THE EVALUATION OF ORDINARY SHARES USING A COMPUTER

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INTRODUCTION

In many respects, this paper may be regarded as a logical sequel to an earlier one(1) presented to the Institute nearly seven years ago. In that paper the assessment of industrial ordinary shares was discussed in considerable detail and methods were suggested which could be used both to make projections of future dividends and earnings for different companies and to utilize these estimates in making investment decisions. Much of the discussion on the earlier paper centred on the latter problem and the present paper is concerned entirely with this aspect of share assessment. The making of future projections will not be considered again; suffice it to say that our methods have remained largely unchanged. The principle on which the technique that we have now developed rests is closely linked with the previous concept of a 'capitalization rate' but in passing we would mention that we have made a brief experiment with what we term 'growth yields', i.e. the compound interest return to be expected from a particular share at a given price on the basis of the forecast future dividends. Our results utilizing this method were in fact significantly better than those obtained using the more familiar concept of relative yields (see §8) but, as the figures presented later in this paper show, they were not as good as those obtained using the method which forms the basis of this paper.

2. The title to this paper includes the word computer and we would stress that the use of the techniques described has only been made practicable by the advent of computers. Over the last decade, their use in business and industry has shown a remarkable increase and more recently this trend has itself been felt in the investment world. In the latter field, the computer has, at least in the U.K., been mainly used for accounting purposes although on the gilt-edged side it has been used for the calculation of redemption yields and, in fitting yield curves, used in connexion with switching techniques.(2) In contrast the computer has, until very recently, made virtually no impact in the field of equity investment and even now little attempt has been made to use it to make actual investment decisions (as opposed to the processing of historical data) in spite of the availability of powerful statistical methods. Our paper is largely concerned with a description of the way in which we have tried to use the computer to take investment decisions and a discussion of the statistical and investment aspects of the problem.
3. Any suggestion of a formalized approach to the evaluation of ordinary shares inevitably meets considerable criticism from those who regard investment as an art and not a science and who ask what help a statistical approach can give to the solution of a problem in which individual judgment plays so large a part. This question can best be answered by breaking the problem down into two parts in order to discover exactly where judgment comes in. In the first place, the investor has to make his own estimate of what a particular company is likely to achieve in terms of growth in the foreseeable future and he then has to consider how the market will react to this progress if it is achieved (or in other words how much of the expected growth is already discounted in the price). Investment analysis already places a great deal of emphasis on the first where personal judgment is admittedly important. The second objective also requires personal judgment but we take the view that much of this work can be undertaken by an unthinking computer which, because it can store and compare a very large amount of data on companies and on the market, possesses considerable potential for making objective comparisons which are, in essence, the basic part of this personal judgment.

4. The crux of the problem is of course the derivation of a relationship between share prices and share data and here we have made use of the technique of multiple regression analysis. The rationale behind the process is to correlate the yields on a large number of shares with quantitative measures of the past record and of our view of the future prospects for each company, having also made allowance for the particular dividend cover on each share. Most investors would normally make such an analysis using purely subjective methods—a particular share has a good record; its future prospects also seem better than average but its dividend cover is relatively low. A comparison is then made mentally with the index yield and a decision as to whether the yield on the share is acceptable follows. The replacement of this subjective approach by a mathematical technique could loosely be considered as a first step in the application of Operational Research to investment and the authors take the view that the investment field could well be wide open to the application of these techniques. In a paper presented to the Institute in 1960(3) Gould discussed the applicability of Operational Research methods in general and he stated that ‘the principle of this method can be applied not only in the Armed Services, but also wherever organization has to make policy decisions where some quantitative knowledge of the factors involved may be used as a basis in place of personal knowledge and opinions’. Gould was in fact referring to applications in industry and commerce generally but it is not difficult to see the particular application to investment. In previous discussions on investment at the Institute it has been suggested that whilst actuaries ought to be able to make a material contribution to the principles of investment, in the selection of shares they may well be no better than the next man. However, their knowledge of both finance and mathematical
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statistics suggest that they could well give a lead in the application of O.R. to portfolio management.

5. Although this paper properly belongs to the sphere of investment, much of the material presented falls in the field of mathematical statistics and in particular regression analysis. In fact the latter technique has not often been discussed at the Institute before, one of the comparatively recent occasions being a paper by Dyson and Elphinstone(4) entitled The Expenses of British Life Offices. In this paper some time was spent in considering the actual regression coefficients produced by the various analyses but in our application we have used multiple regression analysis essentially as a method of prediction and not so much consideration has been given to the meaning and appropriateness of the coefficients themselves, except to reassure ourselves that they were in accord with common sense, e.g. an increase in the payout ratio should be correlated with an increase in the dividend yield. By way of illustration we may cite the use of the technique by the N.I.E.S.R. (5) to predict imports where it was stated that, although the relative importance of the various factors employed (i.e. the coefficients) was sometimes difficult to explain, the agreement between actual and predicted imports was good. A wider experience may shed some light on the meaning to be attached to the calculated values of the coefficients and the manner in which they vary over time (see § 30), but meanwhile we consider that the actual results provide a sufficient justification for continuing our experiments.

6. One of the penalties of introducing a new system (and quite rightly) is that it must be seen to work and produce satisfactory results. We have tested our method of share selection over a period of two and a half years and the results are discussed in a later section. In addition, we also compare the performance of this method with other objective systems of share selection but unfortunately we have little means of comparing any of the results with those obtained by investors who use the subjective approach. We should, however, welcome any data that members feel able to make available.

METHODS OF SHARE SELECTION

7. Investors presumably buy ordinary shares, first, in the hope that the economy will grow in real terms either at a rate of say 3% per annum or some other positive rate and, second, on the grounds that ordinary shares will provide 'a hedge' against inflation, should this continue. Having made the decision to buy ordinary shares, one then tries to find shares whose future growth has not already been fully or over-discounted in the current price. In making this selection one inevitably runs up against the question of how long a view the investor should take and it is here that the authors feel that they may be accused of taking too short a view. The method used actually involves making forecasts over the next five years but the period over which the selection of the individual shares is expected to show is of
the order of twelve months. To some extent this choice was governed by
the fact that it was believed that this was the order of time interval over
which the technique would be judged. However, as will be seen later, the
method is fairly flexible and a time horizon some two and a half years away
could quite easily be used. Before discussing the system in detail we men-
tion two other methods of share selection which have been used in practice.

Relative yields

8. One of the methods used most frequently to evaluate shares is the
relative yield approach. Briefly, the method consists in dividing the yield
on the share under consideration by some index yield—usually the
Financial Times Industrial share index or more recently the Financial
Times–Actuaries Index. The level of the relative yield provides an indica-
tion of the dividend growth prospects for the share as seen by the market,
and the theoretical background to this method is easy to see. If it is
assumed that the average dividend growth rate in perpetuity for the index
stocks is \( g \) per unit and the growth rate for the share under consideration
is \( (1+k)g \) where \( k \) is positive, then if the share is correctly valued in
relation to the index we have:

\[
y + g(1+k) = Y + g
\]

where \( y \) and \( Y \) are respectively the prospective yields on the share and on
the index and \( g(1+k) \) is less than the return on each investment for all
values of \( k \). The relative yield is given by \( y/Y \) or \( 1-kg/Y \). If, for the
purposes of illustration, the yield on the index was 5.0% with a growth
rate of 4.0% per annum, the relative yield of a share with a growth rate of
5% (25% better than the index) would be:

\[
1 - \frac{(0.25)(0.04)}{0.05} = 0.80, \text{ i.e. 80\% of the index yield.}
\]

In practice, few investors would pretend to take a view on growth prospects
over a period of more than, say, five years and relative yields tend to be
used in a more subjective way by following their movement for a given
share to provide an indication of whether the share appears to be cheap or
dear against the current level of the market. Limitations of this method
include the fact that on a short-term view, if an increase in the dividend is
expected in the near future, the relative yield will be low, or on a longer
view, a share with a poor record will have a high relative yield (and vice
versa). In addition, no allowance is made for the fact that the dividend
cover for the share under consideration may, for example, some five
years ago have been twice the average, whereas today it may be the same
as the average. These considerations all suggest that movements in a
share’s relative yield reflect changes in the market’s evaluation of the
future prospects of the company, and relative yield graphs are in fact
made use of, in conjunction with graphs of the percentage change in an
industry's output, as a means of selecting shares. If individual earnings and dividend projections are available the problem can be tackled by the method of capitalization rates advocated in the paper already referred to.\(^{(1)}\) This approach involved selecting a relative yield for the company on the basis of its past yield record and our future projections, and relating this yield to a forecast dividend to produce a suggested value. Once again, however, the selection of the appropriate relative yield contains a large subjective element which in many respects appears to be undesirable.

**Growth yields**

9. In the introduction to this paper mention was made of the technique of 'growth yields' for comparing equities, and in the reply to the discussion on the Weaver and Fowler paper the formula was set out for the calculation of the growth yield \(i\). Very briefly, it is assumed that dividends will grow at an annual rate of \(g\) per unit for the next five years and that thereafter growth will revert to a uniform rate of \(g^{1}\) for all shares. On this basis and knowing the share price \(P\) and the first forecast dividend \(D\), it is possible to calculate the compound interest return or growth yield on the share. Utilizing the dividend projections that we make on the major companies we have used this method to rank ordinary shares. Portfolios selected on this basis have in fact performed significantly better than the index. In some respects, this technique would seem appropriate to the needs of the long-term investor and the extent to which the method tends to be biased towards the high yielding shares seems readily acceptable. Growth yields are, however, subject to a number of criticisms which impose limitations on their practical value:

(a) They are based on too long a view which may be hazardous at a time of great technological changes and when new products are constantly being developed.

(b) They are relatively insensitive to short-term changes which have considerable weight in the timing of new purchases.

(c) They make no allowance for variations in dividend cover between different shares.

The latter is an important disadvantage unless it is assumed that all shares will liberalize their dividend to provide the same payout ratio after five years, in which case the variation in current payout ratios would be allowed for. The growth yield approach does, however, have a permanent value in the analysis of share prices. Thus in the case of low yielding shares, if the question 'what growth is the current share price discounting and for how long' is asked, the growth yield technique will supply a ready answer.

**The computer method**

10. The technique of estimating share values by relating forecast dividends to a capitalization rate (or relative yield) has already been
referred to and it was pointed out that the subjective method by which the capitalization rate was selected seems to us to be the main disadvantage underlying the method. To overcome this, the Computer Method has been developed and the aim has been to use the computer to go through the processes that we believe that the investor carries out in his mind when choosing an appropriate yield for a share. This process would, we think, include the following comparisons:

(a) the dividend cover for the share against the average for all shares;
(b) the future prospects both in the short and long term against the average;
(c) the company's record and that for industry in general.

11. When these comparisons are complete, an estimate is made of the premium (or discount) in yield that the share appears to merit (in comparison with the average for all shares) by placing some arbitrary weight on each deviation. By way of example, if the payout ratio was the only factor affecting dividend yield, investors would presumably have in their mind some relationship between these variables and use this to determine whether the share was cheap or dear. In practice, our experience is that it is exceedingly difficult to be consistent in the weight that one places on the various factors for different shares. However, with a computer, the process of correlating deviations from their respective averages for any number of factors believed to affect the yield on a share with the deviation of a share's own yield from the average, can easily and rapidly be carried out using a multiple regression analysis. This technique determines the 'best' estimates of the regression coefficients for each factor and then applies these coefficients to each share to determine the appropriate yield. In non-statistical language the coefficients may be thought of as 'weights' that the market apparently places on each factor. Comparison of the actual yield with the calculated yield provides a measure of the relative cheapness of each share or, alternatively, the process can be thought of as estimating a computer yield which is then used in conjunction with the forecast dividend to produce a theoretical price. Comparison of this price with the actual price leads to the same estimate of relative cheapness.

12. To summarize, the method consists in estimating the appropriate status for each share and by relating this figure to a forecast dividend a possible value for the share is deduced. If the analysis were to be carried out in terms of current dividends the pattern of computer yields obtained might be considered as providing the static picture, i.e. the position as of today. However, since the change in price of a share over a particular period of time depends upon the change in status and the change in dividend, a dynamic approach seems more desirable and we have estimated yields and dividends that should be obtaining in twelve months' time on the assumption of no change in the general level of the market. As we have stated earlier, there is no reason why one should not take a longer view
and it will be appreciated that the length of the period selected depends on the relative weight it is desired to give to current yield as opposed to future dividend growth. A long period would tend to show as attractive those shares where a high rate of growth was forecast while the shorter period would favour shares with a high current yield and a relatively low rate of growth. To the extent that future estimates of earnings growth must in the nature of things be subject to fairly wide limits of error, the emphasis of the computer method on a rather short view seems to us to be more reasonable.

**THE COMPUTER TECHNIQUE**

*Background to the method*

13. It has already been mentioned that the idea underlying the development of this method is an attempt to parallel, by a mathematical technique, the process that the investor might be expected to go through in evaluating a particular share in the belief that, on the average, the market is consistent in its judgment. Quite clearly this approach is only useful if it is possible to establish correlations between the yield on a share and factors that one might expect *a priori* to exercise an influence on this yield. In this task we have been reasonably successful, as will be shown later, but in passing we thought it might be of interest to refer to the work of other investigators in this field. G. R. Fisher writing in the *Economic Journal*\(^6\) considered the factors exercising an influence on share prices and he found that variations in the last dividend declared per share explained a considerable proportion of share price variation and that, in addition, the last declared undistributed profits per share was also a significant factor. Neither of these conclusions should surprise the investor since they appear to imply merely that shares are valued by the market on the basis of dividend and earnings yields. Fisher also found that company size had a significant effect, but the 'past rate of growth in dividends per share seems to be regarded by the market as a dubious indicator of future prospects, and its effect on share prices appears to be both small and uncertain'. This conclusion has been reached by other workers in this field, and if one accepts the view that the yield on a share should discount the future, it would also follow from the principal conclusion of I. M. D. Little in his now famous paper entitled *Higgledy Piggledy Growth*\(^7\) that past growth performance of companies provides little guide to the future. The statistical analysis underlying this conclusion could in fact be criticized on certain grounds\(^8\) and these have now to some extent been answered in a more comprehensive study by Little and Rayner.\(^9\) On the other hand, the authors of this comparatively recent work have in fact provided evidence that *yields* on shares do appear to reflect both past and future growth records. By correlating relative yields on shares with both past and subsequent dividend and earnings growth rates, they have shown that the past record does appear to exercise some influence on the relative
yield on a share and that future dividend growth is also reflected in a share's present status. In terms of the past record, our own results from the correlation analysis tend to confirm these findings whilst the justification for using forecast dividend growth rate as a factor in our model is also provided by Little and Rayner's analysis. On the other hand, the authors' second main conclusion, that the future earnings growth rate is uncorrelated with relative yield (i.e. that investors are unable to predict the future), is surprising. In the paper some space is devoted to discussing the conflict provided by this conclusion and the fact that relative yields do appear to depend upon subsequent dividend growth rate. However, since the relative yield is a function of dividend cover, an analysis which first adjusted relative yields for this factor before correlating with earnings growth rate would be more likely to lead to meaningful results. In addition, since dividend liberalization accounts for a significant part of growth in dividends this would also explain why dividend growth appeared to be correlated with relative yield.

14. Apart from Little and Rayner's recent analysis, so far as we are aware no work has been done in the U.K. in correlating share yields with an objective estimate of future prospects, possibly owing to the enormous work involved in making these projections, particularly by persons working outside the investment world. Since in the normal course of our daily work we make these projections, it has been relatively easy to carry out a correlation analysis relating yields to both future prospects (as seen by us) and the past record. Moreover, since we have to estimate the dividend expected at the end of the year, we can base the current yield on a blend of last year's dividend and the next. This gives a more realistic 'current' yield than one based on the past dividend and makes due allowance for the varying position of shares in their financial year. This almost certainly leads to better results in the correlation analysis.

15. Our remarks so far on the subject are based on work in the U.K., but in the U.S.A. it is apparent that the computer is already being used to provide a more sophisticated and apparently more successful approach to investment. The Bank of New York has published details of a method by which price earnings ratios (PER) on a large number of companies are correlated with three factors—forecast earnings growth rate, stability index and dividend payout ratio. Having determined the apparent relationship between the PER and these three factors, the relationship is used to estimate the expected PER and thus to enable relative attractiveness of each share to be assessed. It may be argued that this type of analysis, which parallels our own in principle, is much more easily carried out in a country where considerably more detailed information on companies is available. This is true but, against this, it cannot be denied that investors in the U.K. must still of necessity make some estimates before any share is selected, and in any case the new Stock Exchange regulations concerning information required from companies will, one hopes, make
the task of forecasting easier. Further, in using this technique, its virtue of consistency appears to us to become more apparent every day. As an example we may cite the case where the relevant statistics for a particular company are fed into the computer and the share is found to be, say, 10% cheap. The analyst concerned with making projections for the company may sometimes express surprise, since he considered the share to be no more than reasonably valued. In the ensuing period of investigation as to why the analyst and computer disagree it may be found that, in the comparison between the company and the average, the analyst had not been quite fair, or in some cases a simpler explanation offers itself, e.g. the analyst decided, say, four weeks previously that he did not particularly like the company but since then the market has risen 10% and the share has not changed in price. In fact, he was right in his previous assessment, but neglected to change his view at the current price levels obtaining. Further advantages of the computer technique will be discussed later but the advantages of using statistical analysis to determine the rules of the investment game (i.e. the regression relation) and then letting the computer play the game, according to the rules, seem to us to be very real. It is also the only way in which we can use to the maximum advantage all the information coming from a research department.

