THE FINANCIAL TIMES-ACTUARIES FIXED INTEREST INDICES

by


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1. Introduction

1.1. When the F.T.-Actuaries series of indices were first compiled in 1962 the comprehensive representation of ordinary shares quoted on the London Stock Exchange was supplemented by fixed interest yields and price indices.

1.2. The information presented on a daily basis was:

(i) Yield on 2½% Consols;
(ii) Yield on and price index for 20-year British government stocks;
(iii) Yield on and price index for 20-year debenture and loan stocks;
(iv) Yield on and price index for investment trust preference shares;
(v) Yield on and price index for commercial and industrial preference shares.

1.3. The yield calculated for a 20-year British government stock was derived from a weighted average of the mean gross redemption yields of three pairs of stocks, each pair selected from consecutive five-year periods. The price index was derived from the yield by entering bond tables.

1.4. The debenture and unsecured loan stock yield and price index were derived in a similar manner from 1965 onwards, although originally only a directly derived price index was published.

1.5. The single yield and price index presentation was considered in 1964 by Haycocks & Plymen (T.F.A. vol. 28, p. 377, J.I.A. vol. 90, p. 267) to give only a very limited picture of the gilt-edged scene and they questioned "whether the gilt service should not be extended to include a calculation of a yield curve with publication of redemption yields at, say, quinquennial intervals throughout the list".

1.6. The movement in interest rates and the many and varied shapes taken by the gilt yield curve since the original design of the index have detracted from the usefulness of a single figure to represent the gilt market. There have also been theoretical and practical problems associated with the particular 20-year stock indices, which are discussed further below.

1.7. Accordingly much thought has been devoted in recent years to more refined computational techniques and also to how extensive the daily presentation in the Financial Times should be. Computer application could, and clearly should, consider every quoted British government stock in determining the gilt-edged picture. In order to ascertain the opinions of interested parties a memorandum on the proposed indices was prepared by the Joint Index and Classification Committee of the Institute and the Faculty of Actuaries in July 1975 and circulated to professional investors inviting constructive comment. This included a copy of a proposed layout. Reception was at that time highly favourable and accordingly work proceeded with extensive testing. The final layout has been the subject of modest changes.
2. The Original 20-Year Government Stock Index

2.1. The original yield index was a weighted average of the gross redemption yields on six stocks. The stocks were changed periodically, and the weights given to their yields were changed continuously in such a way as to keep the average term to maturity at 20 years. The six stocks used when the index was stopped on 13th May 1977 were:

- 5¾% Funding Loan 1987/91
- 3% British Gas Guaranteed Stock 1990/95
- 8½% Treasury Loan 1997
- 6¾% Treasury Loan 1995/98
- 9½% Treasury Loan 1999
- 8% Treasury Loan 2002/06.

The price index was based on the price at which a 20-year stock of given coupon (finally 6%) would stand to yield the figure given by the yield index.

2.2. On the face of it the yield index met the objective of providing a historical record of yields, at least for one point of the yield structure. Closer examination, however, revealed that it did so in a manner which could be, and probably was, misleading for two reasons:

(i) For the same term and conditions a high coupon stock normally stands on a higher gross redemption yield than a low coupon stock, because the stocks appeal to groups of investors with different tax statuses. The yield on the index therefore depended on the choice of stocks in its composition and the history of the yields could therefore have been different had different stocks been chosen in the past. It is clear that the final set of six stocks was more heavily weighted with low coupon stocks than was the market as a whole, and so the index yield had become rather too low compared with what would have been the case had, for example, all stocks in issue of terms between about 14 and 26 years been used.

(ii) With each day that passed the weight accorded to the higher coupon stocks in the index was increased. Other things being equal, therefore, the index yield would have increased accordingly, giving an entirely spurious indication that yields were rising.

