PRICING AND PROFITABILITY IN A LIFE OFFICE

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1. PHILOSOPHY

1.1. Profits are essential to the economic viability and indeed the survival of any enterprise. Life assurance companies are no exception whether they be mutual or proprietary. Unless profit—or as it is often termed, surplus—is available for distribution to shareholders and/or policyholders, the reasonable expectations of shareholders or policyholders will not be met; nor will the office be able to accumulate and maintain adequate contingency funds.

1.2. The word profit will be used synonymously with surplus throughout this paper. Profit is defined as the excess during any period of income over outgo, where outgo includes the necessary increase in valuation reserves.

1.3. It is essential for an office to define its objectives with respect to profit so as to:

(a) Recognize and reconcile different expectations of interested parties—policyholders, shareholders, employees and sales agents.
(b) Reflect corporate purpose and objectives so as to assist harmony of activities and efforts.
(c) Correspond to the real world economic and risk-bearing characteristics of a life office.

These needs can be fulfilled through a philosophy regarding pricing and profitability.

1.4. By ‘pricing’ this paper refers to the process by which acceptable office premiums and/or bonuses are established. By ‘profitability’ is meant the conceptual definition of ‘profit’ and the measurement of that profit. It is the central theme of this paper that only by analysing both the level and incidence of profits can any firm foundation be laid for a useful definition of philosophy of, or actuarial technique regarding, pricing and profitability.

1.5. This paper will explore the concept and application of a ‘profit-test system’ which takes profits as defined in § 1.2 above and uses the progression of these over the expected lifetime of a policy to analyse that policy. This system in fact embraces a family of specific methods and some of these may be found an acceptable and sufficient basis for measuring profitability and, hence, for defining a useful philosophy of pricing and profitability.

1.6. One of the particular methods of profit testing is based upon the concept of capital utility; the economic value of capital consumed. The concept arises when we observe the typical operation of a life office resulting in a more or
less continuous generation and employment of capital funds. Whether an office is mutual or proprietary in its structure or neophyte or mature in its tenure the process is the same. Specifications—as to amounts of profit and rates of return on capital employed—which are acceptable within a profit-testing system may vary considerably and justifiably between offices and even by lines of business within a single office. Thus no one method or standard can be truly universal but it is suggested that the profit-test system as defined here is universal in its application. Specific methods within the system will be examined and their strengths and weaknesses noted.

1.7. No system or method of pricing, or of testing profitability, can achieve any real useful meaning unless it is ultimately related to the actual results achieved. A profit-testing system provides the process for defining standards of expected performance as to all essential parameters. It also provides an excellent basis against which to measure actual performance as it develops.

2. RELATIONSHIP BETWEEN PREMIUM AND PROFIT

2.1. Any premium however calculated implies assumptions as to the following parameters:

(a) Benefits payable, for example upon death, surrender, maturity and disability. Bonuses also fall under this category.

(b) Decrement rates for death, surrender and disablement at each elapsed duration for each age at entry.

(c) Commissions and expenses payable.

(d) Reinsurance costs.

(e) Rates of return on assets generated. For unit-linked business this includes the growth rate of, and management fees on, unit funds.

(f) Tax status and tax rates.

(g) Valuation basis.

(h) Profit objective.

A scientifically calculated premium would incorporate explicit assumptions for the above parameters.

2.2. In addition to what one might refer to in § 2.1 as expected or conservative values of parameters, incorporation of probability distribution functions for each of the parameters might be worthy of consideration.

2.3. Actual profit will differ from the objective to the extent that actual office premiums differ from the scientific premiums; as well as to the extent that underlying experience differs from assumptions with respect to underlying parameters.

2.4. In determining parameter estimates it is useful to distinguish between those subject to control by the office and those not so subject.

In respect of in-force business no direct control is available over mortality, surrender and disablement rates, tax rates or interest rates. However, a notable
exception exists in respect of those companies issuing that type of unit-linked business characterized by the deduction of fees fixed as a percentage of funds irrespective of actual yields—which fees are then used for actuarial funding. To this extent one may say that complete control exists over that interest rate. Some degree of control exists over margins for contingencies, reinsurance costs, valuation basis, expenses (although not in the short term) and tax state. It may also be noted that some control exists at point of sale over disablement and mortality rates through underwriting and marketing. Lapse rates are subject to some degree of control via mode of payment, market in which policies are sold and control of sales force.

The degree of control that exists over benefit values is quite interesting: on the whole we may separately identify on the one hand unit-linked business and non-profit business with guaranteed surrender values; and on the other hand non-profit business with non-guaranteed surrender values plus with-profit business. Of course the main reason for such control over benefit values in the United Kingdom (U.K.) is the desire to hold investments matched specifically to maturity liabilities and, hence, to eschew intermediate benefit value guarantees in order to minimize exposure to asset price fluctuation. It is almost certainly correct to say, however, that benefit values under the control of the office have not in the past led offices to attempt to achieve profit objectives by significantly worsening the profit distribution to policyholders: although the mechanics of bonus declaration in many offices does appear rather favourable to one or the other of shareholders and policyholders.

At present offices do have control over premium rates and therefore profit expectations. Indeed it would seem that premium rate is the main determinant of profit expectation: although actual profit achievement will also depend upon unplanned parameter variations between actual and assumed.

2.5. Various yardsticks for measurement of profit are in current use and some of these are discussed in Section 3. It is, however, the underlying theme of this paper that life assurance, like any other business, requires capital. Capital funds required for a life assurance company are substantially determined by:

(a) The need to finance working cash balances and non-liquid current fixed assets—including contingency reserves.
(b) The valuation strains caused by the interaction of new business and the valuation basis. This results from the familiar feature that the 'fund' at the early durations of most policies is insufficient to meet minimum acceptable valuation standards.
(c) The need for a source of last resort funds to stand against contingencies both internal and external—these contingencies not being otherwise anticipated by the valuation basis.

Profits have substantial utility to 'investors' whether they be policyholder or shareholder investors, provided they are distributable: in other words when they emerge against the statutory reserve basis. By 'statutory' reserve basis we mean
one which is both acceptable to the supervisory authorities and regarded as making adequate provision for both contractual and reasonable expectations of policyholders.

In order to determine the quantum of capital consumed by a new policy and the adequacy of the premium rates one can use a profit-test system. By projecting expected income and outgo against the background of the reserve basis using assumptions as to all underlying parameters we can test a policy for profitability, which test includes its ability to support bonuses if applicable. This profit-testing system is the only one which permits analysis of both amount and timing of expected profit emergence before and after bonus distribution if applicable. Unless one knows the 'shape' of one's office one clearly understands little—but shape must refer to the size and sign of expected future revenue account items. The profit-test system approach, particularly when applied in the form of a revenue account—that is per policy issued—provides an insight into the office and enables one to introduce both varying and non-standard assumptions in the areas of:

(i) Lapse rates.
(ii) Non-standard mortality rates.
(iii) Surrender values.
(iv) Bonuses.
(v) Interest rates.
(vi) Reserve basis.
(vii) Inflation of maintenance expenses.
(viii) Tax status—for example, to allow for the delay between expenses disbursement and obtaining the associated tax relief when an office is not in an immediately tax paying position.

2.6. In Appendices A and B are contained illustrative profit tests together with the algebra used to develop them. There are many ways of carrying out the basic algebra underlying a profit test and that shown in Appendix A is only one of many possible approaches.

2.7. It is now possible to examine the concept of a 'minimum premium'. Above it has been argued that premium rate is the prime determinant of profit expectations. Profit expectations have been seen to consist of a series of profits or losses measured against the statutory reserve basis—it is a simple matter, having constructed the profit test, to ascertain the rate of return on capital employed by solving for the discount rate which equates the initial capital loss to the subsequent (presumed) stream of profits. The rate used to discount expected future profits is critically important. Unless it exceeds a certain level it may not be in the interest of the providers of capital for such business to be issued.

A number of approaches to determining the minimum level of discount required are possible, for example:

(a) External sources—to the extent that shareholders provide capital,
the rate should be at least comparable with that which can be obtained elsewhere, if there is a comparable risk.

(b) Internal sources—to the extent that policyholders provide capital the rate should be in excess of the rate obtainable from alternative investments with less risk attached.

To anticipate an argument occurring later in this paper, if policyholders provide all of the capital they will need a rate at least as great as that determined by the planned rate of expansion of new business (see Section 7 and Appendix E).

There are two further vital considerations in determining the interest rate applicable in particular circumstances:

(i) Emerging profits may or may not be taxable—depending upon to whom they are distributed and through which funds they emerge.

(ii) There is a significant capital requirement for non-income bearing (or at best low-income bearing) assets acquired when new business is written. This must be allowed for. For example, if low-income bearing assets (say 3% after tax) amounting to 25% of the initial loss must be acquired, and one’s target after tax rate of return is say 12%, then the risk discount rate which equates anticipated future profits to initial loss needs to be 15%.

However determined, such a risk discount rate will enable the fixing of a minimum premium below which the office cannot go without damaging its ability either to attract capital funds from external sources—whether shareholders or reinsurers—or to justify the retention and utilization of capital funds from internal sources—policyholders or shareholders. For an example of the measurement of rate of return we may cite the first two policies in Appendix B. Unless these rates of return are in excess of the judged minimum required then these contracts will need modification.

2.8. A very significant area of philosophy is the extent to which required discount rates may differ between mutual offices, non-profit proprietary offices and ‘quasi-mutuals’—by which is meant proprietary offices issuing both with-profits and non-profit business, the with-profit policyholders being entitled to say 90% of all profits. An office whose only with-profit business consists of policies entitled only to the profits arising from their class may be more effectively considered as two different offices—a mutual and a non-profit proprietary office. It may be that different discount rates should apply to the two parts. This paper does not seek to provide any clear cut answers. It is, however, suggested that the main determinants of critical discount rates in a mutual office are the planned rate of expansion of the business and the yields obtainable on relatively risk-free policyholder fund investments. For a shareholder office the prime determinant may perhaps be the yields required by shareholders. Considerations applicable in quasi-mutual offices are somewhat complicated and perhaps include a judicious mix of both the above considerations.
3.1. The technique to be used for setting the overall level of gross premium will depend upon the philosophy of the office and in particular its method of measuring profits and its standard for acceptance of a given level of profits. While the actual table of office premiums may well be ultimately produced by a formula method reproducing the pivotal office premiums, these pivotal premiums may be determined by considering the interrelationship between the minimum required premium and those premiums offered by other relevant offices. The techniques discussed below relate to the calculation of the minimum office premium or the assessment of the profit level provided by the adoption of any premium. The two main bodies of techniques may conveniently be referred to as Asset Share and Formula.

3.2. Formula techniques correspond to the calculation of a net premium such that, at the interest rate chosen, the present value of death, survival and disablement benefits plus expenses is not greater than the present value of office premiums. This calculation is performed under the influence of mortality, disability and interest rates only. 'Profit' then emerges as follows:

(a) As the excess of the office premium charged over the net premium as calculated.
(b) As deviations between actual experience and the assumptions made in the premium calculation. An additional source of profit is the margin arising from profit on surrender, being the excess of the reserve on the premium basis over the surrender value.

This Formula approach is obviously simple and easy to apply but suffers from three main drawbacks:

(i) No account whatsoever is taken of the incidence of profits: in particular no account is taken of the effect of a statutory reserve basis on the profit flow. It is implicit in this approach that capital is unlimited and providers of capital either do not require or are not entitled to any higher return than the interest rate likely to be experienced from the appropriate matched assets.

(ii) No guidance is provided as to the likely future state of the office in the areas of the need for investment or disinvestment of assets, tax position, size of in-force portfolio and funds at future dates, etc. Each of these, and indeed many other considerations, need to be studied separately and independently of the premium calculation process.

(iii) No account is taken of the impact of surrenders in the premium formula.

3.3. Asset Share techniques equate in general terms with the profit-test system discussed variously in this paper. The Asset Share analyses shown in Appendix B (non-profit endowment assurance and 'Type A' unit-linked assurance) illustrate
the fundamental difference between the Formula approach and the Asset Share approach. Two things strike one immediately:

(a) The greater complexity of the Asset Share approach—incorporating as it does so many explicit parameters as well as the statutory reserve basis. 
(b) The obvious ease with which one can build a bridge between the profit test and projected future revenue accounts.

These are, on reflexion, two sides of the same coin: the use of essentially computer-based techniques to study the existing and potential future development of an office.

If we consider the non-profit endowment assurance profit study we can see that the items produced for inclusion in the book profit formula will, after multiplication by a survivorship factor—allowing for decrements of both mortality and withdrawal—and inclusion of new business figures, produce projected revenue accounts. Actual progress can then readily be monitored against projection in such crucial areas as cash flow, tax position and importance as to investment yield obtainable. Again with modest computing facilities available it is a simple matter to test the sensitivity of results to parameter changes. Information is provided by the process sufficient for the investment yield obtained at each year in the future to be made a function of not only the pattern of yield anticipated but also the actual tax position of the office at that point and the flow of new money for investment.

Special considerations for unit-linked contracts would include the relative effect of growth in management fees and expense inflation together with the size of the sterling (non-unit) reserve. Various growth and expense inflation assumptions might be tested with only modest computing facilities. Unit-linked policies providing for a cash penalty on surrender where appropriate, reflecting this in the statutory reserves, may result in negative sterling reserves at early durations—corresponding to borrowing cash from other policyholders to purchase units. It is important in such a case to recompense other policyholders adequately for such borrowing and the solution finally adopted might well be to debit a higher interest charge when sterling reserves are negative than the credit when sterling reserves are positive. No insight into such problems would of course be provided by the Formula approach.

