EXTREME VALUE TECHNIQUES PART I: PRICING HIGH-EXCESS PROPERTY AND CASUALTY LAYERS

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Extreme Value Techniques
Part I: Pricing High-Excess Property and Casualty Layers

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Abstract. We show how modern extreme value theory concepts for the estimation of long-tailed loss severity distributions and simulation approaches to parameter uncertainty and aggregate loss calculations can be used to create a family of new multiline, multiyear risk transfer products for the Fortune 500 group of large industrial companies. Swiss Re's recently launched "Beta" high-excess property and liability coverage for the Oil & Petrochemicals industry is presented as an example for a successful application of this methodology.

Keywords. Extreme value theory, peaks-over-thresholds model, generalized pareto distribution, reference dataset, risk-adjusted capital, value proposition.

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1. The "Beta" Insurance Coverage

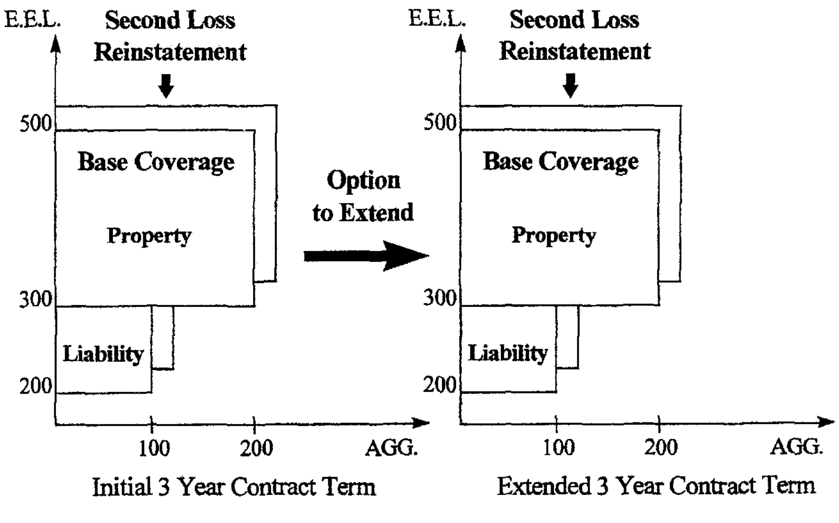
"Beta" provides multi-year, high-excess, broad form property and comprehensive general liability coverage with meaningful total limits for Fortune 500 clients in the Oil & Petrochemicals industry.

Coverage is provided at ***optimal layers*** within prescribed minimum and maximum per occurrence attachment points and per occurrence (i.e., each and every loss: E.E.L., see Fig. 1 below) and aggregate (AGG.) limits, split appropriately between property and casualty. These attachment points and limits are derived from the risk profiles and the needs of the insureds (***Swiss Re's Value Proposition***[1] for the Oil & Petrochemicals industry).

The aggregate limits provide "Beta" ***base coverage*** for one year and over three years. Simply stated, if the base coverage is not pierced by a loss, then its full, substantial limits (USD 200M property and 100M casualty) stay in force over the entire three year "Beta" policy term.

Insureds might be concerned they would have no (or only a reduced) coverage if losses were to pierce the base coverage. Therefore, "Beta" includes a ***provision to reinstate*** all or a portion of the base coverage that is exhausted.

Lastly, the "Beta" design includes an ***option*** at the inception of the base coverage ***to extend*** its initial three year high-excess insurance coverage (i.e., the property and casualty base coverage and the provision for a single reinstatement of the base coverage) for an additional three year policy term at a predetermined price.



[1] See the paper *Extreme Value Techniques - Part II: Value Proposition for Fortune 500 Companies* by Gerhard Geosits, Hans-Fredo List and Nora Lohner, Swiss Re Zurich.

Fig. 1: The "Beta" Insurance Coverage for the Oil & Petrochemicals Industry

2. Risk Quantification and Optimal Layers

The ***risk quantification process*** leading to the above optimal "Beta" layers for multi-year (i.e., three years) high-excess property and casualty Oil & Petrochemicals industry insurance coverage in principle follows standard actuarial tradition - however with some new elements:

(1) Historical loss data are verified and adjusted. Loss adjustments (e.g., for inflation, IBNR, IBNER, etc.) are at the discretion of the experienced Oil & Petrochemicals industry underwriter. The concept of a "Beta" ***reference dataset*** is crucial in this step: the loss information taken into account represents the "Beta" target portfolio in the Oil & Petrochemicals industry over the next six years (normally on a one-year adjustment basis).

