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AN EARLY BOOK ON COMPOUND INTEREST

RICHARD WITT'S *ARITHMETICALL QUESTIONS*

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ONE of the highlights of the year 1613 was the burning down of the Globe Theatre in London, during a performance of Shakespeare's newly-written 'Henry VIII', when the discharge of a piece of ordnance set fire to the thatch.

Another event which took place during the year was less spectacular but, to actuaries, just as noteworthy. This was the publication of a book by Richard Witt entitled: *Arithmeticall Questions, touching The Buying or Exchange of Annuities; Taking of Leases for Fines, or yearly Rent; Purchase of Fee-Simples; Dealing for present or future Possessions; and other Bargaines and Accounts, wherein allowance for disbursing or forebeareance of money is intended; Briefly resolved, by means of certain Breviats.* (See Title Page in Figure 1 on facing page).

There do not seem to be any references to this book in actuarial literature (though it is mentioned by De Morgan(1) and Attwood(2)) and it is doubtful whether many actuaries have previously been aware of its existence. Nevertheless, it is a pioneering work in a subject which forms one of the foundations of actuarial science—compound interest. The author should not be confused with the Dutch Prime Minister, Johann de Witt, whose work on life annuities was carried out nearly 60 years later.

The book delves deeply into the subject in a very practical way, and it is evident from the clarity of expression and the care which has been taken that the author thought in much the same way as modern actuaries. Nothing seems to be known about him, except that he is described on the title page as 'R.W. of London, practitioner in the Arte of Numbers'. We know that he was alive in 1613 because the title page states that the work was 'examined also and corrected at the Presse, by the Author himselfe'. (In those days the author went along to the printing shop and examined the first sheets coming off the press; any corrections were made to the type there and then.) He died before 1634, when a second edition of the work was published. This contained additions by Thomas Fisher, who says in his introduction:

> I shall not need to use the help of words to expresse the praise of the first Author of this Work: the paines & Art therin deciphered being sufficient to manifest his desert and worth; who though being dead, yet (his fame) liveth, and though it alive, yet (by change of times and customes) the Book almost forgot and out of use . . .

In later years the book was completely forgotten. For example, in 1808

The Breuiat of the Table of 10. per Cent.

Yeares.		Yeares.	
1	11000000	16	45949729
2	12100000	17	50544702
3	13310000	18	55599173
4	14641000	19	61159090
5	16105100	20	67274999
6	17715610	21	74002499
7	19487171	22	81402749
8	21435888	23	89543024
9	23579476	24	98497326
10	25937424	25	108347059
11	28531167	26	119181765
12	31384283	27	131099941
13	34522712	28	144209936
14	37974983	29	158630929
15	41772481	30	174494022

These Numbers in this Breuiat must also be esteemed Numerators, each of them hauing for Denominator 10000000.

Now follow the Questions: In working whereof, we will vse this Breuiat.

The

The firſt Queſtion, being an example of the firſt Direction.

If 1^{li}. *be put forth at Intereſt after* 10. per Cent. per Annum, *Intereſt, and Intereſt vpon Intereſt, for* 30. *yeares; Vnto how much will it amount by the end of that time?*

BEcauſe the time in this Queſtion is 30. yeares, looke in the Breuiat next before ſet downe, for the thirtieth number; which you ſhall finde to be 174494022. from this cut off 7. figures, beginning to tell from your right hand towards your left, & then it will ſtand thus: 17|4494022. The 17. that ſtandeth on your left hand is 17^{li}.

Now Multiplie 4494022. (the 7. figures cut off) by 20. and the product will be 89880440. from this alſo cut off 7. figures, and then it wil ſtand thus 8|9880440. The 8. on your left hand is 8 ſh.

Now Multiplie 9880440. (the figures laſt cut off) by 12. and the product will be 118565280. from this alſo cut off 7. figures, and it will ſtand thus 11|8565280. The 11. on your left hand is 11 d.

So haue you found, that if 1^{li}. be put forth at Intereſt after 10. *per Cent. per Ann.* Intereſt, and Intereſt vpon Intereſt for 30. yeares, it will amount by the 30. yeares end, vnto 17 l. 8 ſh. 11 d.

The Worker

Facit	l 17		4494022
	ſh 8		988044
	d 11		856528

The

Francis Baily wrote in the introduction to his *Doctrine of Interest and Annuities:*

> The first express treatise on the subject of Interest and Annuities, that I have been able to find, and that pretends to any degree of merit, is Ward's *Clavis Usurae* published in the year 1709. It had, however, been handled by several authors previously thereto.

