   c) If the cedant is using cession limits, the reinsurer needs to be very confident of the basis before using this as a pricing cap.

   d) Then use normal aggregate or loss based methods, consistent with rating X/L exposure in the same territory.

   e) Often there will be a tariff in the territory for the natural peril concerned, e.g. earthquake in Turkey, and the primary premium will be split between natural peril and fire premium, with different commission scales. The reinsurer should obviously make himself aware of the source of the tariff, and in the spirit of proper pricing be prepared to take a different view.

(iii) Illustrative example

The separate components of basic loss cost, peak loss cost and natural perils cost are all calculated using similar techniques to those shown in the X/L sections.

(iv) References for further reading

"The Rating of Pro-Rata Treaties", LIRMA 1994, gives a clear exposition of the principles, and a long worked example,

"Property Insurance: Data Requirements for Calculating Correct Premiums in Insurance and Reinsurance", Munich Re 1990, covers other topics as well, but shows in great detail the degree of information reinsurers should ideally require from cedants.

(v) Points in practice

Underwriters now find that, given the information requirements and the variety of techniques involved, pricing a proportional treaty is often harder than pricing an X/L treaty. Given the relative premium volumes, and the greater uncertainty on exposures, they should not be surprised!

Other issues include variable commission scales, Loss participation clauses (LPC) and loss corridors, event limits (cf. cession limits), and the cashflow mechanics of different types of commission scale

3. 4. Use of simulation methods

(i) Introductory remarks

Used for calculations in the following circumstances:

Special treaty conditions depending on the aggregate amount of losses:

Aggregate deductibles

Reinstatement premiums

Sliding scale commissions on pro-rata treaties

Sliding scale premiums on risk X/L treaties ("swing rates")

Profit commission,

Treaties with special coverages depending on the aggregate amount of losses:

Loss corridors

Loss participation clauses

Covers with aggregate annual limits

Total loss only covers.

(ii) Basic recipe

Assess the loss cost for the ordinary cover without the special conditions, using the normal techniques,

Analyse the ordinary loss cost into separate components of frequency and severity,

Assume a suitable distribution for each of these components, e.g. Poisson for frequency and Pareto or lognormal for severity,

Using this model of the loss cost, simulate the result of, say, 1,000 years of experience of the coverage with the special conditions included using, say, the @RISK add-on package,

Care should be taken over the treatment of expense and profit loadings:

Calculate amount of loading on the original premium. This should be a cash amount addition to special premium, not a percentage loading.

(iii) Illustrative example

Suppose you have a cover £0.5m xs £0.5m, where the ordinary loss cost (before loadings) is 1 15% of GNPI of £100m, i.e. you expect about 4 to 5 losses to hit the layer, and the cedant asks you to quote for the cover with an annual aggregate deductible of £1m, Assuming the exposure is per risk only with no event exposure, then re-express the premium basis as:

a.) expected frequency of 5.0 losses p.a. to the layer, modelled as having a Poisson distribution

b) severity is Pareto with alpha of 2.3

These parameters might come directly from the original rating calculation. If they are not available, for example, because an experience method was used, it is probably instructive to estimate them from the data as a first check of the result.

Then it is very simple to model the cover with the deductible in @RISK, which shows the new loss cost is £0.34m, i.e. the credit for the deductible is £0.81m, i.e. 81%.

If the original loadings for expenses and profit had been (100/75), i.e. 33%, the new premium should be loaded with £0.38m (33% of £1.15m), not £0.11m (33% of £0.34m).

Further examples are given in section 4.3 on stochastic profit testing

For simulating catastrophic events alpha stable distributions are being considered. See Appendix 1.2

3.5. Stop-loss treaties

(i) Introduction

Burning cost techniques are not to be used, as illustrated by the following example:

		Treaty A	*Treaty B*
		Incurred Loss Ratio	*Incurred Loss Ratio*
Year 1		82%	58%
Year 2		68%	72%
Year 3		80%	60%
Year 4		28%	112%
Year 5		92%	48%
Mean		70%	70%
Standard deviation		25%	25%
BC xs	70%	8.8%	8.8%
	80%	2.8%	6 4%
	90%	0.4%	4.4%
	100%	0.0%	2.4%

The mean and standard deviation are the same, but the skewness is different. The historic burning cost may also zero.

Each treaty needs special consideration and it is hard to create standard approaches. It is vital to understand the FGU exposures and the structure of rest of reinsurance programme, i.e which other covers inure to benefit of stop-loss cover being rated. It is also necessary to decide if the cover to be rated is being bought as a frequency or a severity cover.

(ii) Basic recipe

Fit a probability distribution to the aggregate of total claims amounts,

Use the probability distribution to calculate the loss cost on a mathematical basis,

i.e. integrate the probability of claim times amount of claim to the layer.

Ideally, you would like to create separately distributions for both frequency and severity of the underlying primary experience, but you may have to fit it against as-if historical loss-ratio experience.

(iii) Illustrative example

Consider a medium size specialist UK insurer seeking a stop-loss on its household and small business property account. The natural perils losses were subtracted from the experience of the last 20 years. Once the residual experience was revalued, it was quite clear that there was an underlying cycle. Instead of taking the plain arithmetical standard deviation, the variability about a suitably modelled trend was assessed. This was used to model an expected trend loss ratio for underlying claims, and variability around the trend.

The catastrophe exposure was modelled separately, using a loss-based method giving a distribution of return periods for events of different sizes,

The two separate distributions were combined using @RISK, which allowed the pricing any desired stop-loss cover.

(iv) References for further reading

"On the Rating of a Special Stop Loss Cover", G. Benktander, ASTIN 1974.

(v) Points in practice

Care over treatment of expense and profit loadings.

In excess of loss reinsurance rating, only 5 years of statistical information is normally used. In stop loss, you need a much longer period, desirably 20 to 25 years. It is preferable to have covers where the fluctuation comes from the claims side and not the premium side, e.g. hail or windstorm.

3.6. Credibility Theory

3.6.1 Much has been written about Credibility Theory in Actuarial and Risk Theory Literature. Interested readers can refer to [15] and [16] for further and more detailed information. This section highlights aspects of Credibility Theory that are sometimes used in the London Market albeit often without the knowledge of the user.

3.6.2. Consider the following scenario. You need to charge a premium for a risk where you either have :-

1. No loss data for the risk (e.g. a new building); or
2. Some loss data relating to the actual risk.

In the first case you may end up charging what is perceived to be the "Market Rate" for the exposure unit appropriate for the class with perhaps some subjective allowance for the feel of the risk; for example the quality of the risk management, the quality of the business the broker usually shows you, the current market cycle, and so on. In the second more common scenario we are faced with some further additional questions:-

1. What use are we to make of the loss history presented to us ?
2. Is the data relevant to the loss experience going forward ?
3. Is other collateral data relevant to the risk, and if so, what use can we make of it ?

3.6.3 In essence, we want to incorporate an allowance for both the collateral data relating to other similar risks and to the individual assureds loss experience. Ignoring expenses, commission, profit and other loadings we wish to arrive at a premium based on

$$P = Z * \bar{X} + (1 - Z) * \mu \qquad (1)$$

P = premium
$\bar{X}$ = price based on loss experience only
Z = Credibility weight
μ = price based on collateral data only

3.6.4 Applying equation (1) to the above two examples, Z takes the value 0 in the first case as no weight is given to the individual assureds historic loss experience. Moreover, the absence of any historical loss experience may lead to an extra element of doubt in the underwriters mind and would thus lead to an extra risk loading on the overall premium.

Case 2 requires a lot more effort in arriving at an appropriate credibility factor Z for the risk in question. There are some natural constraints on the value.

1. Z can only take values between 0 and 1.
2. The more data we have for the risk, the more weight we should attach to the insured's historic loss experience, i.e. the larger Z.
3. The less relevant the collateral data, the less weight we should attach to it, i.e. the larger Z.
4. The more the underwriter feels the claims process going forward is likely to be similar to the past, the larger the value of Z.

This naturally leads to the question of how much data is needed before full credibility is ascribed, i.e. Z equals 1.

3.6.5 The criterion used by some North American actuaries to determine the amount of data available from the risk under consideration was sufficiently large for Z to be taken as unity was that the relative error between the true pure premium and that estimated based on historical data alone should be less than some given amount with some given probability. The terminology used is that the data set is Fully Credible (k,p) if the data size is such that the probability of the pure premium estimate is p percent certain of being within k per cent of its "required" value. Numerical examples are given in [16]. When full credibility is not available, partial credibility results in Z being less than unity. There are a few ad-hoc methods employed to assign a value to Z, for example

$$Z = \min(1, \sqrt{\frac{n}{m_0}})$$

n = number of claims in loss history

m_0 = number of claims required for full credibility

This formula has the advantage that it is easy to understand and has some statistical justification if we are looking at a risk whose aggregate loss distribution is compound Poisson.

3.6.6 **Empirical Bayes Credibility Theory**

This is a useful branch of credibility theory which has practical uses in premium rating where you have loss and exposure data for similar risks. Consider the data in the example which relates to the number of losses for various hospital trusts in a particular

American State. We wish to estimate the expected loss frequency for the forthcoming policy year.

Y_{ij} represents the number of losses experienced by hospital i in year j
P_{ij} represents the exposure measure for hospital i in year j
X_{ij} therefore represents the loss frequency per unit of exposure for hospital i in year j and equals Y_{ij}/P_{ij}

We wish to estimate the loss frequency for one or more hospitals in the state for the forthcoming year. The process used will allow us to use the collateral data available from other hospitals which can be expected to exhibit similar but probably not identical loss frequencies.

3.6.7 The assumptions underlying the Empirical Bayes Credibility Theory Model are

1. X_{ij} are independent for j = 1,...,n
   The distribution of X_{ij} *depends on the value of* θ_i *whose value is fixed but* unknown. $X_{i1} | \theta_i$, $X_{i2} | \theta_i$ are independent but not necessarily identically distributed.
2. $\theta_1, \theta_2, \ldots, \theta_n$ are independent and identically distributed.
3. The Pairs (θ_i, X_{ij}) *and* (θ_k, X_{kj}) are independent for $i \neq k$

If we assume that there are functions m( ) and s( ) such that

$$E[X_{ij} | \theta_i] = m(\theta_i) \qquad (2)$$
$$V[X_{ij} | \theta_i] = s^2(\theta_i)/P_{ij} \qquad (3)$$

Rewriting equation (1), our estimate of loss frequency will be in the form :-

$$\text{Loss frequency} = Z_i \times \bar{X}_i + (1 - Z_i) \times E[m(\theta)] \qquad (4)$$

where $\bar{X}_i = \sum_{j=1}^{n} P_{ij}X_{ij} / \sum_{j=1}^{n} P_{ij}$

$$Z_i = \sum_{j=1}^{n} P_{ij} / (\sum_{j=1}^{n} P_{ij} + E[s^2(\theta)]/V[m(\theta)])$$

If we further define

$$\bar{P}_i = \sum_{j=1}^{n} P_{ij}$$
$$\bar{P} = \sum_{i=1}^{N} \bar{P}_i$$
$$P^* = (Nn-1)^{-1} \sum_{i=1}^{n} \bar{P}_i (1 - \bar{P}_i/\bar{P})$$
$$\bar{X}_i = \sum_{j=1}^{n} \bar{P}_{ij} X_{ij} / \bar{P}_i$$
$$\bar{X} = \sum_{i=1}^{N} \bar{P}_i \bar{X}_i / \bar{P} = \sum_{i=1}^{N} \sum_{j=1}^{n} \bar{P}_{ij} \bar{X}_{ij} / \bar{P}$$

It can be shown that the estimates for the credibility frequency are

$$E[m(\theta)] \approx \bar{X}$$
$$E[s^2(\theta)] \approx N^{-1} \sum_{i=1}^{N} (n-1)^{-1} \sum_{j=1}^{n} P_{ij} (X_{ij} - \bar{X}_i)^2$$
$$V(m(\theta)) \approx P^* \{(Nn-1)^{-1} \sum_{i=1}^{N} \sum_{j=1}^{n} P_{ij} (X_{ij} - \bar{X})^2 - N^{-1} \sum_{i=1}^{N} (n-1)^{-1} \sum_{j=1}^{n} P_{ij} (X_{ij} - \bar{X}_i)^2 \}$$

3.6.8 Example

Hospital Y_{ij}		Year Total Number of Claims				
	j	1	2	3	4	5
1		520	464	400	380	425
2		178	188	150	145	200
3		100	88	89	110	125
4		200	189	212	230	225
5		300	250	340	199	288
6		50	80	100	89	110
7		40	60	61	59	62

P_{ij}		Total Exposures (OBU)				
	j	1	2	3	4	5
1		8000	7500	7454	8125	8565
2		3500	4512	4156	4878	4210
3		1322	1566	1787	1714	1500
4		3750	3600	3500	3700	3782
5		6454	6544	4959	5252	5745
6		4000	4152	4565	4755	6500
7		1215	1515	2000	1858	1959
Total		28241	29389	28421	30282	32261

Y_{ij}/P_{ij}	1	2	3	4	5
1	0.065	0.062	0.054	0.047	0.050
2	0.051	0.042	0.036	0.030	0.048
3	0.076	0.056	0.050	0.064	0.083
4	0.053	0.053	0.061	0.062	0.059
5	0.046	0.038	0.069	0.038	0.050
6	0.013	0.019	0.022	0.019	0.017
7	0.033	0.040	0.031	0.032	0.032

