SOME CHANGES IN ACTUARIAL METHODS ARISING FROM THE USE OF A SMALL COMPUTER IN A MEDIUM-SIZED OFFICE

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Many assurance offices are in the early stages of changing to a computer for the mechanization of their business. Each office has its own problems which frequently arise from its historical record of amalgamations, changes in management policy, etc.

2. Probably the biggest problem for the office is the standardization of its data into a form which can be read by the computer. When this has been accomplished the small-size computer can readily take over practically all the work which has already been mechanized, usually giving a better product in a shorter time and with reduction in manual work. There is also a simplification of diverse mechanical systems into a single medium.

3. In general it would seem that the work of the insurance office falls into two categories—the fairly straightforward tasks such as renewal notices, receipts, accounting, listing and other routine work, which are often already on mechanical systems, and the more difficult tasks such as the valuation of life business, the calculation of surrender values, preparation of loan repayment schedules, the matching of the credit notes from the banks for premiums paid by banker's orders with the actual policies due for renewal, examination of switches of British Government securities, policy writing, etc.

4. The cost of a computer depends upon the power of its central processor and the amount and quality of the peripheral equipment. The computer configuration an office requires, and hence its cost, will therefore largely be determined by the most complex of the tasks it will be required to do. Extra care and thought applied to the planning and programming of these more difficult tasks may, therefore, be more than amply repaid by a saving in the size and expense of the equipment. Some jobs may be so complex or infrequent that it would be more economical to use a manual system or buy time on a larger computer.

5. It is frequently found that the existing machinery is used to assist in what is primarily a manual system.

6. The considerable increase in speed and flexibility offered by a computer often means that the existing systems can be greatly simplified and altered to good advantage. Actuarial work is no exception to this. Indeed, much of the work readily lends itself to change and can sometimes be accommodated more easily under the revised systems.
Size of office and computer

7. The methods described in this paper have been developed in a medium-sized office conducting Industrial Branch business (2,000,000 policies), Ordinary Branch business (100,000 policies) and Fire and Accident and Motor business (150,000 policies). The computer is the punched-card version of IBM Model 1401 with an immediate access ferrite core store of 4,000 alphanumeric characters: there is no other form of store. Peripheral equipment consists of an IBM 1402 card reader/punch with punch feed read feature and an IBM 1403 line printer. The office does not put policyowners' addresses on documents. If it did it is probable that some medium other than punched cards (e.g. magnetic tape) would have been considered desirable for maintaining the policy records, but it is not thought that this would vitiate the principles described. In spite of the limited store, every task which has been considered suitable for the computer has been dealt with. This has often demanded ingenuity in both planning and programming.

8. The computer is used for a very wide range of tasks but this paper has been limited to those matters which are mainly or wholly actuarial in nature.

9. The computer undoubtedly has a fascination peculiar to itself. The enormous speed and accuracy with which the machine undertakes its work often masks the extremely simple logic through which it operates. So accustomed are we to our present systems and methods that it is easy to overlook the fact that many involve very sophisticated techniques designed to reduce manual work or to collect data in stages to enable the seemingly inevitable clerical error to be traced. In planning for the computer it is often difficult to clear one's mind of present methods and to reduce the job to its simplest and fundamental form—the simple logic to which the computer is best suited.

Choice of computer

10. A point in time had been reached when the existing conventional punched-card machinery was wearing out and it was necessary to replace it. An estimate was made of the cost of similar machinery with sufficient reserve capacity to meet the increasing demands of a growing business. This cost, plus the cost of conversion of the existing 65-column cards to 80-column cards was regarded as the normal replacement cost. Two tasks were examined in detail, the matching of monthly instalment payments with the corresponding policy record and the preparation of bonus certificates. From these it was considered that for all the office work the excess cost of the computer over the replacement cost would be offset by savings and greater efficiency in a reasonable space of time. Little or no consideration was given to a larger machine. It was felt that the IBM 1401 was so powerful that it would take many years to gain experience and to change methods and that by the time that this had been accomplished the next generation of computers would have arrived.
The main functions of the Ordinary Branch are:

(a) Policy issue.
(b) Premium renewals and accounting.
(c) Bonus certificates.
(d) Valuation and analysis of surplus.
(e) Mortality investigation.
(f) Claims.

In some offices these functions are dealt with by different departments and each or many of the departments may carry their own records. In particular, it is common for the valuation records to be a distinct and separate file from the renewals—often the files being in different media.

The most important development in this branch was that with very few exceptions all the information required for renewals, accounting, bonus certificates, valuation and mortality statistics was condensed into a single 80-column punched card for each policy. The card contained the following details:

(a) Policy number.
(b) Division, district and agency.
(c) Name of life assured.
(d) Premium.
(e) Mode of payment.
(f) Extra or reduction of premium.
(g) Month of maturity (or of policy anniversary for the whole-life cases).
(h) Year of maturity (or of cessation of premiums).
(i) Year of entry.
(j) True year of birth.
(k) Sum assured.
(l) Table number.
(m) Mortality class (including sex, medical, non-medical).
(n) First year commission (coded).
(o) Renewal commission (coded).
(p) Bonus adjustment factor.

Except for the bonus adjustment factor the items are self-explanatory. Until the introduction of the computer, bonuses had been laboriously calculated and checked each year by manual methods. From the records the bonus standing at the date of transfer to the computer was readily available and this was punched in the card as the bonus adjustment factor. This factor is only adjusted if bonus is surrendered, in which case it is reduced by the amount surrendered. If the bonus, including some or all of that declared since the date of transfer to the computer, is surrendered,
the factor is a minus quantity. For policies entered since the date of transfer to the computer the bonus adjustment factor is nil, or if some bonus has since been surrendered, a minus quantity representing the amount surrendered. In all programmes for which the bonus to date is required, a simple routine is included which adds to the bonus adjustment factor the bonuses declared since the date of transfer to the computer or since the date of entry whichever is the later. It is expected that the wear and tear on the cards in the file will be such that they will have to be reproduced about every five years; when this takes place the bonus adjustment factor will be updated to represent the bonus standing at the date of reproduction. This has the effect of limiting the number of rates of bonus (declared annually), which the computer will have to store at any one time, to about five and thereby reduces the amount of core store required for this simple routine.

15. The system permits the policy detail cards (approximately 100,000) to be kept in two files, one file containing about 40,000 policy detail cards for Ordinary Branch monthly instalment policies and the other containing about 60,000 policy detail cards for 'other' renewals. Included in the latter file is a name and address card for each agent. These files are maintained in district and agency order. This has the great advantage that when renewal notices, renewal lists or bonus certificates are produced they are in agency order ready for dispatch and the agent's name can be included. Each month the whole of the 'other' renewals file is passed through the computer to prepare the renewal notices. The computer only prints those cases due for renewal that month. It also picks out next month's maturities (or policies on which premiums cease), counts the number of cards in each agency, for control purposes, and carries out a limited validity check on each card to reduce the possibility that any card has been wrongly punched. The validity check is varied from time to time so that all reasonable checks are included in the course of a year. The number of validity checks each month is limited to the amount of free processing time available between the reading of each card.

16. The two files of policy detail cards are, therefore, the only source of data for all mechanical procedures. They have to be constantly kept up to date. Each card contains all the necessary information for renewal and valuation purposes so that when a card is altered the renewal records and valuation records are dealt with in the same operation. This single file system makes for considerable economy in file maintenance and also ensures that the valuation corresponds exactly with the in force cards.

Valuation

17. For the valuation the cards are updated to the end of the year. However, the work involved in keeping the cards up to date is normally a daily process, and, at most, the files can only be retained for a few days in the
year end position. It is necessary, therefore, to keep to a minimum the time the cards are required for valuation. In the main valuation run the programme deals with every card. Most of the policies are valued, but exceptions are provided for by reproducing the policy detail card when the programme is unable to value the policy or any part of it. At the end of the run details of the cards valued are printed out in table totals in the form required by the Fourth Schedule to the Insurance Companies Regs. 1958. A total of the number of cards reproduced is also given as a control on the punched output. At various stages in the run totals are taken of the number of cards passed. This total is checked to the control total of the number of cards in each agency.

18. It will be noted that the information contained in each card makes no provision for valuation constants. However, no sophisticated valuation techniques are called for. Every policy is valued by the net premium method using the true year of birth, although an adjustment could be made to this for female lives or in other ways if this were considered desirable. A reduction in the age reduces the liability for whole-life contracts but may increase or decrease the liability for endowment assurances according to the age at entry, duration and original term of the contract.

19. A very large amount of store would be required to record all the values of the factors that may be required using the normal valuation formulae. It is essential, therefore, that these formulae be developed into some other form. In general it is found that a reduction in the store required involves an increase in the amount of arithmetic performed by the computer, which may also mean an increase in the time taken. Although the time taken to value a policy may vary by less than a tenth of a second as between one method and another this will make a difference of several hours in valuing the whole portfolio. The formulae must be developed, therefore, to such a form as will give maximum speed consistent with being able to store the programme and factors, to value a high proportion of the policies, to provide for those it cannot value and to store the details for the Fourth Schedule to the Insurance Companies Regs. 1958 in one pass through the computer. The basic table of $l_x$ is unsuitable because the time taken for the multiplication by $v^x$ to secure a series of values of $D_x$ for $N_x$ would be too high. The decision lay between storing the table of $D_x$ or $N_x$. The choice fell on the table of $N_x$ because $D_x$ can be obtained by a single subtraction of $N_x - N_{x+1}$ while $N_x$ required a series of additions of $D_x$. The amount of storage required differs little.

20. The formula for the endowment assurance is:

Net premium for sum assured of 1 = \[ \frac{N_x - N_{x+1} - d(N_x - N_{x+n})}{N_x - N_{x+n}} \]

Value of premium of 1 = \[ \frac{2(N_{y+1} - N_{x+n})}{N_y - N_{y+2}} \]
Value of sum assured of $1 = \frac{2N_y - 2N_y + d(N_y + 4N_{y+1} + N_y + 2 - 4N_{x+n})}{2N_y - 2N_y + 12}

Where $x = \text{year of entry} - \text{year of birth} \quad n = \text{original term} \quad y = \text{year of valuation} - \text{year of birth}.$

21. These formulae are very convenient for programming because, by omitting the $N_{x+n}$ terms from the numerators of the formulae for the net premium and the value of the sum assured, the formulae become appropriate for a whole-life policy with limited payments; and further, by omitting the $N_{x+n}$ terms from the denominator of the formula for the net premium and from the numerator of the value of premium formula, they become appropriate for a whole-life policy. The programme first analyses the type of policy and sets indicators (called switches) accordingly. All these policies are valued in one simple routine which includes or omits the various items according to the indicators. An additional indicator is also used to distinguish the policies with profits from those without profits. After valuing the basic contract the programme will test the indicator and for a without-profit policy it skips over that part of the programme which calculates and values the bonus.