3.4. Formula methods of calculating gross premiums lend themselves readily to the inclusion of a profit margin but less readily to the establishment of logical and consistent profit criteria since no clear view is obtained of either the amount or timing of profit emergence. Furthermore, Formula methods offer no insight into their relationship with other factors—for example, capital employed and degree of risk fulfilment.

3.5. Asset Share methods such as Hoskins's (the Accumulation method) and Anderson's (the Net Present Value method) do not reflect such shortcomings and these methods will be further discussed below.

3.6. Under the Accumulation method, cash flows (ignoring reserve increases)
are accumulated at the earned rate of interest. The resultant asset share is compared with the statutory reserves and/or surrender values at various durations: the excess of the asset share over the statutory reserve reflecting accumulated surplus funds. Profit criteria particularly associated with this method can be expressed as:

(a) Asset share at a certain duration should be at least say 110% of reserves.
(b) Asset share should be at least equal to reserve—or surrender value—by not later than a certain duration.

From this description it is clear that the Accumulation method carried out in this way is little more than a sophistication of the Formula method. Few new concepts have been introduced. However, the method does show more clearly the interaction of the various parameters including lapses.

3.7. The essential concept missing from the Formula approach and from the Accumulation version of the Asset Share approach, is the identification of the impact of the valuation basis on the timing of profit emergence. The need to recover initial capital strain with interest at a rate higher than the assumed earned rate is ignored in these methods. Implicitly capital committed to cover initial strain is recovered with interest only at the assumed earned rate.

3.8. Under the Net Present Value method profits emerging are associated directly with the initial valuation strain at the rate of discount deemed appropriate on that valuation strain.

The stream of yearly book profits emerging against the valuation basis can be examined under the Net Present Value method in either or both the following ways:

(a) The expected yield rate on committed capital funds can be calculated. This is the rate of return which equates at issue the present value of book profits to the first-year loss. Thus it produces a net present value of zero.
(b) If one has identified the required risk discount rate independently from other criteria, the Net Present Value of book profits at that discount rate can be calculated. This Net Present Value can then be tested against a profit criterion. Various profit criteria suggest themselves: an absolute amount, a proportion of sum assured or premium, or perhaps the initial commission paid to the sales agent.

In § 4 will be discussed expense allocation methods. The Net Present Value method fits in well with one of the approaches there suggested—namely, if only those annual expenses and commissions which are 'direct' are included in the profit test then the resultant net present value represents a 'cake' to be bargained for between:

(i) Agents—initial commission.
(ii) Office—overhead expense allowance.
(iii) Shareholders—profit on sale over and above expected return on capital invested.
(iv) Policyholders—benefits including bonuses.

3.9. A possible alternative statement of the Net Present Value profit goal is, by expressing the Net Present Value as a level percentage of premiums to achieve consistency of pricing in a comprehensible way. The Net Present Value would be spread using the risk rate of discount together with mortality and lapse. For example, it is difficult to comprehend the difference between the Net Present Value profit criterion of say 10 per thousand of sum assured at a 15% discount rate, and the same Net Present Value at a 20% discount rate. To say that the profit criterion changes from a margin of 6% of premiums to 7½% of premiums may prove more meaningful.

3.10. It is clear that an internal rate of return method breaks down when there is little or no first-year valuation strain. It also needs to be questioned whether the Net Present Value method has any utility in such a situation. In such a situation profits are still required because of the risks involved in the realization or otherwise of parameter estimates. In the presence of capital needs, the pricing method has to reflect the most significant risk parameters which are the repayment of capital and the remuneration of sales agents. In the absence of capital needs we may still identify remuneration of sales agents as a primary risk parameter but must look further afield for other significant risk parameters. Perhaps the nature of the contract that provides no initial surplus strain needs analysing more carefully. If one postulates that the statutory reserving basis will be more stringent than assumptions necessary to sell policies in a competitive market, it follows that a no-strain product is most likely to arise in an imperfect market. Such a market may not exist very long and perhaps rather than sophisticate pricing one should be rather more strictly commercial in one’s attitude.

4. CONSIDERATIONS AFFECTING THE CHOICE OF ASSUMPTIONS

4.1. The philosophy underlying this section is that a profit-test system is being adopted which consists of examining the amount and incidence of profits emerging against the statutory valuation basis under realistic assumptions. It is implicit in this section that the assumptions are realistic—expected value—assumptions. Risks of adverse deviation are substantially to be dealt with by the use of a risk discount rate appreciably in excess of that obtainable on alternative no-risk investments. The likelihood of achieving parameter estimates—the expanding funnel of doubt—could of course be reflected within the system by using variable rates of discount depending on duration.

4.2. The mortality assumption should cover not only the statistically expected death cost if such is quantifiable but also a contingency margin for catastrophic deviations from such an expected value. Catastrophic deviations are different in kind rather than degree from random fluctuations which should be dealt
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with by the risk discount rate. Catastrophic deviations could result from an epidemic and specific reserves are needed to be accumulated against such contingencies. Costs of reassurance should be separately allowed for. Catastrophe margins can be simply allowed for by increasing the judged expected rates of mortality by an amount equal to the estimated cost of mortality catastrophes—this clearly identifies a margin as additional cost over and above the expected mortality rate, as opposed to an additional premium which will fall into surplus if it is not required. This is consistent with normal practice in the construction of valuation mortality tables.

Some offices will have a clear idea as to the relationship between their actual and anticipated future mortality experience relative to any standard published table. While it is almost invariably necessary for statutory reserve purposes to use the standard tables subject to a simple modification, in assessing profitability the office should use either its own table or such a standard table subject to a more or less extensive modification.

The resultant tables of mortality appropriate for use in pricing and profitability work should be reasonable—that is they should not contain any unreasonable or unrealistic margins in the name of conservatism. Under a profit-test system it is a very simple matter to determine the impact on profits of changes in mortality assumptions and thereby to test the sensitivity of premiums—or bonuses if applicable to such. For example, using a profit-test system it is a relatively simple matter to use a mortality rate \( aq_{x+k} + b \) where \( a, k, \) and \( b \) are variable parameters. For some offices reinsurance costs now form a substantial and perhaps increasing item of expenditure with a resultant impact on profitability. Such costs should be recognized in profit tests. As a result of reinsurance larger policies will be accepted than would have been possible without it. The larger the policy written, the larger is likely to be the contribution of that policy to overhead and profit. Against this must, however, be set the excess of reinsurance premiums payable over commissions and expected claim payments receivable.

Reinsurance for other purposes—such as financing—can also have a major impact on profitability and may be reflected in a profit test. On a strictly practical level, if pricing is done on the basis of certain average policy size which implies the existence of an appreciable amount of risk premium reinsurance, then the excess of reinsurance premiums less commissions over claims should probably be allowed for on the appropriate proportion of such average policy size.

4.3. For almost as long as all actuaries can remember we have been faced by the existence of current interest rates which seem to be at historically high levels and it has seemed appropriate to make allowance for a decline from these levels. It is probable that in deciding upon an actual interest rate for use in a 'commutation function' or formula approach to pricing the actuary will construct an explicit or implicit model as follows: 'current long-term interest rate \( j \) will be likely to fall to a conservative/realistic long-term level \( i \) in \( n \) years: the level equivalent of this is \( i' \), which rate \( I \) will use in factors'. 
Unless a profit-testing model is very detailed it will be unlikely to consider the interaction of:

(a) Different rates of build-up of actuarial reserves on different plans at different ages.
(b) Lapses.
(c) Changing asset and consequent liability values.
(d) Timing of profit emergence.

If pricing is carried out by tracing policies through their currency then a changing interest rate scenario can be encompassed. At a practical level it can be dealt with in at least two distinct ways:

(i) Considering the rate at which new money may be invested each year in the future allowing explicitly for changes in the market value of underlying assets where necessary, as well as asset turnover.

(ii) Considering the total return earned on the fund—allowing for changes in asset values where necessary—on an aggregate basis.

Bearing in mind the nature of the emerging profits distributable to policyholders and/or shareholders, and the need to distinguish between cash and non-cash—or even non-admissible—assets, it would seem that the former method (i) is theoretically preferable.

Similar considerations to those involved in choosing the rate of interest apply in estimating the rate of growth under unit-linked policies. Depending upon the method of product design chosen, many unit-linked policies may result in the rate of interest on 'sterling' funds (being the difference between total actuarial reserves and that part of the reserve which it is necessary to hold in units) may be critical in that sterling reserves might start out to be substantially negative and move to be substantially positive before falling to zero at the maturity of plan. Plainly, reasonable conservatism here must relate to whether the overall sterling funds of the office are positive or negative: overall positive funds may result from some policyholders borrowing cash from others, overall negative funds may result in transfers from shareholders. In the former case considerations of equity may require the use of a high rate of interest, in the latter case shareholders may require the payment of a high rate of interest.

4.4. Since lapses may not result in a 'true profit' to the office it is essential that they be recognized in profit tests and important that they are not materially understated or overstated. 'True profit' is not of course measured by the difference between the surrender value and the valuation reserve at the date of surrender: while no unique definition is discernible the following three quantities may be usefully compared with the surrender value:

(a) Asset share after debiting 'direct' expenses only—i.e. those expenses that are incurred by and only by the issue of a policy.
(b) Asset share after charging an overhead contribution.
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(c) Valuation reserve less discounted value of expected future profits measured against that valuation basis.

A fourth concept is possible and may best be illustrated as follows:

(d) Asset share after deduction of annual planned profits to date.

In any event it is usual for the surrender value to be greater than either (c) or (d) so that surrender values may adversely affect the overall profit expectations of the company for a block of business issued.

In addition lapses may have an impact on the future tax position of the office—this is almost certain to be adverse except in certain exceptional cases since for life fund business issued in a competitive market it may be necessary to fix the office premium on an assumption that expenses are immediately relieved for tax. Most lapses at early policy durations will be on policies upon which expenses exceed investment income to date. The higher level of lapses the less likely it is that the tax assumption will be realized. In the most clear-cut case of a new or fast expanding office with a cumulative excess of management expenses over investment income the lapses may cause such a serious delay in tax relief on those expenses—with a consequent interest-free loan to the revenue—that an office's tax position may be in serious danger.

Particular problems exist at the early durations of most contracts, most noticeably those of a monthly premium nature where surrender values can exceed (a) or (b). In addition one may cite examples where commission is paid in excess of the rate at which it is 'earned'. Almost all lapse studies and standard lapse tables show rates as a function of age and expired duration: increasingly offices are able to generate their own lapse statistics but the selection of appropriate lapse rates for a new company, or for an existing company entering a new market, unfamiliar either by reason of product or sales medium, is a major problem. It is, however, a problem which an office ignores at its peril.

If pricing and projections of operating results are done by a profit-test method then it is a relatively simple matter to use the lapse assumption or standard that one wants to: further, one can consider the sensitivity of results to that lapse assumption. In any event thought will have been given to the problem and the machinery established to subsequently monitor performance against the standard chosen. One of the final features of lapses which must be brought into consideration is the effect on cash flow.

4.5. The area of expenses is perhaps the most difficult of all since an office is required to:

(a) Analyse its own past experience both as regards level and trends of unit cost, by line of business.

(b) Decide upon a method of the allocation/recovery of overheads.

In theory expenses associated with a given product line may be divided into at least four categories—expenses embrace commissions:
The division between direct and indirect expenses is of course the most grey area.

Precise consideration of the use to which the numbers will be put is important: it is vital to realize that overheads will be covered and profit emerge if and only if a sufficient volume of business is sold carrying a known margin between office premiums and marginal unit costs. Any attempt to price by allocating overhead expenses as if they were marginal or 'direct' is doomed to failure unless it stems from this earlier realization. From this consideration an office should seek to include as 'direct' expenses only those which will occur if business is sold.

A radical and indeed interesting idea would be to apply gaming theory to the sale of life assurance: specifically one can view the transaction of life assurance as a game between three players: the prospective policyholder, the sales agents, and the life office. Under this approach commissions would be viewed as an appropriation of margin (struck before deduction of commission) rather than as a direct cost. The overall three-player game comprises two sub-games:

(i) The life office and sales agents combine to sell business to prospective policyholders at the optimum price to maximize the expectation of gain to the coalition -- expectation of gain being measured as the product of the volume of sales and the margin (premium less direct costs excluding commissions) per sale.

(ii) The life office and agents agree on commission and sales attempt levels.

This leads to the theoretical possibility that a life office could adopt a strategy of maximizing profits in absolute terms rather than maximizing unit profits. The rationale for this strategy reflects that the life office must cover its overheads, the higher the level of production achieved by sales agents the higher the absolute level of 'margins' generated and hence the higher the commission the company can pay. Indeed it is apparent that some offices today do adopt such a commission strategy. In any case it is clear that a life office should be aware of the level of sales necessary to cover overheads and make an acceptable profit, and the proportion of total 'margin' the sales medium is requiring.

The above comments tend to support the allocation of overheads in proportion to margins and commissions simultaneously since then, provided the sales medium achieves its income goal, so will the company achieve its overhead contribution and profit goal. It is implicit in the above that margins and commissions are the discounted value of future margins/commissions.

In conclusion it would seem rational for a company to calculate premiums using 'marginal' unit costs and to allocate overheads primarily if not solely in proportion to initial commissions.
4.6. In present circumstances a further issue is the necessary recognition of the effect of inflation on profit-test assumptions. Among the factors that need to be borne in mind are:

(a) In the long run there may be a relationship between the rate of inflation and the rate of interest or capital growth.

(b) While there is little evidence, there may be a relationship between lapse rates and inflation rates.

(c) A relationship clearly exists between non-commission expenses and inflation.