It follows from the above equations that

$$\bar{P} = 148594$$

$$P^* = 3596.93$$

$$\bar{X} = 0.0451$$

Whence $E[m(\theta)] \approx 0.451$

For each hospital we calculate the following

	$\sum_{j=1}^{N} P_{ij}(X_{ij} - \bar{X}_i)^2$	$\sum_{j=1}^{N} P_{ij}(X_{ij} - \bar{X})^2$
1	1.963	5.997
2	1.235	1.690
3	1.189	4.237
4	0.283	3.136
5	3.297	3.468
6	0.207	17.987
7	0.085	1.344

$E[s^2(\theta)] = (1.963 + 1.235 + .. + 0.085)/(7 \times 4) = 0.295$

$V[m(\theta)] = [(5.997 + 1.690 + ... + 1.344)/(7 \times 5 - 1) - 0.295]/3596 = 0.00023$

$E[m(\theta)] = 0.451$

If hospital 1 expects an exposure unit of 9,000 in the forthcoming year, the credibility weighted expected frequency is

$$9000 \times \left\{ \bar{X} \times \frac{E[s^2(\theta)]}{V[m(\theta)]} + \sum_{j=1}^{n} Y_{ij} \right\} \div \left\{ \sum_{j=1}^{n} P_{ij} + \frac{E[s^2(\theta)]}{V[m(\theta)]} \right\}$$

$$= 9000 \times \left\{ 0.451 \times \frac{0.295}{0.00023} + 2189 \right\} \div \left\{ 39644 + \frac{0.295}{0.00023} \right\}$$

$= 9000 \times 2246.84 / 40926.6$

$= 9000 \times 0.05489$

$= 494$

If we rewrite in terms of $Z_i \times \bar{X}_i + (1 - Z_i) \times E[m(\theta)]$

$Z_i = 96.84\%, \bar{X}_i = 0.0552, (1 - Z_i) = 3.16\%, and\ E[m\{\theta)] = 0.451$

Thus in the case of hospital ! we are assigning 96.48% credibility to the historic loss frequency of the hospital and only 3.16% weight to the collateral losses.

The rates per unit of exposure, credibility factors and sample output for all of the hospitals is as follows

Hospital	Rate	Z_i	Exposure	Estimated Frequency
1	0.0549	96.84%	9,000	494
2	0.0408	94.26%	4,500	183
3	0.0621	85.90%	1,650	102
4	0.0568	93.40%	3,450	196
5	0.0475	95.72%	5,000	237
6	0.0193	94.88%	7,000	135
7	0.0346	86.84%	2,100	73

3.6.9 **Robust Credibility**

Another problem in assessing credibility is the existence of outliers, i.e. extreme events which, although appearing in the data, should be initially ignored for the purpose of rating. A paper by Gisler & Reinhard in Bulletin of ASTIN gives a solution to part of the problem. In this paper they deal with loss ratios. Previous methods of treating extreme loss ratios is to apply trimming at some percentage point, and to "distribute" the excess losses over the whole portfolio by an appropriate loading. The question arises as to the level of such trimming.

3.6.10 The real aim is to limit the influence of large claims on the data, and this is the direct aim of Robust Statistics Theory. The combination of Robust Statistics and Credibility Theory gives us Robust Credibility.

In their paper Gisler & Rheinhart apply the mathematics of Robust Credibility to claims which act in accordance with the general Buhlmann Straub model (which is similar to the Bayesian technique given above.

Let X_{ij} be the anticipated loss ratio for policy i in year j.

Let T_i be the credible loss ratio for the upper trimming of data

(Note that the paper does not consider trimming low loss ratios)

The calculations give a loss ratio as follows

$$T_i = \frac{1}{n}\sum_{j=1}^{n} \min(X_{ij}, 2T_i)$$

3.6.11 Alternatively, the estimate for the rate is calculated by trimming the loss ratio at twice the expected value. T_i is solved by iteration.

The paper does not deal with the use of low cut off points (i.e. where loss ratios in one year are exceptionally low) and as far as we are aware there is no solution to the problem. It has been suggested that the truncations form a pair (T_b , T_h) where T_b represents the lower bound and T_h the upper bound. (0, $2T_i$) is just one such pairing.

This sort of approach may be considered against (say) the background of swing rated (or experience rated) business where upper and lower limits are placed on the base rate to accommodate variation in the loss ratios.

3.7 Distribution Calculus methods

Distribution calculus is an important method in assessing rates for risk excess reinsurance. The basis is to start with a number of individual (or grouped) claims over either one or more years. Technical details such as indexation, and how to deal with incomplete sets of data need to be addressed (truncated or censored data).
From this data set an empirical loss distribution is devised. Then, using maximum likelihood methods or similar statistical approach, a Fitted Distribution is calculated which may extend the empirical distribution to allow for the possibility of claims in excess of those recorded. The distribution selected may be more prudent than the empirical data.
The next stage is to calculate the aggregate loss distribution for the layer. This is achieved by recursive algorithms which are documented in many papers, and in particular [6]. The method is as follows

1. Determine the distribution of claim size.
2. Determine the lower limit (truncation point) and upper limit (censor point).
3. Use an appropriate claims distribution , often determined by the first three moments (mean, variance and skewness).
4. The distribution is calculated, and in particular the expected value and variance, and these are then input into the appropriate rating formula to calculate the rate.

The algorithms are readily available in computer programmes, and are relatively easy to apply

3.8 Generalised Linear Interactive Models (GLIM)

3.8.1 Generalised Linear Interactive Models are used in those cases where there is a substantial volume of individual data and well defined rating factors. As such their use in pricing London market business is very limited. The use of the GLIM algorithm is well documented in recent actuarial papers and in text books. Examples of London market business where it is of use include Motor Classes, Marine Hull, Marine Cargo and P& I Clubs.

3.8.2 GLIM models are normally applied to large data sets that can easily be classified into constituent rating groups. The algorithm then seeks to apportion the extent that the expected loss cost was related to the individual rating factors. Individual rating factors could be eliminated from the pricing process if their contribution to the expected loss cost was not significant. i.e. they do not significantly improve the fit of the model. The "best" model would then be the one which produced the nearest approximation to the data based on the least number of rating factors.

3.8.3 As an example , for Marine Hull business, losses could be categorised by Age, Tonnage, Type of Vessel, Location of Loss and so on. A Linear Model is then fitted to this, and careful consideration must be paid to any interactions that may exist, for example between Tonnage and Age, and also the uneven distributions in the database. Oil Tankers below 40,000 tonnes are rare, as are Tugs over 15,000 tonnes.

4. STRATEGIC PRICING

4.1 Introduction

This section of the paper describes the various components of an approach to pricing under the heading of "Strategic Pricing". It is considered that the pricing/underwriting objectives must fall within the company's overall objectives. These are usually formalised when a corporate plan is prepared setting out the direction of the company over the planning horizon. The plan may consider the type of business the company wishes to write and the capital required to support that business. Certain underwriting results after expenses would be required from the business written which together with any investment income should be sufficient to provide the required return on the capital and to generate sufficient surplus to meet any growth targets included in the plan.

The methodology described in this section may be used in many different ways, for example to :

a) Provide input on price determination of large risks
b) To set portfolio pricing objectives
c) Input into strategic planning
d) As part of the control cycle

The above uses of pricing are each considered through the application of profit test methodology allowing for the time value of money, risk/reward and other assumptions to determine the true profitability of a particular tranche of business.

There are two main approaches when considering pricing

Approach 1 - Empirical Pricing by Individual Risk

Take an estimated loss ratio derived using more traditional techniques and then apply profit test logic to determine the true profitability of the business, probably using a Return on Capital (ROC) type measure. Revise underwriting approach if the result id not what is required. This approach is useful when trying to price a particular risk, where allowances for individual risk characteristics may be made, for example premium receipt patterns, level of excess.

Approach 2 - Portfolio Pricing Targets

Make assumptions about the required return, investment return and so on to determine the required 'target' loss ratio which is needed to satisfy shareholders' objectives. This approach is useful when considering a collection of risks together at various levels.

Section 4.2 describes the links between underwriting targets, pricing and reserving. The processes involved in determining the underwriting targets and pricing and accepting new business are outlined. The underwriting performance measurement in the form of reserving and monitoring plan variances are then described. This is followed by description of early warning systems and feedback loops whereby the latest available information is made use of in the planning and pricing decisions for the subsequent periods.

Section 4.3 describes the processes involved in segmenting the business, measuring risk variability of different classes of business and in determining the capital required to support a given level of business. The processes involved in determining risk weighted return on capital are then described. The determination of capital requirement to support business written is related to the concept of risk based capital which looks at the company from the solvency perspective.

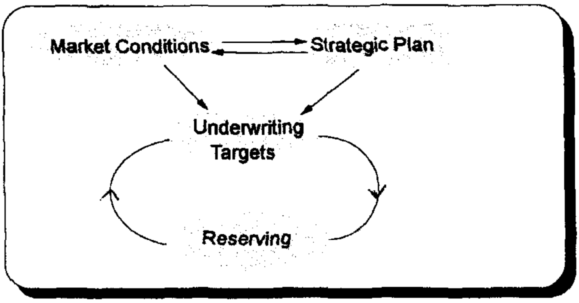
Section 4.4 describes the concept of profit testing. The concept is described in outline including the various assumptions, cash flow projections and scenario testing. The section also illustrates the concept for a simplified class of business.

Section 4.5 covers the topic of underwriting management. This includes problems encountered in managing the underwriting cycle and detailed consideration of factors influencing underwriting targets, balancing the portfolio and risk selection.

4.2 UNDERWRITING TARGETS & CONTROL CYCLE

4.2.1 Introduction

This part of the paper describes the links between underwriting targets, pricing and reserving. The processes involved include reserving, tracking the business written and establishing feedback loops whereby the latest available information is made use of in the planning and pricing decisions in the subsequent periods. The process may be schematically described by the following diagram:



4.2.2 Planning over a time horizon

A reinsurance company may prepare a business plan over one year, three year , or possibly even longer time scales. A one year plan considers the underwriting targets for the forthcoming year and has an immediate impact, whereas a three year plan allows longer term strategy and direction to be considered. A five year or longer term plan may be less common in the London Market, but could be beneficial if it was possible to forecast the underwriting cycles, which have historically tended to repeat over five-six year time periods.

The business plan makes best assumptions about the levels of business expected to be written. These may be based on capital allocations and required return on capital, allowing for market conditions and expected developments over the plan period.

The assumed losses may be based on weighted loss ratios for the significant business within each portfolio allowing for any market trends. The loss ratios may be those given by the pricing models, either conventional or profit testing models described later. It is possible that such loss ratios may be inconsistent with those required to achieve the return on capital, in which case such inconsistency may have been accepted in view of longer term strategy considerations or it may be intended to achieve the target loss ratios by actively managing the portfolio. These issues are considered in detail later on in the paper.

The plan will make other assumptions such as expense levels and investment income. The plan may also consider cash flow projections which involve assumptions about premium and paid loss developments. Any such assumptions should be consistent with those used in the profit testing and pricing models.

4.2.3 Pricing & accepting new business

The underwriter may carry out pricing calculations and accept/reject business using either the conventional methods, profit testing models, application of judgement, or a combination of methods.

He will have the underwriting premium targets in mind since his performance may be measured against these targets. If the market conditions are less favourable than anticipated in the plan then the underwriter will be faced with difficult decisions. It is also possible that the plan assumptions may have been overtaken by subsequent events, e.g. a large catastrophe in the interim significantly affecting the market conditions. There may also be new opportunities not envisaged in the plan.

The underwriter may be constrained by guidelines on limits by contracts, programs (groups of contracts within a class of business), reinsureds, class of business and countries. The underwriter will also have limits by individual events or perils and aggregate exposure to such losses by individual territories. Such limitations may be determined by the existence of outwards reinsurance program and the nature and terms of such program.

The underwriter will want to make effective use of the capacity available and will want to ensure that he leverages the capacity for catastrophe business to get better deals with individual reinsureds on other classes of business

Some of these issues are considered in more detail later in the section 4.5 of this paper on "Underwriting Management".

In any case at the time of underwriting a risk, there will be more up to date information available relating to the market as a whole and the experience of the individual risks being underwritten. There may also be more up to date information available in respect of historic underwriting years relating to the class of business.

More and relevant the information available to the underwriters the better the underwriting decisions that can be made. The quality and the timeliness of the appropriate underwriting information can make the difference between profit and loss when the market conditions are difficult and competitive.

4.2.4 Reserving

The reserving process may be considered as periodic evaluation of the underwriting performance. The process makes use of the known information together with assumptions about the unknown. The experience is then tracked and the assumptions are modified in the light of the emerging experience.

When no historic credible data is available, the reserving process effectively starts with the plan assumptions. The reserves are based on up to date premium estimates, the loss ratios assumed for the plan and allow for the expected development patterns of premiums, paid losses and incurred losses. The development patterns may be based on the company's own experience for previous underwriting years adjusted for any available market statistics.

If the actual reported losses are adverse compared to the expected losses, the adverse movement may be investigated in more detail to determine scope for further deterioration and the plan reserves may be strengthened accordingly.