2.3. It can be seen that there was no easy remedy to either of these objections. To include other stocks of higher coupon would have produced either an abrupt or a gradual increase in the index yield which would have destroyed continuity with the past index. And indeed there is a conflict of objectives which an index constructed in the way the original one was cannot satisfy: if it is to represent the history of a particular group of stocks of a particular coupon level then it cannot represent the current position of a market in which that particular group of stocks is no longer typical, and, contrariwise, if it is to represent the current position of the market then it must also reflect the drift in yields consequent on a change in the level of coupon typically issued.

2.4. The price index was derived from the yield index, the coupon and the term being held constant. As well as the defects that stemmed from those of the yield index it had one of its own. If the yield is constant the price is constant. In conditions of constant yield, however, the experience of gilt-edged investors will be affected by maturity drift as prices move with time towards the redemption price. The index did not therefore provide a standard against which portfolio performance could be satisfactorily measured.
2.5. A further factor which caused difficulty in the construction of the former index was that each group of stocks introduced into the formula, from which yields are derived, was selected from stocks available in the market with final redemption dates lying in the 25 to 30 year bracket. These remained in the formula for 15 years until, with the passage of time, the weight once again became zero. In recent years, however, the new, high coupon, stocks issued by the government have not had original terms as long as 25 years, so have been ineligible for inclusion in the index. This feature, which affected the gilt index, caused even more acute problems in the effort to maintain a yield and price index that was representative in the debenture and loan stock market.

2.6. It was therefore clear to the Joint Index and Classification Committee of the Institute and Faculty of Actuaries that the 20-year government stocks index should be replaced by two new sets of indices, one providing a current and historical description of the market by showing only redemption yields, the other providing a standard for performance measurement by showing the current price of the gilt-edged market together with data to allow overall rates of return to be calculated.

3. The New Yield Indices

3.1. A table of ten yields is quoted daily, with values of gross redemption yields for 5, 15 and 25 year terms, for each of three coupon bands, consisting of low, medium and high coupon stocks, together with a yield for irredeemable stocks. Redeemable stocks are distributed among the bands so that approximately one-third of the stocks, by number, is included in each band; as new stocks are issued and old ones reach maturity, so the constituents of the bands are changed to maintain this one-third split. The use of these bands reflects the fact that stocks of differing coupons stand at differing redemption yields, since they appeal to different groups of investors with different tax statuses.

3.2. The yields are calculated by fitting a curve to the redemption yields for all stocks in the coupon band. Redemption yields are calculated taking accrued interest as part of the price and using a true compound interest formula, i.e. finding the value of \( v \) to give \( f(v) = 0 \) where

\[
f(v) = v^n \left( C_1 + C \frac{(1-v^n)\nu}{1-v} + R \nu^n \right) - P - \sum_i B_i \cdot \nu^p
\]

and \( v \) is the discounting factor per period (e.g. half-year)

\( R \) is the redemption amount

\( C \) is the coupon amount per period

\( C_1 \) is the actual coupon due at the next payment date (which may be zero if the stock is already quoted "ex dividend"), or may be a first fractional payment

\( n \) is the integral number of periods till redemption from the next payment date

\( p \) is the fractional period till the next payment date

\( P \) is the price actually payable (with "accrued interest" not "stripped out", but, for shorts, added in)

\( B_1, B_2, \text{etc.} \) are outstanding calls on a partly-paid stock

\( b_1, b_2, \text{etc.} \) are the fractional periods till these calls are due.
When the root of $f(v)$ has been found the gross yield, $y$, convertible half-yearly, is obtained from

$$y = 200\left(1/v^{k/2} - 1\right) \text{ per cent}$$

where $k$ is the frequency of coupon payment per year.

Where the stock has a range of optional redemption dates the earliest or latest is used, whichever gives the lower redemption yield.

3.3. The curves are fitted to each band independently, using a formula:

$$\text{Yield}(t) = A + B \exp(-Ct) + D \exp(-Ft)$$

where $t$ is the term to redemption.

