THE NET PREMIUM METHOD OF VALUATION

by

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Introduction

1.1. The net premium method of valuation has been controversial ever since it was first advocated over a century and a quarter ago. Its merits and deficiencies have exercised the minds of actuaries whose names are legendary in actuarial science—men such as Finlaison, Jellicoe, Sprague, Manly, Bailey, Coutts, Elderton and Redington; it has been tested in practice against almost every kind of experience; and its demise has repeatedly been predicted. Nevertheless, although it has been modified somewhat over time, in principle it remains today firmly entrenched in life office practice. Consideration of it, even in modern times, has divided actuaries in old ways—the artist from the scientist, the theoretician from the practical man.

1.2. In the light of such prolonged and learned attention to the net premium method a modern student of actuarial science could believe there was little left to be said about it. However, in the course of gathering some notes from the voluminous literature on the subject, principally for the purpose of clarifying my thoughts, I felt that a paper which summarized concisely the development, philosophy, difficulties and mechanisms of the net premium method, as I saw them, might be of use or interest to others.

Development

2.1. Ryan (1) ascribes ‘the formulation of the principles of the net premium system of valuation to a much earlier date than 1845’, and cites evidence of John Finlaison, J. A. Higham, C. Jellicoe, and W. T. Thomas in a ‘Report of the Parliamentary Committee on Assurance Associations (1853)’ in support of this. Nevertheless it appears that up to the mid 1800s most of the prominent insurance companies, and particularly the Equitable which had pioneered modern life insurance since 1762, were using modified gross premium methods based upon the Northampton Table of Mortality, and
2.2. The period from the first publication of the Journal of the Institute of Actuaries (then called the Assurance Magazine) in 1851, to the passage of the first Life Assurance Companies Act (which was held by many to imply official support for a net premium method) in 1870, was arguably the most vigorous in the history of British actuarial science (2), and this vigour was reflected in discussion on valuation methods. The overriding concern of actuaries at the time was solvency, a concern which was given emphasis in a very real manner by the failure of two life companies, the Albert and the European, in the 1860s. Nevertheless this was by no means an exclusive concern: in fact the problem of equitable surplus distribution had been the subject of a paper by Jellicoe (3) in the very first volume of the Journal.

2.3. At that time there were three principal kinds of valuation employed—gross premium methods, the 're-insurance' method, and the net premium method. The only essential difference between gross premium methods and the net premium method employed lay in the premiums valued, but the fact that a number of the offices using a gross premium method apparently took as an asset the value of future office premiums with very small or even no margins for future expenses was reflected in a general feeling of unease among actuaries regarding gross premium methods as a whole. However, as the practice of loading office premiums specifically for bonuses grew, some saw merit in a gross premium approach, and in fact Sprague in one of his earliest papers (4) in 1857 anticipated the bonus reserve method, although he did not use that description.

2.4. The 're-insurance' method was referred to in a number of papers by Jellicoe during the 1850s and appears to have been used by several offices at that time. The method involved calculating liabilities by discounting over the future term of each policy the differences between the office premiums corresponding to the attained ages of the lives at the valuation date, and the actual office premiums charged corresponding to the entry ages. It was apparently common practice to deduce artificial annuity values by entering office premium rates in the premium conversion relationship, and this enabled the calculation of liabilities by reference only to the scales of office premiums, and pure interest functions. The method was strongly advocated by Tucker (5) in 1862, and is of interest today largely
because it provoked an equally strong attack by Sprague (6) in the following year, in a paper which, to quote Cox and Storr-Best (7) '... struck a powerful blow in favour of net premium valuations'.

2.5. Sprague (8) in 1870 introduced in British literature the first major modification of the net premium method, which has survived till today. He argued that no allowance was made in the methods then employed for the uneven incidence of expenses, which could give rise to the problems of new business strain familiar to modern actuaries: he suggested the use in certain cases of the 'Zillmer' modification, which had been developed in Germany, or of his own 'Sprague' modification which minimized the possibility of negative values intruding into the valuation.