22. Pure endowment contracts are valued by a separate routine, storing $\bar{a}_m$.

23. Most offices have in their portfolio a number of joint-life policies. These are troublesome because they may require a separate classification and separate tables for valuation. The liability in respect of these policies is usually very small compared with the single-life tables and the expense involved in a special valuation of these policies is not justified. This is recognized by the use of equivalent equal ages which is only accurate if the mortality follows a special curve. It was found, however, by the examination of a model office, that a satisfactory approximation to the liability was secured by the valuation of joint-life policies as single-life policies with an addition of eight years to the equivalent joint-life age for whole-life assurances and an addition of four years to the equivalent joint-life age for endowment assurances.

24. It will be seen from the above that the programme which includes the routines for valuing life assurance policies and for valuing pure endowments deals with the following tables on one pass through the machine:

<table>
<thead>
<tr>
<th>With profits</th>
<th>Without profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole-life single-life</td>
<td>Whole-life single-life</td>
</tr>
<tr>
<td>Whole-life joint-life</td>
<td>Whole-life joint-life</td>
</tr>
<tr>
<td>Whole-life limited payments</td>
<td>Whole-life limited payments</td>
</tr>
<tr>
<td>Joint-life limited payments</td>
<td>Joint-life limited payments</td>
</tr>
<tr>
<td>Endowment single-life</td>
<td>Endowment single-life</td>
</tr>
<tr>
<td>Endowment joint-life</td>
<td>Endowment joint-life</td>
</tr>
<tr>
<td>Pure endowment</td>
<td>Pure endowment</td>
</tr>
</tbody>
</table>
The Use of a Small Computer in

25. The programme is in two parts—the first the valuation programme, and the second the printing programme. The printing programme is written as a 'sandwich' programme; that is to say it can be inserted anywhere among the data cards. It overwrites the valuation programme in store and causes the computer to print the totals to that point and it then resets the original valuation programme without destroying the totals so that the data continue to be valued cumulatively. This programme values approximately 20,000 contracts per hour.

26. It is desirable to record separate totals for different tables, although in some cases the method of valuation is the same. For example, children's deferred assurances are, prior to the option date, valued as pure endowments for the cash option, but the totals are kept separate from other pure endowments so that a loading can be added for the option. Where a whole-life assurance or endowment assurance has an additional benefit in the form of an income benefit an additional card is necessary to record the income benefit; the basic contract alone is valued and the detail card reproduced so that further attention can be given to the income benefit part of the contract. Mortgage protection policies with an endowment benefit are treated as a separate table and the values for the endowment part valued and totalled. A card is reproduced for these contracts as well as for all temporary assurances. Totals are taken of the number, sum assured, premium and extra premium of the mortgage protection type of temporary assurance for control purposes.

Expense loading for limited-payment policies

27. Single-premium policies and paid-up policies could be valued by the same programme as other assurances by omitting that part which involves the net and office premium. This is not done because it is the office practice to place these policies all on special agency on which no premiums are payable and they are excluded from the normal renewals routine. This is fortuitous, but it does permit a separate programme to be written not only of valuing these policies but also of providing for the loading for future expenses and profits.

28. Under a net premium valuation the reserve for future expenses and profits may be calculated by:

$$P_x a_y - \pi_x a_y = tP_x a_{y:n} - t\pi_x a_{y:n} \quad \text{(Lochhead)}$$

where $P_x$ and $\pi_x$ are the equivalent whole-life office and net premiums as at the date of entry and $tP_x$ and $t\pi_x$ are the actual office and net premiums appertaining to the policy in question. This formula is subject to two criticisms; (a) that no allowance is made for the saving of costs of collecting premiums after premiums have ceased and, more important, (b) a strengthening of a net premium valuation by a reduction in the rate of interest may reduce the amount reserved for future expenses and profits—although such a strengthening may, in fact, be due to a desire to provide for greater profits in future.
29. Moreover, the computation of this function requires an additional constant in the card, namely $P_x - \pi_x$. This factor cannot be accommodated on an 80 column card after provision has been made for all the other information which it is necessary to record. Even if room can be found it is considered wasteful to set aside columns in the card for this factor because the majority of cards would carry blank columns.

30. In respect of a fully paid policy a compound bonus of $i\%$ can be provided for by valuing at a rate of interest of $i\%$ lower than the net yield expected in the future. If a simple reversionary bonus of $n\%$ is to be provided for, the valuation rate of interest need not differ from the net yield expected by as much as $n\%$ but by $j\%$, which will, however, for premium-paying policies be more than the difference between the valuation rate and the net yield because, in these cases, a release of surplus arises each year from the bonus loading.

31. If, therefore, the liability on the normal valuation basis be $V$, then the extra reserve needed for single-premium policies will be $V' - V$ where $V'$ is the liability calculated at a rate of $i\%$ less than the yield expected for compound reversionary bonuses or $j\%$ less than the yield expected for simple reversionary bonuses.

32. A valuation at two rates of interest is no problem to the computer because all that is involved is for the programme to value the data twice on two different sets of $N_x$ in the store. If, however, the fully paid policies are not segregated, in order to avoid a second run of the policies it would be reasonable to obtain the average valuation age $x$ from $A_x = (\text{Value of sum assured})/(\text{Sum assured})$ and using this value of $x$ to calculate $V' = A'_x \times (\text{Sum assured})$.

33. When premium-paying limited-payment policies are involved it is desirable that the provision for expenses and profits build up from zero at entry to $V' - V$ at the date when premiums cease. For whole-life policies $V' - V$ can be expressed as $(\text{Sum assured}) \left( A'_{x+t} - A_{x+t} \right)$ where $x$ is the valuation age and $t$ the number of years to the date when premiums cease. The valuation reserve builds up from zero to $(\text{Sum assured}) A_{x+t}$ so that a convenient expression for the extra reserve is $(\text{Valuation reserve}) (A'_{x+t} - A_{x+t})/A_{x+t}$. Average values of $x$ and $t$ can be obtained from the figures in the Fourth Schedule:

$$A_x = \frac{\text{Value of sum assured}}{\text{Sum assured}}$$

$$1/\overline{\delta}_{x+t} = \frac{\text{Value of net premiums}}{\text{Net premiums}}$$

34. By using this method the premium-paying whole-life limited-payment policies are valued with other premium-paying policies in a single run through the computer.
Temporary assurances and policies with temporary assurance benefit

35. Brief reference has so far been made to contracts containing a temporary assurance. These fall into two classes: (1) those of short duration such as the type used for covering the estate duty under gifts inter vivos, and (2) the longer term contracts such as family income contracts and mortgage protection contracts with or without a basic whole-life or endowment assurance.

36. For the class (1) contracts a reserve based on the office premium is satisfactory. These contracts rarely form a substantial homogeneous group and can, if necessary, be valued individually with little trouble. The computer is programmed to reproduce the data cards for these contracts, thus providing a full record for valuation purposes but leaving the main file undisturbed.

37. Class (2) are a different problem. They frequently fall into two or three tables and under current conditions a substantial proportion of new business is written under them. The problems of a manual valuation are great. They involve the calculation of the net premium and also the calculation of the sum assured currently standing under the contract. Negative liabilities can occur and should be eliminated. In addition the sum assured under the mortgage protection type of policy may have been altered under different prospectuses to keep in line with the rate of interest currently charged by building societies, etc.

38. The programme to value the main tables occupied nearly all the available store in the computer and it was apparent that there was little prospect of being able to value the temporary assurance tables satisfactorily—even by some form of approximate method—with this programme. At the present time these temporary assurance tables, while significant and increasing in numbers, form only a small proportion of the whole. It was decided, therefore, to reproduce the data card for all these contracts along with the other temporary assurances and exceptional types of contract so that they are valued after the main file has been released. If and when their numbers form a substantial proportion of the main file consideration will be given to the alternative of running the main file through the computer again to value the mortgage protection type of contract.

39. The programme to value these long-term temporary assurances accurately is well within the capabilities of the computer. By storing $\ddot{a}_t$, $C_x$ and $N_x$, the schedule of sums assured is calculated from the capital outstanding under a loan repayment schedule, and the numerator of the net premium and of the sum assured are calculated by multiplying the sum assured in the schedule by $C_{x+t}$.

Continuous mortality investigation

40. A second pass of the two Ordinary Branch files is necessary to secure the 'standing' for the Institute and Faculty continuous mortality investigation. In fact the office requires additional information for its internal
mortality investigations. The writing of the programme to classify the data from the main file is quite straightforward. It does involve a strain on the store because over one thousand registers have to be used.

41. As with the valuation programme it was found necessary to write a separate sandwich programme to print out the information stored in the computer. This programme is used at frequent intervals (a) to check the count of the cards against the control totals, and (b) to give a restart point in case of an interruption. The programme does not destroy the totals, so that the result of processing further data gives a cumulative result. The computer deals with approximately 40,000 cards per hour with this programme.

42. The deaths are, of course, extracted from the main in force file. These cards are kept until the end of the year and used for the mortality investigations. For this, and no other purpose, the exact date of death is required. It has not been considered worth while to mechanize this part of the procedure and the records of the deaths are compiled manually.

Analysis of surplus

43. A duplicate card is made of all contracts entering during the inter-valuation period, i.e. alterations on (including paid-up policies) new business and revivals. All cards in respect of contracts leaving during the period, i.e. surrenders, deaths claims, alterations off, and lapses are also retained. With the valuation programme the liability for these policies can readily be calculated.

44. The main difficulty in analysis of surplus by a manual method is the calculation of the death strain at risk and the expected death strain, particularly in connexion with the mortgage protection type of temporary assurance. This problem is met by a special programme similar to the valuation programme but providing for the calculation of the death strain at risk times $q_x$. This programme is run in the middle of the year, the cards in respect of deaths during the first half of the year being included for this purpose. It is recognized that a small error arises from assuming that the business in force in the middle of the year is the average for the whole year.

Fifth Schedule to 1958 Regulations

45. This schedule requires a very great deal of store and it is not possible to accumulate all the necessary totals in one run. The data had to be passed through the computer twice. However, one run of 100,000 cards takes about 3 hours and the task only occurs once every five years.

Punching out totals

46. As a practical point it has been found of great help to have a programme which will reproduce the information in the store of the computer on punched cards. These cards may be used to restore the information into the computer. By this means any job may be broken off at any point and
recommenced later. It also makes it possible to run the valuation just before the year end, for example at the time the January renewals are being prepared. The data cards in respect of movement 'on' and 'off' to the end of the year are reproduced. The results of the valuation are then adjusted by restoring the information into the computer and passing the movement 'on' cards through the normal programme and the movement 'off' cards through a special programme which subtracts the values from the totals. This last task—a small one—is all that needs to be run in the first few days of January when the computer is heavily loaded with other work.

**Office premium rates**

47. The revision of office premium rates is not usually of frequent occurrence. However, when it does occur it may involve many hours of tedious manual work. Often this has to be done by a limited section of the staff in addition to their normal duties.

