JSS 26 (1983) 153-178

TIME SERIES ANALYSIS OF THE SEX COMPOSITION OF THE UNEMPLOYED

BY S. HABERMAN, F.I.A., M.A., PH.D., A.F.I.M.A.

1. INTRODUCTION

1.1 THIS paper describes the use of time series analysis in the solution of a problem arising in social insurance. As part of a model which estimates the future cost of unemployment benefit the Government Actuary's Department (GAD) is required to forecast the proportion of the unemployed in future calendar quarters, who are male. The format of the paper is to describe forecasting in general terms in § 1 and the particular problem under consideration in § 2. In subsequent sections, the data available (§ 3), the existing forecasting model (§ 4) and alternative time series models (§§ 5–8) are described.

1.2 The everyday job of the actuary involves the estimation of a future series of events. Examples include the estimation of future streams of liability outgo and asset income in life assurance, the run-off of outstanding claims in nonlife insurance, and the future numbers of persons in a subgroup of the total population. This estimation can be qualitative or quantitative, short-term or long-term, deterministic or stochastic and will involve the establishment of a mathematical-statistical model, and the determination of the relevant parameters by an analysis of the data available.

1.3 It is important to distinguish between two areas of application of techniques for the estimation of future events. These two fields may be identified by the size of data available and the level of stochastic variability. The problem described in detail in this paper requires forecasting techniques for its solution i.e. the data base is small, medium term forecasts are required and the stochastic variation about any model is anticipated to be large. The label of projection will be reserved for those essentially deterministic models used in the estimation of variables for which the stochastic variation is small and the data base available is extensive.

1.4 The purpose of this paper is to illustrate how some of the statistical methods for forecasting may be used to solve a practical, actuarial problem requiring medium term estimates from a small data base. The basic strategy of these techniques is to utilize all the non-random and consistent information from the past history of the time series under consideration for the purpose of forecasting. The methodology used is based on a combination of finite differences and multiple linear regression, each of which is an established part of actuarial training.

1.5 Statistical methods for the forecasting of time series have been described and used in several cases in the actuarial literature. The references provided are meant to be illustrative rather than exhaustive[(1)–(9)].

1.6 No attempt is made either to describe or to use the projection models referred to above. The recent Continuous Mortality Investigation Bureau Report[(10)] provides an illustration of this approach.

2. INTRODUCTION TO THE PROBLEM

2.1 The Government Actuary's Department (GAD) uses a complex, hierarchical mathematical model for estimating the future cost of unemployment benefit, using as a basis relevant data collected from or provided by other Government departments.

2.2 Figure 1 illustrates the layout of this model and indicates the principal components. At the first stage of the model the proportion of the wholly unemployed in a calendar quarter who are male is required. This is an essential part of the model since the two sexes receive different levels of unemployment benefit on average, experience (significantly) different patterns of duration of unemployment and exhibit (marginally) different levels of entitlement to benefit by duration. As Figure 1 shows, the model subdivides the number of unemployed by sex and then estimates 'survival as unemployed' factors so that the numbers of the unemployed by duration can be estimated, and then the numbers of female unemployed are subdivided by marital status (married and other women). Marital status is important for females because a higher proportion of married women are entitled to benefit. The estimated cost of benefit is derived by using factors representing entitlement to benefit and factors representing the average level of benefit by sex, duration and marital status (for women).

2.3 The structure of this model is not altogether satisfactory in that there are good reasons to expect the determinants of male unemployment over time to differ from those for female unemployment. It might therefore seem more natural to model these two series separately. However, the overall model is constrained externally since it must use as inputs estimates of the total numbers unemployed provided by a separate Government department.

2.4 A time series analysis of the 'survival factors as unemployed' has also been carried out and this is discussed in detail elsewhere[(11)].

2.5 It is useful to set up, *a priori*, an idea of the accuracy needed for the fitting of the model and for forecasting by the model. Currently an error of ·01 in the proportion of the unemployed who are male (*pm*) changes the numbers of estimated males unemployed by 20,000–30,000. The principal error that can arise from the use of *pm* in the costing of unemployment benefit is the result of males having a higher average benefit than females because, for example, they have more dependents and have a higher earnings related supplement (ERS). The first factor is of decreasing importance because of the relative reductions in child dependency additions to benefit and the second factor is of minor importance now because of the phased abolition of ERS.

As shown in Figure 1, *pm* is input at an early stage to the hierarchical model used to make financial estimates for unemployment benefit. The estimation of *pm* is therefore critical to the costing of unemployment benefit, with subsequent items of the model being dependent on it e.g. the duration specific 'survival' factors for the two sexes; the split of women by marital status; the proportion

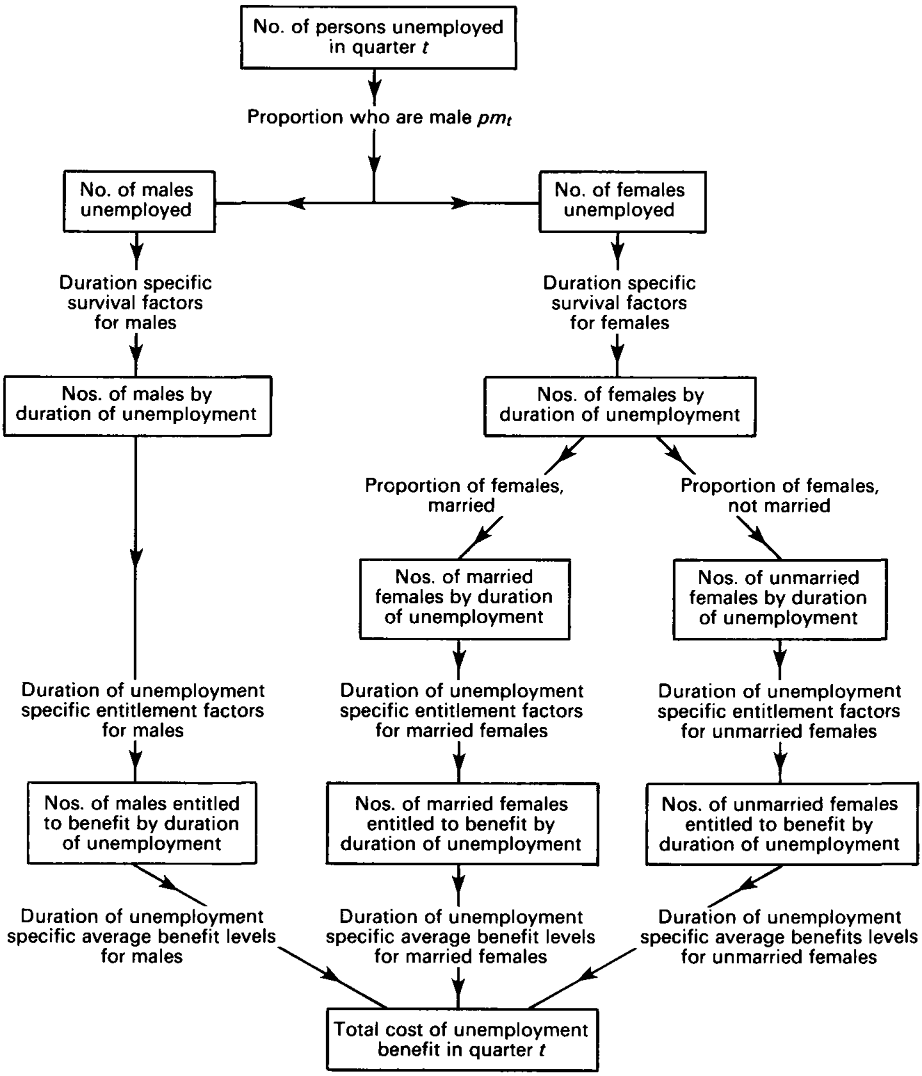

Figure 1. Model for estimating total cost of unemployment benefit in any quarter.

entitled to benefit by duration and sex. When this project was commenced in 1979–80, an error of ·01 in *pm* led to an error of about 2–3% in the financial estimates. Because of the reduction in the sex differential in average benefit levels referred to above (reduced child dependency additions and abolition of ERS), the impact of an error of ·01 in *pm* is currently (1982) believed to lead to an error of 1% in the financial estimates (i.e. about £20M). Despite these changes over time, ·01 has been used as a criterion throughout for identifying an unacceptably high level of error for estimating *pm* in both the fitting and forecasting periods.