2.6. It was the ubiquitous Sprague who also first mentioned in the British literature the second major modification to the method, which still survives—a modification which fundamentally alters the nature of the net premium method. In 1879 in a letter to the Journal (9), Sprague referred to the device of artificially depressing the rate of interest in a net premium valuation basis as a means of deferring surplus release, thereby assisting the emergence of surplus in a manner appropriate for reversionary increments on sums assured. A natural valuation basis would, Sprague argued, tend to release relatively uniform cash surpluses over the life of each contract, which would lead to diminishing reversionary bonuses, whereas the device mentioned would defer some of the early surplus into later durations giving rise to increasing cash surpluses and more uniform reversionary bonuses. Although Sprague's letter to the Journal is the earliest written reference to this device that I have found, it is likely that many offices had retained their old valuation assumptions as interest rates rose during the mid-Victorian period, and awareness of Sprague's point may well have been quite widespread at the time.

2.7. Thus the essential features of the modern net premium method had been recognized by the 1880s, at the latest; while the modifications to meet more realistically the problems of new business strain had been suggested, at the same time the method was departing from reality in respect of its interest assumptions to harmonize with the popular uniform reversionary bonus system; it was by then the widely accepted standard for published valuations and the principal tool used by actuaries to demonstrate solvency and to determine distributable surplus.
Philosophy

3.1. Manly, in an essay which won the 1868 Messenger Prize (10), provided a reference point for all future discussions of valuation methods and bases. His paper systematically described and discussed various different methods, and illustrated in copious tables the effects of differing assumptions upon calculated liabilities. He described the net premium method as (11) ‘That of valuing sums assured and pure premiums only by what is termed a true Table of Mortality and a true rate of interest’. Of particular interest however is his reason for advocating use of the net premium method, namely that, provided the valuation basis was the same as the premium basis for each policy, a comparison of the calculated net premium reserve with the realized accumulation of the assets showed the true realized profit or loss at each duration as the policy experience unfolded.

3.2. Manly was apparently reluctant to adjust a net premium valuation basis in response to shifts in experience: that, he said (12) ...is equal to declaring that the data upon which the premiums were originally calculated were wrong, and therefore, in justice to all the members present and future, the premiums ought to be re-calculated—unless Profits are returned in such a manner as to fully compensate for the difference.

3.3. It is perhaps pertinent that shifts in life office experience were then infrequently recognized—premium bases remained relatively unaltered for long periods—and actuaries were struggling with the problems of equitable surplus distribution in stable conditions, without the added complications of a shifting experience. All the same, Manly’s philosophy, involving the gradual release of profits and losses as they were realized, rather than their capitalization in a change of valuation assumptions, could leave an office vulnerable to adverse changes of experience.

3.4. In 1878 Bailey set out a directly countervailing philosophy to that of Manly a decade before, and was supported, in the discussion of his paper, by the then President of the Institute (J. H. Williams). Bailey said (13) ... it seems to me that valuations will not be made on right principles until it is well understood that no regard is to be paid to the data on which the office premiums were calculated ... an estimate must be made of what future rates of mortality and interest will be.

3.5. Bailey’s general argument was that office premiums were greatly influenced by competition between life companies, and thus their bases were quite inappropriate for valuation purposes. The
crucial difference in philosophy between Bailey and Manly lay in the timing of the release of experience profits and losses: Bailey's philosophy would lead to the capitalization of future profits or losses at each successive adjustment to the valuation basis; Manly's philosophy would lead to the gradual emergence of such profits or losses as they were actually realized against a valuation basis frozen to the original premium basis. Where Manly would fail to anticipate future losses, Bailey was open to the obverse criticism of anticipating future profits before they materialized. Bailey however did recognize the problem, and forestalled criticism by commenting on the revealed surplus (14): '... it by no means follows that this should all be divided. How much should be retained, and how much given away, depends on a number of considerations of prudence and expediency, and in regard to which much thought and judgement have to be exercised'.

3.6. In practice it appears that this conflict of philosophies was resolved in the manner described by Lochhead in 1932 (15): '... the traditional policy pursued has been to face a loss immediately and to reveal a profit slowly'. A similar resolution of the same problem in respect of the assets was also described by Lochhead (16): 'The established practice of British offices is to write down security values when prudence dictates such a course and only in very exceptional circumstances to take credit for capital appreciation apart from actual realized profits on maturity or sale'.