Base Period			**Extended Agreement Period**		
Threshold:	19'000'000		Threshold:	21'000'000	
Displacement:	35'556'727		Displacement:	41'161'356	
	Loss Frequency	Loss Severity		Loss Frequency	Loss Severity
Total	98	11'122'001'288	Total	102	12'960'819'507
Mean	4.9000	556'100'064	Mean	5.1000	648'040'975
Std	3.4473	821'569'868	Std	3.3388	949'459'852
Year of Loss	**Frequency of Loss**	**Severity of Loss**	**Year of Loss**	**Frequency of Loss**	**Severity of Loss**
1972	1	23'958'123	1972	1	27'734'522
1973	2	89'443'793	1973	2	103'542'371
1974	2	253'654'111	1974	3	315'282'920
1975	9	672'734'348	1975	9	778'774'099
1976	7	195'761'373	1976	7	226'618'259
1977	4	172'687'891	1977	4	199'907'820
1978	2	91'544'077	1978	3	127'240'943
1979	2	134'443'858	1979	2	155'635'571
1980	14	828'038'260	1980	14	958'557'791
1981	4	127'521'023	1981	4	147'621'524
1982	3	329'142'562	1982	5	423'822'614
1983	6	282'044'028	1983	6	326'501'218
1984	5	515'671'205	1984	5	596'953'879
1985	10	568'474'190	1985	10	658'079'934
1986	3	102'412'299	1986	3	118'555'037
1987	3	847'656'158	1987	3	981'267'960
1988	5	3'039'409'867	1988	5	3'518'496'847
1989	9	2'627'918'971	1989	9	3'042'144'699
1990	1	27'628'417	1990	1	31'983'346
1991	6	191'856'736	1991	6	222'098'153

Fig. 2a: Oil & Petrochemicals Industry Property Reference Dataset for 1997-1999 (Base Period) and 2000-2002 (Extended Agreement Period)

Remark: The Oil & Petrochemicals industry "reference datasets" presented here are of course just synthetically created examples for this paper. They are however carefully constructed and the results derived with our ***extreme value techniques*** are quite realistic. It should also be noted that the methodology presented here does not, of course, replace traditional actuarial (exposure rating) techniques. It is in fact a ***complementary*** way of pricing high-excess layers[2].

Base Period			**Extended Agreement Period**		
Threshold:	18'000'000		Threshold:	24'000'000	
Displacement:	30'579'545		Displacement:	40'701'375	
	Loss Frequency	Loss Severity		Loss Frequency	Loss Severity
Total	51	4'718'096'481	Total	51	6'279'786'416
Mean	3.4000	314'539'765	Mean	3.4000	418'652'428
Std	3.6801	498'226'908	Std	3.6801	663'140'014
Year of Loss	**Frequency of Loss**	**Severity of Loss**	**Year of Loss**	**Frequency of Loss**	**Severity of Loss**
1979	1	40'365'000	1979	1	53'725'815
1980	0	0	1980	0	0
1981	0	0	1981	0	0
1982	0	0	1982	0	0
1983	1	157'064'531	1983	1	209'052'891
1984	1	109'367'952	1984	1	145'568'744
1985	7	194'027'999	1985	7	258'251'267
1986	2	47'776'295	1986	2	63'590'249
1987	4	210'129'192	1987	4	279'681'955
1988	13	1'632'203'224	1988	13	2'172'462'491
1989	5	1'371'302'207	1989	5	1'825'203'237
1990	4	242'645'679	1990	4	322'961'399
1991	8	357'887'742	1991	8	476'348'584
1992	4	323'024'661	1992	4	429'945'824
1993	1	32'301'999	1993	1	42'993'961

Fig. 2b: Oil & Petrochemicals Industry Casualty Reference Dataset for 1997-1999 (Base Period) and 2000-2002 (Extended Agreement Period)

(2) Anticipated future developments concerning the insured or the entire Oil & Petrochemicals industry are also taken into account in order to be able to quote an overall "Beta" premium that is stable under all conceivable changes in the insured's loss generating process. Therefore, a range of ***scenarios*** specific to "Beta" for 1996 to 2001 (or a few representative annual subperiods thereof) is developed by the experienced underwriter.

[2] For a simplified pricing approach based on increased limits factors techniques, see the paper *Extreme Value Techniques - Part III: Increased Limits Factors (ILF) Pricing* by Hans-Fredo List and Nora Lohner, Swiss Re Zurich.