This formula was adopted after extensive experimentation. It allows for one or no turning points, and the curve tends towards an irredeemable yield $A$. The parameters are chosen by minimising the weighted sum of squares of the difference between the curve yield and the stock yield. The weights are total market capitalisation of the stock. All the irredeemables are included in each coupon band; this gives stability to the long ends of the low and medium curves. The Convertible, sinking fund stocks, variable interest stocks, any other "unusual" stocks, and also stocks under one year to maturity are omitted. A 5-year yield will not be quoted unless there is at least one stock in the coupon band with a term of less than 5 years.

3.4. No corresponding prices are calculated from these yields. It should be noted that they are not equivalent to the yields quoted in the Bank of England Quarterly Bulletin, which are "par" yields, i.e. the coupon that would be consistent with a price of 100. In certain conditions, such as occurred in 1974, the par yields might be higher than any investor could actually obtain. Further, the proposed yields take no account explicitly of tax. To be consistent, the lower coupon stocks should perhaps be dealt with on a net redemption yield, but no assessment is made of what rate of tax is appropriate. A more complete structure might be to fit a total surface to yields on a two-dimensional frame, using term and coupon as independent variables, yield as the dependent. This could perhaps have been studied further, but the present proposals appear to be simple and to give adequate results.

4. The New Price Indices

4.1. The whole gilt-edged market (except the variable interest stocks) is included in the main "all stocks" index and four sector indices are also produced for stocks in the under 5 years, 5 to 15 years, over 15 years redeemable and irredeemable groups. The grouping depends strictly on redemption date, and stocks with a spread of redemption dates slide back and forth between sectors as the price rises above or falls below "par", implying a shift from the latest to the earliest date or vice versa.

4.2. The daily price indices are arithmetically weighted gross prices (with accrued interest added back for the shorts), weighted by market capitalisation. The index is "chain-linked" for new issues and redemptions as they occur and to allow for the movement of stocks between groups. The method of calculation is essentially the same as for the ordinary share indices, and represents the actual price performance for a holder of a uniform percentage of all government stock issues. Further details of the calculation methods are given in §6.

4.3. Issues with sinking funds which are redeemed in the market are
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4.4. Since the prices at present show an upwards drift towards maturity these price indices go steadily upwards if there is no change in the level of yields. The necessary complementary figure of "quantity of interest received" is also published for each sector and for all stocks. On each ex-dividend date the amount of interest for the stock concerned is calculated, and the total for that day is shown, as "xd adj". The cumulative total of such interest for each calendar year to date is also shown.

4.5. From the price and interest figures it is thus possible for users to construct "rates of return", using whatever tax rate they choose, for whatever period they choose. These are total rates of return, including income. A comparison of the price indices alone is inappropriate except for someone who pays 100% tax on income—or for the reversionary beneficiary of a trust fund when someone else gets the income.

5. The Debenture and Preference Indices

5.1. For debentures and unsecured loan stocks there is one price index, constructed in the same way as for British government stocks, and three yields, being yields at 5, 15 and 25 years on a single yield curve, also constructed in the same way as for British government stocks. The principles on which stocks are chosen to be included in the indices are that most stocks of a sufficient size and of a suitably normal type are included in the price index, regardless of "quality", but that, of these, only the stocks with sufficient interest cover should be included in the construction of the yield curve.

5.2. The qualification rules for debentures and loan stocks for inclusion in the price index are:

(i) Outstanding nominal issue not less than £11 million.
(ii) The company registered in the United Kingdom.
(iii) The principal quotation for the stock on the London Stock Exchange.
(iv) Stocks with options such as "renderable at par for exercising warrants" are excluded.
(v) Stocks where a sinking fund exists intended at the date of issue to redeem more than one third of the issue before maturity are excluded.

And for inclusion in the yield index:

(vi) Interest to be covered by the profits shown in previous two years' published accounts. (This cover requirement applies also to new issues.)
(vii) Coupon not unusually low (4% or less at present).

Some 56 stocks meet the price index rules and of these 47 also meet the yield index rules.