Thus in the reserving process, more up to date information becomes available at the portfolio level. This information may shed light on deterioration in any risk factors used in the pricing process. Information may become available on how the historic years are materialising and if any marked changes have occurred in premium and loss development patterns. In analysing adverse loss movements, further information may become available on individual large losses, or losses by regions or for individual reinsureds.

In some instances, such matters may be discussed extensively with the underwriters so that they too are more aware of how the business written in more recent years is performing.

4.2.5 Monitoring variances in Underwriting Targets

The actual experience will be monitored against the underwriting targets. The variance will be analysed in detail and explained so that the information can be used in future plans.

As more up to date information becomes available about the market conditions, terms of trade and the business actually written, it may be possible to draw conclusions from this information about how the current year is likely to materialise.

New information about losses from the current year and historic years will become available from the reserving process described above. In addition, more up to date information about outwards reinsurance costs and expenses will become available as well as any changes in exchange rates and investment returns

Efficient collation and use of such information as it becomes available not only gives advance indicators of the potential profitability of the year as a whole, but also gives information as to validity or otherwise of the assumptions made in the planning and pricing models. This enables any significant changes in the underlying assumptions to be considered and immediately reflected in the subsequent pricing decisions.

Such information would also enable the company to take corrective measures relating to types and levels of business to be written and to achieve more effective control and use of resources.

4.2.6 Early warning systems

In the process of monitoring the variances in the underwriting targets, information will come to light on various risk factors which are likely to have an impact on the business development. Use of the information from the reserving process combined with *effective management information systems will give early advance indicators about* favourable or adverse developments. The use of this information would enable the company to achieve the business development in a controlled environment.

Some of the risk factors which would be closely monitored are aggregate exposures, rates of premium growth, trends in loss ratios and reasons for any adverse movements, effectiveness of the reinsurance program, trends in expense ratios and acquisition costs, and investment returns.

Effective management information systems would enable the above risk factors to be analysed and tracked at a detailed constituent level so that the effect of the changes at individual class/risk level can be assessed as well as the combined effect on the business as a whole.

4.2.7 Feedback loops to Underwriters / Senior Managers

At a micro level, the actuary would talk to the underwriters/claims staff about experience of a class at a portfolio level or of individual risks in order to understand the business, so that the reserving process can be made more effective. When exploring adverse movements, the actuary will gain an insight into the underlying risk factors which can then be incorporated into any pricing models. The actuary can also give the underwriters useful information at the portfolio level so that the underwriters can better appreciate the effect of their underwriting decisions on the financial results.
Any sophisticated pricing models incorporating various risk factors and financial considerations may assist the underwriters in identification of the obviously good and bad risks so that they can focus on the more difficult and complicated risks. The actuary can also assist the underwriters with the larger risks which have significant financial impact and where a different perspective may be useful and justified.
The actuary will also be involved at the macro level in investigating the business experience and presenting the actuarial report to the senior managers. At such presentations the actuary may report on variances against plan, the reasons for the variances and any corrective measures required. The senior management meeting may also address longer term strategy issues on capital allocations, return on equity, levels and classes of business to be pursued, effective resource management, investment and taxation matters, cash flow management and dynamic solvency.
The actuary is well positioned to participate in both micro and macro levels since he has the relevant skills and knowledge to communicate effectively at both levels.

4.3 **RISK & REWARD**

4.3.1 Introduction

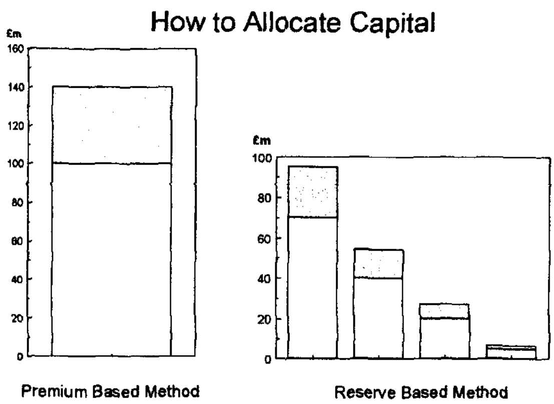
In return for investing capital in a project the investors (often the shareholders in a general insurance context) require an adequate return over the lifetime of their investment. Traditional risk/reward considerations apply in that to entice investment in a relatively risky venture a higher level of return is expected. Thus shareholders will not invest in an insurance venture unless the expected proceeds from their investment exceed the risk free rate available by a margin sufficient to compensate them for the degree of risk undertaken. This immediately gives rise to certain questions, namely:

i) How much capital should be allocated and for how long?

ii) What is the likely risk free rate of return?

iii) What level of return do the shareholders expect?

Below are two methods to allow for riskiness when conducting a profit test:



Method 1 (More risk requires a higher return)

Allocates capital to a tranche of business on a basis which ignores the degree of risk (possibly in proportion to GWP) and then seeks a varying return dependent on estimated risk of writing that block of business.

Method 2 (More risk requires more capital backing)

Allocate variable amounts of capital dependent on the estimated level of downside risk associated with writing a particular tranche of business but seek the same return to shareholders for all capital invested regardless of where.

Empirically Method 1 would appear to be sensible, however under this method it is less clear what the maximum amount of business is which could be supported by a given amount of financing Furthermore the relationship between risk and return for various tranches of business needs to be determined in the form of percentages.

Like Method 1, Method 2 also needs to determine the relationship between risk and return but this seems more straightforward in that the impact on capital of certain downside risks is easier to contemplate (e.g. the impact on capital of a major catastrophe say once in 10 years). As in practice most insurance concerns will know reasonably accurately what their current available capital is, it is straightforward to determine the implications on maximum premium volume of a given capital allocation.

Ideally the absolute level of risk needs to be determined for each class being modelled. This is perfectly possible though a partial solution is to consider relative levels of risk between class.

There is a hybrid approach making use of both Method 1 and Method 2, for example a large multinational may decide that its' insurance subsidiary needs to yield x% and its' non-insurance activities need to yield y% with capital being allocated between the two using a broad brush approach along the lines of Method 1. However within each subsidiary Method 2 could be used.

When considering the return to shareholders taxation should not be forgotten

Determining the absolute level of risk associated with a particular class of business is not straightforward (use of some statistical models to give an estimate of volatility would help). Though statistical models find it difficult to model systematic influences unless explicitly modelled (e.g. prior to the late 1980s what level of risk would have been estimated for MIG business?).

A first cut approach may be determined by considering on a relative basis how risky each class of business, some examples include:

i) Working excess of loss versus high layer protection

ii) Direct hull versus cargo

iii) Treaty versus facultative

iv) Proportional versus non-proportional

There are many reasons why capital is required to back insurance business, the main ones are listed below:

Under-reserving	Poor Investment Performance
Unexpected Catastrophes	Fraud
Absence of claims equalisation reserves	Bad Management
Unanticipated high claims frequency	Statutory Requirement
R/I failure	Unfavourable currency movements
Market Requirement	

It is important to decide how capital should be allocated

a) In proportion to written premium - the traditional approach

b) In proportion to reserves, with subsequent release in proportion to the run off of the reserves.

As most of the reasons that capital is required remain during the run-off of a block of business it appears sensible that capital should be allocated on a basis based on estimate of reserves.

Due to the element of independence of results between different types of business (class, territory etc.) a given portfolio of business may be backed by an amount of capital that is less than the sum of all the capital required if each tranche of business had been written via a separate entity, this is a non-trivial problem which RBC has already begun to tackle.

4.4 PROFIT TESTING

Table of Contents

4.4.0 An Overview

Profit testing falls into three distinct elements and views the interaction of three processes, which are usually analysed separately, and at three (or more) levels.

The three distinct elements are

1. Establish cash flow analysis on an agreed determined set of parameters
2. Review these processes by assessing the impact of changes in the parameter assumptions in a deterministic way (A 1% change in a specific assumption gives an x% change in profit).
3. Review this second process in a stochastic manner by setting distributions to the parameters and simulating the likely outputs.

The three distinct processes are

1. Underwriting
2. Asset assumptions and Management
3. The servicing of allocated capital.

The three levels are

1. The individual policy level (true pricing)
2. The portfolio/account level
3. The whole business level.

This latter should relate to the business plan.

To understand and analyse the cash flows we must first review the three processes.

Underwriting

Here we need to make assumptions on the following

1. The collection of premiums
2. Payments to brokers (or other third parties)
3. Expenses (acquisition and ongoing)
4. Claims reporting pattern
5. Claims payment pattern (including claims expenses)
6. Anticipated Ultimate Loss Ratio (for assessing IBNR)
7. Strength in reserving
8. Rate of interest used to discount reserves.

Expenses may differ between a renewed contract and a new business proposal. It is unusual to take this into account in the individual policy profit test, but when looking at portfolios over time it is vital to get some correspondence between model and real expenses.

Assets

1. Yield on Capital Assets/Free Reserves
2. Yield on Insurance Reserves
3. Cost of borrowing (to finance cash and reserve requirements)
4. Investment expenses

Clearly these assumptions can be complicated by asset mix, tax and so on.

Capital

1. Amount allocated to product
2. Yield on assets
3. Pattern of payment to shareholders.

4.4.1 Introduction

The technique of profit testing is routinely used in the life insurance industry, but less frequently so in general reinsurance. There may be many causes for this, for instance that

1. focus is on determining the expected loss cost and its variability, which is normally less problematic for life business, and that
2. general business is typically not as long-term as life, which reduces the importance of investment income.

In view of the generally slim margins in general insurance, properly taking account of all cashflows caused by the insurance transactions in a profit test is nevertheless very valuable, both for pricing individual pieces of business as well as for business segment studies. In this section, we will develop such a profit test for application to general reinsurance.

The application that most naturally springs to mind is the evaluation of a block of business, for instance a territory or a class of business. In order to fix ideas, let us think about evaluating the profitability of a medium-tailed class of business in sections 4.4.3.-4, and postpone the discussion about the applicability of the model to different aggregation levels to section 4.4.5. The details of an example profit test are discussed in two sections, section 4.4.3, which deals with the underwriting cashflow, and section 4.4.4, discussing the cashflows arising from the capital base and from conservative reserves.

A profit test should by definition carry out a present value calculation of all cashflows caused by the considered business, and subsequently compare this present value against the required risk weighted target return on capital.

A list of the cashflows to consider for general reinsurance would include:

- Premiums
- Claims
- External acquisition costs
- Expenses
- Outwards reinsurance costs
- Potential recoveries
- Capital base allocation
- Technical reserves
- Investment income on technical reserves and capital base

By making assumptions about investment returns, setting the return on capital to the target one, and subsequently solving for the ultimate loss ratio, one could also compute an implied maximum allowed loss ratio for the considered business segment.

Finally, by varying the parameters over which the reinsurer has some control, e.g. acquisition costs, expenses, outwards reinsurance costs and capital base allocation, one can study the effect on profitability of writing varying premium volumes for a business segment, and the effects of using different allocation bases for these parameters. For example, writing a larger book of business for a certain line normally results in reduced profitability for the extra volume, it affects retrocession costs and capital allocation, and often reduces the management expense ratio. A profit test can study the impact of all these changes simultaneously. If this is done as a strategic study for all segments of business simultaneously, a suitable allocation of costs and capital can be arrived at, which would even allow the reinsurer to calculate the overall maximum theoretically achievable expected profit amount, and the optimal line of business premium mix. This will be described in section 4.4.8, by way of a simplified example.

In all the above proposed analyses, it is of fundamental importance also to study the sensitivity in the resulting answers. One can often learn just as much from studying the sensitivity in the profit to for instance interest rate or loss ratio assumptions as from the actual computation of the profitability, and we will give several examples of simple sensitivity tests in sections 4.4.3-4.4.4.

4.4.2 The profit test table

An example of what a six year basic profit test table may look like is shown in Table 1. This is intended to fix ideas for the discussion of individual items in section 4.4.3, realising that there are many different ways of designing the table. In particular, table 1 shows cashflows as percentages of ultimate premiums, and a natural alternative would be to show them as amounts.

"PV" in the table stands for present value at 1/1 of development year 1, and the choice of discounting rate is somewhat involved. Any realised profit or loss must obviously be revalued at the return on investment required from the insurance operations. The return on investment used in the profit test table is 15%.

