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USING OPTIONS TO PRICE MATURITY GUARANTEES

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1. INTRODUCTION

THE discussion on Maturity Guarantees, as applied to unit linked life assurance policies, has followed two quite distinct paths—the conventional or mainstream approach, as exemplified by Benjamin (1976) and endorsed by the Maturity Guarantees Working Party (MGWP 1980) on the one hand, and those who seek to reduce the risks associated with maturity guarantees by using an immunization strategy on the other (Brennan & Schwartz, 1976).

The solution proposed by Benjamin and others is to determine the minimum level of reserves required based on some acceptable 'probability of ruin'. 'Ruin', however, seems a somewhat exaggerated term since, even if the insurance company were technically insolvent, it might continue to be a 'going concern'. In this case, additional equity is likely to be raised from the capital market. The premium for the maturity guarantee, or investment performance guarantee, for that is what it is, would then reflect the cost of servicing these reserves, and their subsequent movements.

Of course, implicit in these calculations are some assumptions about the long term behaviour of the unit price. Protagonists of this approach have long since discarded the simple random walk model of equity prices, in favour of two component models where dividends are assumed to follow a random walk with upward drift, and yields are assumed to fluctuate about a fixed level (MGWP 1980). But if these models should prove incorrect, or equity prices in the future no longer behave as they did in the past then calculations of this kind will be wrong and estimated reserves may prove excessive (and therefore costly) or inadequate to prevent 'ruin'.

As described by Collins (1982) the alternative strategy of immunization draws upon option pricing theory to calculate appropriate reserve levels. In the remaining sections we describe briefly the principles of the strategy, and provide numerical examples of the premia for investment performance guarantees based on the de Zoete stock index.

2. MATURITY GUARANTEES AS AN OPTION

Brennan & Schwartz (1976) have pointed out that asset value guarantees may be priced on the basis of contingent claims theory. Consider the investor who seeks an asset value guarantee of g with respect to a maturity date T. The value of a reference portfolio at time T is denoted by $x(T)$ and its current value by $x(o)$.

Thus, an investment of $x(o)$ at the present time will be worth $x(T)$ on the maturity date. The investor hopes that $x(T)$ will be greater than g, i.e. on maturity the value of his investment will be worth more than the guaranteed amount. To insure himself (or any intermediary that acts on his behalf) against the prospect of $x(T)$ falling below g he may buy a put option which has an exercise price equal to g. This option will be exercised only if $x(T)$ falls below g, in which case the exercise profit is equal to $g-x(T)$.

Thus, at time T the pay-off to the investor is equal to the value of his investment in the reference portfolio plus the exercise profit:

$$V(T) = x(T)+\max(g-x(T),o) \qquad (1)$$

If $x(T)>g$ the put option is not exercised and the investor has surpassed the target value of g. If $x(T)<g$ the put option will be exercised, and the exercise profit will offset pound for pound the shortfall in the value of the reference portfolio with respect to g. In this case $V(T)=g$ regardless of how badly the reference portfolio performs. The downside risk is completely hedged and the maturity value of g is guaranteed. Of course, $V(T)$ may be greater than g: investors naturally have no objection to upside risks.

How much should an investor pay for such a maturity guarantee? The nature of the solution is suggested by equation (1). The premium essentially consists of two elements. First, it consists of the value of the reference portfolio at the present time, $x(o)$. Secondly, it consists of the price of a put option on the reference portfolio with maturity date T and exercise price g. Thus

$$V(o) = x(o)+P(x(o), g, T) \qquad (2)$$

is the equilibrium premium where P denotes the price of the put.

Since $x(o)$ is known, it remains to determine the put price in order to calculate the fair premium for the maturity guarantee. By fair, we mean that risk is efficiently managed and that sellers of maturity guarantees earn competitive and therefore normal profits. Under the assumptions used below to calculate P any other premium would imply either that customers were not getting value for money, or that suppliers were offering maturity guarantees too cheaply. Thus $V(o)$ provides an important benchmark for maturity guarantee pricing.

Although it did not explore this approach to maturity guarantee pricing, the Report of the Maturity Guarantees Working Party (1980) concluded that it was a subject which merits further investigation. This paper partly reflects this interest.