5.3. For preference stocks there is also a price index, on the same basis as the others, and a single yield index, being the weighted mean yield on all the (irredeemable) preference stocks. In view of the limited range of preference stocks now in existence, and the limited market, the commercial and industrial and the investment trust preference indices have been amalgamated into a single category. As for the debentures, most stocks that meet certain qualification rules are included in the price index, and among these, those with sufficient dividend cover are included in the yield index.
5.4. The qualification rules for preference stocks are:

(i) Outstanding nominal issue not less than £3 million.
(ii) The company registered in the United Kingdom.
(iii) The principal quotation for the stock on the London Stock Exchange.
(iv) Normal rights, i.e. cumulative, not redeemable, not convertible, not participating, and with "normal" voting rights (i.e. not in all circumstances).
(v) Dividends paid to date.
(vi) Dividend cover not less than 3 times, calculated on a priority percentage basis.

Some 39 preference stocks meet the price index qualifications, of which 37 are also eligible for the yield index.

6. Construction of the Price Indices

6.1. In principle the price indices are constructed in the same way as the F.T.-Actuaries share indices. If there were no changes in the constituents the index value on any day would be:

\[
\frac{\text{Total market value of stocks on that day}}{\text{Total market value of stocks on base date}/100}
\]

and the quantum of interest received (xd adjustment) on any day would be:

\[
\frac{\text{Total amount of interest payable in respect of stocks that have gone "ex div" on that day}}{\text{Total market value of stocks on base date}/100}
\]

6.2. In practice there are changes in the constituents of different kinds from time to time and the denominator of the above expressions (the "base values") must be adjusted so as to "chain-link" the indices appropriately.

6.3. The types of change for the British government securities indices are in some respects more varied than for the share indices, and since the intention is to represent the market comprehensively they must be taken into account correctly. The possible changes are:

(i) New issues, brought in on the day of issue at the issue price (the initial payment for partly paid stocks).
(ii) Calls, i.e. the amount of a further call on a partly-paid stock, brought in on the due date.
(iii) Amalgamations of further tranches of an existing stock (A or B stocks), which take place normally on the first ex-dividend date, after which the stocks are identical.
(iv) Alterations of nominal amount outstanding (e.g. for stocks with a sinking fund), brought in from time to time as the information is available.
(v) Redemptions, taken out on the day the books close, about three weeks before the redemption date, at the latest quoted price.
(vi) Late entrants, such as the partly-paid stocks being brought in when fully paid, brought in at the quoted price.

6.4. The above changes can apply to any stocks in any of the price indices. In addition a stock may be transferred from one of the sectors to another ("up to 5 years", "5 to 15 years", "over 15 years") depending on its redemption date. The possible further changes are:

(i) "Shorteners", whereby a stock moves from one sector to a shorter sector as its redemption date becomes closer; this occurs on the day the stock has exactly 5 or 15 years to go, at the quoted price.
"Sliders", whereby a stock that has a spread of redemption dates moves from one sector to another (either shorter or longer) as its price moves above or below "par", thus implying the earlier or later redemption date; the criterion used is strictly whether the quoted price is above or below the "equilibrium price", i.e., the price at which the redemption yields to the earliest and latest redemption dates are equal, which is given by:

\[
equilibrium \text{ Price } = \left( \frac{R}{R+C} \right)^p (R+C_1) - \sum_i B_i \left( \frac{R}{R+C} \right)^{b_i}
\]

with the same definitions as in §3.2. For stocks where the earliest possible redemption date has already passed, the next due date is taken as the presumed earliest redemption date.

Sliders are moved from one sector to another on the day the price moves above or below the equilibrium price, and are moved at the equilibrium price; thus, when the quoted price goes up, part of the gain is attributed to the old sector, part to the new, and vice versa when the quoted price falls; neither sector gains at the expense of the other.

6.5. On any one day there may be up to three adjustments to the "base value", i.e., the adjusted denominator of the expressions in §6.1:

(i) Redemptions are taken out at "yesterday's" prices. Thus the base value is multiplied by (Total market value yesterday of all stocks except redemptions) and divided by (Total market value yesterday of all stocks).