3. DATA AVAILABLE

3.1 The following notation will be used. pm_t denotes the porportion of the unemployed who are male and u_t denotes the numbers of wholly unemployed (in thousands) without seasonal adjustment in quarter t. The term 'unemployed' excludes school-leavers.

Table 1 presents quarterly figures for pm_t and u_t from 1974 (2nd quarter) to 1981 (1st quarter). Figure 2 provides a graph of pm_t (left hand vertical scale) and one of u_t (right hand vertical scale) using data from 1968 (1st quarter) to 1981 (1st quarter).

3.2 The shape of the pm_t time series indicates a local trend with a consistent variation about the trend. The trend is approximately flat until 1974 after which it is downward. To explore the properties of the data, the autocorrelation structure of the series was analysed. A numerical measure of the degree of autocorrelation in an observed series, Z_t, of length m, at any lag k (0, 1, 2, . . .) is given by the sample autocorrelation coefficient r_k where

$$r_k = \frac{\sum_{t=k+1}^{m} (Z_t - \bar{Z})(Z_{t-k} - \bar{Z})}{\sum_{t=1}^{m} (Z_t - \bar{Z})^2}$$

and $\bar{Z}$ is the sample mean,

$$\bar{Z} = \frac{1}{m} \sum_{t=1}^{m} Z_t.$$

Table 2 gives values of r_k for $k = 1, \ldots, 8$ for the time series pm_t. The values of r_k decline as k is increased, but, for the range of k shown, do not reach zero. The estimates of the autocorrelation using the above statistic will be biased downwards e.g. with $m = 25$ the maximum numerical vaue r_k is ·9 at lag 4 and ·8 at lag 8.

This decline of r_k with increasing k is typical of time series which demonstrate upward or downward trends[12]. Such trends introduce non-stationarity. For the purpose of understanding the behaviour of time series and analysis and forecasting, before further analysis it is usual to transform non-stationary series

Table 1. *Numbers currently unemployed in thousands (u_t) and proportion of unemployed who are male (pm_t) by calendar quarters: Great Britain*

t	*Year*	*Quarter*	u_t	pm_t
1	74	2	538	·852
2		3	562	·843
3		4	624	·833
4	75	1	747	·827
5		2	799	·825
6		3	937	·805
7		4	1,078	·793
8	76	1	1,217	·787
9		2	1,185	·782
10		3	1,234	·759
11		4	1,257	·752
12	77	1	1,322	·752
13		2	1,259	·750
14		3	1,345	·727
15		4	1,366	·724
16	78	1	1,396	·731
17		2	1,284	·729
18		3	1,307	·709
19		4	1,277	·709
20	79	1	1,366	·721
21		2	1,201	·716
22		3	1,206	·687
23		4	1,247	·691
24	80	1	1,377	·698
25		2	1,403	·701
26		3	1,648	·693
27		4	1,955	·707
28	81	1	2,272	·721

into a form that is stationary i.e. varying approximately about a fixed mean level. A reasonable quantity to examine is the quarterly change in pm_t i.e. $\nabla pm_t = pm_t - pm_{t-1}$. Figure 3 displays the ∇pm_t series and Table 2 displays the sample autocorrelation coefficients for this series. The values of r_k at lags $k=2$, 4 and 8 are large, with other values being small enough to be approximated by zero. The implications of this for forecasting will be discussed in paragraphs 5.6 and 7.4. Transformations of the form $\nabla_4 pm_t = pm_t - pm_{t-4}$ and $\nabla\nabla_4 pm_t$ might also be useful in removing these remaining large autocorrelations.

3.3 The Department of Employment provides labour force projections by age, sex for future calendar years (as at 30 June) for Great Britain. The term labour force represents those in employment, those seeking employment and those unemployed or prevented from seeking work through temporary sickness. It

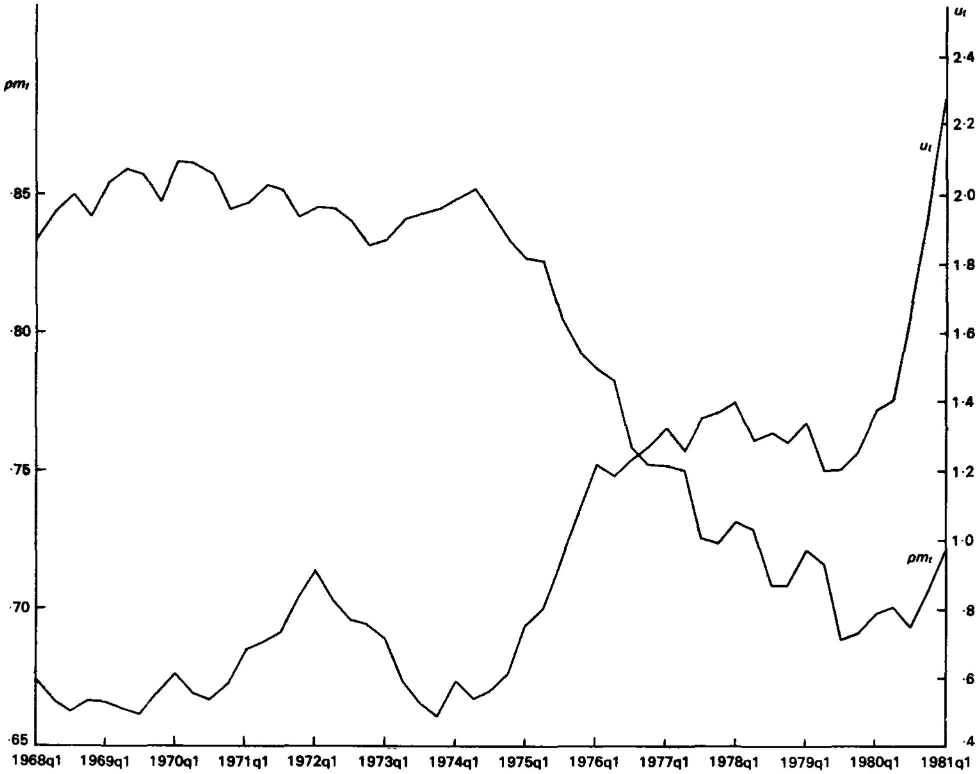

Figure 2. Variation over time (since 1968) in the numbers unemployed (u_t) and the proportion of the total who are male (pm_t).

Table 2. *The sample autocorrelation function r_k for the time series pm_t and ∇pm_t.*

Lag	pm_t	∇pm_t
1	·89	·02
2	·78	−·41
3	·68	·14
4	·58	·71
5	·44	·01
6	·32	−·04
7	·23	·16
8	·15	·53