1. Development year	1	2	3	4	5	6	7
2. Gross Premium	50.0%	25.0%	25.0%	0.0%	0.0%	0.0%	0.0%
3. External Acqisition Costs	-5.0%	-2.5%	-2.5%	0.0%	0.0%	0.0%	0.0%
4. Expenses	-5.0%	-2.5%	-2.5%	0.0%	0.0%	0.0%	0.0%
5. Net Retro Cost	-1.0%	-0.5%	-0.5%	0.0%	0.0%	0.0%	0.0%
6. Paid Claims	-15.0%	-15.0%	-18.7%	-15.0%	-7.5%	-1.9%	-1.9%
7. Net U/W Revenue	24.0%	4.5%	0.8%	-15.0%	-7.5%	-1.9%	-1.9%
8. PV at rsv disc Net U/W Rev	23.6%	4.3%	0.7%	-13.5%	-6.6%	-1.6%	-1.5%
9. PV at rsv disc End year liab	-18.2%	-22.5%	-23.2%	-9.7%	-3.1%	-1.5%	0.0%
10. End yr liab, valued at rsv disc each year	-18.8%	-23.9%	-25.4%	-10.9%	-3.6%	-1.8%	0.0%
11. Fund b/f	0.0%	24.7%	23.9%	25.4%	10.9%	3.6%	1.8%
12. Interest on Net Rev and Fund	0.7%	1.6%	1.5%	1.1%	0.4%	0.2%	0.1%
13. End yr borrowing	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
14. Interest on borrowing	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
15. Realised Profit/Loss, valued at 31/12 of year	0.0%	6.9%	0.7%	0.5%	0.2%	0.1%	0.0%
16. Fund c/f	24.7%	23.9%	25.4%	10.9%	3.6%	1.8%	0.0%
17. PV Realised Profit/Loss	0.0%	5.2%	0.5%	0.3%	0.1%	0.0%	0.0%
18. Capital Base (CB)	60.0%	2.4%	2.5%	1.1%	0.4%	0.2%	0.0%
19. CB Injection/Release 1/1 each yr	-60.0%	57.6%	-0.1%	1.4%	0.7%	0.2%	0.2%
20. Interest on CB	5.4%	0.2%	0.2%	0.1%	0.0%	0.0%	0.0%
21. Tot Cashflow from CB	-54.6%	57.8%	0.1%	1.5%	0.8%	0.2%	0.2%
22. PV Tot Cashflow from CB	-47.5%	43.7%	0.1%	0.9%	0.4%	0.1%	0.1%
23. PV Net U/W Revenue	23.3%	4.1%	0.6%	-12.2%	-5.8%	-1.4%	-1.3%
24. PV End year liability	-15.9%	-20.0%	-20.6%	-8.4%	-2.6%	-1.3%	0.0%
25. End year liab, valued at 31/12 of each year	-16.8%	-22.5%	-24.6%	-10.6%	-3.5%	-1.8%	0.0%
26. Extra Tech Reserve (ETR) 31/12 each yr	1.9%	1.4%	0.8%	0.3%	0.1%	0.0%	0.0%
27. ETR Injection/Release 1/1 each yr	-1.9%	0.5%	0.6%	0.5%	0.2%	0.1%	0.0%
28. Total Cashflow from UW & CB	-54.6%	64.7%	0.8%	2.1%	1.0%	0.3%	0.2%
29. PV Total Cashflow	-47.5%	49.0%	0.5%	1.2%	0.5%	0.1%	0.1%
30. Total PV of Cashflow	3.9%						

However, the underwriting cashflow, resulting in a net underwriting revenue, cannot simply be revalued at the same rate of interest as profits, as premiums are not available for writing new business against until the corresponding claims have been paid. Instead, they are put in a technical reserve, which is typically allowed to grow only at a lower, less risky interest rate, often close to the long bond yield of the currency of the book considered.

In many cases, regulatory concerns or company policy decisions prevents the reinsurer from using even a risk-free rate for the discounting of the technical reserves, and a very conservative rate is used instead, or no discounting at all takes place. The discounting interest for the technical reserves in the example is 3%.

There are several interesting issues arising from the choice of interest rate on technical reserves, most of which are beyond the scope of this paper. However, it deserves to be mentioned that many companies with technical reserves predominantly invested in equities, would still choose to use a low risk/risk-free, notional, discounting interest rate. This is logical as long as the performance is measured against the risk-weighted capital allocated to the insurance operations, as the insurance result on capital should not gain from profits produced by taking investment risks, as long as these are backed up by a capital base of their own. A company that would choose a higher interest based on equity investments would thus also have to increase the capital base and/or the return requirement. In our example profit test, we have chosen a nominal interest rate on technical reserves of 6%. Investment expenses are assumed to have been taken into account into this choice.

The final aspect of interest rates we need to bring up is the case where the cashflow generated by the business does not generate enough funds for the technical reserves. This is often the case for undiscounted reserves for long-tailed business. The required funds can in principle be obtained by borrowing from shareholders capital, or from a third party. The appropriate borrowing yield to apply depends on

1. The risk involved that the borrowed money may have to be used for paying claims. If money is borrowed for a short-tail business being funded for three years for accounting purposes only, this risk is very low, while if the money is for long-tail business being discounted at a slightly conservative rate, the risk is obviously considerably higher.

2. Whether the shareholders capital is seen as an indivisible entity, and an risk-adjusted rate of return has been determined for this capital base, or if different parts of the capital base are seen as being exposed to different levels of risk.

In the example profit test, a borrowing yield of 10% is used.

The calculation of the table is straightforward, but in order to ensure a full understanding for the discussion of the underwriting cashflow items in the next section, we just briefly mention that

- PV End year liability, is the sum of all future PV Net U/W Revenues,
- The End year liability, valued at 31/12 of each year, is the preceding PV End year liability revalued to 31/12 of each year by the technical reserve interest rate,
- Total borrowing, which is the technical reserve minus the available fund, and
- The interest rate on Net Revenue and Fund, is calculated as the interest for a full year on the fund b/f and interest on 6 months on the Net U/W Revenue for each year.

4.4.3 <u>Underwriting Cash Flow Items</u>

The items in the table has, somewhat artificially, been grouped into underwriting items, lines 1 to 17, and capital base (CB) and extra technical reserve (ETR) items, lines 25 to 27. The present section will discuss the items normally directly connected to underwriting, while the details of the CB and ETR's are left for section 2.3.4. It is not unusual to see profit test tables in the London market only covering the underwriting cashflow, and this can to some extent be motivated by the typically quite small effects of the other cashflows. However, for a full understanding of the results of writing reinsurance business they should not be omitted. An additional reason for including them would be, again, that the margins in reinsurance over time are not large enough to ignore even small effects if their effects are negative, which is often the case for the CB and ETR cashflows.

The premiums and claims paid distributions over time must be studied carefully in advance, to determine an underlying development pattern (accumulated proportion paid of ultimate by development year) for each one of them, and a forecasted ultimate claims ratio. The payment patterns can be calculated from classical triangulation schemes, with proper care taken to reflect expected future changes in payment speeds. These triangles reflecting historic results also give ideas for future expected loss ratios, which must be complemented with underwriting information.

For the table 1 profit test, the underlying development patterns are shown in the following table:

Development year	1	2	3	4	5	6	7
Premium payment pattern	50.0%	75.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Claims payment pattern	20.0%	40.0%	65.0%	85.0%	95.0%	97.5%	100.0%
Ultimate loss ratio	75.0%						

Table 2. Payment patterns

Premiums and external acquisition costs are typically the most straightforward items. However, in reinsurance, there are complicating features that make even them stochastic. Firstly, the net premium may be determined by a claims-dependent formula, such as a sliding commission, profit commission, a sliding scale or a reinstatement premium and secondly, the reinsurance premium will always depend on the original gross net premium income, which is only estimated in advance by the cedant. These features will be reflected in triangulations for prior years only as long as the nature of the business is not expected to change much in the near future.

Retrocession costs have to be modelled carefully, if retrocession is not bought only to a negligible degree. As the profit table is concerned with expected cashflows, the net retrocession cost after taking expected recoveries into account, should be given a payment pattern. This pattern will greatly depend on whether the cover bought is proportional or nonproportional. Again, triangulation schemes may be valuable, although the losses to retrocession programmes are typically rare, so triangles would often have to be coupled with sound judgement. A common simplification is to assume that if very high-level covers are bought, the recoveries are so remote they can be ignored, and then the retrocession payment premium is just a proportion of the

incoming premium. This is the simplifying assumption in table 1, where retrocession costs have been set to 2% of the incoming premium.
It is also common to start the profit test table with the premium net after retrocession, which is particularly suitable if the underlying triangulations are done on a net basis, or if the retrocession is proportional.

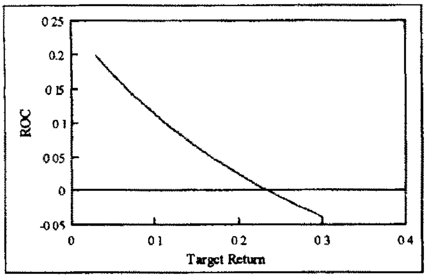
The expense ratio is determined by the allocation basis and the overall expense level of the company. A traditional approach would be to allocate expenses by premium income, maybe modified for proportional/nonproportional and/or fac/treaty, depending on the reinsurer's organisation. A more detailed allocation can certainly be of value for strategic purposes, for instance favouring growth areas.
Paid claims is the most obviously stochastic item in the table. The shown values are expected values, but both the timing and the amounts are stochastic. Variability in final loss ratio can often be calculated by modifying classical pricing formulae. Timing variability has been studied very little in the actuarial literature, although some guidance can be found in for instance in [Mack94]. If one is able to measure the variability, a stochastic model in a simulation package could be set up to give a better picture of the potential expected profits and timings. An example of this is shown in section 4.4.8.
Underwriting profits or losses in this example are booked after two years, with additional profits appearing with the unwinding of the discount. Other common rules are realisation after one year, but with more conservative technical reserves, or realisation spread out over several years. Both rules are easily implemented in the profit test table. The decreased risk of taking out the profits later is matched by the fact that their present value decreases with time, as the profits only earn a low return as long as it resides in the technical reserves, while they are eventually discounted at the return on investment requirement when released.

4.4.4 Capital Base Cashflow and Reserving Strain

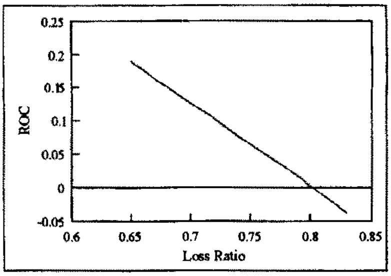
A capital base is allocated to your business when the business is written, and then released as the reserve is run off. The way in which the capital is allocated is determined is not the subject of this section, and often the capital is released after one calendar year, as a simplification. A tentative capital base, with a 60% capital to premium ratio for the first year, and then a 10% capital to liability ratio for the remaining years, is used to the profit test of table 1.

As the capital also produces investment income, this interest needs to be included in the model. The investment policy for the capital base may differ substantially from the policy for technical reserves, as it is often less conservative and contains a higher proportion shares, so care must be taken to reflect that properly. As already discussed in section 4.4.2, a common approach is to include only a nominal growth on the capital base, and consequently keep the risk-weighted target return free from investment risk, although this is by no means the only way to proceed.

One may think that adding the investment income from the capital base may automatically give a higher profit in the profit test, as more income but no new losses are added. However, the required return from underwriting and capital growth is naturally higher than that from underwriting alone. This is reflected in the risk-weighted target return at which the profits are revalued, which now is the company's full return on investment requirement and in the target profit, so it does not necessarily lead to easier targets. The present value profits at varying target returns as shown in graph 1 for the example profit test at a constant ratio.



Graph 1 PV Return on Capital as a function of the target ratio



Graph 2 PV ROC sensitivity to ultimate loss ratio

Thus, we need three lines in the profit test to conveniently describe the effect of the capital base, one for the capital base itself, one for the necessary injection or release of capital to shareholders, and finally one for the nominal interest on the capital base.
To be able to run off our capital base against the technical reserve and to compute interest rates on reserves, we need to know exactly how the reserves are determined. This information could be in the form of an IBNR profile, a reserve discounting policy and a general reserving policy (degree of conservatism, "50/50" reserve etc.). An IBNR profile giving more prudent reserves than the present value of future liabilities, a policy to discount technical reserves at a low interest rate, a general policy to carry 5% more reserves than the expected loss or reserves enough to meet liabilities with a high probability, are all different ways of specifying the extra reserve, which is really just a way to increase the capital base allocated to the line.
In the example profit test, we have as already mentioned used a conservative interest rate for technical reserves as this is the most common way of specifying an ETR, but then we have also calculated the amount by which the reserve is prudent, as this information shows the increase in the actual capital base.

If the introduction of an ETR is for accounting or regulatory purposes, the pattern of emerging profits is more often than not affected. The "unwinding of the discount" is probably the most common example of this. However, if the ETR is a provision against run-off deviations, one could just as well argue that the profits could be realised just as without this extra reserve.

Taking the present value of the injections and releases of capital base amounts gives us an amount to be added to the present value of the underwriting cashflow , giving us the NPV of the business segment considered in the profit test. This value can then, for instance, be divided by the capital base of the first year, giving a return on capital measure. If this value meets the target one, then the studied segment of business is considered sufficiently profitable to take on the books, or to continue to write.

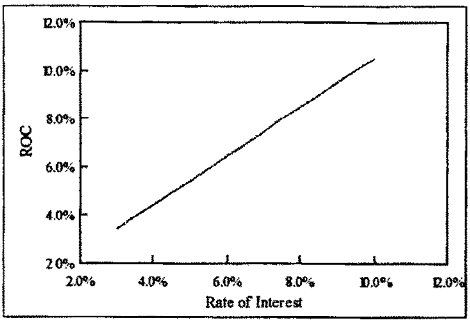
Naturally, the target return may have been set to reflect a long term commitment to a territory or class (it may even be negative!), and therefore meeting the target does not necessarily mean profitability.

In another approach we would set the investment return, return on equity parameters and also the target profit according to our assumptions and requirements and subsequently solve for the highest L/R allowing us to meet the target. This approach has its value in that it provides the underwriters with a parameter that is meaningful to their work, and can be discussed with cedants.