The model

16. The basic assumption underlying the application of the regression approach is that the yield $y$ on a share can be expressed as a linear function of variables $x_1, \ldots, x_t$ with the addition of a random element $e$ by the relation:

$$y = B_0 + B_1 x_1 + B_2 x_2 + \ldots + B_t x_t + e$$

The $B$'s are known as regression coefficients and since the $x$ values may be, and in our application are, interrelated we shall in future refer to them as 'predictor' variables rather than independent variables. In practice, of course, we are unable to isolate quantitatively all the factors which affect the yield $y$ but the assumption made is that those factors that we cannot separate out are small in their effect on the yield in comparison with each $x$ value and may be included with the random element $e$. The lack of independence between the $x$'s in no way invalidates the analysis as a method of forecasting or predicting although it does make it much more difficult to interpret the results in terms of the causal effect of $x$ on $y$. This point came out in Fisher's analysis when he found that inclusion of the past dividend growth rate in the model (after dividend cover) did not provide a significant improvement to the 'fit' of his share price model. If, in fact, companies with high cover tend to have high dividend growth rates (dividend growth lags behind earnings growth) then this result would, as Fisher pointed out, be expected. Reversal of the order in which these two factors were introduced in the model might well suggest that dividend growth had a significant
Mathematical solution

17. For each of \( n \) shares we are given an array of values or observations \( y, x_1, x_2, \ldots, x_t \) and the problem is to obtain estimates \( b_0, b_1, \ldots, b_t \) of the regression coefficients \( B_0, B_1, \ldots, B_t \). Unbiased and minimum variance estimates of each \( B_i \) can be obtained by choosing the values \( b_i \) to minimize \( S \) the sums of squared residuals:

\[
S = \sum (y - (b_0x_0 + b_1x_1 + \ldots + b_tx_t))^2
\]

where the summation takes place over all the \( n \) arrays of values of \( y \) and \( x_i \), and \( x_0 \) is a dummy variable always equal to 1. It can be shown that the least squares estimates of \( B_i \) are given by the matrix equation:

\[
Cb = B
\]

where \( C \) is a square matrix of dimension \((t + 1)\) whose element in the \((i + 1)\)st row and \((j + 1)\)st column is \( \Sigma x_i x_j \)

\( b \) is a column vector of \((t + 1)\) elements \( b_i \)

\( B \) is a column vector of \((t + 1)\) elements \( \Sigma y x_i \)

This part of the analysis makes it possible for the estimates \( b_i \), i.e. the regression coefficients, to be calculated but it is also of interest to calculate or consider the following:

1. The residual sum of squares \( S_r \) for \( y \) (i.e. the sum of squares of the deviations between the actual values of \( y \) and the estimates obtained from the fitted relation).

2. The proportion of the sum of the squares of the yield values about their mean \( S_a \) accounted for by the predictor variables. This proportion is equal to:

\[
(S_a - S_r)/S_a
\]

3. The standard error of the estimates of the \( b_i \) values. Comparison of the actual regression coefficient with its standard error then enables the statistical significance of that factor to be assessed.

4. The analysis of variance table which shows the amount of variation in the yield values accounted for by each predictor variable \( x_i \), in the order in which they are introduced into the model.

The analysis of variance table takes the following form:

<table>
<thead>
<tr>
<th>Variation due to</th>
<th>Sum of squares</th>
<th>Degrees of freedom</th>
<th>Mean square ratio (1)/(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>( S_1 )</td>
<td>1</td>
<td>( S_1 )</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>( S_2 )</td>
<td>1</td>
<td>( S_2 )</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( x_t )</td>
<td>( S_t )</td>
<td>1</td>
<td>( S_t )</td>
</tr>
<tr>
<td>Residual</td>
<td>( S_r )</td>
<td>( n - 1 - t )</td>
<td>( \sigma^2 )</td>
</tr>
<tr>
<td>Total</td>
<td>( S_a )</td>
<td>( n - 1 )</td>
<td></td>
</tr>
</tbody>
</table>
The statistical analysis has been carried out on an Elliott 803 computer but unfortunately the programme available did not include the facility for producing the analysis of variance table. This was obtained, therefore, by carrying out five consecutive \((t = 5\) in our model) analyses on the same data increasing the number of predictor variables by one on each run. The amount of variation in yields explained by each newly introduced factor (say \(x_j\)) was then equal to the reduction in the residual sum of squares, namely, \(S_k\). The significance of each factor in the order in which it was introduced into the model was then judged by referring the ratio of \((S_k \div \sigma^2)\) to the \(F_{1,n-1-t}\) distribution in the normal way. The significance using this method for the last variable introduced is of course equivalent to the \('t'\) test for the regression coefficient \((b \div \text{standard error of } b)\) since the latter test considers the significance of each factor on the assumption that it comes last. Further details of the mathematical background to the analysis are given in Appendix 1.

The variables

18. The variables included in the model fall into two sets—first the dependent variable dividend yield and, secondly, the so-called independent or predictor variables which quantify the share’s payout ratio, past record and future prospects. The predictor variables also fall conveniently into two sets—those that are historical and can be readily calculated and, secondly, those that in some way summarize the future prospects for the shares. Before discussing the choice of the two sets of factors it is perhaps necessary to consider whether or not it is reasonable to make any allowance for record at all in the analysis. On this question we have to confess that we are in two minds since we would agree with those critics who argue that the record should in fact have already exerted its influence when we made our projections. Against this point, we would argue that the company’s record is something that cannot be disputed, in contrast to our projections and it can therefore be regarded as something in the nature of a firm base point. In addition, the statistical ‘fit’ of our model was significantly improved by inclusion of factors representing the record. We feel that the answer to this problem probably depends upon the length of view that the investor is prepared to take. For example, if we consider a share which has a record of declining earnings but for which it is believed prospects are now relatively good, the stigma of the record may in fact remain with the company even after the first signs of recovery are apparent—these results could in fact be a ‘flash in the pan’ and the investor (perhaps rightly so) is still wary. In practice, therefore, the price of this particular share may not at first perform significantly better than the index but as the record wears off and the longer-term prospects move nearer, the performance should improve as investors are more ready to take the risk. This philosophy, which appears also to work in reverse, is contained in our computer technique which tries to strike a balance between past and future in its
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determination of status. In addition, since we incline to the view that
methods of share selection are usually expected to work over compara-
tively short periods of time, our method, involving as it does a certain
degree of ‘inertia’, seems more appropriate.

19. Six variables are used in the regression analysis:

(1) Dividend yield $y$
(2) Dividend payout ratio $x_1$
(3) Forecast earnings growth rate—short-term $x_2$
(4) Forecast dividend growth rate—long-term $x_3$
(5) Historical earnings variability $x_4$
(6) Historical earnings growth rate $x_5$.

20. Since the analysis attempts to place an appropriate value on each
share in 12 months’ time, all the variables with the exception of the two
historical ones are based on forecasts. Thus $y$ is the dividend yield (at
today’s price) in 12 months’ time and $x_1$ is the corresponding payout ratio.
The exact definition of each variable is discussed below, but in the case of
$y$, $x_1$ and $x_2$ it will be noticed that we have introduced the concept of mean
dividend and mean earnings. This has been done to overcome the difficulty
created by the fact that companies have different year ends. In the case of
dividend yields it is usual to base these on the latest known dividend but an
exception occurs when the company has made a forecast. Use of this fore-
cast quite clearly raises an inconsistency making the share appear relatively
more attractive, if an increase is in prospect, than it otherwise would have
been. If the concept of a mean yield is introduced, based on a weighted
average of the latest declared dividend and the next forecast one (according
to the number of months to the next results), all shares may be conveniently
compared on a consistent basis. The same treatment is necessary for
earnings. The exact definitions of the variables used in the analysis are
as follows:

(1) Dividend yield

The dividend yield $y$ based on the current price $P$ and the mean dividend
payable in 12 months’ time is calculated from the relation:

$$ y = \frac{m(tD_1 + (12-t)D_2)}{12P} $$

where $m = \text{nominal value of share}$

$P = \text{current price}$

$t = \text{number of months to next results}$

$D_1$ and $D_2$ represent the first and second
forecast dividend rates respectively.
(2) Dividend payout ratio

The mean payout ratio \( x_1 \) is defined as the reciprocal of the average cover for the mean dividend and is calculated from the relation:

\[
x_1 = \frac{tD_1 + (12-t)D_2}{tE_1 + (12-t)E_2}
\]

where \( E_1 \) and \( E_2 \) represent the first and second forecast earnings rates respectively.

(3) Forecast earnings growth rate—short term

The forecast earnings growth rate over the short term is defined as the change in mean earnings over the 12 months following the next 12 months, and is calculated from the relation:

\[
x_2 = \frac{tE_2 + (12-t)E_3}{tE_1 + (12-t)E_2}
\]

(4) Forecast dividend growth rate—long term

For the longer-term variable, we have used the dividend growth rate rather than the earnings growth rate. The reasoning underlying the preference for dividends rather than earnings is that, whilst it is possible to make what seems a reasonably accurate estimate of the fifth year dividend, in the case of earnings these could have a much wider fluctuation, particularly in the case of cyclical industries. In addition, although part of the dividend growth may reflect liberalization (already allowed for in the payout ratio) on the other hand it is sometimes possible to distinguish between earnings that are or are not likely to be translated into dividends. At the moment, the longer-term dividend growth rate is calculated from the ratio of the forecast dividend for the fifth year \( D_5 \) to \( D_2 \). However, to be consistent with the use of the mean dividend defined by the relation \( D_{12} = (tD_1 + (12-t)D_2)/12 \) for the yield, the long-term growth rate should be calculated from the relation \( x_3 = s\sqrt{D_5/D_{12}} \) where \( s = (3+t/12) \). By way of example, if a company has just declared its results (i.e. \( t = 12 \)) then \( x_3 = 4\sqrt{D_5/D_1} \) and when results are due the growth rate has the value \( 3\sqrt{D_5/D_2} \). This slight sophistication which has since been introduced can be important when a company is expected to cut its dividend to \( D_2 \) from \( D_1 \) but in the longer term to restore it to at least \( D_1 \) again. As the mean dividend moves progressively towards the reduced value of \( D_2 \), the long-term growth rate will increase to compensate this to some extent. Under the present system, however, \( x_3 \) can be quite high representing the full restoration of the dividend but \( D_{12} \) only moves progressively to its cut value of \( D_2 \) and the share therefore appears to be correspondingly more attractive than it should for the first few months after the declaration of the dividend \( D_1 \).
(5) Historical factors

The two historical factors used in our model relate to the company’s past record. In our initial examination of the problem we used the annual growth rate in dividends but, since this might give an advantage to a company which merely had a relatively low payout ratio at the start of the period, we replaced this variable by the earnings growth rate over the past ten years. To avoid any cyclical pattern in the company’s record, the growth rate is based on all ten earnings figures and, although at first we obtain this figure by fitting a straight line through the earnings points, for ease of calculation we now use the following expression:

\[ x_5 = \left( \frac{\sum_{t=1}^{5} E_{t+5}}{\sum_{t=1}^{5} E_t} \right)^{1/5} \]

It may be argued that more weight should be given to recent earnings figures in the calculation of a growth rate by using some exponentially weighted formula but, since the majority of companies tend to exhibit some cyclical pattern in earnings, this would be incorrect and a trend figure is all that is required. The other factor \( x_4 \) which would on a priori grounds be expected to affect an investor’s assessment of a share is the degree of fluctuation in its record about the earnings trend line. Initially, we allowed for this in a very simple manner by introducing a characteristic variable which took the value 0 if a company had had a setback, and 1 otherwise. A setback was defined as a fall in earnings of 20% or more in any one year or between 10% and 20% in two consecutive years. Since this method seemed rather too arbitrary we now make an estimate of the standard deviation of the historical earnings figures over the past five years about the trend line. The use of a ten-year term for the growth rate and five years for the variability may seem slightly inconsistent but the problem really stems from the fact that we are using a straight line to represent a growth trend that may in fact be far removed from this and the variability would therefore contain the trend. By using a shorter period to measure the variability it is hoped that the worst effect of this difficulty will be eliminated. A computer programme has been written to calculate both the earnings growth rate and variability making an automatic adjustment for the advent of corporation tax. A complete set of typical computer input data is shown in Appendix 2(a).

Other factors

21. The mathematical model set up to describe the pattern of yields on ordinary shares assumes that the yield depends upon the dividend payout ratio, the short- and longer-term prospects for the company and, finally, the company’s past record. When these factors have been allowed for in the analysis, the yield anomalies for each company are taken as indications of whether the share in question is cheap or dear. In practice, of course,
Evaluation of Ordinary Shares using a Computer

whilst these variables are probably the main ones uppermost in investors’ minds when ordinary shares are evaluated, there must be many others that exercise an influence and the following are possibilities:

(1) Does the industry under consideration appear to have particularly good long-term growth prospects beyond the next five years?
(2) How does the current share price compare first with the asset value and second with the all-time high?
(3) Has there been a management change? This should, of course, be allowed for in the future projections but, since the result of changes of this nature is often for the company to expand into new fields, it may be very difficult to make proper allowance.
(4) Is a take-over bid likely? This question is usually interrelated with the ratio of the share price to the asset value.
(5) Are there any long-term fears about the industry in question, e.g. the cancer risk for tobacco shares?
(6) Political considerations for certain industries, e.g. nationalization of the Steel Industry.

In our opinion, too much emphasis is often placed on some of these factors—take-over bids may eventually occur at half the current share price, asset values may be dissipated by prolonged periods of unprofitable trading and so-called growth industries may prove to be disappointing in terms of profitability. Management changes are possibly the most important factor for which full allowance cannot be made in our model, whilst asset values may be very important for certain industries, e.g. property companies, which in fact have not been included in our model. On balance, we consider that allowance for some of the more unquantifiable factors can always be made by the individual investor according to his own opinion after the computer has, so to speak, carried out the ‘donkey work’ and allowed for the obvious factors.

22. In the case of objective factors such as the asset value and the all-time high price, both would have to be expressed in terms of the present share price. In fact both these correlations would tend to produce unacceptable results in the sense that a share, for example, showing a large discount on assets may be expected to be on a high yield so, the larger the discount, the higher the yield required (and therefore the less attractive the share). Similar reasoning would apply to the current share price in relation to the all-time high whereas, on commonsense considerations, a share at the all-time low with a large discount on assets would seem to be an attractive purchase. In addition, there is the general point applicable to predictor variables, namely, that they must not be a function of the current share price. This follows from the fact that, if they were included in the model, the resulting change in the share price suggested by the computer to make the share correctly valued would in itself alter the predictor variables and lead to a new suggested price. On the other hand, since
both these statistics could well have some predictive value in terms of future share price movements, we shall refer to them again in a later section in connexion with an alternative but as yet untried system of ranking shares.

Form of the model

23. In describing in § 16 the statistical basis of the model employed, the actual form of the relation used was not discussed since this can only be considered after the actual factors employed in the analysis have been defined. If, in the first place, we consider the relation between the dividend yield and the payout ratio only, the starting point in setting up a model might be to use a relation of the form \( y = a_0 + a_1 x_1 \) where \( y \) and \( x_1 \) actually represent the yield and the payout ratio. However, this model would imply that a fixed change in the payout ratio would have the same effect on the yield irrespective of the level of either variable and, to overcome this, we have adopted a logarithmic model. This argument can in fact be extended to include all the remaining predictor variables and the precise form of the model used in the analysis is as follows:

\[
\log y = a_0 + a_1 \log x_1 + a_2 \log x_2 + \ldots + a_5 \log x_5
\]

where

- \( y \) = mean dividend yield
- \( x_1 \) = mean payout ratio
- \( x_2 \) = the forecast short-term earnings growth rate
- \( x_3 \) = the forecast long-term dividend growth rate
- \( x_4 \) = the historical earnings variability
- \( x_5 \) = historical earnings growth rate.

In addition to the a priori reasons for using the logarithmic model, in practice we have found that a higher proportion of the variation in the logarithms of the yields about their mean was explained by the predictor variables than was the case with the linear model.

THE STATISTICAL ANALYSIS

24. Consideration has been given to the choice of variables which it was thought would, on a priori grounds, influence the dividend yield on an ordinary share. The main task of the statistical analysis, however, is to provide an estimate of the magnitude of the effect of each factor rather than to establish that each can be shown to be statistically correlated with the dividend yield. This approach may, we feel, conflict with that of others who are in the habit of using regression analysis programmes to eliminate systematically variables that do not appear to have a statistically significant effect on the dependent variables. However, on the grounds that a factor, although not significant in the statistical sense, may still have an important
effect (and vice versa), we feel that the former approach is more relevant to the problem that we are faced with. The question of significance will, however, be considered, although in view of the correlations between the predictor variables it is difficult to arrive at any satisfactory conclusions.

**Diagram 1**

Relation between yield and payout ratio.

At present, the regression programme is run once a week but to make the discussion easier to follow, the results from a single run will be considered in detail although these may be regarded as typical of the period since we started these calculations on a routine basis. The results of the multiple regression analysis are summarized in the following table which sets out,
for each predictor variable, the corresponding correlation coefficient, regression coefficient and the standard error of the latter estimate:

<table>
<thead>
<tr>
<th>Predictor variable</th>
<th>Correlation coefficient</th>
<th>Multiple regression coefficient</th>
<th>Standard error of ( a )</th>
<th>Ratio of ( a ) to its standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payout ratio ( x_1 )</td>
<td>+0.654</td>
<td>+0.354</td>
<td>0.064</td>
<td>5.5</td>
</tr>
<tr>
<td>Forecast earnings growth ( x_2 )</td>
<td>-0.124</td>
<td>-0.266</td>
<td>0.161</td>
<td>1.7</td>
</tr>
<tr>
<td>Forecast dividend growth ( x_3 )</td>
<td>-0.649</td>
<td>-2.484</td>
<td>0.568</td>
<td>9.4</td>
</tr>
<tr>
<td>Historical earnings variability ( x_4 )</td>
<td>+0.256</td>
<td>+0.087</td>
<td>0.036</td>
<td>2.4</td>
</tr>
<tr>
<td>Historical earnings growth rate ( x_5 )</td>
<td>-0.347</td>
<td>-0.295</td>
<td>0.168</td>
<td>1.8</td>
</tr>
</tbody>
</table>

25. As one might expect (and Little and Rayner agree with the results for \( x_3 \) and \( x_5 \)) the signs of the regression coefficients for \( x_2 \), \( x_3 \) and \( x_5 \) are negative, indicating that an increase in the growth rate (historical and forecast) is associated with a decrease in yield, whilst for \( x_1 \) and \( x_4 \) increases in the payout ratio or earnings variability lead to an increase in the yield. By themselves, the actual regression coefficients give no indication of the practical importance of the predictor variables—this will depend upon their range. For purposes of illustration, therefore, the accompanying Diagram 1 shows the fitted mathematical relation between yield and payout ratio. Curve A applies to companies for which the values of the variables other than yield and payout are at the average, but curves B and C show the effect on the yield/payout relation for companies having above-average short-term and long-term growth prospects. In both cases, the above-average growth rates represent the mean value of each predictor variable together with twice their standard error.

26. In terms of the statistical significance of each factor in its effect on yield, comparison of the regression coefficients with their respective standard errors shows that three out of five were significant at the 95\% probability level (i.e. the ratio is greater than the 5\% value of \( 't' \)) but the other two were significant at the 10\% probability level. This suggests that all five of the predictor variables have an effect on yield but, in order to judge more clearly the usefulness of including each factor in the model,
the analysis of variance is more informative. The table which is shown above sets out the proportion of the variation in yields accounted for by each factor (in the order in which it was introduced) for an analysis based on 134 companies.

27. It will be seen that nearly 60% of the variation in yields is explained by variation in the chosen factors—the percentage has been as high as 75% but never less than 55%. The best figure was obtained when the number of companies included in the analysis was around 100, the effect of widening the scope of the regression being to increase the unexplained variation, presumably reflecting the greater heterogeneity of the companies. This suggests perhaps that a regression analysis for separate industry groups would achieve a better overall fit to the data, but the following points need to be borne in mind:

(a) Since there appears to be no logical reason why ordinary shares in different sectors should be valued any differently from one another, it would be necessary to combine the results from the separate analyses in some way. Thus although we may be interested to know the cheapest brewery share, it is probably more important to know how brewery shares compare with shares in, say, the heavy electrical industry.

