JSS 21 (2) (1975) 206-245

THE DANISH LIFE ASSURANCE MARKET

by

P. G. A. CAMMIDGE

(*A paper discussed by the Society on* 4 *February* 1975.)

1. MAIN POINTS

'There must be much you can learn from us—and many things we can learn from you.'

Danish Actuary

1.1. HISTORICALLY, as today, the Danish insurance community has been subjected to a far greater degree of official supervision than their U.K. colleagues are accustomed to. Insurance can only be written if the Forsikringsraad (Insurance Supervisory Board) recommends that the applicant office be granted a concession to do so. In particular, the Forsikringsraad will have to be satisfied that the office's premium rates are adequate. At a time when the U.K. is moving towards closer supervision, it is particularly interesting and instructive to examine a functioning supervisory system. (Section 6 of the present paper deals with this aspect of insurance in Denmark.)

1.2. Danish life offices operate a tariff, all charging the same scale of premiums for any given class of business. The offices compete in their bonus systems and service to policyholders. Non-profit classes are not written, except immediate annuities and annuities with a short deferral period. Valuations are made on the current premium bases, which employ $4\frac{1}{2}\%$ interest for assurances and $4\frac{1}{4}\%$ for pension contracts. A net premium method is employed, the trend being towards valuing each policy individually using computers. Surplus arises mainly from excess interest earnings.

Editorial footnote

A Danish actuary who attended the meeting at which this paper was discussed has written to say that mention might be made of the system of selling life assurance in Denmark. He advises that all sales take place through each insurance company's own agents, there being an agreement between companies preventing an agent from writing insurance business for more than one company. There is no equivalent to the English insurance broker. He also felt that we should stress that the bonus system constitutes the only difference between individual life assurance companies and is therefore the most important element in competition between the companies.

1.3. Legal constraints on offices' investments are severe. Of assets held against life and pension liabilities, 85% must be in ultra-safe fixed interest securities (some allowance being made within this percentage for property). Those investments with certain redemption are entered in balance-sheets at book value (if below redemption price), no allowance having to be made for market values being currently much lower. Employed in conjunction with the net premium method of valuation, this procedure leads to a more equitable emergence of surplus at the expense of balance-sheet strength. A consequence of the existence of a supervisory authority is that the offices do not have to publicly demonstrate impregnable strength, rather it is for the Forsikringsraad to satisfy itself on this point.

1.4. Denmark is the home of a remarkable form of inflation-proof annuity policy, the 'indexcontracts'. These are written as conventional policies by offices, the inflation-proofing being added by the state. The issue of new contracts was, however, discontinued when the government belatedly realized how colossal the costs of its guarantees would become should present inflation continue. (Details are to be found in 8.2.)

2. INTRODUCTION AND ACKNOWLEDGEMENTS

'What need have I for life insurance? Here in Denmark we have a welfare state.'
Danish student

2.1. *Introduction*

2.1.1. Denmark provides the world with an outstanding example of how a country with few natural resources can nevertheless provide her people with an extremely high standard of living. Simultaneously, one of the most advanced welfare states ensures that all shall share in prosperity. The student quoted at the head of this section was expressing a common viewpoint that the welfare state should shield citizens from (the financial consequences of) life's ups and downs. Had the present author sufficient knowledge at the time, he could have pointed out areas in which deficiencies in state provision render private insurance essential. The reply would surely have been that the welfare state must be improved in the areas mentioned.

2.1.2. For the actuary, Denmark is a fascinating study of how private assurance can survive—and prosper—alongside an all-embracing welfare state. At the same time, we may examine what policyholders and insurers living in less regulated environments

may expect to come to pass if the present trend towards closer supervision continues.

2.1.3. Unfortunately, we cannot isolate these important aspects of Danish assurance for detailed observation. Both insurers and the welfare state are seriously threatened by a rising tide of inflation; the latter also by a taxpayers' revolt. Scandinavia serves as a social laboratory wherein developments overhanging the larger and less happy world outside occur first under controlled conditions.

2.2. *Acknowledgements*

2.2.1. This paper was made possible by a scholarship generously awarded by the Rentokil Foundation on the recommendation of the Anglo-Danish Society. This stipend enabled the author to spend six enjoyable months in Denmark on research. Grateful thanks are due also to a leading life office which gave him leave of absence and to Michael Ashman for so persuading the company.

2.2.2. Nonetheless, the greatest thanks are due to the Danish Actuarial Society and to the Danish actuaries, too many to name, who gave the author so much of their valuable time and who so patiently answered his numerous questions. International actuarial relations are fostered not by a plethora of papers but by understanding and hospitality in personal contacts. At this the Danes are superlative!

2.3. *Presentation*

2.3.1. To make this paper more readable, wherever possible statistics have been banished either altogether or to appendices. The Insurance Supervisory Board compiles excellent statistics of the Danish insurance industry which may be easily consulted at their source (see 12.1.3). Figures in Danish crowns have been converted into sterling at the exchange rate prevailing on 6 January 1975, 13·28 crowns to £1.

3. HISTORY

'None must dare to insure a person's life.'
Law of King Christian V, 1683

3.1. *Introduction*

3.1.1. In view of subsequent developments, it is entirely appropriate

that the first noteworthy event in the chronicle of Danish assurance should be a law regulating life insurance to the point of banning it altogether—see the quotation above. (The law was a reaction to the practice of gambling on the lives of prominent people.)

3.1.2. The history of Danish assurance reads very differently from that in lands more familiar to English-speaking actuaries. The similarly differentiated present condition of assurance in the little land of plenty can only be understood after examination of that history. The following account is divided at the years 1842, 1901 and 1951, for the important events of those years makes such an arrangement convenient.

3.2. *To 1842*

3.2.1. Shortly after the act of 1683, the sale of annuities got off to an equally inauspicious start when the Danish consul in Holland began to sell life annuities at the rate of 1 unit of annuity for 8 units of capital, irrespective of the age of the prospective annuitant. This venture resulted in heavy losses! At about the same time the idea of the tontine was brought to Denmark, the Director of Posts having met Tonti in Amsterdam.

3.2.2. Widows' funds appeared first among the clergy; pensions depended on the funds' financial state at the date of payment. 1740 saw the foundation of a state-guaranteed widows' fund, open to all, to provide annual widows' pensions equal to 40% of single premiums (invariant with the ages of the married couples in question). The Treasury suffered serious loss. In 1775 another government-guaranteed widows' fund, 'Den almindelige Enkecasse' ('The Common Widows' Fund'), was instituted, based upon the death table prepared by the German cleric, Peter Sussmilch. Ten years later it was reorganized, the state having to make good losses, but it survived to 1845; its demise being due to mortality rates declining below those assumed in its premiums.

3.2.3. The years around 1800 saw the growth of a number of unsound assurance and allied companies. Subsequent collapses caused the government to appoint, in 1810, a committee to which assurers had to submit reports and with powers to order cessation of operations.

3.2.4. It was felt that the Common Widows' Fund was both dear and difficult to join. The German-born actuary Johan Nicolaj Tetens therefore proposed a new state-guaranteed institution to be

based upon mortality tables he had drawn up; this 'Den almindelig Forsørgelses-Anstalt' ('The Common Provision-Institution') came into existence in 1795. It offered life and survivorship annuities and life assurance in return for single premiums. Unfortunately, the Institution received little support and ceased to exist in 1816, its assets and liabilities being taken over by the state. Tetens is remembered outside Denmark principally for his work on commutation functions.

3.3. *1842–1901*

3.3.1. The lack of a true life assurance company continued to be felt. In 1842, the modern era in Danish assurance began with the foundation by the state of 'Livrente-og Forsørgelsesanstalt' ('Annuity & Provision Institution'), to be followed in the next year by Livsforsikringsanstalten i Kjøbenhavn' (The Life Assurance Institution of Copenhagen'). Both the new offices employed Finlaison's mortality table, with $3\frac{1}{2}\%$ interest and accepted annual premiums—important new features. It was intended that the Livsforsikringsanstalt should become a mutual company with a state guarantee expiring after five years; instead it continued as a public corporation. The new institutions were not a great success; however when they merged at the end of 1870, the Liverenteanstalt showed a surplus of $3\frac{1}{2}$ million crowns and the Livsforsikringsanstalt had neither surplus nor deficit. The new company was named 'Livsforsikrings-og Forsørgelsesanstalten of 1871'—'The Life Assurance and Provision Institution of 1871'. 1872 saw the office pay its first (cash) bonus. In 1893 came a change of name to 'Statsanstalten for Livsforsikring' (The State Institution for Life Assurance'). Today it is Denmark's largest assurer. For a number of years following the dissolution in 1861 of the 1810 supervisory committee, the director of Statsanstalten exercised supervisory powers.