48. Each premium rate usually involves a simple calculation based upon a formula which can be readily written in a form using commutation columns. Unlike many of the programmes referred to earlier in this paper this type of programme is usually quite short. Furthermore, no totals are required, so that it is normally possible to store all the commutation columns required. Once the programme has been written, the premium rates are calculated and printed by the computer in a few minutes. In order to gain the best use from the computer the programme is written to quote for as wide a range of ages and terms as possible so that it will not be necessary to run the task again.

**INDUSTRIAL BRANCH**

49. The main features of Industrial Branch business are the large number of contracts and the limited number and age range of the tables; special policies are not permitted, the bonus is not usually surrendered and renewals are not handled centrally. An additional feature is that the sum assured, irrespective of bonus, may have been increased above that written in the contract—especially for contracts entered upon many years ago—because, on the introduction of a new prospectus giving higher benefits, the benefits of earlier policies have been increased to avoid lapse and re-entry.

50. All these features lead to a different approach to the problem of administration of the Industrial Branch. The large number of contracts alone makes an individual valuation impracticable with a card system using a small computer. However, the limited number of tables permits the business to be grouped under year of entry, table, age at entry and original term for endowment assurances. These groups, unlike groups made in the Ordinary Branch for manual valuations, are homogeneous and do not
involve any approximations. It was decided, therefore, to continue the existing system of carrying a summary card for each group.

51. The summary card for business at the year end contains:

<table>
<thead>
<tr>
<th>Table</th>
<th>Year of entry</th>
<th>Age of entry</th>
<th>Term (if applicable)</th>
<th>Number of policies</th>
<th>Total office premium</th>
<th>Net premium per £1 office premium</th>
<th>Sum assured per £1 office premium</th>
</tr>
</thead>
</table>

About 10,000 summary cards are required.

52. It will be noted that the net premium is, in this case, carried in the summary card. This is done to simplify the valuation programmes. If a change in valuation basis were made, the summary cards would be reproduced with a revised net premium. This would be done on the computer and would take about one hour.

53. During the year summary cards are made in respect of each type of movement (new business, revivals, lapses, maturities and deaths). These summary cards are then merged with 'brought forward' summary cards and by one pass through the computer, at the end of the year, up-to-date summary cards are produced. Under this system the factors are not carried on the summary cards representing the movement. The factors are carried from the 'brought forward' summary card to the updated summary card.

54. The summary cards representing the movement during the year are prepared by the computer from detail cards. These detail cards, each representing a policy, are first sorted into type of movement and then into year of entry order. It was found, in spite of the limited size of the computer, that it could accumulate the number of policies and total premium for all the groups, except for a few obsolescent tables, for one year of entry. The data cards, after being sorted into year of entry are fed into the computer which accumulates all the groups within its range and (1) punches a card in summary card format for each detail card outside its range, (2) punches a summary card whenever its store for that particular group is full and zeroizes the store, and (3) on a change in the year of entry punches a summary card for every group in its range which is not zero, sets the store to zero and proceeds to prepare summary cards for the next year of entry.

55. This programme has the great advantage that very little sorting is necessary. Year of entry order and policy number order are the same. The new business detail cards (which may contain some policies backdated to the previous year) may be passed through this programme immediately after they have been punched.
56. There are a number of valuation programmes, one for each group of tables (e.g. one for whole-life, one for endowments, etc.) Each programme is fairly simple in character and does not require any special technique to reduce it to the store available.

57. The summary cards which represent the movement of the business during the year are used for the analysis of surplus. The brought forward cards enable a calculation of the expected death strain to be made.

58. The valuation programmes take less than an hour to run.

Surrender values and paid-up policy values

59. One of the major problems in the Industrial Branch, after the calculation and checking of agent's accounts (which can be readily adapted to the computer but are a subject outside the scope of this paper) is that of the calculation and quotation of surrender values and paid-up policy values for reduced sums assured.

60. Experience shows that in the Ordinary Branch a surrender value is often required for information only. In the Industrial Branch, however, the cash is usually taken. In fact, the rate of acceptance is so high that it has been found more economical to amend the records forthwith. In the event of the surrender not being proceeded with, the policy is revived.

61. The detail card for the policy for which a quotation is required is extracted and punched with (a) the date (week number), (b) the arrears, and (c) a code indicating a surrender. These cards are then fed into the computer which prepares (a) a quotation and (b) an audit card for use by the claims department for their records to remit cash to the district. The detail card for the policy, having been extracted from the main file, is passed automatically into the agent's accounting routine and into the valuation summary card routine as a surrender. If the surrender is not proceeded with the detail card is repunched as a revival and the audit card destroyed.

62. It is common under manual methods to have many tables of paid-up policy and surrender values. It was quite impracticable to include all these tables in the store of the machine. As one might expect, however, the simplest approach was used, that is to say formulae were developed to produce the desired results.

63. The following two points had to be covered:

(1) The values produced must always be at least those prescribed by the Industrial Assurance Acts.

(2) Changes in prospectus had in the past caused lapse and re-entry problems. These had been dealt with by increasing the sum assured of policies existing at the time of the change in prospectus. These changes have not been recorded in either the detail cards or the policy documents. It is necessary, therefore, in respect of certain tables to check the year of entry. If it is prior to a prospectus change a revised sum assured has to be
included in the calculations. This sub-routine is also used in all programmes where the sum assured under these tables may be required (e.g. listing of an agent's debit, claims procedures, etc.).

64. The calculation of the surrender value is similar to the calculation of the liability in the Ordinary Branch but the commutation column $\bar{N}_x$ is stored. For Industrial Branch business, however, continuous functions are used. In order to avoid storing both $\bar{N}_x$ and $D_x$ it was necessary to find a simple relationship between them. The formula was as follows:

$$D_x = \frac{11\bar{N}_x - 18\bar{N}_{x+1} + 9\bar{N}_{x+2} - 2\bar{N}_{x+3}}{6}$$

65. A central difference formula is not convenient in this case because $D_1$ may be required and satisfactory values of $N_0$ and $N_{-1}$ are not available.

66. The full liability is adjusted by a charge. If the charge in the first year is $K$ of the liability, the liability is multiplied by the following factor:

$$(1 - K \left(1 - \frac{t^2}{n^2}\right))$$

where $t$ is the number of years premiums paid and $n$ is the number of years premiums payable. Where premiums are payable through life, $n$ is taken as 100 minus the age at entry. This method is a convenient and satisfactory way of reducing the charge to zero at the end of the premium-paying period.

MORTGAGE REPAYMENT SCHEDULES

67. The calculation of the capital and interest under an annuity certain may not occur with any frequency but is, nevertheless, troublesome.

68. Reference has been made to the programming of this problem by Michaelson(2). It was found in practice, however, that a different approach was advisable. In order that a satisfactory schedule be produced the total of the capital elements must equal the outlay exactly. This usually involves some adjustment to the rounded-off figures. In order to do this the programme was written to calculate the capital element in pence to several places of decimals. The total of the capital elements, each one rounded off to the nearest penny, is then subtracted from the outlay. The difference gives the number of items to be adjusted. If that difference is positive, the programme adjusts by finding the highest decimal, adding one penny to the appropriate capital element and setting the decimal to zero. This is repeated until all the adjustment has been completed. A similar procedure in reverse is followed if the difference is negative.

69. The final step is to calculate the interest element by subtracting the capital from the instalment and printing the results.
70. The calculation of the exposed to risk is usually done by the census method. However, this method is not always suitable and, on occasions, more detailed calculations of exposed to risk have to be made. When this is done by a manual method, an exposed to risk formula is usually evolved which enables the calculations to be made by a continuous method. With a computer, however, such a formula is not necessary. A register can be allocated for each age of exposure \((E_x)\) and for each age at death \((\phi_x)\).

71. A card is punched for each life which will contain:

(a) Investigation year of birth (year of entry—age last birthday at entry) \((YB)\).
(b) Year of entry \((YE)\).
(c) Year of exit (if any) \((YX)\).
(d) Code of mode of exit (if any).

If there is no year and code of exit the life is assumed to be still in force.

72. It may be helpful to consider the following example:

It is assumed the investigation covers the period from 1 January of one year \((YA)\) to 31 December of another year \((YZ)\). Then lives who entered before \((YA)\) are beginners \((b_x)\) and those who are still in existence after \((YZ)\) are enders \((e_x)\). New entrants \((n_x)\) are the only other form of entrant into the experience and other exits will be assumed to be withdrawals \((w_x)\) or deaths \((d_x)\). In each case \(x\) will be taken as (year of entry or exit—investigation year of birth).

73. The normal method of calculating the exposed to risk using an exposed to risk formula requires a substantial amount of sorting, scheduling and arithmetic. With a computer, each card, representing a life, is examined separately to determine the amount contributed to each age of exposure and age of death. For a beginner one is added to the register \(E_x\) corresponding to the age obtained from \((YA)\) minus the investigation year of birth or, for a new entrant \(n_x\) is added to the register \(E_x\) corresponding to the age obtained from the year of entry \((YE)\) minus the investigation year of birth. One is then added to the register for subsequent ages \(E_{x+t}\) and to \((YA)\) or the year of entry until the year of exit or \((YZ)\) is reached. One is then added to the register \(E_{x+n}\) if the mode of exit be by death or an ender but only \(n\) if it be a withdrawal. If it be a death one is also added to the register \(\phi_{x+n}\).

74. The method used by the computer may also be expressed by a block diagram (see diagram). The last item in the block diagram shows that the expected deaths will be printed out. This presupposes, of course, that the values of \(q_x\) have been loaded into the store. In order to leave the maximum amount of store available for the exposed to risk and death registers (particularly important when select rates are being investigated) it is quite practicable to omit the programme for printing the results and the values of \(q_x\) until the last of the data has been dealt with by the exposed
to risk programme. The exposed to risk programme can then be cleared
from store and the programme to calculate the expected deaths and print
the results put in its place. In fact, it has been found convenient to follow
the main run with several programmes each programme calculating the
expected deaths using a different value of $q_x$.

75. No attempt has been made to include in the block diagram details of
the various validity tests that would be included (for example that the year
of cessation is greater than the year of entry and that the code for the mode
of exit is valid).

**VALUATION OF A PRIVATE PENSION FUND**

76. The valuation of a private pension fund differs in many respects from
that of a fund subject to contract. A multiple decrement table is involved—
this is not normally met with in life office work. Furthermore, the basis is
not usually predetermined, as it so often is in the valuation of a life
assurance fund, but may be decided as a result of the experience in the
fund.

77. The fund to be valued consists of pensioners and several classes of
lives in service—chief office staff, outdoor officials and full-time agents.

78. A card is punched for each member of the fund giving the year of
birth, the year of entry and the present pensionable salary or present
pension.

79. The method involves four programmes. The one is a simple pro-
gramme to value the pensions to pensioners in attained age classification,
bringing out, also, the expected deaths.