3.7. The inherently conservative bias that the policies described by Lochhead introduced into valuations could clearly give rise to inequities over generations of policyholders, but Lochhead argued (17): '... the problem is whether it is more desirable to preserve the stability of the company, even at the expense of some injustice to the present policyholders, than to strive after strict equity and thereby, perhaps, jeopardise the whole concern'.

3.8. In 1878, the same year that Bailey's paper was published, Sorley expressed a view which has subsequently pervaded the philosophy of the net premium method. His paper was essentially concerned with the impact of selection and initial expenses upon net premium policy values, and, in demonstrating their counteracting effects, said (18):

... the opinion has been expressed, that the net premium method is one which must inevitably disappear before the advance of true actuarial science. Circumstances, however, seem to favour its continuance; not on the ground of any
algebraic demonstration of its correctness—for the question is not a mathematical
one—nor that it gives the highest reserves, or is in any other respect extreme, but
rather as a compromise or via media (Sorley's emphasis), where selection, ex-
spenses, occupation, residence and other influences—some affecting the result one
way, and some in another—are all alike ignored, and whose defence takes its stand on the broad principle of averages, which lies at the foundation of all
actuarial principles.

3.9. Perhaps the most comprehensive and authoritative explana-
tion of the philosophy of the net premium method in modern times
was that given by Redington in 1952. In a remarkable paper, which
also defined the theory of immunization and established the concept
of a life office's 'estate'—ideas which have permeated much subse-
quent actuarial thought—Redington described the net premium
method, based generally upon the premium assumptions of the
business valued, as a tool for determining what he termed the
'natural revenue surplus'. He stated (19):

It would perhaps be tendentious to apply the label 'true' to any particular figure
for the surplus, but there is one figure for the surplus, which can legitimately
claim the title 'natural', namely, surplus resulting from a valuation on the same
bases as were used in the calculation of the premiums and based on revenue
accounts from which all capital items have been scrupulously excluded. Surplus
calculated by this system can be described as 'natural revenue surplus', and a
valuation on the premium basis can be described as on the 'natural valuation
basis'.

Redington later continued (20):

The great justification for the net premium valuation is that it gives an approxi-
mation to the premium basis of valuation and therefore to the natural revenue
surplus. As such it is relatively fool-proof and cannot easily be abused. It cannot
be used, for example, to capitalise future profits and thereby to bolster a position
where expenses are running at an untenable level. When we consider that the life
assurance fund of a well-established company contains the history of many
years of changing events, there is great virtue in a net premium valuation which
can be made to approximate to all the various premium bases which have been
employed.

3.10. It is interesting to note how Redington's views coincide with
those expressed in Manly's Messenger Prize Essay of 84 years
previously; Redington does apparently recognize that such a philo-
sophy of the net premium method can fail to capitalize future losses,
saying (21): '...there are circumstances when the net premium
valuation fails us, and when this happens there is much to be said
for a bonus reserve valuation', and later (22): '...when we turn to
the question of solvency there is no word to be said in favour of a
passive policy; it is only an active policy, paying full regard to
existing (and estimated future) experience, which has any significance'. It is also of interest that Redington's arguments gave a fresh twist to Sorley's defence of the net premium method, in 1878, as a *via media*, not, as Sorley had expressed it, as a broad representation of the experience at the time of valuation, but rather as representing a kind of average of the various premium bases upon which the business in force had originally been underwritten.

*Criticisms*

4.1. Sprague, who had done so much to popularize the net premium method, was ironically among the first to express doubts about it. In his paper of 1870 (23) Sprague objected to the manner in which the method allowed for future expenses, namely by assuming implicitly that they would be covered by the excess of the actual gross office premiums received over the net premiums assumed in a net premium valuation. That, Sprague suggested, could in some circumstances be inadequate, and he advocated the use in certain cases of gross premiums specifically adjusted to allow for the actual expenses likely to be incurred. Sprague's paper was however more concerned with the incidence of expenses than with the level, and it was left to later events and actuaries to give greater prominence to his criticism.