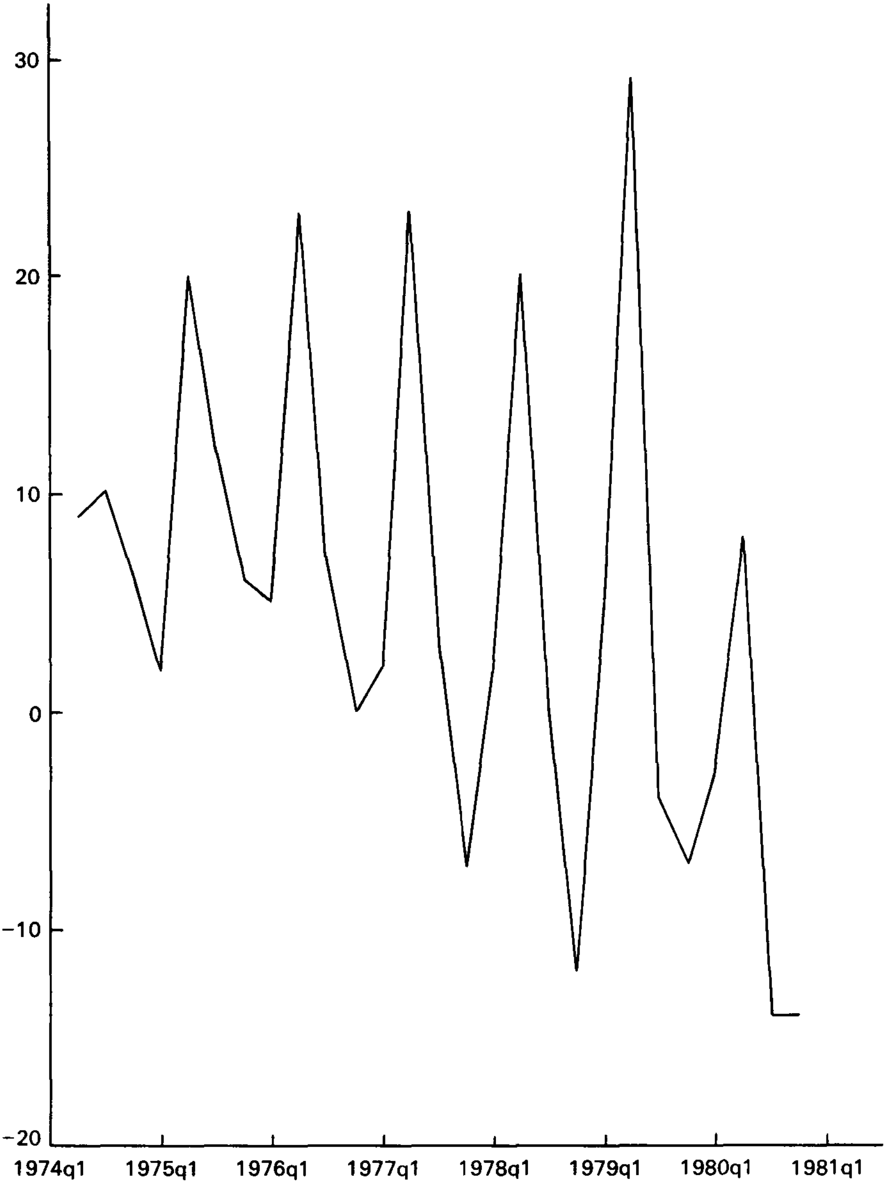

Figure 3. Variation of 1,000 ∇pm_t over time (1974–81).

excludes all students in full time education, although some take jobs during their vacations. A fuller definition is provided by Department of Employment Gazette[13]. The male labour force expressed as a percentage of the home male population gives the male economic activity rate. The female rate is similarly defined. As a means of measuring the changing sex composition of the labour force in future years, two measures were devised for each calendar quarter, t:

(a) The ratio of the male economic activity rate to the female economic activity rate (excluding students) in a calendar year t, g_t;
(b) The proportion of the labour force that is male (excluding students) in calendar year t, h_t.

The Department of Employment[14] provides estimated values of the age/sex specific economic activity rates for 30 June of past and future calendar years, together with corresponding estimates of the economically active population by age and sex. Using these data, values of g_t and h_t were calculated for the period 1971–91, and are presented in Figure 4.

3.4 The variation of g_t and h_t with time shown in Figure 4 indicates that the two factors appear to move in parallel. Thus only one is required as a proxy for the projected changing sex composition of the labour force in any regression model to estimate pm_t. Including both would produce difficulty arising from the presence of collinear variables. It was decided to use g_t, on the grounds that it related sex-specific numbers economically active to the overall numbers exposed to risk of activity. By interpolation, values of g_t were computed for each calendar quarter, and are shown in Table 3 for the period since 1974 quarter 2.

3.5 It was felt, *a priori*, that the most significant 'independent' variable for determining pm_t would be u_t, the numbers unemployed in the same quarter. For, when unemployment rises, one would expect the proportion of males to fall, reflecting the increased flow of women into the unemployed status from 'marginal' jobs affected by the underlying economic conditions. This is borne out by the observations shown in Figure 2. Recent trends indicate that this simple hypothesis no longer holds. This may be because the current historically high number of unemployed and its rising trend involve the loss of jobs other than the abovementioned marginal ones. Further, in any practical projection exercise, future numbers of unemployed are provided for GAD, usually by the Treasury, and so can be used as independent variables in a forecasting equation. Thus the effect of different unemployment assumptions may be quantified. This possible association between u_t and pm_t is felt to be very important. In particular it is felt that any practical method for estimating pm_t should involve u_t as an independent variable, so that different sets of expected future unemployment numbers would produce different estimated values, $\hat{pm}_t$.

3.6 For the period after 1974 quarter 2, the trend in pm_t shown in Figure 2 displays a marked seasonal fluctuation with the first quarter's figure apparently above and the third quarter's apparently below the general time trend. This is highlighted by the r_2 value of $-{\cdot}41$ in Table 2. The reason for these fluctuations

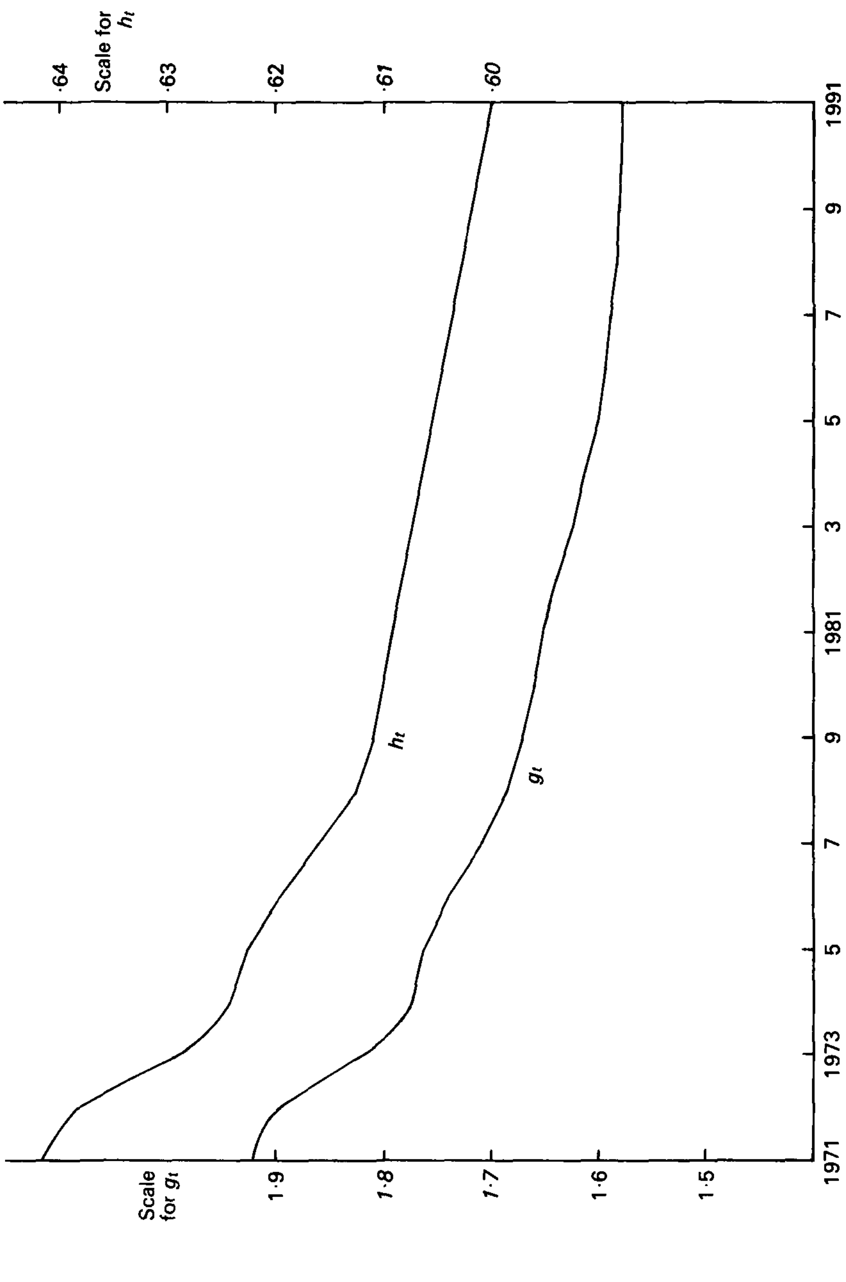

Figure 4. Variation of g_t, ratio of male activity rate : female activity rate (excl. students), and h_t, proportion of labour force that is male (excl. students), over time.