80. The cards for the staff are then passed through the following pro-
grammes:

1. A simple programme to calculate the expected deaths on a trial basis.
2. A programme which lists (a) the numbers of those in service in five-
year age-groups and in select durations 0 to 3, 4 and over; (b) the
number of withdrawals in the four categories resigned, died, retired
and promoted; (c) the proportion which each type of withdrawal
bears to those in service. These figures are then reduced to rates
of resignation, death, retirement and promotion; (d) the average
salary for each age-group.

81. At this point it is necessary for the Actuary to supply in respect of
each class of life:

(a) Rate of interest
(b) Rate of withdrawal at each age in select durations
(c) Rates of retirement at each age
(d) A salary scale
(e) The annuity factor for each pension age (this will presumably
make allowance for ill-health retirements)
(f) Mortality
(g) The proportion of salary eligible for pension according to duration $k$ since entry (as defined by the rules) $f_k$

82. The next programme stores all the above factors and for each card makes the following calculations:

(1) $I_{y+t+1} = I_{y+t} (1-q_{y+t}-w'_{y+t}-r_{y+t})$
where $y$ is the valuation age

$w'_{y+t}$ is taken as $w'_{y+1}, w'_{y+t-1}+1, w'_{y+t-2}+2, w'_{y+t-3}+3$ or $w'_{y+t}$ according to the duration since entry. It is important that $(1-q_{y+t}-w'_{y+t}-r_{y+t})$ should not be negative. The programme was written to adjust this factor to zero if a negative should occur.

(2) As each value of $I_{y+t}$ is calculated the following calculations are also made

(a) $I_{y+t} r_{y+t} (s_{y+t-2}+s_{y+t-1}+s_{y+t}) v t a_{y+t} f_{k+t}$
where $y$ is the valuation age

(b) $I_{y+t} r_{y+t} (s_{y+t-2}+s_{y+t-1}+s_{y+t}) v t a_{y+t} f_{k}$

(c) $I_{y+t} v t$

and (d) $I_{y+t} v t s_{y+t}$

Then liability in respect of all service is:

Present salary $\times \frac{\Sigma(a)}{3I_{y} s_{y}}$

Liability in respect of past service is:

Present salary $\times \frac{\Sigma(b)}{3I_{y} s_{y}}$

Future level contribution is:

Present salary $\times \frac{\Sigma(a) - \Sigma(b)}{3s_{y} 2\Sigma(c)}$

Future contribution related to salary is:

Present salary $\times \frac{\Sigma(a) - \Sigma(b)}{3\Sigma(d)}$

83. By this method a multiple decrement table is built up and used for each member. The average of the last three years' salary is used. This could obviously be modified, as could nearly all the other factors, according to circumstances.

84. Details of the year of birth, year of entry, present pensionable salary, past liability, total liability, future level contribution and future contributions related to salary are printed in respect of each member. Totals are also printed at the end of the run. It is found that the computer is able to deal with about 80 cards per minute.

**EMERGING COSTS**

85. It is not intended to introduce, in this paper, a discussion on emerging costs. This method of valuation, which involves a forecast of the net
outgo from the fund and a comparison with the net income, including redemptions and sales from the assets, is not, as yet, a normal practice. Although the broad principles may have been established, the details of the methods by which such a valuation would be made may be the subject of differences of opinion.

86. Many papers have been written on the subject of valuations of life business and pension business. These papers do not, however, assist in preparing the schedule of liabilities in the form of emerging costs. No doubt sophisticated techniques could be devised which would permit a valuation to be conducted under the principle of emerging costs using simple accounting machine equipment. However, in this paper it is merely intended to illustrate the method that could be used on a computer.

**Ordinary Branch**

87. For the purpose of this illustration, only the income and outgo under the contracts is dealt with and no attempt has been made to examine the method of assessing the income and outgo of the assets. The final schedules will be in three columns:

- **Column (1):** The sum assured and existing bonus, forecast to be payable in each future year under contracts now standing (i.e. payments on death and maturity).
- **Column (2):** The amount forecast to be payable in each future year in respect of 1% simple reversionary bonus (i.e. \(0.01 \times \text{sum assured}\)).
- **Column (3):** The amount of income expected based on the office premium less commission and a level amount in respect of each renewal (monthly instalments being subject to a special rate).

88. The programme would contain the normal life table \(l_x\) in respect of the table of mortality to be used. A set of three registers would be allotted for each year in which a payment could arise (i.e. to the limit of the \(l_x\) table). The programme, on reading a policy detail card would calculate:

\[
\begin{align*}
(a) & \quad (\text{Sum assured plus existing bonus}) \times \frac{l_{x+t}}{l_x} \\
(b) & \quad (1\% \text{ of sum assured}) \times \frac{l_{x+t}}{l_x} \\
(c) & \quad (\text{Premium less expenses}) \times \frac{l_{x+t}}{l_x}.
\end{align*}
\]

89. Items (a) and (b) would be added into the registers for the year \(t\). Item (c) would be added to the register for \(t=1\) as well as \(t\) to provide for the fact that the premium, on average, is payable in the middle of the year. Special provision would be made for \(t=0\) and also to provide for half yearly, quarterly and monthly premiums in the year in which premiums cease.

90. The figures in the registers would not be in the form for printing in the final schedule. After the last data card has been passed the required values for year \(t\) for columns (1) and (2) would be obtained by subtracting
the register $t+1$ from register $t$. The registers for column (3) would merely need to be divided by 2.

**Industrial Branch**

91. In this branch the problem and computation is very similar to that in the Ordinary Branch except that, because the data are on summary cards, a separate programme can be prepared for each table. It is convenient to write these programmes so that they use the same registers. The programmes can then be run successively and the accumulated results for all the tables printed out at the end.

**Private pension fund**

92. The normal valuation programme is in multiple decrement form. The alteration to produce an emerging cost is quite simple. The value $l_{y+t+1} = l_{y+t}(1-q_{y+t}-w'_{y+t}-r_{y+t})$ is calculated successively. When the item $r_{y+t}$ occurs the programme passes into a special routine to calculate

\[
\text{Salary} \times l_{y+t} r_{y+t} \left( \frac{s_{y+t} + s_{y+t-1} + s_{y+t}}{3l_y s_y} \right) \times f_{k+t} \frac{l'_{y+t+n}}{l'_{y+t}}.
\]

$l'_{y+t}$ is based upon the mortality table for pensioners. $n$ takes the values from 0 to $(100 - y - t)$. Each value is added into the register for year $t+n$.

93. This valuation is executed at the rate of about 35 cards a minute for members in service. For pensioners the formula is simply: (pension) \( \frac{l'_{y+n}}{l'_{y}} \) and about 400 cards a minute are valued.

**ACTUARIAL TRAINING**

94. Little, if anything, new in actuarial technique has been described in this paper. Indeed, the main purpose of this paper is to draw attention to the need to set aside existing sophisticated techniques and look to first principles. It may well be that the advent of very fast, automatic calculating machines will cause us to develop new techniques and in particular develop new methods of valuation and presentation; methods which have not previously been possible due to the amount of labour required for their fulfilment.

95. It is unfortunate that the simple methods described in this paper do not mean that the burden of the actuarial student can be lightened by the exclusion of the methods of group valuation from the examination syllabus. It will be many years before it can be assumed that the actuary will have a fast calculator available. In the meantime it seems that the student may have to prepare himself for both types of approach to actuarial problems.

96. An even greater problem may result from the fact that, with the introduction of a computer, the work of the office tends to become
integrated and departmental boundaries moved or removed. In particular, a department which records the statistics and carries out the valuation may be unnecessary. Similarly, the departments which make premium quotations, quote surrender values and terms for alteration of contracts may well find that nearly all their work is taken over by the computer. As a result of these changes one of the normal training grounds for the actuarial student may no longer be available. However, in its place a new department will grow, namely that of systems analysis and computer programming. Nearly every job in the office will be dealt with in detail in this department. The actuarial student in this department will have the opportunity to learn the methods and practice of the whole of the office, including the methods of valuation, etc. Perhaps, however, this is asking the student to learn too much at the same time as his actuarial studies.

CONCLUSION

97. The computer has now been installed for over three years and we are satisfied that our original choice was the correct one. We now have over 200 programmes in regular use. They are very varied, but most of them involve a short run with a printed output. Wherever practicable, plain paper has been used, the programme arranging for the computer to insert the headings, etc. By this means work is dealt with very quickly with little or no delay between tasks.

98. Our policy has been first to transfer the most important part of each area of work on to the computer and to run this for some months to iron out most of the problems. We then extend the area covered by the computer step by step towards our final goal of an integrated system. We have always been ready to alter our programmes to meet any reasonable request from the departments concerned, and to facilitate this we have written our own assembly programmes. As a result we have been able to get work on to the computer before all the problems have been solved and, also, the staff concerned have played an active part in the development of the system. It does, of course, mean that a high proportion of computer time is used in programme testing (over 15% of total time used), but our computer is not yet fully used on a one-shift basis so we do not regard the programme testing time as costly.

99. All the programmes described in this paper, except that for the emerging costs of a life office valuation, which has not been written, and that for the calculation of Industrial Branch surrender values, which is just about to start operation, are in regular use. Taken over the year, these programmes occupy well under 5% of the total production time of our computer.

100. It is very difficult to assess the saving, if any, due to the computer in monetary terms. Many savings can be shown, on paper, to be realizable. However, it is another matter to realize them. Particularly is this so where a
reduction could be made in long-service medium-grade staff, or where there is a reduction in space requirements. On the other hand, we have been able to introduce new terms of remuneration for our Industrial Branch agents under which the whole of the accounting is mechanized. This and many other changes would not have been possible without the computer. There is no doubt in our minds that it has been worth while, particularly if we ask ourselves how we would now manage without it.

101. As stated earlier, our computer is not used to full capacity on a single-shift basis. We are prepared to make arrangements for it to be in use outside normal work hours—even a two-shift system if need be. Because of this degree of spare capacity, coupled with the fact that we have been able to accomplish all the tasks we have set out to do, only little thought has been given to the use of a more powerful machine. We do not believe that we could at present realize any material savings by the installation of a large random access store, certainly not commensurate with the additional cost. Under the present system updating is effected manually on the detail cards. There would seem to be little saving in this manual work if the data were prepared for the updating of a random access store. We are, however, very conscious of the fact that the actual updating of the store would require computer time.

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REFERENCES

Mr W. T. L. Barnard, in introducing the paper, said that one of the problems of senior management was to keep in touch with the various processes of the various departments of the office. They were familiar with the valuation and other techniques designed to reduce the manual work and to permit them to take advantage of the facilities offered by conventional punch card equipment. With the introduction of computers they were faced with substantial changes in their methods. Arithmetic was no longer a major problem and the valuation of each individual case was a practical proposition. The purpose of the paper had been to describe some of the changes made by one office in solving its actuarial problems. He wished to emphasize that the methods had been adapted to a small computer. It was, indeed, surprising how much could be achieved with such a modest machine. He only hoped that people would not be asking why their office was buying a larger one.