3.3.2. The fused Institution of 1871 used tables prepared by Oppermann based upon the experience of its predecessors over 1842–68. These tables, for assured lives, annuitants and women were particularly important. In comparison with Finlaison's, there was a large reduction in mortality, especially for males. Financial calculations were at 2% per half year. Oppermann employed a generalization of Makeham's formula viz:

$$\mu_x = (a+b)e^{-kx} + ce^{lx}$$

To deal with the younger ages where μ_x is a declining function of x, he used (*inter alia*) an expression:

$$\mu_x = \frac{m}{\sqrt{x}} + n + p\sqrt{x}$$

3.3.3. 1872 greeted the foundation of the first modern proprietary assurer 'Hafnia'. (The name refers to the same haven from which Copenhagen takes its name. The City was originally known as 'Kobmaendenes Havn' (The Chapmen's Haven').) Some 4 or 5 years previously there had been an attempt to form a life office subsidiary of the established general insurers 'Nye Danske', but nothing came of it. The (future) founder of The Danish Actuarial Society, T. N. Thiele, was amongst Hafnia's moving spirits.

3.3.4. Autumn 1872 witnessed the birth of Denmark's first mutual assurance office, 'Danmark', an offshoot of the mutual fire office of the same name. More offices followed, some independent, others members of insurance groups. In particular, 1898 saw the foundation of 'Tryg' ('Secure'), and 'Foreningen af danske Livsassurandører' ('The Society of Danish Life Assurers') which in the following year took its present name 'De danske Livsforsikringsselskabers Forening' ('The Danish Life Offices' Society').

3.4. *1901–51*

3.4.1. In 1890 there existed only four Danish life offices, but a further thirteen were founded during 1896–99. The merger of two offices a couple of years later left 16 direct assurers. The ground was thus prepared for the foundation of an actuarial society.

3.4.2. 'Den danske Aktuarforening' ('The Danish Actuarial Society') was founded on 23 April 1901 when T. N. Thiele, Professor of Astronomy at Copenhagen University, invited eight actuaries to meet on that day. Thiele was chosen as President of the Society and fifteen other actuaries invited to join. The first ordinary meeting, on 31 May adopted a set of Articles and the name 'Foreningen af danske Aktuarer' ('The Society of Danish Actuaries'). (In 1924 the name was changed to the present title.) The May meeting included discussion on such subjects as size of subscription—for some members wished the Society to send representatives to actuarial congresses—and the smoothing of mortality tables.

3.4.3. In 1904 the Society published 'Aktuaren' ('The Actuary') —edited by Gram. It was hoped that this would blossom into a nordic actuarial journal, but fourteen years were to pass until in

1918 the Swedish journal (founded in 1914) was, on Danish initiative, amended to 'Skandinavisk Aktuarietidskrift' ('Scandinavian Actuarial Journal').

3.4.4. The question of actuarial education had long occupied actuaries' thoughts. Professor Thiele had given some instruction at Copenhagen University in actuarial mathematics, otherwise actuaries were self-taught, assisted by various articles in English and Danish. The problem was taken up by the Actuarial Society in 1908 when three members prepared a rough draft of a note to government and parliament on the subject. A meeting of the Society produced broad agreement that a chair at the University (then the only one in the realm) would be appropriate. The University was sympathetic, but no action resulted. Five years later, a new initiative from the actuarial side with the less ambitious objective of a lectureship was successful and an actuary, Lars Iversen, was appointed lecturer on 1 February 1916. The syllabus for the new course was to include economics, statistics and law in addition to actuarial mathematics. A royal ordinance of 24 August 1917 established an examination in actuarial science and statistics.

3.4.5. J. F. Steffensen succeeded to the lectureship in 1919 and enjoyed elevation to a chair in 1923. Following his retirement in 1943, the new professor, W. Simonsen, proceeded to exceed his predecessor's tenure, retiring in 1974.

3.4.6. As the insurance market developed, the need for a law to govern life assurance was felt. Early in 1900 a joint Danish-Norwegian-Swedish commission was set up; three of the four Danish members were actuaries. The commission put forward principles for common legislation; Sweden passed a law in 1903 and Denmark in 1904. Containing provisions as to establishment, activities and supervision, it had the same outline as today's legislation. Assurance was to be transacted on the 'concession principle'; the present 'Forsikringsraed' ('Insurance Supervisory Board') was established to do just that. The first chairman of the Board was also Treasurer of the Actuarial Society; two of the Board's three other members were also actuaries.

3.4.7. Danish legislators have felt that the long duration of assurance contracts and the need to protect life savings render special protection of life and pensions policyholders essential. Problems in the general insurance field after the first world war led to an extension of supervision to that branch under a law of 2 May 1934.

3.4.8. Nineteen Scandinavian offices pooled their experience for the years to 1900. From 1905 to 1926 the Hafnia employed tables based upon this, with a margin. The Statsanstalt's experience utilized by Oppermann (see 3.3.2) but extended to 1890 and excluding the first five years in force of each assurance was used to form the very important SAOR 1910 table. (Statsanstaltens Overlevelsesrentetavle 1910, Maend—The State Life Assurance Institution's Survivorship Annuity Table 1910, Males.) Interest was at 4% convertible annually. In 1926 the Statsanstalt introduced premiums derived from SAOR 1910 and the private offices followed suit, though employing higher loadings.

3.4.9. In 1916 five of the most important life offices established 'Livs-og Genforsikringsselskabet Dana' ('The Dana Life and Reassurance Society') with the particular object of reassuring sub-standard lives. 1917 saw the foundation of the first pensions office 'Pensionsforsikrings-anstalten' ('The Pension Assurance Institution') by a group of employers, to be followed by Dansk Pensions-og Livrenteselskab' which was taken over by the former a couple of years later. Two more pension offices followed in 1919, 'Pensions-og Livrente-Institutet af 1919' owned by the offices which established the Dana and 'Andels-Pensionsforeningen' to serve the cooperative movement.

3.4.10. 'Lov nr. 129 af 15. april 1930 om forsikringsaftaler'—'Law no. 129 of 15 April 1930 concerning insurance arrangements' governs insurance policy conditions and similar matters; it is another aspect of Scandinavian cooperation, the Swedish, Norwegian and Finnish laws being broadly similar.

3.4.11. Prior to 1936, pension premiums were based upon German foundations. In that year the offices adopted bases derived from the experience of civil servants by Iversen, employing 3½% interest, compounding semi-annually.

3.5. *1951–Present*

3.5.1. In 1950–51 the Danish life and pensions offices adopted new tables and rates, the HS tables. These were derived from the experience of Hafnia for 1926–41 and the Statsanstalt for 1920–45. Interest was 3¾% convertible annually, mortality a slightly-modified Makeham function. The pension bases, P51 and L51 used 3½%.

3.5.2. 'Lov nr. 147 af 13 maj 1959 om forsikringsvirksomhed'—'Law no. 147 of 13 May 1959 concerning the transaction of insur-

ance' (as amended) governs the establishment, activities and supervision of offices, both life and general, today. The law will be referred to in this paper as the Insurance Act.

3.5.3. The current legislation governing the Statsanstalt is dated 21 December 1965. It supersedes the other legislation so far as that Institution is concerned.

3.5.4. The 1970s have seen a quickening of the pace of events. A number of mergers have taken place, partly caused by a feeling that greater strength would be needed to face foreign competition and partly to secure economies of scale. On 1 January 1972 important groups associated with the names 'Baltica' and 'Scandinavia' joined forces. At the beginning of 1974 the new group entered into a cooperation agreement with a north of England office and minor shareholdings were exchanged.

3.5.5. All but one of the mergers have taken place without incident. The exception was that proposed in 1972 between the Hafnia and the smaller, charmingly-named 'Haand-i-Haand' ('Hand in Hand')—group. Hafnia made an agreed bid for Haand-i-Haand shares, but the Codan group, seeing an opportunity to increase its market share, stepped in with a better offer. The resulting conflict was nicknamed 'the punic war', perhaps with reference to the personal animosities involved or, more picturesquely, as 'the sea-battle in Holmen's Canal' (where Haand-i-Haand and Hafnia have their offices). Much was made of a well-known English office's interest in Codan, when that fact came to light. (It is almost impossible to ascertain who are the major shareholders in a Danish company.) Eventually, victory went to Hafnia, but not before criticisms of the industry had been heard from many quarters. Most serious was that policyholders in the three contending groups appeared to have no control over their fates. (The Forsikringsraad safeguarded their interests—but that is not the same thing.)

3.5.6. On 1 January 1973 Denmark joined the European Community, a move greatly welcomed by Danish insurers, though not without apprehension. To date, direct consequences have not been significant. Increased interest has been shown by foreign insurers in setting up in Denmark, but to date only one assurer has done so—and that a Swiss firm.

4. THE INSURANCE MARKET

'The law makes it difficult for a foreign life office to begin writing business in this country—but no more difficult than for a new Danish company.'
Danish Actuary

4.1. *Classes of Insurers*

4.1.1. At the end of 1972 the market comprised:

	Number
Proprietary Danish life and pensions offices	11
Mutual Danish life and pensions offices	4
Reassurers	8
The State Institution for Life Assurance	1
Foreign life office	1
Pension funds	263
Labour market supplementary pension fund	1
Burial funds	53
Properietary Danish general insurance offices	204
Foreign general insurance offices	84

Further statistical details are to be found in 12.3, comprehensive data are contained in the Reports of the Insurance Supervisory Board (see 12.1.3).