2. After a dedication and introduction, the book begins with a table of $(1+i)^n$ at 10%. Decimals were not then in use and the user of the table was expected to 'cut off' the seven right-hand figures (see Figure 2).

This table is then used in various worked examples to obtain v^n, $s_{\overline{n}|}$ and $a_{\overline{n}|}$ as required, by means of the formulae:

$$v^n = 1 \div (1+i)^n$$
$$s_{\overline{n}|} = \{(1+i)^n - 1\} \div i$$
$$a_{\overline{n}|} = \{(1+i)^n - 1\} \div i(1+i)^n$$

The use of these formulae shows that the process of compound interest was clearly understood.

After some worked examples the author gives tables (or 'breviats') for v^n, $s_{\overline{n}|}$, $a_{\overline{n}|}$ and $a_{\overline{n}|}^{-1}$ at 10%, which was the main rate of interest then in use for financial transactions not involving land. These tables, of course, avoided the tedious calculations resulting from the formulae quoted above—there being no calculating machines.

Other rates of interest were also in use, for the author then goes on to give tables of $(1+i)^n$ at 9%, 8%, 7%, 6% and 5%. The other functions are not quoted, however—a lack which was evidently felt by at least one reader, because the British Museum has a copy of the book in which there have been inserted contemporary manuscript tables that give the missing functions at length.

Apparently it was common in those days for freehold property to be bought on the basis of 16 years' purchase, and Witt goes on to give tables of $(1+i)^n$, v^n, $s_{\overline{n}|}$, $a_{\overline{n}|}$ and $a_{\overline{n}|}^{-1}$ at $6\frac{1}{4}$%.

An extremely interesting aspect of the book is that it deals with half-yearly and quarterly payments. First there is a table of $(1+i)^{n/2}$ for odd values of n and a table of $2s_{\overline{n/2}|}^{(2)}$ for all integral values of n.

This is followed by a table of $(1+i)^{n/4}$ for odd values of n and a table of $4s_{\overline{n/4}|}^{(4)}$ for all integral values of n. These tables for half-yearly and quarterly payments are given for annual rates of interest of 10% and $6\frac{1}{4}$%.

It is clear that the author fully appreciated the difference between interest at 5% p.a. and at $2\frac{1}{2}$% per half-year. To answer questions based on a rate of interest 'such as men have when they buy land at 20 years purchase, and receive halfe the years Rent at the end of each halfe yeare', there are tables of $(1+i)^n$ and $s_{\overline{n}|}$ at $2\frac{1}{2}$% per period. Similar tables are given at $1\frac{9}{16}$% per period for interest based on 16 years' purchase with quarterly rents. It is not known why there are no tables based on 20 years' purchase with quarterly rents or based on 16 years' purchase with half-yearly rents.

3. Having described the tables which are included, we now come to the 124 worked examples which are such an interesting feature of the book. A selection from these is set out below.

Q. 70 One oweth £900 to be paid all at the end of 2 yeares: he agreeth with his Creditor to pay it in 5 yeares, viz. every yeare a like summe. They demaund what each of these 5 payments shall be, reckoning 10 per Cent. per Ann.int. and int. upon int.

The solution is obtained from the expression:

$$900\ v^2 a_{\overline{5}|}^{-1} = £196.\ 4s.\ 3d.$$

which shows a knowledge of a fundamental actuarial concept, the equation of payments.

Q. 72 The Mannor of Dale is out in Lease for 7 years at £10 per Ann. Rent; which Mannor by survey is found to be worth £80 per Annum. This Mannor is to be sold at 16 yeares purchase, according to the said rate of survey: what shall be paid, reckoning such interest as men have when they buy land at 16 yeares purchase, and receive the Rent quarterly?

The solution is obtained from the expression:

$$\{80 \times 16 + 2\tfrac{1}{2} s_{\overline{28}|}\} \div (1+i)^{28} = £885.\ 11s.\ 6d.\ (\text{at } 1\tfrac{9}{16}\%)$$

Q. 73 Sept 1 Anno 1612, A did owe to B £1200 to be paid in manner following:

£200	Sept 1	Anno 1613
£200	March 1	Anno 1613
£200	Sept 1	Anno 1614
£200	March 1	Anno 1614
£200	Sept 1	Anno 1615
£200	March 1	Anno 1615.

