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UNIVERSAL OR VARIABLE LINKED LIFE ASSURANCES AND LIFE ANNUITIES

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1. The Universal or Variable Linked Life Assurance policy has been issued by a number of companies in the United States of America and in the United Kingdom, and it is now reasonably familiar in the marketplace. It does not appear to have received much attention in actuarial literature, perhaps because of its simplicity, though a paper has been presented to the Institute of Actuaries Students' Society (Sheraton, 1984). So far as I know the corresponding Variable Linked Life Annuity has not been issued by any office, and some of its features are slightly different from those of the Life Assurance policy.

2. This note describes both types of contract from a purely technical angle. No consideration is given to questions of legislation, taxation or qualifying policies. Only brief reference is made to expense charges; but some practical features of underwriting are discussed.

3. First some notation and preliminaries: time will be measured in *periods*, not necessarily years. x will denote the age of the life assured, in periods. q_x and p_x will denote the probabilities of (x) respectively dying within and surviving to the end of the appropriate period. If we are working in periods of a month, an estimate of q_x would be one-twelfth of the normal annual q_x from whatever mortality table is being used.

4. It is assumed that the linked part of each policy is invested in 'Accumulation Units', whose price at time t, again measured in periods, is $u(t)$. Strictly there may be two prices, Offer and Bid, used respectively for the purchase or sale of units by the policyholder, or on his account, but this does not affect the principles. By Accumulation Units, I mean units in which any income from the underlying investments is reinvested in the unit fund, and goes to increase the value of the unit. There is thus no explicit income to deal with, which distinguishes them from 'Income' or 'Distribution' Units. Neither are they 'Capital Units', where the income accrues directly to the life office, rather than to the policyholder's account.

5. Let $n(t)$ be the number of units attributed to the policyholder at time t, after all income and outgo at time t has been processed; let $n(t-)$ denote the number of units at time t before such processing. In some cases $n(t-)=n(t-1)$, if the number of units in the policyholder's account is not changed except at the end of each period, but in other cases this assumption is not valid, as will be seen.

6. The value of the policyholder's units at time t, after processing, is given by $V(t)=n(t)\cdot u(t)$. The value of the policyholder's units just before processing is $V(t-)=n(t-)\cdot u(t)$.

7. Let the premium paid by the policyholder at time t be $P(t)$. In general, this may vary as desired, being different for each t, and may be zero in some periods

and positive in others (but never negative). There may, however, be practical constraints on varying the premiums. These may be imposed by legislation, or by the administrative requirements of the office.

8. The premium at time t is used to buy additional units at the (Offer) price prevailing at time t. The number of units so purchased is $P(t)$ divided by $u(t)$. If the only part of the contract is the payment of premiums, so that we have a pure savings contract, then the number of units at time t after payment of the premium is given by:

$$n(t)=n(t-)+P(t)/u(t),$$

with

$$n(t-)=n(t-1),$$

and the value standing to the policyholder's account at this time is:

$$V(t)=V(t-)+P(t).$$

9. In a pure savings contract we also allow the policyholder to withdraw funds. Let the cash amount withdrawn at time t be $B(t)$, which must be zero or positive, and which is produced by the sale of units on the policyholder's behalf at the (Bid) price. The policyholder's account is thus reduced by $B(t)/u(t)$ units. In full:

$$n(t)=n(t-)+P(t)/u(t)-B(t)/u(t),$$

and

$$V(t)=V(t-)+P(t)-B(t).$$

10. There are two ways of treating the benefits on death, both of which may apply within one contract. The first is the payment of a fixed money sum assured on the event of death in period $(t, t+1)$, of an amount of $S(t+1)$. Imagine that this is paid from a separate collective insurance account, which requires a premium for carrying the risk. Assume that the life assured is aged x at time t. Let the premium rate per unit of sum assured per period be q'_x, which is equal to the appropriate mortality rate for the period q_x reduced by discounting for half a period or one period if desired, and increased by any appropriate expense loadings. The mortality charge is therefore $S(t+1) \cdot q'_x$.