There are several other ways to use the results of the profit test. A common method from the life insurance industry would be to actually solve the entire table for the rate of return (the internal rate of return, IRR) giving a nil present value of the total cashflow and compare this to the target rate of return. As Graph1 shows the ROC as a function of the target rate, the IRR can be looked up immediately from the graph. In general business, the total cashflow can in fact change sign more than once, giving rise to the well-known problem of multiple IRR's. We will see two more profit test applications in sections 4.4.6-8.

The sensitivity of profits to small changes in the ultimate loss ratio is typically quite high, and this is depicted in Graph 2. A capital to premium ratio of 60% is assumed for this class. It is worth pointing out though, that the effect often is roughly linear, as long as profits are realised within a few years.

A final example of sensitivity analysis is shown in Graph 3. where ROC is shown for varying interest rates on technical reserves..



Graph 3. ROC as a function of the rate of interest

4.4.5 The Application of Profit Tests at Different Levels of Aggregation

In the preceding sections, we have looked at a generic profit test, without reference to any difference in application depending on the level at which it is applied, or to the relative importance of different problems. In this section, we aim to give some guidance on these issues.

There are three obvious levels of application,

1. the whole account level,
2. the business segment level, and
3. the individual treaty level.

The most common application would be the second one, where many of the parameters are already set, for example capital allocation, retrocession and management expenses, and the typical question is whether a certain book of business is profitable or not, or at what target loss ratio it could be written to be profitable. The business segment needs to be a reasonably homogeneous for the profit test to be valid, as a mix of long and short tailed business could distort the effects of investment income.

In order to answer some of the strategic issues arising a whole account level, for example about optimal premium volumes and suitable expense and capital allocations, the profit test needs to be applied to the whole account. This does not mean just one profit test for all business, especially in view of what has just been said about homogeneity, but rather a number of simultaneous profit tests, run with different scenarios for the studied strategic parameters. In life insurance terminology, this would be a "model office". This is an integral part of the planning process. The practical problems of actually carrying out an optimisation in this environment are enormous, so one would typically have to confine oneself to look at a small number of scenarios.

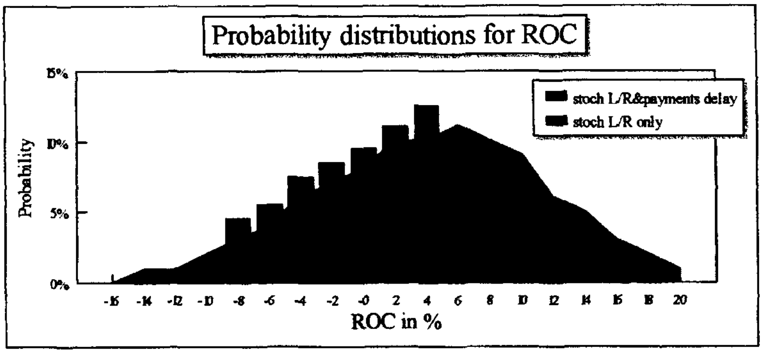
Another issue to consider when using profit tests for strategic purposes, and in particular for multiyear planning, is the issue of tax. If the target ROC is set net of tax, one has to bear that in mind in the profit calculation. In particular, if the target ROC is considered a long-term average target, and if results are volatile, tax will only be paid on profitable years, and hence simulations or scenarios are necessary, rather than just a straight percentage deduction.

Finally, when most parameters have been set in the cascade structure of the whole account and business segment profit tests, we need to perform profit tests for individual treaties, or groups of treaties. Overall targets without proper reference to individual circumstances, or at least territory and class-specific calculations, are seldom accurate enough. Obviously, it is difficult to determine payment patterns for single treaties unless very large, but as far as possible one must try to use similar treaties and to reflect present, not past, market conditions and practices.

4.4.6 Application 1: Distribution of Portfolio Results through Simulations

In this section, we look at the variability in results for a nonproportional, long-tailed line of business, and the effects of uncertainties in some of the underwriting parameters.

The portfolio studied is a book of liability business, where the loss ratio for a specific underwriting year has been estimated to be normally distributed, for simplicity, with an average of 95% and a standard deviation of 10%. The premiums are paid up-front, and the claims are paid with a linear payment pattern over 10 years. Brokerage is 10%, internal expenses 10%, capital base 50%, and the rest of the parameters are as in our example profit test from section 4.4.2. As the result sensitivy to variations in the loss ratio is roughly linear (cf. graph 2), the return on capital (ROC) distribution is approximately normal as well, as shown in graph 5. The expected value and standard deviation are 3.1% and 6.9%. The expected value can be compared to the deterministic profit of 3.5%. 1000 simulations in @RISK were run.



Graph 5. Distributions of results with deterministic and stochastic payment delays.

The next step in our study is to look at the sensitivity to payment delays. What if we have got the average payment delay (5 years) wrong? First of all, let us just look at the deterministic result after changing the pattern to a linear one with a 5.5 year average. The ROC now becomes 6.5%, which is quite a large difference, considering that the future average payment delay is often hard to determine with good precision. If we, for the sake of argument, consider the payment delay to be an uncertain, random variable, with say a uniform distribution between 4.5 and 5.5 years, and run the same simulations as above, the standard deviation now becomes 7.9%, while the expected value is 7.0%. The explanation for this is of course that we add uncertainty in the payment pattern, but the since the effect is roughly linear for a small change in the payment delay, the expected value does not change.

Finally, we are going to try to take into account that this particular book's ultimate loss ratio is sensitive to the average payment delay through social inflation. If we make the simplified assumption that the expected loss ratio goes from 90% to 100% as the average payment delay goes from 4.5 to 5.5 years, due to social inflation on this nonproportional account, we can very easily build this into our simulation model. The standard deviation for the loss ratio is now reduced to 4.0%. The ROC distribution for this case is shown in graph 5. The distribution is slightly more compressed than the one where the average payment delay is deterministic.

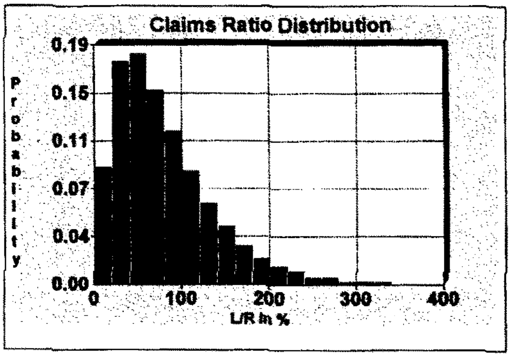
This example illustrates the sorts of analyses you can perform with a stochastic profit test, without going into the decision process, which is not the purpose of this paper.

4.4.7 Application 2: Distribution of Treaty Results through Simulations

Various simple simulations can give additional information about the profitability of a specific treaty. This section will give a few examples of this to illustrate the potential use.

First of all we consider a profit test for a proportional treaty in continental Europe, where the ultimate loss ratio is uncertain, and therefore taken to be stochastic, and gamma distributed with a mean of 80% and mode of 40%. The tail is quite short, say for simplicity linear over 3 years. The remaining necessary parameters have been taken from our exampe profit test of section 4.4.2. This would be typical for say a fac obligue fire treaty, a treaty type with typically a small number of large losses. We would now like to know the implied probability distribution of the return on capital (ROC), as well as the expected value and variation. In particular, we would like to *know if the expected value of the return on capital is approximately equal to the ROC* we get from assuming a deterministic ultimate loss ratio of 80%.

The probability distribution of the ultimate loss ratio is shown as a histogram in Graph 6. The capital to premium ratio used is 50%. Due to the skewness of the distribution, the expected value of the ROC (-1.5%) is noticeably less than the deterministic value 2.8% in spite of Graph 2, and the standard deviation of the loss ratio of 56% transforms into a standard deviation of the ROC of 106%.

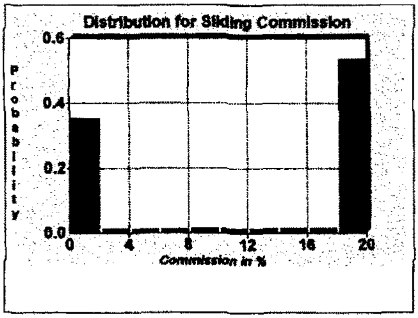


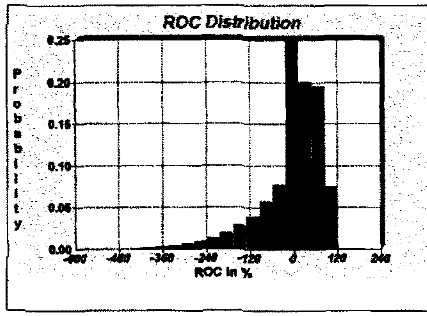
Graph 6. Gamma distributed claims ratios.

A *more interesting question could be the following: What if the commission is a sliding* commission going from 20% to 0% for loss ratios from 60% to 80%? Is the expected commission still 10%? What is the expected ROC?

The distribution of commissions and ROC is shown in a pair of histograms below, Graphs 7&8, the expected commission is 12.0%, which is a not insignificant deviation from the deterministic 10%, and consequently the ROC has lost value to -5.5%. The standard deviation is reduced significantly, which is the very purpose of the sliding scale commission, to 86%.

The questions posed above were just intended for illustration purposes, and were by no means chosen for radical results. There is a multitude of questions to be asked for each treaty of segment of business in questions, and in many cases the answers can be quite surprising, both in terms of giving different expected values than expected, but also by the large variability in the results.





Graphs7&8

Distributions for sliding scale commission and ROC, given a gamma distributed loss ratio.

4.4.8 Application 3: Optimisation of Portfolio Mix

An important application of profit tests is the area of strategic planning for business units writing a more than one line of business. This would be part of the planning process and we are here looking at a one-year planning horison only. The actual profit test is an integral part of the optimisation procedure, and will become important if at least one of the lines of business is a long-tailed one. We will look at a simplified, but non-trivial example of determining the optimal business mix for a book of two lines of business, catastrophe excess of loss, which is very short-tailed, and nonproportional motor third party, which is very long-tailed. To fix ideas, let us assume that the business is written in the UK. For each line, the profits, capital needed and costs will be described, and subsequently the optimisation procedure and results are presented.

The company in question deem it possible to write between 10 and 20 million pounds cat x/l for the year in question, the management expense ratio depends on the income written. There are both fixed and variable costs, and the total expense ratio can be described as

$$m_c = 5\% + 30\%/p_c$$

where p_c is the cat x/l income in millions of pounds. As almost no diversification is achieved by writing more income for this class on its own, all ceding companies are likely to be hit rougly to the same degree by a windstorm or flood, the capital base is decided to be 120%, irrespective of income level. The underwriter also estimates that the expected L/R at which he would write the book at would increase from 50% to 60% linearly as the income rose.Acquisition costs are 10%.

For the motor book, the feasible income range is estimated to between 10 and 30 million pounds and the expense ratio

$$m_m = 3\% + 90\%/p_m$$

where p_m is the motor income in million pounds.

As the treaties are relatively independent, and as most potential treaty shares would be roughly of the same size, it is estimated that a capital base (expressed as a percentage of premium)

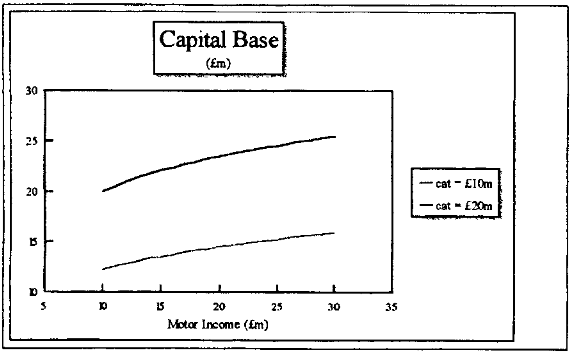
$$c_m = \frac{2}{\sqrt{p_m}}$$

is adequate for the class written on its own. Thus, at minimum income level, the capital base would be 63%, and at maximum level, it would be 37%. For this portfolio, the L/R estimate from the underwriter is that the L/R will increase from 100% to 120% with income, and acquisition costs are 10%.

The capital base advantage of writing the two classes together is felt to be appropriately reflected in a reduction of the overall capital base percentage by the term ("covariance term")

$$r = 0.4c_c c_m$$

This formula could have been arrived at by way of stochastic modelling of the portfolio. The resulting overall capital need is shown in a graph for two different levels of catastrophe business and a range of levels of motor business.



Tail patterns are for simplicity taken to be linear with a 1 year and a 6 year average. No capital is needed for the run-off, and no retrocession is bought. The remaining parameters are as in our example profit test of section 4.4.2, for simplicity.

The final constraint is that the available capital is 20 million pounds sterling, as the shareholders cannot be persuaded to supply more capital than this.

Finding an optimal, or even a good business mix for this simplified book of business is not easily done without some calculations, as even for a two-line exampe, the problem is very complex. The problem is in theory a nonlinear optimisation problem, the solutions of which are given by the Kuhn-Tucker conditions ([Luen84] or an operations research textbook). However, this is not a very practical approach in this case, and an approximate solution can be found by calculating the individual and combined ROC (return on capital) at various feasible income level combinations, taking into account only the combinations satisfying the capital constraint. This is simply done by executing the profit test for each one of the lines. The results are shown as a table of feasible profits for combinations of premiums.

Cat\Motor premium (£m)	10	14	18	22	26	30
10	2.9	3.3	3.4	3.2	2.7	2
12	3.2	3.6	3.7	3.6	3.1	2.4
14	3.4	3.9	4	3.9	3.4	
16	3.6	4.1	4.3			
18	3.7					
20						

Table 2. Expected profits (£m) for various income distributions.