Table 3. *Ratio of male economic activity rate to female economic activity rate, excluding students (Great Britain), g_t*

Time	*Year*	*Quarter*	g_t
1	74	2	1·776
2		3	1·771
3		4	1·770
4	75	1	1·768
5		2	1·766
6		3	1·761
7		4	1·756
8	76	1	1·750
9		2	1·744
10		3	1·737
11		4	1·729
12	77	1	1·721
13		2	1·714
14		3	1·706
15		4	1·700
16	78	1	1·694
17		2	1·688
18		3	1·684
19		4	1·680
20	79	1	1·676
21		2	1·673
22		3	1·669
23		4	1·666
24	80	1	1·663
25		2	1·661
26		3	1·659
27		4	1·658
28	81	1	1·656

probably lies in the different composition of each quarter's male and female unemployed totals by industry (in which last employed) and the seasonal influences that affect particular industries together with the timing of the influx of college leavers into the unemployment stock. These fluctuations indicate the likely value of incorporating a seasonal factor into the estimating equation(s).

3.7 A detailed examination of Figure 2 indicates that the total period may be divided into (at least) four fairly homogeneous sections:

(a) 1968–71 when both pm_t and u_t were fairly stable. The fluctuations present appear to be seasonal in origin.
(b) 1971–74 when u_t followed a cyclical pattern.
(c) 1974–80 when u_t followed a semi-cyclical pattern but pm_t continued to fall.

The full cycle is incomplete, unlike (b).

(d) 1980–81 u_t increased dramatically and pm_t increased, but to a lesser extent.

It is clear from Figure 2 that the two unemployment cycles (b) and (c) are quite dissimilar in their level and slope and in the speed with which pm_t responded to the upward movement in u_t. Hence, great care must be taken in choosing how much of the data to use in fitting and testing any forecasting model. To use as a data base the complete set of pm_t presented in Figure 2 would introduce serious bias into the results, as the period being considered is made up of distinct components. The higher level of pm_t in cycle (b) relative to (c) probably stems from the nature of the unemployment cycle, and, the industries involved and persons affected. There is evidence that in the recent period the age composition of the unemployed has not remained static: in particular the contribution of the youngest age groups has varied. Thus in July 1974 10·3% of the unemployed were aged under 20. This increased to 16·4% in July 1977 and fell to 15·6% in July 1979. Further the male proportion varies markedly with age: for example in July 1977, pm_t was ·533 among the age group under 20, but ·803 at ages 30–44 and ·989 at ages over 60.

3.8 It thus seems reasonable to place more reliance on the data from the beginning of cycle (c) only (i.e. 1974 quarter 2) rather than the whole period shown in Figure 2. Thus the figures in Table 1 refer only to the period since 1974 quarter 2.

3.9 There is an established rule of thumb that 'if you think you can produce good forecasts for a series 'by eye' then statistical projections will probably work well. But if you can't, then they won't!' Given the data in Figure 2 up to 1979, extrapolation 'by eye' from 1979 onwards seems feasible. But the differences between periods (c) and (d) indicate that it might be very difficult to extrapolate graphically from 1981 into the future. So there may be some doubts as to whether any reasonable forecasts can be made using statistical methods. The period after 1974 quarter 2 includes a fourfold variation in u_t between 538,000 and 2,272,000. It is clear therefore that any forecasts made using a subset of this period for estimating parameters will be subject to considerable uncertainty. Estimates of unemployment benefit expenditure at times when unemployment levels are greatly in excess of the recent past should therefore be viewed in this light.

3.10 As indicated in paragraphs 3.3 and 3.5, the estimated number unemployed for future years are usually provided by the Treasury, and for past years by the Department of Employment. In view of the use in the current model of partial sums of u_t as an explanatory variables (described more fully in § 4) the correlation will be investigated between pm_t and u_{t-i} (for $i=0,1,2,3,4,5,6,7$) and the following partial sums

$$a_t = \sum_1^4 u_{t-i} \quad b_t = \sum_1^3 u_{t-i} \quad c_t = \sum_1^2 u_{t-i}$$

$$d_t = \sum_2^4 u_{t-i} \quad e_t = \sum_3^4 u_{t-i} \quad f_t = \sum_2^3 u_{t-i}$$

The set of explanatory variables includes the above variables and their squares, to allow for the possibility of curvilinear trends in the more recent quarters.

3.11 As indicated in paragraph 3.8, there are now 28 observations of pm_t under consideration (25 in period (c) and 3 in period (d)). At the time the work started (in 1979) there were 21 observations available. It was decided to use the first 16 observations for the fitting period, and to use the latest, remaining pieces of data for testing the various methods proposed and comparing their accuracy with the method being currently employed. Sixteen observations in the fitting period means that there are four available for each quarter to measure any seasonal factors. It is clearly reasonable to use the most recent cycle for the fitting period in order to allow for developments like the tendency for more married women to seek employment. Ideally the testing period should include periods of both falling and rising unemployment. However, it is unlikely that any formula or procedure would perform equally well in such diverse economic conditions.

3.12 Broadly, two sets of results are presented. The first relates to the earlier analyses performed when the data available in the forecasting period consisted of 5 quarters only (i.e. up to $t=21$). The second, more recent analyses were performed as part of a regular monitoring of the forecasting model when the most recent data point available was $t=28$ i.e. 12 quarters in the forecasting period.

3.13 Section 4 presents discussion of the method in current use for estimating the proportion of the unemployed who are male. This method was felt by GAD to be in need of improvement. Subsequent sections describe the investigations performed in order to update and improve on this approach. The criteria used for measuring the goodness of fit within both the fitting and forecasting periods are outlined in Appendix 1.

4. CURRENT METHOD OF FORECASTING

4.1 The current method of forecasting uses as possible inputs the current numbers of unemployed and the numbers unemployed in each quarter of the previous calendar year. The equation used to provide an estimate, $\hat{pm}_t$, of pm_t is:

$$\hat{pm}_t = \beta + \gamma \sum_{i \in I} u_{t-i} \tag{1}$$

where β and γ are constants and I is a suitable subset of the integers, 0–4 inclusive. β, γ and I are determined graphically, using the latest data available for β and past data for γ and I. The values of γ and I are monitored regularly as a check that a reasonable fit is obtained. If α represents the latest set of data, taking the difference between equation (1) for $t>\alpha$ and for $t=\alpha$ leads to a model of the following form:

$$\hat{pm}_t = pm_\alpha + \gamma \sum_{i \in I} (u_{t-i} - u_{t-\alpha}) \text{ for } t > \alpha \tag{2}$$

Using the data for the period 1968–76, γ and I were determined graphically giving values of $\gamma = -\cdot000035$ and $I = \{0,2,3,4\}$. The forecasting equation (2) will be called *model 1A*.

4.2 The success of (2) as a predictive equation depends on the choice of α. Thus, we take the period 1974–79, most of which forms part of the fitting period used to determine the constants in (2), and illustrates the accuracy of (2) for four different values of α. The total error and root mean squared error are shown below for each value of α:

	1974q2	*1974q3*	*1974q4*	*1975ql*
Total error $\sum_i^n (pm_t - \hat{pm}_t)$	$-\cdot422$	$-\cdot317$	$-\cdot152$	$-\cdot134$
Root mean squared error $\sqrt{\frac{1}{n}\sum_i^n (pm_t - \hat{pm}_t)^2}$	·023	·018	·012	·011
Length of Period, n	21	20	19	18

where n denotes the number of quarters in the period. This wide variation in performance of (2) suggested that an alternative approach might produce improved estimates.

4.3 A further recognized defect of the current model is the absence of any seasonal adjustment. The inclusion of such an adjustment improves the goodness of fit of (2) in the fitting period and beyond. A simple set of seasonal factors $s^{(j)}(j = 1\text{–}4)$ may be constructed by examining the residuals ε_t $(= pm_t - \hat{pm}_t)$, averaging by quarter and then adjusting (i.e. normalizing) so that

$$\sum_1^4 s^{(j)} = 0.$$

This produces the following seasonal factors, $s = (\cdot003, \cdot006, -\cdot004, -\cdot005)$. Incorporating s into the right hand side of equation (2) gives *Method 1B*:

$$\hat{pm}_t = pm_\alpha + \gamma \sum_{i \in I} (u_{t-i} - u_{t-\alpha}) + s^{(j)}$$

4.4 Table 4 compares the goodness of fit in the fitting and forecasting periods for *models 1A* and *1B* (using the criteria of Appendix 1). Results are presented for two sets of forecasts: firstly using $\alpha = 16$ as the baseline in equation (2), and secondly using a moving baseline so that the forecasts are each time for one step ahead only. The forecasting period has been taken as points 17–21 inclusive and then as 17–28 inclusive as more data have become available.