Of these parcels A paid to B £600 upon March 1 Ann. 1612 and agreed to pay the rest upon Sept. 1 Anno 1613. The Question is, what he shall then pay, reckoning 10 per Cent. per Ann. interest, and interest upon interest?

In interpreting this question it must be remembered that the civil year began on 25 March instead of 1 January. Thus the payment on 1 Sept 1614 occurs 6 months after the payment on 1 March 1613. The solution is obtained from the expression:

$$\{400 s_{\overline{3}|}^{(2)} - 600(1+i)^3\} \div (1+i)^{2\frac{1}{2}} = £439.\ 9s.\ 4d$$

Q. 99 A man hath a Lease of certaine grounds for 8 yeares yet to come; for which he payeth £130 per Ann. Rent, viz. £65 per halfe yeare: which grounds are worth £300 per Ann. viz. £150 per halfe yeare. If this man shall surrender-in his Lease; what ready mony shall he pay with it to his Land-lord for a new Lease of 21 years, not altering the Rent of £130 per Ann. reckoning such int. as men have when they buy Land for 20 years purchase, and receive the Rent halfe yearly?

The solution is obtained from the expression:

$$85s_{\overline{26}|} \div (1+i)^{42} = £1085.\ 1s.\ 6d.\ (\text{at } 2\tfrac{1}{2}\%)$$

Q. 103 A oweth to B £1200 to be paid in 6 yeares, in 12 equall payments, viz. at the end of each halfe yeare £100. They agree to cleare this debt in 3 yeares, in 6 equall payments, viz. at the end of each halfe yeare, one payment. The Question is, what each payment ought to be, reckoning interest after the rate of 10 per Cent. per Ann. and int. upon int.

The extremely elegant solution is:

$$100 + \{100 \div (1+i)^3\} = £175.\ 2s.\ 7d.$$

Q. 123 Certaine Lands stand charged for payment of 40 sh. per Annum for 13 yeares, and afterwards for payment of £10 per Annum for 17 years. The owner of the Land agreeth with the partie to whom hee should pay these Annuities, to bring them both to one; so, as he shall pay throughout the whole 30 yeares a like yearly summe. The Question is, what that new Annuity ought to be, reckoning 6¼ per Cent. per Ann. int. and int. upon int.

The solution is:

$$2 + (8s_{\overline{17}|} \div s_{\overline{30}|}) = £4.\ 15s.\ 10\tfrac{1}{4}d.$$

4. The second edition of 1634 gives in addition a wide range of tables at 8% per annum, because in 1625 an Act of Parliament was passed which provided that this should be the maximum rate of interest. These tables include one showing $[(1+i)^n - 1]$ at 8% for periods of less than a year, and it is pointed out that compound interest is *less* than simple interest in these circumstances. Thomas Fisher implies that those who charged simple interest at 8% for periods of less than 1 year might for this reason be infringing the Act.

There is an extensive range of Tables at 8%, including the following functions: $(1+i)^n$, $s_{\overline{n}|}$, $a_{\overline{n}|}$; $(1+i)^{n/2}$ and $(1+i)^{n/4}$ for odd integral values of n; $2s^{(2)}_{\overline{n/2}|}$, $2a^{(2)}_{\overline{n/2}|}$, $4s^{(4)}_{\overline{n/4}|}$ and $4a^{(4)}_{\overline{n/4}|}$. In each case there is a separate table giving the reciprocal of the function, to avoid division. These are followed by 'ready reckoners' giving the interest or discount for periods of months and days up to a year.

The ready reckoner for calculating interest makes use of the formula

$$k(1+i)^n = \{k(1+i)^t\}\ (1+i)^{n-t}$$

where k is the principal,
n is the period of accumulation (<1 year),
and t is the number of complete months in n.

Thus **Q. 8** reads 'If £1950 bee forborne for 18 months 10 dayes, how much will it amount unto at the end thereof, reckoning 8 per centum per annum, interest upon interest?'