From the table it can be seen, for instance, that the maximum profit amount is £4.3m, and that this is achieved for a mix of about £16m cat x/l and £18m motor business. The empty lower part of the triangle reflects that the capital constraint is violated for these income combinations. Whether the shareholders would prefer the maximum profit amount or the maximum ROC would probably be determined by the alternative investments available to the shareholders.

It should be obvious by now, that

- optimisation of business mix even in simplified cases becomes very cumbersome, that
- many of the constraints and relations are difficult to model, and that
- a profit test is an integral part of the strategic planning, although the investment effects are less crucial than for life business.

It is thus very seldom that one will be able to carry out such a straightforward study as in this example for business mix optimisation, and even more so when one has more lines of business than two to consider, and one will normally have to confine oneself to study a handful of scenarios in more detail. In addition to studying the expected profits, one should also give attention to the sensitivity to assumptions, especially rate of interest ones, and to the variability in the results, which can often be modelled in ways similar to the example in section 4.4.6.

4.4.9 References

[Luen84] David G Luenberger, "Linear and Nonlinear Programming", Addison-Wesley, 1984

[Mack94] Thomas Mack, "Measuring the Variability of Chain Ladder Reserve Estimates", CAS Forum Spring 1994.

4.5 UNDERWRITING MANAGEMENT

This section describes the problems encountered in managing the underwriting cycle and possible approaches to overcoming the problems. The section covers topics such as determining the underwriting targets, balancing the portfolio of business and individual risk selection.

4.5.1. Managing the Underwriting Cycle

It could be argued that how can one manage the underwriting cycle if it can not be measured or predicted. Furthermore, if the cycles were predictable, then people would act accordingly, and make the cycles unpredictable.

What we get is extremes of cycles. The cycles are caused by actual underwriting experience and perception of how that experience is expected to develop over the next few years. As a result of adverse claims experience the capacity for writing new business diminishes putting upward pressure on rates. Then the ensuing hard market leads to a period of profitability attracting new capital and capacity to the market which in turn put downward pressure on the rates. The extent of rigidities in the ability to increase rates or attract new capital affects the degree to which the cycles are extended and accentuated.

As for predicting the cycles, the reinsurance cycles may precede the insurance cycles. The extent of the availability and cost of reinsurance may affect the rates insurers can charge the primary insureds. Conversely, the demand for insurance may be affected by the economic climate. Thus, the supply and demand for reinsurance is very difficult to forecast.

Therefore, managing the underwriting cycle may be less concerned with forecasting the cycles accurately, and more concerned with the ability of the company to respond flexibly, speedily and profitably to an eventuality falling within a reasonably acceptable range of scenarios.

In order to achieve this, the company may consider the likely economic climate and market trends over the foreseeable future and take a view about a range of expected market conditions. It is possible that different conditions may be expected for different segments of the market (e.g. by class of business or territories). The company would then consider a range of business strategies appropriate to the range of expected scenarios.

A detailed audit of the company's resources and capabilities will enable the company to identify any needs in terms of capital or resources to adopt a given strategy and to be in a position to respond to changes in market conditions.

Thus, managing the underwriting cycle is about being in a state of preparedness to respond to a given change in market conditions within an expected range of scenarios. The extent of analysis carried out internally and externally, and the flexibility of the organisation structure to respond to a given change would differentiate between the companies which are able to survive and prosper in the current competitive climate of global reinsurance and the companies which are not.

4.5.2 Underwriting Targets

The underwriting targets may be discussed and agreed when the corporate plan is prepared. The targets may be broad in terms of premium income, target loss ratios, and target operational ratios for the business units, or they may be more detailed in terms of units of risks within a class of business in a given territory.

The targets may be constrained by limits on aggregate exposures which may be determined in the light of the company's outwards reinsurance program. There may also be limits on risk size for an individual risk, groups of risks, or exposure to a given reinsured.

The underwriter will want to ensure that he makes full use of the available capacity to achieve not only his immediate targets but also to develop long term relationships with the reinsured for mutual benefit. He will want to ensure that adequate returns are made in relation to the risks accepted without penalising the reinsured for an exceptional bad experience.

The underwriter will have an understanding of the portfolio of business he has written in terms of the risk profile of the business. He will perform an analysis of the portfolio by the number of risks and program of risks with a given reinsured and how that program has behaved over the past and is continuing to behave. Such portfolio analysis may be carried out periodically at a detailed level to identify the significant risk exposures. The underwriter will also consider the cost and benefit of such analysis in relation to future profitability.

The detailed portfolio analysis at periodic points will enable the underwriter to consider the current shape of the portfolio and the desired shape of the portfolio in order to meet his underwriting targets. In this context, he may consider the current security rating of the reinsured in order to take a view as to continued existence of the reinsured in the foreseeable future. The underwriter will be effectively building a portfolio which is expected to be durable and profitable over the longer period.

4.5.3 Balancing a Portfolio

It is a fundamental concept of insurance business to spread the insurance risk over a large portfolio of insureds to obtain some stability and predictability to the emerging losses. It is therefore important to ensure that any insurance operation has a large and diversified portfolio of risks both within class and over the whole company. Against this is the constraint that the number of such risks need to be correctly underwritten and managed.

The underwriting targets at the corporate plan level will attempt to balance the portfolio at the highest level to achieve a desired risk/reward trade off. At a business class level, a portfolio analysis will highlight the current risk profile and the desired risk profile in the future. In this context, the underwriter will be concerned with benefit of diversification compared to the potential cost of diversification from additional resources required to service that portfolio. Conversely, a fewer number of risks well researched and costed may well generate less volatility.

The risk profile and the balance of a portfolio could be investigated by formulation of a model office for the classes of business which are currently being written. The model office will include specimen policies representative of the portfolio and can attempt to simulate losses for those policies on a range of assumptions. The model can be expanded to incorporate various assumptions associated with the insurance operation The effect of changing the risk profile on the stability can be investigated. A balanced portfolio is one where the aggregate expected losses are stable over time and do not demonstrate large volatility.

The concept of the model office can in fact be significantly expanded to model the company as a whole. In this case all the constituents such as premiums , assets, expenses, investment returns can be incorporated within the model office. The model office would then consider the volatility of the emerging profits on current conditions and perform various scenario testing to determine the effect on profitability and the solvency position on different sets of assumptions.

4.5.4 Risk Selection

In selecting an individual risk the underwriter may use techniques such as those described in this paper but ultimately he will exercise underwriting judgement on whether the risk contributes towards his overall underwriting targets.

The underwriter will be constrained by what risks are presented to him. This will depend on his standing in the market and the perception of his company's image and security rating. If the underwriter is considered a lead underwriter, then his past performance may well be under regular scrutiny. In order for a program to be fully subscribed by the market, the lead underwriter's rating has to be respected and acceptable to the following underwriters.

The underwriter will select an individual risk not only because it is technically priced adequately, but also whether it falls within his general underwriting strategy and underwriting authority. Typical considerations are:-

1 The limits of liability.
2. Any potential impact on the outwards reinsurance treaty.
3. *The Annual Premium Income Limits set by his Board.*
4. The minimum premium limits per risk
5. The minimum rate on line.

The *underwriter will also be looking to develop and maintain a long term relationship* with the reinsured. In any business, it is more expensive to generate new business than to renew existing business. The underwriter will therefore wish to select those reinsureds who are likely to be profitable, show good risk management skills and are expected to survive the peaks and troughs of underwriting cycles.

The underwriter will consider the group of policies with a given reinsured to ensure that the overall returns from these policies are adequate. He will also want to ensure that he makes good use of his capacity to write catastrophe business by leveraging this against other classes of business relating to the reinsured. There are other considerations.

1. Exclusivity of risk

The underwriter may exercise commercial judgement when considering certain risks. For example, he may take a higher percentage line with the intention of keep the risk exclusive. A 100% line may be written with the intention of preventing competitors having access to the risk. Writing at a 75% level will open the door to other insurers and could readily result in the signing down of the risk, as well as a potential loss of information.

2. New line of business.

When presented with a new or unusual line of business, the underwriter may initially accept a small line in order to gather the underlying experience before fully committing himself..

3. High Profile Risks

The underwriter may also at times take on a high profile risk for reasons of prestige.

4. Cross subsidy within a risk

An underwriter may accept a lower rate for a particular layer to access a more attractively priced layer in the same program. In this case it is the overall combines profit that is important.

Finally, underwriting should be considered as much as an art as science. It is important to ensure that all the technical aspects are fully explored and correctly costed. However, ultimately the skills of underwriting may be in understanding in depth qualitative information relating to the nature of an individual risk and the quality of reinsured, as well as in negotiating the best price for the risk.

5. The Role of Actuaries in Pricing

5.1 This paper has broached the subject of Actuaries becoming more involved with the underwriting process. This naturally prompts the question "Would involving actuaries in the underwriting process be of benefit to the business?" There are clearly mixed reactions to this type of proposal as some may view this as be taking over the underwriting role. Qualified actuaries have even progressed to underwriting lines of business both in U.S. and in Europe.

5.2 When we started to write this paper, the actuarial role in most London Market Companies was essentially one of reserving, and only occasionally were they asked for their opinion on rating. These risks tended to be those with a large volume of historic loss data which requires techniques similar to reserving to project the profitability of these past years. During the ensuing months there was a clear trend to actuaries being employed in a pricing role, particularly in those companies with European or American parents.

5.3 The skills of the actuary and underwriter are complementary. The actuary has considerable technical skills, and in this respect he is better equipped than the underwriter. This paper has dealt with those skills by illustrating the calculation of the technical rate for the risk. Having computed this rate, the underwriter imputes his skills which takes into account market conditions, handling negotiations with brokers, and so on. The skills required in the underwriting process are:-

1. Risk selection and assessment
2. Product design (coverage, terms and conditions)
3. Treaty wording and legal skills
4. Contract negotiation with cedant or broker
5. Questions of balance of portfolio
6. Longer term strategy.

We have shown that actuarial skills could assist in the management of this process, particularly in the first and last two area.

5.4 The crucial point is that underwriters must first have confidence in the actuary. For this condition to hold actuaries certainly need to understand the underlying business and have a working knowledge of the various contracts. This extends to the standard policy wording and the slip details and market conditions

5.5 Factors affecting the future exposure change. The risk management features of the account needs to be understood for the actuary to translate from the historic loss experience (and any collateral data employed in the rating) to the new exposure period being assessed. Experience in this process could be gained by the actuary spending a portion of his training period in the underwriting room assisting in the day to day underwriting. The average period to qualify for a good student actuary is between 4 to 7 years, so a period of 1 to 2 years in that function is not unrealistic.

5 6 One by product of this training strategy is that the student would be in a position to add considerable value to the continuing reserving process of the account. The role of the Chief Executive of a London Market operation is also as Chief Underwriter, and if actuaries harbour ambitions to achieve such a status then it is essential that they have a sound grasp of the day to day operations in the underwriting room and the difficulties that underwriters have in pursuing their objective of writing profitable business. Can actuaries achieve this goal while pursuing a purely reserving role.?

5.7 As actuaries become more involved in this role there is clear potential for a conflict between a pricing and reserving role, not least in the remuneration is contingent on the performance of a specific account. There is a clear need for professional guidance with the possibility of divorcing the actuarial pricing and reserving roles. Such a divorce between sales and control is extremely common in other financial service industries.

5.8 In conclusion, this paper illustrates the skills that have evolved and continue to be developed as actuaries become more involved with the pricing role. These new skills are becoming actively used in many companies, and will give companies who use actuaries in pricing a distinct competitive advantage.

Appendix 1 More Advanced Techniques

The purpose of this appendix is to explore new techniques that may be applied to London Market business or new approaches that are being developed. Some of these techniques have been applied in the past, but are complex mathematically and esoteric in places

These can be classified into three types of development.

1. Direct Modelling.

In section 3 we had mention of Generalised Linear Models. These type of models are being slowly introduced into London Market rating. On top of these lie Fuzzy Sets and Neural Networks. Fuzzy Sets apply when there is uncertainty about the choice of the rating parameter. Neural Networks apply when there is uncertainty about the model. Neural Networks are described in more depth in Appendix 1.2

2. Modelling of Extreme Events.

The buzz words of the moment are Chaos Theory and Self Similarity. Some of the Distributions generated by this theory give a better fit to both frequency and size of some of the catastrophic events we have recently had than the more conventional theories and distributions.

3. The use of Financial Mathematics to rate policies.

An Excess of Loss Policy has the same pay off structure as an option so can't we price them in a similar way? The Return on Equity approach to pricing which is found in US rating has its foundations in Financial Mathematics.

Appendix 1.2 Direct Modelling - Neural Networks.

Trying to model the underwriting process using a mathematical black box is not new. In the early 1980's the Government, ABI and a number of Insurance Companies tried to create an expert system that would emulate the underwriting of UK Commercial Business. The derived model was not used in practice. Ayling [18] undertook an analysis of a London Market underwriter in the Aviation Excess of Loss Market. He attempted, by standard statistical methods, to identify and measure the components used to rate such business. The results of the analysis indicated the task was extremely difficult .