The paper did not present any new actuarial techniques. The formulae illustrated, however, that the actuary must for efficiency be prepared to depart from the conventional methods. The formulae were not the only ones that could be used. Most actuaries would find it necessary to devise their own to accommodate their basic data and machine. For the best use to be made of the computer it was necessary for the actuary to be conversant with the whole of the data processing system and also with the programming of the computer, so that he could obtain his results with the minimum of computer time and inconvenience to the main flow of work.

Mr P. A. Taylor, in opening the discussion, said that the paper ranged over many of the aspects of actuarial work to be found in a life office and described in some detail the way in which the computer had been used to take over that work. The amount of work that had been mechanized was very considerable, and it was striking that, as pointed out by the author in §99, the machine time involved was less than 5% of the total time available on the computer.

There were two ways in which a computer could play a part in actuarial work. First, it could handle the existing work more quickly and efficiently. Secondly, new techniques could be developed which would give the actuary a much greater insight into the working of the office. Most offices using a computer were still in the first stage and the author had mainly confined his remarks to that aspect of the work. In transferring to the computer it was essential to forget the old methods and to look at the basic data available and the results required. As a consequence the author had abandoned a number of well-known actuarial techniques and in a number of cases had simply returned to first principles.

The author had passed lightly over one of the largest sources of saving that the computer could bring to an office, namely, the maintenance of the data for most if not all of the functions of the office on one file, thus eliminating the necessity for updating two, three or even four different sets of records. The fact that data for actuarial purposes would in future come from a general file was bound to produce some substantial changes in actuarial methods. The order in which the data would be kept on the general file was most unlikely to be the same as for the existing actuarial records. In the past, the actuarial records could be altered to facilitate valuation requirements, and techniques were devised to reduce the work involved in handling data and to enable the required results to be calculated swiftly. When devising a new formula, it had often been necessary first to obtain the theoretically correct formula and then look around for some approximation that would simplify the work and allow it to be carried out quickly by unskilled
staff. Those conditions had changed. Even a small computer could handle complicated actuarial formulae swiftly and accurately. The new problem was to develop formulae which would enable the actuarial work to be done from data which from the point of view of valuation were in random order. That was not the only requirement, however. It might well be that part, if not the whole, of the valuation could be carried out when the computer was carrying out another routine run, such as that required to update the file or to produce renewals. In fact, the most efficient system was one where the figures for the valuation, analysis of surplus and other actuarial work were derived as a by-product of some normal production run.

Another point which might have an important bearing on the method to be used was the question of checking the results. In §9 the author mentioned that under a manual system very sophisticated techniques had been designed to trace the seemingly inevitable clerical error; but he was sure that the new system would derive its own form of errors—perhaps there would be an error in the critical data or in the punching. Furthermore, there was always the possibility that the computer itself might make an error, although fortunately that seemed to be a very rare event. Usually a computer was powerful enough to carry out the work required in a number of ways, but the difference in the time involved in tracing an error could vary considerably between the various methods, and that might therefore be an important factor in determining the method to be chosen.

The author had shown that it was quite a straightforward matter to take the basic data in district and agency order, and to carry out the valuation by dealing with each policy individually and summing the results. Indeed, that method had a number of advantages. First of all, it meant using a file which was being worked continually for renewals, etc. and the data were likely to be more accurate than under the conventional continuous system whereby, if a mistake were made, it was liable to be perpetuated. Secondly, by valuing each policy individually the correct age at maturity could be used for every endowment assurance. The problem of obtaining an average maturity age therefore disappeared, and Lidstone's Z-method and the fixed-maturity-age method might well become obsolete. Furthermore, although it had not been shown in the paper, it was quite a simple matter to make the correct allowance for frequency and incidence of premium payment in respect of every policy. That was a point of topical interest since there seemed to be an increasing tendency for premiums to be paid monthly by banker's order which, if continued, would substantially change the average frequency of payment of premiums, which in turn could affect to some extent the valuation liability, as new business was not often spread uniformly throughout the calendar year.

In §100 there was a brief reference to the saving produced by the computer. He knew that it was difficult to cost some aspects of the work accurately but it was essential in running an expensive machine such as a computer to carry out as accurate a costing as possible. In a commercial concern a fast and accurate return of data was an effective aid to stock control, and that could substantially reduce costs. In a life office, however, the production of a surrender value in fifteen seconds rather than fifteen minutes, or the completion of a valuation on the 5th of a month rather than the 15th produced no real saving. It seemed to him, therefore, that the only real justification for introducing a computer into an office was that it would reduce management expenses. It would have been interesting if the author had estimated the cost of valuation by the new method and compared it with the old, allowing, of course, for depreciation and maintenance of the computer. He would have expected the saving to be substantial, bearing in mind the fact that little or no allowance need be made for time spent in obtaining the valuation data under the computer system since the data were taken straight from the file and
would automatically be up to date if the renewals were being processed. On the other hand, under the old system a considerable amount of time was spent bringing the actual records up to date before the valuation could be started.

In §4 the author mentioned the initial cost of a computer and went on to say: 'The computer configuration an office requires, and hence its cost, will therefore largely be determined by the most complex of the tasks it will be required to do.' He did not agree with that statement. The most complex tasks a computer would be called upon to perform might be the annual valuation of the life branch, the valuation of a pension fund or perhaps the calculation of premium rates for special policies. Those were all jobs which were carried out infrequently. A computer of any size could usually carry out that work, although a small machine might need to process the data severeral times before it could produce the desired results. Since, however, the work of that nature was infrequent, the time spent on it was relatively immaterial and an office was not justified in purchasing a larger machine just to carry out those functions. If the mechanized work of a life office were assessed in terms of computer hours used, then the bulk of the work would usually be data processing rather than calculation. He would have expected most of the work to have been taken up by the production of renewals and updating the file. He felt, therefore, that the choice of a computer should depend to a large extent on its ability to handle data rather than on the power of its central processor.

In considering the type of machine that should be installed, an office might be faced with the problem that its work could not be handled by a small machine on a 40-hour week. Two possibilities then arose. Either the office could look for a larger and faster machine or it could introduce shift working. If shift working were introduced it meant that high salaries had to be paid because of the awkward hours involved. Furthermore, it might be essential to keep a programmer available in case a programme ran into trouble; but, although the salaries paid in that way might seem high, they could be dwarfed by the very substantial saving obtained by stepping up the output of the computer. As a corollary to that, there was no doubt that time wasted on the computer was very expensive. For an efficient system it was essential to have staff of good quality, especially good systems analysts and programmers. If a programmer could save just 2½ minutes on one programme used in a daily run he was probably saving the office about £100 a year on a system of the size described in the paper; and the saving would be much larger on a larger machine.

He agreed with the author's statement in §10 that it was a good idea to start with a small system, but he thought that magnetic tape was probably the medium of the future and would prefer to have a tape system installed as soon as possible in order to gain experience. The cost of adding tapes on the IBM 1401 system outlined in the paper would probably add about 40% to the initial cost but it created a much more powerful system and substantially reduced data processing time. In §14 there was a description of the system used for calculating bonuses. That system, however, seemed rather complicated. For an office declaring quinquennial bonuses or an office using a tape system it would be easy to keep the actual bonuses vested on the file but in that case he supposed that it had not been found worth while to keep the vested bonuses on the cards as it would mean reproducing the whole file every year.

He did not propose to comment on the valuation formulae set out in §20 since they would vary from office to office according to the methods of grouping and the assumptions made in respect of age. There was, however, one general point of importance and that was that the formulae derived in the actuarial textbooks were designed to save calculation and involved a large number of different types of factors. To store all those
factors in a computer took up space and it was more economic to store only one set and let the computer derive the other factors required. That new requirement would tend to make parts of the standard textbooks obsolete.

In §38 the author had illustrated a technique which enabled a small computer to handle quite complicated calculations. In the main programme, if the computer could not calculate part or all of the function required in the first run, the data could be set aside by punching it out on a card which would be put aside until the first run had been completed, and then fed in again and the remaining calculations finished off. By that method, a small machine could handle complex jobs, although it could be a relatively slow process. The method was, however, much more effective if tape units were available.

It was suggested in §48 that when a revision of premium rates was required a programme was written and the values calculated. He did not think it was even necessary to write a programme every time the premium formulae were changed. A general programme could be written to calculate a formula of the form, \( \frac{a \cdot b + c \cdot d}{e \cdot f + g \cdot h} \), so that in practice the formula could be much more complicated. To calculate the premium the actuary specified the values of the constants. Those were punched together with the commutation functions at the desired rate of interest and fed into the computer to produce the new rates. That meant that several sets of premium rates could be given to the actuary to examine, without any necessity to change the programme even if there were variations in the premium formulae.

In §60 there was a description of the method used for dealing with surrender applications in the Industrial Branch under which each application was treated as an actual surrender. That meant that when the valuation was carried out the 'in force' would be understated by the number of surrender applications received but not accepted. He would expect to find that about 15% to 20% of surrender applications in the Industrial Branch did not result in a subsequent surrender. The average surrender value of an Industrial Branch policy was, of course, low and the effect on the liability might possibly be ignored in practice, but he personally did not like undervaluing any section of the business. Furthermore, the number of surrender applications substantially increased just before Christmas so that, at the valuation date, the number of outstanding surrender applications would be at a peak.

Another change from the existing method was demonstrated in the section on exposed to risk in §§70-75, where it was shown that a return to first principles was often the best way of carrying out a process on the computer. Again it would have been interesting if the author had given a comparison between the cost and the time spent on the new and old methods of obtaining the exposed to risk and the net saving involved. Of course, that description showed only one way of obtaining the exposed to risk but the method could be just as easily applied when other assumptions were made in respect of the ages involved.

Most of the work so far discussed had involved the transferring of current actuarial work on to the computer but he thought that in future computers might lead to a fundamental change in actuarial technique, particularly in respect of valuation. Existing methods of valuation produced a result at one point in time. In future, it might be possible to ascertain what sort of bonus would emerge year by year if their assumptions as to future experience were realized, and then to show what would happen to the bonus if there were a variation in one or more of the factors affecting the experience. Of course, there were a large number of factors that would affect the future bonus, and the permutations involved could be so numerous that to interpret the results the actuary might
need another computer—a much larger one! But it was possible to go a stage further and ask in what circumstances would the current rate of bonus have to be reduced. If the computer could be programmed to supply a concise answer to that question, then not only would the actuary have information which would be extremely useful in deciding what the current rate of bonus should be but also—and what was probably more important—he would be warned of the most likely dangers for which to look out. A considerable amount of research would be necessary before those techniques would be of practical use, but since the vast amount of calculation required for that type of work could be quickly performed by the computer, and, furthermore, since that work could be done at off-peak periods, it seemed likely that in future actuaries would be able to obtain a much better insight into the future prospects of the office at very little extra cost.