(ii) "Sliders" are transferred at equilibrium prices. The adjusted base value is multiplied by (Total market value at today's prices of yesterday's stocks plus sliders-in at equilibrium price) and divided by (Total market value at today's prices of yesterday's stocks, minus sliders-out at equilibrium price).

(iii) All remaining changes are allowed for at today's quoted prices. Thus the third adjustment factor is (Total market value at today's prices of stocks after changes, i.e., plus new issues, calls, etc.) divided by (Total market value at today's prices of stocks before changes).

"Today's price" for a stock where a call amount is due today is taken as "quoted price—call amount" before change, and "quoted price" after change.

Finally an amount of interest received is calculated, based on the interest amount due shortly on any stock which has gone "ex-dividend" today for the stock as it was in force yesterday. This is divided by the "base value" brought forward from yesterday, before adjustment for changes, to give the "interest received" index figure. The cumulative "interest received" index figure is simply the sum of these amounts.

7. The Purpose and Use of Fixed Interest Indices

7.1. Before describing what the Joint Index and Classification Committee considers are the possible uses of fixed interest indices it is necessary to state certain assumptions, which, although perhaps obvious, nevertheless are the justification for the indices. It is assumed that whereas redemption yields on fixed interest stocks at any point in time may vary by term to redemption, coupon, security and other conditions such as tax status, they do so in a consistent and reasonably uniform manner. That is, stocks with
identical term, coupon, security and conditions will stand at prices such that they carry identical or almost identical redemption yields, and stocks whose term, coupon, etc. are only slightly different will carry only slightly different redemption yields. It is thus possible to discuss the structure of yields in the market. It is assumed further that British government stocks have the same "gilt-edged" security, and that their other conditions are sufficiently similar for the structure of their yields to depend almost solely on coupon rate and term to redemption.

7.2. The first objective of fixed interest indices is to provide a description of the level and possibly also the shape of the yield structure at any one time. The redemption yield for the 20-year government stocks index indeed attempted to do the former, by providing one average yield for a fixed term to redemption of 20 years. Immediately consequent on the first objective is that of providing a historical record of the yield level and yield structure, or at least those characteristics of the yield structure that have been determined.

7.3. In reality an investor will hold one or more specific stocks, and does not in practice hold a specific cross-section of the yield structure. The term of actual stocks reduces as time passes, and their prices tend towards the redemption price. Any change in the capital values of a portfolio includes some portion of the redemption yield (unless stocks are always at par), and the whole concept of a redemption yield implies an equivalence between income and increase in capital. A second major objective of fixed interest indices is therefore to provide a standard against which the "portfolio performance" of investors in the fixed interest market can be measured.

7.4. There are many other possible uses for the fixed interest yield and price indices and some of them are listed below.

Uses for yield indices:

(i) Description of the level and shape of the yield structure and of how this changes with time.
(ii) Comparisons of yield on British government securities with yield on ordinary shares as a measure of the "yield gap", "yield ratio" or the "level of confidence".
(iii) Assist choice of the appropriate yield, and hence price, for dealing in quoted fixed interest securities.
(iv) Assist choice of the appropriate yield, and hence price, for valuing or dealing in unquoted fixed interest securities.
(v) Assistance in fixing the terms of new issues.
(vi) Guide for the actuary of a life assurance company for determination of premium rates etc.
(vii) Guide for actuaries (in particular for the Government Actuary) in various pension scheme calculations.
(viii) Study of variations in the relationship between low, medium and high coupon stocks, and between short, medium and long dated stocks.

Uses for price indices:

(i) Standard against which "portfolio performance" of investors in fixed interest securities can be measured.
(ii) Approximate valuation of fixed interest portfolio.
(iii) Determination of terms of entry into and exit from an "internal fund" deemed to hold fixed interest securities but to which no specific assets are hypothecated.
7.5. When using the indices for any of the above purposes careful and precise formulation of the question to be answered will greatly reduce the likelihood of misleading "assistance" or invalid "comparisons". This general warning about care perhaps applies particularly in the third, fourth and fifth uses suggested for the yield indices.