3. PUT PRICING

The put in equation (2) is a 'European' rather than 'American' option since early exercise before time T is ruled out. This is convenient because closed-form formulae for put prices only exist in the 'European' case, see e.g. Parkinson (1977). Thus, the put may be priced on the basis of the put-call parity condition according to which

$$P = C - x(o) + ge^{-rT} \tag{3}$$

where

$$C = C(x(o), g, T)$$

is the price of the corresponding 'European' call option and r denotes the instantaneous default free rate of interest on a bond which is due to mature at time T.

Black and Scholes (1973) argue that C is determined according to equation (4).

$$C = x(o)N(d_1) - ge^{-rT}N(d_2) \tag{4}$$

where

$d_1 \quad = \dfrac{\ln(x(o)/g) + rT}{\sigma\sqrt{T}} + \dfrac{\sigma\sqrt{T}}{2}$

$d_2 \quad = d_1 - \sigma\sqrt{T}$

$\sigma \quad$ = instantaneous standard deviation of the rate of return on the reference portfolio.

$N(\cdot)$ = cumulative probability for a unit normal variable, e.g. $N(-\infty) = 0$, $N(0) = 0{\cdot}5$, $N(\infty) = 1{\cdot}0$.

Equation (4) assumes that the price of the reference portfolio may be described by a geometric Brownian motion process

$$\frac{dx}{x} = \mu dt + \sigma dz \tag{5}$$

where

μ = instantaneous expected rate of return (it measures the drift in the Random Walk through time, dt)

dz = a Wiener process with properties $dz^2 = dt$, $dzdt = 0$ and $E(dz) = 0$

Equation (4) further assumes that

(i) r is deterministic as well as default free,

(ii) a hedge portfolio which is long on one unit of the reference portfolio and short on $N(d_1)^{-1}$ of call options is continuously rebalanced to ensure that there are no arbitrage profits. This implies that the return on the hedge portfolio is equal to the default free rate of interest,

(iii) the transactions implied by this rebalancing are costless.

Over relatively short periods of time it may be safe to assume that r is constant and deterministic. Brennan and Schwartz (1976, 1979) make this assumption. However, over the long time spans of maturity guarantees it might be more sensible to assume that interest rates are stochastic and variable, rather than deterministic and constant. Merton (1973) has suggested that under these assumptions equation (4) becomes

$$C = x(o)N(h_1) - ge^{-r(o)T}N(h_2) \tag{6}$$

where

$$h_1 = \frac{\ln(x(o)/g) + (r(o) + \bar{\sigma}^2/2)T}{\bar{\sigma}\sqrt{T}}$$

$$h_2 = \frac{\ln(x(o)/g) + (r(o) - \bar{\sigma}^2/2)T}{\bar{\sigma}\sqrt{T}}$$

$$\bar{\sigma}^2 = \sigma^2 + \sigma^2(r) - 2\sigma(x,r)$$

$\sigma^2(r)$ = instantaneous variance of interest rates.

$\sigma(x,r)$ = instantaneous covariance between the returns on the reference portfolio and interest rates.

Thus, to calculate the competitive price of a maturity guarantee equation (6) or (4) may be substituted into equation (3) and the result substituted into equation (2).

4. CAVEATS

This methodology is as robust as the assumptions upon which it is based. Beenstock (1982) argues that the continuous time assumption is not vital and that the discrete time case with which in practice we have to deal poses no problems. Of greater importance is the assumptions of equation (5). Inevitably, different stochastic processes generate different results. Indeed, one of the criticisms of the immunization strategy is that equity prices do not follow a Wiener process. However, Beenstock reports that in most cases equation (4) provides broadly similar results although it may go seriously wrong if the price of the reference portfolio jumps discretely and the option is 'in the money'. But even here the problem is not insoluble and Cox & Ross (1976) have demonstrated how the Black–Scholes formula may be adapted to accommodate jump processes. If the stochastic process has been mis-specified then the investment performance guarantee will be incorrectly priced. But this is not a criticism of the immunization strategy alone. The same requirement is made of the conventional approach—a wrongly specified stock price model will lead to inadequate/costly reserves.