4.5 The values of U indicate that *models 1A* and *1B* are inferior to the naive random walk model described in Appendix 1. The values of the other indices (*mape* and *mse*) in Table 4 will be benchmarks against which the performance of other models will be compared. As one would expect the one-step ahead forecasts show a better fit (i.e. smaller *mape*, *mse* and U values) than those using a fixed baseline.

Table 4. *Goodness of fit statistics using current models 1A and 1B*

Goodness of Fit Statistic (see Appendix 1)	Fitting period 1–16		Forecasting period 17–21				Forecasting period 17–28			
			Fixed baseline at $t=16$		One step ahead forecasts		Fixed baseline at $t=16$		One step ahead forecasts	
	A	B	A	B	A	B	A	B	A	B
mape	2·35	2·34	2·21	2·38	1·31	1·42	3·78	3·94	1·88	2·02
mse ($\times 10^6$)	401	360	307	337	112	137	971	977	322	333
Number of large ε	15	16	4	5	3	3	10	11	6	7
U	1·790	1·695	1·638	1·714	0·989	1·093	2·666	2·673	1·535	1·561

Note: Model 1A uses 1 parameter, Model 1B uses 4 parameters.

4.6 In §2 the criterion of a maximum tolerable error of ·01 in pm_t was mentioned. The estimates provided by equation (2) show that the errors exceed this criterion even within the fitting period. For example, taking the data between 1974 and 1979 the proportions of errors exceeding ·01 for different starting dates (α) is:

Starting date, α	*1974q2*	*1974q3*	*1974q4*	*1975q1*
Proportion of large errors	19/21	14/20	8/19	8/18

4.7 When the forecasting period is extended up to $t=28$ so that period (d) is included, the goodness of fit of the *models 1A* and *1B* changes markedly. From the results in Table 4 it can be seen that the period (d) of rapidly rising unemployment numbers means that the predictive *models 1A* and *1B* are using unemployment numbers as input which are much higher than the values used to determine the parameters in the respective fitting periods. This extreme degree of extrapolation causes such models to perform poorly in the forecasting period $t=17-28$, with, for example, $U>1{\cdot}5$ in all four cases.

4.8 Throughout the fitting procedures described here and in §§7 and 8 normalized seasonal components have been calculated that do not vary with time, on the grounds that the paucity of data does not justify the complications of, for example, the full Holt–Winters procedure.[15] Further, there is no evidence to suggest that any of the seasonal components have been changing significantly with time. The inclusion of seasonal components markedly improves the fit of most of the equations quoted herein. For many of the models the signs of the seasonals were as one would expect (paragraph 3.6) namely positive for quarters 1 and 2, negative for quarters 3 and 4.

5. TIME SERIES METHODS

5.1 In attempting to forecast a univariate time series there are broadly two different methodologies that may be used. We will describe time series methods first and then consider econometric or regression methods.

5.2 Given a time series Z_t, time series methods analyse (and possibly transform) the data and then derive models to forecast Z_t often without introducing exogenous variables. As mentioned in paragraph 3.2 it is often necessary to difference the series (perhaps several times) in order to reduce it to stationarity, in which case we aim to forecast $W_t = (1-B)^n Z_t$ where $BZ_t = Z_{t-1}$ and B is the back-shift operator and n denotes the number of differences taken. The projection process relates Z_t (or W_t) to a linear combination of values of Z_{t-i} (or W_{t-i}) for $i \geqslant 1$.

5.3 There are three commonly used time series procedures in the literature:

(i) Box–Jenkins procedures.[9],[12],[16]
(ii) Holt–Winters procedures: a generalization of exponential smoothing.[15]–[17]
(iii) Stepwise Autoregression.[16],[18]

The literature on this subject is voluminous, both theoretical and of the case study type.

5.4 Analysis using Box–Jenkins or Holt–Winters procedures has not been carried out for the following reasons.

(i) It is the consensus of the literature that a good deal of skill and experience is essential if Box–Jenkins procedures are to be used to best effect. These procedures are not automatic. Also, at least 50 items of data are needed.[19]
(ii) The Holt–Winters and Stepwise Autoregression procedures can be made automatic, but when the number of data points available is fewer than 30, Chatfield[15] and Newbold and Granger[18] suggest that the parameters estimated will be too unstable. Some simple autoregressive methods including an exponential smoothing approach will be described in paragraphs 5.5 and 5.6 and used in the analysis of the current problem. However, the problem of instability will always be present with any method given the small and dynamic data set.
(iii) The Box–Jenkins procedures are less effective when trend and seasonality (rather than randomness) are the dominant sources of variation as is the case with pm_t.[19] In this case, Holt–Winters methods can produce as good results as the others with considerably less effort.
(iv) The complexity of the Box–Jenkins approach has not produced forecasts which are significantly better than those derived from simpler models in various reported case studies.[16]

5.5 There are insufficient data available to justify the use of the Box–Jenkins or Holt–Winters procedures. However, it may be possible to proceed using a simplified version of the Holt–Winters procedure viz. exponential smoothing. It is apparent from Figure 2 that, since 1974 quarter 2, pm_t has experienced a downward trend, with seasonal fluctuations. However, if we ignore these features, we can try to fit a simple exponential smoothing model of the form

$$\hat{pm}(t,1) = k.pm_t+(1-k)\,\hat{pm}(t-1,\,1) \qquad (3)$$

where $\hat{pm}(t,1)$ is the estimate of pm_{t+1} made at time t, and k $(0<k<1)$ is to be determined from the data,[20] by minimizing the sum of squared one-step forecast errors:

$$\text{i.e. minimize } E_k = \sum_{t=0}^{15} (\hat{pm}(t,1)-pm_{t+1})^2$$

5.6 The stepwise autoregression procedure is based on the following model:

$$Z_t = q+\sum_{m=1}^{N} V_m\, Z_{t-m}+\text{error term} \qquad (4)$$

where $Z_t = pm_t$ or $Z_t = \nabla pm_t$ introduced to reduce the series to stationarity. Given the comments in paragraph 3.2 on the autocorrelation coefficient for pm_t and the non-stationarity of the series, it is anticipated that taking $Z_t = \nabla pm_t$ should give improved results although this series is itself not stationary. The model may be fitted to the data available using a stepwise multiple regression procedure, which introduces into equation (4) the lagged variable from the set, $Z_{t-m}\, m = 1,2, \ldots N$, which contributes most to the variation in Z_t. The regression procedure involves the minimization of the sum squared one-step errors within the fitting period. This may be extended to estimating the best m-step ahead forecasting equation (for $m>1$) but because of paucity of data this is not applicable here. Clearly, conventional hypothesis testing procedures are not valid. Newbold and Granger[18] suggest using an arbitrary F ratio of 4·0 to determine whether variables should be added or dropped from the model, and this has been adopted. The use of this F ratio will be discussed more fully in paragraph 8.10.

6. ECONOMETRIC OR REGRESSION METHODS

6.1 The current method of estimating pm_t, based on equation (2) permits the use of a set of u_{t-i} as explanatory variables in a particular combination. This is restrictive in that it does not allow u_t and u_{t-1} to make different marginal contributions to the estimates and further it prevents the use of other explanatory variables e.g. economic activity rates. Regression-type procedures will enable these problems to be resolved and will also formalize the fitting methods and permit inferential statements to be made about the parameters, subject to certain assumptions about the error structure.

6.2 In the econometric or regression approach[21] the linear model

$$\mathbf{pm} = \mathbf{A}\,\mathbf{b}+\mathbf{e} \qquad (5)$$

will be used as a means of representing the given data, where $\mathbf{pm}$ is an r dimension vector representing the r observations pm_t. **A** is an $r \times n$ dimension matrix representing the observations on n independent variables believed to be of importance in describing **pm**. **b** is an n dimension vector representing the coefficients to be derived by some statistical procedure, and **e** is an r dimension vector representing the errors.

6.3 The nature of the pm_t series, its clear relationship to the u_t series (which is estimated independently by econometric models) and the desire for any estimating process to involve u_t as independent input (paragraph 3.5) together suggest that the econometric approach may be preferable to the time series approach of § 5.