4.1.2. Section 5 of the Insurance Act stipulates that 'An insurance company must not carry on any other kind of business than insurance agency' and 'Life assurance must not in the same company be combined with other kinds of insurance'. Offices transacting both life and general insurance when the Act came into force were permitted to continue doing so. Today only one company writes both life and general business. However, reassurers often transact both classes; some companies do life and non-life reinsurance together with general insurance. (Thus there is an element of double-counting in the table in 4.1.1.) Insurance companies may form groups, thereby covering all classes of business. As elsewhere, it is felt advantageous to offer a full range of contracts.

4.2. *Life and Pension Offices*

4.2.1. No Danish assurer is a member of a commercial group operating in non-insurance fields. Very close ties with non-commercial organizations do, however, exist. The business of cooperative

societies is handled by 'Andels-Pensionsforeningen' ('The Cooperative Pension Society'), that of local authorities (kommuner) by 'Kommunernes Pensionsforsikring' and 'Alka' serves the trade unions.

4.2.2. In general, Danish offices do not write both individual and group business, although there is no legal bar to this. Some do a little group business in addition to their portfolios of individual business; only one, the Statsanstalt, a law unto itself, transacts both classes on a sizeable scale. Excluding the special connections mentioned in 4.2.1, almost all group pensions business is divided between Pensionsforsikringsanstalten, Pensions-og Livrente-Institutet and Statsanstalten. The other offices do, however, write sizeable amounts of individual pension business.

5. RATES AND BASES

'We all charge the same premiums.'

Danish Actuary

5.1. *Introduction*

5.1.1. We saw in Section 3 that the Danish assurers traditionally have a common tariff. This is not the case in the general branch; the position is almost the reverse of that which once prevailed in the U.K. Assurer's offer only with-profit contracts, except for immediate annuities and deferred annuities with less than ten years to the first payment. Although offices charge the same premiums, bonus systems vary considerably; this is a principal area of competition. The surprising absence of non-profit classes is a feature of the Scandinavian scene. It is a consequence of the view that the premiums an assured can afford to pay determine the amount of assurance he will take out (rather than his need for cover being the determinant). Thus it becomes the task of the office to provide the maximum cover for that premium. This view links logically with the tariff, the diversity of bonus systems ensuring diversity of policy-proceeds.

5.1.2. Further, a life office's rates must be approved by the Insurance Supervisory Board. It is doubtful whether that body would be happy with non-profit rates, fearing both their being too low and jeopardizing solvency, and their being too high and not giving policyholders a fair return.

5.1.3. The present bases (see 12.4) were adopted by all offices from 1 January 1966. Mortality is expressed using Makeham's formula:

$$\mu_x = A + Bc^x$$

5.1.4. Work has begun on a new common tariff. This is only at an early stage, therefore it is impossible to comment regarding likely results.

5.2. *Mortality and Interest*

5.2.1. Mortality in Denmark has been falling in recent years for the same reasons as that in the U.K. Women appear to have enjoyed the greater improvement, that for males being confined to ages under 55. In 12.4 may be found the Makeham constants employed in the 1966 tables, together with some values of the more important functions. The radix $l_0 = 1$ and $\log c = \cdot 042$ in the life tables.

5.2.2. Since the Second World War, interest rates have moved generally upwards. The Danish economy is heavily dependent upon that of the outside world and any attempt to isolate the country from international interest rate changes would be hopeless.

5.2.3. It will be seen (in Section 9) that the investments of Danish offices are overwhelmingly of the fixed-interest type. At the close of 1965, bank rate stood at 6%. After falls in 1957, interest rates available on bonds, insurers' preferred investment medium, had been rising until at the end of 1965 first mortgage bonds yielded some 8.7% effectively, second mortgage bonds 9.3% and third mortgage ones about 10·3%.

5.2.4. Danish assurers value their liabilities on the current premium bases; the need for caution and profits led to the choice of 4½% for life and 4¼% for pension rates, in view of the longer average term of the latter contracts. Rates used for immediate annuities are frequently changed, reaching 17% at one point in 1974.

5.3. *Pension Bases*

5.3.1. To produce the pension bases, it was necessary to segregate the mixed mortality into that of active lives, force μ_x^a and disabled lives, force μ_x^b. The force of disablement is μ_x^i. The mortality of active lives is represented by Makeham functions with $\log c = \cdot 044$ for males and ·046 for females. Constant A is kept identical with the corresponding value in the mixed mortality, and constant B is calculated so as to equate each active mortality with the corresponding mixed one at age 85.

5.3.2. The force of disablement is a Makeham function, the con-

stants being the same for all tables, varying only with sex. It is interesting to observe that the force of disablement is only 50% higher for females than for males, whereas previously it was taken as 125% higher. The force of mortality for disabled lives is calculated from

$$\mu_x^i = \frac{l_x\mu_x - l_x^a\mu_x^a}{l_x - l^a} \qquad (x \geqslant 11)$$

where the superscript a refers to the active tables.

5.3.3. Danish actuaries employ continuous functions widely. The value of a pension from age n, together with a disablement benefit of equal size may thus be expressed as

$$\bar{a}_x^{ai} + \frac{\bar{N}_{x+n}^a}{D_x^a} = \frac{1}{D_x^a}\int_0^\infty D_{x+t}^a \mu_{x+t}^\beta \bar{a}_{x+t}^i dt + \frac{1}{D_x^a}\int_n^\infty D_{x+t}^a dt$$

and a temporary disablement benefit to age n as

$$\bar{a}_{\overline{xn}|}^{ai} \qquad \frac{1}{D_x^a}\int_0^n D_{x+t}^a \mu_{x+t}^\beta \bar{a}_{x+t:\overline{n-t}|}^i dt$$

Hence useful new commutation functions have been defined and tabulated:

$$\bar{F}_x^{ai} = \int_0^\infty \frac{D_{x+t}^a}{D_{x+t}^i}\mu_{x+t}^\beta dt = \int_0^\infty \frac{l_{x+t}^a}{l_{x+t}^i}\mu_{x+t}^\beta dt$$

and

$$\bar{F}\bar{N}_x^{ai} = \bar{F}_x^{ai}\left\{\bar{N}_x^i - N_x^{ai}\right\}$$

where

$$\bar{N}_x^{ai} = \int_0^\infty D_{x+t}^a \mu_{x+t}^\beta \bar{a}_{x+t}^i dt$$

5.3.4. Structurally interesting is the transformation given by Kühle for y, the age of a wife of a man aged x:

$$u_y = \frac{\text{arc tan}\,\dfrac{y-x+k_x}{10} - \text{arc tan}\,\dfrac{-m_x+k_x}{10}}{sx}$$

where parameters k_x, m_x and s_x are simple second degree polynomials of x, and u_y is a truncated normal distribution. This is of use in the valuation of a collective widow's pension benefit.

5.4. *Expenses*

5.4.1. Traditionally, Danish life offices have loaded for expenses according to the formula

$$P^{(1)} = k \cdot \frac{\bar{A}_{x\overline{n}|} + \beta \bar{a}_{x\overline{n}|}}{(1-\gamma)\bar{a}_{x\overline{m}|}}$$

where k is a continuity factor converting the continuous premium into an annual one. In the 1966 rates the constants are given the values

$$k = \cdot 995;\ \beta = 2‰;\ \gamma = 11\%.$$

(With the HS tables the Statsanstalt used

$$k = \cdot 97;\ \beta = 1{\cdot}75‰;\ \gamma = 10\%.$$

The private offices had the same loadings for sums assured of 15,000 crowns and more but used higher gross premiums below that level.)

$$I = \frac{A_{x\overline{n}|} + \cdot 002\bar{a}_{x\overline{n}|}}{\cdot 93\bar{a}_{x\overline{m}|}}$$

is applied to compute single premiums on the 1966 rates.

5.4.2. For a number of years offices have waived premiums in the event of total disability. Formerly no specific allowance was made, the ample margin in the mortality being more than adequate. For the 1966 bases, mortality being more closely calculated, it was essential to meet the cost of this benefit by an increase of premiums of the order of 1%, in fact by multiplication by ·995/·985.

5.4.3. The continuity factor is given by

$$k = \frac{(1+i)^{1/6}}{(1+i)^{1/2}} = \cdot 985 \ @ 4\tfrac{1}{2}\%,$$

the annual premium being due, say, six months before the continuous one, but a delay of two months being experienced, on average, before the annual premium is received. (Mortality may be ignored;

premiums paid but relating to a part-year after death are returned.) Continuity and disability waiver factors are combined to give

$$k = \frac{\cdot 995}{\cdot 985} \times \cdot 985 = \cdot 995$$

for many cases.

6. THE SUPERVISION OF INSURANCE

'We (actuaries) are so few that the system works very well on an informal basis.'
Danish Actuary

6.1. *Principles and Foundations*

6.1.1. The Danish system of insurance supervision is based on the 'concession principle'. Prospective insurers must obtain a recommendation from the Insurance Supervisory Board to the Minister of Commerce. In the case of assurers, the Insurance Act places upon the Forsikringsraad the duty of ensuring that 'the basis of calculation, both in its entirety and within the particular elements of calculation, e.g. the mortality, the probability of invalidity, the rate of interest and the addition to the net premiums which are made the basis of computation, are adequate'. The 'concession principle' stands in contrast to the 'publicity principle' which has been the basis of British regulation. Supervision in Denmark today has its legal basis in the Insurance Act and the Forsikringsraad as its executive arm. The Insurance Act does not apply to assurance not carried out for gain nor to burial societies; other legislation brings pension funds and burial societies under the Forsikringsraad's surveillance.