Abstraction from transactions costs is inevitably a short-coming of the methodology. The true cost of the maturity guarantee is the solution to equation (2) plus the present value of transactions costs incurred through rebalancing the hedge portfolio. Brennan and Schwartz (1979) report that if transactions costs are 2% of the value of the reference portfolio transacted equation (2) understates the true cost of a ten year maturity guarantee on a single-premium contract by about 3%. If these estimates are representative it most probably means that in practice the solutions from equation (2) lie within the margin of error. Alternatively, it would be necessary to add approximately 3% to the solutions implied by equation (2). Thus, equation (4) provides a reasonable starting-point in relation to its assumptions about transactions costs, although it is worthwhile noting that in practice life offices in the United Kingdom are currently constrained by the valuation regulations from rebalancing their hedge portfolios since they are required to hold 100% of their reference portfolios at all times.

5. THE DATA

Because maturity guarantees extend over decades it seems sensible to obtain parameter estimates for σ etc. that are estimated over relatively long time periods. Brennan and Schwartz (1976) use a 5 year estimation period with respect to Canadian data which seems rather brief. However, in their subsequent work on United States of America data their observation period is 1926–1974.

Our own estimates of σ are based upon the de Zoete rolled-up index over the period 1919–1979 provided in the Report of the Maturity Guarantees Working Party (*J.I.A.* **107,** 154). However, we have carried out all our analysis in real terms. Thus x refers to the real price of the reference portfolio and is defined as

$$x = \frac{\text{de Zoete index}}{\text{consumer price index}}$$

This reflects our view that individuals are interested in real maturity guarantees rather than nominal ones and that equation (5) may be applied on this basis. Indeed, an investment performance guarantee in nominal terms would provide no practical guarantee at all. A real guarantee implies that the holder of a portfolio buys not only the right to sell the reference portfolio for a given amount, but also the right to use the proceeds from that investment to buy goods at a particular price.

We proxy r by the gross redemption yield on index linked gilts of similar maturity to the asset value guarantee. Thus, r is defined in real terms as is appropriate for our analysis. As is well known, gross redemption yields on index linked gilts depend on the rate of inflation. This is unsatisfactory, but we assume for purposes of illustration that the rate of inflation remains constant at 5% per year. In any case, it turns out that the price of the maturity guarantee is hardly sensitive to the errors in r that are generated by uncertainty about inflation.

Because index linked stocks are recent developments we do not have sufficient time series data to estimate $\sigma(r)$ and $\sigma(r,x)$. Instead, we proxy $\sigma(r)$ and $\sigma(r,x)$ by using the *ex post* real returns on Treasury Bills. Strictly speaking, we should have performed these calculations by taking account of the fact that over the life-time of the option the maturity date of the default free bond shortens. In other words, the correct concept is a real gross redemption yield on a maturing basis. Thus, at $t=0$ the variance is for a T period bond; at $t=1$ the variance is for a $t-1$ period bond, etc. etc. Since the variance of gross redemption yields tends to vary inversely with the term to maturity this approach would have implied that $\sigma(r)$ was not constant. In this case equation (6) cannot be applied.

By using Treasury Bill rates to estimate $\sigma(r)$ we are therefore overstating the volatility of interest rates. However, below we perform sensitivity analyses with respect to $\sigma(r)$. Happily, it turns out that the maturity guarantee price is not greatly affected by alternative estimates of $\sigma(r)$. An alternative proxy, the *ex post* real rate on Consols was considered but again this was not strictly correct for our purposes and made little difference to the results.

A further detail is that in equations (3), (4) and (6), it is assumed that bonds have zero coupons i.e. they are deep discount bonds. This is necessary so that the bonds do not generate any cash flows before the option matures. Coupons would have to be invested at uncertain returns and this would complicate the analysis. Index linked gilts have positive coupons. However, the difference between the gross redemption yields of actual stocks and hypothetical deep discount stocks is likely to be small. In any case, as has already been stated, it turns out that maturity guarantee prices are insensitive to even fairly large changes in gross redemption yields.

Finally, we consider only the single premium case, not the more usual regular premium contract, to emphasize the basic principles involved. It turns out that the option pricing solution for a regular premium contract is the same as that obtained by Merton (1973) for the value of an option which pays a regular dividend only now the dividend takes a negative sign. The numerical solution is complex but involves no new theoretical principles.