(b) For a particular industry sector it would be necessary to have sufficient companies to provide enough degrees of freedom for the residual variance to stand a reasonable chance of establishing that a particular factor has a real effect.

(c) The accuracy of each regression coefficient depends upon the spread or variability of each predictor variable and in a single sector it is possible that a large proportion of the companies may, for example, have payout ratios between 0.8 and 1.2. A single company having a payout ratio of 0.5 in the same sector would therefore require a considerable degree of extrapolation in the estimation of the appropriate computer yield. This is much less likely to happen with a group of 150 companies although it still does occur with, for example, Jaguar*, which has an exceptionally low payout ratio.

28. Reverting once again to the analysis of variance table, the normal F variance ratio test shows that all variables are significant at the 5% probability level (5% value of $F = 4$) with the exception of the historic earnings growth rate. Since this variable comes last its significance level agrees with the earlier 't' test. The table above shows that a high proportion of the explained variation in yields is accounted for by variation in the payout ratio and that, of the remaining four factors, those representing future prospects are relatively more important. In passing, however, we would stress that the payout ratio, although obviously correlated with the current

* This company has since become part of the British Motor Holdings group.
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historical ratio, is based completely on forecast dividends and earnings, so that in fact our forecasts play a larger part in the determination of the attractiveness of the shares than would seem apparent at first sight. As we have stated, too much cannot be deduced from the various percentage contributions of each factor to the explained variation in yields, owing to the correlations between the predictor variables. The following correlation matrix illustrates this point:

<table>
<thead>
<tr>
<th>Variable</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$x_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>1.00</td>
<td>0.07</td>
<td>-0.58</td>
<td>0.21</td>
<td>-0.20</td>
</tr>
<tr>
<td>$x_2$</td>
<td>0.07</td>
<td>1.00</td>
<td>0.23</td>
<td>0.22</td>
<td>0.06</td>
</tr>
<tr>
<td>$x_3$</td>
<td>-0.58</td>
<td>0.23</td>
<td>1.00</td>
<td>-0.06</td>
<td>0.35</td>
</tr>
<tr>
<td>$x_4$</td>
<td>0.21</td>
<td>0.22</td>
<td>-0.06</td>
<td>1.00</td>
<td>-0.22</td>
</tr>
<tr>
<td>$x_5$</td>
<td>-0.20</td>
<td>0.06</td>
<td>0.35</td>
<td>-0.22</td>
<td>1.00</td>
</tr>
</tbody>
</table>

As an approximation, any correlation coefficient greater in absolute size than 0.17 is statistically significant (the correlations are based on a sample size of 134 companies) and the highest correlation coefficient $r_{13}$ between the long-term dividend growth rate $x_3$ and the payout rate $x_1$ is -0.58. This extreme result would as we have previously pointed out be expected owing to the potential for liberalization contained in the payout ratio. Of the other correlations, the value of $r_{35} = +0.35$ suggests (contrary to Little perhaps) an association between past earnings growth and future dividend growth, whilst the value of $r_{15}$, which is just significant, suggests that a high past earnings growth rate is associated with a low payout ratio. On the grounds that dividend growth normally lags behind earnings growth this result seems reasonable. Reverting once again to the effect of the correlation between future dividend growth and payout ratio, if earnings growth rate was used in the model in place of dividend growth, the net effect would probably be only slight, with the regression coefficient for the payout ratio increasing in importance at the expense of the coefficient for the long-term growth factor—the argument for this is that future dividend growth is a function of future earnings growth and potential for liberalization contained in the payout ratio. Rearrangement of the order in which the factors are introduced into the model alters the analysis of variance table in terms of the figures for the percentage variation accounted for by each factor. If, for example, forecast dividend growth is introduced second and payout ratio last, the percentage contribution from the latter variable falls from 42.8% to 10.1% and the contribution from dividend growth increases to 40.6%. This further analysis of the data provides some indication of the significance of the various factors introduced into the model. However, given that the factors discussed may on a priori considerations be expected to influence the yield, the calculations are in practice carried out weekly in order to estimate the regression coefficient or weight which the market places on each factor. The weights vary from week to week both because they are only an estimate from a particular set of data.
and because there may be trends in their value over time. For example, in a period of industrial downswing, when prospects of general growth are poor, more emphasis may be placed by the market on dividend yield with the result that low-yielding, well-covered shares may become less attractive and accordingly the weight for the payout ratio becomes smaller.

30. It may be argued (and we would agree with this point of view) that any system of share price forecasting should estimate the weights that will apply at some future date rather than now but we feel that, in our present state of knowledge and experience of the method, this is not yet possible. An indication of the variation from week to week in the individual weights is provided in the graph shown in Appendix 3, which gives the values obtained in consecutive analyses over approximately a five-month period. The vertical lines attached to each point measure the approximate 95% confidence limits for each weight and, as one would expect, most of the confidence limits exclude zero as a possible value since the regression coefficients were usually found to be statistically significant. Examination of the variations in the weights for each factor in fact shows little evidence of any trend but a longer period of study would provide more information on this point. Unfortunately, however, although the method as a whole has been running for some 2½ years now, we have over the course of this period made a number of changes to the variables which would in themselves alter the weights. Examples are the introduction of mean yields, the definition of earnings growth rate and the measure of historical earnings variability. In practice, since the estimates of the weights will also vary owing to random fluctuations, we carry out the regression analyses weekly to estimate the weights but actually calculate the computer yields by using weights, which are the mean values of those obtained in the six previous weeks. This method reduces the fluctuations in the calculated computer yields that are due merely to alterations in the weights themselves and, by using a relatively short (in time) moving average, it also makes some allowance for any possible trends.

COMPUTER METHOD IN USE

31. When the computer method of ranking ordinary shares was first introduced, the appropriate input data were analysed on an Elliott 803 computer to estimate the regression coefficients. Having determined the yield equation, it is then necessary to ‘back substitute’ the predictor variable data in the calculated relation to produce the estimated computer yields for each company. Surprisingly, the regression programme that was available did not contain the back substitution facility, so this part of the programme had to be specially written. The final figure in the output represents the ratio of the actual mean yield for the share as defined in §§ 19 and 20 divided by the computer yield and is termed the ‘computer ranking’. A figure of 1.25, for example, indicates that a rise in the share price of 25% would be required for the computer to reckon that the share
was correctly valued (or no better than the average). However, the intro-
duction of the idea of using mean weights necessitated a change in the
system, since it separated the analysis of the data to produce the regression
coefficients from the ‘back substitution’ routine. At the same time, our
office acquired an IBM 1440 computer and, although this is too slow to
carry out the fundamental analysis, it is admirably suited to the relatively
simple task of estimating the computer yields, given the mean weights and
the individual input data for each share.

32. The organization of the system for producing the computer ranking
list each week may be summarized as follows:

(a) Companies are divided into industry groups on the basis of the F.T.-
Actuaries share index and each group is allotted to one of eight
security analysts. During the course of a week, the analyst is required
to review his forecasts or projections for every company that he covers
and to amend them should this be necessary. At the end of the week,
all projection changes are collated and the data for each company are
brought up to date. Where companies have had annual results, the new
historical earnings growth rate and the variability are also recalculated.

(b) The information on the companies is punched on to two packs
designated Data 1 and Data 2 cards respectively and the individual
items of information that they contain are shown below:

Data 1 card:

(i) Company code number, name, nominal value of share
(ii) Calendar month in which next final results are due
(iii) Latest historical earnings and dividend
(iv) All forecast dividend and earnings figures
(v) Long-term forecast dividend growth rate ($x_3$)
(vi) Historical earnings variability and growth rate ($x_4$ and $x_5$)

Data 2 card:

(i) Company code
(ii) Share price
(iii) High/low price

In addition, there is a third type of card known as the Weight card
which contains the mean weights to be used in the calculation of the yields
and also states the month in which the regression run is being carried out.
This latter figure combined with item (ii) on the Data 1 card enables the
computer to calculate the number of months ($t$) for each company to the
next set of final results and hence, using item (iv) on the Data 1 card and
item (ii) on the Data 2 card, the values of $y$, $x_1$, and $x_2$ (forecast mean
dividend yield, payout ratio and short-term earnings growth rate) are
determined.
(c) From the information contained on the Data cards, the IBM computer carries out the following operations:

(i) Calculates the computer yield and the actual mean yield 12 months hence for each share.

(ii) Calculates the ratio of actual mean dividend yield to the estimated yield and hence gives the percentage cheapness \(= 100 \times (\text{Yield Ratio} - 1)\).

(iii) Sorts shares according to percentage cheapness and places them in categories, in order:

<table>
<thead>
<tr>
<th>Percentage cheapness</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>More than +15%</td>
<td>Category 1</td>
</tr>
<tr>
<td>+15% to +7\frac{1}{2}%</td>
<td>Category 2</td>
</tr>
<tr>
<td>+7\frac{1}{2}% to -7\frac{1}{2}%</td>
<td>Category 3</td>
</tr>
<tr>
<td>-7\frac{1}{2}% to -15%</td>
<td>Category 4</td>
</tr>
<tr>
<td>Less than -15%</td>
<td>Category 5</td>
</tr>
</tbody>
</table>

(iv) Calculates the actual current mean dividend yields (based on the latest known dividend and our first forecast dividend) and the \(P/E\) ratios.

33. Part of a specimen ranking list is shown in Appendix 2(b). It will be seen that the first section shows the shares arranged in categories in descending order of attractiveness, the price, yield and percentage cheapness. We also indicate whether the ranking has changed from the previous list. The second part shows the same companies arranged in \(F.T.-\text{Actuaries Index}\) order and the computer also prints out the high/low price range and the date of the next final results. In addition to the actual share rankings, the computer has a secondary output consisting of the values of the variables \(y, x_1, x_2, \ldots, x_n\) for each share and these are then analysed on the Elliott 803 computer to produce the latest estimates of the weights. It would perhaps seem more logical to calculate the current set of weights first so that they can be incorporated in the mean weights used, thus avoiding the time lag that our method introduces. In practice, since the regression analysis is the most vulnerable part of the exercise (a gross error in a yield for example would require a complete re-run) and since the variation in mean weights is only slight, we have found it more convenient to accept the time lag.

34. Apart from the weekly routine production runs, we find it necessary to carry out individual share rankings between computer runs—the most frequent reason for this occurs when a company has had results and of course it is at this time that there is usually the maximum interest in a share. Using the latest set of mean weights and adjusting the constant term in the yield relation for any change in the \(F.T.-\text{Actuaries share index}\), these rankings were worked out on a hand calculating machine. This calculation,
however, is slightly tedious and liable to error and we have therefore
developed a small analogue computer S.E.E.C. (Share evaluation electronic
computer) to carry out the calculation.

35. In this computer, precision resistance networks provide voltages
which are analogous to the variables of the equation. These voltages are
then fed to operational amplifiers which sum them in a manner prescribed
by the equation. The computer is desk mounted, about the size and weight
of an office typewriter, and is operated from the mains supply. It is capable
of dealing with regression relations containing up to ten logarithmic vari-
bles. Two control panels are provided, one containing the coefficient setting
controls, the other the logarithmic variables. Coefficients are set by adjust-
ing individually calibrated knobs to the desired values, logarithmic inputs
are monitored on two large-scale meters. An interlocked push button
system is used to select each variable for presentation on the meters and
associated control knobs set the variable to the correct value. The solution
to the equation appears on the meters once all the variables have been
inserted and the solution button pressed. The machine is accurate to within
3%; the time taken to evaluate a share is about 2 to 3 minutes.

PERFORMANCE OF THE METHOD

36. Following the introduction of the computer system, portfolio
performance programmes have been set up in order to test the effectiveness
of the method. At the same time it was thought to be of interest to compare
the performance of the computer rankings with that of other methods of
share evaluation and the following additional methods were examined:

(1) Relative yields
(2) Growth yields
(3) The ‘100’ Companies.

Both relative yields and growth yields have been referred to as possible
methods of share evaluation in §§ 8 and 9 respectively but the ‘100’
Company evaluations were incorporated in a publication introduced by
our firm, whereby each of the large companies at present included in the
computer system was assessed by the analysis department once a month
and a subjective evaluation using a numerical scale from one to six was
made for each share. In fact, the ‘100’ Company system was, over the test
period, superior to the alternative objective methods. In practice, however,
the production of this publication occupied the department for up to a
week (the computer evaluation probably takes less than one day using
relatively unskilled labour for the data preparation) since each analyst
had to rethink his evaluation in the light of his forecasts and current yields.
In addition some difficulty was usually experienced by the analysts in
separating the evaluation for a share relative to the market from their view
of the market as a whole, with the result that at times there were very few
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companies in the extreme categories—for example in the January 1965 list only 2% of stocks were in category 1 with 92% concentrated in categories 2, 3 and 4 and the remaining 6% in category 5. The computer, of course, has the virtue of producing a symmetrical distribution of shares by category.

**Diagram 2**

Performance of computer rankings. Values of portfolios compared with all shares ranked.

**Test (a)**

37. The actual performance test was designed to see if the shares ranked at the top of the list in order of attractiveness rose faster on average than those at the bottom and to compare each with the index. The system used was to set up a portfolio at the beginning (November 1963) and at the end of each month the shares were all sold and the proceeds reinvested in equal amounts in each share in the new portfolio (no dealing expenses were allowed for). Whilst this test is not intended to be practical in the sense that the benefits of any of the particular systems of share evaluation would be materially reduced by switching expenses, it is, however, a fair test of the rankings. At the time that this exploratory test was carried out, the shares in each system were placed in one of six categories (to fit in with the ‘100’ Companies) and the six portfolios set up for each method contained shares in category 1, shares in categories 1 and 2, and so on down to the last portfolio which contained all shares ranked. The results are summarized in Table 1 from which it will be seen that the best performance was obtained with the computer and ‘100’ Company methods.
Table 1.

Comparison of methods of share evaluation.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shares in categories</td>
<td>1</td>
<td>1, 2</td>
<td>1, 2, 3</td>
<td>1, 2, 3, 4</td>
<td>1, 2, 3, 4, 5</td>
</tr>
<tr>
<td>Evaluation method:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computer rankings</td>
<td>15.2</td>
<td>10.7</td>
<td>5.1</td>
<td>1.8</td>
<td>0.8</td>
</tr>
<tr>
<td>Growth yields</td>
<td>7.9</td>
<td>11.6</td>
<td>5.3</td>
<td>2.4</td>
<td>0.4</td>
</tr>
<tr>
<td>‘100’ Companies</td>
<td>20.7</td>
<td>14.9</td>
<td>4.9</td>
<td>1.7</td>
<td>0.3</td>
</tr>
<tr>
<td>Relative yields</td>
<td>5.6</td>
<td>4.8</td>
<td>2.8</td>
<td>1.0</td>
<td>0.2</td>
</tr>
</tbody>
</table>

* Aggregate portfolio contains all shares ranked—i.e. those in categories 1, 2, 3, 4, 5, 6.

38. On the basis of our portfolio tests, the computer system has been adopted and the performance of the method has been continuously monitored since November 1963, although the shares have in fact been placed in five categories as already described in § 32(c). The results of this investigation are shown on Diagram 2, but the presentation has been altered from the initial investigation by choosing two portfolios A and B. Portfolio A contains the most attractive 30% of shares and portfolio B the least attractive. The value of the portfolios as a percentage over or under the index (which was the performance of the aggregate of all shares ranked) are plotted each month, from which it can be seen that the divergence from the index performance of the two portfolios has been both continuous and consistent.

Test (b)

39. As we have stated, test (a) is not, of course, a practical proposition but a system approximating much more closely to practice has been evolved and the results of two such tests have been calculated. In the first case equal amounts of money were invested in the most attractive 20% of shares in November 1963 and, whenever a share fell into the least attractive 20% category, the holding was sold and the proceeds reinvested in the most attractive share not already held, after allowing for full switching expenses. A portfolio obeying these rules was started each month and the percentage by which the performance of each portfolio exceeded that of the F.T.-A. 594 index is shown in the column headed ‘Buy and switch’ in Table 2. In addition we have made calculations allowing in a simplified manner for capital gains tax by taking 30% of net profits on shares sold in each portfolio. Surprisingly, the effect of allowing for this factor was small, since an examination of the individual portfolios suggests that computer switching over the period tested is largely a question of selling those shares which have performed badly rather than taking profits on successful selections. A different result, however, might well appear
when the test has been continued for a longer period. Finally, since the larger funds would probably find a high degree of switching unworkable, due to difficulty in dealing at those prices in large amounts, and since in

any case it is of interest to investigate the benefits to be obtained by switching, we have tested the method on the assumption that the original selections had been held throughout. A ‘hold’ portfolio was started each month and the performance of each portfolio is also shown in Table 2. Once again, most of the ‘hold’ portfolios beat the index (there were four

Table 2.
Comparison of performance of test portfolios with F.T.-A. 594 index

<table>
<thead>
<tr>
<th>Portfolio started in</th>
<th>Number of months (Final valuation date May 1966)</th>
<th>Buy and hold</th>
<th>Buy and switch</th>
<th>Buy and switch with straight 30% gains tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>November 1963</td>
<td>30</td>
<td>+6.2</td>
<td>+8.4</td>
<td>+8.4</td>
</tr>
<tr>
<td>December</td>
<td>29</td>
<td>+14.3</td>
<td>+15.7</td>
<td>+14.3</td>
</tr>
<tr>
<td>January 1964</td>
<td>28</td>
<td>+8.2</td>
<td>+9.5</td>
<td>+9.5</td>
</tr>
<tr>
<td>February</td>
<td>27</td>
<td>-1.0</td>
<td>+4.2</td>
<td>+4.2</td>
</tr>
<tr>
<td>March</td>
<td>26</td>
<td>+6.5</td>
<td>+10.4</td>
<td>+10.4</td>
</tr>
<tr>
<td>April</td>
<td>25</td>
<td>+8.8</td>
<td>+14.4</td>
<td>+13.7</td>
</tr>
<tr>
<td>May</td>
<td>24</td>
<td>+7.3</td>
<td>+12.5</td>
<td>+11.7</td>
</tr>
<tr>
<td>June</td>
<td>23</td>
<td>+9.1</td>
<td>+16.9</td>
<td>+16.7</td>
</tr>
<tr>
<td>July</td>
<td>22</td>
<td>+9.3</td>
<td>+17.7</td>
<td>+17.5</td>
</tr>
<tr>
<td>August</td>
<td>21</td>
<td>+4.3</td>
<td>+11.4</td>
<td>+11.4</td>
</tr>
<tr>
<td>September</td>
<td>20</td>
<td>-0.5</td>
<td>+8.7</td>
<td>+8.7</td>
</tr>
<tr>
<td>October</td>
<td>19</td>
<td>+8.8</td>
<td>+13.9</td>
<td>+13.9</td>
</tr>
<tr>
<td>November</td>
<td>18</td>
<td>-0.3</td>
<td>+6.7</td>
<td>+6.7</td>
</tr>
<tr>
<td>January 1965</td>
<td>16</td>
<td>-0.8</td>
<td>+13.6</td>
<td>+13.6</td>
</tr>
<tr>
<td>February</td>
<td>15</td>
<td>+3.6</td>
<td>+9.2</td>
<td>+9.2</td>
</tr>
<tr>
<td>March</td>
<td>14</td>
<td>+6.5</td>
<td>+9.5</td>
<td>+9.5</td>
</tr>
<tr>
<td>April</td>
<td>13</td>
<td>+9.4</td>
<td>+10.1</td>
<td>+10.1</td>
</tr>
<tr>
<td>May</td>
<td>12</td>
<td>+6.4</td>
<td>+5.9</td>
<td>+5.9</td>
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<tr>
<td>June</td>
<td>11</td>
<td>+8.0</td>
<td>+9.4</td>
<td>+9.0</td>
</tr>
<tr>
<td>July</td>
<td>10</td>
<td>+4.4</td>
<td>+3.7</td>
<td>+3.7</td>
</tr>
<tr>
<td>August</td>
<td>9</td>
<td>+5.1</td>
<td>+5.1</td>
<td>+5.1</td>
</tr>
<tr>
<td>September</td>
<td>8</td>
<td>+2.9</td>
<td>+2.9</td>
<td>+2.9</td>
</tr>
<tr>
<td>October</td>
<td>7</td>
<td>+2.4</td>
<td>+2.4</td>
<td>+2.4</td>
</tr>
<tr>
<td>November</td>
<td>6</td>
<td>+2.4</td>
<td>+2.4</td>
<td>+2.4</td>
</tr>
<tr>
<td>December</td>
<td>5</td>
<td>+2.4</td>
<td>+2.4</td>
<td>+2.4</td>
</tr>
<tr>
<td>January 1966</td>
<td>4</td>
<td>+2.1</td>
<td>+2.1</td>
<td>+2.1</td>
</tr>
<tr>
<td>February</td>
<td>3</td>
<td>+1.1</td>
<td>+1.1</td>
<td>+1.1</td>
</tr>
<tr>
<td>March</td>
<td>2</td>
<td>+2.7</td>
<td>+2.7</td>
<td>+2.7</td>
</tr>
</tbody>
</table>

Average annual performance: +3.7 +6.1 +6.1
exceptions) but the gains were on the average only just over half those of the switch portfolios indicating that even over the comparatively short test periods investigated, the costs of switching have been more than offset by the benefits from computer selection.

