Contributions to the History of Insurance, and of the Theory of Life Contingencies, with a Restoration of the Grand Pensionary De Wit's Treatise on Life Annuities. By FREDERICK HENDRIKS, ESQ., *Actuary to the Globe Insurance Company.*

[Concluded from No. VI.]

(§ 16.) We now pass to the traces of the early practice of LIFE INSURANCE, a subject which has in general been but very slightly noticed.

The estimation of the value of human life in its rudest approxi-

mation—that is, of the term of years which, on the average of a sufficient number of observations, would be observed to fall to the share of each of a given number of individuals born under certain circumstances, and continuing in certain climates or ranks—was to all appearance a subject the consideration of which, not only the speculations of the remoter ages had no impelling cause to enter upon, but one which, if pursued, would have led not only to scepticism, but even to persecution, for entering on a tacitly admitted *forbidden* ground. As to the refinements of either the average duration or expectation of a life's survivorship at any given age, a still greater obstacle intervened; the numerical expression for weighing of probability, or *chances*, being quite beyond the purview of the older classical writers, and the term itself unused except as signifying *destiny*, or as a rhetorical definition of the value of opinion. That death did *not* preserve an order of difference in its devastations amongst the young and the old, was an express *dictum* amplified in poetry and in prose. The lessons of Nature's teaching, and personal hopes and fears, led the ancients to study "vital statistics" in one direction, that of *longevity*, and to their making comparative observations, rough though they may have been, on the prolongation of some assumed *ordinary* term of life. For instance, Eusebius quoted from Plato to the effect that many persons had lived to a great age, by reason of the serenity of the air and the almost continuous summer prevailing in Egypt and Syria; and observed, that therefore, from the earliest times, the *experience* of a long series of years had led to the *examination* of the facts, which had thus been handed down to his time with greater exactness. And the remarks of a similar kind on the relative mortality from different climates, and in various nations, occur in many other writings. Pliny, for example, in his well known observations on the census of Vespasian, and the enumeration of the inhabitants of the country between the Apennines and the Po, takes special care to distinguish the number of the persons living, at ages above 100. And although there are some fabulous accounts inserted in certain narratives of places and times distant from the personal knowledge of the authors, a close agreement is seen between the ancient and modern definitions of the terms of middle, old, and extreme old age. Scriptural allusions also confirm this with much precision; and, contrary to the opinion both of perfectibilians and deterioration-ists, we may infer that there was not that difference either way which they would have the world suppose.

(§ 17.) Notwithstanding the early neglect of the subject of

Life Contingencies, the Roman lawyers, at least about the time of the division of the Empire, found it necessary to consider and frame a table by which annuities could be valued, so as to meet the requirements of the Falcidian law, which prevented the testator from leaving more than three-fourths of his property to any others than legally constituted heirs. The ingenuity of the jurist was thereby taxed to supply some means for the valuation of such legacies as were charged upon the succession for limited terms and as annuities *for life*. One of the most eminent commentators on the Justinian Code, the Prætorian Præfect *Ulpianus*, gave a table of the estimated present worth of such life annuities. In modern times such valuations require, in the nature of things, as direct a consideration of the element of interest of money, as of the probabilities of survivorship; but although one or two writers have expressed an idea that some assumption of interest realizable on the testator's estate may have had its influence in fixing the values given by Ulpian, an argument the opposite way might perhaps be better supported; and particularly for this reason, that as it was the special duty of the jurist to look to a sufficient fund being created for the discharge of the obligations under such annuities, he is not likely in those times to have taken much account of *interest*, which at best could only have been continuously obtained with great difficulty. Taking, then, the meaning of the table to have been simply the *expectation of life*, it is manifest that the old Roman jurisprudence gave far more correct views of the comparative value of life at different ages than the moderns possessed in a popular way, until nearly the close of the Seventeenth Century!*

Ulpian's estimate betokens no mean skill and discrimination in the subject, and seems to bear intrinsic evidence of some careful collection or observation of facts. These may either have been obtained from inquiries on the results of like annuity engagements, or from returns of the number of deaths occurring within a given time at various ages; the former method would seem to have been the most likely to be available, but the other was quite within the

* In the *History of Life and Death*, it may be perceived that Lord Bacon has not given the most obscure hint of an attempt to discover an approximation to the laws of vitality or mortality, such as they are now understood. He restricts himself entirely to a curious collection of instances of human longevity, and to dissertations (after the fashion of the old thaumaturgists) respecting the prolongation of life by their usual specifics, some of which are common-place and indubitable, whilst the more extraordinary are replete with that degree of absurdity, that it is a marvel how they could have obtained further credit in the progress of the Elizabethan age.

bounds of possibility as the foundation of an approximate computation, for there is ample record of a kind of registration, or *ephemeris*, of deaths having been observed by the ancients.

Let us compare Ulpian's estimate with the first deductions ever made on the value of life in a city, by comparison of the deaths at all ages for a certain length of time with the *enumerated population* living at corresponding ages, and exposed to the risk of mortality during such term. I refer to Dr. Price's Table, No. XLVI., showing the probabilities of the duration of human life among males and females in Stockholm, formed from the proportions of the living to the numbers who died in Stockholm at all ages for 9 years, from 1755 to 1763. From Ulpian's calculation, which I have annexed, the reader will have the best opportunity of judging for himself how far the preceding observations, or inferences respecting it, can be borne out.

Ages.	Expectation of Males. Stockholm Life. (Dr. Price.)	Expectation of Females. Stockholm Life. (Dr. Price.)	Ages.	Expectation of Males and Females. Roman Life. (Ulpian.)
Birth	14·25	18·10	Birth to 20	30
5	31·05	37·12	20 to 25	28
10	30·00	36·89	25 „ 30	25
15	26·74	33·43	30 „ 35	22
20	23·85	30·01	35 „ 40	20
25	21·40	26·80	40 „ 41	19
30	19·42	23·98	41 „ 42	18
35	17·58	21·62	42 „ 43	17
40	15·61	19·25	43 „ 44	16
45	13·78	17·17	44 „ 45	15
50	11·95	15·12	45 „ 46	14
55	10·30	12·89	46 „ 47	13
60	8·69	10·45	47 „ 48	12
65	7·39	8·39	48 „ 49	11
70	5·81	6·16	49 „ 50	10
75	4·09	4·39	50 „ 55	9
			55 „ 60	7
			60 & upwards.	5

If the proper averages of columns 2 and 3 of the above table be compared with the figures in column 5, the results will be in close enough agreement to justify the inference that Ulpian's estimate must have been based on observation of actual results, though, it may be, limited to a small number, or inartificially combined.

(§ 18.) In an author of the Sixteenth Century I find a very curious example of the rough application of the probabilities of

survivorship to the discounting of sums receivable on such a contingency at the end of a given term. It is of the class of endowments for children, where the purchaser becomes an insurer to the extent of the premium which he pays, and which he loses in the event of the child's death; whilst, on the other hand, he is the insured, so far as respects the gain which will accrue in the event of the child outliving the specified term. Dr. (or Sir) Thomas Wilson, writing in 1569, observes,—*

"A merchant lendeth to a corporation or companie an hundred pound, which corporation hath by statute a grant, that whosoever lendeth such a summe of money, and hath a childe of one yeere, shall have for his childe, if the same childe doo live till he be full fifteene yeeres of age, 500 *li.* (£) of money; but if the childe die before that time, the father to loose his principall for ever: whether is this merchant an usurer or no? The lawe saith, If I lend purposely for gaine, notwithstanding the perill or hazard, I am an usurer."

To show the singular casuistry which could bias the judgment upon such a contract as the above, another example from the same writer may be adduced, though it is difficult to divine how in one case the transaction can be called a *purchase*, and in the other branded as of usurious character. The favourable view is in the following:—

"A corporation taketh a 100 *li.* of a man, to give him 8 in the 100 *li.*

* *Vide* "A Discourse upon Usurie, by waie of dialogue and oracions, for the better varietie and more delight of all those that shall read this treatise: by *Thomas Wilson*, doctor of the Civil lawes, one of the maisters of his maiesties honourable courte of requests. Imprinted at *London* by Roger Warde, dwelling neere Holburne Conduit, at the signe of the Talbot." 1584. (12mo. litt. goth.) Fol. 104. 6.

This work was composed at least 15 years previously; the Epistle dedicatory to "his most especiall and singuler deere Lord, the Earle of Leicester, &c. &c." being dated "From the Queenes maiesties Hospitall at Saint Katreynes, thys twenty of July, 1569." The author (Sir Thomas Wilson) was Tutor to Henry and Charles Brandon, sons of the Duke of Suffolk, and to the Princess Mary; Ambassador from Queen Elizabeth to Mary Queen of Scots, Secretary of State, Privy Councillor, and Dean of Durham. He died in 1581. *(See Bibliotheca Grenvilliana.)* In another work it is stated, that "in the reign of Mary he lived abroad, and was seized by the Inquisition at Rome, but escaped in consequence of a fire, which induced the populace to force open the dungeon that the prisoners might not be burnt." I am sure that the Holy Brotherhood could not fairly have taken him to task, if it had but known how zealous and orthodox a defender of its own views on the subject of interest of money existed in the person of Wilson, who took quite the Romanist and Canonical side of the question, and in his treatise does not so much as refer to the amended and faithful teaching on the subject by the great promoters of the Reformation. The discourse is one of great prolixity, occupying upwards of four hundred pages, in a Dialogue entitled "A Communication or Speeche betweene the riche worldlie merchant, the godlie and zealous preacher, the temporall and civil Lawiers, touching usurie, or the lone of monie for gaine." It will well repay perusal, and its earnestness is worthy of attention. It forms moreover a tolerably complete manual of the extraordinary quibbles and make-shifts by which the money trade of the 16th and preceding centuries was conducted; and all this is seasoned with a running artillery of theological denunciation, *supposed* allusions of Holy Writ, and direct attacks and anathemas of popes and councils.

during his life, without restitution of the principall. It is no usurie, for that here is no lending, but a sale for ever of so much rent for so much monie. Likewise it is, if a private man hath a thousand pound lieing by him, and demandeth for his life and his wive's life, a 100 *li.* by the yeere, and never to demand the principall. It is a bargaine of sale, and no usurie, for that the principall is not to be restored againe at anie time. And therefore no lending can be pre-supposed."