In Figures 1 and 2 we show the time paths of the real rolled-up value of the de Zoete index and our proxy for the real risk free interest rate.

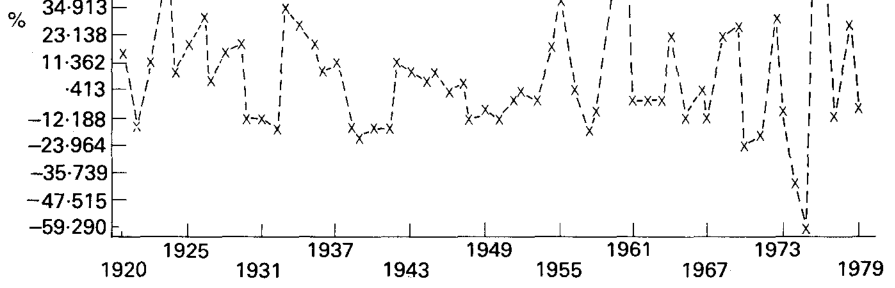

Figure 1. Annual real returns on the de Zoete index

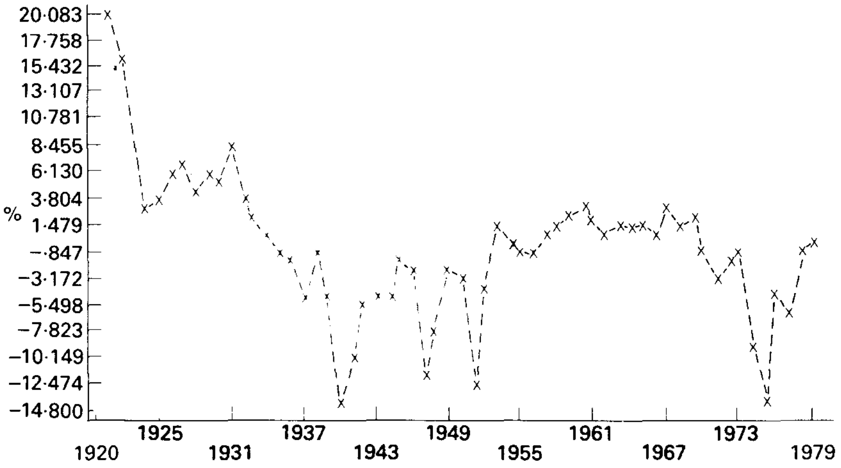

Figure 2. Real Treasury Bill rate 1919–1979

6. PARAMETER ESTIMATES

The Black–Scholes option pricing formula assumes that the return to the reference portfolio can be decomposed into a trend, or expected mean rate of return plus a random component that is both serially uncorrelated and of constant variance. An ordinary least squares regression of the percentage change in x on time over different sample periods revealed the following (where t values are shown in parentheses and Q_x is the Box–Pierce statistic which tests for randomness in the correlogram of residuals).

(I) 1919–79

$$\frac{\Delta x}{x} = \underset{(\cdot 566)}{212{\cdot}78} - \underset{(-\cdot 548)}{\cdot 1057T}$$

$$Q_x = 14{\cdot}38$$

(II) 1919–49

$$\frac{\Delta x}{x} = \underset{(2\cdot 14)}{1470{\cdot}41} - \underset{(-2\cdot 139)}{\cdot 7564T}$$

$$Q_x = 21{\cdot}02$$

(III) 1950–79

$$\frac{\Delta x}{x} = \underset{(\cdot 034)}{46{\cdot}509} - \underset{(-\cdot 029)}{\cdot 021T}$$

$$Q_x = 14{\cdot}64$$

For the whole sample period and the postwar period the trend coefficient is not significantly different from zero, in other words the real returns on the de Zoete index are stationary. Moreover, the errors meet the Box–Pierce criterion for non-autocorrelation (the critical Q value at 95% significance level is 15·51) and pass the Breusch–Pagan test for heteroskedasticity. Note this is in direct contrast to Wilkie (1978) who found significant negative autocorrelation of about $-\cdot 3$ in the second lag of the raw de Zoete series. Using the real value of the rolled-up index the significant autocorrelation in the error structure of equation I disappears; the value of the AR2 coefficient using data for the whole sample period is $-\cdot 1549$.