**FURTHER WORK**

40. The object of the work described in this paper has been to develop a computer technique for ranking ordinary shares in order of attractiveness. As a first step, we are quite encouraged by our results to date but it seems probable that the technique can be improved both as regards the length of view that one is taking in selecting the shares and in the type of share and the factors included in the analysis. Later in this section we shall consider possible ways in which this may be accomplished but in passing we should like to refer to the more general problem of the application of computers (and more particularly statistical methods) to the problems of investment analysis and investment management as a whole. Earlier in this paper we mentioned the possible application of Operational Research techniques to investment problems and it has to be admitted that in an 'industry' which abounds with good quality data requiring continuous decisions it would be surprising if this science could make no contribution. The authors would not make any claim to be O.R. men but the above application of the methods of mathematical statistics (on which many O.R. techniques are based) suggests that the potential could be immense. Some of the possible directions in which this work could proceed are as follows:

(i) The determination of the relative attractiveness of fixed-interest securities, equity shares and cash both on a short- and long-term view. In the former case it should be possible to arrive at a solution using the various economic indicators available.

(ii) On the assumption that it is possible to attach a specific degree of risk to an equity investment, it should be possible to set up portfolios where the return is maximized subject to an acceptable degree of risk.

(iii) On a purely management question the aim would be to relate the share selection mechanism to clients' portfolios, providing a much closer degree of control.

41. The general theme running through the above suggestions is of course a much closer degree of 'true management' than has hitherto been possible. This results from the advantage of being able rapidly to relate conclusions or decisions reached by the computer (modified by subjective judgment if necessary) to 'actualities' and in an earlier section we referred to the advantage of the computer ranking method in maximizing the use made of information coming from a research department. Of course, 'all the way along the line' one is attempting to make quantitative judgments where previously a general feeling of rightness or otherwise (based on long
experience) was used. Our decision-making methods would be based in part on an analysis of past experience (e.g. fixed interest versus equity investment) and it is here that one perhaps can make an analogy with industry. At one time crude oil was distilled into its various fractions by persons of 'long experience' in the industry and certainly by those who had seen many years overseas actually drilling for the oil. Today a large part of the necessary decisions are undertaken by computers using linear programming techniques, the calculation being made so that the whole process is optimized to minimize costs. One can also take heart from developments in investment methods in the U.S.A. to which we have already referred. Thus regression analysis was being suggested as a method of analysing stock market investments as long ago as 1935, although it would be wrong to conclude from this that computer evaluation methods are in general use in the U.S.A. On the other hand, there is considerable evidence to suggest that, with investment analysis practised there on a wide scale, the evaluation and management problem is receiving close attention by the large institutions. In November 1965, Dr Pierre Rinfret addressed the Society of Investment Analysts on the subject of *The use of computers in investment analysis* and although this was stated to be an interim report (the computer's advice was not necessarily being acted upon) the scale of operations was impressive—since 1957 annual expenditure had averaged $200,000. At the other end of the scale our attention has recently been directed to a computer study carried out by a private individual in the U.S.A. and although his share selection mechanism was relatively unsophisticated—it consisted essentially of correlating P/E ratios for different shares with past earnings growth rates—simulated portfolio studies showed that even this system would have produced beneficial results. Against this background we take the view that the days when investment was considered to be solely an art are now passing. Personal judgment in the making of forecasts must of course remain an important factor but from there on the computer appears to possess exciting possibilities. As a by-product, the use of these techniques may well place more emphasis on the performance of share selections and it is interesting to appreciate that our limited success so far must in part reflect favourably on the forecasts which play a vital part in the computer technique. The supporters of the 'random walk' approach to share selection would have us believe that the pin is still the most efficient tool but they concede that it is possible that this is an overstatement. Thus Fama\(^{(11)}\) in an article entitled *Random Walks in Stock Market Prices* (December 1965) ended on the note that the 'analyst cannot merely protest that he thinks the securities he selects do better than the randomly selected securities, he must demonstrate that this is in fact the case'.

42. We shall now consider first further possible developments in the regression model and, second, the application of alternative techniques to the share ranking problem.
A. The regression model

43. Future developments in the regression or the computer ranking technique seem likely to fall into two parts. First, the aim must be to increase the scope or number of shares analysed and second, if possible, to make improvements to the model. The latter may include increasing the number of factors and altering the period over which a view is being taken of the probable change in share price, i.e. the period of selection.

44. As far as the first possibility is concerned, it is hoped gradually to increase the number of shares ranked towards a target of 300, thereby both improving the usefulness of the system and at the same time obtaining improved estimates of the regression coefficients or weights. To a large extent, this increase in share coverage will come from increasing the number of shares within each industry group but there are, however, certain industry groups that at the moment do not appear to lend themselves to our computer technique. Thus, all composite insurance and most property shares would appear to be relatively dear and, whilst it is possible that they are, it also seems likely that the market is placing a much greater emphasis on asset values for these shares than it does for industrial ordinary shares as a whole. At the moment, therefore, neither property shares nor insurance shares are included in the analysis and, to the extent that both classes represent rather specialized investment trusts, we feel that any valuation system devised ought to place considerable weight on asset values (where it is possible to calculate or estimate them). By way of example, in the particular case of composite insurance shares, it may be possible to value the underwriting part of the business by using the computer technique, i.e. treating it as a normal trading company and, secondly, valuing the investments as an investment trust.

45. With regard to the form of the model, it is possible that more factors may be included, although as we mentioned in §§ 21 and 22 some factors would be unsuitable. The second question of how long or short a view one takes is probably of more importance and this point is underlined by Little and Rayner who ask 'over what length of time is the investor expressing his expectations when he thinks that a certain yield is the correct one for a particular firm? Is he predicting the figures for the following year, or is he trying to forecast over a period of five or six years?'. The computer method tries to forecast the relative yield for each share that will be obtaining in twelve months' time on the basis of five-year earnings and dividend projections and, by combining this with the forecast dividend, a suggested price is determined. It would, of course, be of more use to the long-term investor to determine the relative yield status for the share in five years' time and, by combining this with five-year forecast dividend growth, a long-term view on the possible movement of the share price would be determined. This method would have the advantage of giving the fast growing shares an improved ranking since their above-average dividend growth makes five, as opposed to one year's contribution to the price change.
the other hand, to forecast a relative yield five years away inevitably means
that one needs to make earnings and dividend projections over a longer
period. The authors feel that this is unlikely to be possible with any degree
of accuracy although it would perhaps be practicable to extend our time
horizon to a point 2½ years from now. This question of striking a balance
between the short- and long-term view is mentioned briefly in § 12 and it
may well be, of course, that one needs to vary the method according to
the type of investor being advised. For example, the relatively active
private client would be more interested in the short-term view (he would
be disappointed if his chosen share fell 15% before rising say 40% in total
over five years). On the other hand the long-term investor, while wishing to
know how dear a particular share was on immediate yield considerations,
might be prepared to act on the apparent long-term growth potential of the
share, bearing in mind the present difficulties for a large fund in building up
an adequate holding in a stock. We would suggest, however, that whichever
way the problem is approached, today's growth industry may be 1970's
'shipbuilding' and the investor taking a five-year view on relative yields is
risking too much.

B. An alternative share ranking method

46. The method underlying the computer ranking technique has been
to set up a theoretical model whose form is based on a priori consider-
ations (i.e. how it is believed that investors make their investment decisions)
and then test it to see if the highest ranked shares perform better than the
average. The alternative method of predicting share price movements is to
correlate them over a given time period (once again the problem of the
length of view one is taking occurs) with various statistics for each com-
pany. By way of example, we might correlate the share price movement
with:

(1) Dividend and earnings yield
(2) Asset value
(3) Earnings on capital employed
(4) Historical earnings growth rate and variability
(5) Current share price in relation to the all-time high.

It will be seen that variables (2) and (5) which we rejected for reasons set
out in §§ 21 and 22 can now be included. A technique that seems appro-
priate to this problem is known as 'discriminant analysis'. Briefly, without
going into too much mathematical detail, if we assume that our problem
is to sort shares into two categories (those that are cheap and those that
are dear) we then choose a discriminant function \( L \) defined to be a linear
function of the various statistics that we can calculate for each share and,
according to the value of \( L \), we place a particular share in one of the two
categories. The calculation would be based on an analysis of the price
performance of \( n \) shares over a given time period and those that performed,
say, 10% better than the average would be designated as cheap and those that performed 10% worse would be dear. The idea is of course to choose $L$ such that it takes widely differing values according to whether the share comes from the cheap or dear population, and so that the chances of misclassification are minimized. The method can easily be extended to cover more than two categories (or populations) and, although the fact that one is 'jobbing back' in terms of prices suggests that future projections are out of place, on the other hand there is no reason why actual growth achieved over the period studied should not be included. If this approach was used, it would mean that whether or not the system worked in the future would depend upon the accuracy of forecasting (perhaps not surprisingly) and this would need to be separately checked. In passing, although a share that achieves a high rate of earnings growth would on \textit{a priori} considerations seem most likely to rise in price by an above-average amount, this, as many investors know to their cost, may not be so if the dividend yield is too low and this latter factor would receive due weight in the discriminant function. If future projections were left out, we would in effect have a system in which one needed to have faith that it would work rather than one like the regression or growth yield technique, both of which attempt to parallel the evaluation process that the investor goes through in his mind in selecting shares. On the whole we feel that the discriminant technique is less likely to work than our present system although it is interesting enough to merit investigation.

\textbf{SUMMARY}

47. In conclusion, it is reasonable to ask at this stage, first, what we have achieved in terms of the application of computers to investment analysis and, second, where do we go from here? In answer to the first question, the computer method appears so far to have proved itself as a method of selecting shares and has provided a formal basis for looking at the evaluation problem separately from the question of forecasting. As is usual when computers are used, one needs to provide explicit answers to the questions involved in selecting shares and the main one in this application is the necessity for deciding on the length of view that one is taking in assessing a share. The other interesting by-product following the introduction of a computer method is the need for placing emphasis on testing the computer selections. In general terms it is probably fair to say that little emphasis has so far been placed on this aspect—the investment analysts, as a body, have found it difficult to refute the claims of the 'random walkers'. However, in the U.S.A. it is believed that much greater weight is placed on the value of share selections from investment advisers and we feel that this trend will develop here. If it does, the usefulness of the computer could well be shown to be high, in the sense that some measure of the results from conventional methods would be forthcoming.
48. We now come to the second question of where we go from here. We freely admit that we are probably only 'scratching at the surface' in the application of computers to the problems of share evaluation. Here, we still feel that the potential is immense and would stress that, whilst the application of computers to data processing is important in providing tools for the analyst, the main value must be in carrying out exercises which are arithmetically not possible by hand. It is not difficult, for example, to envisage a system of share selection which is continually available and being updated and linked completely with existing portfolios. This ideal may well be some years off but it is hoped that the present paper will provide the basis for an interesting discussion by members on both existing practice and future trends in investment work.

ACKNOWLEDGMENTS

49. Finally, we would like to express our appreciation of the help and criticisms received from colleagues and in particular from Messrs P. M. D. Gibbs, M.A., F.C.A. and C. J. Lewis, F.I.A.

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(2) Pepper G. T. The selection and maintenance of a gilt-edged portfolio. J.I.A. 1964, 90, 63.
(8) Hall M. G. Some thoughts on 'higgledy piggledy growth'. The Investment Analyst, 1963, No. 5.
APPENDIX 1

MATHEMATICAL DERIVATION OF LEAST SQUARES SOLUTION TO MULTIPLE REGRESSION ANALYSIS

For each of $n$ shares we are given an array of values or observations $y, x_1, x_2 \ldots x_t$ and the problem is to obtain estimates $b_0, b_1, \ldots b_t$ of the regression coefficients $B_0, B_1, \ldots B_t$ given by the linear regression model:

$$y = B_0 + B_1 x_1 + \ldots + B_t x_t + e_t$$

where $e_t$ represents a random variable having a variance of $\sigma^2$.

Unbiased and minimum variance estimates of each $B_i$ can be obtained by choosing the values of $b_i$ to minimize $S$, the sum of squared residuals given by the expression:

$$S = \sum_{r=1}^{n} (y - b_0 x_0 - b_1 x_1 - \ldots - b_t x_t)^2$$

where the variables $y, x_0, x_1 \ldots x_t$ have particular values for each value of $r$ (i.e. each array).

Differentiating with respect to $b_i$ we obtain:

$$\frac{dS}{db_i} = 2 \sum_{r=1}^{n} (y - b_0 x_0 - b_1 x_1 - \ldots - b_t x_t) x_i$$

for $i = 0, 1 \ldots t$

Equating to zero:

$$b_0 \sum_{r=1}^{n} x_0 x_i + b_1 \sum_{r=1}^{n} x_1 x_i + \ldots + b_t \sum_{r=1}^{n} x_t x_i = \sum_{r=1}^{n} y x_i$$

This set of equations may be written in matrix notation:

$$\begin{bmatrix}
\Sigma x_0^2 & \Sigma x_0 x_1 & \ldots & \Sigma x_0 x_t \\
\Sigma x_1 x_0 & \Sigma x_1^2 & \ldots & \Sigma x_1 x_t \\
\vdots & \vdots & \ddots & \vdots \\
\Sigma x_t x_0 & \ldots & \ldots & \Sigma x_t^2
\end{bmatrix}
\begin{bmatrix}
b_0 \\
b_1 \\
\vdots \\
b_t
\end{bmatrix} =
\begin{bmatrix}
\Sigma y x_0 \\
\Sigma y x_1 \\
\vdots \\
\Sigma y x_t
\end{bmatrix}$$

i.e. $Cb = B$

where

(i) $C$ is a square matrix of dimension $(t+1)$ whose element in the $(i+1)^{st}$ row and $(j+1)^{st}$ column is $\Sigma x_i x_j$

(ii) $b$ is a column vector of $(t+1)$ elements $b_i$

(iii) $B$ is a column vector of $(t+1)$ elements $\Sigma y x_i$
<table>
<thead>
<tr>
<th>Company</th>
<th>Company number</th>
<th>Forecast dividend growth rate</th>
<th>Mean dividend yield</th>
<th>Variability</th>
<th>Forecast change in earnings in 12 months</th>
<th>Mean payout ratio</th>
<th>Growth rate of past 10 years' earnings</th>
<th>Growth rate of past 5 years' earnings</th>
</tr>
</thead>
<tbody>
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<td>Bristol Aero</td>
<td>2.01</td>
<td>4.82</td>
<td>0.900</td>
<td></td>
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<td>1.00</td>
<td>1.142</td>
<td>1.127</td>
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<td>1.120</td>
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<td>6.30</td>
<td>0.872</td>
<td></td>
<td>1.140</td>
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<td></td>
<td>1.138</td>
<td>1.06</td>
<td>1.120</td>
<td>1.120</td>
</tr>
<tr>
<td>Marley Tile</td>
<td>2.12</td>
<td>5.65</td>
<td>0.811</td>
<td></td>
<td>1.138</td>
<td>1.06</td>
<td>1.120</td>
<td>1.120</td>
</tr>
<tr>
<td>Redland Holdings</td>
<td>2.13</td>
<td>6.01</td>
<td>0.811</td>
<td></td>
<td>1.138</td>
<td>1.06</td>
<td>1.120</td>
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<tr>
<td>Rugby Portland Cement</td>
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</tr>
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<td>5.67</td>
<td>0.811</td>
<td></td>
<td>1.138</td>
<td>1.06</td>
<td>1.120</td>
<td>1.120</td>
</tr>
<tr>
<td>Wimpey</td>
<td>2.17</td>
<td>5.65</td>
<td>0.811</td>
<td></td>
<td>1.138</td>
<td>1.06</td>
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<tr>
<td>A.E.I.</td>
<td>2.18</td>
<td>6.01</td>
<td>0.811</td>
<td></td>
<td>1.138</td>
<td>1.06</td>
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<td>1.120</td>
</tr>
<tr>
<td>B.I.C.C.</td>
<td>2.19</td>
<td>5.60</td>
<td>0.846</td>
<td></td>
<td>1.125</td>
<td>1.08</td>
<td>1.120</td>
<td>1.120</td>
</tr>
<tr>
<td>Crompton Parkinson</td>
<td>2.20</td>
<td>5.02</td>
<td>0.684</td>
<td></td>
<td>1.125</td>
<td>1.08</td>
<td>1.120</td>
<td>1.120</td>
</tr>
</tbody>
</table>
## APPENDIX 2(b)