The introduction of a computer, even a small one, meant that the work of the actuarial department, as far as premium quotations, surrender values, valuation, etc., were concerned, would be taken over by the computer. The work upon which most actuaries in the United Kingdom had been brought up would probably be done in future on the computer and, quite obviously, similar experience would not be available to future students. Furthermore, knowledge of the methods of operation of the computer might be important background, in much the same way as it was essential for an actuary to know the working of the commutation functions of the life table. Future new entrants to the profession might start their careers in the programming section of the computer department, progress to systems analysis and then transfer to what was left of the actuarial department. Since a large part of the work of the office would concentrate round the computer, it would give a student a good background knowledge of the whole working of the office which would be of considerable use when, as an actuary, he came to deal with the administrative rather than the actuarial problems of the office.

Mr F. E. Guaschi's experience lay with a computer very similar to the author's, although his work was concerned mainly with valuation in a re-assurance office. It certainly was a feature of small computers that programming became a test of a programmer's ingenuity, especially when a complicated job was being handled. As the author had pointed out, there was room to store only one column of commutation functions and it was therefore important to choose that function carefully; \( N_x \) ultimate took up 810 positions of storage, nearly one-quarter of the total. In the end he too had chosen \( N_x \), although their valuation basis demanded the calculation of future bonus factors. That meant that \( S_x \) was also a possibility, but it was decided that the functions for the calculation of simple factors for valuing the sum assured and premium would become unnecessarily complicated. In a re-assurance office, not only were there more varied policy classes than was the case with the ordinary direct office, but there were also the problems of differing rates of bonus and a large number of currencies. It was therefore impossible to produce a valuation result in one run, or even to calculate the net premiums at the same time, so the net premiums were fed in at the same time as the new business tabulation was being prepared. There were a few classes where the net premiums were calculated by hand but that task was considered to be an interesting exercise for actuarial students.

The author had mentioned that two runs were required for a valuation in the form required by the Fifth Schedule to the 1958 Regulations and it was probably there that the critics pointed to one of the shortcomings of the computer. But even two runs were within the limits of tolerance when considering times and costs and in his office three runs were
required mainly to cope with the varying rates of bonus. The main valuation, however, was performed in one straight run with a punched output for each policy, showing the valuation details. Because the net premiums were already on the cards it was possible to value nearly every class of policy by the one programme. The other runs were just concerned with conversions into sterling, the bonus question and the calculation of expected death strains. Any portfolio had to contain a certain number of altered classes and where those were standard types, such as where the option had been exercised to convert a whole-life policy to an endowment after five years, it was a simple matter to programme for that; but there would still remain a number of policies which, however few in number, could be troublesome. He knew of some offices where the number of such cases could reach awkward proportions and it would be interesting to hear how the author coped with them in his valuation. As the author implied, the small computer forced the actuary to consider his valuation formulae in terms of one function, for example, $N_x$. That could be found to be very revealing, since very interesting patterns emerged according to which type of policy was being considered. The suppression of an $N_{x+n}$ could transform an endowment assurance factor into a whole-life factor. The surprising thing about the small computer was what could be done with it rather than what could not be done with it.

Mr T. M. Springbett mentioned two points primarily of an actuarial nature. In his comments on the valuation of paid-up and limited-payment policies in §29 the author had pointed out that the constant $P_x - \pi_x$ could not be used as there was not sufficient room for it on the card, and he had proposed instead an approximate method of valuing the future loading required. That called for manual adjustments after the valuation had been completed—a procedure to be avoided if at all possible—and it also made the accurate calculation of the expected release of reserves difficult. There was no need to store the constant $P_x - \pi_x$ on each card as tables of the current loadings could be stored in the computer and the appropriate values selected when required. For endowment assurances the loadings were sensitive to the original term of the policy but were comparatively insensitive to changes in maturity age. It was therefore possible to use loadings for a fixed maturity age for that purpose rather than a full double-entry table of endowment assurance loadings. The simple procedure would then be to value the gross premium, if any, by the appropriate annuity value and to reserve as well the whole-term loading for the whole term of the policy. The fact that in the past there had been different scales of premium rates, and might be again in the future, was a difficulty but that could be overcome by carrying a constant equal to the difference between the loading on the original premium basis and on the current basis. A similar principle could be used also for the valuation of altered policies, the gross premium being valued and the whole-term loading reserved. It was only necessary to code such policies to show that they were altered policies, as an indication to the computer that it should take the appropriate action. He was not very clear how the author valued his altered policies but he took it that item (f) of the information listed in §13 had some connexion with the valuation of such policies.

The author did not touch on the question of the valuation of annuities, but for some offices, especially those transacting group pension business, immediate annuities were an important but rather troublesome class of business to value accurately. The situation was considerably eased with modern computers which were able to value each case individually and there would therefore no longer be any need to value certain classes of annuities by approximate methods—for instance, joint-life and last-survivor annuities.
The complications of immediate annuities were, however, considerable. A joint-life and last-survivor annuity might be guaranteed for a minimum period of years and on the death of the husband during the guaranteed period the annuity to the widow might become payable immediately and not at the end of the guaranteed period, or there might be no duplication during the remainder of the guaranteed period. A further complication might be that the annuity decreased by, say, one-third on the husband's death or perhaps on the first death of either spouse. To cover all those cases by separate programmes would absorb considerable planning time and storage space, and it was desirable to have a comprehensive programme which would deal automatically with all such cases.

A rewarding approach was that used by Elphinstone and Lindsay in their paper to the Faculty as long ago as 1939 (T.F.A. 17, 44). They suggested that, for an annuity reducing on the first death, three amounts of annuity should be carried in the annuity data, and those amounts could be either positive or negative. The first was valued by the annuity value applicable to the male life, the second by the annuity value applicable to the female life, and the third by the joint-life annuity value. Thus, if the annuity were for £150 a year reducing to £100 on the first death, the three amounts of annuity to be carried would be +£100, +£100 and −£50. That principle could readily be extended to include annuities with a guaranteed period, either with or without duplication, by using four amounts of annuity instead of three, the fourth amount being valued by the annuity certain for the outstanding guaranteed period. A very flexible and workable system could thus be built up in a form well suited for computer working; it also had the advantage that the expected release of reserves could be calculated easily in all such cases.

Mr H. H. Scurfield thought that an exciting, though difficult, time had been reached for actuarial science. The arrival of computers in life offices meant that the actuary could obtain many more statistics both easily and quickly. The problem would be to know what to ask for and then what to do with all the figures produced. One of the great advantages of the computer would be in enabling the actuary to see very much more clearly into the future. Investigations which at one time would have involved a prohibitive amount of time, labour and expense would in future be done at the push of a button.

The first life assurance premium rates were very large because there was no experience to draw upon, and ample safety margins had therefore to be included. As experience was gained those margins had been reduced and, when the full power of the computer was made use of, he thought they would be reduced even further. That would only be the continuing process of evolution or another chapter in a book entitled Three Hundred Years of Actuarial Advance.

The author had mainly been concerned with adapting his punch card system to fit into the faster and less cumbersome computer. It was disappointing that he did not go further than that. In §17 it was implied that a copy of the state of the Ordinary Business cards as at the end of year was not kept. By keeping such a duplicate set of cards a lot of interesting work could be done when the pressure of producing the net premium valuation had passed. Valuations, both net and gross premium type, could be carried out on various bases and, to help in the consideration of the bonus earning power of the fund, the liabilities at the end of the previous few years could be revalued on new bases. Magnetic tape was ideal for keeping the old records because of the small amount of storage space required.

In §§95 and 96 the author had spoken of the problems facing actuarial students of the future. They were to be envied; no longer would they be faced with the calculation of
tables of premium rates and surrender and paid-up values, nor with the routine of valuation. They would have the more interesting problems with which the computer could not cope and the analysis of the statistics which the computer could produce. They would be fortunate enough to be brought up in the atmosphere of the computer and would not have to make the change from orthodox actuarial techniques. Existing actuaries could be consoled that they did not have to sit examinations on computer programming; but he did not think that students would regard that as a compensation for the removal from the syllabus of methods of group valuation.

The author had mentioned that point in §95 and had gone on to say, 'It will be many years before it can be assumed that the actuary will have a fast calculator available.' The speaker thought that that stage had already been reached; a computer could calculate and print out a complete set of with-profit endowment assurance premiums for all ages and terms in 58 seconds (and at the same time be doing something else). The only information input required was a series of values of \( I_x \), select and ultimate, a rate of interest, and bonus and expense loadings. In less than another minute the whole-life limited-payment premium rates could be printed. It was of interest that, since the rate of mortality at age 106 was taken as unity, the rate for premiums ceasing at age 106 was the throughout-life rate even though it was produced on the 'limited-payment' programme. Two such programmes meant that the effect of using a different mortality basis in the premiums could be tested simply by feeding in a new set of tables of \( I_x \); within two minutes the complete result would be produced and without the actuary having had to use a single commutation function. That he regarded not only as an advance but also as fast calculating.

Mr A. D. Wilkie, F.F.A., remarked that by keeping a main file in punched card form the author had given himself (or his staff) the trouble of doing all the necessary updating of the file by hand-punching and hand-picking of cards. There were quite a lot of opportunities for clerical error throughout that procedure and by using a card input and output system for the computer there were considerable further opportunities for errors in the handling of cards. He would be much in favour of a system which kept the main file on magnetic tape, as the opener had suggested, though he did not entirely agree with him that magnetic tape was the storage medium of the future. He would have thought that magnetic discs or data cell units were much more flexible for general programming purposes than magnetic tapes. The author had not mentioned how much time was required for setting up those systems but the speaker thought that the cost involved in the systems analysis and in the programming required was, for many extensive computer systems, just as great as the capital cost of the computer itself, and it would be interesting to know whether that had also been the author's experience.

All the information the author required for each policy was listed in §13 and he was very lucky to be able to get it all on one card. In the speaker's own case not 80 but 160 columns were needed to get the required information on to punched cards. He had noticed that there was no provision for such information as the year of cessation of premiums for an endowment assurance by limited premiums, nor for the date of last extra premium if the extra premium were payable for only a limited period. Without such data it was difficult to pick out alterations automatically. Maturities could be picked out, of course, and normal whole-life limited-payment cases, but there were other types of alteration that it was useful to get the computer to notify.

For the continuous mortality investigation the author succeeded in getting all his results into 1,000 registers. Using the same sort of basis of calculation, the speaker had worked out that he would need 4,800 registers; but perhaps the author had some method...
of obtaining different tables in successive runs by inserting the policies in duration order so that the table for each duration could be brought out and printed and the totals cleared before going on to the next duration. He also gathered from the paper that, for the internal mortality investigation, all the calculations of expected deaths, and of actual over expected deaths, had to be done by hand. Perhaps the author had some system for using the computer with summary cards to do those calculations, but that had not been mentioned.

He was particularly interested in the section of the paper on emerging costs. It was something to which he had given a little thought, for it was such a complicated task that the time taken was extremely long compared with any other job and he was not certain whether the benefits were commensurate with the expense. In the author's explanation of how an emerging costs valuation would be done he had simplified the calculations almost too much. For example, the very neat system mentioned in §90 of avoiding the use of $d_x$ by subtracting one year from the next all the way back did not cater for term assurances of any kind and it would seem that those had to be dealt with separately.