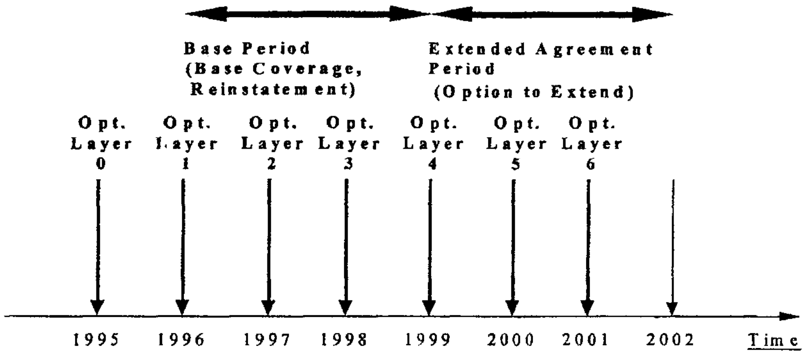


Fig. 3: Oil & Petrochemicals Industry "Beta" Scenarios

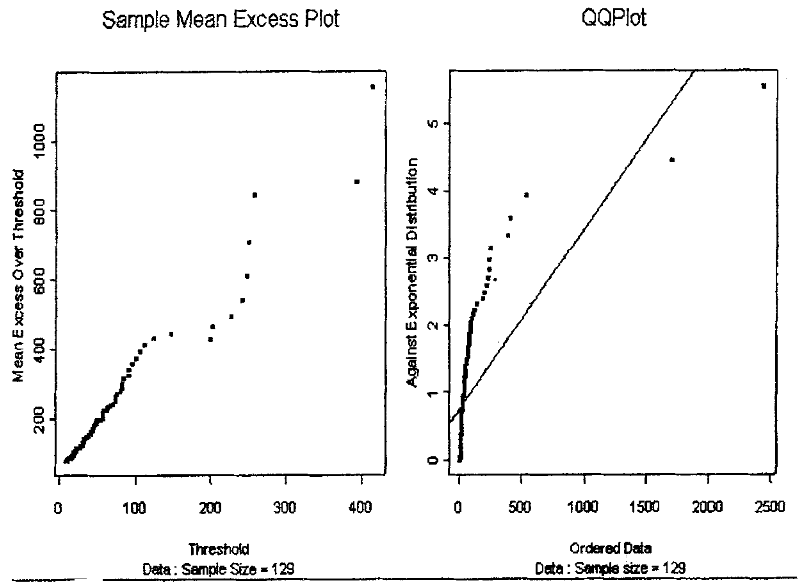
(3) The standardized and adjusted loss information (both historical and scenarios) is summarized by annual loss frequency and annual aggregate loss severity (see Fig. 2 above). Any ***trends*** in the insured's claims patterns can be recognized and carefully evaluated at this point.

(4) The individual standardized and adjusted losses are used to develop statistical/actuarial ***models*** describing ***analytical loss severity distribution functions***. The severity models provide mathematical approximation and extrapolation, at the discretion of the experienced Oil & Petrochemicals industry underwriter, of historically observed as well as anticipated (scenario) loss dynamics. The "Beta" implementation team (consisting of Swiss Re and ETH personnel) has developed and implemented a consistent and stable (with respect to small perturbations in the input data) actuarial and Value Proposition based modelling approach for "Beta" high-excess property and casualty layers. This new methodology is based on ***Extreme Value Theory*** (***Peaks-Over-Thresholds Model***[3]) and fits a ***generalized Pareto distribution***[4] to the ***exceedances of a data-specific threshold*** (see Fig. 2 above and Fig. 4

[3] It has to be noted that ***claims histories are usually incomplete***, i.e., only losses in excess of a so-called ***displacement*** δ are reported. Let therefore (X_i) be an i.i.d. sequence of ground-up losses, (Y_i) be the associated loss amounts in the "Beta" layer $D \leq x \leq D+L$ and $Z=\sum_{i=1}^{N} Y_i$ the corresponding aggregate loss. Similarly, let $(\tilde{X}_i)$, $\tilde{X}_i = X_i 1_{X_i > \delta}$, be the losses greater than the displacement δ and $\tilde{Z}=\sum_{i=1}^{\tilde{N}} \tilde{Y}_i$, $\tilde{N}=\sum_{i=1}^{N} 1_{X_i > \delta}$, the corresponding "Beta" aggregate loss amount. Some elementary considerations then show that $F_Z \equiv F_{\tilde{Z}}$ holds for the aggregate loss distributions, provided that $\delta < D$. ***The Peaks-Over-Thresholds Model (Pickands-Balkema-de Haan Theorem)*** on the other hand says that the ***exceedances of a high threshold*** $t < D$ are approximately $G_{\xi,t,\sigma}(x)$ distributed, where $G_{\xi,t,\sigma}(x)$ is the ***generalized Pareto distribution*** with ***shape*** ξ, ***location*** $t \equiv \mu$ and ***scale*** $\sigma > 0$. The threshold $t < D$ is chosen in such a way that in a neighbourhood of t the MLE-estimate of ξ (and therefore the "Beta" premium) remains reasonably stable (see Fig. 4 below). For more details, see the paper *Extreme Value Theory in the BETA Product* by Paul Embrechts and Alexander McNeil, ETH Zurich.