7.6. However, more specific guidance on the use of the price indices in portfolio performance computations is appropriate. The basic requirement of any system operated is that the standard with which the portfolio performance is compared is appropriate to the user's investment strategy (which should depend on the nature and length of the fund's liabilities).

7.7. The user, bearing this in mind, can select his own "standard" adopting, if he wishes, a mix of the short, medium, long and irredeemable indices.

It is also desirable that the system should:

(i) Allow properly for the high rate of turnover resulting from active management.
(ii) Permit interim assessment readily.
(iii) Permit income and capital to be traced separately (to allow study of taxation effects).

7.8. It is widely accepted that a money-weighted rate of return is not, other than in a closed fund, a statistic which can fairly be compared with the rate of return on an index. The reason for this is that the money flows to and from a fund are an influence usually outside the control of the portfolio manager, and particularly in volatile markets will tend to invalidate comparisons with the index (which has no cash flows). Two methods of removing this distortion are available: the use of time-weighted rates of return or the construction of a standard portfolio.

7.9. Under the first method, the portfolio must be valued on every injection and withdrawal of funds and the rate of return since the last injection or withdrawal calculated. The quotient of the return factors \((1 + \text{period rate of return})\) of the portfolio and the index (or mix of indices) gives the performance ratio for the period.

7.10. Under the second method, the value of the actual portfolio at the opening of the study is deemed to be invested in the index (or indices, in allotted proportions), the index level being regarded as the price of £100 nominal of stock, while the income for the standard portfolio is derived from the "xd adjustments", each daily adjustment being the income per £100 nominal of such stock for the day. On each injection or withdrawal of money, stock of equal value is injected into or withdrawn from the standard portfolio. A valuation is performed only when the performance ratio (which is the quotient of the values of the actual and standard portfolios, adjusted for income) is required.

7.11. Under both methods, care must be taken to ensure that the treatment of income as to deemed time of receipt, deemed reinvestment and tax is the same for both the actual portfolio and the index or standard portfolio. As the sums involved in purchases and sales of British government securities will usually be much higher in relation to the total fund than would occur if an equity portfolio were under consideration, the accuracy of the method used in adjusting the standard fund is especially important. If a very bearish view is taken of the market switching shorter might take place. If securities are sold for cash does one regard this as "ultra-short" and therefore still in the market? The answer must of course depend on the precise function of the work being carried out but in many cases it will be correct to assume that the money is invested ultra-short.
7.12. In terms of ease of operation, the time-weighted return method will probably be preferable for funds with few injections or withdrawals. The standard portfolio method will probably be easier to operate for active funds, and is more flexible in that the effects of different tax and dividend assumptions may be more readily derived.

7.13. However, there is also a basic difference in principle between the methods. The standard portfolio method is an adjusted money-weighted method; moderately good performance when the portfolio is large can compensate for markedly bad performance when the portfolio is small. The time-weighted return is unaffected by the size of the portfolio and spotlights the relative performance of the portfolio manager rather than of the portfolio itself. Nevertheless in practice unless the portfolio changes markedly in size over the period of review either method will be a good guide on both matters.

8. Experience

8.1. The base date for the British Government Securities Indices was 31st December 1975. The indices were first published on 16th May 1977, but data for the period from commencement to publication is available from the Financial Times. Notes on the indices during 1976 and 1977 follow this paper.

8.2. During 1977 there have been two complications not foreseen when the index was first designed: the government has issued variable interest rate stocks, and has also issued stocks where the issue price is payable in separate “calls” on a series of dates. It was clear that the variable interest stocks were of different kind and should be omitted from the index entirely (as would stocks whose coupon and redemption amounts were altered in accordance with e.g. the retail price index). But the partly-paid stocks could be included on a wholly correct basis—once the computer programs were altered to allow for them! This could not be done in time between the announcement of the first such stock and its date of issue (less than a week), so the published indices include these stocks only when they are fully paid. A correct series of indices has also been constructed; the historic data, when published, will contain the correct series; and at a suitable date the published indices will be adjusted to bring them into line.