Whatever inflation assumption is made—and one certainly should be made—it should not be unrealistic in the sense that 20% inflation in perpetuity seems unlikely to co-exist with the insurance market as we know it today, and long-term maintenance expenses very large in relation to premiums should perhaps cause the office to recommend surrender to the policyholder on terms more favourable than the policyholder could reasonably expect.

4.7. Apart from difficulties in establishing what tax rates will apply to taxable income whether profits, I-E, etc., there exist other problems:

(a) Does one use marginal or average tax rates?

(b) How will the use of the profit/surplus impact tax rates?

(c) How are unrelieved management expenses to be treated?

In theory at least it is a relatively simple matter to determine the appropriate marginal rate of tax to be assumed when pricing: one projects the office both with and without the marginal policy and deduces the marginal impact.

This of course over-simplifies the real world situation since, first, the marginal tax rates will depend on relative volumes and characteristics of existing and new business—unrestricted writing of new business will cause even the most mature of offices eventually to move from an I-E position to an excess E position. Secondly, new contracts are not written in isolation—a variety of contracts are being written all the time and the marginal combined tax rate is by no means certain to be the combination of the marginal tax rates.

No simple solution exists. Perhaps the most practical answer is to price each product in isolation and then to see if the resultant total office when projected produces satisfactory results when measured against possible alternative strategies.

In certain situations there is no tax penalty attaching to the distribution of profits to shareholders: the most common no-penalty situation is where the I-E tax charge exceeds the notional Case I charge and the shareholder distribution is less than franked income attributable to shareholders.

In other situations, however, distribution penalties may exist. In these cases profits may be ploughed back into writing new business even if the return on such investments is below shareholder normal expectations. In the extreme the profits may merely be left as life fund investments.
To the extent that the existence of unrelieved management expenses can be reflected in the reserve basis, such excess $E$ has considerable value. To the extent that this is not possible the value of the expenses is substantially reduced on two counts:

(i) On a closed fund basis annual profits will emerge in the future to the extent that future $I$ exceeds future $E$. Thus the value of future unpaid tax is correspondingly reduced by the need to discount at a shareholder rate of interest rather than a valuation interest rate.

(ii) Until the excess $E$ is reduced to zero there is an opportunity cost in that new business will be written on less favourable terms than would be the case if an excess of $I$ over $E$ already existed which would permit some or all of initial expenses to be immediately relieved by tax.

In any event the company is plainly deprived of cash equal to the tax relief not yet received. This is of particular importance to an office issuing mainly linked business since while unit reserves may be zillmerized so as to reflect the presence of the excess $E$, nevertheless units will have to be purchased. In this case the 'loan to the revenue' may be a crippling tax disadvantage since insufficient cash may exist elsewhere in the company to be temporarily loaned to permit unit purchases.

4.8. Outside this paper it has often been stated that reserve bases affect only the timing of profit emergence and not the level of ultimate profits. This can only be true if the time value of money be taken as the rate of interest on accumulated assets. It is certainly not true if one's judgment as to profitability reflects discounted emerging annual distributable profits at a rate of discount greater than the asset return rate.

Of the several methods of profit measurement and hence premium calculations, the Formula method and the accumulation version of the Asset Share method ignore the impact of reserve bases on the incidence of emerging profits. Indeed the Formula method is a shorthand version of Hoskins's method ignoring lapses. Only the Net Present Value version of the Asset Share method does reflect the effect of reserves on emerging profits.

It is the fundamental theme of this paper that any method of pricing or assessment of profitability must take account of both level and timing of realized profits: whether distributed to policyholders and/or shareholders, or redeployed in supporting new business. Since the only situation in which the value of profits is independent of the reserve basis is when the rate at which profit is discounted is equal to the assumed asset interest rate, and since the realization of expected future profits carries a greater degree of risk than the realization of expected income on invested assets, it is unrealistic to adopt a standard or measure of profitability for pricing which is independent of the reserve basis. This leads us to identify two distinct viewpoints regarding the appropriate level of valuation reserves:
Pricing and Profitability

(a) The actuary's viewpoint: the reserves must be adequate to demonstrate not only solvency and adequacy but also the ability to meet policyholder reasonable expectations. In particular the reserves must at least meet minimum statutory standards.

(b) The finance director's viewpoint: over-strong reserves increase the need for capital funds and can inhibit the expansion of the office which would otherwise be attainable.

The extent to which these two considerations raise questions regarding professional judgment is not tackled in this paper.

4.9. Random fluctuations in such parameters as interest, capital values, mortality and surrenders are dealt with in the long run by the proper estimation of values of those parameters and are explicitly provided for by the use of an adequate statutory valuation basis. Catastrophe margins essentially have nothing to do with random fluctuations: they are best regarded as providing a fund against such catastrophic events which cannot be regarded as part of the normal year-to-year variation in parameters—an epidemic was cited above when considering mortality assumptions. Such catastrophe margins cannot be allowed to fall into surplus if in a given year they are not required. Instead they should be regarded as an item of outgo in assessing profitability and assumed to build up in a catastrophe or contingency fund.

5. IMPLICATIONS FOR THE EVALUATION OF A COMPANY

5.1. Evaluation of a life company is a mixture of art and science. It is the purpose of this section purely to examine the application of methods discussed within this paper to the scientific evaluation of a life company.

5.2. There are many reasons why the scientific evaluation of the worth of all or part of a life assurance company is made. The four which lend themselves most readily to application of the methods of analysis discussed elsewhere in this paper are:

(a) Purchase or sale.
(b) Determination of an equitable price for the raising of new capital.
(c) Evaluating a strategy in relation to new business.
(d) Determining the performance of management in maintaining or increasing the value of a particular sector of the company. For example, to assess whether the money spent in developing a new sales outlet has been repaid in terms of the worth of business already sold plus the ability of the sales outlet to deliver in the future.

The Net Present Value method applies naturally and logically to the evaluation of the worth of any enterprise, or indeed any part of any enterprise. In general the approach is to estimate future profit flows after tax and discount these at a
specified risk rate of return judged appropriate in the circumstances. The resultant worth is the amount which could be paid for the enterprise to realize the specified rate of return.

5.3. In a valuation of a life company the following parts would be considered:

(a) Statutory net worth—adjusted for such items as non-admitted assets.
(b) The value of existing business.
(c) The value of 'sales plant'—productive capacity.
(d) Any other items.

Existing business has a value only in terms of profits emerging against the valuation basis. A gross premium valuation in order to estimate the size of the office's 'estate' has no part in a scientific valuation process since the 'estate' has no objective reality.

5.4. Often it is necessary to evaluate an office before it has reached a 'mature state'. Mature state in this context refers to whether or not the company is living within its long-term acquisition and maintenance expense standards. If the office is not in a mature state and one has confidence that the company will in due course reach maturity, perhaps the simplest approach is to calculate future profits as if the mature state expenses were being currently incurred. It is then relatively simple to make specific provision for excess expenses incurred—for example, the current level of expense overrun may be readily established and this projected to decrease in a manner and over a period which can be established based on trends and following discussions with management and knowledge of the industry. Such an approach to expense overruns clearly has its dangers—in particular it is important not to underestimate the difficulty of achieving the transition from an expense overrun position to a mature state position.

5.5. Provided that an office is currently in the position of having investment income in excess of management expenses in the life fund, and making Case VI profits in other funds, there is little difficulty in determining the appropriate tax treatment of investment income and expenses—unless of course there is good reason to anticipate a change in the tax status. Such a clear-cut tax position is, however, rare: the most common situation in practice is an excess of management expenses with a consequent delay of several years anticipated before the life fund may be expected to be taxed on an I-E basis.

The existing carryforward of unrelieved management expenses can be straightforwardly valued if the company is regarded as closed to new business. To the extent that the statutory reserves can be based on a gross rate of interest until the carryforward expenses are exhausted, the nominal tax relief expected can be discounted at the net valuation interest rate. Any excess management expenses beyond those capable of reflexion in the statutory reserve basis must effectively be discounted at the shareholders' required rate of return. If carryforward expenses are $E$ and $(I-E)$, is the excess of investment income over management expenses each year, while the appropriate discount rate, $j$, is such that
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\[ v = 1/(1+j) \] (where \( j \) is either the net valuation interest rate or the shareholders' required interest rate as appropriate) then

Value of \( E \) is

\[
\sum_{t=1}^{n-1} (I-E)_t T_t v^t
\]

Where \( n \) is such that

\[
\sum_{t=1}^{n-1} (I-E)_t < E < \sum_{t=1}^n (I-E)_t
\]

and \( T_t \) is the tax rate applicable in year \( t \).

This approach, that of a closed fund, may not be appropriate in reality. Viewing the company as a going concern it is clear that the existence of carry-forward expenses has an adverse effect on new business priced on a net interest/net expenses basis—since an assumption that expenses are relieved immediately necessarily implies an excess of \( I \) over \( E \) of at least the new business expenses. Indeed, if there were no carryforward expenses, then new business would be more profitable than if there were such carryforward expenses. We may say that there is an opportunity cost in an unrelieved management expense asset.

Two approaches would then be possible:

(a) To value existing unrelieved management expenses on a first in first out basis, taking credit for tax relief on new business at some reduced rate reflecting the period before relief is obtained taking account of the existence of the expense asset.

(b) To value new business on the basis that there is no carryforward expense asset and then reducing the value of the carryforward expense asset by discounting it to the point at which there is sufficient excess of investment income over expenses to cover not only new business expenses but also the expense asset.

In practice, the first in first out approach may be found to be both more comprehensible and more practicable.

5.6. There are innumerable methods for evaluating productive capacity, including:

(a) To forecast new sales for the company by line of business and product mix over some future period—either just for existing sales agents or including new agents and new lines. Determine the net present value of each year's sales at the point of sale using the profit-test system. Discount each year's sales net present value to the current date at the discount rate of interest. The result is the value of future sales. It may be appropriate to use a greater discount rate to discount back the Net Present Value of each year's sales than to discount back the profits arising from a year's sales. One might also introduce 'funnel of doubt' criteria.

(b) To estimate the multiple 'number of years sales' to be applied to the value obtained for a given year's sales. The resultant amount value of
sales times multiple—produces an estimated value of the sales plant. Naturally, different marketing media are associated with different multipliers and indeed these multipliers also vary by degree of maturity of that marketing medium. For an example of direct sales force see Appendix C.

Also of significance may be contingency reserves, reinsurance arrangements—in particular coinsurance arrangements—and potential sales areas—for example, unexploited franchise groups, associated unit trust management companies, banking connexions, etc.

5.7. While in the final analysis the relevant criterion of worth is the net price realizable in practice, a 'scientific' valuation along the lines described does provide a useful and independent bench mark of value and perhaps negotiating position. A much simplified valuation of a company is contained in Appendix D in order to illustrate the practical application of the methods discussed.

6. THE PROFIT-TEST SYSTEM REVISITED

6.1. Apart from the uses discussed elsewhere in this paper of a profit-test method in the areas of calculation of minimum premiums and scientific office evaluation, there are many other possible applications of the method and consequent benefits from its use.

6.2. For mutual and quasi-mutual offices a profit test can be particularly valuable in (a) comparing the incidence of annual bonus costs with the incidence of pre-bonus expected book profits, (b) comparing the present value of bonuses based on current bonus levels with the present value of pre-bonus expected book profits, and (c) providing a realistic basis for the projection of operating results. It is then feasible to examine equity between different generations of policyholders, to determine under what conditions current bonus can be supported in the future; examining the scope for, and level of, bonus costs for a single plan/age cell or an entire line of business against emerging profits, etc. A profit-test method, by its very nature, compels management to set standards of expected performance as to all relevant parameters—the Formula method does not do this. The explicit mapping of anticipated progress provided by the profit-test method may be likened to flying an aeroplane over well-charted territory. The lack of such explicit mapping under the Formula method is tantamount to taking off in an aeroplane without even knowing one's destination—even if objectives set are reasonable the method provides little or no definition of individual parameter standards or incidence of profit emergence.

6.3. The profit-test system successfully builds a bridge between statutory returns and meaningful management financial reporting—any system of financial reporting that is to be meaningful to parties other than the regulatory authorities may perhaps have to include the following features:

(a) Acceptability and comprehensibility.
6.4. The basic logic, and specific algebra, underlying the profit-test method lends itself readily to the projection of operating results in order to assist actuaries to produce information in a form readily comprehended by non-actuaries—in particular the concept of a life assurance contract as a sequence of modules of a revenue account is particularly helpful in this respect. The approach lends itself well to incorporation as a building-block in a total model of an office, such a model embracing:

(a) Sales forecast model.
(b) Manpower model.
(c) Tax model.
(d) Asset model.
(e) Revenue account model.

7. PRICING CONSIDERATIONS REVISITED—MUTUAL/QUASI-MUTUAL OFFICES

7.1. If there were no 'new business strain' then there would be no technical constraint on the volume of business that an office could transact, the resultant bonuses would be fully determined in terms of investment return, mortality and expenses—leaving aside surrender profits and losses—using traditional expected value commutation columns at the actual investment returns achieved.

7.2. Let us symbolize as follows: if no new business strain exists then premium $P$ and investment return $j$ support bonuses $B$ irrespective of new business growth rate. If we now introduce new business strains and attempt to maintain $B$ for given $P$ and $j$ we shall find that the estate is impaired if the attempted new business growth rate exceeds the implicit return $j$ on new business strain: impaired to the extent that new business strain less releases of new business strain—i.e. that surplus not used to increase bonuses because it is amortizing the initial loss—keep on increasing forever. (See Appendix E.)

7.3. Thus if new business strain exists and a sharp and sustained increase in new business growth is either necessary or desirable, it is likely that unless the valuation basis or bonus scale is altered—for example, valuation basis weakened or bonus scale 'tilted'—then the overall bonus rates will have to decrease (or premium rates increase without a corresponding increase in bonus rate). Revisiting pricing considerations in respect of 'rates of return' we may see that in a quasi-mutual office the concept of return on capital is as relevant as in a proprietary office and that the minimum discount rate is closely related, if not actually equal, to the planned rate of growth of new business.