(§ 19.) Temporary insurances *on lives* were clearly not unfrequently effected about this time, or previous to it, to provide against hazards of a definite duration, and, like the contract of marine insurance, were doubtless little better than rough estimates of odds; but laboured under a disadvantage which was never the fate of the marine policy, viz., of being considered reprehensible and illegal. An interesting chapter of the *Guidon* is devoted to this subject, and is of importance as offering proof of much earlier traces of the various applications of insurance than are generally supposed to have existed. The work referred to can scarcely have a less antiquity than three hundred years. The following is the substance of the remarks made by Cleirac (Bourdeaux, 1661) :—

"The treatise entitled *Le Guidon* is a French production, and was formerly compiled for the benefit of the merchants trading in the noble city of Rouen. . . . Its author, in explaining the contracts or policies of insurance, has inserted much on other maritime contracts, so that he has omitted nothing except his name, which would have kept him in remembrance, and preserved him the honour which he deserves for having so much obliged his own country, and all other nations of Europe. But, as it is the ordinary result of the best productions to contract faults and obliterations from time, and that principally from the want of care, or the small intelligence of copiers and correctors of impressions, so this work had become so cut up with errors, faults of omission, and transpositions, that it was disregarded like a diamond in the rough, and quite obscure and unrecognizable."

The following is a full translation of the chapter, omitting Cleirac's notes, which are not material :—

"XVI.—1. In other countries, where the bodies of people may be captured, and reduced to bondage, there are various usages for the insurance of the body and life of men, whether they be of free condition, or slaves; which customs will not be mentioned here, because, in France, men of whatsoever nation are of frank and free condition.

"2. Notice only will be taken of what is practised in this

country, by those who undertake distant voyages, as to the coast of Italy, Constantinople, Alexandria, or other like voyages in the Mediterranean and Atlantic Seas, on account of the fear which they have of the galleys, *fustes*, and frigates of the army of the Turk, or corsairs, who make a traffic of the sale of Christians, whom they capture as well on sea as on land; which creates occasion for the masters and captains of this country, when they undertake such voyages, to stipulate with their merchant-freighters, or others, for the restitution of their persons, in case they are captured; and this they can do even for the people of their crew.

"3. In such a case, the master must in the policy estimate his ransom, and that of his companions, at so much per head; declare the name of the ship, the stay or touchings which it will make, the duration of each stay, and to whom the ransom is payable. The insurer is bound to pay the sum insured for the ransom 15 days after verification and certification of the captivity, without waiting for the usual two months' delay; and without other formality of seeing freightage, bill of lading, or charter party, it will suffice to produce the attestation of capture and the policy.

"4. Pilgrims going to the Holy Sepulchre of Jerusalem, or on other distant voyages, may effect insurance for their redemption, valued at a given amount. Description shall besides be made of their persons, names, surnames, country, abode, age, and rank; and, moreover, limit shall be made as to within what time they undertake to make and accomplish the voyage: the longest period shall be of three years inclusive, without admitting excuse of illness, or other detention whatsoever. In imitation of the preceding, those who undertake journeys or vows for a lengthened period, or a passage from one country to another, may insure for their ransom.

"5. Another kind of insurance is made by other nations upon *the life of men, in case of their decease upon their voyage*, to pay certain sums to their heirs or creditors. Creditors even may insure their debts, if their debtor *remove from one country to another*; the same can be done by those having rents or pensions, so as, in case of *their decease, to continue to their heirs such pension or rent as may be due to them.* Which are all stipulations forbidden, as against good morals and customs, from which endless abuses and deceptions arose, whence they have been constrained to abolish and prohibit the said usages; which is also to be prohibited and forbidden in this country."

(§ 20.) It was not only in France that life insurance was assumed unrecognizable by the law. The same is observed,—

1st. In the Netherlands Ordinance of Philip the Second, dated 1570.

2nd. In the civil statutes of Genoa, dated 1588; and the latter prohibition is of so peculiar a character in its information upon the practice of wagers on fortuitous events, that we venture to encroach on our small remaining space with a translation:—"Assurers are not held responsible for the barratry of the captain of the vessel, except it were otherwise made a condition. Securities, bonds, or wagers may not be made, without the licence of the senate, upon the *life* of the pope, nor upon the life of the emperor, nor upon the life of kings, cardinals, dukes, princes, bishops, nor upon the life of other lords or persons in constituted dignities, ecclesiastical or secular. Neither may they be made upon the acquisition, loss, or change of lordships, governments, kingdoms, provinces, duchies, cities, lands, or places . . . . . nor upon expected famine or war, or the contrary, nor upon the election of the governor or magistrates of the republic, and, in fine, upon all other transactions having this species or form of a bond, security, or wager (*vadimonii, securitatis, seu partiti*); but all are understood and are forbidden." (The remainder of the statute is occupied with an account of the attendant penalties.)

3rd. The 24th Article of the Amsterdam Ordinance of 1598 recites,—"We expressly prohibit insurance of the life of any person, and likewise wagers upon any voyage or frivolous purpose; and where they are made, we declare them void."*

4th. The Rotterdam Ordinances of 1604 and 1635 repeat the latter injunctions.

5th and lastly. The 10th Article (*Titre* 6) of the great French Marine Ordinance of Louis XIV., dated 1681, says, "We forbid the making of any insurance on the life of men." The 11th Article gives a qualification:—"Nevertheless, those who shall redeem captives may have the price of the redemption assured upon the persons whom they withdraw from slavery, which the assurers are bound to pay, if the redeemed on his way back is retaken, killed, drowned, or if he perish *by other means than natural death.*"

Even later than the Seventeenth Century, life insurance was regarded in France as obnoxious. In 1783, there remained a spirit of opposition to it. Émerigon, whose work on assurance (several times noticed in this inquiry) comprises more than 1300 quarto

* Cleirac's comment on another prohibitory clause, that against insuring perishable property, runs thus:—*Nec mors humano subjacet arbitrio.* Contre les effets du temps, de l'âge, et de la nature, toutes les asseurances et précautions humaines sont vaines et fort inutiles."

pages, devotes *one page* to the subject of life insurance, and that short space to the purpose of attacking the system. His remarks, omitting references, are as follow:—"At Naples, Florence, in England, and other places, assurances on the lives of men are allowed to be made; but this kind of assurances are not assurances properly so called, they are true wagers. These wagers, improperly called assurances, are prohibited in Holland and in several other countries. For a long time they have been prohibited in France; and this prohibition has been renewed by the marine ordinance. Man is beyond price. The life of man is not an object of trade, and it is odious for his death to become matter for mercantile speculation. And, as observes Grivel, these kinds of wager are of sad augury, and may occasion crimes. Such assurances are therefore absolutely void. The premium stipulated is not even due."*

(§ 21.) Here we get rather beyond our chronological order. We must retrace our steps to the Seventeenth Century. At that period, commercial pre-eminence had passed from the Southern nations of Europe to England and Holland. In the theory and practice of commerce and its institutions, France was under a disadvantage. The faculty of theology had power enough to prevent a minister of state establishing a bank; and in many other ways such adverse influence was used to a marked extent, and with important results. But all this led the van in the promotion of increased energy in other nations less trammelled by the oppressive force of dogmatic opinion; and a Frenchman, the great *Pascal*, not only cleared away many clouds of pseudo-philosophic obscurity, but established the first principles of that doctrine of chances, which was as necessary in the laying of any foundation of the theory of vital statistics, and its application to the calculation of life contingencies, as to the other greater sciences to which it has since been applied in its advanced perfection. Pascal, in his Provincial Letters, completely annihilated the traditionary definitions which the Jesuits were too willing to attach to the term *probability*, as suiting their own purposes; but it was in his mathematical correspondence that the true germs of scientific inquiry on the subject were embodied. Some questions put to Pascal by an ingenious

* The business of *Marine Insurance* also met with opponents in France in the last century. The anonymous author of "*Les Intérêts de la France mal entendus*" (a work which attracted much attention, and in other countries, and which was highly lauded for its patriotic views) observes, "Quoi qu'il en soit, je dirai hardiment que les assurances chez nous ont mis des entraves aux progrès de notre Commerce, et que si notre Administration n'y remédie, elles sont à la veille de causer son entière ruine." He goes on to argue that the world had done without assurances for 6000 years, and therefore might just as well dispense with them in his time. (!!) (See pages 46 to 59, Edition of 1757.) The reputed author was, I believe, the Chevalier Ange Goudard, of Montpelier.

man, the Chevalier de Méré, respecting the relative chances in certain conditions of games of hazard, led to his devoting his attention to sketching the goundwork of the science, in solving the cases proposed. This took place in 1654; but the letters to Fermat, in which the results were communicated, did not appear in print till 1679. The continental mathematicians were, however, informed of what was going on, long before the latter date. In Holland, the celebrated *Christian Huygens* wrote a treatise in Dutch, extending the circuit of Pascal's inquiries. The treatise was translated into Latin by *Francis Schooten*, who inserted it at the end of his *Mathematical Exercises*, printed at Leyden in 1657. These labours were quite sufficient to point out the fundamental proposition, that the probability of any event happening (or not happening) might be expressed by the ratio of the number of chances for its happening (or not happening, as the case may be,) to the total number of the chances for its happening *and* for its not happening. With the help of this discovery, and of some experimental observations of facts on the average duration of life among persons to whom the States of Holland had previously and occasionally granted annuities, the prime minister, John de Wit, prepared a report and treatise on the terms of such transactions. On the 25th April, 1671, it was resolved by the States General to negotiate funds by life annuities. On the 30th July, 1671, the resolution was affirmed; and on the same day, De Wit's Report was, at the request of several members, distributed to them. In presenting the reader with a restoration of the import of this Treatise, I have appended a few notes where it has seemed advisable, and, in all respects, the Grand Pensionary's own arrangement is preserved. On the importance of the original document,—on the accounts respecting it by various authors,—and on its having remained as good as lost for now one hundred and eighty years,—it will be my duty to speak in the subsequent part of this paper. Here I need only observe, that in its perusal the reader must particularly hold in recollection that it is entitled to be considered as the first known production of any age treating in a formal manner on the valuation of life annuities. The careful, anxious explanation entered upon by De Wit, respecting each step of the process by which he arrived at his conclusions, cannot fail to be worthy of notice, even apart from the practical importance and peculiar history of the Treatise, and the interest attaching to it from the deservedly honoured memory of its author. Without further preface, I annex the following English version:

(§ 22.)