**SPECIMEN COMPUTER RANKING LIST**

*F.T.-Actuaries 594 share index, 15 August 1966: 94.1*

Percentage change since previous ranking list 1 August 1966: -2.8

(i) **Ranking of shares in order of attractiveness (first four companies in each category only)**

<table>
<thead>
<tr>
<th>Company number</th>
<th>Company name</th>
<th>Price</th>
<th>Mean dividend yield</th>
<th>Mean P/E ratio</th>
<th>% relative cheapness</th>
<th>Previous rank if changed</th>
</tr>
</thead>
<tbody>
<tr>
<td>13-01</td>
<td>Hoover A</td>
<td>30/4</td>
<td>7.41</td>
<td>9.67</td>
<td>44</td>
<td></td>
</tr>
<tr>
<td>25-02</td>
<td>Gallaher</td>
<td>22/10</td>
<td>8.31</td>
<td>7.96</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>6-01</td>
<td>Babcock &amp; Wilcox</td>
<td>28/3</td>
<td>7.08</td>
<td>8.63</td>
<td>26</td>
<td></td>
</tr>
<tr>
<td>3-03</td>
<td>BPB Industries</td>
<td>16/9</td>
<td>6.57</td>
<td>10.10</td>
<td>26</td>
<td></td>
</tr>
<tr>
<td>19-01</td>
<td>A. B. Picture</td>
<td>24/9</td>
<td>6.06</td>
<td>8.95</td>
<td>15</td>
<td>1</td>
</tr>
<tr>
<td>19-02</td>
<td>Granada A</td>
<td>33/9</td>
<td>7.41</td>
<td>11.29</td>
<td>15</td>
<td>3</td>
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<tr>
<td>14-02</td>
<td>Jaguar A</td>
<td>32/-</td>
<td>8.81</td>
<td></td>
<td>14</td>
<td>1</td>
</tr>
<tr>
<td>7-01</td>
<td>John Brown</td>
<td>22/6</td>
<td>9.78</td>
<td>55.00</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>20-07</td>
<td>Spillers</td>
<td>9/10</td>
<td>6.58</td>
<td>11.87</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>3-01</td>
<td>Allied Ironfounders</td>
<td>15/10</td>
<td>7.74</td>
<td>9.43</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>21-01</td>
<td>I.P.C.</td>
<td>17/10</td>
<td>5.87</td>
<td>12.51</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>25-01</td>
<td>B.A.T.</td>
<td>74/6</td>
<td>5.37</td>
<td>9.66</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>12-03</td>
<td>Elliott-Automation</td>
<td>10/7</td>
<td>4.94</td>
<td>21.76</td>
<td>-7</td>
<td>3</td>
</tr>
<tr>
<td>23-12</td>
<td>Woolworth</td>
<td>16/3</td>
<td>6.15</td>
<td>14.44</td>
<td>-8</td>
<td></td>
</tr>
<tr>
<td>30-07</td>
<td>English China Clays</td>
<td>22/3</td>
<td>3.82</td>
<td>14.05</td>
<td>-8</td>
<td>3</td>
</tr>
<tr>
<td>26-09</td>
<td>Unilever</td>
<td>29/1</td>
<td>4.29</td>
<td>9.56</td>
<td>-8</td>
<td>3</td>
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<tr>
<td>23-03</td>
<td>British Home Stores</td>
<td>20/-</td>
<td>4.25</td>
<td>17.49</td>
<td>-15</td>
<td>4</td>
</tr>
<tr>
<td>2-03</td>
<td>Rolls Royce</td>
<td>42/10</td>
<td>4.98</td>
<td>14.68</td>
<td>-15</td>
<td></td>
</tr>
<tr>
<td>21-02</td>
<td>Thomson Organization</td>
<td>20/9</td>
<td>6.75</td>
<td>19.26</td>
<td>-16</td>
<td></td>
</tr>
<tr>
<td>23-09</td>
<td>W. H. Smith A</td>
<td>39/3</td>
<td>4.59</td>
<td>14.32</td>
<td>-16</td>
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</tr>
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</table>
(ii) *List in industry order of shares ranked (first twenty companies only)*

<table>
<thead>
<tr>
<th>Company number</th>
<th>Company name</th>
<th>Price 15 Aug. 66</th>
<th>High</th>
<th>Low</th>
<th>Mean dividend yield</th>
<th>Mean P/E ratio</th>
<th>Ranking</th>
<th>% relative cheapness</th>
<th>Next final results due</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.01</td>
<td>Bristol Aero.</td>
<td>28/-</td>
<td>31/7½</td>
<td>17/1½</td>
<td>5.22</td>
<td>15.75</td>
<td>5</td>
<td>-33</td>
<td>May 67</td>
</tr>
<tr>
<td>2.02</td>
<td>Hawker Siddeley</td>
<td>36/6</td>
<td>42/4½</td>
<td>35/1¾</td>
<td>6.58</td>
<td>10.91</td>
<td>1</td>
<td>16</td>
<td>May 67</td>
</tr>
<tr>
<td>2.03</td>
<td>Rolls Royce</td>
<td>42/10</td>
<td>53/-</td>
<td>38/3¾</td>
<td>4.98</td>
<td>14.68</td>
<td>5</td>
<td>-15</td>
<td>May 67</td>
</tr>
<tr>
<td>2.04</td>
<td>Westland Aircraft</td>
<td>16/9</td>
<td>20/10½</td>
<td>14/6</td>
<td>4.48</td>
<td>11.63</td>
<td>4</td>
<td>-13</td>
<td>Dec. 66</td>
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<tr>
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<td><strong>Building</strong></td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>3.01</td>
<td>Allied Ironfounders</td>
<td>15/10</td>
<td>23/9</td>
<td>15/9</td>
<td>7.74</td>
<td>9.43</td>
<td>3</td>
<td>7</td>
<td>June 67</td>
</tr>
<tr>
<td>3.02</td>
<td>A. P. Cement</td>
<td>47/7</td>
<td>58/3</td>
<td>46/7½</td>
<td>4.62</td>
<td>14.58</td>
<td>4</td>
<td>-10</td>
<td>April 67</td>
</tr>
<tr>
<td>3.03</td>
<td>BPB Industries</td>
<td>16/9</td>
<td>26/-</td>
<td>16/6</td>
<td>6.57</td>
<td>10.10</td>
<td>1</td>
<td>26</td>
<td>June 67</td>
</tr>
<tr>
<td>3.04</td>
<td>Derbyshire Stone</td>
<td>14/3</td>
<td>20/-</td>
<td>14/-</td>
<td>5.26</td>
<td>13.90</td>
<td>2(3)</td>
<td>11</td>
<td>Aug. 66</td>
</tr>
<tr>
<td>3.05</td>
<td>Goodlass Wall</td>
<td>19/10</td>
<td>25/6</td>
<td>19/9</td>
<td>6.29</td>
<td>9.17</td>
<td>2</td>
<td>12</td>
<td>April 67</td>
</tr>
<tr>
<td>3.06</td>
<td>Hall &amp; Harri River</td>
<td>9/-</td>
<td>13/0½</td>
<td>8/10½</td>
<td>6.07</td>
<td>12.00</td>
<td>1</td>
<td>19</td>
<td>Apr. 67</td>
</tr>
<tr>
<td>3.07</td>
<td>London Brick</td>
<td>17/7</td>
<td>22/-</td>
<td>17/7½</td>
<td>5.67</td>
<td>16.19</td>
<td>5</td>
<td>-17</td>
<td>April 67</td>
</tr>
<tr>
<td>3.08</td>
<td>Marley Tile</td>
<td>15/6</td>
<td>23/10½</td>
<td>15/6</td>
<td>5.91</td>
<td>15.79</td>
<td>2</td>
<td>9</td>
<td>Dec. 66</td>
</tr>
<tr>
<td>3.09</td>
<td>Redland Holdings</td>
<td>13/10</td>
<td>22/1½</td>
<td>13/6</td>
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<td>16.57</td>
<td>4</td>
<td>-10</td>
<td>July 67</td>
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<tr>
<td>3.10</td>
<td>Rugby Portland Cement</td>
<td>14/-</td>
<td>18/7½</td>
<td>13/9</td>
<td>4.04</td>
<td>13.91</td>
<td>3</td>
<td>0</td>
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<tr>
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<td>40/-</td>
<td>31/3</td>
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<td>12.15</td>
<td>4</td>
<td>-10</td>
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<tr>
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<td><strong>Contracting &amp; Construction</strong></td>
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<td></td>
</tr>
<tr>
<td>4.01</td>
<td>Taylor Woodrow</td>
<td>20/4</td>
<td>25/1½</td>
<td>19/9</td>
<td>4.91</td>
<td>8.79</td>
<td>3</td>
<td>0</td>
<td>Apr. 67</td>
</tr>
<tr>
<td>4.02</td>
<td>Wimpey</td>
<td>26/7</td>
<td>35/-</td>
<td>25/9</td>
<td>2.82</td>
<td>11.73</td>
<td>4(3)</td>
<td>-10</td>
<td>April 67</td>
</tr>
<tr>
<td></td>
<td><strong>Heavy Electricals</strong></td>
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<td>55/6</td>
<td>43/3</td>
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<td>B.I.C.C.</td>
<td>62/6</td>
<td>75/-</td>
<td>62/3</td>
<td>5.60</td>
<td>12.41</td>
<td>3</td>
<td>-3</td>
<td>Mar. 67</td>
</tr>
<tr>
<td>5.03</td>
<td>Crompton Parkinson</td>
<td>8/7</td>
<td>12/0½</td>
<td>8/6</td>
<td>6.32</td>
<td>12.65</td>
<td>3</td>
<td>1</td>
<td>Oct. 66</td>
</tr>
</tbody>
</table>
APPENDIX 3

REGRESSION COEFFICIENTS

Payout ratio

Forecast earnings growth rate

Forecast dividend growth rate

Regression analysis - run no.
Earnings variability

Earnings growth rate

Regression analysis – run no.
ABSTRACT OF THE DISCUSSION

Mr D. Weaver, in presenting the paper, said that it had its origin early in 1963 when he had asked a group of his colleagues if it were possible to devise a set of sieves that would enable them to select more effectively shares of investment merit from the very large number quoted on the Stock Exchange. It was fortunate that he had made the request in the hearing of one who had the necessary statistical knowledge and investment experience to produce an acceptable answer, even if it had led to a somewhat unequal collaboration.

The method, as it finally evolved, turned out to be oddly similar to the 'capitalization rate' approach of the 1960 paper but with the capitalization rates reached by a more logical and better documented route.

There were a number of different aspects which had had to be covered in the paper, which accounted for its being rather longer than intended. First, there were the variables put together to form the model. All practitioners had strong views on the factors influencing share prices, and the authors expected considerable discussion on that aspect should there be any investment experts present. Then there was the statistical solution and analysis which contained much of technical interest and the authors hoped that the various tables given would supply sufficient material for statisticians to assess their methods. Those two parts taken together could well provide the basis for a discussion of the several academic reviews of investment methods that had appeared in recent years and especially, of course, those associated with the name of I. M. D. Little.

Ordinary share data were somewhat intractable to handle statistically and on a number of occasions in the paper it had been necessary to break off to deal with practical problems which had to be solved if the statistical methods employed were to yield the best results. They had also been anxious to give enough information to enable anyone who so wished to repeat their calculations.

In passing, the authors wished to draw attention to the fact that the whole statistical analysis had been carried out using logarithms of the input variables and that where, for example, they referred to $x_t$ as the variable in the correlation matrix in § 28, it should be understood that the analysis was in fact based on $\log x_t$.

The authors had sought a practical method and it was by their results that they should be judged. The performance of the test portfolios, as given in Table 2, was not unsatisfactory, and those tests had been carried further. They had since had the results down to the end of 1966 for 36 'buy and hold' portfolios started at monthly intervals between November 1963 and November 1966. The average gain over the F.T.-Actuaries Index still came out at 3.6% per annum, although the gain on the 'buy and switch' portfolio had fallen to 4.1% per annum.

There were those who did not believe that there could be a rational system of share selection. Those were the 'random walkers'. If actuaries could not answer their case they would have to abandon their claim to investment skill and merely invest in the average. The authors had given their answer but they hoped that there were others present with better answers. All that was asked was that those others should describe their methods and set out their results.

Mr A. T. Grant, in opening the discussion, said that the paper opened up a wide field which was currently under intensive cultivation. It contained glances to the horizons of scientific investment management but also contained a good deal of grass-root detail,
Evaluation of Ordinary Shares using a Computer

and for critical commentary he was not sure whether the better tool was a telescope or a lawn-mower.

Perhaps it was best to try to achieve a sense of perspective by considering the approach outlined in the context of the overall problem of investment in equities. The investment manager considering equity investment had to confront several levels of problem. He had to consider general economic and political forces and broad sea-changes in the market standing of ordinary shares. He might have to specify preference for certain types of equities, for example preferring immediate income stocks to low yielders or possibly preferring non-cyclical stocks. He had to evaluate individual equities, and apart from the selection process he was likely to attempt to benefit from careful timing of purchases and sales. He had to construct an equity portfolio with a sensible overall balance and he had the considerable administrative problem of continuous scrutiny of the shares in his portfolio, including the practical supervisory problem of imposing consistency on the views of any subordinates responsible for watching specific parts of the overall portfolio.

It was a tribute to the authors that it was hard to think of any function which their method did not cover. It did not cover economic or political changes, nor yet investment evaluation 'sea-changes'. It did not differentiate between different types of share on behalf of different types of investor. It was both an asset and a disadvantage of the system that it summed up the assessment of a share in a single figure. To that extent it was best used as a guide rather than as a complete answer. It did not attempt to make up a balanced portfolio, nor did it directly assist analysts to make forecasts of future earnings and dividends.

Although it was described as a method of evaluating shares, it should also be given credit for the assistance it provided on timing and for the continual scrutiny of shares which it provided automatically, and also for the fact that it forced individual analysts towards consistency in half the evaluation process of share selection. That last point was important. With that system, once an analyst had fixed on his forecasts of future earnings and dividends for each company, the computer would fix on a suitable market price for that company's ordinary shares. That led to a valuable improvement in consistency between different analysts. But the forecasts of earnings and dividends made by the analyst were still subjective assessments. He could give full weight to all the knowledge available to him, whether qualitative or quantitative, and the use of the computer should not affect his trading forecasts. It was equally true that he would make mistakes in his subjective forecasts, and those mistakes would work through to the final share evaluation. The authors did not claim that the computer method was a magic formula which infallibly picked the right shares but it removed from the analyst the task of fixing realistic market prices for shares once he had completed his dividend and earnings estimates, and to that extent it reduced his task and eliminated one source of inconsistency between analysts.

Bad subjective forecasts of earnings and dividends should certainly produce bad results. If the authors' methods were efficient, good forecasts should produce good results. It would be most valuable to hear from the authors how close the forecasts used—say in 1964—had proved so far in practice, and what degree of variability of error was involved. Currently opinion in the U.S.A. seemed to be divided on the level of accuracy which could be attained in earnings estimates. One group had found that satisfactory estimates could not be made even of earnings for the current year of trading, despite an intensive analytical effort. Other groups claimed to have been fairly efficient in the
making of forecasts. It would be interesting to know how critical those estimates were in terms of overall results.

Over the very short term, in theory the forecasts need not be any more accurate than those of the general body of investors. Indeed, it could even be that better than average forecasting accuracy could be detrimental to share price performance in the short run (simply because it would differ from the average), although that should give way to improved performance fairly quickly. That could readily be seen, in principle, by considering what a multiple regression approach sought to achieve.

At Staple Inn they were familiar with the concept of expected yield, and once future dividends were forecast for a share those future dividends could readily be capitalized, allowing for whatever compound interest assumptions they wished, to derive an intrinsic value for the share, or alternatively, they could deduce the expected yield to a buyer at the current market price. That involved assuming that the buyer did not sell the share, or alternatively that he sold it at some assumed terminal value. Since terminal values were difficult to guess accurately enough in practice, such an approach was most suited to a large institution with growing funds, where the emphasis was on buying shares much more than on selling them. The authors had been associated with that method in the past, as they had mentioned in the first paragraph of their paper.

The multiple regression approach was different. It did not purport to calculate intrinsic values for shares. It gave expression as to how the stock market appeared to be valuing shares at the time, regardless of whether it was right or wrong; and when a share was brought out as 25% cheap relative to the overall regression equation formula, the implication was that the market was being inconsistent in its evaluation of that share. It did not mean that the share was 25% cheap in terms of intrinsic value to a long-term holder.

In his view that was a perfectly valid concept provided it was borne clearly in mind just what the method set out to do. As an analogy, in the gilt-edged market a yield curve could be fitted to gross redemption yields and stocks adjudged to be dear or cheap according to whether their yields were lower or higher—or unusually lower or unusually higher—than the curve. While profits could be made at times by tactical switching on such yield differences, money could also be made from a judgment that the overall shape of the current yield curve was wrong by engaging in strategic switching.

The multiple regression analysis of equities was very similar to a yield curve analysis of gilts. It was at the same time a description of the whole market at one point of time and a base from which to measure individual stocks. It would be interesting to know whether one feature of yield curve analysis existed in regression analysis, namely, the persistent tendency of certain stocks to stand well above or below the norm. As mentioned in the paper, G. R. Fisher had found that within individual industries large companies were preferred by investors to small companies, and they were familiar with the company which commanded a loyal following for its shares for several years at a time. If it were a noticeable feature that certain stocks were persistently dear or cheap, perhaps some allowance should be made for such cases. Victory bonds would not be expected to return to the average according to the yield curve. Should the same be expected for Marks and Spencer or Tesco? It might be that such companies were inconsistently valued, but that the inconsistency was itself regular over successive regression analyses.

In the first sentence of § 13 the authors mentioned that the idea underlying the computer method was to parallel the mental process of the investor in evaluating a share, 'in the belief that, on the average, the market is consistent in its judgment'. He
Evaluation of Ordinary Shares using a Computer

wished to underline that assumption of the market's consistency. It was a major assumption underlying the use of multiple regression analysis as a method of prediction, rather than as a method of spotting major inconsistencies in evaluation at one particular point of time. In § 30 the authors noted that point but explained that, until the present detail of the system had been in force for a longer period than five months, it would not be possible to analyse the statistical backing for that assumption. Appendix 3 displayed some variability in the regression coefficients over 22 weeks and, since it was somewhat difficult to judge from those coefficients how much that degree of variability affected the final cheapness or dearth of individual shares, perhaps the authors would consider running through the input data for say run 22 combined with the regression coefficients derived from all runs 1 to 22 in turn.