7.4. Two further matters arise for consideration: first, the time horizon of
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with-profit policyholders is very specific—the lifetime of their contracts—but shareholders' time horizon may be either shorter or longer than this. Furthermore, sales agents will wish to see evidence of a company's ability to sustain its bonus forecasts. Secondly, if planned growth rates are sustainable without use of external capital either because of size of estate or pricing practice, it will be found that since new business strain is an investment for policyholders, the relevant rates of return are related to alternative policyholder asset investments at that time, allowing for different degrees of risk, and hence are likely to be lower than shareholder target rates of return. One might speculate, therefore, that in low growth mutual or quasi-mutual offices one could anticipate either higher bonuses (or lower premiums), or stronger valuation bases than in proprietary offices.

8. SUMMARY AND PURPOSE

The purpose of this paper has been to examine briefly the financial framework within which life assurance companies operate with primary reference to methods of setting premiums and measuring profitability. It has been argued that the concept of profitability expectation embraces both with-profit policies and non-profit policies. The author's aim has been to place the main emphasis of the paper on a series of techniques being increasingly widely used within the U.K. as well as elsewhere. Perhaps a consensus of methodology and techniques will eventually emerge: this paper claims no originality; it is hoped, however, that it will assist the formation of such a consensus.

A short bibliography is given below but the author would like to express his gratitude to all the many people who have taken the time to discuss their ideas and review his approaches.

BIBLIOGRAPHY


A.1. ALGEBRA FOR PROFIT TESTING

Below is shown the type of algebraic formulae that may be used in profit testing two typical contracts:

(a) Orthodox non-profit endowment assurance (A.2).
(b) Type A unit-linked policy (A.3).

The algebra can be done in a variety of ways and can become quite complex. That shown here is merely one relatively simple version of the algebra possible for an annual premium policy. A similar method of analysis can be used to develop the algebra for monthly premium policies and master revenue accounts for incorporation in model office projections.

A.2. NON-PROFIT ENDOWMENT ASSURANCE

All items assumed to arise at the beginning of policy year \( t \), unless otherwise stated.

\[ P_t \quad \text{Office premium.} \]
\[ C_t \quad \text{Commission.} \]
\[ E_t \quad \text{Expense.} \]
\[ DB_t \quad \text{Death benefit payable—assumed to occur mid-year.} \]
\[ T_t \quad \text{Tax rate on } I \text{ and rate of relief on } E. \]
\[ i_t \quad \text{Interest rate, before tax, earned on total assets.} \]
\[ S_t \quad \text{End-year reserve per unit in force at end of year } (i_t \text{ and } S_t \text{ based on unchanging asset values).} \]
\[ CV_t \quad \text{Surrender value payable—assumed to occur at end-year.} \]
\[ q_{it}, w_{it} \quad \text{Dependent rates of mortality, lapse.} \]
\[ D_t \quad \text{Cost of waiver of premium or other ancillary benefits, assumed to occur at mid-year.} \]
\[ AS_t \quad \text{Excess of asset share over statutory reserve at end-year per unit in force at end of policy year.} \]
\[ B_t \quad \text{Surplus or deficit in year taken as at the beginning of the year.} \]
\[ v_t \quad 1/(1+i_t(1-T_t)). \]
\[ v_t' \quad v_t^{\frac{1}{2}}. \]
\[ y_t \quad \text{In force at end of policy year } t, \text{ per unit issued.} \]

We then have, taking begin-year values for \( B_t \):

\[
B_t = P_t - \{S_t(1-q_t-w_t)v_t-S_{t-1}\} - (C_t+E_t)(1-T_t) - q_t DB_t v_t' - w_t CV_t v_t - D_t v_t'.
\]
\[
AS_t = (AS_{t-1}+B_t)(1+i_t(1-T_t))/(1-q_t-w_t)
\]
The excess of investment income over expenses, \((I-E)_t\), per unit in force begin-year may be identified as:

\[
(I-E)_t = i_t \{S_{t-1} + P_t - (C_t + E_t) (1 - T_i) - \frac{1}{2}q_t \ DB_t - \frac{1}{2} \ D_i \} - (C_t + E_t)
\]

if \(B_t\) is assumed to remain in the fund until end-year.

Hence a policy can readily be treated as self-supporting in the sense that:

\[
T_i = 0 \text{ until } z_t > 0 > z_{t-1}
\]

where

\[
z_t = z_{t-1} + (I-E)_t y_{t-1}
\]

and

\[
y_t = y_{t-1} (1 - q_t - w_t)
\]

\[
y_0 = 1
\]

We can now calculate various items, for example \(PV_t\), the present value at time \(t\) (prior to the \(t\)th premium payment) per unit in force at that time, of future profits \(B_t\), as follows:

\[
PV_t = \frac{1}{ny_{t-1}} \sum_{k=t} B_k y_{k-1} / (1+j)^{k-t}
\]

where \(j\) is risk discount rate.

In Appendix B, § B.1 illustrates the algebra in action.

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**A.3. Type A Unit-Linked Contract**

- \(a_t\): Allocation to unit fund as proportion of premium.
- \(u_t\): End-year unit reserve—the meaning of \(S_t\) now becomes end-year 'sterling' or 'non-unit' reserve—unlike orthodox business, \(S_t\) may be negative provided in the case of statutory reserves the overall reserve does not fall below any guaranteed surrender value.
- \(g_t\): Unit-price growth rate.
- \(f_t\): Annual management charge on unit fund, assumed to arise at end-year on end-year unit fund.
- \(b_0\): Bid/offer spread on unit fund.
- \(b_1\): Either discount given to company by unit-trust managers plus proportion of eventual 'churning' profit, or initial charge accruing to company, as appropriate.
- \(SA_t\): Basic sum assured.
- \(L_t\): Cash deduction from value of units on surrender, so that surrender benefit is \(u_t - L_t\) assumed to arise at end-year.
- \(G_t\): Guarantee cost as a proportion of premium.
If the death benefit is taken as the greater of the basic sum assured and the unit fund, then:

\[ B_t = P_t - a_t P_t (1 - b_t) \]
\[ - \{ S_t (1 - q_t - w_t) v_t - S_{t-1} \} \]
\[ - (C_t + E_t) (1 - T_t) \]
\[ - G_t P_t \]
\[ - q_t v_t \{ \max (0, SA_t - u_t / (1 + g_t)) \} \]
\[ + w_t L_t v_t \]
\[ + f_t u_t v_t (1 - \frac{1}{2} q_t) \]
\[ - D_t v_t' \]

where \( u_t = (u_{t-1} + P_t a_t (1 - b_0)) (1 + g_t) \)
\[ u_0 = 0. \]

This would need slight modification if the link is to an internal fund rather than a unit trust.

The definition of \( S_t \)—the sterling reserve—requires further consideration. Initially the intention may be to hold reserves equal to the surrender value, i.e. \( S_t = -L_t \).

This is possible provided (a) sufficient cash exists elsewhere in the company and it is appropriate to lend it to fund unit purchases (if this is not possible then \( S_t \) must be set to zero), and (b) no losses are likely to occur in any future year.

Thus the procedure is as follows:

(i) Set \( S_t \) equal to either \(-L_t\) or 0 as appropriate.
(ii) Choose appropriate sterling reserve assumptions—these constitute the statutory reserving basis and may or may not be somewhat constrained.
(iii) Carry out a profit test to determine whether on these assumptions any \( B_t \) is ever negative after the first year.
(iv) If any negative \( B_t \) exists, say \( B_r \), then set

\[ S_{r-1} = S_{r-1} - B_r \]

so that \( B_{r-1} = B_{r-1} + (1 - q_{r-1} - w_{r-1}) B_r v_{r-1} \) (in practice \( w_{r-1} \) will almost certainly be zero under statutory assumptions).

This process is continued backwards until all negative book profits (after the first year) have been zeroed.

A final step is then necessary in that, using these sterling reserves, a further profit test will be run on ‘realistic’ assumptions to give the actuary comfort that these reserves are indeed appropriate: such is the interaction of changing assumptions on a unit-linked contract that it is by no means certain that a set of assumptions, each of which is appropriate, are appropriate overall.

In Appendix B, §§ B.2, B.3 and B.4 illustrate the algebra in action.
ILLUSTRATIVE PROFIT TESTS USING THE ALGEBRA DEVELOPED IN APPENDIX A

Column definitions, using the Appendix A symbols, are

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALLOC</td>
<td>(a_t P_t(1 - b_t)).</td>
</tr>
<tr>
<td>PREM</td>
<td>(P_t).</td>
</tr>
<tr>
<td>NET EXP</td>
<td>((C_t + E_t) \cdot (1 - T_t)).</td>
</tr>
<tr>
<td>G+WOP</td>
<td>(D_t \cdot v_t).</td>
</tr>
<tr>
<td>DEATHS</td>
<td>(DB_t \cdot q'_t \cdot v'_t) in B.1, otherwise (q_t \cdot v_t \cdot \max{(0, SA_t - u_t/(1 + g_t)^t}}.</td>
</tr>
<tr>
<td>Lapses</td>
<td>(CV_t \cdot w_t \cdot v_t).</td>
</tr>
<tr>
<td>FEES</td>
<td>nil in B.1, otherwise (f_t \cdot u_t \cdot (1 - \frac{1}{2} u_t) q'_t v_t).</td>
</tr>
<tr>
<td>↑RES</td>
<td>(S_t \cdot (1 - q'<em>t - w_t) v_t - S</em>{t-1}).</td>
</tr>
<tr>
<td>PROFIT</td>
<td>B.1: ((\text{PREM}) - (\text{NET EXP}) - (G + \text{WOP}) - (\text{DEATHS}) - (\text{LAPSES}) - (\uparrow \text{RES})).</td>
</tr>
<tr>
<td></td>
<td>B.2 to B.4: ((\text{PREM}) - (\text{ALLOC}) - (\text{NET EXP}) - (\text{DEATHS}) - (\text{LAPSES}) + (\text{FEES}) - (\uparrow \text{RES})).</td>
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<tr>
<td>DEATH</td>
<td>(1,000 \cdot q_t (q_t = q'<em>t[x</em>{t+1} - t - 1] in standard mortality table notation)).</td>
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<td>LAPSE</td>
<td>(100 \cdot w_t).</td>
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<td>1-Q-W</td>
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<td>I/F END</td>
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<td>STERL</td>
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<tr>
<td>PVFP</td>
<td>(PV_t).</td>
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</table>
B.1. Non-profit endowment assurance

Age 35
Term 10
Premium 85
Sum assured 1,000
Reserve 4·5% Interest, 2·5% Zillmer, 95% limit
Valuation premium 80·75, Unlimited net premium 81·11
Cash value 5% Interest, 3% Zillmer
Reserve and cash value mortality A 49/52
Commission: initial 25% year 1, 0% year 2
renewal 2·5%
Expenses: initial: 0 per mille
27·5%
0 per policy
maintenance: 6 per policy
Inflation 5% throughout
Experience mortality A 67/70 select
Lapses, 15, 10, 5, 5, 5, 5, ...
Sterling interest 9·6% throughout
Tax 37·5% throughout
Risk discount rate 15%

<table>
<thead>
<tr>
<th>YEAR</th>
<th>PREM</th>
<th>NET EXP</th>
<th>G + WOP</th>
<th>DEATHS</th>
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PV book profits at 10% = 14·18; at 20% = 6·92.
B.2. Type A unit-linked: reserve = surrender value. (need for extra reserves shown by existence of losses in years 2 to 5 inclusive)

Age 55
Term 10
Premium 200
Sum assured 1,500
Allocation as % premium 50, 100
Growth 3%
Commission: initial 25% year 1, 0% year 2
renewal 0%
Expenses: initial 25%
maintenance 16 per policy
Inflation 0% throughout
Fees: initial 5.5%
annual 0.5%
Mortality A 67/70 ultimate
Lapses 0, 0, 0, 0, 0, 0...
Sterling interest 6.4% throughout
Tax 37.5% throughout
Risk discount rate 15%

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PV book profits at 10% = 18.68: at 20% = 14.08.
B.3. As B.2, but with a sterling reserve set up so as to eliminate losses

Most columns are identical to B.2.: the differences are as follows:

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PV book profits at 10% = 16.52; at 20% = 9.11.
**Pricing and Profitability**

**B.4. Uses B.2 sterling reserves but with ‘realistic’ assumptions rather than the ‘statutory’ assumptions used in B.2, B.3**

- **Age 55**
- **Term 10**
- **Premium 200**
- **Sum assured 1,500**
- **Allocation as % premium 50, 100**
- **Growth 6%**
- **Commission:** Initial 25% year 1, 0% year 2
- **renewal 2·5%**
- **Expenses:** initial 25%
- **maintenance 8 per policy**
- **Inflation 10% for 10 years then 5%**
- **Fees:** initial 5·5%
- **annual 0·5%**
- **Mortality A 67/70 select**
- **Lapses 20, 10, 7·5, 5, 5...**
- **Sterling interest 10% throughout**
- **Tax 37·5% throughout**
- **Risk discount rate 15%**

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PV book profits at 10% = 29·34; at 20% = 25·47.
In general the evaluation approach is two-pronged: (a) calculate present value at issue of book profits on business sold by a typical agent; (b) calculate expected future sales from a typical agent at different points in his career.