"NOBLE AND MIGHTY LORDS,

"In so extensive an administration as that of the united country of Holland and West Friesland, it is better, as I have several times stated to your Lordships, for several reasons perfectly well known to you, to negotiate funds by life annuities, which from their nature are infallibly terminable, than to obtain them at interest, which is perpetual, or by redeemable annuities; and that it is likewise more useful for private families, who understand economy well, and know how to make a good employment of their surplus in augmenting their capital, to improve their money by life annuities, than to invest it in redeemable annuities, or at interest at the rate of 4 per cent. per annum; because the above-mentioned life annuities, which are sold even at the present time at 14 years' purchase, pay, in fact, much more in proportion than redeemable annuities at 25 years' purchase. I have consequently respectfully to submit to your Lordships the unchallengeable proof of my assertions, and at the same time to respond to the wish manifested by the members of this body to have such proof in writing. That proof, founded on a solid basis, is proposed to your High Mightinesses in the following manner:—

"*Value of Life Annuities in Proportion to Redeemable Annuities.**

"I lay down the following presuppositions in order to determine the proportion of a life annuity to a redeemable annuity. For example, in presupposing that the redeemable annuity is and will be current at 25 years' purchase, or at the rate of 4 per cent. per annum, we must find at how many years' purchase the life annuity should be sold, to be in proportion to the aforesaid redeemable annuity, in such manner that the life annuity may, if not with mathematical precision, at least in its discovered value, be more advantageous to the purchaser than an annuity redeemable with the same capital.

"FIRST PRESUPPOSITION.

"I presuppose that the real value of certain expectations or chances of objects, of different value, must be estimated by that which we can obtain from equal expectations or chances, dependent

* The title, in the original, is here "*Waardye van Lyf-renten naer proportie van Losrenten.*" Baily and other English writers, who have called it "De Vardye van de Lifrenten," &c., seem to have followed the French misquotation.

on one or several equal contracts. Let us take, for example, a small matter, and under circumstances intelligible at first sight: —A person has 2 different expectations or chances, which may easily lead, the one to nothing, the other to 20 stuyvers. If, by one or several equal contracts, he can obtain for 10 stuyvers 2 like expectations or chances, we must estimate that the 2 aforesaid chances are worth to him exactly 10 stuyvers, because he can really obtain for 10 stuyvers these 2 expectations or chances, by making an agreement with another person that each of them should stake 10 stuyvers, and then gamble or draw lots, by *odd or even, head or tail, blank or prize,* or in some such way, to determine which of the two should have the 20 stuyvers; thus by the said contract, equal in every regard, he evidently finds himself in the position of having in reality the 2 expectations or chances, the one of nothing, the other of 20 stuyvers.

"SECOND PRESUPPOSITION.

"That in taking at pleasure some years of a man's life, limited to the time when he is in his full vigour, and neither too young, nor too advanced in age (this space of time shall be here 50 years, namely, from the third or fourth year of his age, up to the fifty-third or fifty-fourth year), it is not more likely that this man should die in the first half-year of a given year, than in the second half: similarly, it is not more likely that he should die in the second half-year of the aforesaid year than in the first half. But although it depends entirely on chance whether this man, after having lived to the given year, and dying in the course of that year, should demise in its first or in its second half, one finds nevertheless in this regard an equality of likelihood or chance similar to the case of a tossed penny, where there is an absolute equality of likelihood or chance that it will fall *head* or *tail,* although it depends entirely upon chance as to the side on which it shall turn, and this to so high a degree that the penny may fall *head* 10, 20, or more times following, without once falling *tail;* and *vice versâ.*

"THIRD PRESUPPOSITION.

"That a man having passed the aforesaid vigorous time of his life, namely the fifty-third or fifty-fourth year of his age, it begins to be more likely that he should die in a given year or half-year of the second period than has previously been the case; or that it is not likely, with respect to another man of like constitution or state of body, that the latter should die in less than a year or half-year

of the said vigorous time of his life; whilst this likelihood or chance of dying in a given year or half-year of the 10 first following years, namely from 53 to 63 years of his age, taken inclusively, does not exceed more than in the proportion of 3 to 2 the likelihood or chance of dying in a given year or half-year during the aforesaid vigorous period of life: so that, taking for example two persons of equal constitution, one aged 40 years, and the other 58 years, if these two persons made such a contract, that in case the person of 58 years should happen to die in less than six months, the one aged 40 were to inherit a sum of 2000 florins from the property of the defunct; but that if, on the other hand, the person aged 40 years should die in less than six months, the one aged 58 years were to have 3000 florins from the property of the deceased; such a contract cannot be considered disadvantageous for the person who would have the 3000 florins, if the event were favourable to him, and who, in the contrary event, would only lose 2000 florins.

"I then presuppose that the greatest likelihood of dying in a given year or half-year of the second series of the 10 following years (that is, from 63 years to 73, taken one with the other, rather than in a given year or half-year of the period of the vigour of life), cannot be estimated at more than double, or as 2 is to 1; and as the triple, or as 3 is to 1, during the 7 following years, that is, from 73 years to 80.

"Finally, in supposing that life necessarily ends at the twenty-seventh year after the expiration of 50 years of age above presupposed, this time is neither assumed at too high nor too low a standard, as experience manifestly teaches us that the life of some men exceeds by a considerable period the age of 80 years, the age of 81 years, and even more.

"These three articles being presupposed, we have, by a demonstrative calculation, mathematically discovered and proved that the redeemable annuity being fixed at 25 years' purchase, as above, the life annuity should be sold at 16 years' purchase, and even higher, to be in equality one with the other; so that in the purchase of 1 florin of life annuity, on a young and vigorous nominee more than 16 florins should be paid, as is proved by the following demonstration:—

"First Proposition.

"The value of several equal expectations or chances, a certain sum of money or other objects of value pertaining to each chance,

is found to be exactly determinable by adding the money or other objects of value represented by the chances, and by then dividing the sum of this addition by the number of chances: the quotient or result indicates with precision the value of all these chances.

"To give greater clearness to the demonstration, let a person named John have, for example, 3 equal expectations or chances,—one of a certain pearl, or of 2000 florins; the second of a certain ruby, or of 3000 florins; and the third of a certain diamond, or of 4000 florins; as beneath.

Chances.			
1	pearl,	or 2000	florins.
1	ruby,	or 3000	,,
1	diamond,	or 4000	,,
3		3)9000	,,
		3000	florins.

I say that the 3 above-mentioned expectations or chances are together worth to him precisely the third of the 3 above-named jewels taken together, or 3000 florins, resulting from the 3 above-mentioned objects or sums of money, first added up, and then divided by 3, which is the number of chances.

"DEMONSTRATION.

"In the first place, let John have purchased in community with two other persons, namely Peter and Paul, and let each of the two have paid one-third of the value of the 3 jewels before mentioned; or rather that John with Peter and Paul have made common purse, by each contributing 3000 florins, which has evidently been an equal contract.

"In the second place, let John have agreed to cease his communityship with his two partners, or for other reasons to draw lots by three tickets, namely, two tickets *blank*, and one ticket *prize*, for the 3 above-named jewels, or the aforesaid common purse of 9000 florins of capital, so that each of them may draw one of the aforesaid tickets, and that Fortune may thus point out to which of them she assigns the above-named jewels or whole purse; which is again evidently an equal contract.

"In the third place, let John have agreed with Peter in particular, that if fortune favour one of them, in drawing the 3 aforesaid jewels or whole purse, the winner should give the loser the

pearl, or 2000 florins out of the purse; which is likewise evidently an equal contract.

"In the fourth place, let John have agreed with Paul in particular, that if the jewels or purse should fall by lot to one of the two, the winner should in compensation give the loser the ruby, or the 3000 florins out of the purse; which is again indisputably an equal contract.

"The four conventions or contracts being thus entered upon, the matter as concerns John is reduced to this: that he has 3 easy and equal expectations or chances; that is to say, one chance of the pearl, or of 2000 florins, if fortune favour Peter, who, in compliance with, and in virtue of, the third above-named contract, made with him in particular, must give up to John the pearl or 2000 florins;—one chance of the ruby, or of 3000 florins, if fortune favour Paul, who, from the tenor of the fourth contract, made with him in particular, must give John the ruby or 3000 florins;—and, lastly, one chance of the diamond, or of 4000 florins, if fortune favour himself (John), since, by virtue of the two aforesaid particular contracts, John having to hand over to Peter the pearl or 2000 florins, and to Paul the ruby or 3000 florins, yet retains for himself the diamond or 4000 florins;—which chances all proceed from the aforesaid jewels, or from the purse of 9000 florins, drawn by lot: so that, because John can obtain the proposed expectations by a third share of the 3 jewels, or by a capital of 3000 florins, such third of the 3 aforesaid jewels, or the capital of 3000 florins, is the real value of the expectations or chances proposed in the first presupposition. We will in the same manner demonstrate the proposition when there are 2, 4, 5, 6 equal expectations or chances, and even more, of objects of different value, provided that we assume in greater or lesser proportion as many contractors with or partners of John, as also in greater or lesser proportion as many particular contracts made with each of his partners; therefore the proposition is generally demonstrated.