The evidence from Little and Rayner that past records of earnings and dividend growth affected share prices gave some support to the belief that the market was consistent in its judgment.

If the coefficients remained unchanged and the analyst's forecasting ability was of the same order as the market's, then the inconsistencies of the market thrown up by the regression method might afford a profit. (Persistent inconsistencies and random fluctuations were the main remaining dangers in that case.) If the coefficients remained unchanged and the analyst's forecasting ability were better than that of the market, so long as the regression coefficients were of the correct signs then an improvement in profit should be achieved. In others words, profits could be made from that system, with average or above-average forecasting ability, so long as the coefficients remained stable, even if the coefficients were wrong in terms of intrinsic value.

How dangerous were random fluctuations? They ought to be cautious on that score. Although in terms of the example on p. 182 it was creditable to have accounted for 58% of the total variance, the 42% still outstanding left room for other sources of variation, including random variation, which could be important quite apart from any errors of forecasting.

In the table in § 26 the total sum of squares of variations from the mean for 134 stocks was 5.044 and Mr Weaver, in introducing the paper, had explained that that related to variations in logarithms of yield, not to variations in the yields themselves. The speaker had himself made some rough calculations, working on yields rather than their logarithms and, making a mental adjustment to the logarithms, the sort of error to be looked for might be on the average 10%. Perhaps the authors could comment on the sort of random error as it worked through to final prices, as his mentally adjusted figures might be wrong.

It should be noted that the work done on random walk theory of stock exchange prices was not inconsistent with the idea of statistically significant systematic changes with random variations superimposed. A useful final chapter in Higgledy Piggledy Growth Again by Little and Rayner could be recommended as a succinct commentary on the subject. In particular, where earlier researchers had looked at individual share prices rather than the movement of overall indices, they had detected some significant systematic changes, although of a small order.

A feature of the paper was the provision of figures showing that over the past 2½ years the method outlined had produced positive results. Of the profit measurements quoted, he preferred those of Table 2 to those of Diagram 2. Since the latter worked on a month by month basis, it was possible that the publication of the lists of recommended purchases and sales by the authors' colleagues might itself have influenced prices over the following month. However, the tests of Table 2 applied over longer periods and that was
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less likely to have happened; it was comforting to see in Table 2 itself that the active investor had consistently surpassed and, according to Mr Weaver’s introductory remarks, was still surpassing his inactive brother.

Could the reason for that good performance be positively pinned down? In particular, was it due to success from good analysis, to alterations in the coefficients used by the market, or to the eradication of inconsistencies?

If the method were successful in principle, were the variables suggested the best ones? Certainly objections were felt to the fact that of the five ‘regressor’ variables used by the authors, no less than three were estimates, and in the example quoted those three accounted for 54-2% of the total variance, while the two hard figures based exclusively on past data accounted for only 3.6%—although those two were admittedly introduced last. The independent variable was also strictly an estimate. However, given the current state of financial reporting in Britain, it had to be recognized that the authors were right in accepting a degree of subjective estimation rather than use figures which did not represent the current state of business knowledge. That again was a question of the intractability of data.

An alternative approach was to job backwards through time to examine whether share prices had moved in accordance with such variables as changes in earnings and dividends. In his own office they had tried to do that and for the previous 10 years had examined percentage movements in price for individual companies, correcting those movements by the Financial Times Index movements. They had decided to use the dates at which the accounts for individual companies were published and tried to co-ordinate those with the actual changes in earnings and dividends which had happened, again allowing for changes in dividends and earnings on the Financial Times Index. They had also tried co-ordinating relative price movements with the dividends and earnings at the outset. He had been very excited on getting the first results appertaining to 1958–59, which showed that about 96% of the increase in prices was explained by the increase in dividends. That afternoon the figures for two other years had become available and for those years the position had changed entirely. The dividend changes accounted for very little in those years, and overall that approach was disappointing, with changes occurring in the coefficients on trial multiple correlations. Those vast changes in simple correlation showed that caution was necessary, and the results produced from that analysis in terms of multiple regression seemed to bear that out. On the whole that would have been expected to be a reasonable line of approach because perfect forecasting ability could be assumed and the market reaction to the results could be seen as they emerged. The other attraction of the method would have been that the persistent inconsistency of a Tesco would have been less worrying inasmuch as they would just examine the percentage variation in the share over the year.

It was a major disadvantage of a straightforward multiple regression analysis using least squares that, if the regressor variables were correlated with each other, no real importance could be attached to the regression coefficients. The authors recognized that point (for example, in § 28), but in his own view it should be made more strongly. In § 5 they expressed the hope that time would help to explain the meaning to be attached to the calculated values of the coefficients—but even with time the calculated values would only be of indirect causal value (because of interaction). That was a great pity because it would be very valuable to have a continuous method of watching for ‘sea-changes’ in the weights given by the market to different factors.

In the multiple regression analysis, the basic assumptions were that the regressor or predictor variables were independent of each other, and that the residual errors were
independently distributed random variables, constituting a sample from a probability distribution with mean zero. The residual errors had to be independent of the regressor variables. Given those conditions, the regression coefficient of, say, (log) payout ratio appearing in the equation measured the change in (log) dividend yield if (log) payout ratio were altered, with the other variables being kept unaltered.

When some of the regressor or predictor variables were highly correlated, as the authors pointed out, the coefficients became meaningless in themselves—they could even have the wrong sign. It was tempting simply to drop one of the variables, but that could produce a quite different regression equation. That problem, known as multicollinearity, could be attacked by a secondary analysis, or by confluence analysis or by principal component analysis. Being no expert in advanced statistical theory at that level, for the mechanics of such techniques he was content simply to make reference to Kendall’s monograph, *A course in multivariate analysis*—which also included a chapter on discriminatory analysis—and Chapter 27 of *Advanced Theory of Statistics*, Vol. 2, by Kendall and Stuart. It would, however, be most interesting to know whether the residual errors brought out in practice an independently distributed random variable, stemming from a population with mean zero.

The authors’ suggestion that discriminant analysis would be a fruitful approach was appealing. In that context it might be of interest to know that standard programmes were available from at least some computer service bureaux, which should take some of the hard work out of discriminant analysis. In practical terms it might be that the results of discriminant analysis would allow more confidence in the performance of an individual chosen security, while it might prove that the multiple regression technique was more suitable for categories of shares taken as groups. Examination of the current prices of the shares graded into categories as at 15 August 1966 in Appendix 2, showed large individual variations in price performance since that date. That was, in his view, to be expected, and gave no cause for complaint. It was a question of selecting groups and allowance should be made for individual variations.

In the United States very considerable sums of money were being spent by investment houses and brokers on attempts to use operational research methods and computers in improving investment management. Since the authors had invited discussion of the other fields selected, he might mention the considerable amount of work done on portfolio profiles. For example, it seemed reasonable to say to an investment manager, ‘Put it through the computer and we will tell you what total assets you have and how these assets are made up’, etc. The whole portfolio could be expressed then in terms of one conglomerate stock. The same approach had been used in conjunction with linear programming. One of the attractions in that was that different strategies could be offered to different individuals, and even a ‘strategy of strategies’ might be a sensible thing for an investor to have, adopting one position in the ‘bull’ market and one in a ‘bear’ market. In one particular house a very large analytical effort had been made covering 60 analysts and 59 industries. Forecasts were made by them for a large number of individual stocks and industries, and it was decided to add together all the forecasts to see how they compared with the national economic forecasts, amendments being made in the individual analysts’ forecasts accordingly.

The authors, in their approach, had opened up a highly interesting field. Although they claimed only to have scratched the surface, that was surely what a good farmer did when he wanted to plant some seeds. He felt sure that the authors’ seeds would germinate well. Theirs was a practical paper of great interest and was likely to be the forerunner of others following in their trail.
Mr W. J. Bishop said that his comments would relate to the investment rather than the statistical aspects of the paper. He had always been basically indifferent as to whether the yield finally obtained on a particular ordinary shareholding had arisen by way of income or through capital appreciation, although, of course, certain types of fund might have a requirement for immediate income. Therefore he had usually regarded current yield as a factor of marginal, rather than central, importance in assessing ordinary shares and had tended to quarrel with the share rankings produced by the computer method on the grounds that they seemed to pay too much regard to current dividend yield. He was therefore very interested to note the possibility mentioned in §12 of the paper that that bias could perhaps be corrected by taking a longer view.

The performance achieved by the present computer method had taken place in a period when the prices of many shares in fast-growing companies had been undergoing a correction from the overvaluation accorded by the market to such situations in the early 'sixties. On the other hand, the proliferation of high-yielding unit trusts, the tendency of companies to over-distribute in front of the introduction of corporation tax, and finally, the advent of the dividend freeze, had been factors encouraging investors to overrate the importance of immediate yield. He doubted whether, against a reversal of that market background, the current system would enjoy similar success and he would, in spite of fully appreciating the fallibility of longer-term forecasts of earnings and dividends, again favour taking a rather longer view.

There were some dangers in evaluating the results of a system such as that in terms of market performance of shares over a relatively short period. The performance shown in Table 2 might largely be produced by successfully picking only a few shares whose market performance had been outstanding, and he felt that that would be much less convincing statistically than if the performance derived from an above-average showing by a larger number of shares. It would be interesting to know what underlay Table 2 in that respect.

A further factor was that if the results of an apparently successful share-picking system were circulated widely it would acquire a following among investors. The result of that would be that, by the operation of supply and demand in the market, the system would produce initially excellent results in terms of share price movements against the index, though he doubted whether many investors would succeed in dealing at the prices in force when the selection was made. He was tempted to suggest that through that there might be introduced something which could be called 'spurious share selection'. For example, in a widely circulated market review published some ten days before, it had been pointed out that the authors' computer had beaten the index by 8-6% on portfolios selected a year previously. There then followed a list of the top fifteen shares in the computer rankings as at 3 January 1967. He was interested to discover that, as at the close of business on Friday, 20 January, the average rise in price of the fifteen shares since 3 January had been 5-8% against a rise in the F.T.-Actuaries 594 share index of 1-4%, and that only two of the fifteen shares had failed to rise by more than the index. He would be extremely cautious in concluding that, in the short space of just over two weeks, the market had suddenly appreciated the attractions of that selection of shares on the one-year view taken by the system. To be fair, though, the system might there be partly the victim of its own uniqueness. If a number of systems were in operation, each involving different subjective assumptions, and therefore picking different shares, the results of any single system might well not affect market prices so sharply.

That did, however, draw attention to the potentially self-defeating aspects of too great a concentration by investors on short-term market performance. That had become
very apparent in the United States where, following publication of a report which brought to light poor portfolio performance by many mutual funds (or unit trusts), a number of such funds had, with typical transatlantic enthusiasm, gone to the opposite extreme from their previous inactive policy. Those funds were no longer prepared to buy shares they considered intrinsically cheap and wait for their market performance to improve. They preferred to buy shares which were already on the move and to get out as soon as the movement stopped. The results of those tactics were, in certain favoured shares, violent fluctuations in share prices as funds jumped on and off the band wagon en bloc and, he suspected, an environment in which even the most agile found it impossible always to avoid being trodden on.

He felt that there was some danger that a system which appeared to be a successful indicator of short-term market performance could be employed as a basis for a similar approach to equity investment in the London market, and, in a market which was less free than that in New York, the results could be still more chaotic. He felt sure, however that the authors would deprecate such a development no less strongly than he would.

In putting forward his views he had tended, as was natural, to concentrate on the points where he had doubts on the methods and results set out in the paper. He would not like it to be thought, however, that he was not fully appreciative of the efforts of the authors to put the assessment of ordinary shares on a more scientific basis, nor that he was not frequently both intrigued and stimulated by the practical results of their system. It was of great value to the investment world that such pioneer work was being brought forward for wider discussion, as it was probably the only computer project in the investment field in the United Kingdom operating at as high a level of sophistication as could be found in the United States.

Mr T. H. Beech said that he entirely concurred with the concluding remarks of the previous speaker. A tremendous volume of results had by now emerged from the authors’ intriguing approach to portfolio selection. In view of that, it seemed a pity that it had been found possible to devote less than four pages (§§ 36–39) to those results and comparison with the performance of other methods. Even within such a space limitation, however, more could have been revealed, perhaps, with a different choice of presentation.

His main remarks would be concerned with methods of presenting the results of portfolio selection procedures but first he would like to make a comment on the ‘100’ Companies method. The superiority of that purely subjective method, if it should turn out to be a genuine superiority rather than a quirk of the particular period of comparison, surely called for more consideration than was given in the paper. He would have thought that any advantage in forecasting performance, provided it was genuine, would be of sufficient value to those responsible for the management of major funds and their advisers to justify a reasonable amount of extra work.

Presumably Diagram 2 and Table 2 had been based on square tables in which each row represented a portfolio started in a particular month (as in Table 2), the figures in that row in successive columns, say, 100, 100·8, 101·5, 101·0, 102·7, etc., showing the relative value (normalized against the share index) in successive months after the start of the portfolio using a common radix 100 at the start of each portfolio. Such a square table would, at a given point of time, have the boxes on the rising main diagonal and all boxes in the triangle to the upper left of it completed, and all the boxes below the main diagonal blank. Diagram 2 contained only the information in the boxes in the ‘one-month after selection’ column, while Table 2 contained only the information in the boxes
on the main diagonal. Thus, the great majority of the data in the completed triangle was ignored entirely.

Although the matter of presentation in summary form of such extensive data was, to some extent, a matter of personal preference, he would describe what seemed to him to be the optimum, in the sense of giving a simple appreciation of the practical value of any selection method in a minimum space while making use of all the data in the square table. His proposal should have some intuitive appeal to members, being really only an application of familiar exposure-to-risk and mortality-table ideas. It was very analogous to a method used in preparing machine-life tables where new batches (portfolios) of machines were being delivered each month. First, the data in the boxes in the square table needed to be in slightly different form from that mentioned. The box for month \( n \) for any particular portfolio commencing in calendar month \( k \) (row \( k \)) would contain the ratio of the value of the portfolio (normalized in terms of the share index) at month \( k \) to its corresponding value at month \( k - 1 \). For each column of such ratios an average would be taken; that would be for the particular method of portfolio selection, the average relative improvement in value to be expected in a single month commencing \( k - 1 \) months after a portfolio was selected. By chain-linking such average relative improvement ratios for each month of age of portfolios, a characteristic curve could be produced for the selection method. (Curve \( OAP \) in the diagram above.) That contained essentially all the information available. Because the number of months' exposure on which the characteristic curve was based reduced gradually down to one month as they moved to the right away from the origin, the curve was of course subject to an expanding funnel of doubt—but that was inevitable. A similar situation occurred with machine-life tables. Of particular interest from a theoretical point of view was the asymptote of the characteristic curve which, for a 'buy and hold' selection procedure, might be
expected to be horizontal. Suppose, for instance, that shares in category 1 had an average cheapness of 20%. Ideally, the asymptote should correspond to a relative value of 1.20. If the actual asymptote turned out to be 1.10, the ultimate rise in value was obviously 50% of what the model led them to expect and they might call 50% the internal efficiency of the selection method in terms of its own assumptions.

The asymptotic relative value, $R$, was also of some practical importance as it was only if that lay above $M$, the appreciation required to cover the average margin of expenses and jobber's turn, that the selection procedure could pay off.

From a practical aspect, however, a more important—indeed vital—parameter to be derived from the characteristic curve (ignoring the possibility of switching shares individually) was the optimum incremental yield which he would define. In the diagram $OM$ represented, on a logarithmic scale, the increase in relative value necessary to recover buying and selling expenses and margins, and $MA$ was a straight line drawn tangentially to the characteristic curve. If they were only prepared to re-select the portfolio as a whole, not in terms of individual shares one at a time, $OK$ defined the time cycle at which such a re-selection should take place to obtain on average the optimum incremental yield, being that determined by the slope of $MA$; that was the maximum extra rate of appreciation the system could provide over and above that offered by the underlying index against which all portfolio values were normalized.

A still more practical presentation, provided they were prepared to make the extra effort of calculating out a ‘buy and switch’ policy for individual shares, as the authors had done in Table 2, was to plot the characteristic curve for such a policy (shown as $OQ$ on the diagram). Bearing in mind the logarithmic scale of relative value used, an asymptote might also be expected but in that case it would not be horizontal. The slope corresponding to that asymptote defined the optimum incremental yield of an overall procedure incorporating both a portfolio selection method and an appropriate associated switching policy, provided that the expenses and margins were taken into account every time a switch was made.

The latter presentation could, of course, only be applied to a system where all shares were continuously under review. The interesting point about the simple ‘buy and hold’ version, however, was that it could be applied very easily to compare the merits of any investment selection procedure whatever, not excluding that under which each portfolio consisted of a single share recommended by a particular adviser, or even by a particular newspaper! The results of such a comparison might prove interesting.

As far as could be judged from Table 2, it appeared that the asymptote for ‘buy and hold’ was of the order of 8% to 10%, thus comfortably exceeding the average margin required for buying and selling expenses and the jobber’s turn. (Could the authors, incidentally, suggest a standard percentage for analysts to assume for that overall margin? Standardization of that point would assume major importance if the optimum incremental yield should become widely used as a means of summarizing the performance of a selection procedure.) Furthermore, the ‘buy and switch’ policy underlying the penultimate column of Table 2 seemed to lead to an optimum incremental yield of the order of 6% per annum, which, if confirmed over a longer period, was a very satisfactory result indeed, bearing in mind that the selection consisted of the top 20%; presumably still better results would ensue if, say, the top 10% or 5% were used.

The matter of dividend did not seem to have been discussed. Was it right to assume, as the authors had, that the dividend did not make much difference? Unless he had misinterpreted them, they were working purely on capital values, but dividend could affect the picture in two ways. There could well be some sort of correlation between computer
rankings of shares and the phase of shares in their respective dividend cycles. One easy check that might be made would be to see whether portfolios with switching had a superabundance or an underabundance of dividends; if either effect were present it would be necessary, though somewhat tiresome, to bring in the effect of dividend increment as well as improvement in capital value. That would also be the case if it should be found that the rankings tended to favour high- or low-yielding shares.

Finally, on a philosophical point, it seemed to him that there would, in the nature of things, be an inherent instability in the coefficients of any multiple regression forecasting system which included a mixture of objective and subjective parameters. That instability arose out of psychological feedback. The analysts responsible for the subjective parameters would subconsciously (sad to say, consciously as well, if his remarks came to their notice) regard the existence of non-zero coefficients attaching to the objective parameters as a reproach to their subjective ability, and there would be a tendency for them to correct that. The simplest example of that was seen in considering $x_2$ and $x_5$. Would not the fact that the regression coefficient of $x_5$ was negative lead the analysts inevitably to infer that they could not be taking sufficient account of historical earnings growth rates when forecasting future earnings growth, and consequently to take more notice of historical earnings growth when making earnings forecasts in the future? In that way he would expect the coefficients of the ‘historic’ parameters $x_4$ and $x_5$ to approach zero over a period, quite possibly in a damped oscillatory manner, as there might well be a tendency to over-correct.