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<td>452</td>
<td>3.7</td>
</tr>
<tr>
<td>Year 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-705</td>
<td>0.333</td>
<td>450</td>
<td>106</td>
<td>421</td>
<td>4.0</td>
</tr>
<tr>
<td>5</td>
<td>-613</td>
<td>0.250</td>
<td>480</td>
<td>74</td>
<td>315</td>
<td>4.3</td>
</tr>
<tr>
<td>6</td>
<td>-533</td>
<td>0.200</td>
<td>500</td>
<td>53</td>
<td>241</td>
<td>4.6</td>
</tr>
<tr>
<td>7</td>
<td>-464</td>
<td>0.167</td>
<td>514</td>
<td>40</td>
<td>188</td>
<td>4.7</td>
</tr>
<tr>
<td>8</td>
<td>-403</td>
<td>0.143</td>
<td>525</td>
<td>30</td>
<td>148</td>
<td>4.9</td>
</tr>
<tr>
<td>9</td>
<td>-351</td>
<td>0.125</td>
<td>533</td>
<td>23</td>
<td>118</td>
<td>5.1</td>
</tr>
<tr>
<td>10</td>
<td>-305</td>
<td>0.111</td>
<td>540</td>
<td>18</td>
<td>95</td>
<td>5.3</td>
</tr>
<tr>
<td>11</td>
<td>-265</td>
<td>0.100</td>
<td>545</td>
<td>14</td>
<td>77</td>
<td>5.5</td>
</tr>
<tr>
<td>12</td>
<td>-231</td>
<td>0.091</td>
<td>550</td>
<td>11.6</td>
<td>63</td>
<td>5.5</td>
</tr>
<tr>
<td>13</td>
<td>-200</td>
<td>0.082</td>
<td>554</td>
<td>9.1</td>
<td>51</td>
<td>5.6</td>
</tr>
<tr>
<td>14</td>
<td>-174</td>
<td>0.077</td>
<td>557</td>
<td>7.5</td>
<td>41.9</td>
<td>5.6</td>
</tr>
<tr>
<td>15</td>
<td>-152</td>
<td>0.071</td>
<td>560</td>
<td>6.0</td>
<td>34.4</td>
<td>5.7</td>
</tr>
<tr>
<td>16</td>
<td>-132</td>
<td>0.067</td>
<td>563</td>
<td>5.0</td>
<td>28.4</td>
<td>5.7</td>
</tr>
<tr>
<td>17</td>
<td>-115</td>
<td>0.063</td>
<td>565</td>
<td>4.1</td>
<td>23.4</td>
<td>5.7</td>
</tr>
<tr>
<td>18</td>
<td>-100</td>
<td>0.059</td>
<td>566</td>
<td>3.3</td>
<td>19.3</td>
<td>5.8</td>
</tr>
<tr>
<td>19</td>
<td>-087</td>
<td>0.056</td>
<td>568</td>
<td>2.8</td>
<td>16.0</td>
<td>5.8</td>
</tr>
<tr>
<td>20</td>
<td>-075</td>
<td>0.053</td>
<td>569</td>
<td>2.3</td>
<td>13.2</td>
<td>5.7</td>
</tr>
</tbody>
</table>

Column 1: $v^r$ at 15% where $r$ is mid-period.
Column 2: survival table for agents per 1 recruited at time 0.
Column 3: illustrates rising productivity of agents over time, index could be interpreted as, for example, monthly rate of basic commission production.
Column 4: value of each period sales (product of first 3 columns), again as an index.
Column 5: 'back-sum' of column (4) making an arbitrary allowance for the 'over 20 years service' agent productivity.
Column 6: ratio column (5) divided by column (4), which illustrates how the ratio of 'the value of future sales' to 'the value of this period sales' changes with growing agent 'maturity'.

Consider an agent who has been with the company for about seven months and producing business with a value, at issue, to the company at a rate of...
£1,000 p.a. A value of £3,600 would be placed on that agent according to the above theory.

For an example of the incorporation of this theory in a company evaluation, see Appendix D.
D.1. Data provided
(a) Balance sheet—all figures in £000:

<table>
<thead>
<tr>
<th>Share capital</th>
<th>Units</th>
<th>Establishment account</th>
<th>Sterling assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,500</td>
<td></td>
<td>(720)</td>
<td>1,348</td>
</tr>
<tr>
<td>Life fund</td>
<td>3,102</td>
<td></td>
<td>250</td>
</tr>
<tr>
<td></td>
<td>3,882</td>
<td></td>
<td>3,882</td>
</tr>
</tbody>
</table>

The balance sheet asset value reflects the reduction of £100,000 as a result of the asset valuation regulations. Unrelieved management expenses carried forward amount to £800,000.

(b) Sales information. Two-thirds of sales made through brokers, one-third through a 100-man direct sales force whose lengths of service with the company are distributed as follows:

<table>
<thead>
<tr>
<th>Length of Service</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-3 months</td>
<td>50</td>
</tr>
<tr>
<td>3-6 months</td>
<td>35</td>
</tr>
<tr>
<td>6-12 months</td>
<td>10</td>
</tr>
<tr>
<td>over one year</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

Current rate of production, in £000 annualized:

<table>
<thead>
<tr>
<th></th>
<th>Annual premium</th>
<th>Basic commission</th>
<th>Present value at issue of profit, at 15%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orthodox</td>
<td>400</td>
<td>100</td>
<td>47</td>
</tr>
<tr>
<td>Unit-linked</td>
<td>2,000</td>
<td>500</td>
<td>271</td>
</tr>
<tr>
<td></td>
<td>2,400</td>
<td>600</td>
<td>318</td>
</tr>
</tbody>
</table>

D.2. In-force calculation

<table>
<thead>
<tr>
<th>Class</th>
<th>Duration</th>
<th>Annual premium (£000)</th>
<th>Unit (£000)</th>
<th>Sterling (£000)</th>
<th>PVFP (£000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EA non-profit</td>
<td>1</td>
<td>300</td>
<td>--</td>
<td>230</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>200</td>
<td>--</td>
<td>356</td>
<td>60</td>
</tr>
<tr>
<td>Type A unit-linked</td>
<td>1</td>
<td>1,500</td>
<td>752</td>
<td>164</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1,000</td>
<td>1,532</td>
<td>58</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3,000</td>
<td>2,284</td>
<td>818</td>
<td>320</td>
</tr>
</tbody>
</table>
D.3. Value of company

<table>
<thead>
<tr>
<th></th>
<th>(£000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net worth</td>
<td>780</td>
</tr>
<tr>
<td>Non-admitted asset</td>
<td>100</td>
</tr>
<tr>
<td>Adjusted net worth</td>
<td>880</td>
</tr>
<tr>
<td>Existing business (1)</td>
<td>320</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1,200</td>
</tr>
<tr>
<td>Broker outlets (2)</td>
<td>424</td>
</tr>
<tr>
<td>Direct sales force (3)</td>
<td>371</td>
</tr>
<tr>
<td>Tax asset (4)</td>
<td>100</td>
</tr>
<tr>
<td>Total company value</td>
<td>2,095</td>
</tr>
</tbody>
</table>

Notes. (1) See in-force calculation.
(2) $2 \times \text{annual profits (see (6))} = 2 \times \frac{2}{3} \times 318 = £424,000$.
(3) $3\frac{1}{2} \times \text{annual profits (see (6))} = 3\frac{1}{2} \times \frac{1}{3} \times 318 = £371,000$.
(4) £800,000 \times 0.375 \times \frac{1}{3} = £100,000 to allow for delay in obtaining tax relief and the fact that new business is not yet obtaining immediate tax relief on expenses despite assumption in profit testing.
(5) No expense overruns assumed—i.e. maintenance and issue expenses incurred are in line with assumptions used in profit testing.
(6) Factors of 2 and 3\frac{1}{2} are essentially judgment items: in practice a range would be displayed—although the direct sales force factor might be derived as in Appendix C.
Consider an office selling policies with an emerging profit pattern, per unit issued, of:

\[
\text{year 1} : \quad F
\]

\[
\text{year } t (>1) : \quad y_{t-1}B_t
\]

such that

\[
F = y_1B_2v + y_2B_3v^2 + \ldots
\]  \hspace{1cm} (1)

where

\[
v = 1/(1+i)
\]

If new business growth rate is \(g\) then eventually the result of a single year's revenue account will be (in year \(n\)):

loss \(F(1+g)^{n-1}\) from this year’s new business

profit \((1+g)^{n-2}y_1B_2 + (1+g)^{n-3}y_2B_3 + \ldots\) from previous years’ issues

overall result is \((1+g)^{n-1}\left\{-F + y_1B_2v_1 + y_2B_3v_1^2 + \ldots\right\}\ldots\) \hspace{1cm} (2)

where \(v_1 = 1/(1+g)\).

We see from (1) that (2) is zero if \(v = v_1\), i.e. \(i = g\), and that if \(g > i\) losses will always be made.

**Conclusion.** Unless the rate of return earned on new business strain is at least as great as the long-term new business growth rate, the office will eventually run out of either estate or capital as appropriate.
Mr D. Shore: In his synopsis the author explains that the paper results from an apparent gap in the United Kingdom actuarial literature dealing with the concept of profit in life assurance. He discusses two important concepts. First, the philosophy of return of capital in life assurance and, secondly, techniques for evaluating that return. The nature of the techniques explains why there has been a gap in the philosophy: whereas generations of actuaries have used commutation functions to limit the volume of calculations, the computer now allows the modern actuary to carry out investigations which would have been beyond the dreams of his predecessors.

The Formula or commutation approach provides no insight into the emerging effect of the many variables affecting a life office. In relatively stable conditions and with low inflation that may not have been crucial as the experience of the actuary could provide the insight. However, with the level of inflation over the last few years, it is difficult to see any realistic alternative to the techniques in the paper.

My only criticism of the paper is that I do not feel it is quite in a form which the majority of the students could refer to and readily understand. I would have preferred to have seen a less condensed paper with more attention paid to the very important appendices.

I should like to make a suggestion about terminology. Whilst I appreciate the author’s reasons and agree with his methods, I dislike the term ‘net present value’. I think it is confusing: it sounds like a Formula or commutation function approach. I would prefer the techniques to be referred to as ‘emerging cost methods’ even although the results may then be translated into a net present value. I first started using emerging cost techniques to price unit-linked contracts because I believed them to be the only realistic way to allow for the many variables involved, and to see what the effects would be year by year, at different rates of inflation, unit growth, dividend yield and required yield on capital together with assumptions as to when an office in a gross tax position might start to pay tax.

Having developed the programs, it seemed to be totally logical to apply similar tests to pricing non-unit linked business. That also provides a new approach to the problem of what rate of interest to use for an office in a gross position in a period of historically high interest rates. The ‘emerging cost’ approach can take full credit for current high level of interest rates, and let the expanding funnel of doubt reduce the interest rate year by year to an envisaged long term rate. Either the office can be considered to go net in a given year or, alternatively, a policy can be considered in isolation and tax allowed to become payable when the policy’s accumulated income exceeds its accumulated expenses.

The author states that the reserve basis can affect the level of profits. I would take that one step further and suggest that statutory valuation on a net premium basis can itself be a direct cause of loss to an office. For example, the investment dating requirements for a net premium valuation can be somewhat shorter than for a corresponding gross premium valuation, and if the actuary attempts to match on a net premium basis, then the office in real terms is vulnerable to a fall in interest rates. Perhaps we ought to be considering statutory valuations on an ‘emerging profit’ basis.

The question is raised whether shareholders and policy holders should expect different yields on their capital. My first reaction was, why should they? On reflection, perhaps there is a distinction in that shareholders may be willing to accept a lower yield on the capital rather than see an office stop trading. That leads directly to the old point that the management of a mutual office ought regularly to consider whether the interests of the with-profits policyholders would be better served by ceasing to take on new business. That point is particularly relevant in a period of high rates of interest and inflation.

I question the application of gain theory as used on expenses, as it seems to me to be an over-simplification. The theory appears to assume that the policyholder has only two options, to buy or not to buy; but of course he has the option to buy elsewhere, through another agent or with another company. Similarly, the agent has the option to sell the product of another
company, and the company has the option to use other agents or, alternatively, to go direct to the policyholder, through direct salesmen or newspaper advertising.

However, I think that the conclusion is very interesting, and now that most offices use a premium-related initial commission structure geared to premium terms, relating indirect expenses to initial commission may provide a useful compromise in practice between relating expenses to sum assured, and relating them to premiums.

The suggestion of calculating pivotal premiums by emerging cost techniques and using the Formula method to calculate intermediate values relegates the Formula method to a graduation function. It may be sufficient simply to equate the pivotal premiums to a pure net premium and solve for the rates of interest. A simple interpolation function can then be applied to the pivotal rates of interest to produce the graduated premium rates. Refinements of that method can very easily be introduced assuming that all the calculations are on the computer.

If as a profession we accept those techniques, as I am sure we must, we must also accept the fact that in the examination courses we are teaching our students history and not actuarial science, and that radical changes of syllabus are required. This is a paper of major significance, and the author is to be congratulated for guiding actuarial science into new and extremely interesting directions.

Mr J. M. Hill: There is a danger of confusion about the nature of the investment which a life office makes when it writes new business, and hence a danger of adopting unsound criteria by which to judge the profitability of a particular tranche of potential new business.

For simplicity, I wish to consider a proprietary life assurance company writing only non-profit business, which has both tangible and intangible shareholders' assets. The intangible assets are its organization and expertise which, as is recognized in § 5.3, have a real value, and which would normally have cost substantial sums to establish and to maintain. In writing a new policy of a particular class the office will not only be committing tangible assets, that is capital, but also the efforts of its organization which might have been more profitably employed in selling business of another class. Thus the office's investment is not just of capital, and in measuring the attractiveness to the shareholders of writing a particular policy, the use of what the author calls the 'internal rate of return method', that is, expressing emerging profits merely as a rate of return on capital employed may be materially misleading.