"COROLLARY.

"From that which precedes, we may easily conclude that the before-described rule is not the less decisive, although some of the expectations or chances be of zero or nothing; because in such case the demonstration requires no further change than then to suppose one associate or partner more than the number of objects of value, relatively to the expectations; and further, that no contract like the above is made with the partners or associates.

"If, for example, John has the following expectations or chances, namely,—

Chances.		
1 of zero or nothing . . .	0	florins.
1 of a certain pearl . . .	2000	„
1 of a certain ruby . . .	3000	„
1 of a certain diamond . .	7000	„
4	4)12000	„
	3000	florins.

—I say that the 4 above-mentioned chances are together worth to him precisely the quarter of the 3 above-named jewels, or the sum of 3000 florins; for supposing that John, having bought with Peter, Paul, and Nicholas the 3 aforesaid jewels, or that each of them having furnished 3000 florins, they have made a common purse of 12,000 florins, and then that he makes a general agreement with them, and with Peter and Paul, each separately, but *not* with Nicholas, a special contract (similar to that made by him above), the matter as concerns John is reduced to this, that he has 4 equal expectations or chances; namely, one chance of zero or nothing, if fortune favour Nicholas, with whom he has not entered into a special agreement relative to any reciprocal reimbursement; —one chance of the pearl, or of 2000 florins, if fortune favour Peter, who, in such case, and by virtue of the special contract made between the two, has to give up the pearl to him, or make good 2000 florins;—one chance of the ruby, or of 3000 florins, if fortune favour Paul, who, in such case, and according to the special contract, must hand over to John the ruby or 3000 florins; —and, lastly, one chance of the diamond, or of 7000 florins, if fortune favour himself (John), because, by virtue of the above-mentioned special contracts made with Peter and Paul, to whom respectively he has to hand the pearl or 2000 florins, and the ruby or 3000 florins, he yet retains for himself the diamond or 7000 florins;—which chances all proceed from the aforesaid jewels, or from the purse of 12,000 florins, drawn by lot.

"And it is to be observed, that I have here expressly made use of an example or case of three objects of value, without expression of any sum, as in speaking of a pearl, a ruby, or a diamond, so as to cause the demonstration to be applicable to all sorts of numbers, to fractions as well as integer numbers, to irrational as well as to rational numbers, since all imaginable numbers may be applied to the value of these jewels.

"SECOND PROPOSITION.

"If any one has different equal expectations or chances, of which some will cause him to obtain each a certain sum of money or other object of value, and the others will produce him nothing at all; if, besides, he possess several other chances, each of a certain sum of money or object of value; and further, if he has some other chances, each of a certain sum of money or object of value, and so on;—we find the actual value of all the aforesaid chances, by multiplying each item or sum of money, relative to each expectation in particular, by the quantity or number of existing chances, then adding the products of the resulting multiplications of these partial operations: we finally divide the sum, or mass of partial products, by the collective number of chances; and the quotient indicates exactly the value of all these chances.

"Suppose, for example, that a person has the following chances of the objects or values annexed:—

Chances.	Each Chance of	
6	0	0
6	1,200	7,200
4	2,100	8,400
3	3,600	10,800
2	4,200	8,400
21		34,800

$$21\overline{)34{,}800}\left(1{,}657\tfrac{1}{7}\right.*$$

I say that all the above-mentioned chances are together worth to this person exactly $1{,}657\frac{1}{7}$; a value which we find, as is mentioned in the proposition, by multiplying each item, — namely, 1,200 by 6; 2,100 by 4; 3,600 by 3; and 4,200 by 2; then adding the products of these multiplications, that is to say, 7,200, 8,400, 10,800, and 8,400, and dividing the sum total, or 34,800, by 21, which is the collective number of chances.

"DEMONSTRATION.

"Because we can represent the above chances reduced to their unities, as well as their values, in the following manner:—

* In the original, the old galley form of Division (or, as the Dean of Ely terms it, the scratch method of division) is here employed, and also in the subsequent examples. I have not, however, thought it worth while to trouble the printers with the now strange and obsolete type it requires.

Chances			of
6	1	. . . . .	0
	1	. . . . .	0
	1	. . . . .	0
	1	. . . . .	0
	1	. . . . .	0
	1	. . . . .	0
6	1	. . . . .	1,200
	1	. . . . .	1,200
	1	. . . . .	1,200
	1	. . . . .	1,200
	1	. . . . .	1,200
	1	. . . . .	1,200
4	1	. . . . .	2,100
	1	. . . . .	2,100
	1	. . . . .	2,100
	1	. . . . .	2,100
3	1	. . . . .	3,600
	1	. . . . .	3,600
	1	. . . . .	3,600
2	1	. . . . .	4,200
	1	. . . . .	4,200
	21		34,800

And being so represented, according to the solution in the first proposition, their value would be found by adding all the values, *i. e.* zero or 0 six times; 1,200 six times, which is the same thing as if the number 1,200 were multiplied by 6; 2,100 four times, or as if 2,100 were multiplied by 4; 3,600 three times, or like that number multiplied by 3; and, lastly, 4,200 twice, or as though 4,200 were multiplied by 2; then by dividing the sum total by the number of chances, namely 21; and as the above summary addition of all the items, individually treated, evidently does not differ from the addition resulting from the multiplication of the items in the aggregate, but is identical, namely 0; 7,200; 8,400; 10,800; 8,400 (the sum of these final items being divided by the same number,—that is, by the sum of the collective chances, or 21), the same quotient must necessarily be obtained, which proves that the proposition is true.

"COROLLARY.

"It plainly results from the foregoing proposition, that in a strict sense it is not the number of chances of each value which we

must consider, in the application of the aforesaid rules, but son their reciprocal proportion. For it is manifest that the divisor augments or diminishes by the result of the addition of increase or decrease in chances, in the same proportion as the dividend or number to divide increases or diminishes by multiplication, when the ratio between the chances remains the same; so that, as in the aforesaid case, the divisor and dividend remain reciprocally in the same ratio or proportion, and the quotient remains unchanged, being the real value of all the chances together. This will be better understood by the preceding example, treated as follows in three different methods, with their solution:—

I.

Chances		of	
6	. . .	0	. . . 0
6	. . .	1,200	. . . 7,200
4	. . .	2,100	. . . 8,400
3	. . .	3,600	. . . 10,800
2	. . .	4,200	. . . 8,400
21, divisor.			34,800

$$34{,}800 \div 21 = 1{,}657\tfrac{1}{7}$$

II.

Chances		of	
1	. . .	0	. . . 0
1	. . .	1,200	. . . 1,200
$\frac{2}{3}$	. . .	2,100	. . . 1,400
$\frac{1}{2}$	. . .	3,600	. . . 1,800
$\frac{1}{3}$	. . .	4,200	. . . 1,400
$3\frac{1}{2}$, divisor.			5,800

$$5{,}800 \div 3\tfrac{1}{2} = 1{,}657\tfrac{1}{7}$$

III.

Chances		of	
18	. . .	0	. . . 0
18	. . .	1,200	. . . 21,600
12	. . .	2,100	. . . 25,200
9	. . .	3,600	. . . 32,400
6	. . .	4,200	. . . 25,200
63, divisor.			104,400

$$104{,}400 \div 63 = 1{,}657\tfrac{1}{7}$$

"From the reasons before mentioned, we obtain in the above three examples, by means of the operation of the rule, one and

the same quotient to determine the total value of all the chances, namely $1,657\frac{1}{7}$. (It would be the same in every similar case.)

"THIRD PROPOSITION.

"Each half-year of life is equally destructive or mortal to a person aged 3 or 4 years, to 53 or 54 years; in such period he is neither too young, nor too aged, to be wanting in the vigour needful for the prolongation of his days: so that there is not greater hazard nor likelihood that the day of his death should arrive rather in the first than in the second half-year of this vigorous period, and *vice versâ;* nor that the day of his decease should occur rather in these two aforesaid half-years, considered each in its individuality, than in the third half-year, and *vice versâ.* And thus with the other half-years during the aforesaid space of time.

"DEMONSTRATION.

"Any year of the vigorous period of life of the aforesaid person, being taken at pleasure, the first half of that year, or the first six months, is as destructive or mortal to him as the second six months. (According to the second presupposition.)

"And taking a second or other year of this period of the vigour of his life, in setting out from the second half-year of the first year taken, which ends consequently just six months after the expiration of that first year, the first half of the second year, which thus becomes the second half-year of the first year, is quite as destructive or mortal to him as the second half of the second year, which is thus the third half-year, reckoning as before. But, as the first half-year, as well as the third, is as destructive or mortal as the second, the first half-year and the third, compared with each other, are so likewise, since each of them in particular is as destructive or mortal as the second half-year; therefore, the aforesaid half-years, namely, the first, the second, and the third, each separately considered, are equally mortal.

"We might also demonstrate in the same manner that the second half-year and the fourth, when the one is compared with the other, are equally mortal; and again, that consequently the first half-year, the second, the third, and the fourth, each considered by itself, has the same chance of destructiveness: it is the same thing for all the preceding or subsequent half-years, comprised in the above time of the vigour of life;—which was to be demonstrated.

"COROLLARY.