11. The second consideration in the event of death is whether or not the policyholder receives the value of the units standing to his account. This is most neatly dealt with by assuming that the policyholder forfeits his units on death, and that there is also due a sum assured on his death in the period $(t-1, t)$ which is a multiple, $k(t)$, of the value of units to his account. For an ordinary annuity, $k(t)=0$, and for a pure savings account, $k(t)=1$. The number of units available for survivors is $n(t-)$ given by:

$$n(t-1)=q_{x-1}\, k(t)\, n(t-)+p_{x-1}n(t-),$$

in which the $n(t-1)$ units at the start of the period have to be subdivided to give $k(t) \cdot n(t-)$ units to each of the deaths and $n(t-)$ units to each survivor. Hence:

$$n(t-) = \frac{n(t-1)}{p_{x-1} + q_{x-1}\,k(t)}$$

For an ordinary annuity, $k(t)=0$, $n(t-)=n(t-1)/p_{x-1}$, and we would say that the annuitant 'gets the benefit of survivorship'. For a pure savings plan, $k(t)=1$, the denominator is also 1, and $n(t-)=n(t-1)$. The formula for an assurance depends on whether the sum payable on death is just $S(t)$, in which case $k(t)=0$, or whether it is $S(t)$ plus the value of the units, in which case $k(t)=1$.

12. We can now write down the full recurrence relation that determines the number of units to the policyholder's account:

$$n(t) = \frac{n(t-1)}{p_{x-1} + q_{x-1}\,k(t)} + \frac{P(t)}{u(t)} - \frac{B(t)}{u(t)} - \frac{S(t+1)q'_x}{u(t)}$$

or in terms of money values at time t:

$$V(t) = \frac{V(t-1)\,u(t)}{u(t-1)\,(p_{x-1} + q_{x-1}\,k(t))} + P(t) - B(t) - S(t+1)q'_x,$$

so that the policyholder first gets the benefits of survivorship, if any, adjusted for the units on death in the previous period, then adds a premium, deducts any withdrawals, and pays for the next period's fixed sum assured.

13. This rather complex contract seems to do everything, hence its description as 'universal'. For assurances there is normally a positive premium, and no withdrawal, except perhaps the occasional partial surrender. The benefits on death may be wholly fixed in money terms, being wholly determined by $S(t)$, with $k(t)=0$; or more usually they may involve a refund of the units to the policyholder's account, together with a supplementary sum assured to make up some desired approximate total.

14. In the immediate annuity, there would normally be no sum assured on death, and the policyholder would abandon his units on death; hence $S(t)=0$ and $k(t)=0$. After the first period there would be no premiums, but there would be a benefit, $B(t)$, which in theory could be a variable amount on each occasion, though in practice it would be more administratively convenient for it to remain constant for, say, a year at a time.

15. With both the assurance and the annuity styles of contract there are underwriting considerations. The policyholder on his deathbed might increase his desired sum assured, or withdraw the maximum possible annuity benefit. In order to avoid such options against the office, the office would have to restrict adverse changes in the sum assured to policyholders who could give satisfactory evidence of good health. Similarly, they would have to restrict or delay increases in the amount of annuity benefit beyond some reasonable figure. It might be satisfactory to pay, for example, an extra year's benefit on death (charging for this in the formula), delaying any large increase in the benefits for a period of one

year. The appropriate mortality rates to use would depend on the precise underwriting conditions imposed.

16. A deferred annuity during the deferred period could be treated as a pure savings contract, or as an assurance with a sufficiently small sum assured for a significant fund to have built up by the date on which it is converted to an immediate annuity. Indeed, were it not for legislative and taxation considerations, a single contract could start effectively as a term assurance, with a very high sum assured for a young policyholder, turning to a savings contract as he gets older and can afford larger premiums, and on his retirement be converted into a variable annuity.

17. At each stage, it is desirable to advise the policyholder of the 'level maintainable benefit', which could be determined either in fixed money terms or in approximate index-linked terms. Let me explain this: if the initial premium rate is low and the sum assured high, the contract is effectively a term assurance, and in due course, the cost of the sum assured will exceed the value of units available to pay it, and the policy will expire with zero value. On the other hand, if the chosen sum assured is very low, the value of units will eventually rise so that a very large amount is available in old age. If it is assumed that premiums are held constant in money terms, and that the unit value grows at a fixed rate of interest, then there is some level sum assured which can be maintained indefinitely.

18. If it is assumed that the premiums are index-linked with price inflation, and that the unit value grows at a real rate of interest in excess (or below) this, then a corresponding level maintainable index-linked sum assured can be calculated. This involves practically the same calculation as the first, at a different rate of interest.