6.4 Before considering the theoretical form of **b** we need to decide what variables to incorporate into **A**. In fact, two questions must be answered: what factors are expected to correlate highly with **pm** and can the factors be used as independent variables in a regression equation such as (5) to estimate **pm**?

Those factors believed to correlate highly with **pm** are the following:

(a) The numbers of unemployed u_t, and the change in u_t, for the reasons described in paragraph 3.4. Future numbers are available from the Treasury estimates as working assumptions. In the current model used by GAD, u_t is the only independent explanatory variable.

(b) The structure of u_t by age. It has been noted that pm_t varies with age being lower at the younger ages. Estimates of the age structure are currently not available for future dates from the Treasury and, in any event, such estimates would be highly speculative.

(c) The structure of u_t by duration for similar reasons to those advanced for the breakdown by age.

(d) The relative supply of people seeking employment and in particular the increasing economic activity of married women. Data are available from labour force projections provided by the Department of Employment (for example the series published in April 1978).

Of these (a) and (d) appear usable.

6.5 Further, the possibility of transforming the series pm_t before forecasting should be considered. There are no intuitive grounds for considering a more complicated function of pm_t in the left hand side of equation (5) e.g. $\log_e pm_t$ so that a multiplicative model is postulated. Preliminary tests with ∇pm_t and pm_t/pm_{t-1} were not successful. Investigation of the relationship between the age specific components of pm_t and the variables (a) and (b) (mentioned in paragraph 6.4) did not reveal a consistent association. Such an approach would also involve unwarranted complexity. It was decided therefore to use pm_t as the function to be estimated by the regression approach.

6.6 From the viewpoint of statistical theory, it is unsatisfactory to use a proportion as the dependent variable in a regression equation because its variance is not independent of its mean. The homogeneity of variance assumption of the regression model is violated. The use of the logit transformation of the proportion i.e. log $(pm_t/(1-pm_t))$ as the dependent variable stabilises the variance. Since the values of pm_t being used in fitting and testing lie between ·68 and ·85 (Table 1) little difference in the final results is produced by this more sophisticated approach since for this range of values of pm_t the variances are approximately equal. But in the event of the proportion males pm_t taking more extreme values than over the last 5 years it would be necessary to use the logit function to transform pm_t.

6.7 As an alternative in setting up least squares estimation, it would be appropriate to weight the sums of squares by weights inversely proportional to the variances of the dependent variables. These variances are not known and cannot be estimated from the data that are available. Plausible substitutes might be $1/u_t$ or u_t/pm_t, if the time series of the proportion of wholly unemployed who are males is taken to be the result of a sequence of independent Bernoulli trials, and we approximate the variance by these quantities. Using $1/u_t$ as weights produced little effect on the final result, so that the approach has been to use unweighted least squares to determine the parameters.

7. RESULTS: TIME SERIES MODELS

7.1 In the exponential smoothing model (3) the values of E_k for different k were examined and the minimum E_k was found for $k=1$. When $k=1$ (3) reduces to $pm(t,1)=pm_t$ which is the naïve approach described in Appendix 1 in connection with the U statistic.

7.2 A trend factor p and a set of seasonal components $s^{(j)}$ may be calculated from an examination of the residuals ε_t in the fitting period. The estimating equation (3) then becomes

$$\hat{pm}(t,1)=p+pm_t+s^{(j)} \tag{6}$$

where $p=-·007$ and $s^{(j)}=(·006, ·005, -·012, ·001)$.

This method of estimation of seasonal factors has been adopted for simplicity and is less optimal than a procedure using dummy variables.

The goodness of fit of (6) is assessed in Table 5. In the forecasting period the results are presented for forecasts made using a fixed baseline at $t=16$ and also for forecasts made one step ahead.

A comparison of the indices in Table 5 with those given in Table 4 for the current model being used (*model 1A* or *1B*) shows the superiority of the exponential smoothing approach—which has lower values of *mape*, *mse*, U and *number of large errors* in the fitting and both forecasting periods considered.

Equation (6) contains a fixed negative trend factor which is likely to lead to poor fits over long periods of extrapolation. With a forecasting period of $t=17$ to 28 therefore equation (6) with p constrained to be zero has been tested and proves to be superior to equations (6) and (2). Use of the full Holt–Winters procedure would enable the trend component to be updated continuously and hence would avoid this problem.

Table 5. *Goodness of fit of exponential smoothing model: Equation (3)*

Goodness of fit statistic	Fitting period 1–16	$p \neq 0$: 5 parameter model, Forecasting period 17–21, Fixed baseline $t=16$	One step ahead	Forecasting period 17–28, Fixed baseline $t=16$	One step ahead	$p=0$: 4 parameter model, Forecasting period 17–28, Fixed baseline $t=16$	One step ahead
mape	·44	1·37	·84	3·35	1·16	3·14	·92
mse ($\times 10^6$)	15	170	68	1053	98	593	62
Number of large ε	1	2	1	8	4	11	3
U	·350	1·217	·768	2·776	·848	2·082	·671

7.3 For the autoregressive model represented by equation (4), two sets of data were used in the fitting period namely $t = 1 - 16$ and $t = 7$–16. The latter restricted set was introduced as it was felt that if the lag m becomes large, values Z_t relating to the previous cycle of unemployment will affect the values of the parameters. A maximum of three terms was included on the right hand side of (4) predicting Z_t. This principle of parsimony is described more fully in paragraph 8.6. Normalized seasonal components $s^{(j)}$ were calculated from the residuals and included in equation (4). With $Z_t = \nabla pm_t$ as 'dependent' variable, the values of the goodness of fit statistics i.e. *mape, mse, U, number of large* ε were calculated for pm_t, the original series, rather than for ∇pm_t.

7.4 Table 6 gives the form and the value of goodness of fit statistics in the fitting period for those models satisfying the principle of parsimony and giving the most satisfactory fits to the data. The independent variables that appear in Table 6 for ∇pm_t indicate an important correlation with values 4 quarters ago. This effect is similar to a seasonal effect and is not unexpected given the autocorrelation coefficients observed in Table 2. For *model* (*d*) the seasonal components computed were little different from zero i.e. (·002, $-$·003, 0, ·001), and so *model* (*d*) has been repeated with zero seasonals to give *model* (*e*). As in Tables 4 and 5, Tables 7 and 8 give the goodness of fit statistics for the models in

Table 6. *Functional form and goodness of fit in the fitting periods of the autoregressive time series models based on equation (4)*

Model	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
Fitting period	1–16	1–16	7–16	7–16	7–16
Dependent variable	pm_t	∇pm_t	pm_t	∇pm_t	∇pm_t
Independent variable	Z_{t-1}	Z_{t-4}	Z_{t-1}	Z_{t-4}	Z_{t-4}
Seasonal components	Yes	Yes	Yes	Yes	No
Number of parameters fitted	5	5	5	5	2
mape	·44	·49	·20	·33	·37
mse ($\times 10^6$)	20	21	2	8	11
U	·404	·414	·152	·239	·277
Number of large ε	1	0	0	0	0

Table 7. *Values of goodness of fit statistics in the forecasting period 17–21 of the autoregressive time series models based on equation (4)*

	Fixed baseline at $t=16$				One step ahead forecasts			
Model	*mape*	*mse* ($\times 10^6$)	*U*	*number of large* ε	*mape*	*mse* ($\times 10^6$)	*U*	*number of large* ε
(*a*)	·67	37	·570	1	·81	43	·610	1
(*b*)	1·79	208	1·349	3	·56	21	·427	0
(*c*)	1·09	70	·780	1	·67	32	·525	1
(*d*)	·28	6	·229	0	·42	11	·310	0
(*e*)	·14	7	·254	0	·36	10	·326	0

Table 8. *Values of goodness of fit statistics in the forecasting period 17–28 of the autoregressive time series models based on equation (4)*

	Fixed baseline at $t=16$				One step ahead forecasts			
Model	*mape*	*mse* ($\times 10^6$)	*U*	*number of large* ε	*mape*	*mse* ($\times 10^6$)	*U*	*number of large* ε
(*a*)	1·43	242	1·330	3	1·04	74	·737	4
(*b*)	5·24	2,230	4·039	10	1·01	84	·784	2
(*c*)	1·88	248	1·457	6	·83	43	·564	3
(*d*)	·97	98	·845	3	·82	66	·696	2
(*e*)	1·72	248	1·348	8	·80	63	·678	2

the forecasting periods 17–21 and 17–28 respectively with a fixed baseline at $t=16$ and with one-step ahead forecasts. Allowing for the principle of parsimony, the most satisfactory fit is provided by *model* (*e*)

$$\nabla \hat{pm}_t = \cdot 003570 + 1\cdot 2055\ \nabla pm_{t-4} \tag{7}$$

which requires only two parameters.