4.2. A period of declining interest rates, around the turn of the century, provided the net premium method with perhaps its severest test. In 1902 Warner was quite categorical in his views (24):

... our criticisms of the net premium method centre themselves in the charge that it withdraws our observation from the actual premium we are entitled to receive, and fixes it upon something else, which is not that premium, and which only on certain rigid and definite assumptions bears the relation to it that the method implies. These assumptions may in course of time cease, even approximately, to represent the facts. The system however, has from its nature no room to make allowance for such changes, and may thus eventually produce an altogether arbitrary and artificial result.

Warner stressed the point that a reduction in the valuation rate of interest, reflecting a fall in the experience rate, resulted in a consequential increase in the valuation net premiums and hence eroded the provision for future expenses. Thus a fundamental characteristic of the net premium method—the nexus between the net premiums valued and the valuation assumptions themselves—meant as Redington later said (25): '... a net premium valuation is only partially sensitive to a change in the rate of interest because the
change in the rate is accompanied by a change in the net premiums—a technical idiosyncrasy which has no counterpart in the facts. Thus to allow adequately for changes in experience interest rates, particularly downwards, exaggerated adjustments were required to the valuation rate, and in extreme conditions one could even envisage a need for a negative valuation interest rate.

4.3. Warner was also unconvinced by Sorley’s earlier defence of the method as a *via media* arguing (26): ‘... the method will only be of value so long as variations of its assumptions from fact keep within certain manageable proportions ... now those variations seem to present possibilities so large and complex as to demand more definite recognition’.

4.4. The discussion of Warner’s paper was comprehensive and evidenced a widespread feeling that the net premium method ought soon to be superseded by some kind of gross premium method. Berry (27) pointed out the failure of the net premium method to allow properly for expenses on paid-up and limited payment policies, and Manly (28), stating explicitly that his views had changed since 35 years previously, advocated a gross premium method with specific reserves for future expenses and bonuses. Levine however took issue with other speakers who criticized the method as unscientific (29): ‘... if the method by which the office premiums were calculated, and the method by which bonus was distributed, were in harmony with each other, then the net premium method was absolutely scientific’, he claimed—a view which has been sustained to the present day.

4.5. In 1903 Ryan summed up feelings which must then have been common in actuarial circles (30):

Look at it how we will, there is something in modern tendencies to weaken the foundations of the net premium system of valuation ... it seems quite within the bounds of possibility that the twentieth century will record the supersession of the net premium system by some other method, more elastic, flexible and adaptable to changing conditions ...

4.6. Although the elements of the system which many saw at the time as fulfilling Ryan’s prophecy, had been mentioned by Sprague (31) in 1857 and by Manly in a letter to the Insurance Record in 1901 (32), it was in a paper to the Journal in 1908 that Coutts coined the phrase ‘bonus reserve’ valuation (33). Coutts’s paper described the elements of the method, and provided the opportunity for its full consideration and discussion by the profession. Although it is beyond
the intention of this paper to examine the features of bonus reserve valuations, Coutts's paper and its discussion are of relevance in that they crystallized the dissatisfaction which had built up with the way the net premium method had been increasingly departing from its original concept of valuing on a 'true' basis. The method had become artificial first to meet the needs of the uniform reversionary bonus systems, and secondly because of the exaggerated and complex patchwork of adjustments that had been necessary to accommodate the exigencies of a more volatile environment. Gross premium bonus reserve valuations permitted a return to natural assumptions, and made clearly visible all the hidden features of net premium methods. The new approach was not however without its critics: Allin warned of the risks of revealing future bonus reserves to policyholders (34); King raised the thorny problem of fitting the new system into the format required for Board of Trade returns (35); and Manly (36) and Woods (37) foresaw the dangers of capitalizing future profits and respreading them in an arbitrary fashion. The discussion was also of interest because it contained one of the earliest references, as far as I can tell, to the relationship between the valuation assumptions and the value placed upon the assets (38).