That accounts for the author's dilemma in § 3.10 where the 'internal rate of return method' is seen to break down if applied to a policy which creates no capital strains. I do not regard the apparent resolution of that dilemma, namely, the rejection of the policy as being a freak product of an imperfect market, as being legitimate. It is the method of measuring profitability, not the policy, which requires analysing more carefully. Indeed, it is well known that some companies—involved mainly in selling unit-linked business—operate successfully with the use of very little capital and presumably sell large volumes of business which involve very little or no capital strain, and the policy projected in Appendices B.2 to B.4 appears from those projections to be in this category.

In § 3.2 the author comments on the Formula approach: "It is implicit in this approach that capital is unlimited and providers of capital either do not require or are not entitled to any higher return than the interest rate likely to be experienced from the appropriate matched assets." I do not regard that as being a valid inference. So long as a policy is itself profitable shareholders will recover their capital plus whatever return has been obtained on the assets in which it was invested plus—and this the author appears to have ignored—the profit on the policy. Thus, considered purely as a return on capital employed, the total emerging profit represents a yield in excess of that which would have been obtained merely by investing the capital. So long as adequate profit margins are incorporated in the premium the Formula approach recognizes the need to obtain a proper return on the shareholders' investment. Moreover, I do not regard it as necessarily correct to assume that a proprietary life assurance company can, or in any sense should be able to, obtain a substantially higher rate of return on marginal capital than that available on a portfolio such as the office might regard as suitable for its Long Term Business Fund. Indeed, if a proprietary life company were confident of
obtaining such a higher rate of return on shareholders' capital, it should raise fresh capital since the rate of return it could offer on the new money would be attractive to its shareholders. The truth of the matter is that more than capital is needed for the profitable operation of a life office, the other ingredient being an efficient organization, and it is therefore incorrect to measure profitability merely by reference to capital employed. A correct measure of profitability would relate emerging profits to both capital and intangible assets employed and would reflect the relative scarcity in the particular office of these two items.

As Mr Shore pointed out, the paper is essentially about two things: the concept of profitability and techniques used to measure that profitability. The techniques which the author is advocating are basically model office techniques and their uses, as he recognizes, extend far beyond the assessment of suitable premium rates for new policies. These uses were admirably set out in the unpublished paper by Messrs Rathbone and Ross to the Students' Society in April 1975. I would like to add one further use to their list, namely the examination of the matching position of an office, which would involve not only a projection of liability cash flows on a suitable range of assumptions, but also a projection of asset proceeds.

The use of 'model office' techniques has grown in importance in recent years for two reasons. First, computers have made the production of cash flow projections on a range of assumptions a much less onerous task. Secondly, the commutation function (that is, Formula) approach, because of its sophistication, is restricted in its application and cannot cope satisfactorily with many types of unit-linked product which can, therefore, be examined fully only by cash flow projection techniques.

I am a firm believer in the use of cash flow projections as an additional tool in life assurance work, but their interpretation requires great care and an uncritical approach is dangerous. To take what is perhaps a rather obvious example, Appendix B.1 shows the projection of an endowment policy with a first year strain. It would be quite incorrect to assume from that projection that no other capital is required to support that policy. A substantial increase in interest rates some time after the policy has been issued resulting in a reduction in asset values, combined with the use of a conventional net premium valuation method, will normally result in substantial further capital requirements which must be met from some form of contingency reserves within the overall net liability.

To anyone contemplating the setting up of a computer system to produce cash flow projections of life assurance contracts, I would strongly advise against any preconception as to how it is to be used. Such preconceptions will not only limit its use, but may pre-empt subjective judgments which can only be properly made when some experience in the use of cash flow projections has been gained.

I believe that to a significant extent use of computer produced cash flow projections will replace traditional methods of setting premium rates, surrender values and determining bonus systems and levels.

Mr S. Benjamin: It is a pity in a way that the paper did not come before my own (J.I.A., 103, 233), because I felt then that some of the discussion was missing the point. A number of people talked about estimating outstanding claims, and so on, but really that was not the object. It is interesting that the author traces his idea back to Anderson's paper as early as 1959. I had not read that paper until after I had presented my own, and I traced the basic ideas back to Beard. I think that Beard presented those ideas in a rather more generalized form than Anderson. I included one appendix on the life assurance problem to cover the question of the valuation basis in calculating profitability, and what problems that would cause the profession.

The author would have done better to have used the phrase 'Anticipated actual experience', because that makes it very clear that you are talking about two bases interweaving all the time. One is the valuation basis, which the author calls the 'statutory' valuation basis, and the other is the anticipated actual which is the expected experience. I found it very difficult to sort out the bases in the appendices. I wanted to see from the headings exactly which part was the valuation basis and which was the anticipated actual experience.

No clear statement of the method appears until § 4.1, whereas as early as § 2.1 the author
states: “Any premium however calculated implies assumptions as to the following para-

meters . . .” and he lists them (a) to (h). Item (g) is the valuation basis. But that is not true: the traditional methods do not bring in the valuation basis.

Again § 1.2 states that profit equals surplus. That is not really true even on the author's terms. It is actually 'surplus for the year', and it is a pity to use that phrase, because the first year's strain becomes a loss, and loss is a very emotional word! Readers would have found it helpful if the author had said essentially what was happening. Graham Stacey, the accountant who spoke on my paper, was very worried indeed about those terms of 'profit and loss', 'surplus and strain', and in private discussion with him afterwards we made the closest contact when I said that what we were considering was returning the surplus to the proprietor at any point of time, at least notionally, and the proprietor saying “I will decide whether to invest it in the new business or to put it somewhere else.”

Section 4.7, the purchase of units for matching purposes, takes a peripheral subject into some depth. The 'scientific' premium is not a useful concept. The author states that profit will be affected if in fact actual premiums differ from the scientific: if the actual premiums do differ merely because they are out of date, it would be better to say so. If the volume of new business is brought into the calculations as yet another direct parameter, it would have to be said, that the office premium is the scientific premium. The author himself recognizes that: he says you need a model of a whole office, and he also discusses how one tries to allocate expenses from the point of view of a unit policy. He says that profit will be affected if the underlying experience differs from the assumed parameters. The underlying experience means the actual experience, and his use of the word 'parameters' there is misleading, because the valuation basis is not really one of the parameters, and it would have been more helpful to call it again the anticipated actual experience.

In §§ 2.2 and 4.1 it is stated that there exists a probability distribution of each of the parameters. That is not true for the valuation basis. Even if valuation is performed using the ruin probability approach, the ruin probability used has to be specified, and that parameter does not really have a probability. The probability distribution really only exists in the anticipated actual experience basis. The author says, that the assumptions are intended to be realistic. They are expected value assumptions, and the deviations are dealt with by using the risk rate of return. I did exactly the same in my paper, and I cannot see any alternative to that way of dealing with it. But there is a problem when that approach is used. Suppose we concentrate on an anticipated actual experience where we consider that the rate of interest is the most import-

ant thing. We all agree that there is a funnel of doubt concerning the distribution. For every value of rate of interest that might be earnt in the future, there is a corresponding profit, so there is a distribution of profit, but the average profit over that distribution is not the same as the profit on the mean rate of interest multiplied by some risk discounting rate. Faced with this sort of intellectual background model problem, I really felt it necessary to devote one section of my paper to the fact that the statistical model was being 'collapsed', where one talked about an anticipated actual experience.

In § 5.3 the author states that the gross premium valuation (in order to estimate the size of the estate) is not scientific and the estate has no objective reality. I agree: if he is saying that the estate is probably not a useful concept, I would also agree. Unfortunately, in § 7.2 he goes on to talk about the rate at which new business impairs the estate. I hope that was only a slip of the tongue, and that he is really agreeing. In his excellent Appendix E he calls it the estate or capital. Let us call it capital, or net asset value but avoid the word 'estate'. The word has been used against the background in the past in actuarial literature where the valuation basis has appeared to be a sort of moveable thing. It is not. It is needed nowadays to demonstrate that thing called 'adequacy' or sometimes 'strong solvency', and we had better make up our minds as a profession what we are doing. We are having to demonstrate, under model conditions, to the DoT and the public, adequacy excluding future bonuses. A basis which is weaker nominally but counter-balances by bringing in future bonuses does not demonstrate adequacy. After we have demonstrated adequacy, we then need to go on to show how we can smooth the bonuses over the future by holding back surplus—we cannot bring it forward. Then perhaps we can
Pricing and Profitability

start to discuss why we are holding back bonus declarations, and then we start to arrive at some of the fundamental questions posed in this paper.

Once the fundamental position of the valuation basis is accepted in the calculation of profitability, then we are led on to professional problems. In Appendix B an example is given of a valuation basis for unit-linked business, growth of unit 3% per annum, inflation 0%. That determines the management charge to support expenses. Now in fact a growth rate of 10% gross, or, say, 6½% net, and inflation of 8% is a very much stronger basis. Those are not the sort of figures to which we are accustomed when thinking of a valuation basis, but it may be that for unit-linked we have to start thinking in that way. The result is very sensitive to inflation, but is not very sensitive to growth rates.

Mention is made of the need for last resort funds and contingencies not otherwise anticipated in the valuation basis. Mr Hill talked of the possibility of having to strengthen the valuation basis in the future and recognizing that now. If we are really going ahead on bases which we recognize are going to need strengthening, then we shall need to ask now how much they need strengthening. If the possible strengthening is held in a contingency reserve, then that reserve should be inside the net liability, as the author says, otherwise what happens if there is not enough capital lying around to strengthen later on? If any further reserves are going to be needed outside the actuarial reserve, then the professional position of the actuary is weakened. If someone else, for instance the DoT, is going to say "It is all very well, the actuary has done his little calculations in a back room, but we want some solvency margins on top", we are losing our professional position. The only reason for the further funds is to stand the strain of writing further business.

In § 5.5—it is almost slipped in as an aside—it is stated that, when dealing with the mechanical problems of unit linked business, if there exists a carry-forward of excess of expense over interest, then it can be discounted at a net valuation rate of interest. Although I believe this is right, I have never seen a convincing demonstration. One instinctively feels that it ought to be a gross rate of interest.

One insurance office says quite openly that it tries to calculate its commission as a constant proportion of the profitability on the policy. I am not sure whether the author had that in mind when he was talking about the game in § 4.5.

One large firm of accountants has accepted the following argument on profitability. A parent company owns an insurance company, which as part of its business issues endowments and linked mortgages. The parent had the problem of carrying the profits of the insurance company into its own balance-sheet. The accountants treat it exactly as if it were just a straight mortgage. The end result is quite a high rate of interest, and they will amortize each loan by the actuarial method on that high equivalent rate of interest and that interest will be carried into the parent account year by year.

We have published a history of ‘Surplus in British Assurance over 200 years’ without any mention of profitability as developed in Mr Smart’s paper. It is true that actuaries dealing with the opening of overseas branches, especially those who have had to deal with the problems of Australia, have been doing something very similar to that; but they have not been bringing the subject right back into the premium calculation in the integrated approach displayed in this paper. This method requires a great deal more work than the old Formula method, but it gives a financial analysis and a pricing of the product and a pricing of the company in such a way that, it is very difficult to see how we ever managed before without it.

I do not think Mr Hill took the argument far enough. It is true that an office has intangible assets, but I do not think the author is saying (and I certainly would not say it) that by looking at the inherent yield in the design of a policy you are deciding what the office should do, and what total return it is getting. The answer can be found by looking at the price of the company, either by the simpler approach in my paper or by the more detailed analysis in this paper. Looking at alternative new business strategies, from the point of view of what would be the inherent price of the company at the end of the year, you can only decide which of those strategies increases the value of the company most by using both profitability and the inherent rate of interest. Whether or not the policies cause a new business strain is immaterial.
I remember reading a Faculty discussion in which Redington was asked "If I can sell policies on profitable premium rates but your valuation basis prevents me from doing so, who is wrong—my premiums or your valuation?" Unfortunately, Redington had already spoken in the discussion, so he could not reply. The premium rates and the valuation basis are inextricably bound up together: they are part of the same calculation.

Mr R. J. Squires: Most Institute papers in the past have assumed a mature office and non-inflationary conditions. Even if both assumptions are true, cash flow studies are both interesting and instructive. If either assumption is untrue, such studies are highly desirable. If neither is true, the studies are imperative.

I should like to consider the special case of the office that operates with a direct sales force, because there we have a double deferment of the return on the investment. The author, when he considers the division between fixed costs and marginal costs, and between continuing expenses and new business expenses, implies that all expenses can be related to one of those four groups. But an investment in recruiting, training and financing new salesmen is not one that can be related to the current year's expenditure. It is an investment in future productivity and, conversely, the current year's productivity and the production of new business should be related to past investment in recruiting and training. That is, of course, a special case of the general point that Mr Hill was making. I believe what is necessary is a separate cash flow model showing the expenditure that is required to recruit, train and finance a certain number of new salesmen, and the amount of production that can be expected from their successors in the future, allowing for turnover rates. It is then possible to determine the amount to be loaded into policy rates to repay that investment, and that figure can then go into the cash flow calculations for the individual policy. But we must never lose sight of the fact that that figure does not represent expenditure in that year. There is a deferment from the time expenditure on recruiting and training is made to the time when the policy is sold; but the sale of that policy does not itself mean a return of cash. It means a further investment, so there is a second deferment from the time the policy is sold to the time the profit comes through on that policy to help repay the investment in the sales force. That model is necessary not only to price the product, but also to monitor the experience of recruiting and training salesmen, and, indeed, to value an existing sales force as part of the problem of putting a price on the company.