Over the period 1919–49 there is a significant downward drift or trend in the returns to the reference portfolio and the errors are no longer serially uncorrelated.

We therefore concentrate on the whole sample period and the postwar thirty year period, which is still sufficiently long for the purpose of estimating realistic put prices for maturity guarantees.

Table 1. *Parameter estimates*

	σ	$\sigma(r)$	$\rho(r,x)$	$\sigma(r,x)$
1919—79	·257	·064	·2125	·350
1950—79	·320	·043	·3305	·455

In Table 1 we report some summary statistics for both the rolled-up returns to the investment portfolio and interest rates over these two periods. In both periods $\sigma(r)$ is significantly lower than σ and the correlation between the two is both low and positive. The Black–Scholes formula assumes that the variance of returns to the investment portfolio stays constant. The observed increase from 25·69% to 32·05% although not significant, will raise the value of the call as the upside risk increases and this will drag up the put price.

Indeed, as constant volatility of returns is a crucial assumption in the Black–Scholes formula, we divided the whole sample into six periods (covering roughly ten years each) and performed a triangular F test on the data to see whether there were significant changes in volatility over time. In other words, we compared the variance of returns over the period 1919–29 with the variance of returns over the period 1919–39 and so on each time adding another ten years of data and testing the hypothesis that the two samples were drawn from a population of the same variance. The statistics below suggest that we could not reject the hypothesis. Of course, the cut-off points for each data sub-sample is quite arbitrary. Different sample sizes and a different analysis of time periods will lead to different results. Thus, in a comparison of volatility of returns in the years 1970–79 with almost any other decade we find a significant change occurs. Our discussion of immunization strategy is therefore only experimental at this stage.

F values for different sample periods and their appropriate critical values.

$F^{1920\text{–}39}_{1920\text{–}29} = 1{\cdot}032 \qquad F_c = 2{\cdot}77$

$F^{1920\text{–}49}_{1920\text{–}39} = 1{\cdot}118 \qquad F_c = 1{\cdot}93$

$F^{1920\text{–}59}_{1920\text{–}49} = 1{\cdot}087 \qquad F_c = 1{\cdot}79$

$F^{1920\text{–}69}_{1920\text{–}59} = 1{\cdot}018 \qquad F_c = 1{\cdot}63$

$F^{1920\text{–}79}_{1920\text{–}69} = 1{\cdot}356 \qquad F_c = 1{\cdot}60$

7. THE DETERMINISTIC CASE

We make the following assumptions:

1. That the real value of the investment or reference portfolio today is £1.
2. That investors choose both the time to maturity of the option (i.e. the period over which the maturity guarantee will run) as well as the desired real growth rate in the value of the guarantee. For instance, a high real rate of growth of, say, 3% per annum over 20 years implies an exercise or guarantee value of £1·822 for each £1 invested in the reference portfolio today.

In Table 2 we present the put prices, calculated according to equations (4) and (3) for different annual growth rates and different number of years to maturity. The correct premium for a maturity guarantee is equal to the price of the put plus the initial investment in the reference portfolio. Thus, an investor who wishes to guarantee the real value of his portfolio at £2·226 or 2% p.a. in forty years must pay a premium today of 30·54p on top of his £1·00 initial investment in the reference portfolio.

For any given T we see that the put price increases with the real rate of growth of the guarantee (see Figure 3). This makes intuitive sense. The larger the sum to be guaranteed the higher must be the premium to assure that sum. However, it may be argued that few investors would choose to protect their wealth at such a

Table 2. *Deterministic case*
($\sigma = {\cdot}257$)

(put prices in pence)

Years	0%	½%	1%	2%	3%
10	13·35	15·03	16·90	21·20	26·40
20	12·15	14·73	17·77	25·55	36·08
30	10·64	13·76	17·65	28·57	45·09
40	9·03	12·40	16·90	30·54	90·84