The solution runs as follows, from the ready reckoner:

Intr.	of £1000 for 12 months, is	£80
	of £ 900 for 12 months, is	£72
	of £ 50 for 12 months, is	£ 4
	of £1950 for 12 months, is	£156

156

Then £2106 for 6 months, vizt:

		£	s	d
Intr.	of £2000 for 6 months, is	39	4	7.2
		39	4	7.2
	of £ 100 for 6 months, is	3	18	5.5
	of £ 6 for 6 months, is		4	8.4
	of £2106 for 6 months, is	82	12	4.3

82. 12. 4.3

Then £2188. 12. 4.3 for 10 days, vizt:

		£	s	d
Intr.	of £2000 for 10 dayes, is	2	2	2.5
		2	2	2.5
	of £ 100 for 10 dayes, is		4	2.6
	of £ 80 for 10 dayes, is		3	4.5
	of £ 8 for 10 dayes, is			4.0
	of 10s. for 10 dayes, is			.2
	of 2s. for 10 dayes, is			0
	of £2188. 12. 4.3 for 10 dayes is	4.	12.	4.3

4. 12. 4.3

facit £2193. 4. 8.6

Right at the end of the book, Fisher slips from the high standard set by Witt. He solves several questions involving the present value of a series of payments by assuming that the payments all fall due on the weighted average (arithmetic mean) date of payment. He quotes the results to 1/10th of a penny but does not point out the considerable error involved in his method.

He gives a table of weighted dates of payment for a level annuity, based on the formula:

$$e = \tfrac{1}{2}r(n+1),$$

where e is the number of months' time in which the 'equated' payment must be made,
r is the number of months between each payment, and n is the number of payments.

We can guess that Fisher intended the book to be mainly for the use of merchants, because he includes a table which shows 'the just Tret [to a quarter of an ounce] of any number of pounds suttle, from 10,000*L.* to halfe a pound'. The Tret was an allowance of 1/26th part of the whole weight of certain kinds of goods, which was granted to the purchaser free of charge to allow for waste.

5. Now that the contents of the book have been described, some attempt may be made to assess its importance.

First of all, it is clear that by 1613 the techniques of compound interest were no longer still in their infancy. It was accepted that compound interest should be allowed in ordinary business and legal transactions, and the methods of carrying out the arithmetic were clearly understood. The differences between simple and compound interest were fully appreciated, as well as the difference between, for example, a rate of 10% per annum and a rate of $2\frac{1}{2}$% per quarter.

It is possible to trace the development of some of these ideas in arithmetical text-books of the preceding century. The writer has not made a full study of these books, but two examples may be cited.

In 1566, in his book *L'Arithmetique* published at Lyons, Jean Trenchant included a chapter on simple and compound interest, which dealt with the subject in a rather mathematical way. Only a few tables were given: these were $(1+i)^n$ and $s_{\overline{n}|}$ at 4%, and a table of $(1+i)^n$ at 10% for periods of less than a year (in complete months) and for $n = 1, 2, 3, 4, 5, 6$. As an example, he discusses whether it is better to receive 4% per quarter interest on a loan or an annuity of 5% per quarter for 41 quarters. The latter is shown to be marginally worse, because $(5-4)\ s_{\overline{41}|}$ at 4% is slightly less than 100. Another example quoted concerns a merchant who bought goods for £1,548 on the basis of payment at £100 per annum till the debt was discharged. If he wished to pay ready money instead, how much should he give? (discounting at 17% p.a.) The solution is obtained by finding the present value of each payment, thus:

$$\sum_{r=1}^{15} (100 \div 1{\cdot}17^r) + (48 \div 1{\cdot}17^{16}) = £536.$$

Hardly a convenient calculation for the merchant!

Trenchant points out that to receive 4% per quarter is better than 16% p.a. He quotes a case where the rent of a farm is £500 per annum for 3 years. If the rent is to be paid in advance by a single payment, this would amount to:

$$(500 \div 1{\cdot}04^4) + (500 \div 1{\cdot}04^8) + (500 \div 1{\cdot}04^{12}) = £1105.$$

Another of his examples, which is particularly interesting, reads as follows: 'If for 6000 one receives 7000 at the end of 3 years, how much would 100 increase by in a year?' In effect the solution is obtained by the expression:

$$100\ \{\sqrt[3]{7000/6000} - 1\} = £5.\ 16s.\ 4d.$$

There is no evidence, however, that he could solve more difficult problems of this nature.

In 1585 a more detailed work on interest was published by Simon Stevin of Bruges as part of his textbook *La Pratique d'Arithmetique*. A number of worked examples were included for problems of both simple and compound interest, together with a set of tables showing v^n and $a_{\overline{n}|}$ for values of n from 1 to 30. These tables were calculated for rates of interest from 1% to 16% (in steps of 1%) and also for rates of interest corresponding to 15, 16, 17, 18, 19, 21 and 22 years' purchase of freehold properties.