1. Claims in the underwriting year could not be reasonably be assessed from slip details at the beginning of the year.

2 Price variations over a period were being reasonably estimated

3. There was no strong relationship between premiums charged and the ensuing claim costs.

4. The premiums charged by the underwriters could not be reasonably estimated from details on the slip.

This work was done in 1984 on data then 10 years old, but is still a slightly disconcerting. The years being reviewed were the profitable years!

One option to this problem is to use neural networks to try and understand what is happening. There are currently a number of software houses selling neural network systems. They tend to be black box in nature, but have the potential advantage in that they can capture the essentials of what is happening in certain systems, particularly those involving human behaviour, over the more traditional models. They act by learning what the underwriter has done before to come up with an answer consistent with recent proposals. To be a good system, it is important to have rejected contracts in the data base, and this is often a barrier to practical application.

There are three types of systems that might be considered

System Type 1.

Under this type contract information is input, and the system decides whether the contract is likely to be underwritten or not. Rejected policies are not processed fu

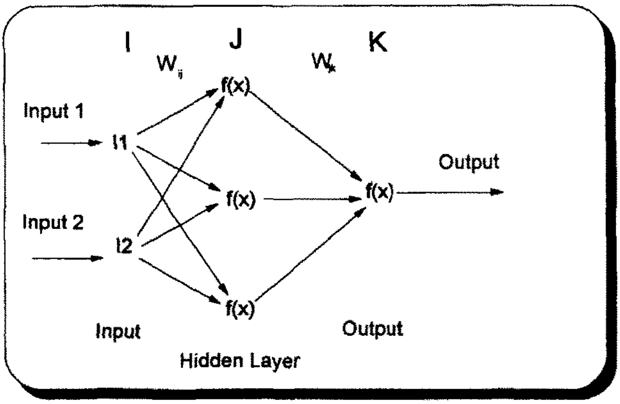
System Type 2.

Based on recent policies that have been underwritten, what is the likely premium r for the contract under review.

System Type 3.

How much of the risk do I want to take.

Clearly non-neural network systems can be devised to solve these problems.



Input to Hidden Layer	f(x) a transfer function
	h(j) = Sum (W(i,j)xI(i))
	S(j) = f(h(j))
Hidden to Output	h(k) = Sum (W(j,k)xS(j))
	O(k) = f(h(k))

I(i) are the network inputs

O(k) are the network outputs

W(i,j) represent the weight connecting neuron i in layer 1 to neuron j in layer 2

W(j,k) represent the weight connecting neuron j in layer 2 to neuron k in layer 3

f(x) is the neuron transfer function, for example 1/(1+exp(-x)).

This function is not dissimilar to the inverse link function in Generalised Linear Models. However GLIM models are usually fitted by maximum likelihood methods, neural networks are trained by least squares. Neural networks can model non linearity.

Appendix 1.2 Chaos Theory and New Distributions

Alpha Stable Non Gaussian Distributions.

In 19th Century mathematicians discovered that the best fit of a curve to a series of points was achieved by the method of least squares. The distribution of errors emphasised the importance of the Normal or Gaussian Distribution. At the same time Fourier invented his famous series and transforms and this was applied to probability distributions to generate characteristic functions. Laplace noted that the the Gaussian density function was its own Fourier transform.

In 1850 Cauchy extended this to generate further distributions.

$$f_N(x) = \frac{1}{\pi}\int_0^\infty \exp(-ct^N)\cos(tx)dt$$

This essentially replaced the Fourier integral using t^2 with t^N

When N=2 we have the Gaussian case

When N=1 we have the Cauchy Distribution

$$f_1(x) = \frac{c}{\pi(c^2+x^2)}$$

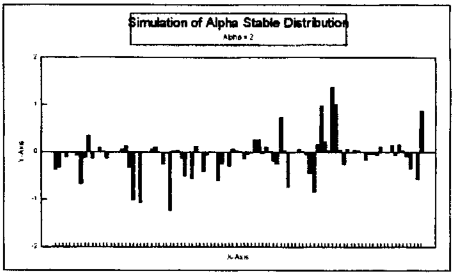
In 1919 Bersham showed that f_N was probability distribution if $0 < N \leq 2$.
Traditionally N is replaced by α

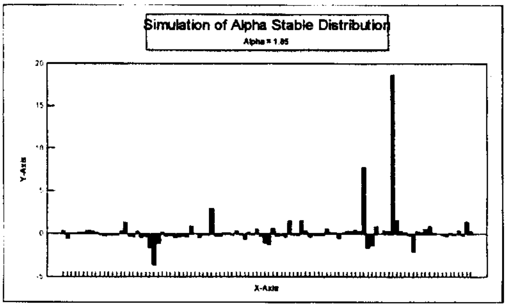
These distributions have special features.

1. The central limit theorem does not apply for $\alpha < 2$
2. They are stable in the sense that if X_1, X_2 are random variables with the same distribution then $CX = C_1X_1 + C_2X_2$ has a similar distribution with the same index α for any C, C_1, C_2 when $C^\alpha = C_1^\alpha + C_2^\alpha$.

These distributions have many interesting features, with high value radical jumps.

Having said this the mathematics of emulating the process is difficult,


Simulation of Alpha Stable Distribution
Alpha = 2
Y-Axis
X-Axis


Simulation of Alpha Stable Distribution
Alpha = 1.85
Y-Axis
X-Axis

Appendix 1.3 Modelling of Extreme Events

For certain classes of reinsurance the modelling of extreme events is of paramount importance. In modelling claims distributions it is often found that whereas the distribution gives an excellent fit to the bulk of the data, the extrapolated tail distribution may appear to differ significantly from what few data points that may exist. In modelling extreme events then there are often few or even no data points in the range where the quotation is required. This problem may be solved by the use of fitting extreme curves.

The Fisher Tippet Theorem comes, in part, to the rescue. This states that data will trend to one of three distributions, subject to an affine (linear with positive slope) transformation.

Let $\quad M_n = \max(X_1, \ldots\ldots, X_n)\ as\ n \to \infty$

This gives rise to a class of distributions known as MAX-STABLE providing they satisfy $\max(X_1, \ldots\ldots, X_n) \stackrel{d}{=} c_n X + d_n\ , c_n > 0$

The Fisher-Tippet Theorem then states

Let $M_n = \max(X_1, \ldots\ldots, X_n)$ where $\{X_k\}$ is a sequence of independent random variables.

If, for some sequence of normalising constants, $c_n > 0, d_n$ there exists a non-degenerate random variable Y such that

$$c_n^{-1}(M_n - d_n) \to Y$$

then the distribution of Y is one of the three following types.

Frechet	$\Phi_\alpha(x) = \exp(-x^{-\alpha})$	$if\, x > 0$
	$= 0$	$if\, x \le 0$
Weibull	$\Psi_\alpha(x) = \exp(-(-x)^\alpha)$	$if\, x \le 0$
	$= 1$	if x >0
Gumbel	$\Lambda_\alpha(x) = \exp(-e^{-x})$	

Hence	$M_n \stackrel{d}{=} n^{\frac{1}{\alpha}} Y$	in the Frechet Case
	$M_n \stackrel{d}{=} n^{-\frac{1}{\alpha}} Y$	in the Weibull Case
	$M_n \stackrel{d}{=} Y + \ln n$	in the Gumbel Case

Example

Let $\{X_k\}$ be Cauchy, i.e. $f(x) = (\pi(1 + x^2))^{-1}$

As $\lim \frac{1 - F(x)}{(\pi x)^{-1}} \to 1$ (by l'Hopital)

Hence $F(x) \sim (\pi x)^{-1}$

$$\Pr(M_n \leq \tfrac{nx}{\pi}) = (1 - \bar{F}(\tfrac{nx}{\pi}))^n \sim (1 - \tfrac{1}{nx})^n \to \exp(-\tfrac{1}{x})$$

i.e. Frechet with $\alpha = 1$

The types of distributions are as follows

FRECHET	Cauchy, Pareto, Loggamma, Burr, Benktander Type 1
WEIBULL	Uniform, Polynomial Growth, Beta
GUMBEL	Generalised Weibull, Exponential, Gamma, Normal; Lognormal

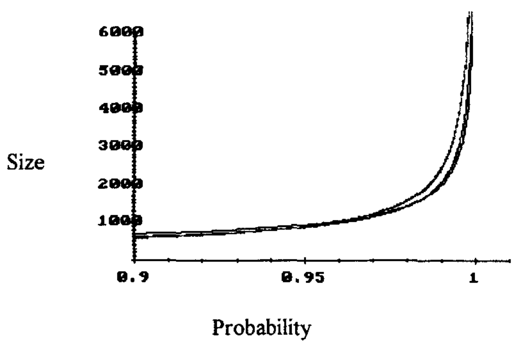
The estimation of the parameters is undertaken using a variety of estimators, of which the most useful is possibly the Hills estimators. The software package XTREMES, which is associated with the book "The Law of Small Numbers: Extremes and Rare Events" by Falk et al and published by Birkhausen concentrates on this approach. In addition there are approaches known as Quick estimates and Mean Estimates.

Attached is an example and data set of Hurricane Data from Hogg & Klugman

This uses the Xtremes software and assumes different numbers of extreme points..

In addition a book entitled "Modelling of Extreme Events in Insurance" by Embrechts et al is soon to be published by Springer. There is a bibliography of over 340 references.

Fitting of Hurricane Projections Using Hill Estimators



HURRICANE DATA

Total damage done by hurricane (in $1,000,000)

Source: Hogg R.V. and Klugman, S.A. (1984). Loss Distributions. Wiley, New York.

History: Total damages above $5,000,000 caused by hurricanes from 1949 to 1980 compiled by the American Insurance Association. A trend factor is included which is based on the Residential Construction Index.

Hogg R.V. and Klugman, S.A. (1984): Suggest, according to the empirical mean residual life (excess) function, that the Weibull and lognormal distributions may provide good models for these observations.

Data Set

5, 6.766, 7.123, 10.562, 14.474, 15.351, 16.983, 18.383, 19.030, 25.304, 29.112, 30.146, 33.727, 40.596, 41.409, 47.905, 49.397, 52.600, 59.917, 63.123, 77.809, 102.942, 103.217, 123.680, 140.136, 192.013, 198.446, 227.338, 329.511, 361.200, 421.680, 513.586, 545.778, 750.389, 863.881, 1638.000.

Appendix 1.4 Relativistic Methods- Options and Curves

One of the questions that can be raised is whether a contract has been priced well with respect to other contracts in the same programme or with peer contracts in other programmes. These have been viewed by two approaches, namely Option pricing Theory and Curve Fitting techniques.

Option Pricing

In [20] Sanders noted the similarity between the pay off pattern of an option and a stoploss policy. The question was explored whether the theory of option prices could assist in either the absolute or relative pricing of aggregate excess of loss contracts and catastrophe excess of loss. In other words have the layers been priced consistently

The Black-Scholes formula, calculated in Option Price theory, makes assumptions of lognormality in the incremental price jumps.

The formula is $C = SN(d_1) - EN(d_2)\exp(-R_f T)$

Where $d_1 = (\ln(S/E) + (R_f + \frac{1}{2}\sigma^2)T)/\sigma\sqrt{T}$

$d_2 = d_1 - \sigma\sqrt{T}$

N(*) = cumulative normal distribution.

C = cost of a call

S = current price

T = time

E = exercise price

σ^2 = volatility

R_f = risk free rate of interest

This calculation is derived from Stochastic calculus, although an alternative approach in the paper is to use a binomial model.

The equation is normally used to calculate the option price, and for this we need to calculate the volatility. The trick is to reverse the issues; for a given option price there is an implied volatility.

In the paper Sanders used this approach to calculate the reinsurance cost based on the assumed volatility of a particular portfolio, given specific quotes, and to simulate costs when the claims distribution when not log normal. There showed a remarkable consistency between the results of the simulation and the pricing using the Black Scholes equation on a stop loss model.

Finally the steps were taken to determine the impact on a single event excess of loss model. In the case of aggregate excess of loss models, only 1 calculation is needed, the volatility established, and all other rates flow from the equation. In catastrophe excess of loss model, three factors are needed, based on deductible, penetration of layer and return period The catastrophe rating models using a Pareto type distribution also need 3 parameters and there is a remarkable similarity between the results.

The approach here differs from that in Financial Theory

The Pareto Model for Excess of Loss Rating is described more fully in two Swiss Re publications "Property Excess of Loss Rating by means of the Pareto Model" by Hans Schmitter and "Use of the Pareto Model for Quoting Proerty Excess of Loss"

Consider a portfolio of policies. The number (n) of claims in a year which exceed a given amount A, the observation point, is distributed as Pareto if:

$$\Pr(x \le S \le x + dx) = \alpha A^{\alpha} x^{-\alpha-1} dx$$

α is the Pareto parameter

Consider an Excess of Loss policy for cover E = U-D

Here U = upper limit of cover

D = deductible

E = Excess Layer

K = U/D = relative length of layer

The penetration frequency is given by $f(D) = nA^{\alpha}D^{-\alpha}$

This is the number of times the layer is penetrated

The risk premium = P

$$= n\frac{A^{\alpha}}{D^{\alpha}}\frac{D}{(\alpha - 1)}\frac{K^{\alpha-1} - 1}{K^{\alpha-1}} \qquad \alpha > 1$$

$$= n\ A\ \ln(K) \qquad \alpha = 1$$

= penetration frequency x average excess cover.