[4] The ***generalized Pareto distribution (GPD)*** is defined by

below). Maximum Likelihood Estimation (MLE) and the corresponding Kolmogorov-Smirnov (KS) goodness-of-fit test are applied to get the associated optimal parameters. The above outlined scenario techniques provide an indication of the parameter uncertainty inherent in the estimation process.



$$G_{\xi,\mu,\sigma}(x) = \begin{cases} 1-\left(1+\xi\dfrac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}} & \xi \neq 0 \\ 1-e^{\frac{x-\mu}{\sigma}} & \xi = 0 \end{cases}$$

where $x \geq \mu$ for $\xi \geq 0$ and $\mu \leq x \leq \mu - \dfrac{\sigma}{\xi}$ for $\xi < 0$. Compare this with the ***ordinary Pareto distribution (PD)***:

$$F_{\theta,a}(x) = 1-\left(\frac{a}{x}\right)^{\theta}, \; x > a.$$

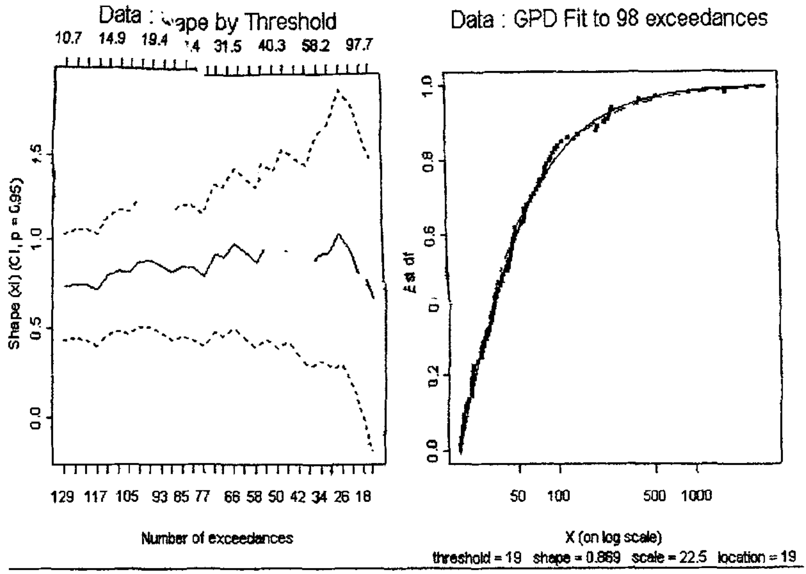
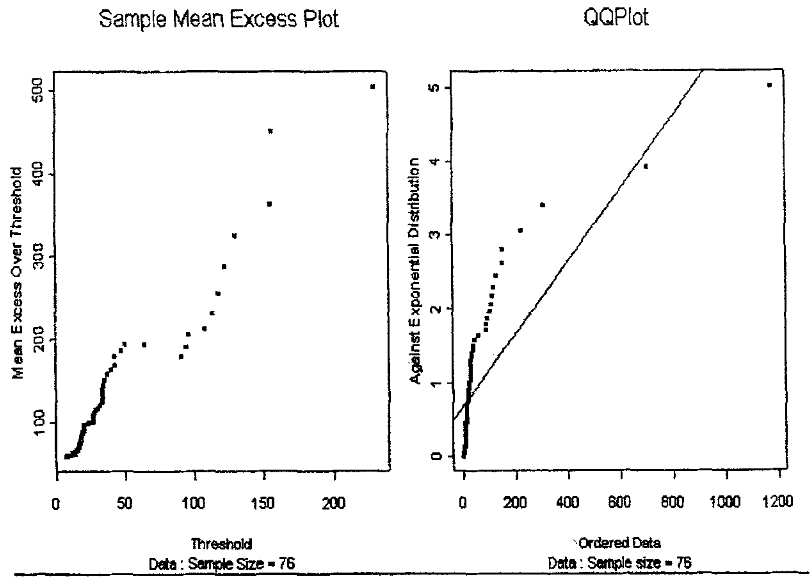