6.1.2. The supervision of monopolies and market-sharing arrangements, within the insurance industry, is entrusted to the Forsikringsraad by Law no. 102 of 31 March 1955.

6.1.3. There is provision for the Supervisory Board to act as referee in case of a dispute between a policyholder and his insurer. Its conclusions, though influential, are not binding on the courts.

6.2. *The Forsikringsraad—The Insurance Supervisory Board*

6.2.1. The Board comprises a Chairman and a Vice-Chairman, both nominated by the crown, and at least three other members appointed by the Minister of Commerce. Either the Chairman or the Vice-Chairman must be an actuary; one member must be a lawyer and one an economist. Members of the Board are prohibited from taking part in the management of any insurance company.

Currently an actuary serves as Vice-Chairman and the professor of statistics at Copenhagen University, a qualified actuary, also sits on the Board.

6.2.2. Appeals may be made against decisions of the Board through the Minister of Commerce to the 'Forsikringsnaevn' ('Insurance Tribunal'). Three members of the Tribunal are appointed by the Minister for five years at a time, one by the appellant and one by the Forsikringsraad.

6.2.3. The Forsikringsraad annually submits to the Minister of Commerce a report of its activities and a summary of the accounts of insurers (see 12.1.3). An account is given of the changes approved by the Board and the comprehensive tabulations have appended English and French translations of the main column headings.

6.2.4. The Chairman of the Forsikringsraad is responsible for keeping an Insurance Register. Registration is a precondition of transacting insurance. The Register, which is open to public inspection, contains reports and accounts submitted by insurers.

6.2.5. The salaries of members of the Insurance Supervisory Board are voted in the annual Finance Act. The general expenses of the Board are met by levies on insurers, rates of contribution being fixed in the Finance Act and contributions assessed by the Forsikringsraad from offices' latest accounts.

6.2.6. The Board is assisted by an office divided into four departments; actuarial, legal, inspection and financial and statistical. It enjoys the services of eight actuaries (including the two on the Board) and has some fourteen full-time and about ten part-time staff.

6.2.7. Relations between the Forsikringsraad and the insurance offices are handled in an informal manner; the small size of the actuarial profession and the insurance world make this easy. Senior actuaries at the Board and elsewhere have known each other throughout their professional careers. Thus offices discuss proposed changes with the Board and ascertain its attitude before making a formal approach to it.

6.3. *The Responsible Actuary*

6.3.1. Section 76 of the Insurance Act requires that every life and pension office must have a responsible actuary, whose appointment needs the ratification of the Forsikringsraad. Offices have, however, a free hand in the selection of other actuaries either subordinate to the responsible actuary or elsewhere in the organization. A company

seeking recognition by the Board of a new responsible actuary must advise the Forsikringsraad of the reason for the change. In turn, the Board must inform the office of its grounds for refusing to recognize a responsible actuary.

6.3.2. The duties of the responsible actuary are described in the Act as 'the performance of the technical calculations and examinations necessary for the conduct of assurance business. . . . The actuary shall be responsible for the company observing its basis of calculation, including the rules governing the surrender of insurance policies and policy loans; he shall be entitled to demand all such information from the company as may be necessary for undertaking this responsibility. It is incumbent on the actuary to report to the Forsikringsraad forthwith any transgression by the company of its basis of calculation.'

6.3.3. Concern has been expressed that the responsible actuary is unprotected in a position where his two masters, the Forsikringsraad and the office's directors, may come into conflict. The Supervisory Board regards an ability to withstand his directors as an essential qualification for a responsible actuary.

6.4. *Reports and Valuations*

6.4.1. Within eight months of the end of the financial year, a life office must submit its annual report and accounts to the Insurance Supervisory Board. Included in these must be a report by the responsible actuary on the calculation of the premium reserve. The forms in which accounts and attendant information are submitted, together with the manner in which published accounts are drawn up, are prescribed by the Forsikringsraad, The Board is empowered to demand such further information as it deems necessary and to conduct examinations at a company's offices.

6.4.2. The liabilities of assurers active in Denmark can be analysed into:

	%
Share (and guarantee) capital	·1
Insurance Funds	78·6
Security Funds (see 8.4.2)	2·2
Bonus Funds	11·4
Other	7·7

(end-1972 figures)

6.4.3. An assurer must include an Insurance Fund amongst his balance sheet liabilities. The Insurance Act specifies this to be the sum of the reserve for unsettled claims and the premium reserve.

6.4.4. The premium reserve must not be less than the difference between the capital value of the company's life insurance (etc.) obligations (including any vested bonuses) and the capital value of future net premiums. Note that many Danish offices have bonus systems which do not involve bonuses vesting before policies become claims. Such non-vesting bonuses are not valued. (The reserve for unsettled claims must, of course, include bonuses.)

6.4.5. Since 1966, Danish offices have been valuing on the basis of the rates introduced that year, i.e. at $4\frac{1}{2}\%$ in the case of life and $4\frac{1}{4}\%$ in the case of pension contracts. Whereas collective methods were formerly used, today computers have allowed a return to the valuation of individual policies. (This development is one reason for the number of changes in bonus systems which have taken place in recent years.)

6.4.6. Any modification of an office's rules for the calculation of its premium reserve must be sanctioned by the Forsikringsraad. If a change results in a fall in the reserve, the difference must be allocated to the Insurance Fund. Should an increase in the reserve be required, the Board may permit the office to achieve this over not more than fifteen years.

6.4.7. The problem of new business strain is resolved by allowing offices to enter in their balance sheets so-called 'legal assets'. These may be the capitalized value of up to 30% of future additions to net premiums for new policies. The additions capitalized in this way must not exceed 7% of the net premiums. In subsequent years the percentage of the additions capitalized must not increase.

6.5. *Offices in difficulties*

6.5.1. The distasteful, but topical subject of offices in financial difficulties is dealt with at length in the Insurance Act. As elsewhere, the concern of legislators has been to protect life and pension policyholders (as far as possible). Should a company fail to comply with the Act or its own Articles, depart from the bases approved for its operations or have inadequate assets, the Forsikringsraad may order necessary measures to be taken within a fixed time limit. If the measures are not implemented and the Supervisory Board deems a

peril to policyholders to be present, it can take over the administration of the office's life (etc.) portfolio.

6.5.2. Should a company go into liquidation, the Board ha powers to place the life portfolio under administration. A company may, of course, be wound up for reasons other than financial difficulties. However, a company cannot be wound up without the consent of all policyholders (unless the business be transferred), nor can a policyholder, in that capacity, petition for the winding-up of a mutual office.

6.5.3. Once a portfolio has been placed under administration the company must cease transacting life assurance. The Board takes possession of all securities held against the Insurance and Security Funds. These securities may only cover policy liabilities. Should the Bankruptcy Court be involved, it must hand over the securities to the Forsikringsraad. Winding-up orders issued after the commencement of the administration have no effect on the administered estate.

6.5.4. The first task of the Supervisory Board is to have a valuation made of assets and liabilities, employing for the latter the bases previously in force, or others if those are not suitable. Negotiations are then to be entered into with Danish life offices with a view to their taking over the portfolio. Resulting proposals may entail a reduction in benefits, changes in conditions, bonuses, etc. If a scheme is agreed to, it is placed before policyholders and put into effect if not more than one fifth (by number) express objections.

6.5.5. If negotiations with other offices prove abortive or the scheme is rejected by policyholders, the Board will attempt to transform the portfolio into a mutual company, having first fixed what policy benefits are to be. Should this fail, it is left to the Forsikringsraad to decide whether further attempts are to be made to transfer the policies to a new or another company.

7. SOCIAL SECURITY, TAXATION AND POLITICS

'A welfare state which can only be supported by a 15% tax on food (purchased in supermarkets) has been taken beyond the point of absurdity.'

Danish conservative

7.1. *Introduction*

7.1.1. The Danish welfare and social security system, one of the most advanced in the world, is a remarkable structure—and a building the cost of the upkeep of which threatens to bankrupt its proud inhabitants. Advances in this sphere in one Scandinavian coun-

try give rise to pressures for similar improvements in the others. Denmark, in particular, suffers from a desire to keep up with wealthier neighbouring Sweden, whose third city, Malmø, is but 16 miles from Copenhagen.

7.1.2. In recent years combined public expenditures have amounted to over 45% of GNP (at factor cost). The biggest single item is social welfare (including the health service) consuming 47% of public expenditure in fiscal 1971–72, followed by education needing 19%.

7.1.3. To maintain a comprehensive social security system requires heavy taxation. Costs rise more rapidly than the general rate of inflation and do not easily allow savings. The public may come to feel the burden too great to be borne.