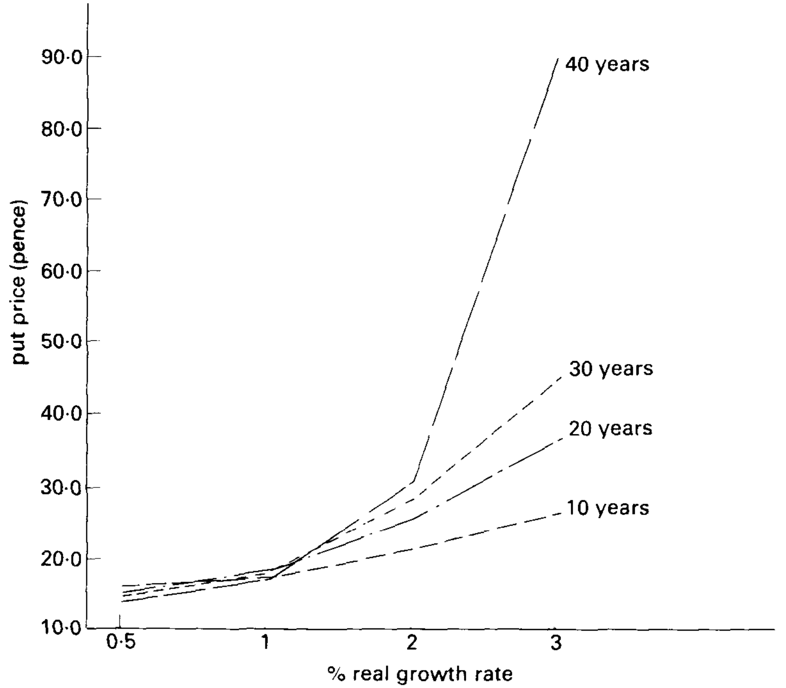

Figure 3. Deterministic case

high cost. Rather they would 'insure' their investment only to the extent that inflation, or possibly disastrous fund management might erode its worth in real terms. Realistically, we might expect investors to purchase maturity guarantees with a growth rate of perhaps $\frac{1}{2}\%$ p.a. or less, at a considerably lower cost, in the knowledge that the value of *most* portfolios is likely to grow by more. (Hence we include the put price for a 0% rate of growth in *real* terms.)

For any given sum assured the further the time to maturity the higher will be the price of the put—at least for short-dated options. For long-dated options the outcome is *a priori*, uncertain. This is so because the greater likelihood of a favourable outcome (i.e. the profitable exercise of the put) may be more than compensated for by the fall in the present value of the exercise price received by the put holder. Thus, in the first column of Table 2 the price of the put falls continuously from 15·03p over ten years to 12·40p over forty years.

However, as the time to maturity lengthens so, too, does the absolute sum guaranteed and this has the effect of raising the put price. For instance, if the maturity guarantee grows at 2% p.a., the put price rises from 21·20p over ten years to 30·54p over forty years, and at 3% per annum from 26·40p to 90·84p. At

Table 3. *Sensitivity analysis (deterministic case)*

(put prices in pence)

	σ			
r	·10	·26	·32	·70
0·02	17·70	43·91	52·58	88·26
·0359	5·28	25·55*	32·73	62·78
·05	1·36	75·25	21·01	46·29

* Base case assumes 2% growth in real value of maturity guarantee over 20 years.