Fig. 4a: Oil & Petrochemicals Industry Severity Parameters (Property, Base Period)
Solid Line: GPD, Dotted Line: PD



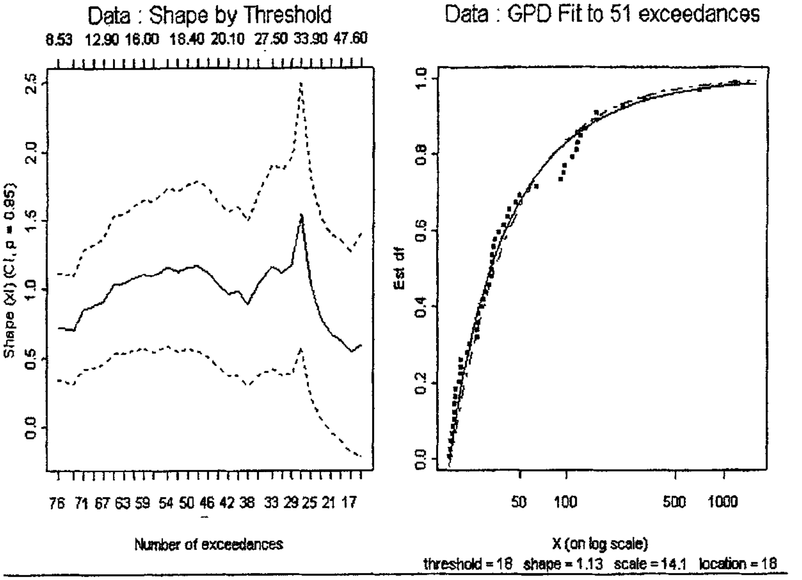


Fig. 4b: Oil & Petrochemicals Industry Severity Parameters (Casualty, Base Period)
Solid Line: GPD, Dotted Line: PD

(5) The ***frequency distribution model (excess of the data-specific threshold)*** is selected by estimating the mean and standard deviation from the annual frequency trends (see Fig. 2 above), with judgment modifications by the experienced Oil & Petrochemicals industry underwriter. Typically, the frequency distribution models utilized are either ***Poisson*** or ***negative-binomial*** (which allows recognition of significant changes in annual frequencies), whereby the parameters are estimated by MLE or by the method of moments. In developing the frequency models, relative changes in the exposure base (i.e., annual revenues or tangible assets) should also be recognized, where warranted.

				mean	std		shape	scale	location
Basic Scenario									
Property				mean	std		shape	scale	location
BP	Threshold	19.00	Frequency	4.90	3.45	Severity	0.8690	22.5000	19.0000
EAP	Threshold	21.00	Frequency	5.10	3.34	Severity	0.8710	25.0000	21.0000
Onshore									
BP	Threshold	15.00	Frequency	3.65	2.96	Severity	0.8430	25.7000	15.0000
EAP	Threshold	18.00	Frequency	3.65	2.96	Severity	0.8790	28.0000	18.0000
Offshore									
BP	Threshold	13.00	Frequency	2.00	1.30	Severity	0.5280	22.0000	13.0000
EAP	Threshold	15.00	Frequency	2.00	1.30	Severity	0.5250	25.5000	15.0000
Casualty									
BP	Threshold	18.00	Frequency	3.40	3.68	Severity	1.1300	14.1000	18.0000
EAP	Threshold	24.00	Frequency	3.40	3.68	Severity	1.1300	18.6000	24.0000
Adjustment Scenario									
Property				mean	std		shape	scale	location
BP	Threshold	32.00	Frequency	5.90	3.65	Severity	0.7830	44.5000	32.0000
EAP	Threshold	40.00	Frequency	6.10	3.70	Severity	0.7650	59.3000	40.0000
Onshore									
BP	Threshold	30.00	Frequency	3.80	2.78	Severity	0.7990	53.6000	30.0000
EAP	Threshold	40.00	Frequency	3.80	2.78	Severity	0.8010	71.1000	40.0000
Offshore									
BP	Threshold	33.00	Frequency	2.20	1.54	Severity	0.6890	31.7000	33.0000
EAP	Threshold	44.00	Frequency	2.20	1.54	Severity	0.6930	41.9000	44.0000
Casualty									
BP	Threshold	44.00	Frequency	3.47	3.60	Severity	1.2500	28.1000	44.0000
EAP	Threshold	70.00	Frequency	3.53	3.68	Severity	1.0300	64.1000	70.0000