7.1.4. The social security system hence has two opposite effects on assurance (in the widest sense.) First, the benefits provided circumscribe the classes of business that may be written by private concerns. Secondly, the state, wishing to minimize the calls on its resources, encourages private provision by tax reliefs, etc., reckoning the revenues forgone or deferred to be money well spent. Naturally, the higher the rate of tax, the more valuable the reliefs. Thus it is that an assurance industry can flourish alongside an advanced welfare state.

7.2. *Social Security*

7.2.1. The present state pension scheme is based upon a law of 1965 (as amended). All citizens over the age of 67 are entitled to the so-called 'full' state pension. For a married couple, this is of the order of one-third of the national average wage. There is no income-related reduction and the pension is automatically adjusted in line with changes in the cost of living every six months; supplements are paid to those who have deferred drawing their pension, to those with no other income and to pensioners aged over 80. Various categories of citizens not yet eligible for the full pension may claim the 'income-related' state pension, which is the same size but subject to a reduction equal to 30% of any income above a certain limit. Both pensions are financed by 'contributions' and taxation on what is, broadly, a pay-as-you-go basis.

7.2.2. Laws are being formulated regarding a 'social pension fund'. Since 1971, contributions (currently 2% of taxable income) have been levied to build up a fund to supplement the state pension. The fund is invested in bonds; in due course the interest will be used both to increase the fund and to supplement pensions.

7.2.3. There is no public provision for wage-related pensions in Denmark. Such plans have been opposed as not being egalitarian, but now that Sweden has a scheme, pressures mount for Denmark to follow suit. Considerations of cost are, however, likely to inhibit progress on this front.

7.2.4. The 'labour market supplementry pension scheme' dating from 1964 is the legislative consequence of an agreement between the employers' and the trade unions' confederations to provide a further uniform pension for all employees. Each wage or salary earner pays three crowns and his employer six crowns per week on his behalf to the fund. In turn, it will provide him with a pension of 4,008 crowns p.a. on attaining age 67. The fund receives no financial assistance from the state, saving the tax concessions available to other pension funds. However, the fund's legislative backing enabled it to be set up with a deficit; benefits promised to those joining at inception exceeded the actuarial value of the relevant contributions. The legislation also gave the fund a special dispensation whereby an employer may borrow from the fund up to 50% of his contributions, subject to a bank guarantee being available.

7.3. *Taxation*

7.3.1. The Danes may be the most highly-taxed nation, even after substantial tax cuts made in the autumn of 1974. Value-added tax is levied at 15%, even on food for home preparation. Income taxes produced some 51·3% of public revenues in fiscal 1971–72 and are levied by the state for itself and for its pension and sickness funds, and also by local authorities. Before the 1974 cuts, a young actuary, without a mortgage, could find himself losing nearly two-thirds of his marginal income in income taxes.

7.3.2. The Tax Department is the insurance industry's best salesman. Every tax-payer may deduct up to 3000 crowns (£226) p.a. from his taxable income if the monies are used to pay life or general insurance premiums. Any premiums in excess of that sum, with the following exceptions, must be met from net income. Group pension premiums, whether the benefits are annuities or capital sums payable by instalments over not less than ten years, are fully deductible from taxable income, whether the premiums are paid by employer, employee or shared. They do not count against the 3000 kr. limit. Similarly treated are premiums for:

Life annuities with payments commencing after age 60;
Survivorship annuities payable to a bereaved spouse;
Disablement pensions;
Orphans' pensions terminating on or before age 24;
Sickness policies;
Index contracts (see 8.2);
Personal accident insurance.

7.3.3. The proceeds of policies with fully-deductible premiums are, conversely, taxable as income on receipt. The proceeds of other insurances, even if the premiums come within the 3,000 crowns limit, are tax-free. A tax-payer may elect to count certain otherwise deductible premiums against his 3,000 crowns limit and enjoy in return more favourable tax treatment of the policy-proceeds.

7.3.4. An annual wealth tax is levied on that part of a fortune which exceeds 450,000 crowns (£33,886) at a rate of nine per mille rising to eleven per mille. Assurance and pension policies, etc., are excluded from taxable wealth.

7.3.5. Companies suffer tax at 36% on their profits after a deduction of 50%, which latter may not exceed 2½% of the company's paid-up capital. Dividends suffer a 30% withholding tax which can be regarded as (part) advance payment of the recipient's tax liability. Assurers are not subject to tax on any profit other than that appropriated to the shareholders.

7.4. *Politics*

7.4.1. The most interesting recent development in Danish politics has been the articulation of public discontent with high and rising taxation. This brought into existence a brand-new political party, the Progress Party, whose disparate members are united only by a common determination to reduce taxes. After contesting its first election on 4 December 1973, it secured 28 seats in the 179-seat Folkting parliament, making it the second largest party there after the Social Democrats. (With only 22 seats in the House, the Liberals formed a government!) That the Progressives had touched a raw political nerve was shown by the hysterical over-reaction of established politicians to their success. Further evidence of public discontent with rising taxes was a widespread communist-led strike in May 1974, protesting against the imposition of further taxes on luxuries (cars, some consumer durables, alcohol and tobacco)! In the follow-

ing autumn the Folkting bowed to public opinion and reduced taxation, particularly on incomes. In this context it must be noted that the Danish welfare state is far from complete; in particular there is no public wage-related pension scheme.

7.4.2. Insurers have in the past been worried by threats of nationalization; today economic problems are pre-occupying politicians. Public criticism is voiced of offices expense ratios and bureaucratic procedures. The 'sea battle in Holmen's Canal' (see 3.5.5) has been the most serious cause of public criticism of the industry in recent years.

8. POLICIES AND BONUSES

'Codan . . . sent an actuary to England in the autumn of 1969 to make a detailed examination of the unit trust and allied movements.'

Publicity for unit-linked assurance

8.1. *Personal Policies*

8.1.1. English assurers will find in Denmark all the traditional classes of policy. We have noted the absence of non-profit classes. The taxation and investment-oriented policies recently so popular in the U.K. are not to be found in the regulated Danish market, though an equity-linked policy was launched in April 1970. Industrial life policies were formerly written; high expense ratios caused their demise. Inflation has given rise to greater public interest in protection as opposed to savings contracts. It has also led to contracts whose premiums rise with the cost-of-living.

8.2. *Indexcontracts*

8.2.1. From 1 April 1957 until 24 November 1971 every Danish citizen (aged between 18 and 57) had the right to take out up to six indexcontracts, a form of personal pension policy unknown (in practice) in the U.K. The annual annuity under each policy is 500 (July) 1956 crowns the actual money payments in any year being that sum multiplied by the ratio of the price index in (July of) the previous year to that in (July) 1956. The annuities are payable from age 67. The initial annual premium for each policy was calculated on conventional principles, assuming that the annuity payments would remain constant at the level appropriate to the year of proposal. Premiums in subsequent years would be escalated by the price index, as described for annuity payments. Each year's additional premiums are used to purchase additional annuities on conventional lines.

8.2.2. When a policyholder reaches 67, there will be a shortfall between the total annuity purchased by the premiums and that due under the terms of the contract. Further, this shortfall will escalate as continuing inflation triggers increases in the annuity after retirement. This deficit is (to be) made good by payments by the state to the institution issuing the policy (at the dates of payment).

8.2.3. It became clear to the authorities that 1 April had been a most appropriate day on which to commence the issue of index-contracts. Continuing inflation could lead to the position where deficits under the contracts demanded the major portion of the state's revenue! The issue of new contracts was terminated, but existing ones continue. Fears have been expressed that the terms of these will not be completely fulfilled. (The author has been unable to discover what calculations were performed by the authorities to establish the viability of the scheme.)

8.2.4. Some 2·6 million contracts were taken out by about 600,000 citizens. They were issued by banks, life and pension offices and savings banks. (Banks and other savings institutions could offer the indexcontracts as contractual savings schemes with no element of assurance. These, like the pension contracts assumed interest at 3½%, the rate in the HS tables.) Regrettably, the life and pension offices secured only around 20% of this business. Indexcontracts could be in the form of capital sums, the capital payments being spread over ten to fifteen years.

8.3. *Equity-linked Assurance*

8.3.1. The only equity-linked contract is offered by Codan Life; the sales literature prominently proclaims that one of its actuaries visited its parent company in England to study British methods in this field. The Danish soil is not easy for the equity-linked assurer to till. In part this is because the share market is small and thin, but the principal obstacle is the Insurance Act, which prohibits offices from investing more than 15% of the assets covering Insurance Funds in anything but bonds of the highest security (or property). An elaborate scheme was therefore necessary to obtain the approval of the Forsikringsraad.

8.3.2. An investor takes out an ordinary life policy with Codan. He signs an agreement with Codan Invest(ment Trust Co.) under which the latter borrows from Codan 50% of each premium paid (as soon as permissible). These borrowed moneys are invested in shares; the

investments of Codan Life being of the pattern common to all life offices. Savers are members of Codan Investeringsforening (Codan Investment Society) which controls Codan Invest. Note that the Codan group does not own Codan Invest—as indeed it may not legally do so. The saver receives an appropriate number of Certificates of Participation in Codan Invest's fund in exchange for passing over his successive policy loans. The number of Certificates varies inversely with the value of the fund's units at the time of each loan.