4.7. The question raised by King regarding published valuation returns was followed by W. E. Norton, who, in a paper written in 1923 from which notes only were published in the Journal in 1931 (39), referred to advantages arising from the use of fairly uniform net premium bases in published valuation returns by most offices. Such uniformity was unlikely if life offices generally started publishing bonus reserve valuations, thereby making the acceptance of a common standard, and the chance of meaningful comparisons between offices, most unlikely.

4.8. In 1931 Elderton criticized the use of the net premium method because, due to its deliberately artificial assumptions, it gave rise to severe distortions in an associated analysis of surplus, and consequently masked the factors contributing to real profits and losses. He suggested that the method had become so distorted in practice that an actuary would require a repeat valuation by a bonus reserve method in order to decide what bonuses could properly be declared (40). In the discussion Lochhead was even blunter (41):

... he imagined most companies employed a bonus reserve method, and made an investigation into the profit earning power of their premium scales as a check on the published valuation figures. That fact seemed to him in itself to condemn
the net premium method, the results of which were put forward largely as a kind of decoy...  

Lochhead continued, almost with a note of desperation: ‘Nevertheless it was so firmly entrenched, screened as it was by the barbed wire entanglements of the Board of Trade returns and sheltered by a positive barrage of misunderstanding, he felt its position was almost impregnable’.

4.9. Warner, it has already been mentioned, had pointed out, in 1902, that to reflect changes in experience interest rates, exaggerated adjustments were necessary to the assumptions of a net premium valuation. Suttie, in 1945, considered the further complications arising from associated fluctuations in the values of the assets. It would now be clearly recognized that the long run situation of a life office in the face of a shift in interest rates would depend upon the degree to which the terms of the assets and liabilities were matched, and that in the likely event that an office is invested ‘short’ it is only falling interest rates that are of long term concern. Nevertheless there will always remain the problem of making adjustments to new circumstances in the short term, and of equitably distributing the effects of the changes over the business. Suttie’s analysis was not favourable to the net premium method, and he concluded that the most equitable solution in the case of rising interest rates lay in the use of a method (42) ‘... which writes down the assets to their new market values, increases the valuation rate of interest to the new experience rate, and values as a liability a future bonus... which the existing premium scale will support under the new conditions’.

Solvency

5.1. The use of a net premium method specifically to test the solvency of a life office is inappropriate for two reasons: first it may not necessarily take account of unrealized profits or losses; secondly it will incorporate in the liabilities a reserve for future benefits which are not guaranteed, to an extent which varies with the valuation assumptions and the age and composition of the business.

5.2. Redington’s 1952 paper, already referred to above, discussed the distinction between capital and revenue items in a life office’s profit and loss account, for the purpose of distinguishing items which should be permitted to be reflected in a net premium valuation result. The distinction was not by any means always clear, but a useful criterion was whether the item in question pertained to the inter-
valuation period or could more appropriately be allocated over some period in the future. Thus, for example, increases in dividend or interest earnings actually received since the previous valuation would represent revenue profit, whereas a change in the market value of the assets reflecting their future earnings would represent capital profit. A similar distinction could arise also in respect of the liabilities: for example a general improvement in mortality could give rise to revenue profits over the year preceding the valuation date, and also capital profits in respect of years after the valuation date. One of the problems associated with capital items is their destabilizing effect if they are immediately brought into account for the purpose of determining current bonuses, and Redington’s philosophy of specifically recognizing revenue items only in the assets and liabilities, with capital items reflecting in changes in the life office’s estate, implied the use of the estate as, among other things, a surplus stabilization reserve. This however presupposes the existence of the estate, a supposition quite inappropriate in the circumstances of a solvency investigation.

5.3. A more liberal interpretation of Redington’s views would involve recognizing capital losses immediately in the valuation, and taking credit for realized capital profits, and placing only capital profits yet to be realized to the estate, to be released if and when they emerge through the revenue accounts. In any event it is clear that a net premium valuation which does not recognize all kinds of profit and loss is inappropriate for testing solvency: the fact that a profit or loss may not be allocated to a particular inter-valuation period does not mean that it is any less real.