Fig. 5: Oil & Petrochemicals Industry Parameters (Property and Casualty, Base Period: BP and Extended Agreement Period: EAP, all Scenarios[5])

(6) With the mathematical models describing loss severity and loss frequency distributions (see Fig. 5 above), ***annual aggregate loss calculations*** are performed, usually in constant dollar terms where the ***reference period is the middle of a "Beta" contract period (e.g., 1998/2001)***. Annual aggregate losses are described in terms of expected value and standard deviation (as well as higher moments where necessary). The calculations may be further extended to investigate annual aggregate loss potentials within high confidence levels (i.e., by considering the entire corresponding probabilistic loss distribution). Generally, annual aggregate loss estimates have more meaning at higher percentiles (e.g., the 90th, 95th and 99th) since these percentiles reflect the potential for adverse loss experience (over and beyond expected value).

[5] To make this presentation simple, we only consider the ***basic scenario*** and an ***adjustment scenario*** (see p. 15 - 17 for more details on the general classes of "Beta" threat scenarios identified).

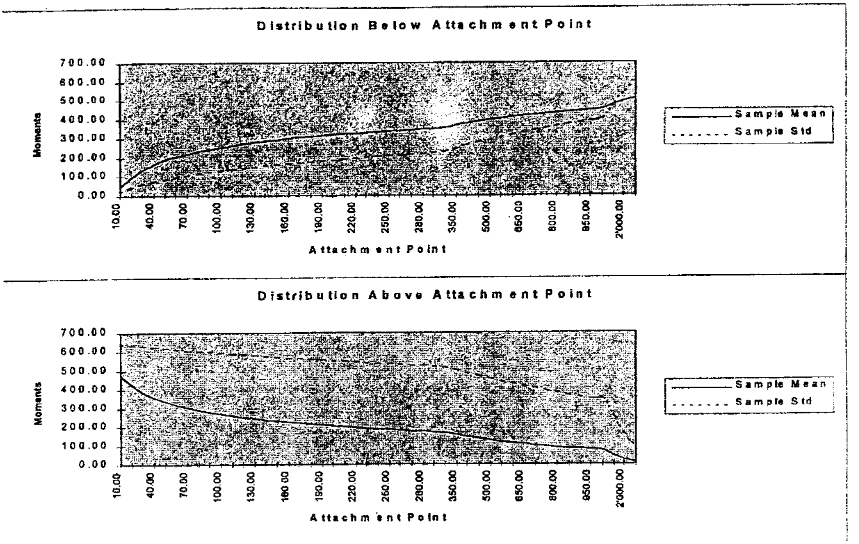


Fig. 6a: Oil & Petrochemicals Industry Annual Aggregate Losses (Property, Base Period)

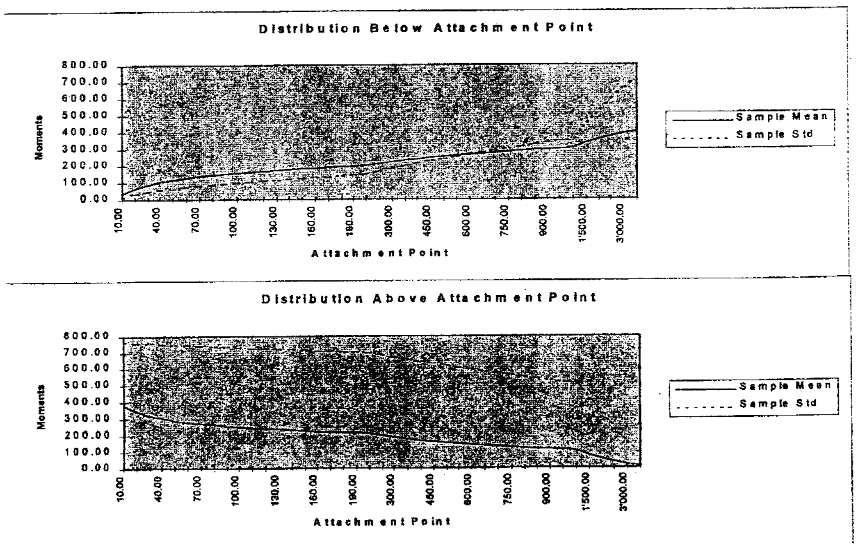


Fig. 6b: Oil & Petrochemicals Industry Annual Aggregate Losses (Casualty, Base Period)