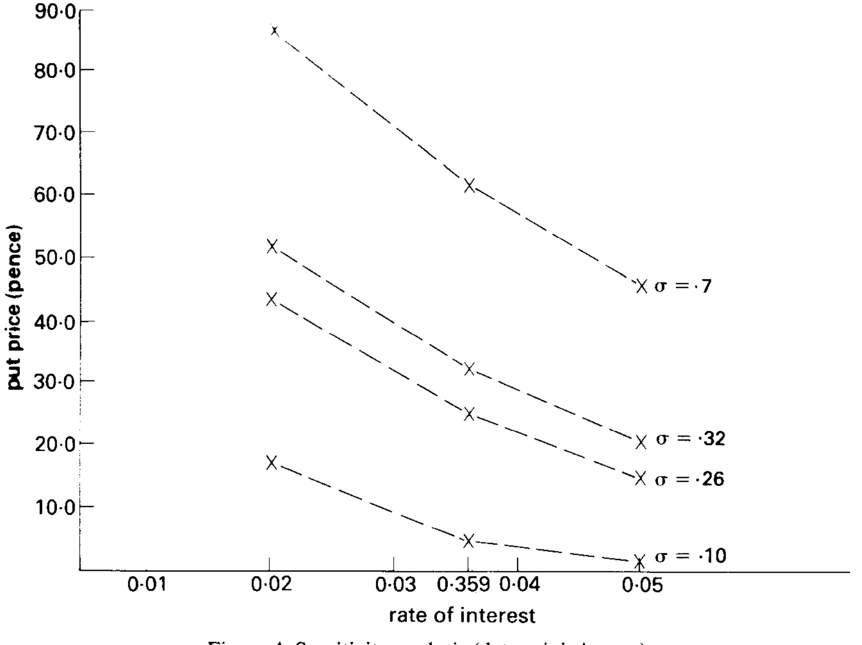

Figure 4. Sensitivity analysis (deterministic case)

1% p.a. growth the higher sum guaranteed more than compensates for the negative effect of time on a 20 year guarantee, but the converse holds for a thirty and forty year guarantee. Thus 'catastrophe' insurance can be relatively cheap if taken out over long periods of time.

In Table 3 and Figure 4 we examine the sensitivity of the Black–Scholes formula to changes in the variance of returns and to different interest rates. Using as our base case a 20 year maturity guarantee of £1·492 (or 2% real growth), we re-estimate the put prices for different σ (including the postwar variance) and different r. For any given interest rate a rise in the variance of return will always raise the put price as the upside risk increases while the downside risk is completely hedged. For any given volatility in the stock return the put price falls as interest rates rise and vice-versa, reflecting the opportunity cost of the deferred right to sell the investment portfolio. Moreover, from the slopes of the loci in Figure 4 we see that put prices, unlike call prices, are relatively sensitive to interest rate changes, (again for long dated maturity options), particularly in the realistic lower end of the range.

8. STOCHASTIC CASE

The Black–Scholes formula assumes interest rates are known with certainty, but this is clearly not the case. To accommodate stochastic interest rates Merton (1973) has adapted the basic option pricing formula (equation 6) and the relevant arguments now include both the variances of interest rates and the covariance between these rates and the returns to the reference portfolio (see Table 4). *A priori*, we should expect that the additional riskiness introduced by these uncertain interest rates will reduce the hedging properties of the investment (the 'hedged' portfolio no longer guarantees a fixed rate of return) and the price of the option will fall. The exact effect will depend on the magnitude of $\sigma^2(r)$ and $\sigma(x,r)$ relative to σ^2.

But the evidence in Table 4 suggests that it makes little difference to the put price if interest rates are known or not—at least on the basis of a relatively low level of interest rate volatility. Each entry in the table is only marginally lower than its deterministic counterpart in Table 2. For instance, the premium on a maturity guarantee of £1·822 over 30 years is £1·2857 with interest rates certain, and £1·2781 with interest rates uncertain.

But just how sensitive are these estimates to *changes* in the variance of interest rates and changes in their covariance with portfolio returns? An increase in $\sigma(r)$ will automatically raise $\sigma(r,x)$ unless ρ, the correlation coefficient falls. Similarly, an increase in ρ will raise the covariance $\sigma(r,x)$ unless either $\sigma(r)$ or σ falls. In the

Table 4. *Stochastic case*

$\sigma = \cdot257 \quad \sigma(r) = \cdot064 \quad \sigma(r,x) = \cdot0035$

Real growth rate years	0%	½%	1%	2%	3%
10	12·87	14·52	16·37	20·62	25·77
20	11·66	14·19	17·17	24·85	35·28
30	10·18	13·23	17·05	27·81	44·17
40	8·63	11·92	16·33	29·76	89·66

Table 5. *Sensitivity analysis—stochastic case*

(put prices in pence)

$\sigma(r)$	$\rho(x,r)$ ·8	·2125	−·6
·01	24·56	25·31	26·31
·064	19·53	24·85	30·69
·25	12·95	32·74	46·69
·40	23·98	44·11	57·11