7.5 Further insight into the interpretation of (7) may be gained by introducing the error term e_t and letting $b = \cdot 003570$ and $c = 1\cdot 2055$: for convenience. Then we have

$$pm_t - pm_{t-1} = b + c\,(pm_{t-4} - pm_{t-5}) + e_t$$

i.e. $\quad pm_t - pm_{t-1} - c(pm_{t-4} - pm_{t-5}) = b + e_t$

i.e. $\quad (1-B)\,(1-cB^4)\,pm_t = b + e_t$ using the back-shift operator B.

i.e. $\quad (1-(c-1)B^4)\,(1-B)\,(1-B^4)\,pm_t = b + e_t$

Viewed as a Box–Jenkins model this equation says that after removing trend $(1-B)$ and seasonality $(1-B^4)$, the model has some seasonal 'noise' around $(1-\cdot 2055B^4)$ and a small amount of random drift, represented by $\cdot 003570 + e_t$. It is, therefore, not surprising that the resulting seasonal coefficients for this model are very small.

7.6 *The goodness of fit statistics in Tables 6, 7 and 8 are lower than those in* Tables 4 and 5 for the current by used models and the exponential smoothing model respectively. This indicates that the autoregressive models, and in particular equation (7), provide a superior method of forecasting pm_t.

These results suggest that *model* (*e*) which requires only two parameters is the best forecasting equation for making one-step ahead estimates. When forecasting from a fixed baseline is required, then *model* (*d*) appears marginally better than *model* (*e*) but requires the inclusion of three further seasonal parameters.

8. RESULTS: ECONOMETRIC/REGRESSION MODELS

8.1 In the linear model $\mathbf{pm} = \mathbf{ab} + \mathbf{e}$, the parameter vector b was estimated using multiple linear regression. For example, the equation might be of the form

$$pm_t = b_0 + b_1 u_t + b_2 u_{t-1} + e_t \quad (8)$$

In this form multiple regression analysis can be viewed both as a means of evaluating the overall contribution of the independent variables and as a means of evaluating the contribution of a particular independent variable with the influence of other independent variables controlled. The estimates $\hat{b}_i$ of the partial regression coefficients b_i in (8) can be used as indicators of such contributions.

8.2 Using the SPSS computer package[22] three types of statistical hypothesis may be investigated and tested, namely

(a) the overall test for goodness of fit of the regression equation;
(b) the test for a specific partial regression coefficient;
(c) the test for a subset of regression coefficients.

For the overall test the statistic employed is the usual

$$F = \frac{SS_{reg}/l}{SS_{res}/(r-l-1)} = \frac{R^2}{1-R^2} \cdot \frac{r-l-1}{l} \quad (9)$$

where SS_{reg} is the sum of squares explained by the entire regression equation, SS_{res} is the residual (unexplained) sum of squares, l is the number of independent variables in the equation and R is the multiple correlation coefficient, and r is the number of observations.

The F ratio is distributed approximately as the F distribution with degrees of freedom l and $r-l-1$.

8.3 The overall null hypothesis, H_0: $R = 0$ is equivalent to the null hypothesis that all the partial regression coefficients are equal to zero in the population, i.e. $H_0\ \hat{b}_1 = \hat{b}_2 = \ldots = 0$. If the overall null hypothesis is rejected, one may conclude that one or more of the population regression coefficients b_i has an absolute value exceeding zero. But the overall test does not indicate which specific b_i coefficients are nonzero. So additional tests for specific regression coefficients are usually made. Such tests may be used in deciding whether certain of the independent variables can be deleted from the regression equation or in deciding how much confidence can be placed on the size of the sample regression coefficients. The strategy used in testing the significance of the regression coefficients involves a decomposition of the explained sum of squares into components attributable to each independent variable in the equation. The hierarchical method of decomposition was used with variables being added to the regression equation in a predetermined order.[22] Variables are added individually and the increase in explained sum of squares at each step is taken as the component of variation attributable to the variable added on that step. Each variable would be tested by calculating a suitable F ratio. This method has the advantage that the independent contributions of each variable sum to the total variation explained by all the variables.

8.4 In this decomposition method the independent variables entered into the regression equation at a particular step are determined by a pre-established statistical criterion based on the respective contribution of each possible variable to the unexplained variation. This stepwise approach is used to identify the combination of independent variables most likely to produce a good representation of the series in both the fitting and forecasting periods. Multiple linear regression was then carried out between **pm** and the sets of variables thus chosen.

8.5 *A priori*, it was decided that an equation giving a satisfactory fit with at most four terms would be the aim, subject to the requirement that the residuals provide no evidence of cyclic or non-random behaviour. Since an examination of the residuals necessarily involves a search for visual patterns, it is best accomplished when residuals are presented in a plot of ε_t (against time t or $\hat{pm}_t$). A full description of residual analysis can be found in Draper and Smith[23]. Thus a linear or curvilinear pattern for ε_t plotted against $\hat{pm}_t$ can be corrected by adding further terms, perhaps of a polynomial nature, to the regression equation or by transforming the dependent variable pm_t; a pattern which expands in width as $\hat{pm}_t$ increases indicates that the variance of the residuals is dependent upon the expected value of pm_t.

8.6 Since the results are to be used in practical estimation, the fewer terms in the estimating equation the better, subject to the need for an adequate representation (this principle of parsimony is a modern restatement of Occam's Razor that 'entities should not be multiplied without necessity'). The model constructed is required to be as efficient as possible. By adding a large number of parameters to the model the residuals in the fitting period can be made effectively zero. But the data would then contain so little information about these parameters that they could not be estimated with any degree of confidence. Furthermore, there is the danger of overfitting which leads to poor estimates when the models are tested in the forecasting period. In addition, with so much uncertainty about the source of the estimated increases in unemployment (for example, long-term unemployment among school leavers or middle-age redundancies), any formula with too many terms and parameters is likely to be criticized for spurious accuracy or to give the misleading impression that the problem has been fully solved (because a complex formula inevitably implies greater precision than a more simple one).

8.7 Normalized seasonal components $s^{(j)}$ were calculated for each possible regression relationship and included, thus the model form is

$$\hat{\mathbf{pm}} = \mathbf{A}\hat{\mathbf{b}} + \mathbf{s} \qquad (10)$$

The equation (10), with values for $\hat{b}$ obtained from the fitting period, was used to provide estimated values, $\hat{pm}_t$, in the forecasting period. The method of 'recursive residuals' could have been used as an alternative approach. In this method the equation (10) based on the fitting period 1–16 would be used to estimate $\hat{pm}_{17}$, and the error ε_{17} calculated. Then equation (10) based on the fitting period 1–17 would be used to estimate $\hat{pm}_{18}$ and the error ε_{18} calculated, and so on. This procedure would have the advantage of utilizing more fully the data available. However, it was not followed because of the excessive computational work involved, and more significantly, because it was felt that the predictive equation (10) should be tested as a means of forecasting up to 5 steps ahead since. In any practical exercise, the GAD are required to produce estimates with a time horizon of 20 steps (not 1) ahead.

8.8 Table 9 gives results for model (11) below, which is of the functional form given in (10). The model examined is that providing the most satifactory fit in both the fitting and forecasting periods and having at most two independent variables (i.e. $n \leqslant 2$ in the notation of paragraph 6.2). The equation for this regression model is

$$pm_t = \cdot 6002 + \cdot 8343 g_t - \cdot 00006571\, u_t + s^{(j)} \qquad (11)$$

where $s^{(j)} = (\cdot 007, \cdot 003, -\cdot 005, -\cdot 005)$, and g_t is defined in paragraph 3.3. The model uses six parameters and has a multiple correlation coefficient in the fitting period 1–16 of ·992.