8.3.3. The policy-proceeds are the original sum assured, together with bonuses calculated as for other Codan policies, less the loan made to Codan Invest. The policyholder also has his certificates, which he may redeem at the then market price or retain. Codan Invest guarantees that the redemption price will not be less than the sum invested in the certificates with 4½% interest. (This is the rate used for calculating the premiums.)

8.3.4. The investment of the policy loans is restricted by the limited Copenhagen share market. Further, Codan Invest cannot follow an active switching policy for fear of being regarded as a dealer in shares and losing its tax-free status. Like assurance policyholders in general, the investor may deduct interest on his loans from taxable income and the loans themselves from his taxable capital. When the certificates are realized, the proceeds suffer a 30% capital gains tax.

8.3.5. The only Danish equity-linked assurance has enjoyed favourable comment and popular demand. Undoubtedly, it helped Codan Life to occupy a much higher place in the new business' league than it did in the assets' league. The plan is sold by a specially-trained sales force, quite distinct from the group's other salesmen.

8.4. *The Law and the Profits*

8.4.1. Section 71 of the Insurance Act contains detailed rules regarding the allocation of what may be described as first charges upon or first slices of any surplus emerging. In the first place, an office, after covering any deficit brought forward, may employ its surplus to pay a dividend of up to 5% on its share capital. (Preliminary expenses, etc. must be written off in full before any dividend can be paid.)

8.4.2. Of the surplus remaining, 50% is to be allocated to a Security Fund until the latter has reached the sum of:

1% of the premium reserve for endowments plus ½% of the

difference between the sum assured and the premium reserve for those (parts of) policies where the office itself bears the risk;

1% of the premium reserve for deferred annuities plus ½% of the difference between ten times the annual annuities and the premium reserve for those (parts of) such annuities where the office itself bears the risk;

2% of the premium reserve for current annuities.

8.4.3. Should an allocation of less than 50% of the surplus be sufficient to bring the Security Fund up to the size set out in the previous paragraph, the Fund need only be brought up to that limit. In any event, however, at least 10% of the surplus must be so allocated until the Security Fund reaches twice the size specified. (The limits of once and twice the size of the Security Fund set out in 8.4.2. may be reduced by any part of the legal asset mentioned in 6.4.7 not shown in the balance sheet.)

8.4.4. After the allocations to the Security Fund, a company may freely dispose of the remaining surplus in accordance with its Articles (which require the approval of the Forsikringsraad). Until the Security Fund has reached the size stipulated in 8.4.2, the company must not exceed 5% in its dividend (or payment to guarantors).

8.4.5. Any amount by which the Security Fund exceeds the 'once times' limit may be allocated to the premium reserve should a change of valuation basis necessitate this. Otherwise, the Fund must not be drawn upon without the consent of the Insurance Supervisory Board, except to cover a deficit.

8.5. *Group Pensions*

8.5.1. It has been the policy of successive Danish governments to encourage occupational pension arrangements. Private pension plans developed—as far as benefits are concerned—as imitations of that for civil servants. The first private arrangements appeared at the end of the nineteenth century; we have already noted the establishment of pension offices around 1920. Today employers have the choice of privately-funded plans—'funds'—or arrangements insured with (life or) pensions offices—'schemes'. In addition to funds and schemes covering the servants of a particular company, there are also funds covering everybody in a particular occupation, e.g. all civil engineers. (Compare the scheme for U.K. university academic staff.)

8.5.2. A pension of 60–66% of 'final' salary is thought a desirable objective. Other benefits commonly provided are widows' and

orphans' pensions and invalidity pensions. Naturally, expense often prevents benefits being as generous as employer and employees would like to see. A law of 1941 gives divorced wives the right to collect the widow's pension in certain circumstances. If the assured remarries, the pension is apportioned between the women, normally in proportion to the duration of the marriages. The widow's pension is commonly half that due to the insured, but may be more. Orphans' pensions might be 10% of the insured's, doubling if both parents are lost. Additional voluntary contributions are rarely encountered.

8.5.3. In contrast to English practice, Danish private pension arrangements normally cover only white-collar employees. Many actuaries might feel that the mortality rates in the 1966 tables are unlikely to be entirely appropriate for blue-collar employees.

8.5.4. Every three years, more often if the Forsikringsraad considers it necessary, every pension fund must be actuarially valued. The actuary concerned must be acceptable to the Board. In practice, the actuary is likely to be a full-time employee of a life office.

8.5.5. Pension plans are usually costed on an annual premium basis, contributions being expressed as a percentage of salary, 12% being probably about the average. Normally no account is taken of future salary progression (unless guaranteed) though improved computational facilities are coming into use for this purpose. Schemes are often valued annually and the contribution rates changed as appropriate. It is assumed that future expenses will be equal to those allowed for in the premium bases and valuations are performed on current rates. (Expenses have been falling as a proportion of pension premiums. Funds generally have their expenses met by the employer.)

8.5.6. Failing to allow for future salary increments can cause the cost of a pension plan to escalate, particularly at a time of accelerating inflation. To avoid this problem, plans often stipulate premiums, not benefits. This is particularly appropriate as the pension offices pay annual cash bonuses on pensions in course of payment. Interest surplus is the main source of bonus; this revenue may be expected to move up and down with inflation.

8.5.7. Taxation legislation of 1958 prevented an employer's pension contributions reverting to him when an employee left his service. As contributions are always allocated to the employee in question, it then became the norm for employees to enjoy vesting of rights, without regard to length of service. Should an employee

transfer from a scheme with one office to a scheme with another, an agreement between offices allows the employee to continue his policy with the first office, only increases in benefits being assured at the second. The latter receives all premiums, remitting appropriate moneys to the first office. As schemes can be very different, policies must frequently be rewritten. Nevertheless, the tariff clearly greatly expedites matters.

8.5.8. Group life assurance became popular only recently, but today is a common feature. Indeed it is even being extended to workers who are not members of private pension plans but are, say, members of particular unions.

9. INVESTMENTS

'It's bonds, bonds and more bonds.'

Danish Actuary

9.1. *Investments of Insurers*

9.1.1. At the end of 1972 the investments of insurers comprised:

	All Assurers %	Pension Funds %	Danish General Insurers %
Fixed Interest	87·4	75·6	45·0
Shares	2·9	2·0	8·8
Property	3·3	·1	12·2
Other	6·4	22·3	34·0
	100	100	100

Policy loans, property and cash excepted, almost all the investments of insurers are quoted on the børs—the Copenhagen Stock Exchange —which is also the channel through which most of their new money is invested.

9.2. *The Copenhagen Stock Exchange*

9.2.1. Constructed between 1619 and 1640, the old Stock Exchange may claim to be the most elegant work of Denmark's master-builder, King Christian IV. Certainly it is the most aesthetically pleasing of market-places as well as that which has enjoyed the longest use. Like other exchanges, however, the børs felt a need for larger, modern

premises—and perhaps a desire to escape from the shadow of the massive government buildings dominating the old Exchange. So, on 1 April (!) 1974, trading began on new floors, appropriately overlooking Copenhagen's main shopping street.

9.2.2. On five mornings a week trading opens with the calling-over of listed obligationer—bonds—and aktier—shares—on separate trading floors. (One reason for the change of venue was a need to be able to call-over bonds and shares simultaneously rather than consecutively.) These main lists comprise some 27 government loans, a very large number of bonds issued by mortgage societies, some local authority and semi-public body securities and about 142 shares. Six insurance shares are quoted, being grouped with communications companies for statistical and other purposes! The calling of the main lists is followed by dealings in a special list, which takes place on the share trading floor. Whereas securities on the main list are auctioned in a certain order, dealers simply call bids and offers for special-list securities. Trading in any securities may subsequently take place by personal contact or telephone. A typical day might see the exchange of 33 million crowns of bonds, 600,000 crowns of shares and 33,000 crowns of securities on the special list.

9.3. *Types of Investment*

9.3.1. In contrast to the situation prevailing in most (capitalist) countries, the Danish market for fixed interest securities has as its most important component not government stock but the obligations of mortgage societies. Indeed, since 1959 the state has issued no new loans to the market, preferring to borrow abroad with a view to assisting the balance of payments. The National Bank of Denmark makes monetary policy effective by the purchase or sale of non-government securities.

9.3.2. As in recent years more than two thirds of the money life offices have had available for new investments has gone into the bonds of mortgage societies, a few words on the nature of these securities would seem to be appropriate. Prospective purchasers of property (be it domestic, commercial, agricultural, industrial or whatever) arrange to mortgage the asset in question, receiving in return debentures of the mortgage society. Money to complete the purchase is then raised by the sale of these bonds on the børs. The purchaser of the paper has not only the security of an individual property but also a joint liability of all the members of the mortgage society, i.e.

the borrowers. The borrower obtains money at the best rates available and need fear no rise in the interest he must pay. (Mortgages are of two types, first and second priority. The relationship between the nominal value of a mortgage and the value of the pledged asset is governed by law and varies with class of property.) Thus, home buyers, in particular, can tap a stable source of finance and one cause of instability is lifted from the building programme.