If that thesis were correct, it would appear that if a number of organizations were to set up computer rankings on the general lines described by the authors, each organization would ultimately be using only subjective parameters in their regressions, a piquant thought that might give some long-range comfort to those in the ‘hunch’ game. The comparative results achieved by a particular organization would depend not only on the degree of brilliance of its subjective forecasters but also on the success achieved in separating out, in as independent a manner as possible, the different subjective parameters to be forecast.

Since that possible future, even if theoretically sound, had a somewhat unrealistic ring about it, should the argument be used in reverse? It might be that the unstable mixture of objective and subjective parameters would eventually be resolved in the opposite direction; a study of correlations between subjective forecasts and observable facts available to the forecasters might enable the judgment element gradually to be rationalized into objective components. He left those more abstruse considerations to the philosophers.

Mr L. Ginsburg said that it had always been a source of irritation to him that although they had available an enormous volume of financial data which should be amenable to statistical analysis, it always seemed to defy treatment. That was partly because they did not know how, and partly because until the computer came along the necessary sums were beyond human capability. The authors, however, had gone a long way towards reducing that unmanageable mass into an orderly and coherent shape. Whether or not they agreed with the authors’ conclusions was not important; what was important was that the authors had devised an analytical signpost which should point the way towards further substantial progress.

Like the alchemists, the authors had tried to transmute lead into gold. They took an inert mass of financial statistics and sought to convert some part of it into a profitable financial operation. Although time had proved him correct in principle the alchemist had
lacked the economic foresight to question what would happen to the price of his gold if transmutation could be achieved as cheaply as he hoped. Obviously gold would revert to the price of the baser metal. In a very mild way the authors had fallen into the alchemist’s trap. Had they considered how long their top twenty shares would remain relatively cheap when investors were made instantaneously aware that they were cheap and by what margin? In the limited markets of the day cheapness would have a very short life indeed; possibly measured in microseconds while the jobbers got out their chalk and marked the prices up. That same self-destruction had been apparent in the gilt-edged market ever since the ultra-sophisticated detection devices succeeded in ironing out the anomalies almost before they had a chance to exist. The true jobbing profits of yesterday had as a result been reduced merely to what could be made out of a friendly exchange of coupons between gross and net funds.

Going on from there, a very substantial jobbing attitude had become embedded in the paper; sell dear, buy cheap and reverse later on. He did not say that in any way disrespectfully. Provided they got off the mark quickly enough and within the limits imposed by marketability, it could be a profitable exercise, as the authors demonstrated. Nevertheless, as an instrument to be used in making long-term investments, it seemed to him that those methods had very serious limitations. In particular, no reference was made to any form of investment other than ordinary shares, beyond saying in § 40 that a possible direction in which further work could be done was an investigation into the relative attractiveness of fixed-interest securities, equity shares and cash. That was much the most serious problem facing the long-term investor who had to deal with substantial investible funds. Compared with that problem, jobbing exchanges in shares seemed merely an interesting and occasionally profitable pastime. But that was perhaps unfair. What the authors were in effect saying was, ‘You must decide for yourself when it is correct to enter the ordinary share market. When you have done so, we shall help you to select the right shares.’ It had to be remembered, however, that a mistimed entry into the market was vastly more expensive than selecting shares which might not prove to have been the cheapest at that time.

In § 21 the authors enumerated the ‘other factors’ which might have been brought into the analysis but which, for reasons they considered adequate, had been discarded. There he disagreed on fundamentals. First the list was altogether too restricted. He would add some other ‘other factors’ such as the likelihood of changes in the social and political structure of the community, and changes in the direction of the economy, so that what was once regarded as the inevitability of inflation no longer appeared to be so; and for particular industries the threat of increased competition attracted by large profit margins or of changing technology and so on. He believed that that extended list of ‘other factors’, both singly and in the aggregate, could have a much greater effect on relative share values than the five predictor variables to which the authors confined themselves. Secondly, it was implicit to the paper that ‘other factors’ would affect all shares equally, which he found very hard to accept. He suspected that the authors were in fact aware of that weakness because they said that individual investors ought to make their own allowances for some of the more unquantifiable factors.

The authors had been a little hard on the subjectivists. After all, the belief that investment was an art should have died when War Loan reached 108. He did not believe that the subjectivists wielded pins better than anyone else, nor did they gaze into crystal balls; they were modest people who did their expected-yield or break-even calculations and then exercised their judgment to decide whether sufficient allowance had been made for the awful uncertainty which surrounded investment forecasting. Uncertainty could be
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reduced by statistical techniques, as the authors had rightly sought to do, but there was such a cold-blooded precision in their results that in the eyes of the bewildered recipient they might appear to have the certitude of holy writ. That would be a pity because he was sure that no such thing was intended by them.

The authors themselves might not be completely free from the taint of subjective reasoning, at least in so far as $x_3$, the long-term dividend growth rate, was concerned, and to a lesser extend also in $x_1$ and $x_2$, the payout ratio and the short-term earnings growth rate, respectively. It was stated that their analysts were locked in for a week, during which time they were required to review the forecasts on which $x_3$, $x_2$ and $x_1$ were based, but no indication was given of the processes of thought or calculation which led them there. The dividend yield, $y$, was obviously susceptible to changes in $x_1$, $x_2$ and $x_3$, and there could clearly be a considerable subjective content in the final result. It might well be, therefore, that the paper merely purported to arrive at certain subjective conclusions regarding a particular share and then used the computer to indicate whether those conclusions had already been discounted in the market price. He saw no harm at all in that, but that was a much more modest objective than might be intended. He considered that the use of dividend growth in $x_3$ and not earnings growth was suspect, if for no other reason than that dividend growth also came into $x_1$, the payout ratio. It was a moot point also whether the 'appropriate' yield was a derivative of long-term dividend expectations or earnings expectations. Possibly there was a good bit of both and it was therefore a weakness that earnings growth had been given so very little weight in the long term.

His mathematical equipment was exceedingly rusty but it seemed to him that the results exhibited showed a strong bias in favour of high-yielding shares. For example, in Appendix 2(b), comparing the cheapest category with the dearest, i.e., 1 with 5, it would be seen that the average mean dividend yield in the cheapest category was 7.3%, and only 5.1% in the dearest. The respective earnings yields were 11% and 6%. Furthermore, in the building industry group the five cheapest shares yielded 6% on dividends and 8½% on earnings, while the five dearest ones yielded 5½% and 7% respectively. There seemed to be a systematic bias somewhere. Maybe it was built into the mathematical techniques employed or maybe it meant that high yielders happened to be cheap on 15 August 1966. That required further investigation.

He hoped that he had not left the impression that he was exceedingly critical of the paper. If he happened to be a little sceptical here and there it was only because he believed firmly that there were many 'other factors' which might have a much greater influence on relative share prices than the five predictors used. He was puzzled too by what might be a systematic bias towards high yielders. Nevertheless the investor could not avoid doing his sums, and the authors had quite brilliantly shown the way in which to do a certain number of them.

Prof. P. G. Moore said that he would address himself to the statistical and computer side and not to the pure investment side of the paper. He accepted the fact that the use of statistical methods and computers in that area of application presented certain novel features. Such techniques, however, had been used a great deal in other areas and it seemed incumbent on them to learn as much as possible from experience garnered elsewhere. § 5 explained that the authors saw their work as basically providing a tool for prediction and that the meaning and interpretation of the regression equation and its coefficients were of secondary importance. That was quite justifiable but could be misleading in that later in the paper, in § 11, the authors stated in non-statistical lan-
guage that ‘the coefficients may be thought of as “weights” that the market apparently places on each factor’. Further on, they made quite a bit of the sign and magnitude of the individual coefficients and attempted some interpretation of them. Luckily, they found that all the coefficients worked in what they regarded as a sensible and expected manner. That would not necessarily always be so. For example, in the last paper presented to the Institute which used multiple regression to any considerable extent, namely, that by Dyson and Elphinstone on ‘Office Expenses’ in 1959 (J.I.A. 85, 211), the coefficient for annuity consideration was negative, thus suggesting that the higher the annuity consideration the lower the office expenses! Although in the present case the standard errors of the coefficients were all such that that danger did not seem to exist to any great extent, nevertheless some of the standard errors were over 50% of the numerical values and had to be interpreted with extreme care. For instance, if \( y \) and \( x_1 \) alone were considered, the regression coefficient was about 0.5; it was down to 0.35 in the table in § 24 when five independent variables were considered and, if more of the variation could be accounted for by the regression equation, it could well be further changed. Hence there was a danger of equating a change of 1 in payout ratio with a change of 0.35 in dividend yield.

That led him to consider the equation itself in a little more detail. One thing missing in the paper was any consideration of the residuals about the regression equations. He would have thought that quite a lot might be learned from that. For example, in multiple regression techniques it was assumed that the residuals were uncorrelated, that they had an expected value of zero and were Normally distributed about the expected value. He wondered whether all those conditions were encountered. If in fact the variation of the residuals about the expected value according to the regression equation were in some way linked with the dividend yield, then the regression equation obtained was being weighted rather differently according to the different classes of shares arranged by dividend. That in turn could produce the effect that the different classes of shares would be considered on slightly different bases. He found it surprising, following that line of reasoning, that the authors argued it was wrong to include an industry term. If the residuals were available it could be said rightly or wrongly whether that was a correct remark or not, but the authors argued that in principle it was wrong to split up the analysis into an industry basis. An examination of past records suggested that there was a correlation over time between industry dividend yields which was considerably greater than the correlation within an industry or indeed over all industries. For example, the correlation between the 23 groups of the F.T.-Actuaries index at 1 January 1966 and 1 January 1967 was +0.78 (which was, incidentally, a higher correlation than that achieved in the multiple regression equation considered in the paper). Thus some form of industry term would seem desirable. Furthermore, it might be better still to have the regression coefficient terms within industries. There were many degrees of freedom available so that it would not be unduly restrictive. Indeed, the situation would seem to be an appropriate one in which to carry out a generalized six-variable form of the analysis of a set of \( k \) regressions along the lines discussed for \( k \) two-variable regressions by Tippett in 1952 (J.I.A. 78, 387), Moore and Edwards in 1965 (J.I.A. 91, 357) and others. Although, incidentally, the title of the paper suggested that the evaluation depended upon the computer, surely the method of evaluation depended upon multiple regression and the computer was merely used to save blood and tears in computing the multiple regression equations.

Turning back to the regression equation used in the paper, it was interesting to speculate on the order in which the variables were put into the regression in the analysis of
variance in § 26. The order appeared to be an arbitrary one, and thus the relative importance of the individual contributions could not be satisfactorily deduced. Many computer programmes were written so as to select the best variable first, then the second best given the first, the third best given the first two, etc. It would be interesting to see whether all the five variables were still significant if that were done, bearing in mind that the authors claimed that the equation was basically for prediction purposes. His guess was that they probably would not all be significant, especially bearing in mind that the significance levels used for the $F$ tests should really take into account the fact that five independent $F$ ratios were being tested and hence the most significant should be tested at the 1% level, etc., if the overall significance level were to be kept at 5%.

His final point related to the use of regression analysis, particularly when there were, as in the case under consideration, significant cross correlations between the so-called independent variables. Although the authors disclaimed any real desire to find out which factors affected yield and by how much, that surely was at the back of their minds and of financial analysts in general. Given all the possible variables that could be used for share price prediction, was there not a case for considering the use of component analysis in order to determine a series of linear components of the many variables that were orthogonal to one another. An attempt could then be made to interpret the components obtained and thus decide those variables upon which it was best to lavish time and effort?

Mr G. Mills said that he would cross swords with some previous speakers. The problem was the difficulty of placing a value on the share, and the paper which had in the past summarized his own views most succinctly was that by Springbett, given to the Faculty in 1964 (T.F.A. 28, 231), when two appendices were devoted to that particular subject. True, Springbett was speaking about valuation in the traditional actuarial context, whereas the authors were concerned with valuation for marketing purposes, but when dealing with a fairly large fund, as Mr Ginsberg had said, they ran up against problems in connection with marketability. It would be all very nice to be able to forecast short-term price movements, but the thing which was of more importance to such a fund was the trend of future income from the shares and that was brought out so well by Springbett.

He would not like anyone (particularly any stockbrokers present) to go away with the thought that he was disinterested in short-term price movements; he was as keen as anyone else to see superb judgment exercised in the timing of purchases and sales, and equally conscious that a balance sheet had to be prepared at the end of the year which did not create a problem for the auditor!

Perhaps his views on short-term price movements would to some people be almost heresy. He was far more interested in the anticipated changes in dividends over the next 5, 10, 15 years. It was a difficult problem and the authors were well aware of it. But he wished to underline that philosophy towards ordinary shares in a life office in order to set in perspective his reactions to the authors' paper.

It had been mentioned already that what the authors were saying was briefly, that anyone wishing to buy ordinary shares could be told what was the best buy. It was helpful for an Investment Manager to have that information but before he could use it he had to decide whether he should be buying ordinary shares at all.

He wished to outline an idea for future research which sprang from a remark by Mr Grant in his contribution to the discussion on Springbett's paper. He too had emphasized the difficulties of forecasting what was going to happen in the future to any
particular company's dividend or price, and said that it was an impossible task to deal with individual companies and look forward 20 years. To quote Grant, 'We could be much more accurate in assessing the future income of our whole ordinary share portfolio than we can be of assessing any individual share.' What he was seeking was a method which looked backwards for a long-term view of the entire portfolio and provided an assessment of each individual ordinary share on a consistent basis, so that the short term and the long term were blended together; for he firmly believed the truism that the long term consisted of a series of short terms.

The lines along which the inquiry should go should be concerned primarily with the return which managements might obtain on the capital entrusted to them. He realized that to some extent the authors' method made some allowance for that, in that they were making forward estimates of dividends over the next five years. The key to the situation, he believed, was to give more weight to the return a company could obtain on its capital.

He concluded with two separate points. First, in the historical earnings record which was part of their input the authors did not refer to the variations which could occur from year to year in those figures on account of the changes in practice, particularly in respect of depreciation or of the values of stocks, and he would be very interested to hear from them, perhaps in the written reply, whether they had explored that. Secondly, like previous speakers, he speculated as to what would happen when everybody was waiting avidly for the authors' next paper to see what predictions were given and their effect on the market!

In the discussion of Springbett's paper it was mentioned that someone had once been given the task of comparing the recommendations from two trustworthy stockbrokers. On their lists, issued simultaneously, there were 12 shares, but there was agreement in only 7 cases. When there were a few computerized forecasts available some divergence in the recommendations could be expected and that would add to the interest of life.

Mr C. G. Lewin said that the authors seemed in effect to be recommending shares on the basis of the prices other people might be prepared to pay for them in 12 months' time. In a relatively short period of years such as that over which the authors' method had been tried it might be possible to get reasonable results, but if a very marked change occurred in the level of the market, perhaps due to a very sharp decline or rise in interest rates, or due to a basic change in the economic or political outlook for the country, then the prices people were prepared to pay for shares might change very greatly. They might suddenly change their ideas as to the relative weight they attached to the various factors taken into account in the regression analysis. He would not necessarily purchase the shares recommended by the authors but might be more inclined to look at the intrinsic value of the share itself and perhaps pay more regard to such factors as the quality of management and a general impression with regard to the company, which factors were unashamedly subjective.

Mr P. E. Moody, in closing the discussion, said that he felt that the authors could be happy with the reception given to their paper. The discussion had been informative and he had personally enjoyed it very much. There had been a certain amount of criticism, but that was to be expected with any original thought. One or two hard knocks had been given but a good deal of the comment had been in the area of suggestions for further developments or suggestions for further investigations into the methods used.

He was not able to comment very much on the mathematics. To say that he had not followed all the details would be to flatter himself! It would be fairer to ask whether he
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had followed any of the details at all! But one or two mathematicians had spoken and they had made their comments and suggestions, and the basic method did not seem to have been unduly challenged. He was interested that there was an assumption that there would be a linear relationship between the yield the authors were trying to determine and the various regressor variables. He had done some sums just to see what would be the yield of stocks according to growth rates and found that for the appropriate logarithms they did not fit very well. When doing that he was probably testing his own prejudices, but he had found that if he determined the constants in the equation to fit high growth-rates satisfactorily, the fit for low growth-rates was poor, and vice versa. Was the linear relationship something to be accepted without argument or was it a defensible assumption?

They had in the course of the discussion defined the areas in which that method of analysis was going to be useful. Several speakers had pointed out that in making investments, particularly for institutional funds, a good many factors not present in the regression analysis had to be brought in. It had also been pointed out that the analysts' forecasts of future dividends and future earnings were basic to the method. As he understood it (turning back to the 1960 paper, which was the forerunner of the one they were discussing), the method used by the analysts, in a preferred case where a company published turnover figures, was that the forecasts were made of future sales and profit margins, and those took into account all that could be forecast about the state of the economy and of the particular industry or industries being studied. The projections thus obtained were then checked partly against the earnings on capital employed and partly against the possibility of a company being able to raise the capital it needed in order to achieve the sales figure projected. They all knew that there was a very substantial margin of error in that type of analysis. It would be interesting to know how successful the forecasts by the analysts were. It might be that the true comparison was not so much between the analysts' forecasts and the company's results as between the analysts' forecasts and the market expectations. If the analysts could be more right than the market, a good result could be obtained. It seemed that one of the great benefits of that method of analysis was where short-term anomalies arose in the price of a stock for various reasons. The most common was where a block of stock was being liquidated. There were other occasions where shares could begin to look dear for one reason or another. Six months after the passing of the 1961 Trustee Act, various high quality shares began to look very dear. The method should throw out anomalies of that kind.

Several speakers had mentioned that, as far as longer-term anomalies were concerned, it was too early yet to give a judgment on the success of the method. In the main the years 1964, 1965 and 1966 had been studied—a difficult time for the equity market—and over that period prices had tended to fall. It would be interesting to see what happened to the regression coefficients over the period of a major 'bull' market. The results provided by the authors had been valuable in enabling the benefits of their methods to be assessed.

Table 1 was interesting, particularly the line concerning computer rankings. Groups 1 and 2 both showed substantially better results than the other four groups. It did not seem, looking at those figures, as though there was a great deal to choose between the performance of the other four groups. It might be due to the fact that he had not the full data. Did the full figures bear that out? For those who had to buy outside the top 20% it was a weakness if the method of selection did not work well over the other 80%.