He also thought that, from an actuarial or managerial point of view, it would be useful to see what the expected claims by maturity were in each successive year and the expected claims by death. Those figures should therefore be given separately.

He agreed with the opener that it was highly desirable to have a large generalized programme to cope with any sort of premium calculation, but it took much more programme and programming time. It might therefore be more practical to write a programme for each particular set of calculations as it came up. The opener also suggested (based really on the emerging costs valuation) that actuaries should go ahead and consider the ways in which possible changes in future conditions could affect bonus rates, and that that could be done provided the computer could do the calculations. He was quite sure that if the actuary could ask the questions, the programmer could write the programmes, but it was necessary to be quite clear what the questions were before handing the job over to the programmer, who could not decide from non-actuarial principles what were the questions that the actuary should be asking.

Mr S. Benjamin said that the paper was almost complementary to a recent Students' Society paper, 'Maintaining Life Office Records', by A. J. Williams (J.S.S. 17, 522) and formed an interesting contrast.

The office under consideration clearly conducted both Industrial and Ordinary Business and its system for the latter seemed to be geared very closely to its agency system; for instance, presumably renewal notices were not sent out direct but through the agent. He thought that the system was generally simpler than that of most offices, both in the type of policies issued and in the handling of policies. The agents were well disciplined if, as was claimed by the author, the checking of the Industrial Business agents' accounts was easily adapted to the computer. It would be wrong if such an office did not make the maximum possible use of its agency system. The punched cards, on which there were no addresses, could be kept in agency order and that made agency accounting reasonably simple. When there was neither a single-card system nor comparatively straightforward numerical data, a card system was barely adequate and magnetic tape was the only alternative medium currently available. A magnetic tape system was more rewarding in the end but it produced its own peculiar difficulties which the author's team certainly did not have to meet. On magnetic tape, to start with, the movement could not be 'pulled out', and that really complicated the system beyond belief. The programming of variable-length items had to be dealt with, and much more sophisticated programming
and systems analysis were necessary. Those apparently small points made all the difference between whether an office could use a fairly simple system like the one described in the paper or whether it could not. He only knew of one other office (a London office) that used a similar machine and a similar system, and at the time that he was familiar with it they were using normal and inter-stage cards so that they could get 160 columns.

He was interested that one speaker mentioned the use of $S_x$ because he had always held the view that $S_x$ was the best function to use if $R_x$ was required, but he gathered that the author did not need it very often.

Rounding errors, which were mentioned in §68, were always troublesome on a machine. A system of dealing with them had to be worked out unless financial statements could be sent out that did not add up to the nearest penny. (It might be that for the good of the country they ought to cease worrying about odd pennies).

The data for the calculation of the exposed to risk were obtained by running the main file through the machine and that was clearly a case where the simplest approach of unit additions was the best.

The author's company had developed its own assembly system and he thought that it would be extremely interesting to all other IBM 1401 users since it could well be an improvement on the system supplied by the manufacturer.

The general method of processing files serially was straightforward for most actuarial work because the results would not be put back in to amend the file from which the data were taken. Unfortunately, the people who were dealing with the data processing itself often encountered the difficulty that if they were going to take the item out to deal with it separately, because they could not deal with it by programme at that point of time, they had to get it back in again properly before the next run. In practice 'before the next run' might mean 'during the next run' but inserting the data a few milliseconds before the main process; however, that was a considerable complication.

It was extremely useful to find re-start procedures mentioned by the author throughout the paper. One of the differences between a good and a bad programmer was that a good or experienced one started his programme by writing the re-start procedures and then wrote the rest of the programme round them.

It was worth trying to distinguish as clearly as possible between new techniques and new specifications of old techniques. Before writing a general purpose pension scheme programme it was necessary to define the different types of scheme and their properties by various parameters which had to be set before the programme was used and, the more schemes which were included, the greater was the number of parameters to be set. The problem then became one of trying to steer a general purpose programme to deal with a particular case, and after a time the steering programme became longer than the general purpose programme and the two together were not only completely unwieldy and inefficient but they also took a very long time to write. It might be an advantage to deal with that problem by specifying the pension scheme in terms of emerging cost; in other words, the decrements that were going to occur should be specified and so also should the mathematical formulae for calculating the amount of the premium or the amount of the appropriate benefit in each future year. In that way it was probably easier to specify the problem and to put it on to the machine. It required extra internal calculation, but machine speeds were going up. It was possible to buy time at 1d. for 20,000 instructions, as opposed to the author's machine which was of the order of about 1d. for 1,000 instructions, and the internal machine time did not cost much. But the specifying of the problem in that way was quite different from developing a new technique such as trying to discover what they would do if they had emerging costs. There was often
some confusion there, and certainly some people were busy writing programmes for emerging costs when they would do well first to decide what they were really going to do with the calculations when they had done them.

One thing that was a little worrying about small machines was that new techniques were unlikely to come from their use. It was only by experimenting with the big machines that the scope of things to come might be seen. With the small machines there was the perpetual worry of running out of registers, and satisfaction was often limited to the successful management of a job within the facilities available. Actuaries had to have computers because as a profession they should be numerate, and in the very near future numeracy was bound to include a very close knowledge of computers, almost certainly down to school O-levels. It would have to be included in the actuarial syllabus at some stage, but how much of it and when and how was another problem. The methods of teaching programming were under active discussion and there were no agreed methods at all. He felt that they ought to try to teach it in the way that some of the more progressive schools were teaching it to their sixth forms, through a high-level language which avoided the sometimes complex details of addressing various registers and got on with teaching what could really be done. High-level languages could only be used with a large machine and he thought they would all be much better off professionally if they were to experiment more with the large machines in high-level languages. There were high-level languages being developed by and for other people. There were none for actuarial work, which fell into the gap between pure data processing and pure scientific work, and as a result the languages being developed for both of those were not useful enough for actuaries. In his own office they were trying to deal with any type of problem that came along by accepting the fact that the programming language most suitable for actuarial work was the one already in use—"I want column numbered m multiplied by column n as far as age z and the result summed from the bottom up"—and so on. In that way, like many other people, they had taken an existing high-level language, because it was easier to write in, and had superimposed their own language on top. It was a fairly small system but it did seem to be worth it since the alternative was to spend something like £50,000 as a minimum on developing a compiler. It might be that for the future they ought to look, both from an office and a professional point of view, to offices having their own small machines, but actuaries should be much more readily prepared to take their jobs to the big machines and to hire time. It was very cheap to hire, between £350 and £750 an hour, and 15 seconds on one of those machines could produce a great deal of work.

Mr R. C. Gilder's first thoughts on a programme for a pension fund had been on the lines of the author's formulae in §82. However, he had come to the conclusion that there was still some virtue in the commutation column technique, at least in functions of the Cz type. Perhaps he should apologize for even considering the use of old techniques in the age of the computer, but they could save a very large number of multiplications, and multiplication still took considerably longer on those machines than selecting a number from a store. For example, in the author's formula (2) (a) if the product of the first five terms, substituting vy+t for vt were calculated and stored for every age, it appeared to him that up to 160 multiplications would be saved for each card on that formula alone. The author had considered only the principal benefit in a pension fund. Had he wished to programme for all the usual benefits, including those to widows, and perhaps also some unusual ones, many more multiplications would be saved. Moreover, it seemed to him that probably no more storage space would be needed than was used by
the author, because the stores containing the rates of decrement (which the author
retained throughout his programme) could be over-written by the $C_x$ functions which
the computer calculated. Probably the author had considered that approach and re-
jected it and if so, it would be interesting to hear why.

The author's device of introducing a pension fraction, $f_{k+t}$, into the $C_x$ function,
had, however, considerable advantages over trying to work in $M_x$ and $R_x$, in that com-
plications such as waiting periods and limitation of pensionable service could be more
easily programmed. By calculating $f_{k+t}$ in a distinct sub-routine it should be possible
to write a general programme suitable for a number of pension funds, even if the sub-
routine had to be altered for each fund.

Mr P. W. Sharman, in closing the discussion, said that the provision of life assurance
cover, combined with investment facilities, ought on the face of it to be a simple matter,
but the industry seemed to have succeeded in building around it a highly complicated
administrative maze, and, of course, each company had its own pet complications, so
that some computer programmes for life office administration were longer and more
complicated than those for any other industry. The difference in the methods employed
by different companies was being accentuated by the great variety of computer equip-
ment which had been installed or ordered during the previous few years. A survey had
been made of 25 leading offices which had ordered computers and it had been found that
the 25 orders included 16 completely different models of computer made by 7 different
manufacturers. Each company was, of course, trying to do the same basic job and each
company believed that it had made the right choice, but inter-company comparison of
methods which had formed much of the subject matter of the discussion became in-
creasingly difficult. All that was perhaps not surprising in a competitive industry operat-
ing in a free society, but one point he wished to make was that, however much they
talked about sweeping away old methods and bringing in new, and however genuine
they were in their intentions in that respect, they were severely limited by their individual
office tradition and background and by the complicated nature of some of the existing
business which they had to continue to administer. That background, in turn, had con-
siderable effect on the size and type of the equipment used.

He agreed with Mr Benjamin that, from the evidence in the paper, the author seemed
to have been more fortunate than most in the simplicity of the administration he had
inherited, or at least he had succeeded in avoiding many of the difficult situations. For
instance, a major simplification was the fact that the author did not have to concern him-
self with policyholders' addresses, and, although, it might seem odd to some to suggest
that the choice of method had been influenced by that fact, he did admit in §7 that, had
he been concerned with addresses, he would probably have decided in favour of a mag-
netic tape system. Had the paper been based on a magnetic tape system it would cer-
tainly have looked very different.

He had the impression that a note of regret had been sounded by some of those
speakers who had touched on the need to look again at basic principles, and who some-
how felt that it was a retrograde step to value policies individually rather than in groups.
The opener had mentioned in that connexion that parts of many of the existing text-
books might rapidly become obsolete. There was no place for sentiment in the change to
computer methods and, as Mr Guaschi suggested, if anyone was looking for mathemat-
ical exercises he would find them in plenty when he tried to get a quart size programme
into a pint size computer store. He felt that, as actuaries, they should take comfort
in the increasing scope which computers provided for the application of actuarial
The Use of a Small Computer in methods over a wider field of life office administration than heretofore. He had in mind particularly subjects such as expense analysis and budgetary control, and the provision of scientifically based management statistics. In a number of composite offices actuaries were taking a leading part in the development of computer systems, with the obvious opportunity to extend their spheres of influence.

As for the training of actuarial students, he agreed very much with the author that the student working in the computer department would have a much better opportunity of becoming familiar with the 'complete life office practice', as it might be called, rather than with the 'actuarial life office practice'.