"It results from what precedes, and from the third presupposition, that as life annuities are paid in all the Offices of Holland and West Friesland by half-yearly instalments, or from six months to six months, that the annuitant loses all his capital, and receives no return whatever from it, if the life upon which the annuity is sunk happen to die in the first half-year after the purchase, or do not live six whole months. The annuity sunk is here supposed to be 1,000,000 of florins, or 20,000,000 stuyvers, per annum, in order that an exact calculation may be made without fractions: therefore, if the above-mentioned life survive a complete half-year, and do not die until in the course of the second half-year, the annuitant has then drawn 10,000,000 stuyvers, from which a deduction being made of 4 per cent. per annum for a half-year, it would have been worth to him in ready cash (that is to say, on the day of purchase of the said annuity) 9,805,807 stuyvers, which he would have had to pay, if taken at the true value. If the above life survive so long as two complete half-years, and die in the third half-year, the annuitant has then drawn 10,000,000 stuyvers after the expiration of the first half-year, and after that of the second half-year likewise 10,000,000 stuyvers; which sums, deduction being made at 4 per cent. per annum, one for a half-year or six months, and the other for a complete year, would have been worth to him in ready cash, or upon the day of purchase of the said annuity, 19,421,192 stuyvers, and so on, according as the day of decease were to occur in the fourth, fifth, sixth, or further number of half-years, which would have been worth to him each time as many terms or half-yearly sums of 10,000,000 stuyvers as complete half-years had elapsed from the time of the purchase of the annuity, deduction being made as above of the respective discounts. The computed amounts are specially given in the following table:—

If the Nominee survive the following Term of Life.

Half-years.	Stuyvers.
0	0
1	9,805,807
2	19,421,192
3	28,849,853
4	38,095,415
5	47,161,435
6	56,051,398

Half-years.	Stuyvers.
. . . . .	. . .
98	431,055,833
99	432,490,825
100	433,897,951
101	435,277,751
. . . . .	. . .
118	455,030,042
119	455,999,472
120	456,950,076
121	457,882,220
. . . . .	. . .
138	471,226,168
139	471,881,080
140	472,523,275
141	473,152,998
. . . . .	. . .
152	479,322,884
153	479,820,563
. . . . .	. . .
199	494,754,836
200	494,952,836

['The above table having been calculated very accurately by us the undersigned, Bookkeepers to My Lords the States-General, each separately, and having been collated by us, we find that a perfect agreement exists, without there being any errors in the figures.

(Signed) 'T. BELLECHIERE.—JACOB LENSE.' *]

"Thus, then, since an annuitant, having purchased and sunk a life annuity upon a young nominee, has in possession, or in his favour, as many different expectations or chances as there are half-years in which the death of the nominee may occur;—since the first 100 different expectations or chances (comprising the term of 50 years, reckoning from the day of the constitution or purchase of the annuity,) may result with the same facility, and relatively

* The above Table, computed to such a nicety by De Wit's directions, is composed of the progressive summations of the present values of 1 Million Florins or 20 Million Stuyvers per annum, receivable in 100 half-yearly instalments for 50 years. The second and every even term will be found correct, on the supposition of discount at 4 per cent. per annum; but the first and every odd term erroneous, in the same way that the remark is applicable to Smart's and Tetens' (or Von Drateln's) Tables, at intermediate half-years, by reason of the interest being reckoned by a geometric instead of by an arithmetic mean. —In the original a complete table is given from 1 to 200 half-years, which, however, it is useless to repeat in full, as the even terms may be obtained by an easy process from the data in other works, and the odd terms are inapplicable to modern purposes.

Struyck, in his *Uitrekening van de Lyfrenten*, has some remarks on the "prodigious labour" of the two Bookkeepers who calculated the Table, although when we compare it with similar ordinary computations of more modern times it is relatively not worthy of such an appellation. At the present date, the tendency is certainly to underestimate such labours; a reaction to the *juste milieu* may, however, take place after a surfeit of Statistics.

to their probability are equal;—since during this term each half-year of the aforesaid nominee's life is equally destructive or mortal (which is demonstrated in the third proposition);—since the following 20 chances or expectations (comprising the first 10 years after the expiration of the 50 years above cited), considered one with the other, each in proportion to each of the first 100 chances, are not in a lower ratio than 2 to 3 (according to the third presupposition);—since the 20 expectations or chances of the 10 following years (comprising the second series of 10 years after the expiration of the first 50 years), also considered one with the other, each in proportion to each of the first 100 expectations or chances, are not in a lower ratio than 1 to 2 (according to the third presupposition);—since the 14 following expectations or chances (comprising the 7 years after the expiration of the two preceding decennial terms, the epoch at which we here suppose the man to terminate his life), taken one with the other, each in proportion to each of the first 100 expectations or chances, are not in a lower ratio than 1 to 3;—it follows that the aforesaid annuitant has in possession, or in his favour, more chances or expectations than there are in the following table:—

Chances		of Stuyvers.		The Life to survive Half-years.
1	. .	0	. .	0
1	. .	9,805,807	. .	1
1	. .	19,421,192	. .	2
1	. .	28,849,853	. .	3
1	. .	38,095,415	. .	4
1	. .	47,161,435	. .	5
1	. .	56,051,398	. .	6
.	. .	. . .		7 to 97 given in original
1	. .	431,055,833	. .	98
1	. .	432,490,825	. .	99
	Sum	28,051,475,578		Once = 28,051,475,578 *
2	. .	433,897,951	. .	100
2	. .	435,277,751	. .	101
.	. .	. . .		102 to 117 in original
2	. .	455,030,042	. .	118
2	. .	455,999,472	. .	119
	Sum	8,911,946,713		Two-thirds = 5,941,297,809
			Carried forward .	33,992,773,387

* The addition here presents a clerical error. It should give 281, &c., instead of, as

Chance	of Stuyvers.	The Life to survive Half-years.
	Brought forward . .	. . 33,992,773,387
$\frac{1}{2}$	. . 456,950,076	. . 120
$\frac{1}{2}$	. . 457,882,220	. . 121
.	. . . . .	122 to 137 in original
$\frac{1}{2}$	. . 471,226,168	. . 138
$\frac{1}{2}$	. . 471,881,080	. . 139
	Sum 9,297,075,282	One-Half = 4,648,537,641
$\frac{1}{3}$	. . 472,523,275	. . 140
$\frac{1}{3}$	. . 473,152,998	. . 141
.	. . . . .	142 to 151 in original
$\frac{1}{3}$	. . 479,322,884	. . 152
$\frac{1}{3}$	. . 479,820,563	. . 153
128	Sum 6,668,408,125	One-third = 2,222,802,708
		Total . 40,864,113,736

*40,964,113,736, divided by 128, gives 320,032,130 $8\frac{9}{16}$, which divided by 20 gives 16,001,606 18 9.

"Whence it follows that we can immediately determine, by a mathematical calculation, according to the principle of the second proposition above enunciated, the worth to the aforesaid annuitant of all the above-mentioned chances, taken together, always presupposing that such value is payable in ready money on the day of purchase of the annuity; and the method is as follows:—

"Since the first 100 items, each taken once, or each multiplied by the number 1, form the sum of 28,151,475,578 stuyvers;—since the 20 following items, two-thirds of each being taken, or each multiplied by $\frac{2}{3}$ (or, which is the same thing, two-thirds of the sum of the aforesaid 20 items), produce a sum of 5,941,297,809 stuyvers;—since then the half of the 20 following items gives a sum of 4,648,537,641 stuyvers, and the third of the 14 following and last items that of 2,222,802,708 stuyvers;—these sums, being

above, 280, &c.; and the general summation 409, &c., instead of 408, &c. This proceeds no further; for in the working of the annuity valuation, the total is correctly given by De Wit.

I particularly refer the reader to a further note on the subject, at the end of this treatise of De Wit; where I have thought it right to make a few observations on the Burgomaster Hudde, and his certificate, which bears in rather a singular manner on the above casual mistake in the addition.

* 40,964,113,736 is here correctly given by De Wit.

combined, amount together to the sum of 40,964,113,736 stuyvers; which being divided by 128 (the number of chances added together), we find for a quotient (that is to say, the real and exact value of all the collective chances,) the sum of 320,032,130 9 stuyvers, or 16,001,607 florins: so that 1,000,000 per annum of life annuities, sunk or purchased on a young life, is worth in fact more in ready money, and should consequently be sold for more than 16,001,607 florins,* preserving the right proportion above mentioned; *i. e.*, that proportionately each florin per annum of life annuity is worth more at 16 florins than the interest of a redeemable annuity at 4 per cent. per annum,—and consequently the person who for 16 florins has purchased 1 florin per annum on a young, vigorous, and healthy life, has made a remarkably advantageous contract;—I assert it to be remarkably advantageous for the following reasons:—

"Because, in the first place, we have not been able to rate at a certain price, by perfect calculation or correct estimation, the power which the annuitant possesses (power which is of very great value

* De Wit's calculation may be simplified and explained as follows:—*Firstly.* Out of 128 lives, aged say 3 years, 1 is supposed to die in every half-year of the first 100 half-years, or 2 per annum for 50 years, leaving 28 alive, aged 53 years, at the end of the term; out of whom 1 dies in every 9 months, being $0\dot{6}$ per half-year during the next 20 half-years, or $1\cdot\dot{3}$ per annum for 10 years, leaving $15\frac{1}{3}$ alive aged 63 years at end of the second term; of whom 1 dies in every year for 10 years, being 0·5 per half-year during the next 20 half-years, leaving $5\frac{1}{3}$ alive aged 73 years at end of the third term; of whom 1 dies in every year-and-a-half for 7 years, being $0\cdot\dot{3}$ per half-year during the next 14 half-years, leaving 1 alive aged 80 at the end of the fourth term; which survivor does not live over another half-year. *Secondly.* Out of the 128 lives, those who die in the respective half-years between the ages of 3 and 80, will receive an annuity certain in half-yearly instalments, for a term equal in continuance to the number of *completed* half-years elapsed between age 3 and the date of their death; therefore, the sum of the present values of half-yearly annuities certain, for the corresponding terms multiplied into the numbers *dying* within such respective terms, gives the present worth of all the annuities which will be enjoyed by the 128 lives, $\frac{1}{128}$ of which represents the present value of the single-life annuity at age of, say, 3 years. The system of valuation is therefore identical with the fifth method described by *Tetens*, whose formula I have had the pleasure to refer to on a previous occasion. (See the *Assurance Magazine*, No. I., p. 9, and 18; and No. II., p. 18.)