With $n=3$, the models tried all provided, of course, a closer fit in the fitting period than the one described in Table 9 but were all less successful in the forecasting period. Comparison of these indices in Table 9 with those for the currently used models, the exponential smoothing model and the autoregressive models indicate that the autoregressive model is superior, with the regression model of Table 9 second-best over the 17–21 forecasting period. However, when the forecasting period is extended up to $t=28$, the equation (11) performs poorly. This phenomenon has already been noted for the currently used models (paragraph 4.7). It can be attributed to the fact that values of u_t are considerably higher in the most recent quarters, than in the fitting period where the parameters are estimated.

Table 9. *Goodness of fit of the regression model: equation (11)*

Goodness of fit statistic	*Fitting period 1–16*	*Forecasting period 17–21*	*Forecasting period 17–28*
mape	·36	·62	2·38
mse ($\times 10^6$)	11	22	914
U	·299	·434	2·567
Number of large ε	0	0	5

CONCLUSIONS

9.1 An example of short-term forecasting has been discussed using time series and regression methods in the context of social insurance. The problem has required the estimation of the proportion of the total unemployed in calendar quarter t who are males, pm_t. This is part of the GAD model for the estimation of costs of unemployment benefit. Using both autoregressive and multiple regression techniques prediction equations for estimating future pm_t have been developed.

9.2 The goodness of fit results for the various methods tried have been compared with those for the method in current use which is of a restricted regression type. Use of multiple regression analysis permits more general equations of this type to be considered and permits independent variables apart from the numbers of unemployed in current and historic quarters to be incorporated. Autoregressive methods take note of the characteristics of the time series itself, and in particular the significantly non-zero values of the sample auto-correlation coefficient. Over the period under consideration (since 1974) the time series pm_t is not stationary. But the transformed series ∇pm_t has improved stationarity properties (although it is not stationary) and features large non-zero autocorrelations at lags 4 and 8. Both multiple regression and autoregression methods can be adapted to include four normalized seasonal factors.

9.3 As discussed in paragraph 6.3, the *a priori* view of these models was that a regression type with u_t as an independent variable would be the most satisfactory so that changes in the levels of forecasts of u_t would be directly reflected in the forecasts of pm_t produced. When the most recent data point available was 1979 quarter 2 ($t=21$), there was little to choose between the autoregressive and regression methods leading to equations (7) and (11) respectively. The similarity covered the goodness of fit in the fitting and forecasting period. At this time equation (11) was used, therefore satisfying the criterion of paragraph 6.3, but the forecasting performance of this equation turned out to be poor. When more data became available and the most recent set was 1981 quarter 1 ($t=28$) the autoregressive approach of (7) gave a superior prediction equation. This superiority has extended up to the current time ($t=33$).

9.4 These changes reinforce the need for continuous monitoring of the performance of forecasting methods for time series, in particular economic time series. The conclusion that equation (7) provides the most satisfactory forecasts raises doubts regarding the prerequisite that a regression-type prediction equation based on u_t should be sought (paragraph 6.3). In view of the improved forecasts obtained by using (7) (relative to (2) or (11)) and the failure of the pm_t curve to respond strongly in either direction to the recent dramatic rise in u_t during period (d) (Figure 2), such a requirement appears both unrealistic and detrimental.

ACKNOWLEDGEMENTS

I should like to acknowledge the invaluable assistance provided by members of the Government Actuary's Department while the work reported above was being undertaken. Further I am grateful to a referee for helpful comments on presentation of material.

REFERENCES

(1) MILLER, R.B. and HICKMAN, J.C. (1974) 'Time Series Analysis and Forecasting' *Trans. Soc. Act.* **26**, 267.
(2) BENJAMIN, S. (1976) 'Maturity Guarantees for Equity Linked Policies' *Proc. 20th Int. Cong. of Actuaries*, **2**, 17.
(3) WILKIE, A.D. (1976) 'The Rate of Interest as a Stochastic Process—Theory and Applications' *Proc. 20th Int. Cong. of Actuaries* **2**, 325.
(4) FAGAN, J.C. (1977) 'Maturity guarantees under investment linked contracts'. Occasional Actuarial Research Discussion Papers No. 22.
(5) POLLARD, J.H. (1978) 'Premium Loadings for Non Participating Business' *J.I.A.*, **103**, 205.
(6) SCOTT, W.F. (1977) 'Reserve Basis for Maturity Guarantees in Unit Linked Life Assurance *T.F.A.* **35**, 365.
(7) WILKIE, A.D. (1978) 'Maturity and Other Guarantees Under Unit Linked Policies' *T.F.A.* **36**, 27.
(8) FORD, P.E.B. and MASTERS, N.B. (1979) 'An Investigation into the Financing of Flexible Endowment Business' *J.I.A.* **106**, 149.
(9) FORD, A., BENJAMIN, S., GILLESPIE, R.G., HAGER, D.P., LOADES, D.H., ROWE, B.N., RYAN, J.P., SMITH, P. and WILKIE A.D. (1980) 'Report of the Maturity Guarantees Working Party' *J.I.A.* **107**, 101.
(10) CONTINUOUS MORTALITY INVESTIGATION BUREAU (1978) Report No. 3: Proposed Standard Tables for Life Office Pensioners and Annuitants. Institute of Actuaries.
(11) HABERMAN, S. (1982) 'Time Series Analysis of the Probability of Survival as Unemployed' Unpublished.
(12) BOX, G.E.P. and JENKINS, G.M. (1970) 'Time Series Analysis, Forecasting and Control' Holden Day. San Francisco.
(13) DEPARTMENT OF EMPLOYMENT GAZETTE (1977) **85** No. 6, 587.
(14) DEPARTMENT OF EMPLOYMENT GAZETTE (1978) **86** No. 4, 426.
(15) CHATFIELD, C. (1978) 'The Holt–Winters Forecasting Procedure *Applied Statistics* **27**, 264.
(16) MAKRIDAKIS, S. and HIBON, M. (1979) 'Accuracy of Forecasting: An Empirical Investigation. *J.R. Stat. Soc. A* **142**, 97.
(17) WINTERS, P.R. (1960) 'Forecasting Sales by Exponentially Weighted Moving Averages. *Management Science* **6**, 324.
(18) NEWBOLD, P. and GRANGER, C.W.J. (1974) 'Experience with Forecasting Univariate Time Series and the Combination of Forecasts. *J.R. Stat. Soc. A* **137**, 131.
(19) CHATFIELD, C. (1977) 'Recent Developments in Time Series Analysis' *J.R. Stat. Soc. A.* **140**, 492.
(20) PLYMEN, J. and PREVETT, R.M. (1971) 'The Computer for Investment Research' *T.F.A.* **33**, 143.
(21) JOHNSTON, J. (1972) *Econometric Methods*. (Second edn). McGraw Hill, New York.
(22) NIE, N.H., HADKIN HULL, C., JENKINS, J.E., STEINBRENNER, K. *Statistical Package for the Social Services*. Second Edition. McGraw Hill, New York.
(23) DRAPER, N.R. and SMITH, H. (1966). *Applied Regression Analysis*. Wiley, New York.

APPENDIX I

Criteria for measuring goodness of fit

The errors of fitting are denoted by $\varepsilon_t = pm_t - \hat{pm}_t$.
Following Makridakis and Hibon[16] the following criteria for measuring goodness of fit will be used; with a fitting period consisting of points $t = 1$ to 16 inclusive:

(i) mean average percentage error:

$$\frac{100}{16}\sum_1^{16}\frac{|\varepsilon_t|}{pm_t}$$

(ii) mean squared error:

$$\frac{1}{16}\sum_1^{16}\varepsilon_t^2$$

(iii) number of ε_t such that $|\varepsilon_t| \geqslant \cdot 01$

(iv) Thiel's U Statistic $u = \sqrt{\dfrac{\sum_1^{16}\varepsilon_t^2}{\sum_1^{16}(pm_t - pm_{t+1})^2}}$

U is based on a quadratic loss function and provides a comparison over a 'no change' model i.e. the naïve or random walk approach of setting $\hat{pm}_{t+1} = pm_t$. $U > 1$ indicates that this naïve method performs better than the forecasting method with which it is compared. $U < 1$ indicates that the method under consideration does better than the naïve approach.

In the forecasting periods ($t = 17$–21 or 17–28) the above definitions are suitably modified, with summations

$$\sum_{t=17}^{21} \text{ or } \sum_{t=17}^{28}$$

respectively.