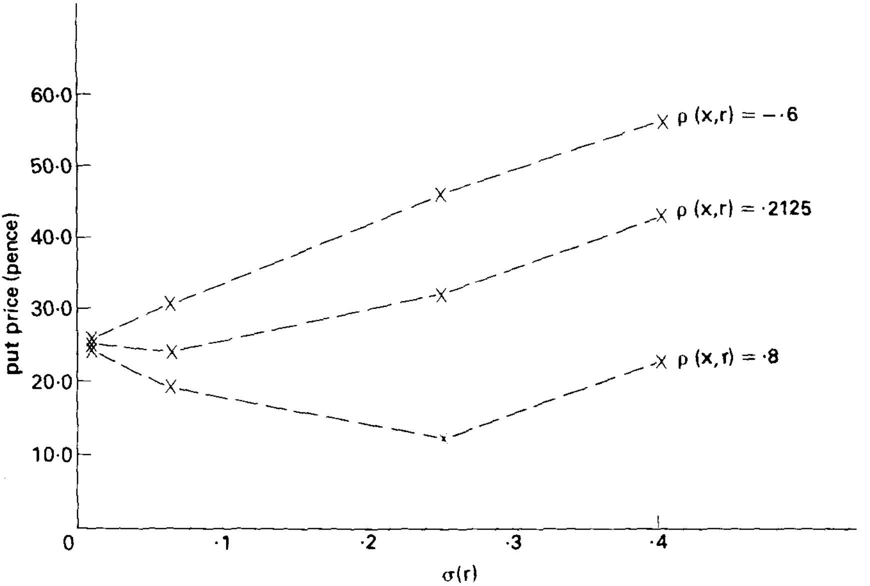

Figure 5. Sensitivity analysis (stochastic case)

sensitivity analysis that follows we again use as our base case a 20 year maturity guarantee of £1·492 and hold σ constant at ·26.

As the correlation coefficient changes from high and positive to low and positive and finally negative, so too does the covariance and the price of the put rises. The less the covariance between the two series, the greater will be the potential for risk diversification and the higher the price of the put for any given $\sigma(r)$. The effect of changes in $\sigma(r)$ for a given ρ are not, however, clear cut. From the nature of equation (6) we see that $\bar{\sigma}^2$ is determined by $\sigma^2+\sigma(r)-2\sigma(x,r)$ and since changing $\sigma(r)$ also changes $\sigma(x,r)$ the total impact on $\bar{\sigma}^2$ and hence on the put price may be positive or negative.

9. SUMMARY AND CONCLUSIONS

We have shown how the theory of option pricing may be effectively applied to the pricing of maturity guarantees. The premiums calculated are expensive, not least for 0% growth in real value, but this still remains the fair and efficient price of such an investment guarantee, irrespective of how small the eventual commercial market may prove to be for the contract.

We have also demonstrated the properties of the pricing structures of these guarantees. Briefly these are, *ceteris paribus*,

(i) for a given sum assured the longer the time to maturity the lower will be the cost of the guarantee,
(ii) the larger the sum to be guaranteed the higher will be the cost of the guarantee,
(iii) the more volatile the returns to the investment portfolio the higher will be the cost of the guarantee,
(iv) the higher the level of interest rates the lower will be the cost of the guarantee,
(v) the higher the correlation between investment stock returns and interest rates (where these are uncertain) the higher will be the cost of the guarantee.

Again we emphasize that our calculations are only valid as long as the assumptions underlying equations (3), (4) and (6) hold. Certain caveats have already been discussed, and perhaps the most important, the correct measure of the volatility of returns to the investment portfolio may yet prove a stumbling block for both the conventional approach *and* the immunization strategy.

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APPENDIX

1.

Index linked gilt edged stock	prospective real redemption yields*
T_r 2 p.c. 96	3·90
T_r $2\frac{1}{2}$p.c. 03	3·59
T_r $2\frac{1}{2}$ 11	3·34
T_r $2\frac{1}{2}$ 20	3·18

* Assumes 5% inflation: Source F.T. 17.3.84.

2. Total final expenditure deflator: C. H. Feinstein (1972) and *National Income and Expenditure*. Central Statistical Office, 1980.