9.4. *Legal Restrictions on Insurers' Investments*

9.4.1. In Section 68, the Insurance Act requires that not less than 85% of assets held by an assurance company against its Insurance Fund (see 6.4.3) must be in ultra-safe investments, i.e. gilts, obligations of mortgage societies, local authority loans, bank deposits, policy loans, mortgages and real property up to four-fifths of the official valuation (for rating purposes). Of these investments, only property has an equity element, the others being entirely-fixed-interest.

9.4.2. The remaining 15% of assets held for the Insurance Fund may be freely invested in shares and other types of assets. In no case may an office have shares in excess of 15% of the share capital of one and the same company.

9.4.3. Section 5 of the Act, while otherwise restricting the commercial activities of insurers to insurance (see 4.1.2) states that its provisions 'shall not prevent an insurance company, as a permanent investment of the company . . . from building, owning and managing real property'. As only four-fifths of the official valuation of a property is eligible to be held for the 'narrow range' 85% of an Insurance fund, the other fifth—if it is to be reckoned an asset at all—most be set against part of the 'wider range' 15% or held against liabilities other than the Insurance Fund.

9.4.4. The assets of an assurance company in excess of those held against the Insurance Fund may be invested at will.

9.4.5. Pension funds are governed by the 'Law concerning the supervision of pension funds'. That legislation contains restrictive provisions regarding the investments of pension funds which are almost the same as those just outlined for assurance companies. Naturally, as the funds' obligations cannot be pledged, there is no provision for policy loans. It is also laid down that a fund must not invest more than 1% of its assets in the shares of any one company. A most important phrase prohibits any investment in the shares of

the company with which the fund is connected 'without permission of the Minister of Commerce, given only in special circumstances and after consultation with the Insurance Supervisory Board'.

9.4.6. Securities held against an Insurance Fund's or a pension fund's liabilities must be deposited in a place of safekeeping from which they may only be removed with the consent of a trustee appointed by the Forsikringsraad.

9.4.7. General business funds enjoy freedom of investment. However, the Articles of a company, which are subject to the approval of the Supervisory Board, must make provision regarding investments. The Board has prepared model articles whereby assets are to be invested in the manner required for assurers (excluding the possibility of policy loans).

9.5. *The Valuation of Insurers Investments*

9.5.1. The Insurance Act provides, in Section 63, that bonds of the highest rating may not be entered in assurers' accounts at above purchase or redemption price (whichever is the lower), assuming the bonds to have a certain final redemption date. In current circumstances life and pension offices normally enter these bonds in their annual accounts at book value, even if this greatly exceeds the market price. Bonds with no final redemption date are valued at the average of the prices at the end of the last ten financial years (assuming them to have been owned throughout that period), but not above par.

9.5.2. At first sight it may be a cause for concern that Danish assurers can—and do—enter the major portion of their investments in accounts at values above those prevailing in the market. However, it is reasoned that a decline in asset values which is unaccompanied by a change in the probability of receipt of the asset-proceeds is irrelevant to a life office (assuming the portfolio to be matched by term to the liabilities). Its ability to meet its commitments is unimpared, indeed it can make new investments on more favourable terms and so enjoy an enhanced interest surplus in future years.

9.5.3. So, there is no need to reduce bonus declarations just because the value of certainly-redeemable assets has fallen. Yet, if the reduced asset values are reflected in the balance-sheet (the value of the liabilities remaining unchanged), an office cannot but reduce the sums available for bonus distributions below what they would otherwise have been. Thus, one comes to the view that certainly—

recoverable depreciation of assets should be included in balance sheets in the interests of preserving equity between generations of policyholders. Naturally the overriding requirement of security makes this impossible unless offices are closely supervised. For a full analysis of the consequences of declining asset values, see 'The treatment of appreciation or depreciation in the assets of a life assurance fund', by J. R. Suttie in *J.I.A.*, LXXII.

9.5.4. The liabilities of general insurers being very short, these offices may not enter bonds in their accounts at above the last quoted purchase price of the financial year. (Mortgage and other top-quality bonds may be entered at the average price at the end of the last four years if the office has owned them that long.)

9.5.5. Other investments are valued in the same way for both life and general funds. Shares are not to be valued at above the last quoted price in the financial year and property at not above the official valuation.

9.6. *Investment Policy of Danish Offices*

9.6.1. The preferred investment media of Danish offices continue to be the obligations of mortgage societies. Government securities, lacking liquidity, are not competitive, for the mortgage bonds have higher yields. Switching is rare, partly because of the thin market but primarily on account of the adverse effects of this on balance sheet values when sale proceeds are so far below book values.

9.6.2. The Insurance Act of 1959 was the first to allow life offices to purchase shares. The majority of their acquisitions have been of banking, insurance and other high-quality issues. The reverse yield gap has made mortgage bonds more attractive than shares. Interest has been greatest in shares supplies of which are frequently available on the børs and which can be traded in some size without causing large adverse price movements.

9.6.3. Property investments take place whenever opportunities present themselves. General offices have shown more interest in property than life offices, apparently contrary to sound investment practice (see the table in 9.1.1). This is actually a reflection of their greater investment freedom. While insurers have recognized the suitability of property as an investment medium, few are experts on property and they are wary of the possibility of political attacks if their holdings are great.

10. THE ACTUARIAL PROFESSION

'In a country whose university courses are noted for their stress on theoretical aspects, that of actuarial studies is distinguished by its rigour and emphasis on theory. Actuarial science attracts the most able mathematicians and the profession enjoys the highest respect on account of its intellectual attainments.'

Danish educationalist

10.1. *The Division of Responsibilities*

10.1.1. In contrast to England where responsibility for all aspects of the actuarial profession rest with a single body, in Denmark responsibility is shared between The Danish Actuarial Society, the University of Copenhagen and the Insurance Supervisory Board. The latter is responsible for certain aspects of professional discipline and recognition of qualifications whilst the university educates young actuaries and conducts research.

10.2. *Den Danske Aktuarforening* (*The Danish Actuarial Society*)

10.2.1. Broadly speaking, the activities of The Danish Actuarial Society parallel those of The Institute of Actuaries, omitting those of formal professional education. Half a dozen meetings are held each winter half-year. Meetings begin with the transaction of formal business, followed by a lecture and/or a discussion; they close with an informal smørrebrød supper. The Society enjoys living up to the Danish reputation for hospitality.

10.2.2. It is not unusual for meetings of The Danish Actuarial Society to be attended by two-thirds of the membership, which latter stood at 147 on 1 November 1974. Attendance is facilitated by all Danish life offices having headquarters within commuting distance of Copenhagen. Beginning with 25 members in 1901, the Society gathered strength to 54 in 1926, 117 in 1951 and reached 148 members in 1973.

10.3. *The Employment of Actuaries*

10.3.1. Full-time salaried employment is the norm for practising actuaries in Denmark. Work like the valuation of pension funds, usually undertaken by consultants in the British Isles is, in Denmark, performed by actuaries otherwise employed by a life office. Not surprisingly, in view of the Statsanstalt's position as the leading assurer, a high proportion of actuaries are employees of the state or its agencies. More remarkable is the very high proportion of lady actuaries, one being numbered amongst the 19 responsible actuaries.

10.3.2. The employment of actuaries in the wider field is almost unknown in Denmark. Recently the position has shown signs of change; actuaries are becoming very involved in computation. Particularly noteworthy is the effort the profession is making to put non-life insurance on sound scientific foundations; the work of Scandinavian actuaries in this field is well known. Nonetheless, even today no actuary is engaged in full-time investment work, though several devote some time to it. (These remarks must be qualified by noting that until the establishment of a separate statistics course at the University of Copenhagen, would-be statisticians trained as, and hence are formally-qualified actuaries.)

10.3.3. Danish living standards are, in general, far above those in the British Isles. The gap between rich and poor is far less in the little land of plenty than we in Britain are accustomed to. So, whilst working people there enjoy a vastly superior standard of living to those here, senior members of the professions actually have a lower standard of living in Denmark than in England. In the summer of 1974, a young, newly-qualified Danish actuary might have expected a salary of 70–85,000 crowns (£5,270–6,400) p.a. (Costs of living and taxation are much higher in Denmark than in the U.K.)

10.4. *Actuarial Education*

10.4.1. All Danish actuaries are trained by the Department of Actuarial Science at the University of Copenhagen. On qualification, graduates receive the degree of 'candidatus actuarii', abbreviated to 'cand. act.' (The term 'candidatus' is of latin derivation and implies that the holder is qualified to be a candidate for a post in the public service.)

10.4.2. In common with all but one of the other Danish Universities, Copenhagen University does not award bachelor's degrees, students proceeding to the 'candidatus', a combined first and research degree. Students enter university aged about 19 and their studies take, in theory, 5–7 years, in practice often longer (to which must be added a period of compulsory military service). University studies in Denmark are far less structured and supervised than in England; a student proceeds at his own pace, may change course or go down for a long period and presents himself for examination when he feels himself ready. Students are supported by special state loans and their own wage-earning.