I should like to make one minor criticism of the author's method on the cost of the death benefit of the unit-linked contract shown in the Appendix. Possibly for simplicity, he shows the calculation of the amount at risk, assuming that the unit price grows on a steady trend line. If the unit price fluctuates about that trend line, the overall cost will be greater than that assumed by using the trend line. Fluctuations that take the unit value over the death benefit contribute no saving, because they accrue to the policyholder, whereas fluctuations that take the value downwards increase the amount at risk.

I noted without surprise that the author has not tried to tackle the problem of what to do when the minimum premium calculated according to his method exceeds the maximum premium that the market will bear. The obvious answer is to get out of the kitchen, as the saying goes, but that is not always easy for an established company with an established staff and an established sales force who expect to have a product to sell in that area. The position of the single premium investment bond that appeared in the British market five or so years ago is a good example. The mature office looking at the terms offered might well have decided that such terms were unjustifiable on the basis of the minimum premium method. So they might be for that office, but those terms were largely dictated by the new offices and, given the overall position of those new offices needing investment income to accelerate the tax relief on the expenses of their regular premium business, the rates could be justified. The author does make a passing reference to that, but I think it is a very important point. Initially the premium basis for a single class of business is considered, but the final decision has to be made by reference to the effect on the office as a whole.

Mr F. R. Wales: The aspect that causes me concern is the treatment of contingency reserves.
In some cases I feel that Mr Smart's treatment is not consistent with his general theme. In § 2.5(c), he suggests that capital funds are needed by a life assurance company as "a source of last resort funds to stand against most contingencies both internal and external—these contingencies not being otherwise anticipated by the valuation basis". In § 4.2 he discusses the problem of mortality and proposes a fairly sophisticated treatment of mortality assumptions. He suggests that the mortality assumption should include a contingency margin for catastrophe deviations as opposed to random deviations from the expected death cost, that the catastrophic deviation could result from an epidemic and specific reserves need to be accumulated against such contingency. He also says that catastrophe margins can be allowed for simply by increasing the judged expected rates of mortality by an amount equal to the estimated cost of mortality catastrophes. A third reference to that contingency problem is made in § 4.9 where he suggests that these margins cannot be allowed to fall into surplus in a given year if not required, but instead should be regarded as an item of outgo in assessing profitability, and should be allowed to build up in a catastrophe or contingency fund.

Those three references suggest that Mr Smart adopts the following approach to the problem of catastrophes and other unexpected contingencies: that an estimate should be made of the likely catastrophes and an expected cost of catastrophe included in the premium rates, and then a catastrophe reserve should be held equal to an accumulation of premium less claims.

That seems fundamentally wrong and is indeed the same point at issue as in the current controversy regarding maturity guarantees under equity-linked policies, that is, he is reserving on an 'expected value' basis and not using the 'risk of ruin' approach. However, the problem is more serious for mortality risks. Receipt of one year's 'expected cost of contingency' premium on a large block of term assurance business would be of little use to the office if the mortality catastrophe were to occur shortly after inception. I suggest that the author's approach is not correct. It is not possible to estimate or forecast the likely cost of catastrophe. Such catastrophes occur rarely if at all and basically the problem is to ensure that if the unlikely does occur, the office will not be rendered insolvent. I have never experienced a mortality catastrophe that is of significance in the overall mortality experience of the offices with which I have been associated. The expected cost in respect of the many term assurance policies that have run their allotted span during my working life is therefore zero or very small indeed. On the other hand, the offices I have been connected with have always held substantial contingency reserves. My suggestion is therefore that contingencies such as catastrophes should be allowed for through the reserving basis, as called for by Mr Benjamin, and that the reserves set up for a policy should include a specific allowance such as the European proposal for a one per mille margin. If this is done, the cost of the contingency is reflected in the profit study as a charge for the investment of shareholders' capital in the reserves. In that way all contingencies are covered by the valuation basis, and the proper contingency reserve is set up at the outset, reserving on the basis of maximum possible loss rather than expected loss.

Mr C. B. Russell, F.F.A.: Maintenance expenses seem to have been included in year one at the beginning of the year which is perfectly legitimate but not the normal practice.

My first difficulty in the discussion of the merits of the formula and asset share method is that I can find no definition of an asset share method. The fact that the word 'reflection' is spelt with an 'x', leads me to wonder whether an asset share method is simply an American term for the emerging cost method, or does it mean specifically an emerging cost method which introduces the valuation basis?

The main advantage of the emerging cost basis is probably the ease with which one can deal with varying elements in the calculation, particularly with investment income and tax rates that vary either by the year in which income is earned, or by the year in which the investment is made. Matters such as lapse, surrender, currency devaluation are equally easily dealt with.

I agree entirely with the author that there will be many cases where it is appropriate to bring the valuation basis into premium calculations. I do not, however, accept his suggestion that such an action is always appropriate or that those who have not done so are to be criticized. The author's argument is based on an error in his suggestion that it is implicit in the
Formula approach that capital is unlimited. If the statement were true, it would indeed condemn the Formula approach, but in my view frequently the position is that although capital is not unlimited, it is not in fact the limiting factor on which sales development depends. The number of salesmen may be limited by other considerations, particularly in the case of a composite company. The number of policies issued may be limited by administration systems, or merely by a desire to maintain business at a specific level where it is efficiently managed. The management structure may not be equipped to deal with too many lines of business. If an office is seeking to maximize profitability, it should first decide what its limiting factors are, and then express the profitability of intended contracts as a function of that limiting factor. For instance, if the number of policies it can or wishes to write is limited, then it should seek the maximum profit per policy issued regardless of the capital required for that policy, or of its type or size. If the limiting factor is the availability of capital, then the introduction of the valuation basis into the calculation of the premium rates is a necessity to determine the return on that capital. Even then it could be argued that premium rates should not vary as much with the valuation basis as the author suggests. If the statutory basis is very strong, and in particular stronger than the office considers necessary for solvency, it follows that the additional slice of capital required is scarcely at risk, and need not earn a higher rate of return than the assets of the policyholders' fund.

It can therefore be argued that an unduly weak valuation basis implies lower premium rates, which is sensible—for how else could a weak company sell—while an unduly strong basis does not necessarily imply higher premium rates.

Mr P. N. Downing: I believe that profits, surplus and profitability are distinguishable and, especially at the general stage of the debate, should be treated separately. On the traditional definitions, it has been essential for students to distinguish between surplus and profit. For a given block of business, 'surplus' is the year-by-year excess of net income over outgo, including supporting the increase in the statutory reserve. 'Profit' on the other hand is the accumulation of the annual surpluses and strains without adjustment for their time incidence. 'Profitability' is the relation of profit to some predetermined criterion taking into account the level and timing of the annual surpluses. That certainly leads to the conclusion of the author, namely, that it is unrealistic to adopt a standard or measure of profitability which is independent of the reserve basis, but it does retain the essential of distinction between annual surpluses and total profit.

A year ago I was suggesting that insurance companies had to compete for the capital resources available within the economy for development purposes, the essential concept underlying the whole of this paper, particularly since in an inflationary period it was unlikely that life insurance products could be priced so that all finance could be self-generated. That approach leads to the author's measurement of profitability by the return on capital employed. However, if that is the only yardstick, I face a conceptual difficulty. Is the rate of return the only criterion of profit to be adopted? Or should shareholders be looking for profits in excess of a lower rate of return? In other words, rather than discounting the future annual surpluses at a risk discount rate, deducting the amount of capital employed, i.e., the initial strain, and testing for that net present value against zero, should we not use a lower discount rate but test against some predetermined positive value? Difficulties arise in fixing the right value of profits in excess of the return on capital employed, but this would overcome the problem identified in § 3.10, where it is said that it is clear that a rate of return breaks down where there is little or no first year valuation strain.

In § 4.8 the possible conflict between the actuary's viewpoint and that of the finance director is discussed. There is no doubt that if the 'return on capital' concept is accepted, it is increasing the point of conflict for the appointed actuary, particularly if he also happens to be the executive responsible for the life division, as the return on the capital employed is probably as sensitive, if not more so, to the reserving basis as to the profit or surplus streams expected. The finance director's viewpoint is not only that over-strong reserves increase the need for capital funds, thereby possibly inhibiting the expansion of the office which would otherwise be
obtainable, but (if he is in the position of having to compete for development funds) tend to reduce the so-called return on capital employed for a regular product.

In § 4.4. the significance of the lapse assumptions in any profit-testing model is discussed, and the comment made that the surrender value is usually greater than both the valuation reserve less the discounted value of expected future profits and the asset share after deduction of annual profits to date. That leads straight to an approach I have often found of assistance, namely, that when looking at a lapse or surrender, rather than adopting the traditional approach of merely having regard to the loss arising from the excess of the surrender value (if any) over the net cash flows produced by the policy, one considers the financial impact of a termination in excess of the number expected on the underlying assumptions. Such a termination must be regarded as depriving the office of the profit margins in the future premiums which had been expected and which will no longer be received. The financial consequences of excess lapses and surrenders are thereby thrown into greater relief, and seen in their correct perspective.

I am satisfied that the return on capital employed, having determined the statutory valuation model and the gross premiums chargeable, whilst being sensitive to any variation in the underlying assumptions, is most sensitive to the persistency assumed. Indeed, when applying the profit test methodology my worksheets are so arranged that the persistency of the business is not incorporated until the penultimate column in order to facilitate more readily a study in the change of the rate of return for alternative termination assumptions.

If I might marginally digress for a moment, but still on the subject of lapses, the concept of reinsurance, through reinsurance costs, is raised at several points throughout the paper. I believe that the traditional role of reinsurance is to reduce the volatility of annually emerging surpluses. Traditionally attention has been focussed on the mortality experience; it is virtually impossible to insure against an adverse investment performance. That has probably been appropriate, in viewing the financial result of a life insurance company in the traditional way, but if the return on capital shows a high sensitivity to the lapse experience, should the possibility of reinsuring the persistency risk also be considered? Due to the method of taxation of a life account in the United Kingdom, every company in an I-E tax position does in large measure co-insure the financial effect of bad persistency with the Inland Revenue, through the facility of only having to incorporate an after-tax zillmer in the surrender value computation. Whether, however, management would be right to rely on the same degree of long-term continuity normally expected of the professional reinsurance market applying equally to the Inland Revenue, is clearly a matter for speculation.

I found the author’s approach to the question of catastrophe margins acceptable and understandable, if his payment for the catastrophe is regarded as a reinsurance premium. I have found difficulty with some of the arguments previously presented by Mr Benjamin, and repeated by Mr Wales, namely that a contingency reserve should be incorporated within the normal reserves of the company. I do not know how to use contingency reserves: when has a catastrophe of sufficient magnitude arisen which enables me justifiably to call on my contingency reserve and to what extent? I find the author’s approach more satisfying.

In § 5.2 the suggestion is made that the ‘return on capital’ concept might be a management tool for assessing whether money spent in developing a new sales outlet has been repaid in terms of the worth of the business already sold plus the ability of the sales outlet to deliver in the future. A new product could be similarly tested. The concept has, of course, wider application and, indeed, I am surprised that the author did not proceed one stage further, namely, to suggest that his concepts and methodology provided a quantitative tool to the management of a composite company to assess the relative merits of demands by the life and non-life divisions for an allocation of shareholders’ development capital.

The most controversial statement in the paper is possibly that contained in the final sentence of § 5.3, “a gross premium valuation in order to estimate the size of the office’s ‘estate’ has no part in a scientific valuation process, since the ‘estate’ has no objective reality”. Eventually I believe that history will show the author to have been essentially correct.
Mr L. M. Eagles: The author has adopted the same definition for both profit and surplus, and that is what in the North American context would be termed "operating profit", in other words, the change in the surplus for the year after making any necessary additions to the reserves. However, I would go further and consider that it may be useful to distinguish profit from surplus and both from operating profit. It appears to me that surplus is strictly the excess of assets over net liability and thus, to the extent it is not immediately distributed, it is cumulative. Profit, which I agree is usually defined in actuarial literature as the sum of operating profits, might more usefully be regarded as the present value at the risk return required of distributions from surplus, allowing for tax. That definition underlies § 3.9 of the paper.

I suggest that the net present value does in fact have utility where there is no first year valuation strain, though possibly the conditions under which that is possible apply only to non-commission paying mutual offices. Certainly the type of policy to which that could be applied would almost certainly be a with-profit policy. The initial valuation strain is partly determined by the shape of the bonus scale. A bonus scale steeply tilted towards terminal bonuses may well be reconciled with acceptable premiums and nil valuation strain. Experiments are being made to construct with-profits rates subject to the constraint of nil valuation strain using profit test techniques.

In Appendix A.1 the point is made that the profit test algebra can easily be applied to monthly premium cases. Because of the incidence of lapses and expenses, particularly during the first year, it is normally advisable to construct an asset share study on a monthly time unit, being the smallest time unit during which it is possible to receive premiums. That is true even when considering an annual premium version of a contract since it is very likely the office will also be operating a monthly premium version. It is in fact very instructive to study the differences in profitability between annual and monthly policies, and that can only be done if the profit test has been constructed on a monthly interval basis.

Mr P. G. Meins: Churchill, I think it was, said that men will forgive a man anything except bad prose. I support those sentiments, but in view of the excellence of Mr Smart's paper, I am prepared to forgive him for his style, which I think originates somewhere in mid-Atlantic! Perhaps he could consider how his zeroed words will impact the real world situation!

Regarding the concept of the uniform distribution of profits, Appendix B shows that profits can emerge from that particular unit-linked policy and, I am sure, most unit-linked policies in rather an erratic manner. When accountants see that type of profit schedule, they wonder what sort of control and logic is behind the emergence of profit in the company. What is going to replace the discipline of the net premium valuation? Possibly the only thing one can do is to ask for full disclosure of that sort of schedule in the valuation returns.