8.3. The effects of including the partly-paid stocks are:

(i) To alter the yield indices slightly in either direction while they are in the partly-paid period, but to have no long-lasting effect.

(ii) To make the price index more volatile during the partly-paid period, with a possible permanent shift in the index by the end of the period.

(iii) To increase the value which is used as a denominator for interest received, and hence to reduce the quantum of interest received during the partly-paid period and also permanently.

None of these adjustments is of great significance except where a very precise portfolio performance comparison is being made.

8.4. The debenture index has a base value of 100.00 at 31st December 1976: the preference index commenced at a value of 70.25 on 30th June 1977, which was the value of the former commercial and industrial preference index on that date. The preference yield is derived from the actual rate of dividend grossed up at the current rate of A.C.T., but the preference “xd adj” is based on the quantum of actual dividend.
9. Conclusion

9.1. The practical experience of users of all the indices may indicate improvements that could conveniently be made, either in their construction or their presentation. The Joint Index and Classification Committee will keep the indices under review, and is willing to consider carefully any suggestions that may be put to it.

ADDENDUM

10.1. In the course of performing the calculations for the British Government Securities yield indices certain apparent anomalies have been observed. These result from the formula that has been used for the yield curve:

\[ y(t) = A + B \exp(-Ct) + D \exp(-Ft). \]

As noted in §3.3 this curve has either one or no turning points, and tends towards an irredeemable yield \( A \). It is sufficiently flexible to be able to represent the yield curve successfully in a variety of circumstances. However, it is possible for two fairly similar curves, with different values of the parameters, to represent the data equally well.

10.2. This can be seen from the method of fitting the curve, which is to minimise

\[ S = \sum_i w_i(y_i - A - B \exp(-Ct_i) - D \exp(-Ft_i))^2 \]

where \( w_i \) is the market value of stock \( i \), \( y_i \) is its redemption yield and \( t_i \) is its term to redemption. Since this is linear in \( A, B \) and \( D \) we can calculate directly the values of \( A, B \) and \( D \) that minimise \( S \) for any given combination of \( C \) and \( F \). But there may be, indeed commonly are, more than one combination of \( C \) and \( F \) that give local minima of \( S \), when we treat \( S \) as a function of \( C \) and \( F \). The method currently being used to find a minimum of \( S \) is in fact the Nelder–Mead simplex method (Nelder, J. A. and Mead, R. (1965); A simplex method for function minimization: *Computer Journal*, 7, 308–313 and O’Neill, R. (1971); Algorithm AS47: Function minimization using a simplex procedure: *J.R.S.S. (Series C)*, 20, 338–345), but any non-linear minimization method would produce a similar answer. The starting point for the search is given by the values of \( C \) and \( F \) for the previous day’s optimum curve, which normally ensures that the curves, and hence the quoted yields, are similar on successive days. On occasion, however, the “hollow” in which a minimum has previously been found fills in, and the search procedure moves to an alternative minimum, which at this stage always produces a curve with an improved fit, but which may produce an apparent discontinuity in the quoted yields.

10.3 The solution to this problem is not obvious, and the Committee is giving urgent consideration to it. To choose the lowest of several local minima might produce an alternation between two of them if they had nearly equal values of \( S \); but in any case it is not practicable to search every day the whole \( C\)–\( F \)-plane for possible new minima. It has been the case over 1976 and 1977 that a minimum is often found near \( C = 0.06, F = 0.061 \), say, and a “probe” in that neighbourhood every day is practicable. The Committee is reluctant to abandon a curve that gives generally a very satisfactory fit, but an equally well fitting curve that had a unique minimum would be welcome.