19. Thus, if we assume level money premiums of P per period, and assume that the value of units increases at the rate i per period, then the level maintainable fixed money sum assured $\bar{S}^M(t)$ is given by:

$$\bar{S}^M(t) = \frac{V(t) + Pa_x}{A_x}$$

where A_x and a_x are calculated at rate i. If we assume that the value of units increases at a real rate j per period, then the level maintainable index-linked sum assured $\bar{S}^I(t)$ is given by:

$$\bar{S}^I(t) = \frac{V(t) + Pa_x^i}{A_x^j}$$

If premiums are also assumed to be index-linked we replace a_x^i by a_x^j.

20. Correspondingly for the annuity: on the assumption of an appropriate money rate of increase of unit value, a level maintainable money annuity can be calculated. If the policyholder chooses a benefit greater than this, he has a chance of running out too soon; and if he chooses a benefit less than this, he has a chance of abandoning too much on his death. The same calculations can be carried out

on the assumption of a given real rate of growth of unit values, with an index-linked level maintainable benefit, or in terms of units themselves.

21. Thus, if we assume no future premiums, then the level maintainable fixed money annuity is given by:

$$\bar{B}^M(t) = V(t)/a^i_x,$$

the level maintainable index-linked annuity is given by:

$$\bar{B}^I(t) = V(t)/a^j_x,$$

and the level maintainable annuity fixed in number of units is:

$$\bar{B}^U(t) = n(t)/e_x,$$

i.e. calculated at zero rate of interest.

22. If the units are invested in a mainly money-based investment, such as fixed interest stock or deposits, then it would generally be appropriate for the sum assured or annuity benefit to remain fixed in money terms, and the level maintainable benefits to be defined in money terms. If the units are invested in such a way that they may be expected to maintain their real value, at least in the long run, such as by investment in ordinary shares, property or index-linked stocks, then it would be appropriate for premiums, sums assured and benefits to be index-linked. However, such matching is not essential to the contract. Whatever the actual investments, if the unit values increase more than expected, it will be possible to increase the level maintainable benefit, and if the value falls below what was expected, then the level maintainable benefit may reduce.

23. If the policyholder chooses benefits that are greater than the level maintainable ones, then he must expect the level maintainable benefit to fall, and vice versa if he chooses benefits less than the level maintainable ones. The choice is to a great extent up to him, subject to the restraints that the office may have to put on him.

24. Universal contracts may be written on two lives, whose ages at time t we may assume to be x and y. If the contract is designed to cease wholly on the first death, then we have a straightforward joint life assurance or annuity, almost identical to the single life contract already described, but with joint life mortality factors replacing single life ones. Thus the premium rate per unit of fixed sum assured q'_{xy} needs to be based on $q_{xy} = q_x + q_y - q_x \cdot q_y$, and the factors for survival or death in the formula in §§ 11 and 12 above are $p_{x-1,y-1} = p_{x-1} \cdot p_{y-1}$ and $q_{x-1,y-1} = 1 - p_{x-1,y-1} = q_{x-1} + q_{y-1} - q_{x-1} \cdot q_{y-1}$, respectively. The level maintainable benefits are calculated similarly.

25. If the contract is designed to continue until the second death, the position is more complicated. The policyholders may choose to have no adjustment to the contract on the first death. Until this occurs, the premium rate per unit of fixed sum assured would then be based on $q_{\overline{xy}} = q_x \cdot q_y$ and the factors for death and survival would be $q_{\overline{x-1,y-1}} = q_{x-1} \cdot q_{y-1}$ and $p_{\overline{x-1,y-1}} = 1 - q_{\overline{x-1,y-1}}$ respectively. The level maintainable benefits would be calculated using the appropriate last

survivor factors. While this is a possible form of the contract, there is the disadvantage that the level maintainable benefit jumps (down for an assurance, up for an annuity) on the occurrence of the first death. This is probably not what the policyholders really desire.

26. An alternative is to arrange the contract so that the number of units changes discretely on the first death. For an assurance, an appropriate increase in the number of units on the first death will allow the level maintainable sum assured to be the same before and after the first death, and for an annuity, an appropriate decrease will allow the level maintainable annuity to continue unchanged.

27. Let us again assume that the policyholders first forfeit all the units if they both die in the appropriate period, and their account receives a multiple, $k(t)$, of the value of the units if they both die (as in § 11); also that if (x) alone dies the multiple of units is $k_x(t)$ and if (y) alone dies the multiple of units is $k_y(t)$. We then get

$$n(t-) = \frac{n(t-1)}{p_{x-1}\,p_{y-1} + q_{x-1}\,p_{y-1}\,k_x(t) + q_{y-1}\,p_{x-1}\,k_y(t) + q_{x-1}\cdot q_{y-1}k(t)}$$

The values of $k(t)$, $k_x(t)$ and $k_y(t)$ could be chosen by the policyholders, but if they wish to maintain a particular level benefit they need to choose particular values which the life office has to advise.