The author could have qualified the agency evaluation in Appendix C a little more. To some extent one arbitrary assumption as to what value to place on agents is being replaced with two or three arbitrary assumptions. More wood is being created which makes it more difficult to see the trees behind. For a small company especially, that sort of exercise can be a little dangerous. If the company is trying to increase its net worth, the simple thing to do is to recruit a few more agents.

In Appendix D the author brings in the shareholders' funds at their straight asset value. That may need to be reduced if earnings on assets are less than the purchaser's required rate of return.

Mr N. D. Freethy: The techniques described in the paper have become vital to determine the viability and capital requirements of a new office. But established offices are also wise to employ this profitability projection technique. Such offices, after all, have far more access to the capital needed to exploit the results of using such projection techniques than the newer office with very limited capital.

It is important to remember how critically dependent profit is on future assumptions. That emerges clearly from the figures in the Appendix. Admittedly they are based upon somewhat arbitrary assumptions, but presumably the assumptions bear a reasonable relationship to
reality. Any adverse variation in the assumptions made would clearly wipe out the profit completely. In particular any variation in the rate at which it is assumed expenses would inflate would do that, and a profit would be turned into a loss almost overnight.

One speaker mentioned that it was possible to project from a gross tax position using the methods in the paper, so that each policy became net at the point at which its own total attributed expenses exceed the amount of total income that is also attributed to it. That is a quite legitimate technique, but it is admittedly arbitrary, and any variation will affect the overall picture. So there are two examples of the way in which a fairly small variation in any given assumption can alter the picture a great deal.

So my fear is not concerned with the techniques themselves which have become virtually a standard method of pricing unit-linked contracts. My misgivings are rather that techniques like those—which reveal in increasingly naked fashion what is going on—will constantly tempt the actuary to weaken his assumptions to the point at which the resultant premium rates become marketable. I sympathize with the predicament so eloquently expressed by Mr Squires. The actuary's capacity to step back and look carefully at the reasonableness of the assumptions he is actually making and to have the strength of character in standing by his judgment of those assumptions, these are qualities which will be more thoroughly tested than ever before.

Mr M. J. Burns: In § 1.4 of the paper the author says that its central theme is that only by analysing both the level and incidence of profits can any firm foundation be laid for a useful definition of philosophy of, or actuarial techniques regarding, pricing and profitability. Speakers in the discussion have endorsed this central theme, and so do I.

There is one passage with which I do not agree. In the first two lines of § 4.3, the author says: “For almost as long as all actuaries can remember we have been faced by the existence of current interest rates which seem to be at historically high levels . . .”. It is, in fact, almost exactly thirty years since gilt-edged yields reached their low point. So either the author is trying to flatter the senior members of the Institute by suggesting that almost all of them are under, say, 50, or else on the contrary he is implying that those of us who are over, say, 50, are not able to recall the events of thirty years ago!

One of the main objections to the Formula method for calculating premiums is that the margin for profit is for the most part implicit and not explicit, since it is derived from cautious assumptions in the bases. As a result of using a cautious, that is, a low rate of interest, the initial expenses are not charged for adequately, although other margins may compensate for that. That may be particularly significant if interest rates are high when the policy is issued.

When using a profit test method realistic assumptions must be used. It is an elementary point but actuaries in their training and in much of their work learn to be cautious, and many of us when asked for a realistic basis will, I suspect, produce a basis which is less unrealistic than we would otherwise have given, but which is still not realistic. For a profit test method it is important that we should discard our caution, and produce what really is our best guess. We will normally use more than one basis in order to see how sensitive the results are to changes in the assumptions. Thus we would determine first our best guess and then consider bases on either side of that best guess.

In Appendix B.1 there is an illustration of a non-profit endowment assurance. I am surprised that there has been little reference to with-profit endowment assurances, and I would be interested to know how the author would deal with those in a similar illustration. If no allowance for bonus is included the emerging profits will be very large, and the deduction will be made that that type of business should be sold to the exclusion of most others. If, on the other hand, bonus is included at the rate currently illustrated, then the profits emerging would generally be negative. In the light of what he says towards the end of § 2.7, the author would not, apparently, write such business at all.

The traditional view is that a balanced portfolio of with- and without-profit business is advantageous to both classes of policyholders. The cushion provided by the with-profit business enables keener rates to be quoted for non-profit business, and in normal times the non-profit business provides additional profits for with-profit policies. The conclusion is that, at least for
an office writing with-profit business, the mix of new business of the office, and not just individual classes, must be looked at.

In § 6.2 the author states that a profit test can be particularly useful *inter alia* in providing a realistic basis for the projection of operating results. I agree strongly with this: it is especially useful at the present time because of two sets of circumstances—the very high rates of inflation which we have seen in the last few years, and the impending introduction of a statutory minimum valuation basis for liabilities. As high rates of inflation continue, all offices, including old-established ones, are likely to find it becoming increasingly difficult to finance the volume of new business which they would like to write.

In Appendix F the author demonstrates the relationship between the rate of return on new business finance and the maximum rate of expansion of new business. That can be ignored in the short term, but not in the long term. With very high rates of inflation and a highly competitive market for new business, the time may come when offices cannot finance from existing resources all the new business which they would wish to write. So far as mutual offices are concerned, there seems to be no way in which additional capital can be provided, and the only answer I can see is to increase the profit margins in the premium rates for new business, or to pay lower bonuses than would otherwise be paid. For proprietary offices, including what the author describes as quasi-mutuals, it is possible for additional finance to be raised, and I think it is noteworthy that for many years now no old-established United Kingdom life office has turned to its shareholders for additional capital. I suspect that this is partly because the mutuals play such a significant part in the market. Since they have been holding back profits for financing new business, there has been little need competitively for the proprietary offices to raise additional finance. That situation is in marked contrast with some Continental countries where rights issues by proprietary life offices have been common.

The advent of a statutory minimum net premium valuation basis for liabilities linked to a market value basis for valuing assets introduces novel problems for United Kingdom actuaries. Unless an office is very strong, it will have to change the rates of interest used for valuing its liabilities if market values of assets fluctuate as sharply as they have done in recent years. A drop in the rates of interest used in the valuation basis for liabilities increases the new business strain and holds up the emergence of profit. One of the most valuable aspects of the profit test system is that it enables projections to be made on different sets of assumptions and such projections should be done on sets of assumptions involving changes in the rates of interest in the future, taking into account the corresponding changes in the values of assets. Use of the method in this way will not only show the amount of finance required for the new business written in each of the next few years, but will produce different results for the emergence of profit from business written in earlier years from those produced if constant rates of interest are assumed. Delay in the release of new business strain arising from reducing rates of interest can tie up substantial amounts of capital. When there is a statutory minimum valuation basis it is obviously important to examine the effects of likely (and even unlikely but possible) changes in investment conditions to see what effect these would have on the finances of the office as a whole.

The profit test method set out in the paper is a powerful tool. I hope that work will also be carried out and published giving the results of applying the method, for example, in considering the effects of the statutory minimum valuation bases under changing investment conditions.

It cannot be stressed too strongly that however refined the method, the results cannot be any better than the assumptions made. The bases must be decided by the actuary, and here we have difficult problems. Part of the prescribed examination reading for our students is the paper on premium rates by Asbury (*T.F.A. 27*, 289). I noticed that the eminent opener of that discussion commented that he was not particularly happy about a certain assumption in the paper since it implied that at the end of 15 years, that is in January 1977, the gross rate of interest will only have fallen to just a shade under 5 per cent! I have not quoted this to poke fun at the speaker: what he said probably represented the views of most actuaries at that time. I have quoted it because it is a striking reminder to me of how wrong our forecasts can be.
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The President (Mr C. M. O'Brien): I have been quite fascinated by the discussion, because the paper did three very simple things. First of all it reminded us yet again that the normal way in which we presented a picture of an office by a single figure—the present value of the liabilities—is nothing but an extreme form of shorthand. The reality, of course, is the future flow of premiums, investment income, and so on, and we cannot be reminded of that too often.

The second element was the analytical approach it takes to the new business strain or, alternatively, the capital that is needed to finance new business; again something I would not imagine is an unknown thought to members. Thirdly, it makes clear that in a method of emerging costs you can see very clearly the timing of the emergence of strains and releases, and in one sense perhaps the newest thing he has said. The effect on that of the reserving basis.

How very simple are all these three things, but when put together they become so important. Accordingly, both from that point of view, and also because of its clear interest to all of us as shown by the standard and length of the discussion, that we are greatly indebted to Mr Smart.

The author (Mr I. C. Smart): Several people have commented on the rather condensed form of this paper. As a result perhaps I have failed to satisfy both those people to whom the method is relatively new, and those people to whom it is relatively old hat. I hope that other people will build on the concepts and nomenclature of this paper.

I got the impression from one or two of the earlier speakers that it was felt the method provided instant solutions, and they quite rightly criticized any instant solutions for running a business. I think that they would agree on reflection that the paper does not in fact provide simple solutions. The theme of the paper is that one important criterion in running a life insurance company is the ability to service the capital that is in the company, whether it is shareholders' capital or whether it belongs to other policyholders. I agree very strongly with a remark that Mr Russell made that one needs to relate profits to scarcity. I think that was the foundation of the so-called science of economics; without scarcity one would not have economics.

The emerging cost method, as Mr Shore would have us call it, does show critical parameters, and this I think is very important. Mr Meins asked why it was, when evaluating a direct sales force for instance, that we needed to introduce three assumptions when one would do perfectly well. I think the answer is that although in the early stages the parameter values of those critical assumptions are unclear, at least one has thought about the problem, and has developed a set of standards one can use to monitor subsequent performance; for example, wastage rates in direct sales force are of critical importance.

A number of people—and I particularly noted contributions of Mr Downing and Mr Benjamin—pointed out the validity of the method in communicating. This is very important, and perhaps we have not paid enough attention in the past to communicating to non-actuaries just what it is we are doing; how the company will develop, where the profits come from, and so on. The emerging cost method provides a tool for communication.

Mr Burns raised the question of how the method would handle with-profit assurances. One can actually project the current level of future bonuses and any associated shareholder transfers, and perhaps the most important thing that one could do with those results is not to say that they look super-profitable or they look terrible; but to say what assumptions caused that bonus scale to be realistic? I think that is the main value of the method.

WRITTEN CONTRIBUTIONS

Mr C. O. Beard: The author demonstrates the use of a profit test system to analyse the financial working of a policy. Whilst the method starts from the annual incidence of strains and surpluses, and therefore shows what is happening, I feel that it is desirable to know more as to why it is happening. In Appendix B that could be seen on a given assumed valuation basis by considering the results of varying one of the parameters of the hypothesized experience, although it would be rather laborious in practice. Greater insight would be more easily obtained if the computer programs for Appendix B also analysed each year's surplus into its
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basic constituents, new business strain, mortality surplus etc. On the type A unit-linked contract
one item in the analysis of surplus for a given year could be taken as the difference between the
actual allocation to units and the theoretical allocation which should be made if the experience
were as the assumed valuation basis; it is, of course, that item which leads to the negative
'profit' figures in Appendix B.2.

The author subsequently wrote as follows: Mr Hill emphasized the investment on the sale of a
policy of both shareholders' capital and intangible assets, the latter being the 'organization'.
The point is related to Mr Russell's regarding scarcity, and putting the two arguments to-
gether indicates the need to relate profits to those scarce resources being used—see § 4.5 of the
paper. This point also applies to the arguments regarding policies that apparently require no
capital resources.

Mr Hill's defence of the Formula method—that it works provided adequate profit margins
are incorporated—is circular, since the ability of the profit margins to provide a proper return
on a shareholder capital can only be tested using a profit study system. He is quite correct in his
emphasis of the importance of model office projections, but it is not always practical to do an
entire projection to test one set of ideas—what is required is a form of 'shorthand' model
office and it is in this light that the profit study system may best be viewed.

Among the wide range of topics covered by Mr. Benjamin is one which deserves further em-
phasis: the need for the actuary to select a stronger basis for his valuation than the 'traditional'
basis. Mr Benjamin rightly points out that a basis of 6½% net growth, 8% expense inflation is
stronger than 3% net growth, 0% inflation.

Mr Squires is a man after my own heart when it comes to model building—but we are still
using different words for similar concepts: a textbook or another paper is the only solution to
this. He is quite correct regarding the over-simplicity of a 'steady trend line' approach to growth:
it is common in practice to incorporate 'waves' about the trend line.

Mr Wales raised the question of contingency reserves and Mr Downing's later remarks were
much to the point: essentially one may either reinsure or not against contingencies. If one
reinsures, then each year a premium is paid out; if one does not reinsure, then one needs some
free reserves which should be augmented out of surplus in those years where no catastrophe
occurs. As Mr Wales said, 'expected cost' is totally inappropriate as a reserve basis.

Mr Russell and I both recognize the need to relate profits to scarcity: where we differ is only
in the case of a company with capital which is more than ample for expansion needs. My
reaction, if I were a shareholder, would be to ask for whom management were working and
could I please have my capital back in order to have the option of using it elsewhere.

Mr Downing, like Mr Benjamin, touched upon the possible conflicts between the actuary
quà financial director and the actuary quà appointed actuary. In the belief that to clarify an
issue is a contribution to the resolution of that issue, the profit study concept where profits
and valuation basis are explicitly related seems preferable to (unstated) conflicts resulting
from a formula approach to pricing.

Mr Meins' comment regarding bringing in shareholders' funds at 'straight asset value' is a
difficult point and I think his remark should be read in the context of § 2.7(ii) where the rate
of return required by a purchaser is less than the rate used to discount emerging profits to allow
for just this point. The argument essentially is concerned with how one allows for the need to
earn more on assets committed to a risk venture than on similar, unencumbered, assets.