If arranged in the modern form of a life table, the following abstract would represent the course of the results of De Wit's suppositions as to mortality:—

Half-year Number.	Age.	Number of living.	Decrements.
1	3	128	1
2	$3\frac{1}{2}$	127	1
. . . . .	. . . . .	. . . . .	.
99	$52\frac{1}{2}$	29	1
100	53	28	$0\cdot\dot{6}\dot{6}$
101	$53\frac{1}{2}$	$27\cdot\dot{3}\dot{3}$	$0\cdot\dot{6}\dot{6}$
102	54	$26\ \dot{6}\dot{6}$	$0\cdot\dot{6}\dot{6}$
. . . . .	. . . . .	. . . . .	.
120	63	$15\cdot\dot{6}\dot{6}$	0·50
121	$63\frac{1}{2}$	$15\cdot\dot{1}\dot{6}$	0·50
. . . . .	. . . . .	. . . . .	. .
140	73	$5\cdot\dot{6}\dot{6}$	$0\cdot\dot{3}\dot{3}$
. . . . .	. . . . .	. . . . .	.
154	80	1 00	1·00

to him) of choosing a life, or person in full health, and with a manifest likelihood of prolonged existence, upon whom to constitute or purchase his annuity; and there is much less risk or danger of a select, vigorous, and healthy life dying in the first half-year than in some of the following half-years at the beginning of which the aforesaid life might perhaps prove to be in a weak state of health or even in a fatal illness; and such greater likelihood of prolongation of life in the purchase of an annuity upon a select, healthy, and robust life, may further extend itself to the second, third, and some other following terms or half-years.

"In the second place, the advantage resulting from the aforesaid selection is so much the more considerable, as one half-year of life, at the commencement of and shortly after the purchase of the life annuity, is of greater value to the annuitant, with respect to the price of such purchase, than eighteen half-years during which the person upon whom the annuity is purchased might live after the said purchase, from the age, for example, of 70 to 79 years,—a circumstance which, although at first sight it appears strange and paradoxical, is nevertheless real and susceptible of demonstration.

"In the third place, although each of the first 100 half-years expiring after the purchase be considered as equally destructive or mortal, according to the principle of the before-established calculation, by reason of the scarcely appreciable difference existing between the first and second half of each year, it is however certain, when we examine the matter very scrupulously, that the likelihood of decease of the nominees upon whom life annuities are usually purchased is less considerable, and smaller in the first years after the purchase than in the subsequent years, seeing that the said life annuities are oftenest purchased and sunk upon the lives of young and healthy children of 3, 4, 5, 6, 7, 8, 9, 10 years, or thereabout. During that time, and for some years ensuing, these young lives, having become more robust, are less subject to mortality than about 50 years afterwards, and than for some years anterior to these 50 years; and so much the more, as during the first aforesaid years they either are not, or are but little, exposed to external accidents and extraordinary causes of death, such as those from war, dangerous voyages, debauch, or excesses of drink, of the sex, and other dangers;—for females, there are also confinements and other like causes;—so that the first years after the purchase or foundation of the annuity are the least dangerous, which is a considerable advantage for the annuitant, particularly if we reflect, as I have above stated, that one of the said first years may, as

regards the original price of purchase, balance a great number of the subsequent years.

"Finally, and in the fourth place, it might also evidently occur, that the life upon which the annuity has been sunk were to live more than 77 years after the purchase, being the time supposed in the above calculation as the term of human life, although such considerations cannot be of much importance; for, notwithstanding that by presupposing the aforesaid nominee living still longer than the expiration of the said term, and preserving life up to the hundredth year inclusive, so that the annuitant or his heirs were to receive 46 more entire half-years of annuity, after the expiration of the term of the aforesaid 77 years, this could not, however, increase the price of the life annuity (calculated, as precedes, at above 16 years' purchase, *i. e.*, at more than 16 florins of capital for 1 florin of annuity per annum) by more than $14\frac{1}{2}$ stuyvers of the same capital; and even if the annuitant could be assured that his heirs were, after the expiration of the above 100 years, to enjoy the life annuity from half-year to half-year, and that perpetually, the value of the capital at the time of first purchase would not thereby be increased by 10 stuyvers.

"Whence likewise, although it may be considered that the latter years are not established as sufficiently destructive and mortal in the aforesaid presuppositions, and in the calculations upon which I have based them, when compared with the anterior years and the time of life's vigour, we easily conclude that it could not cause an appreciable rise in the price of the purchase found by the above calculation, which in fact is true, even on the presupposition of each half-year of the 10 years after the sixtieth year of the purchase being, instead of twice, three times more destructive and mortal than each half-year of the first 50 years, and of each half-year of the 7 subsequent years being, instead of three times, five times more destructive and mortal than each of the aforesaid first years; and even on the presupposition again, as above, that the said nominee would not survive beyond 77 years after the first purchase. All these presuppositions (which, however, manifestly represent the life as subject to too high mortality) could scarcely reduce by 6 stuyvers the aforesaid 16 florins or value of the before-described annuity. In consequence of all these reasons, we may assume it as established and demonstrated, that the value of a life annuity, in proportion to the redeemable annuity at 25 years' purchase, is really not below, but certainly above, 16 years' purchase; so that a person, wishing to purchase a life annuity in such proportion and

according to its real value, ought to pay more than 16 florins for 1 florin of annuity per annum.

"Besides the consideration that this calculation has been made on the principle of a deduction of 4 per cent. per annum, at compound interest, and this with such benefit to the purchaser of the life annuity that he would realize not only the interest per annum, but also, without any intermission, interest upon interest at 4 per cent. per annum, as though he could always thus advantageously make use of his money in purchase of annuity; it is constant that one could not always find such opportunity of investing it, and that one is sometimes obliged to let it lie fallow for some time, and often to lend it at a materially smaller interest, to provide against a greater loss.

"Even besides this, as the capital of life annuities is not subject to taxation, nor to a reduction to a lower amount of annuity or interest, it follows, that if the blessing of the Almighty continue to be vouchsafed to this country, we may consider the life annuity as much more advantageous to the annuitant than the redeemable annuity, as may manifestly be judged by the example of foregoing times,—by reflecting, in fact, that My Lords the States of Holland and West Friesland have in the course of a few years not only increased the charge for life annuities from 11 years' purchase to 12 years' purchase, and from 12 years' purchase to 14 years' purchase, but that these annuities have been sold, even in the present century, first at 6 years' purchase, then at 7 and at 8, and that the majority of all life annuities now current and at the country's expense were obtained at 9 years' purchase; which annuities, by reason of the successive reductions of the rate of interest from $6\frac{1}{4}$ to 5 per cent., and then from 5 per cent to 4 per cent., produce to the annuitants an actual profit of nearly one-half of each half-year's payment, and of more than one-half in the case of those annuities which were obtained at 8 years' purchase or under.

"J. DE WIT."

"I, the undersigned, declare, that having attentively read and examined, at the request of My Lord the Grand Pensionary of Holland and West Friesland, the above propositions, and the conclusions thence derived, for finding the value of a life annuity, compared with that of a redeemable annuity at 4 per cent. per annum, I am of opinion that the method employed for that effect is perfectly discovered, and that the conclusion made therefrom (namely, that the purchaser of a life annuity still makes a gain in

stipulating for 1 florin of annuity per annum for 16 florins of capital) depends upon solid and incontestible mathematical foundations, provided that some fault of a figure has not been involuntarily made in the calculation of the table, or in the copy and addition of the items, which is an ordinary and well-known computation, but which I have not made.

(Signed) "J. HUDDE." *

(§ 23.) Leibnitz is the first author known to have drawn attention to the researches of De Wit on life annuities. We are told in Montucla's "*Histoire des Mathématiques,*" *that when Leibnitz was in Holland* (which was not many years after De Wit's death) *he made every effort to obtain a copy of the Grand Pensionary's Treatise, but without success.* Whether the circumstance of this search is mentioned more at length in the Baron's voluminous correspondence I have not an opportunity of learning, but, in his reply to the Reflexions of Bayle, written about the year 1694, an interesting general notice occurs on the important application, by De Wit, of the doctrine of chances to life annuities, and mention is made of Hudde's labours on the same subject.†

* It is difficult to attempt to fathom the inducements for the latter clause of limitation in the Burgomaster Hudde's certificate. The clerical error in the mere addition, and *not* the application or final result of De Wit's table for the valuation of an annuity, seems to be what is there referred to by Hudde. We have briefly mentioned this lapse of the pen, in a former note; and it is here necessary to state, that not only is it shown to be no more than that, by the present test of a fresh addition of the items, but the accuracy of the applied or amended final total is amply proved by the application of two formulæ for the summation of temporary annuities increasing in the order of the natural numbers. The motive for such a trial has been simply the apprehension that the terms in which Hudde's certificate is couched might, in the absence of explanation, throw a *primâ facie* doubt on his concurrence in the absolute result, though *not* on the principle of De Wit's calculation; or, otherwise, it might lead to the idea that it was an invidious remark on the part of Hudde, who could have *tested* the result. Of De Wit's accurate application of the data laid down, the examination of the figures leaves no question; and respecting the terms employed by Hudde, we ought to be willing to suppose an explanation, in the likelihood of his having been requested by the Grand Pensionary to give his certificate on this new and very remarkable computation, *at the last moment*, before its presentation to the States-General; thus not leaving him the needful time to examine it in all its details. This view becomes the more reasonable, when we discover what is said of the busy responsibilities of Hudde's municipal duties, the anxieties of which, in those trying times of the history of the Netherlands, must have been of pressing importance. As confirming this, I extract a passage which occurs in a recent German Biography of Leibnitz:—"*Ich habe, in Amsterdam, mit Hudden gesprochen*, welchem aber die Geschäfte für den Staat alle Zeit wegnehmen. *Denn er ist einer von den zwölf Bürgermeistern der Stadt, welche nach einander die Regierung führen; kurzlich war er regierender Bürgermeister, jetzt verwaltet er das Amt eines Schatzmeisters. Es ist gewiss, dass seine Papiere noch sehr vortreffliches aufbewahren. Die von Slusius bekannt gemachte Methode der Tangenten war ihm lange bekannt gewesen*," *&c.* (Extract from letter of *Leibnitz* to Oldenburgh, dated $\frac{18}{28}$ November, 1676, and quoted in a letter from Collins to Sir Isaac Newton.) See *Guhrauer's Gottfried Wilh. Freiherr von Leibnitz.* Breslau, 1846. 1 vol. p. 183.