(7) Following the above annual aggregate loss calculations, per claim loss layers are selected and aggregate distributions both within the selected layers and excess of those layers up to the maximum potential individual loss (MPL) in the Oil & Petrochemicals industry (e.g., USD 3 billion for property and USD 4 billion for casualty) determined. This procedure is repeated for sequential layers (usually chosen at the discretion of the underwriter to approximate the anticipated "Beta" program structures reflecting the needs of the insureds or the entire industry), thus mapping out the "Beta" ***risk potential***. The resulting probabilistic loss profiles ("Beta" ***risk landscapes*** or ***risk maps***) can in a second step also be complemented by selecting appropriate aggregate loss limits in addition to the each and every loss limits and superimposing them on the potential losses within the chosen layers, thus further improving the flexibility of "Beta" program designs in the direction of combined single limits/deductibles.

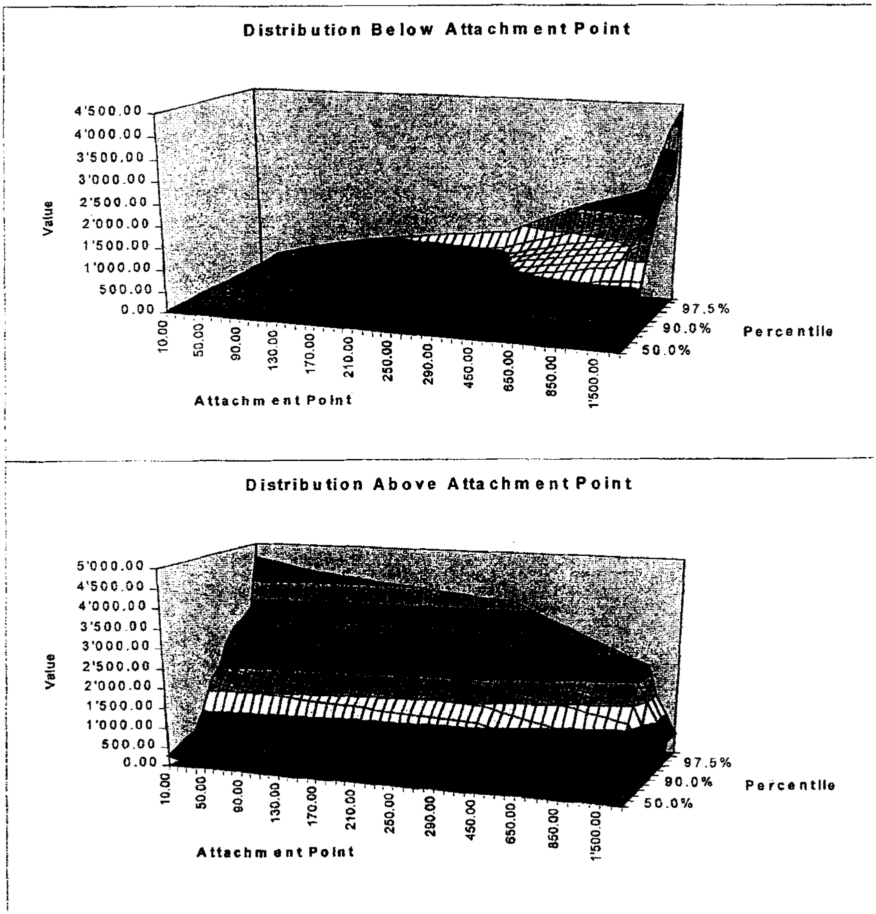
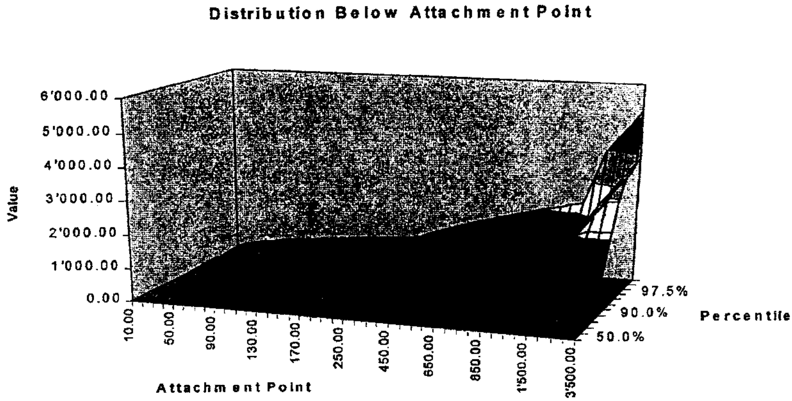