5.4. Implicit in a net premium valuation is a reserve for future expenses and bonuses. This comprises in the one part the present value of the differences between the office premiums receivable in future, and the valuation net premiums assumed, and in the other part the increment in the calculated liabilities by virtue of any artificial reduction in the valuation rate of interest assumed. Relative to the contracted liabilities, this reserve will be largest in respect of newly-written participating business, and smallest in respect of old non-participating business: the presence of this reserve explains why most growing offices writing preponderantly participating contracts can be quite confident of their solvency so long as they continue to display surpluses on a reasonable net premium basis. It is nevertheless clear that the extent to which a net premium basis is too strong depends very much upon the valuation assumptions and the age and
composition of the liabilities, and in an extreme case one can envisage a declining office, with largely old non-participating business on its books, displaying a surplus on a fairly stringent net premium basis, yet nevertheless insolvent because of its expense situation or because of unanticipated losses of a capital nature.

**Distribution of Surplus**

6.1. Referring to the use of the net premium method of valuation for controlling the release of surplus, Coe and Ogborn stated (45)

... in suitable conditions, the margins in the loadings in the premium scale and the interest margin on the valuation liability will produce the surplus at about the rate of release required to provide the uniform reversionary bonuses.

Lidstone gave a numerical example of this feature, but it can be very easily proved from a simple transformation of the policy value, though we do not remember having seen it before. We have

\[ b \cdot A_{x+t} = b \cdot v_x (1 - A_x) + b \cdot A_x. \]  

(14.1)

Suppose the earned rate of interest is \( i'' \) but that the premiums and the valuation liability have been calculated at a rate of interest \( i \) such that

\[ i = i'' - b (1 - A_x). \]

It is clear that an interest surplus will emerge equal to \( b (1 - A_x) \) on the valuation liability. Should the premiums also be loaded by an amount equal to \( b \times A_x \), which is the cost of the first year's bonus, equation (14.1) shows that the combined interest and loading surpluses will emerge in just the right amounts to provide the desired bonus...

... Strictly speaking... the interest margin would also produce a surplus in respect of the value of the existing declared reversionary bonuses. Since this would be at the rate \( b (1 - A_x) \) and not at the rate \( b \) on the existing declared reversionary bonuses the actual bonus from (14.1) would be between a simple and a compound bonus.

6.2. A more thorough analysis of the mechanism of surplus release through the net premium system of valuation follows from the following general relationship, which relates the reserve at time \( t \), immediately before receipt of a premium, to that at time \( t+1 \) immediately before receipt of the next succeeding premium:

\[
\left\{ v_{[x]:[m]} - \left( \frac{I}{\delta_{[x]:[m]}} \right) \delta_{[x]+t:m-n} + B \cdot A_{[x]+t:m-n} \right\} \\
+ \left\{ \left( \frac{1+i}{1+j} \right) \left( \frac{I}{\delta_{[x]:[m]}} + \frac{I}{\delta_{[x]:[m]}} + b \cdot A_{[x]:[m]} \right) + I \left( \frac{j-i}{1+j} \right) \right\} (1+j)
\]
which is true when

\[ i = j - \frac{b \cdot (1 - A^i_{x, \infty})}{(1 + I)} \]  

Thus, if the loadings for renewal expenses just meet those expenses when they are due, and hence a zero reserve is required in respect of them at all times, if \( j \) is the experience rate of interest, if \( I \) is the Zillmer constant, usually corresponding to the loading for initial expenses, per unit of sum assured, and if \( i \), the valuation rate of interest, is given by the formula shown, then surplus will be released each year at a rate just sufficient to provide a reversionary bonus of \( b \) per unit of sum assured, and of \( \left( \frac{j - i}{1 + i} \right) \), that is of \( \frac{b(1 - A^i_{x, \infty})}{(1 + I)} \) per unit of existing reversionary bonus; provided that the office premium, before loading for renewal expenses and any initial expenses not covered by \( I \), is calculated at rate \( i \) by the formula

\[ \left( \frac{1 + i}{1 + j} \right) \left( P^i_{x, \infty} + \frac{I}{\ddot{a}^i_{x, \infty}} + bA^i_{x, \infty} \right) + I \left( \frac{j - i}{1 + j} \right). \]  