In his examples, Stevin does not compound any interest which is payable in respect of a fraction of a year. He quotes the following problem:

> One wishes to know how much £800 capital, with its compound interest at a rate corresponding to 15 years' purchase, would amount to in 16½ years.

The formula he uses is:

$$800 \div (v^{16} \times 15 \div 15\tfrac{1}{2})$$

i.e.

$$800 \times (1+i)^{16} \times (1+\tfrac{1}{2}i)$$
$$= £2{,}321.$$

Among the arguments he puts forward for using simple interest rather than compound in respect of fractions of a year are the following:

(i) All compound interest is intended to be more useful to the creditor than simple interest. But in the case of interest for fractions of a year the reverse would be true. Hence such interest should not be compounded.

(ii) Interest for a single year is not compounded. Hence interest for only part of a year should not be compounded.

Stevin then gives a table of $s_{\overline{n}|}$ for a rate of interest corresponding to 15 years' purchase (i.e. $6\frac{2}{3}$%). However, he does not give similar tables for other rates of interest, but instead demonstrates how the tables of v^n and $a_{\overline{n}|}$ given earlier can be used for the purpose, by means of the formula $s_{\overline{n}|} = a_{\overline{n}|} \div v^n$.

Of particular interest are the problems in which Stevin finds the yield involved in transactions. For example:

> Someone owes £1500 p.a. to be paid over the next 22 years, and he pays his creditor £15,300 in lieu; what rate of interest does this represent?

The solution is obtained by inspection of the tables and is found to be a little greater than 8%.

Even before Trenchant's and Stevin's books were written, compound interest had been charged in business transactions for a very long time.[3] Indeed, the compounding of interest is said to have been known to the Romans, who prohibited it.[4] In England, however, it was illegal to charge any interest until 1545, when an Act was passed which allowed interest of up to 10% per annum (although there is evidence of illegal usury before 1545). Public opinion was not yet ready for the change, probably because religious teachings had condemned usury for centuries, and the Act was repealed in 1552. It was re-enacted in 1571 with the proviso that lenders would be unable to enforce the payment of interest which had been agreed upon, even though this was calculated at 10% or less. The proviso turned out to be ineffective in practice and 10% became the accepted normal rate.[5] The Act was still in force in 1613.

We have seen that Witt was not the first writer on compound interest. However, he was probably the first to produce a practical text-book on the subject, with an extensive range of tables which readers could apply to their own problems. His book may be regarded as the contemporary equivalent of the modern text-book for Intermediate (3) of the Institute's examinations by D. W. A. Donald. The Bibliography at the end of this article lists the other works on the subject which are known to the present writer.

Perhaps the most important limitation on Witt's work is that he does not deal with problems involving the need to find the yield in a transaction where the amounts of each payment are given. The nearest he comes to this is in **Q. 100**:

If a man pay £500 and at 12 moneths end take for it £80 and at the end of each halfe year after, £40 til 10 years in all be expired: whether doth he take more or lesse then after the rate of 10 per Cent. per Ann. interest, and interest upon interest?

The solution is obtained by the formula:

$$80s^{(2)}_{\overline{9}|}-[500(1+i)-80]\,(1+i)^9 = £4.\ 12s.\ 8d.$$

i.e. 'He taketh more than 10 per Cent. per Annum by £4. 12*s*. 8*d*.'

There is no attempt to interpolate in the tables to find the yields involved in transactions, and it may be that there was little call for this in practice.

Would it be true to describe the work as 'the first actuarial text-book'? From the point of view of its date, it certainly has a strong claim to this distinction—though the discovery in future of still earlier works cannot be ruled out completely. As far as its subject matter is concerned, however, some doubts arise as to whether the book can truly be described as 'actuarial'. Although compound interest is important to actuaries, it is nevertheless only one element of their science. The other main element is probability, and there is no evidence that Witt gave any thought to this—all the payments in his problems were certainties, to be received on definite dates. It is possible, of course—though this is conjecture—that the element of risk was allowed for by an arbitrary increase in the rate of interest: this would explain why leases (which were fairly safe) were sold on the basis of 5% or 6¼%, at a time when ordinary commercial transactions (which were more risky) were taking place on the basis of 10%.

Whether the book can be classified as actuarial, or whether this description should be reserved for the later works which combined finance and probability, is not of great importance. What matters is that it shows how far one of the basic actuarial techniques had developed at such an early date—when even logarithms had not yet been invented. The book is unquestionably a landmark in the history of compound interest.

(*Editorial Note*: A copy has recently been added to the Institute Library).

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