† I here quote the original passage, apprehending that it would lose much of its epigrammatic terseness in any translation:—"J'ai presque ri des airs que M. le Chevalier de Méré s'est donné dans sa lettre à M. Pascal, que M. Bayle rapporte au même article.

Leibnitz's fullest remarks on De Wit are to be found in his *Nouveaux Essais sur l'Entendement Humain*, a posthumous work, but which was chiefly composed before the year 1700. He there observes:—"The mathematicians of our day have begun to estimate chances in relation to games. The Chevalier de Méré, whose *Agrémens* and other works have been printed, a man of subtle wit, and who was a gamester and philosopher, gave a start to the matter by framing questions upon stakes, in order to ascertain how much the game was worth if interrupted in such or such a state. By this he induced his friend M. Pascal to examine a little into these things. The question spread, and gave M. Huygens occasion to compose his treatise *De Aleâ*. Other learned men entered upon the subject. Certain principles were established, which were also made use of *by the Pensionary De Wit, in a short discourse, printed in Dutch, upon life annuities*. The foundation built upon amounts to a *prostrapherese* (so in original), that is to say, to the taking of an *arithmetical mean* between several equally admissible suppositions; and our peasantry have made use of such for a long time according to t heir *natural mathematics*. For example, when any heritage or land has to be sold, they form three companies of valuers; these valuers are called *Schurzen* in Low Saxon, and each company makes a valuation of the property in question. Suppose, then, that one estimates it to be of the value of 1000 crowns, the second of 1400, the third of 1500,—they take the sum of these 3 valuations, which is 3900; and because there have been 3 valuations, they take the third, namely 1300, for the mean value required; or, which amounts to the same thing, they take the sum of the third parts of each valuation. It is the axiom *æqualibus æqualia*, for

Mais je vois que le Chevalier savoit, que ce grand Génie avoit ses inégalités, qui le rendoient quelquefois trop susceptible aux impressions des spiritualistes outrés, et le dégoutoient même par intervalles des connaissances solides, ce qu'on a vu arriver depuis, mais sans retour, à Messieurs Stennonis et Swammerdam, faute d'avoir joint la métaphysique véritable à la physique et aux mathématiques. M. de Méré en profitoit pour parler de haut en bas à M. Pascal. Il semble qu'il se moque un peu, comme font les gens du monde, qui ont beaucoup d'esprit et un savoir médiocre. Ils voudroient nous persuader que ce qu'ils n'entendent pas assez est peu de chose. Il auroit fallu l'envoyer à l'école, chez M. Roberval. Il est vrai cependant que le Chevalier avoit quelque génie extraordinaire, même pour les mathématiques, et j'ai appris de M. des Billettes, ami de M. Pascal, excellent dans les Méchaniques, ce que c'est cette découverte, dont ce Chevalier se vante ici dans sa lettre. C'est qu'étant grand joueur, il donna les premières ouvertures sur l'estime des paris; ce qui fit naître les belles pensées *De Alea*, de Messieurs Fermat, Pascal, et Huygens, où M. Roberval ne pouvoit, ou ne vouloit rien comprendre. *M. le Pensionnaire de Wit a poussé cela encore davantage, et l'applique à d'autres usages plus considérables, par rapport aux rentes de vie; et M. Huygens m'a dit, que M. Hudde a encore eu d'excellentes méditations là-dessus, et que c'est dommage qu'il les ait supprimées avec tant d'autres.* Ainsi les jeux mêmes mériteroient d'être examinés; et si quelque mathématicien pénétrant meditoit la-dessus, il y trouveroit beaucoup d'importantes considérations; car les hommes n'ont jamais montré plus d'esprit que lorsqu'ils ont badinés."

equal suppositions we must have equal considerations. But when the suppositions are unequal, they are compared with each other. Let it be supposed, for example, that with two dice one person is to gain if he throw 7, the other if he throw 9: it is asked what proportion is found between their chances of winning? I say that the chance for the latter is only worth two-thirds of the chance for the former, because the former may throw 7 in three ways with two dice, namely, by 1 and 6, or 2 and 5, or 3 and 4; and the other can only throw 9 in two ways, by 3 and 6 or 4 and 5. And all the above ways are equally possible. Therefore, the probabilities (*les apparences*) which are equal, like the numbers of the possibilities, are as 3 to 2, or 1 to $\frac{2}{3}$. I have said more than once that a *new kind of logic* is wanting to treat upon the degrees of probability, since Aristotle in his *Topics* has fallen short in nothing more than that, having contented himself with arranging, in some measure, certain popular rules, distributed in their ordinary position, which may serve upon any occasion where it is required to amplify discourse and to give it some probability, without taking the trouble to give us a needful balance to weigh the probabilities (*apparences*) and to form a solid judgment thereupon. It would be well that he who wishes to treat on this matter should pursue the examination of *games of chance;* and, generally, I wish that a skilful mathematician would compose an ample work, well elaborated and well digested upon all kinds of games, which would be of great use to give perfection to the art of inventing, the wit of man appearing better in games than in more serious matters."

This idea of the utility of investigating games of chance, was a systematic hobby of Leibnitz. On being asked wherein its utility consisted, he replied, "*C'est pour perfectionner l'art des arts, qui est l'art de penser.*" These and similar observations (see last note), which he lost no opportunity of enforcing on the notice of the mathematicians of his time, induced *Rémond de Montmort* to pursue the investigation, and the result was his well-known work, the *Essay d'Analyse sur les Jeux de Hazard.* In the advertisement to the second Edition (*Paris*, 1713), De Montmort observes:—

"I have learnt that M. Hudde and the famous M. Wit, Pensionary of Holland, gave some calculations of the interest, by way of life annuities, suitable to persons at various ages. It would seem that what they have given differs little from that which I have related of Mr. Halley, and supposes a knowledge of the different degrees of mortality of the human race. What there is of calculation is very easy, and depends almost entirely on the solution of

the following problem, which M. Leibnitz solved in 1683 in a very elegant manner, viz. :—'*To find the present value of any sum payable at the end of any number of years.*' The great difficulty is to have exact tables, like those which Mr. Halley made use of for the foundation of his calculations. It is to be wished that the observations should be continued for a greater number of years, and that the like should be made in several great cities of Europe."

The *Marquis de Condorcet* is one of the next writers who noticed De Wit's Treatise. In the "*Discours Préliminaire*" of his great work (*Paris*, 1785), (p. 183), he says:—

"The first mathematician who thought of applying calculation to political questions, is the celebrated John de Wit, Grand Pensionary of Holland. His wise and courageous conduct in this important position, his virtues, his patriotism, his unhappy end, have made his name dear to all those who love their country, or whom virtue affects. There were greater names in the last century, but no more respectable name could perhaps be cited.

"John de Wit had been a disciple of *Descartes*, and one of his best disciples. Before being Grand Pensionary, he had published a work upon Curves, in which, ingenious and novel views are to be found. *It was he who first essayed to fix the rate of life annuities according to the probabilities of life given by the Tables of Mortality.*

"Upon politics, upon the true interests of nations, upon the freedom of trade, he had very superior ideas to those of his age; and we may say that his premature death was a misfortune to Europe as well as to his country."

In 1802, *De La Lande*, in the third volume of Montucla, observed:—

"The problem of life annuities was treated by Van Hudden, (*sic*) who, although a geometrician, was also Burgomaster of Amsterdam, and by the celebrated Pensionary of Holland, John De Wit, one of the first promoters of the geometry of Descartes. I am not acquainted with the title of Hudden's composition, but that of John De Wit was entitled *De Vardye van de Lyfrenten na proportie van de Losrenten*, or the value of life annuities in comparison with free or redeemable rents. (*The Hague*, 1671). Each of the two was more in a position to know the importance of the subject and to procure the necessary examinations of the registers of mortality; Leibnitz also, being in Holland some years afterwards, used his every endeavour to obtain the work of John De Wit, but he could not succeed: it was not however absolutely lost, for M. Nicholas Struyck informs us that he had a copy of it in his hands; and he

gives us a *précis*, from which is seen how rightly John De Wit reasoned upon this subject."

The reference here is to Struyck's work, which appeared at Amsterdam in 1740 (p. 345).* Now, it is quite true that Struyck mentions De Wit's labours, and the date of their publication; but it is by no means so certain that he saw the work,—in fact, he does *not* give a *précis* of its contents, but solely a reference or two to its final results.

In England, any notice of the work of De Wit has necessarily been but very slightly touched upon. Baily, in his *Doctrine of Assurances*, has merely said—"The subject of life annuities was treated by Van Hudden of Amsterdam, and likewise by the celebrated Jean De Wit, in his treatise entitled *De Vardye van de Lifrenten, &c.* (1671.)" (*See previous note on this title of the treatise*).