The corresponding premium without the bonus loading is

\[ \left( P'_{x, \infty} + \frac{I}{\ddot{a}^i_{x, \infty}} \right), \]  

thus the bonus loading, before adjustment for renewal expenses, is the difference, which reduces to

\[ (1 + I) \left[ \left( \frac{1 + i}{1 + j} \right) P^i_{x, \infty} - P'_{x, \infty} \right] + \left( \frac{1 + i}{1 + j} \right) bA^i_{x, \infty}. \]  

The results are quite general and hold for \( n = \infty \) (whole-life contracts) and for \( I = 0 \).
6.3. Relationship (2) above, relating \( b, i, j, I, x \) and \( n \), is crucial, for upon it all the other relationships depend. In practice a particular value of \( j \) will usually be deemed to apply, if not to all the business of an office, then to broad sections of it, and \( x \) and \( n \) are determined in each case by the contracts being valued. Thus \( I, i \) and \( b \) are in practice the variables available to the actuary to harmonize the valuation with the bonus system. It is instructive therefore to examine how each in turn of \( I, i \) and \( b \) should vary with \( x \) and \( n \), while holding all the remaining variables constant.

6.4. The Tables below are based upon the A1949–52 Select Tables, taking as the basic values for the variables in relationship (2) above the following:

\[
\begin{align*}
  b & = 0.25, \\
  i & = 0.35, \\
  j & = -0.5, \\
  I & = 0.03.
\end{align*}
\]

6.5. Table A. Values of \( I \), as \( x, n \) vary; and \( b, i, j \) constant

<table>
<thead>
<tr>
<th>( n )</th>
<th>10</th>
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<th>30</th>
<th>40</th>
<th>( \infty )</th>
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It is apparent from Table A that the effective use of \( I \), the Zillmer constant, as a variable to be adjusted for different sectors of business in order to distribute surplus equitably is out of the question. On the quite reasonable assumptions made above the variation required would be over the range \(-0.5\) to \(+0.3\): in practice only a very small portion of this range could be seriously considered appropriate.

6.6. Table B. Values of \( 100i \), as \( x, n \) vary; and \( b, j, I \) constant

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<td>55</td>
<td>4.19</td>
<td>3.85</td>
<td>3.55</td>
<td>3.72</td>
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<tr>
<td>65</td>
<td>4.24</td>
<td>3.99</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

From Table B it appears that quite equitable results could be
achieved by employing differing valuation interest rates according to the original terms of the contracts. In the example given the rates required range from about 4¼%, in the case of 10 year endowment assurances, to approximately 3¼%, for whole-life assurances. In the case of whole-life assurances there appears also to be some value in sub-dividing the business by age at entry and, in the example given, increasing the valuation interest rate by ¼% for each 10 years by which the entry age exceeds 35 years. In practice however, it is inconvenient to have to apply different valuation bases to various sub-divisions of the business, and it may be the case, as it is in Australia, that the maximum valuation rate of interest is set fairly low by law, thereby severely restraining the actuary's freedom in choosing a range of valuation assumptions for the purpose of statutory returns. Legal minimum bases however are usually a constraint on the aggregate distributable surplus only, and do not prevent the use of different bases for internal investigations for the purpose of allocating distributable surplus equitably among the various classes of business.

6.7.

Table C. Values of \(100b\), as \(x\), \(n\) vary; and \(i\), \(j\) and \(I\) constant

<table>
<thead>
<tr>
<th>(x)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>(\infty)</th>
</tr>
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<tbody>
<tr>
<td>25</td>
<td>5.15</td>
<td>3.03</td>
<td>2.36</td>
<td>2.07</td>
<td>1.89</td>
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<tr>
<td>35</td>
<td>5.16</td>
<td>3.06</td>
<td>2.42</td>
<td>2.18</td>
<td>2.09</td>
</tr>
<tr>
<td>45</td>
<td>5.21</td>
<td>3.18</td>
<td>2.62</td>
<td></td>
<td>2.44</td>
</tr>
<tr>
<td>55</td>
<td>5.39</td>
<td>3.49</td>
<td></td>
<td></td>
<td>3.04</td>
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<tr>
<td>65</td>
<td>5.83</td>
<td></td>
<td></td>
<td></td>
<td>4.12</td>
</tr>
</tbody>
</table>