28. For example, if they wish to maintain a level money sum assured $\bar{S}^M$ calculated initially from

$$\bar{S}^M(t-1) = \frac{V(t-1) + Pa_{\overline{x-1,y-1}}}{A_{\overline{x-1,y-1}}},$$

and so that if $U(t) = U(t-1)\ (1+i)$, i.e. the unit value actually grows at the assumed rate,

$$\bar{S}^M = \bar{S}^M(t) = \bar{S}^M(t-1),$$

then the values of $k_x(t)$ and $k_y(t)$ are given by

$$k_x(t) = \frac{\bar{S}^M\ A_y - Pa_y}{\bar{S}^M\ A_{\overline{xy}} - Pa_{\overline{xy}}}$$

and

$$k_y(t) = \frac{\bar{S}^M\ A_x - Pa_x}{\bar{S}^M\ A_{\overline{xy}} - Pa_{\overline{xy}}}$$

Thus if (x) dies, the level maintainable benefit for (y) is again $\bar{S}^M$, and if (y) dies the level maintainable benefit is similarly $\bar{S}^M$.

29. If a fixed money annuity B^M is the desired level maintainable benefit, given by

$$\bar{B}^M(t-1) = V(t-1)/a_{\overline{x-1,y-1}},$$

which equals $\bar{B}^M(t) = V(t)\ a_{\overline{xy}}$ if the value of the units grows at the assumed rate i, then

$$k_x(t) = \frac{a_y}{a_{\overline{xy}}},$$

and

$$k_y(t) = \frac{a_x}{a_{\overline{xy}}}.$$

30. It will be seen that the appropriate values of $k_x(t)$ and $k_y(t)$ depend on the ages of the lives during each period, and also on the basis on which the level maintainable benefit is calculated (i.e. on the rate of interest in the assurance and annuity factors). For the annuity, the values of $k_x(t)$ and $k_y(t)$ do not depend on the unit values, but for the assurance they do, since $\bar{S}^M$ depends on the relative value of unit reserve and the (assumed level) periodic premium. Thus, a change in the unit value different from what is assumed in the calculation of the level maintainable benefit requires a change also in the values of $k_x(t)$ and $k_y(t)$. This would all appear rather complicated to the policyholders, but it should be within their ability to ask the life office to maintain the benefit at the level maintainable one (on any chosen basis) and to advise the policyholders of the amounts accordingly.

31. It will be seen that the approach adopted here is Sheraton's method (b); neither of the other approaches he describes seems to me to be actuarially sound. It is necessary, however, for the office to ensure that it is notified of the occurrence of the first death. Sheraton's methods (a) and (c) represent attempts to avoid the necessity of requiring this information.

32. Nothing in the universal design of the contracts stops the policyholders choosing whatever values they wish (within limits) for the benefits. Thus, a separate cash sum assured could be chosen for each life, or the supplementary units could be chosen so that the level maintainable annuity was different after the death of each life, and so on.

33. As noted in § 1, the Variable Linked Life Assurance, is now offered by a number of companies. So far as I know, the Variable Linked Annuity has not been. Yet, ignoring any complication of tax, which is an appropriate approach for a pension benefit in the U.K., subject to any revenue limits on the total amount of annuity, the Variable Linked Annuity has considerable advantages. A fixed money annuity offers no protection against inflation. A variable annuity of the type once issued, where the benefit is defined as a fixed number of units, suffers from the disadvantage that the amount of benefit payable each period depends wholly on the value of the units at each payment date, and the fluctuations in this value may be unacceptable to the annuitant. An index-linked annuity offers protection against inflation, and a constant real benefit; but because the office has to calculate the benefit per unit of purchase price on the basis of the yield on index-linked stocks, this rate may be uncomfortably low. The Variable Linked Annuity I have described allows the annuitant to keep his

savings invested in the investment medium he prefers, including the possibility of investing in ordinary shares, and yet to keep his periodic benefit relatively constant. When will a life office offer it?

REFERENCE

SHERATON, D. (1984). Recent Developments in Unit-Linked Whole Life Policies, *J.S.S.* **27,** 105.