10.4.3. The actuarial course divides naturally into two parts, the

first of which is the same as for the degree in statistics and extends over two years. (Historically the Department of Statistics is an offshoot of that of Actuarial Science.) Subjects in Part I, all of which are compulsory, comprise numerical analysis, probability and theoretical statistics, stochastic processes and various mathematical subjects. In Part II are actuarial mathematics, including life and other contingencies and compound interest, economics and certain aspects of Danish statistics, Danish civil laws, insurance laws, and some aspects of the law of property. All subjects are compulsory; students must also pass a test in book-keeping and accountancy. In both Parts students face both written and oral examinations—the latter being open to the general public (as spectators).

10.4.4. In common with other students preparing themselves for the candidatus, a student actuary must prepare a thesis, to which he will expect to devote three months full time work. The choice of subject is the only discretionary element in the course; it maybe suggested by his experience with a life office or proposed by the professor.

10.4.5. The Danish scheme of actuarial studies is a much more mathematical one than that pursued by many U.K. actuaries; one could describe it as a course in actuarial science; certainly it bears all the hallmarks of a course designed by academics. Conversely, it is weak on finance.

10.4.6. An actuarial student might study full-time until he passed Part I, thereafter averaging 25–30 hours per week with an office, to obtain both practical experience and cash.

10.4.7. The academic establishment of the Department of Actuarial Science comprises a professor, a lecturer and a part-time lecturer (who otherwise practises with a life office). Three students graduated in the summer of 1974; at the beginning of the 1974–75 academic year there were some twenty-five students actively pursuing the course. Three are thought to be within a year of graduation, two have studied for two or three years, four entered in 1973 and the remainder in 1974.

10.4.8. The ease with which students may transfer between courses and the common Part I makes it difficult to generalize about the attractiveness of the actuarial profession to school leavers. It is clear that overproduction of specialists in certain disciplines has led students to look with renewed interest at fields like the actuarial one where there is a substantial unsatisfied demand for talent.

10.4.9. The retirement in the summer of 1974 of Professor W. Simonsen from the chair of actuarial science resulted in the elevation of a Norwegian, Professor J. M. Hoem, who will doubtless exercise the prerogatives of all new brooms. The following ideas are being considered for restructuring the actuarial course; it is in no way certain which will be put into practice. Naturally, The Danish Actuarial Society will be asked to comment at the relevant stage and can submit proposals, but it has no power of veto.

10.4.10. The possibility of giving students a range of options is being considered, opening the way to some strengthening of the non-mathematical element, particularly economics, finance and management. It is hoped to be able to design a course giving a unified view of the field and an opportunity for reflection on fundamental aspects of actuarial science so easily lost sight of in the pressure of day-to-day work. Aspects thought to need more emphasis are observational work, tests of hypotheses, graduation etc., and computational studies.

10.4.11. Concern has been expressed over the time taken to complete the course. It has been finished in four years, so an improvement can be looked for. However, the average student can only devote more time to academic work at the expense of the time spent working for his office. This will clearly make his financial position more difficult.

11. OBSERVATIONS

'Poverty, dirt, disease, ignorance and squalor have been banished from Denmark.

Visitor to Copenhagen

11.1. The present author went to Denmark with it in mind to investigate two particular facets of the Danish assurance scene; how it could survive in a comprehensive welfare state and what a system of supervision is really like. On both these points, glad tidings are brought back. A fully-comprehensive welfare state cannot be supported by taxation and transfers of income alone (in a democracy). The state will find it expedient to encourage private provision to reduce the calls on its resources and individuals will continue to seek better benefits than those provided by the state.

11.2. The supervision of insurance need not be a hamstringing, bureaucratic operation. It may be so, but intelligent design of a system and selection of personnel can thwart this. In addition to protecting policyholders from knaves and fools, a supervisory

authority can protect the industry—and policyholders—from short-sighted politicians.

11.3. Political pressures are building up in the U.K. for a public 'stake' in various industries. An English state assurance (and, perhaps non-life) institution would represent a compromise between the wishes of politicians and those of the insurance industry.

11.4. In the interests of equity between successive generations of policyholders, it would be worth exploring ways in which temporary falls in the market values of gilt-edged securities can be prevented from having too adverse an effect on bonus rates in Britain.

11.5. The English system of actuarial examinations tests a candidate's theoretical knowledge but not his practical ability. This could be remedied, to some extent by setting projects or requiring theses.

11.6. The issuance of marketable securities by Danish mortgage societies enables property purchasers to tap a steady source of new money, the assurance industry. Such a system, applied in the U.K. would help to iron out the costly booms and slumps in housebuilding and protect borrowers from rising interest rates.

12. APPENDICES

12.1. *Bibliography*

12.1.1. Transactions of International Congresses of Actuaries:

15th: 'Group coverage for life insurance and pensions', C. A. Busch-Petersen and K. Rostrup.

17th: 'The development of the actuarial profession in Denmark', Sv. Andersen and J. Kaspersen.
'Developments in methods of pension provision during the past twenty years', C. A. Busch-Petersen.

18th: 'Participation in profits in Danish life and motor insurance', National Report.

19th: 'Supervision of private assurance operations in Denmark', P. W. Nielsen.

Scandinavian Actuarial Journal:

1966. 'New Danish bases of calculation in life and pension insurance', H. Buus and E. Rosendahl.

12.1.2.

'Denmark—An official handbook', Ministry of Foreign Affairs: 1974 edition.

12.1.3.

'Beretning fra Rorsikringsraadet', Annual report of the Insurance Supervisory Board in two volumes (dealing with life and general insurance respectively).

'Law no. 147 of 13 May 1959 concerning the transaction of insurance.'

'Notification no. 163 of 26 May 1959 concerning the law relating to the supervision of pension funds.'

All three items are obtainable from the Insurance Supervisory Board.

12.2. *Land and People*

	Area (*sq. miles*)	*Population*
Metropolitan Denmark	17,028	5,000,000
Greenland (Grønland)	840,000	46,500
The Faroe Islands (Faerøerne, Foroyar)	540	40,000

Capital:

Copenhagen (København) 1,400,000

Status: Constitutional Monarchy within the European Community. (The Faroe Islands form a self-governing region outside the Community.)

Language: Danish (and Faroese; both languages are Scandinavian).

12.3. *The Insurance Market*

12.3.1. The life and pension offices (including reassurers):

Year	*Number*	*Premium Income* (*m. Crowns*)	*Total Income* (*m. Crowns*)	*Assets* (*m. Crowns*)	*Assets** (*m. Crowns*)
1926	35	75	120		469
1930	36	93	147		614
1935	31	146	232		969
1940	32	164	267		1358
1945		256	413		1993
1950	31	333	527		2760
1955	31	415	666		3574
1960	30	555	929		4642

Year	*Nnmber*	*Premium Income (m. Crowns)*	*Total Income (m. Crowns)*	*Assets (m. Crowns)*	*Assets* (m. Crowns)*
1965	27	876	1508		6789
1966	27	1020	1676		7431
1967	27	1225	1985		8150
1968	27	1261	2124		8999
1969	27	1396	2360	13440	10156
1970	27	1549	2709	14909	11232
1971	27	1720	3007	16515	12430
1972	25	2184 (£164 m)	3691 (£278 m)	18355 (£1382 m)	13918 (£1048 m)

* Excluding Statsanstalten and the foreign life office.

12.3.2. The life and pension offices, together with the pension funds, the burial funds and the labour market supplementary pension fund:

Year	*Premium Income (m. Crowns)*	*Total Income (m. Crowns)*	*Assets (m. Crowns)*	*Assets* (m. Crowns)*
1945	281	464		2536
1950	397	624		3594
1955	489	795		4795
1960	703	1177		6528
1965	1504	2311		10160
1966	1708	2603		11566
1967	2007	3005		13104
1968	2034	3288		14938
1969	2395	6088	20523	17239
1970	2465	4204	23209	19533
1971	2698	4695	26153	22069
1972	3322 (£250 m)	5688 (£428 m)	29651 (£2233 m)	25214 (£1899 m)

* Excluding Statsanstalten and the foreign life office.

12.4. *The 1966 Life and Pension Bases*

12.4.1. The Makeham constants:

$\mu_x = A + Bc^x$

Table		$10^3.A$	$10+\log B$	$\log c$
Life assurance:				
Males	F66M	·625	5·67167	·042
Females	F66K	·625	5·54567	·042

Pension:				
Males	P66M	·625	5·54567	·042
Females	P66K	·250	5·37767	·042

There are also tables for separate and combined survivorship annuities, with- and non-profit annuities.

F66M: $\mu_x = \cdot 000625 + 10^{5 \cdot 67167 - 10 + \cdot 042x}$

12.4.2.

		F66M:		HS:
Age	l_x	μ_x	q_x	q_x
0	1·0000	·000672	·000674	
20	·9847	·000950	·000966	
30	·9733	·001479	·001521	·00319
40	·9534	·002872	·002980	·00514
50	·9122	·006536	·006808	·01024
60	·8205	·016173	·016806	·02354
70	·6274	·041520	·042627	·05766
80	·3129	·108189	·107349	
100	·0004	·744786	·542490	