There are many methods by which frequncies, premiums and expected excess claims can be compared. The rate on line = P/D. Another variavble is the risk rate on line which assumes the penetration equals 1

The problems with this type of model are that it only deals with risk premium, and there is no allowance for the necessary risk and contingency margins that are needed in underwriting the higher layers

The approach requires the estimation of 3 parameters,

1. The number of claims (n)
2. The Pareto Parameter α
3. The Observation Point A

In addition we need U and D

The estimates of α have been sugested as follows

Fire	1.0 to 2.0 *in general*	
	1 0 for gross accounts	
	1.5 for net accounts above the underwriting limit	
Catastrophes	Windstorm	0.7 to 0.8
	Earthquake	0.3

In the BS model for ths stop loss case it was assumed T = 1, R_f =0,

Consider a series of Excess of Loss Premiums P(1), P(2), P(3),... relating to different layers D(1) to U(1), D(2) to U(2), D(3) to U(3). Furthermore assume U(1) = D(2) and so on, i.e. the cover is continuous. For Catastrophe Excess of Loss theretwo problems

Firstly the "expected loss" should an event arise can only be estimated with great uncertainty, that to a large extent it is unknown. Secondly , if an event happens , then a layer may or may not be penetrated. We need in the B-S model to estimate 3 things

1. The expectation of the lower layer being penetrated, or more specifically the reciprocal of this , the return period.(T(i))
2. The estimate of the expected claim based on this return period. B(i)
3. The B-S implied volatility σ

Consider the first layer where payment is made from D(1) to *infinity*

Assume the return period is T(1) = T

The B-S premium is calculated as

$$P(1)xT = B(1)N(d_1)-D(1)N(d_2)\exp(T)$$
$$d_1 = [\ln(B(1)/D(1)) + \tfrac{1}{2}\sigma^2 T]/\sigma\sqrt{T}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

Consider an Excess of Loss with a return period of 4 years

Consider the following 4 layers

Layer 1	150 xs 100
Layer 2	750 xs 250
Layer 3	3000 xs 1000
Layer 4	6000 xs 4000

The results of the calculation by the Pareto formula and BS is given below

Alpha	B	BS Volatility	All Layer	Layer 2 to Layer 4	Layer 3 to Layer 4	Layer 4
2			99	39	9	1.5
	185	0.8	98	40	154	n/a
1.5			180	106	43	12
	270	1	183	109	42	3,938
1			460	369	230	91
	550	1.25	460	366	173	316
0.8			*755*	*655*	*463*	*210*
	850	1.38	758	656	398	227
0.6			1,327	1,216	949	484
	1,420	1.51	1,327	1,216	890	475
0.4			2,474	2,352	1,977	1,117
	2,570	1.66	2,476	2,358	1,966	1,297

The calculations are made for different alphas. The first premium is that calculated by assuming a Pareto distribution, and the second by the distribution free B-S approach. This assumes n = 25, i.e. a return period of 4 years. Note the consistency in the low alpha range for catastrophes.

Note

1 The B's are intependent of the period of return and depend solely on alpha. They are approximately equal to :

Deductible plus $\sum$ *Penetration x Expected Excess Claim*

2. There is a consistent relationship between the BS volatility for all layers . This is given by

$$\text{BS Volatility (Alpha 1)} = \text{BS Volatility (Alpha 2)} \text{ X[} 1+\frac{[Alpha2 - Alpha1]}{2} \text{]}$$

Again this is independent of the return period.

What is perhaps surprising is the consistency in the results.

Pareto Calculations for T=4

	Alpha	Layer 1	Layer 2	Layer 3	Layer 4
Penetration	2 0	1.0000	0 1600	0 0100	0 0016
Expected Excess Claim		60	187.5	750	2,400
Penetration	1.5	1.0000	0.2530	0 0316	0.0040
Expected Excess Claim		73	250	1,000	2,940
Penetration	1.0	1 0000	0.4000	0.1000	0.0250
Expected Excess Claim		92	347	1,386	3,665
Penetration	0.8	1.0000	0.4804	0.1585	0.0523
Expected Excess Claim		101	399	1,597	4,022
Penetration	0.6	1.0000	0.5771	0.2512	0.1093
Expected Excess Claim		111	463	1,852	4,427
Penetration	0.4	1.0000	0.6931	0.3981	0.2287
Expected Excess Claim		122	540	2,162	4,885

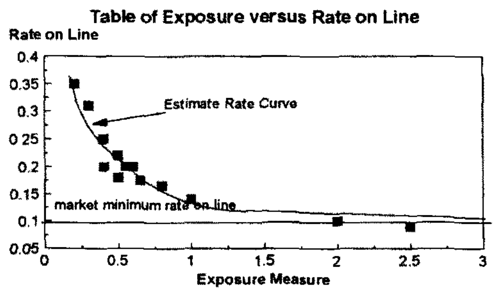
Curve Fitting

This method is fairly simple.

Rates from a number of contracts are represented as a rate on line versus the exposure calculated on a consistent basis. An example might be as a percentage of the expected loss should a storm like 90A repeat itself based on the current portfolio. The points may be based on deductible. mid point of range, and upper limit.

Once these are graphed a curve of a sepcific type is plotted to get a good fit and statistical limits are placed on the curve to identify outliers.

Examples of curves include negative exponentials with an upward shift (to represent a minimum rate on line) although other curves may also be used. There is a balance between goodness of fit and the number of parameters



Appendix 1.6 Financial Models

Financial models have been used in practice to determine premium rates for a number of classes in the United States. They have been used to determine the appropriate rate required to give an equity holder in the company an appropriate "rate of return". This return is also supposed to take in the degree of risk. They do not appear to have been used for reinsurance pricing, but there must be a strong possibility that the techniques are explored for appropriate application in the future.

The key work is The Financial Theory of Pricing Property Liability Contracts by D'Arcy and Doherty[]. They explore three models, the Capital Asset Pricing Model, the Arbitrage Pricing Model and the Option Pricing Model. Note that the Option Pricing model differs from that used in the relativistic pricing in section. Other financial 'theory approaches include discounted cash flow, return on equity and other issues dealt with elsewhere in this paper.

<u>The Capital Assets Pricing Model.(CAPM)</u>

This was applied in 1976 to automobile insurance rates in Massachusetts.

In Capital Asset Pricing Theory, the return required by an investor is expressed as a linear function of the security BETA

$$E(r) = r_f + \beta(E(r_m) - r_f)$$

r_f is the return on the security

r_m is the market (or portfolio) return

β is the measure of market (or systematic) risk

The return on equity r_e on an insurance contract is given by

$$r_e = [1 + k(\frac{P}{E})(1-x)]r_i + (\frac{P}{E})r_u$$

r_u is the underwriting return

r_i is the investment return

P are the written premiums

X are the costs (excluding capital costs) and $x = X/P$

k is the fund generating coefficient and is a corrective factor to allow for the period over which funds earn interest.

Taking expectations we have

$$E(r) = [1 + k(P/E)(1-x)]E(r_i) + (P/E)E(r_u)$$

The investment returns are assumed to be in equilibrium, that is

$$E(r_i) = r_f + \beta[E(r_m) - r_f]$$

When β_i is the Beta of the insurance portfolio, substituting we get

$$[1 + k(P/E)(1 - x)]\{r_f + \beta_i(E(r_m) - r_f)\} + (P/E)E(r_u)] \quad = \quad r_f + \beta[E(r_m) - r_f]$$

Define $\beta_u = cov(r_u, r_m)/\sigma_m^2$ and substitution gives

$$E(r_u) = -k(1 - x)r_f + \beta_u[E(r_m) - r_f]$$

Note if the underwriting results are correlated to the investment market, that the shareholders requires a higher return for the additional correlated risk. This is of importance in rating in the Lloyd's market.

If the investment rate is restrained then

$$E(r_u) = -k(1 - x)r_f + \beta_u[E(r_m) - r_f] + [E/P + k(1 - x)]m$$

Where m = the reduction in rate below the equilibrium premium level.

The formula can be adjusted to allow for the impact of taxes.

CAPM is more generally of use for portfolio management as opposed to assessing individual risks. It can be extended for correlations between lines of business It relates the underwriting risks to the investment and other returns and helps focus on other issues, for example

1. What are the investment returns and how do they correlate with other lines of business/investment returns.

2. How are all the risks correlated (clearly of relevance to risk based capital).

Arbitrage Pricing Model (APM)

This represents the first improvement in determining the rate of return over the more traditional methods. It is a less restrictive theory than CAPM in that CAPM makes the assumption that investors make decisions based on expected values and variance (or semi-variance) Investors are generally more concerned about systematic risk.

The base assumption is that there exists a set of factors, or economic variables, that commonly determine the return on assets, and that the process to generate an individual risks return is a linear combination of these factors

$$R_i = a_i + \sum_{j=1}^{n} b_{ij} I_j + e_i$$

R = realised rate of return

a = expected rate of return if all indices were zero

b = sensitivity of asset returns to specific index

I = value of index

e = residual error term

The pricing relationship is

$$E(r_i) = r_f^1 + \sum_{j=1}^{n} b_{ij}\lambda_j$$

where r_f^1 = expected return on zero systematic risk

λ_j = excess return factor with only one unit of systematic risk of that factor and no other systematic risk

These factors are determined by factor analysis as the b_{ij} from source data

For underwriting

$$E(r_u) = -k(1-x)r_f^1 + \sum_{j=1}^{n} b_{uj}\lambda_j$$

$$b_{uj} = cov(r_{uj}.I_j)/var(I_j)$$

These results look remarkably similar to many linear models used in alternative rating strategies

Option Pricing Model

This is put forward as an extension of the above to be used primarily in rating aggregate excess of loss contracts.

The formula is

$$P_R = R_F^{-1} X Nor(X/\sigma(L)) + R_F^{-1}\sigma(l) nor(X/\sigma(L))$$

Where $X = P[1 + ((1-\theta t)/(1-t))kr_f - \frac{1}{s}\frac{\theta_t}{(1-t)}r_f] - D$

P_R = competitive price for reinsurance contract

r_F = riskless interest rate

$R_F = 1 + r_f$

t = corporate marginal tax rate

θt = *effective tax rate on reinsurers investment income*

k = funds generating coefficient

S = premium to surplus ratio for the reinsurer

D = deductible on the reinsurance contract

P = premium on direct contract (which is assumed to be competitive)

Nor(*) = cumulative standard normal density at *

nor(*) = standard normal density at *

$\sigma(L)$ = standard deviation of loss distribution.

Data comes from four sources

1. Not specific to the firm r_f, t
2. Financial data specific to the reinsurer θ, S
3. Data specific to the loss $k, \sigma(L)$
4. Data on direct insurance contract P

Using similar techniques as in Sanders[20], this approach could be extended to event excess of loss contracts using return periods.

The formula assumes that the Central Limit Theorem applies. Non normal distributions can be handled be these techniques, but some care is required.

Appendix 2.

Output from Simulation on Household Property

Postal District	House Hold Buildings	House Hold Contents
ABERDEEN	482,409,184	111,457,936
BATH	72,673,008	24,490,412
BELFAST	486,422	103,851
BIRMINGHAM	383,212,896	150,098,816
BLACKBURN	132,446,560	50,701,332
and soon		

The effect of 1000 simulations was then analysed on a £10 million excess of £30 million layer with no reinstatements or coinsurance using aggregate information

Expected Loss	£ 421,695
Standard Deviation	£ 1,891,774
20 Year Storm Loss	£ 2,434,775
50 Year Storm Loss	£10,000.000

Appendix 3

Tables From

"An Exposure Rating Approach to Pricing Property Excess of Loss Reinsurance"

These tables give the Loss Distribution as a Percentage of Insured Value

The distributions are split into the following Policy Limit Ranges

1. $1,000 to $25,000
2. $ 25,000 to $100,000
3. $100,000 to $300,000
4. $300,000 to $1,000,000
5. In excess of $1,000,000

Losses are expresses as a percent if insured value at the levels of 5%, 10%, 20% and in 10% increments to 100%

The Distribution given are

Retail/Wholesale Risk, Fire Losses Only, Contents Only	
Retail/Wholesale Risk. All Other Policies	
All Commercial Combined Classes	- Fire Losses Only
	- Wind Losses Only
	- All Other Losses
Retail/Wholesale Risks Only	- Fire Losses Only
	- Wind Losses Only
	- All Other Losses
Services/Office Risks	- As above
Restaurant Risks	- As above

Example

All Commercial Property Classes - Wind Loss Only

Insurance Range

	1,000 to 25,000	25,000 to 100,000	100,000 to 300,000	300,000 to 1,000,000	Greater than 1,000,000
5 %	29.0	63.3	84.3	82.5	99.1
10 %	45 2	74.5	91.9	87.0	100.0
20 %	65.0	82.8	97.3	92.5	100.0
30 %	76.8	87.4	99.5	95.8	100.0
40 %	84.7	90.4	100.0	97.6	100.0
50 %	89.3	92.8	100.0	99 3	100.0
60 %	93.1	94.8	100.0	100.0	100.0
70 %	95.6	96.6	100.0	100.0	100 0
80 %	97.3	97.9	100 0	100.0	100.0
90 %	98.7	99.2	100 0	100.0	100.0
100%	100.0	100.0	100.0	100.0	100.0

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1981	Monograph - Premiums and Rating	Chaired by Bennett and Orros
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1984	Some Notes on the London Market	
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