In practical terms Table C is the most promising. Valuing at 3½% with a 3% Zillmer adjustment will, if the earned rate is 5% and the bonus loadings are appropriate, release surplus which will equitably support bonuses ranging from about 2% p.a. for 40 year endowment assurances, to a little over 5% p.a. for 10 year endowment assurances. In the case of whole-life assurances the appropriate rates of bonus rise with entry age from under 2% p.a. to over 4% p.a. It is usually easier administratively to cope with a range of bonus rates for the different classes of business, than with a range of valuation bases, and furthermore the actuary's freedom of action in this regard is rarely hampered by legal requirements. Even so it appears from Table C that a considerable amount of sub-division of the business is necessary and the range of bonus rates required is quite broad.
There may in fact exist competitive market factors which constrain bonus rates, and which make infeasible the range of bonus rates necessary to make the net premium method work equitably.

6.8. It remains to examine the equity implications of 'painting with a broad brush', that is, examining the earnings rates implied by using common valuation assumptions together with a common rate of bonus across all classes of business.

Table D. Values of $100j$, as $x$, $n$ vary; and $b$, $i$, $I$ constant

<table>
<thead>
<tr>
<th>$n$:</th>
<th>10</th>
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<th>$\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$:</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>4.23</td>
<td>4.74</td>
<td>5.09</td>
<td>5.31</td>
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<tr>
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<td>4.57</td>
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<tr>
<td>65</td>
<td>4.14</td>
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<td></td>
<td>4.41</td>
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</tr>
</tbody>
</table>

Table D above shows for an office valuing at 3½% with a 3% Zillmer adjustment, and which loads for and grants reversionary bonuses at 2½% p.a., the earnings rates actually required to support the bonus for the various classes of business. It is clear that, particularly at younger entry ages, the longer term policies do relatively well with an implied earnings rate of about 5½%, while the short term endowment assurances do relatively badly with, in the example, an implied earnings rate of less than 4½%. The practical effect of this difference is quite substantial: the interest earnings on annual instalments accumulating at 5½% for 10 years exceed those on equivalent instalments accumulating at 4½% for 10 years by about 35%. These inequities would be slightly mitigated if the bonus was simple, and slightly exaggerated if it was compound.

6.8. It is evident also from relationship (2) above that special valuation assumptions are required in respect of non-participating business (or special business with low bonus rates) in order to maintain equity. The appropriate valuation rate of interest for non-participating business is readily seen to be $j$, the experience rate, by putting $b = 0$ in relationship (2). Where an office uses a valuation rate $i$ less than $j$, appropriate to its mainstream participating business, for valuing non-participating business, the total surplus displayed at each valuation will be less than would be the case in the absence of the non-participating business. In effect a 'loan' is made from the surplus to bolster the reserves of the non-participating business. In such circumstances it is clear that, even if the initial establishment
of such a loan was financed without imposing costs on the participating business, the surplus released in each subsequent inter-valuation period will reflect changes in the amount of this loan, as the amount, and the age and composition of the non-participating business vary. Such variations could well disturb the equitable release of surplus for the participating business.

6.9. It has commonly been remarked that the net premium method is inflexible in the face of unstable conditions: the foregoing analysis exposes a strong rigidity in the method, even in ideally stable conditions, which I have not previously seen mentioned explicitly. It is simply that for a given experience interest rate and valuation interest rate, there is a unique rate of bonus, for each kind of contract, which can be equitably released by the net premium method of valuation. These uniquely equitable rates of bonus vary considerably with the original terms and entry ages of the contracts issued. Thus, even in ideal conditions, the use of the net premium method to release surplus equitably requires the inconvenience of a considerable degree of fragmentation of the valuation assumptions employed, or a considerable degree of fragmentation of the bonus rates loaded for and declared. An interesting implication for a potential policyholder, particularly if young, in a country where valuation assumptions are severely constrained by law (as in Australia), is that he should, other things being equal, place his long term business with an office which does not differentiate its bonus rates for different classes of business, and his short-term business with an office which does!

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