And Milne, in the *Encyclopædia Britannica*, (Article *Annuities*, see Supp. edit., 1842), mentions it with a passing observation, referring to the passage in Montucla, with the addition that it "appears to have been very little known or read, and to have had no sensible influence on the subsequent progress of the science, the origin of which may be properly dated from Dr. Halley's Paper in the Philosophical Transactions for the year 1693 (No. 196)."

The following is a specimen of perhaps the most perfect view which is now entertained upon the subject on the Continent. The passage which I translate is from *Dr. Gouraud's "Histoire du Calcul des Probabilités."* (Paris, 1848.)

"In 1671, the Grand Pensionary John De Wit, who was not only a politician of the highest character, but also a geometrician of the first distinction, pupil of Descartes, and quite at the summit of the mathematical intelligence of his time, thought of directly applying calculation to the determination of the likelihood which there might be for a man, in each year of his life, to die in a given lapse of time. Consulting, for that result, the registers of the deaths and births of different towns in Holland, he thence drew the ne-

* *Inleiding tot de Algemeene Geographie, benevens eenige Sterrekundige en andere Verhandelingen.* Door Nicolaus Struyck. 4to. Amsterdam, 1740. The author was a physical geographer and astronomer, and Fellow of the Royal Society of London. His work embraces a variety of subjects; and about 50 pages *at its conclusion* are devoted to that of life annuities. It has, therefore, been misquoted in those instances where we see this section mentioned as comprised in Struyck's *Introduction* to his Universal Geography. In the year 1753 he published a continuation of his researches in political arithmetic, under the title of *Nader Ontdekkingen noopens den Staat van het Menschelyk Geslagt*, in his work *Vervolg van de Beschryving der Staartsterren, &c.* W. Kersseboom, of the Hague, published some strictures on the first work, in 1740. (*Eenige Aanmerkingen op de Gissingen over den staat van het menschelyk Geslagt, Uitreekening van de Lyfrenten, &c.* 'S Gravenhage, 4to., pp. 18.)

cessary elements for the formation of an extraordinary table, and of a nature until then unknown, where the probability of life of a man of his country and of his time was, at each age, mathematically estimated; and on the basis of this comparative state of the number of years of life which still remained to the different members of the society, whose probable mortality he had calculated, he deduced therefrom the actual value of life annuities constituted upon nominees of different ages in such society. This astonishing invention, destined for so enlarged a subsequent use, was then in no way noticed, and the composition of De Wit (*De Vardye van de Lifrenten, &c.*, always a rare work, and which has never been translated) waited twenty years for a reader.

"And again, when in 1693 an English mathematician of the highest order, proceeding in turn to study, in the obituary returns of London and Breslau, the general laws of human mortality, published upon this great subject a Memoir, which is read even to this day with admiration, absolutely no one took any heed of it. Halley, however, in this memoir, taught with equal clearness and exactness the conditions needful for the formation of those tables which are now called Tables of Mortality, the manner of framing them with complete geometrical precision, of deducing therefrom a corresponding table of the present state and annual movement of the population, of reading in them the probability of survivorship of a person taken at random in a given society, of, in fine, concluding upon the probable duration of the co-existence of several individuals, from the sole knowledge of their age. Useless instructions! Buried in the vast and rich collection of memoirs of the Royal Society of London, the admirable labours of Halley were only to be discovered there by posterity."

(§ 24.) Now, the information conveyed in the preceding passage, as far as relates to De Wit, would have to be modified after perusal of the Grand Pensionary's Treatise, which of course Dr. Gouraud had no opportunity of consulting, and which does not contain, as has been seen, any estimated value of life annuities at *each age*, but purely the type of valuation, and the illustrative praxis for one age. On the subject of Dr. Halley's paper, the truth is neatly expressed in the Essay just quoted; and indeed no one, with a candid and unbiassed opinion, would in the present day attempt to deny that Dr. Halley was the discoverer and scientific arranger of what are termed Life Tables, in the full and highly important modern acceptation of the term, and that in his paper (*An Estimate of the Degrees of the Mortality of Mankind, &c.*) he taught the world the best initia-

tory and theoretical form for the computation of life annuities, and of survivorships from and to given ages.

The reader being now enabled to form an idea of what De Wit's Treatise really is, it will be equally incumbent on him to admit that the Dutch Statesman, twenty-eight years before our countryman the Astronomer-Royal Halley, presented to notice at least *a* method by which the value of life annuities could be accurately calculated, different from and inferior in convenience to the perfectly independent solution of the latter, but in no wise less applicable as a method of operation in such valuations, and, if needed, a basis for insurance estimates; though, unlike Halley's, it was not available for the different other forms of life contingencies, or for the purposes of the statistician and political economist, to which that can always be applied, and which would have been sufficient to establish Dr. Halley's fame, had it not been already gained by his brilliant discoveries in the spangled vault of heaven. Neither does the renown of De Wit stand in need of adornment, but historic justice should not be neglected, even in the smallest particular; and, keeping that in view, we ought to avail ourselves of every occasion which can further or maintain it.

It is pleasant to observe that, notwithstanding a few captious remarks "on Halley's method existing in the nature of things," the continental public has been ready to accord the meed of praise to which his useful labours entitled him. Let us do the same with regard to De Wit. The two treatises have in different fashions had the greatest influence upon the subjects whereon they treat. Let each retain its proper place, which does not interfere with the relative degree of credit due to both!

I had the intention of extending the present remarks somewhat further, to an account of the progress of the reciprocally dependent theories of life insurance and annuities, simultaneously with that of political arithmetic or applied statistics; but the preceding topics have occupied too much space to allow this to be done, and it will be preferable to conclude with a few remarks which belong more particularly to the last subject we have been considering. It is requisite that the circumstances under which De Wit's Treatise has been introduced should be briefly detailed. They are as follow:—I had for some time been desirous of endeavouring to clear away the veil under which the work in question had been concealed. After fruitless attempts to attain this object here, it occurred, as a matter of conjecture, that the Treatise, not included in such collections of De Wit's works as could be referred to, was,

in all likelihood, written by him for some government purpose, and consequently might be found in the State archives of Holland, either in a manuscript or printed form. Supplying an esteemed correspondent with some collateral details in support of that view, researches were, through his aid, made at Amsterdam and the Hague. At length conjecture was justified by the ascertained fact of De Wit's Treatise on Annuities having been inserted in the "Resolutions of the States of Holland and West Friesland" of the year 1671; and, being furnished with a transcript of the original in Dutch, and with the more immediate assistance of *a French version*, the difficulty was at an end.

With respect to the causes or effects on which depended the loss (if it may be so termed) of the document, and which loss has endured so many years, I can only suggest the following: that the paper, on being distributed to the members of the States-General, did not, in the peculiar pressure of affairs, get any public attention, or that it was wilfully suppressed. On no other supposition is it easy to account for what is recorded of Leibnitz having in vain used endeavours to obtain it. No person ought to have been better able to procure its inspection than Leibnitz. When he visited Holland about the year 1676, his own writings record, not only that he had interviews with Huygens, but, as it happened, with the Burgomaster Hudde, the very man who employed himself on these subjects (as Leibnitz knew), and who certified, at De Wit's request, the computations in the latter's Report on Life Annuities. (*See Certificate annexed to the translation in the present paper, and the foot-note respecting Hudde.*)

Whether Hudde had his own incentives for not further divulging De Wit's views, it is, perhaps, out of the question to surmise; but there is fairer ground for imagining that there were State reasons for their suppression. The brothers Cornelius and John De Wit met their melancholy death on the 20th of August, 1672. At the very time when Cornelius was in prison, when John De Wit was bowed down under the load of a cruel oppression, the States of Holland were becoming involved in fresh imperative necessities for the raising of funds to carry on immense naval operations against England, and military defences against the beleaguering forces of France. Money had to be raised at the utmost sacrifice. The States might have been fully impressed with the truth of the theory of annuities presented in De Wit's Report, and perfectly ready to avail themselves of the calculations he was at the pains of making; but at the same time they must on deliberation have per-

ceived the impracticability of negociating funds by this means at the low rate of interest there exemplified. The fundholder in perpetual rents or ordinary stocks might have been content with a lower rate of interest, as the basis of terms, than the annuitant. The former would reason—"The Government may fall into difficulties, and might not pay me interest during my lifetime; but it may be different afterwards; the tide of affairs will turn, and my successors thus regain the advantage." Not so with the life annuitant: his loss in such a case would have been either nearly or quite complete; and in times of extreme low credit we may take the investment to have been a speculative one, which called for the otherwise extravagant terms that were offered. The circumstances quoted by Adam Smith, with regard to the practice in this country in the reign of William the Third, can be cited as an instance in point; for although the English Government held out the inducement of 14 per cent., at any age, by way of life annuity, the public were very loth to embark their money in what would, in fact, have been a fortune to them, and held such views of the instability of the times, that these terms procured but few purchasers. Similar reasons very satisfactorily account for the graduated scale of life annuities which the Dutch authorities published at Amsterdam in July 1672 (only a few days before De Wit's death), and which offered from 10 to 11 per cent., at ages where he had computed that $6\frac{2}{3}$ would be the maximum rate at 4 per cent. The new scale, it is very likely, was calculated by Hudde, following out De Wit's method, for each age, and reckoning at a higher rate of interest; and these calculations may have been those which are described as having been kept back from publication by Hudde (see *ante*). The States-General can then be easily imagined to have been desirous of giving no publicity to De Wit's researches, as, among other effects, the difference in the increased rates would have led to unpleasant remarks from the financial economists of the day. Besides this, the capitalists, being the gainers, were slow to disabuse the Government of its continuance in error; and the terms seem to have been adhered to with very slight amendment, and not in fact to have been criticised, until the exact rate of mortality experienced upon the lives to whom the States had granted annuities was made the subject of minute investigation by writers of the succeeding century.

London, December 1, 1851.