One of the important benefits of the paper—and it was something of which he was conscious—was that it had made him examine the fundamentals of his investment think-
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The authors had claimed that the method tried to replace the subjective approach by a mathematical process. In the earlier paper they had said several times that the methods used were aiming to replace the procedures adopted by management. It had occurred to him to wonder whether it was a virtue to translate traditional procedures and habits of thought into mathematical terms, for thinking in the investment world was currently in something of a turmoil. There was the conflict between those who believed that skill could be exercised in choosing investments and the minority who had proved by statistical methods that a process of random selection was equally good. One aspect of that was that they were being forced to pay less and less regard to the company's past record and more and more to their own estimates of the future. The paper bore out the suggestion that past records were not of great predictive value in selecting investments. Certainly it was not possible to take market prices as a reliable indicator of values. They had only to remember that in December 1957 the old Financial Times index was 165; two years later it was 338. They could all explain how it came to double itself, but, however they looked at it, either prices were too cheap in 1957 or too dear in 1959. There never could be any great precision in the valuation of ordinary shares. There were so many random factors, such as the death of a key man in the company—the chance idea that led to a take-over bid or battle, and various other pieces of good and bad luck which could never be fully represented in the mathematically predicted sense.

Several speakers had pointed out that most actuaries were having to choose between cash, medium-term and long-term fixed-interest stocks, and equities, and property as well in many cases. The basic assumption that a decision to invest in equities had been made, limited the value considerably. They were after the evaluation of an ordinary share which resulted in the kind of statement that 'Below price $x$, this share should be bought; above price $x+y$ this share should be sold.' Between $x$ and $x+y$ was a zone where the uncertainties of assessment prevented a definite view being taken. He suspected that $y$ would be of the order of 50% of $x$, so that if a share were bought below 20s. it should not be sold unless the price were over 30s. and possibly far more. That kind of statement should be made against the background of a yardstick which would include such things as the yield on gilts and perhaps their own view on the economic outlook on equity investment.

The authors were prepared for criticism of the fact that the time scale they were using was too short, and from an institutional investor's point of view it could certainly be said to be too short. Even with a relatively active fund it was likely that most shares would stay in the portfolio for 5 or 10 years. Those with larger funds or less activity could well have investments in the fund for much longer periods than that. They could not help having a lot of sympathy with the authors' approach, but there was need for a much longer view.

There seemed to be little danger of investment becoming more of a science than an art. A certain amount could be done in identifying long-run industrial trends or forecasting the prospects of an individual company for some little time ahead, and the authors were doing their best to make investors consistent in their judgments, but there was a great deal of room for the old-fashioned hunch.

The President (Dr B. Benjamin), in proposing a vote of thanks to the authors, said that he welcomed their paper warmly because it seemed to be an honest attempt to make mathematics work in a practical situation. He recalled that Dr Maurice Kendall, some years previously, in his Presidential Address to the Royal Statistical Society, had shown that, in the social and economic field no less than in the physical sciences, there were 'natural
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laws' to be discovered and that a little patient observation and careful analysis by statisticians had shown relationships to exist in areas where previously there seemed to be no systematic behaviour at all. Even in a world which was one part sense and several parts nonsense it was the task of the statistician at least to probe the area of sense and by the same token to delimit the area of nonsense.

Mr M. G. Hall, in reply, referring to Mr Moody's closing remarks, said that he felt that on the whole the period of one year for forecasting share price movement was on the short side. On the question of the relative attractiveness of fixed-interest stocks, equity shares and cash, he agreed with Mr Ginsburg that that was a more important problem than merely selecting shares but he thought that unfortunately the profit to be gained was related to the difficulty of the exercise.

On the question of the general application of principal component analysis, he was completely in sympathy with what had been said. That was a nice way of doing things but in applying that form of analysis to the five \( x \) values, they might be unlucky and find that each of the principal components was still a function of all the \( x \) factors and the subsequent regression analysis on the components might well produce similar results.

Another speaker had mentioned the question of the period of selection. The authors were writing a computer programme to test that. They were examining the performance of shares in the various categories over different time periods and were doing analyses of variance along the way, and asking whether there was anything significant in the performance differences shown by the five categories or whether they had just been lucky.

Some time had been spent in discussing the question of forecasting, on which their methods had been set out in the previous paper. Those methods had not been changed very much and they in fact monitored the results in the short term. Whether or not the figures sounded devastating would depend on whether they had also heard Sir Paul Chambers's address to the Institute (J.I.A. 92, 117) commenting on forecasting in the chemical industry, where standard deviations of the order of 75% were mentioned. They had found in 1966 that their bias—the average percentage difference between the actual change in earnings and their forecast change—was \( \frac{1}{2} \% \), the standard deviation of the individual figures was 10\%, so 95\% of their values were within \( \pm 20\% \) of the true earnings for the year. They hoped to improve on that, but using those forecasts in the system still provided some modest improvement in terms of share performance over the index.

He agreed with Prof. Moore's point about the care required in the case of multiple regression analysis and agreed that it was worth looking at other techniques. Currently quite a lot was being written about people using computers to produce various ratios, and it was very interesting to read that in the previous three months shares with particular characteristics had risen by some 10\%. However, what was needed was to use multivariate techniques to determine what ratios were important in relation to share price movement.

On the question of carrying out a regression analysis for each industry separately, he was against that because there were not enough companies to provide reliable estimates of regression coefficients. Prof. Moore had made the point that the correlation between relative yields in January 1966 and January 1967 was of the order of 0.8. The authors had gone back three or four years and found that it was still 0.8. If the conclusion was drawn from that that the best estimate of the current relative yield for an industry was that of four years ago, they would come sadly unstuck, and the reason was fairly simple. Although the correlation coefficient was high, at 0.8, the fact remained that the standard
deviation or spread of relative yields four years previously was much greater, and the net result was that they had to consider the relationship between the two relative yields. He had done that and found that the slope of the relation was in fact 0.3, demonstrating that there had been a real regression towards the mean. That was how the term 'regression' had been coined originally, when Galton studied the correlation of the heights of fathers and sons and found that the heights of sons were less exceptional than the fathers'. Taking the alternative hypothesis that the best estimate of relative yields was currently 100%, the results would be better than using the hypothesis that they were unchanged from those obtaining earlier, as Prof. Moore had suggested.

The following written contributions were subsequently received:

Mr G. B. Hey: I am particularly pleased that the authors have spent some time in considering whether their method is effective or not. It is in my view a great defect of investment advisers generally, as well as of the papers read to the Institute and the course of reading for Fellowship (7), that little or no regard is paid to the consequences of following various theories or advisers.

It must be admitted at once that the measurement of investment advice and performance is far from easy, and I think there is room for much discussion on the subject. It is not rendered any easier by the fact that different types of investor are interested, at least to some extent, in different features of investment performance, just as different types of fixed-interest stocks are appropriate for different classes of investor.

I hope the authors will not, therefore, think it unfair if I express the view that the measure of success which they adopt (namely, the performance on capital account) is by no means wholly satisfactory, and that the measure of 'the' yield has its disadvantages. I think the prospective yield of a share to someone who is probably going to hold it for a long time (and this must be true of a large number of investors) is more concerned with the ultimate growth of dividend income and, to some extent, of earnings: it may be very different from the yield calculated on short-term market performance, although it is true that if earnings and dividends grow, then to some extent prices will follow suit, at least over the very long term. It is unfortunate that this measurement can only be made accurately after a long interval, by which time the information is of little value. The alternative which the authors adopt of considering the capital performance is very much a second best. Over a period of one month (as in diagram 2) it may well be affected by purely chance events and quite probably by the mere publication of the computer rankings. I must say that whilst I did not automatically accept these rankings I was reluctant to buy anything in the bottom half of the list and usually found enough stocks in the top 30% with which I felt I could agree with the brokers. If this is followed by people who were in the same position as I then was, it could well produce the whole effect shown in diagram 2. The test in Table 2 over a period is certainly better, but since the computer did not apparently (see § 44) rank insurance and property shares, and the 594 index did include them, the results are open to a certain amount of bias in view of the performance of those two classes of shares in the last two or three years. There is a further objection which may or may not have had any influence in that the computer ranks more or less the top hundred, i.e. the very large companies, which could well have behaved rather differently from the 594 which include a much wider range. It is in fact singularly difficult to compare the performance of a portfolio subject to any restraint with an index which is not subject to the same restraint and the comparison is further complicated by the effect of different yields on the portfolio and the index.

The only conclusion which I draw from Table 1 is that the authors and their colleagues
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are rather good in share selection, at least on the short term, and it seems a pity that the computer method depends so much on subjective estimates. If a system could be devised for detecting suitable investments based only on existing information it would be more satisfactory, although if it were accurate it would presumably restrict the range of possible prices to that determined by the differing needs of different classes of investors just as happens in the gilt-edged market. One speaker envisaged a number of computer systems. However, if they differ they could not all be right, and if they agree they would presumably result in a 'known' proper price and no scope for further investigation. I do not envisage this position arising unless Little and Rayner are proved wrong. If, on the other hand, they are ultimately found to be right, it would be interesting to know if an investment analyst could thereafter justify his existence.

Mr J. R. Hemsted: The assumption that there is a linear relationship between the logarithms of the input factors deserves further consideration. If, as the author suggests, the coefficients are regarded simply as weights given by investors to the various factors, then the assumption appears reasonable. But, in practice, investors may better be thought of as selecting shares in the expectation of obtaining a satisfactory return of interest and capital, over a period of time, and the market is undoubtedly supported by long-term investors who, I suggest, establish the basic pattern of the market, and thus define a relationship between certain input variables. The authors' statistical technique may alternatively be regarded as an attempt to discover this relationship by an empirical approach.

In my paper on expected yields (J.S.S. 16, 6) I developed a formula expressing expected yield \( G \) in terms of dividend yield \( y \), payout ratio \( x \) and net equity profitability \( p \) and I discussed the pattern of market assessment. The latter two factors are in the nature of equivalent long-term average values (rather than current values), and subjective estimates must be used in practice. The formula is:

\[
G = y + p(1 - x)
\]

and may be rewritten as an expression for dividend yield

\[
y = G - p + px.
\]

It thus bears some resemblance to the multiple regression formula used in the paper. In a near-perfect market of large and equally stable companies \( G \) should lie within close limits, since selection of the best long-term investment would, as expectations changed, bring prices constantly back into line.

However, since \( x \) and \( p \) are independent variables the expression is not linear, nor is an equation relating logarithms of the factors linear, since \( G - p \) will only be zero in exceptional cases (because \( p \) is independent of the market while \( G \) varies with price).

If the shares are plotted in a three dimensional diagram \((y, x, p)\) they should tend to lie on surfaces of a hyperbolic form (for different values of \( G \)), with their relative cheapness or dearness simply determined by the value of \( G \). Higher degrees of risk can be taken into account by requiring higher values of \( G \).

Since the authors have used \( y \) and \( x \) (slightly different definitions) as their main variables it seems likely that their scatter diagram will be somewhat similar and for the reasons given above a linear relationship is not likely to be completely suitable.

In practice, \( p \) may approximately equal \( G \) in value, on the average, and quite a good fit should be obtained in the logarithmic form over a normal range of \( y \) and \( x \), but I think it would be wrong to conclude that shares can then be ranked, as the authors have done, by applying average coefficients to individual shares. Since the authors' constant term \( a_0 \) may well largely represent an average log \( p \), variations in \( a_0 \) for individual shares may well be justified.
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This means, as I see it, that although the system is in practice successful to a degree, the individual share rankings may in fact be quite a long way from the 'true ranking' as seen by an all-knowing long-term investor with a complete insight into the probabilities attaching to all possible future dividends.

The authors subsequently wrote as follows:

One of the most important questions raised in the discussion was the length of view taken by the method in the ranking of shares. We would agree with both Mr Bishop and Mr Moody that one year was probably too short. The point was discussed in § 45 of the paper and it has to be admitted that the assumption that dividend yield anomalies would be corrected over the chosen period may be far from the truth. The method suggested by Mr Beech for investigating the efficiency of the system would also shed some light on this point. We found Mr Beech's approach interesting and we have ourselves planned an investigation into the relative performance of shares in the different categories according to the period from the initial selection. In addition, the programme will calculate the standard deviation of the performance figures in each category and carry out the appropriate analysis of variance to test whether or not the mean performances for each category is significantly different. There is, however, one point raised by Mr Beech with which we do not agree, namely that the shares in the computer ranking list should be redistributed according to any particular '100' Companies distribution in order to give a fairer basis for comparison between the two methods. The fact that we are only attempting to compare shares, one with another, must mean that any system that at times places only 2% of the shares in category one is unsatisfactory as a selection mechanism (in other words, it cannot always be relied upon to sort the shares).

With regard to the model used, several speakers, including Mr Grant and Mr Lewin, queried the stability of the regression coefficients over time and raised the question of possible changes which might occur should we enter a real 'bull' market. Our experience was limited to around three years during which no noticeable trends in the value of the coefficients emerged. Prof. Moore suggested the inclusion of an industry factor in the model and, whilst this would undoubtedly improve the statistical fit of the model to the data, the assumption of a maintained average status for any industry would, on the other hand, strike at the roots of the evaluation of ordinary shares on the basis of a prospective valuation of dividends and earnings. Experienced investors must be only too well aware of the loss in status experienced in the shares of some industries (computer manufacturing, to take a topical example) and as Mr Moody pointed out, it is not possible to take market prices as a reliable indicator of values.

Both Mr Moody and Mr Grant raised the question of the accuracy of the forecasts and we have already given an indication of their accuracy for 1966. Mr Grant also asked how critical these forecasts were in the individual rankings and both he and Mr Hey suggested that a technique which was based more on known information and less on forecasts would be more satisfactory. We find this latter point difficult to accept, as forecasts of future earnings and dividends are fundamental to share evaluation. Moreover, the less the subjective element the more would assessments by different advisers tend to agree, leading to the difficulties mentioned by Mr Ginsburg and others.

While on the subject of forecasts, it is interesting to note that if, for example, one forecast the next earnings figure for the company correctly but the directors cut the dividend by more than expected (to conserve liquid resources), the resulting fall in dividend yield would in part be offset by the lower payout ratio. In addition, the long-term dividend growth-rate would probably be raised to allow for an element of recovery and so the
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net effect of the computer ranking for the company would quite possibly be small. This seems to be a logical result and demonstrates that it is important to forecast the trend of earnings correctly, since the effect of arbitrary adjustment to dividends is reduced by the system.

Mr Grant raised an interesting point when he asked whether there was a persistent tendency for some stocks to stand well above or well below their norm. We had already met this question from our colleagues, the examples of Hawker Siddeley and Marks and Spencer being quoted to support Mr Grant's point. It is, of course, always easy to be over-influenced by the experience of individual stocks and to fail to perceive the broad overall pattern. The following table sets out the mean ranking of shares at the end of periods of one, two and three years after initial selection. In addition, we show the percentage of shares in each initial category that remained unchanged over the periods covered.

<table>
<thead>
<tr>
<th>Initial category</th>
<th>Mean category at end of period</th>
<th>Percentage of companies still in initial category at end of period</th>
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<td>Years after initial selection</td>
<td>Years after initial selection</td>
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<td>36·4 66·7 14·3</td>
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<td>2·6 3·4 2·5</td>
<td>40·7 14·3 33·3</td>
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<td>2·8 2·8 2·9</td>
<td>39·0 23·5 41·9</td>
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<td>3·6 3·4 3·3</td>
<td>23·3 18·8 37·5</td>
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<td>3·9 3·6 4·3</td>
<td>35·7 28·6 71·4</td>
</tr>
</tbody>
</table>

The table shows that there is only a slight tendency for stocks to remain either well above or well below the norm and the reason for this tendency is probably associated with the fact that the system appears to prefer high yielders, a point commented on by both Mr Bishop and Mr Ginsburg. However, there are many investors who would support the computer in its preference. Why there should be this difference of emphasis between the system and market is not easy to answer but subjective judgment of shares could well associate high yield with risk and low yield with growth, without giving adequate consideration to the underlying factors. By separating forecasting from evaluation the computer avoids this pitfall and, given two shares with the same growth prospects, quite reasonably prefers the higher yield.

Mr Ginsburg also raised the question of the attractiveness of other types of investments vis-à-vis ordinary shares and here the authors entirely agree with his comments and are looking at this problem in some detail. Mr Ginsburg, however, made a criticism of the ranking system when he asked how long their top twenty shares would remain relatively cheap when investors were made instantaneously aware that they were cheap and by what margin. The implication appears to be that it is wrong for an investment adviser to have a system and this we find difficult to accept. In fact, of course, there is the additional point that the shares are only attractive to an investor if he believes the forecasts made for the company in question, relative to industry as a whole. It is also pertinent to add that the rather extraordinary variation in share prices seen in the short term (which rather suggests that, contrary to views expressed in the discussion, investors take a very short view in evaluating shares) might be ironed out to some extent if more investors used some rational system for choosing shares. Is it reasonable, for example, for the price of B.M.C. to vary between 100% and 200% of the asset value over the cycle
of motor car production? On the subject of share price variation, Mr Grant mentioned the question of random errors. He had carried out some calculations based on the total sum of squares of yields shown in the table in § 26 and he invited our comments. In fact as we pointed out in our introductory remarks, the analysis was in terms of the logarithms to base $e$ of the yields and on this basis the sum of squares of 5.044 produces a variance of 0.0376. This leads to a standard deviation of around 21½% for actual yields whilst the residual standard deviation is 14%.

Both Prof. Moore and Mr Hemsted (in his written comment) have referred in some detail to the statistical aspects of the computer evaluation model. Some of Prof. Moore's points have already been referred to but his suggestion for picking out the best combination of predictor variables is an interesting one although it has perhaps objections on a priori considerations that we are setting up a model which contains factors that most people would almost certainly agree would influence the status of a share. Indeed, contributors to the discussion suggested that many more variables should be included and the existence of these latter variables may in fact reduce the statistical significance of those factors chosen.

As far as Mr Hemsted's contribution is concerned, he makes the point, working from theoretical considerations, that the assumption of a linear relation between the logarithms of the input factors is unlikely to hold in general. We would accept this criticism, but we emphasize that our method has a different approach in that, whilst introducing factors that we would expect to have an effect on yields, we have not attempted to derive the form of the distribution from theoretical considerations. Common sense would suggest, as we have said, that a logarithmic model is more likely to work than a simple linear one, and it is possible that by introducing powers of the various input variables and interactions terms, a better statistical fit might be obtained. Mr Hemsted's method appeals on the ground of its theoretical appropriateness but in fact, since both the form and the parameters of the distribution of $p$ are unknown, its practical use seems limited. However, a similar approach has been described by Mr J. R. Marshall in a recent Faculty paper and we would be interested to see evidence of benefits to be gained in the use of this system.

In conclusion, we wish to express our thanks for the way in which the paper was received and regret that we cannot at this stage follow up all the interesting points raised in the discussion. There is clearly a very wide interest in applying more scientific methods to share evaluation now that the computer can take out so much of the drudgery of the arithmetical work. The body of data that is being built up by attempting to evaluate shares in an organized way will enable other investigations to be made, giving experience in the use of other statistical techniques and perhaps shedding more light on a fascinating problem.