Mr Benjamin had raised the question of punched cards v. magnetic tape systems, and like him he had expected that subject to receive more attention in the discussion than it did. In fact, the author had rather laid himself open to criticism by his comment in §101 that little thought had been given to the use of a more powerful machine, although, to be fair, the order for his computer would have been placed some four years earlier, when considerations and prices may have been considerably different. He thought it unlikely that an office would currently embark upon a system that was not planned ultimately to lead to access to policy records at least once a day, and he believed firmly that substantial savings in cost would not be made until that was done. He did not intend to develop further the arguments for or against using a punched card computer system as an intermediate stage before going to magnetic tape or some other more comprehensive system, as Mr Wilkie had covered that point adequately. However, he did think that the transfer from a card computer to a larger machine was at least as difficult as the initial change that the author had already made.

He would have liked to have seen investments mentioned amongst the main functions of the Ordinary Branch in §11. He did not foresee the early use of computers in connexion with the daily administration of a life office portfolio—not, at least, until there was a machine capable of reading the Stock Exchange Daily List—but there was an excellent computer application in the regular analysis of a large portfolio, and when the author got more deeply involved in the study of emerging costs he would find it essential that the computer be used to evaluate income from interest, dividends and capital profits.

Several speakers had mentioned the questions raised in the paper about the formulae to be used; the author relied on $N_x$ in every case. That was very much a function of the size and type of equipment being used. No doubt the author had made the right decision in his own case, but the speaker was personally more concerned with a computer which had about thirty times the size of store of the author's, and he was also much more concerned with time than the author was. Therefore in general his approach would be to include in his store as many functions as were necessary, e.g. $N_x$, $D_x$ and $S_x$, rather than to spend valuable computer time in calculating them from some of the more extensive formulae used by the author.

The opener mentioned the fundamental changes of technique that were taking place, and Mr Scurfield mentioned the fundamental changes in the statistics which would be produced in the future. When the more extensive statistics became available in future, it was possible that the figures might suggest that their bonus distribution methods were inequitable and they would be forced to make changes. Mr Scurfield thought that the author had not been particularly ambitious from the point of view of making use of new techniques, and personally he agreed that, as a profession particularly concerned with probability and statistics, they were not yet being very ambitious in that direction. However, the situation should be looked at through the eyes of the
actuary who was trying to organize and manage a new computer department. Everywhere the pressure was on to try to reduce costs, and generally speaking the savings to be made on the actuarial side of the office were less than those to be made from the application of the computer to premium collection and accounting processes. That was evidenced by the author's experience that only 5% of computer time was taken up by the actuarial side of the process. Whether as individuals they liked it or not, it was inevitable that at the early stage of development the emphasis should be placed upon those applications where savings could be most quickly realized, but there was another point. Before they could adopt refined statistical techniques they had to have the data which were to be the subject of analysis in a form easily digestible by the computer and many of them were still trying to achieve that. He certainly thought that in setting up their records, either on cards or magnetic tape, they should endeavour to include all the information which they felt would subsequently be required for analysis, even though it was not all necessary for their immediate day-to-day administration. He agreed with Mr Wilkie in thinking that the author's system was too restrictive in that respect. Whilst the author had certainly achieved the desirable objective of getting all information necessary for his computer operations on to one 80-column card, he had obviously had to cut his information down to the bare essentials, and so had limited scope when it came to wider statistical investigations. One of the biggest restrictions of the system described arose from the use of a one-card file which was in a constant state of flux, so that it was available for valuation purposes for only a few days each year. With conventional punched card systems, where the valuation cards were usually independent of cards used for accounting purposes, the cards were usually available for investigation purposes generally, and for bonus reserve valuation purposes in particular, for several months in the year. One of the long-term advantages of magnetic tape systems would be the ability to retain the computer end-of-the-year records in processable form together with annual movement, so that the investigation of trends over long periods would be possible retrospectively using census methods.

The President (Mr H. Tetley, C.B.), in proposing a vote of thanks to the author, said that it was a commonplace that everybody dealing with electronic computers was constantly craving for bigger and better machines, and, above all, for machines with bigger storage capacity, particularly of the quick-access type. Obviously a great deal depended on what was available and what could be afforded, and it was rather interesting that in his department, where those computers had been used, it had been found that, with the limited resources available, \( N_x \) seemed to be the most appropriate commutation function to use. He had noticed that G. C. Phillip, in a paper delivered to the Faculty in 1957 (T.F.A., 25, 225) came down in favour of the same function. There was a danger that the impression might be given that \( N_x \) had some particular property which made it obviously stand out as the most suitable. He was not at all sure that it was. If, as Mr Sharman indicated, the machine was an extremely elaborate one in which many other functions could be stored, well and good; but he remembered very clearly that the first time he came across electronic computers was when the Institute and Faculty were preparing tables based on the \( a(55) \) experience of last-survivor annuities and joint-life annuities, and the data fed into the machine were of the very simplest type possible. There was the \( l_x \) column for male lives, the \( l_y \) column for female lives, and values of \( v \) at various interest rates, but nothing else. The machine did all the rest of the extremely complicated calculations at fantastic speed; in that context and with that particular machine, that was the best way to operate.
He was rather envious of the speed of the author's machine, which apparently handled about 48,000 cards an hour. That struck him as a very high speed for a relatively modest machine. One of the machines his office used (not their own machine) was technically supposed to be able to handle 12,000 cards an hour, but it had been found that in order to keep down the number of cards wrecks (cards slightly damaged so that they were not reliable in the future) and the number of misreadings, the speed had to be reduced to something like 7,500 to 8,000 an hour; that was very slow by modern standards.

With regard to the calculation of exposed to risk, his department followed (subject to some fairly considerable modifications of detail) much the same programme as the author; but they had found it most convenient to use central exposed to risk. That had the advantage that all exits and entrances (on the usual assumptions, at any rate) could be treated alike as taking place in the middle of the year, and he hardly needed to point out that that greatly simplified the subsequent manipulation of the functions if independent probabilities or a multiple decrement table were to be derived.

He congratulated the author and all those who had taken part in the discussion on having avoided two great dangers of discussions on electronic computers. It was all too easy to deal in generalities which usually involved platitudes and got nobody very far. At the other extreme (for it seemed to be difficult to steer a middle course) it was very easy to go into the kind of detail which applied to one particular machine but to no other. They were all familiar with papers of that type, which were of great importance to the person using that machine but of very little use to other operators. The paper itself and the discussion that evening seemed to him to have been admirable in the way they had gone into enough detail, without introducing too many points peculiar to one particular machine.

The author made a brief acknowledgment and subsequently wrote as follows:

The subject matter of the paper was carefully limited to the changes in actuarial methods arising from the use of a small computer because, as the President mentioned, it is easy to make generalizations which are of little real value. My self-imposed limitations were, however, severe and I was not surprised to find that many speakers drew attention to subjects they would have liked to have seen included in the paper.

A number of speakers referred to the matter of cost. We have made several attempts at calculating the cost and savings of the installation and on each occasion we have found many factors which are almost indeterminate. For example the demand for space at Chief Office was such that ideas were being put forward for additional buildings. Since the installation of the computer the number of staff has decreased and space has become available. Had we not had a computer we do not know whether we would have, by now, had to increase the office space or what the cost would have been. Not all the saving in staff can be attributed directly to the computer although its presence has meant that many office systems have been looked at with a view to using the computer. In some cases this has resulted in the system being revised, without the use of the computer. The cost of programmers is difficult to assess because the programmers have, in the main, been drawn from Chief Office staff and have not been replaced there. We have two persons engaged in planning systems and they are supported by four programmers.

In all our tasks we have checks on the card input and the output. It has been interesting to note the number of errors in the manual system shown up on conversion to the computer. Our checks have also brought out a number of errors on the computer system. We believe them to be smaller in numbers and magnitude than those that existed under the manual systems. There are, no doubt, some errors which are as yet undetected.
Mr Wilkie referred to the trouble of updating a card file and the possibility of error. Unfortunately my experience is limited to a card system. As I mentioned in the paper the cards are checked by the computer as far as possible and I cannot appreciate how a tape system would be any less liable to error. In fact, we have found that policies are often misquoted, particularly in the Industrial Branch where there are often several policies on the same life. It is quite possible that manual methods employing experienced filing clerks are more reliable than a tape system—bearing in mind that any updating starts in the form of manual input at some stage. In fact we have had less trouble from card handling than we had expected.

As I stated in the conclusion, we are satisfied that our choice of machine was the correct one at the time. This must be read in conjunction with many factors, including the machines on the market at the time we placed our order, the experience available at that time in the use of computers, and the size of the office. Even at the present time our capital outlay in computer equipment as measured against our premium income is relatively high. We are sure, however, that had we had a substantially more powerful machine we would not have covered as much ground as we have.

Referring to the more detailed points raised in the discussion, I would mention that in our office the proportion of Industrial Branch surrenders not taken up is negligible. This may be because the cases are vetted carefully both locally and at Chief Office before a quotation is made. The calculation of surrender values has not yet been transferred to the computer because certain administrative problems remain outstanding.

The proportion of difficult cases requiring manual valuation is about 3 per thousand. These are mainly risk-premium or temporary assurances—often with variable premiums which are nearly all anticipated and the values calculated manually in advance.

Mr Springbett drew attention to the calculation of the extra reserve for limited-payment policies. In fact it is not possible to store any additional factors in the computer due to the severe limitation on the store. The number of limited-payment endowments in the portfolio is negligible. Endowments payable in instalments are valued for a reduced cash sum at the end of the premium term. The calculation of the extra reserve could, of course, be programmed but as the calculation takes about 15 minutes each year it is not likely to be worth while in the lifetime of the present machine. Altered policies are dealt with by including in the card as a reduction the difference between the revised premium and the premium that would have been charged on a new policy effected to mature at the revised date. The net premium is calculated proportionately.

Mr Scurfield referred to the need for a record of the position at the end of the year. A record of all movement is kept so that it is simple matter to obtain the year-end position by valuing the cards at any date and ‘jobbing’, backwards by valuing the ‘ons’ and ‘offs’. Before any such action is taken, however, it is important to decide why we should do this and what purpose it will serve. With regard to the matter of actuarial training, I do not think we can assume a computer is always available because we must bear in mind that a number of actuaries are working overseas where computer facilities may be very limited. In any event a substantial capital cost is involved in setting up a programme and, if the task is only a small one, such as valuing a Friendly Society, it is unlikely to be economic.

Mr Wilkie referred to the mortality investigation requiring 1,000 registers. Actually the figure is 1,794. The total possible number of registers is much higher but, in order to fit the task within the capacity of the computer a careful examination was made of the distribution of ages. It was found for example that a range of ages from 15 to 70 was adequate for select durations; the few cards outside this range were reproduced.
It is certainly a practical proposition to use the computer to calculate the expected deaths from the punched output of the store at the end of the run. This, like so many other tasks, remains to be done. We have found, however, that the expected death strain is a more valuable figure and the actual mortality experience has been given low priority.

Mr Gilder referred at some length to the method used for dealing with a pension fund. In this connexion it must be borne in mind that select withdrawal rates are involved. If the number in the fund were large it might be more efficient to prepare commutation columns.