Fig. 7a: Oil & Petrochemicals Industry Risk Landscape (Property, Base Period)



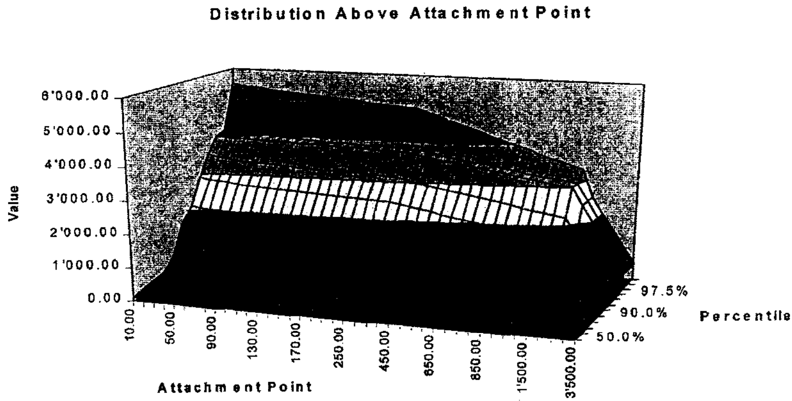


Fig. 7b: Oil & Petrochemicals Industry Risk Landscape (Casualty, Base Period)

(8) The same approach is finally also used to build probabilistic profiles of entire "Beta" (three year aggregate) ***loss portfolios***[6]. These optimal risk portfolios are structured in three dimensions: (a) across various exposures (e.g., property and casualty), (b) across time periods

[6] This is for the "Beta" ***standard layers*** USD 200M xs 300M property and USD 100M xs 200M liability. The parameters are taken from Fig. 5 and a ***normally distributed parameter uncertainty of 25% at the 95th percentile*** around these expectations is assumed for both frequency (Poisson) and severity (GPD). We also assume independent risks. The "Beta" implementation team has however looked into the issue of correlated risks and has developed corresponding models and pricing tools. Little can be done directly with existing historical loss information; ***scenario techniques*** have to be used instead. For an overview on the subject of correlated coverages and their rating, see the paper *Multiline Excess of Loss Rating* by Erwin Straub, Swiss Re Zurich.

(e.g., three years), (c) across insureds or groups of insureds (e.g., selected companies or industries).

	Basic Scenario		Adjustment Scenario	
	BP	EAP	BP	EAP
Sample Mean	182.96	224.50	418.41	656.91
Sample Std	168.27	184.96	252.18	308.83
%iles				
50.0%	175.30	200.00	400.00	626.11
66.7%	200.00	300.00	500.00	769.50
75.0%	300.00	313.32	582.27	849.76
80.0%	300.00	397.63	611.17	904.20
90.0%	400.00	500.00	760.46	1'071.08
95.0%	500.00	591.07	885.29	1'205.87
96.0%	507.73	600.00	904.56	1'249.56
97.0%	578.90	624.88	961.92	1'300.00
97.5%	600.00	665.49	995.49	1'333.30
98.0%	600.00	700.00	1'013.17	1'373.16
99.0%	700.00	778.39	1'110.58	1'487.60
99.9%	900.00	1'000.00	1'400.00	1'823.44

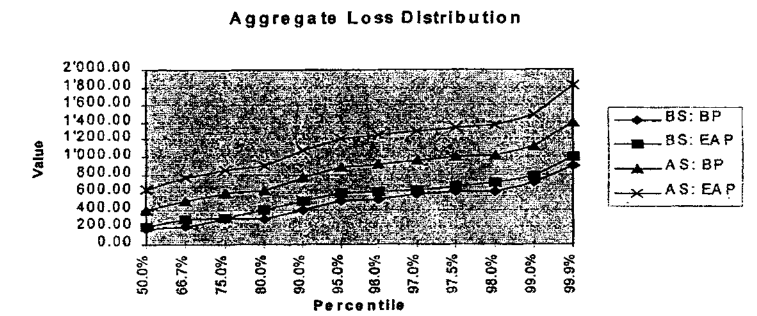


Fig. 8: Oil & Petrochemicals Industry "Beta" Loss Portfolio (3 Year Aggregate Loss Distribution, Property and Casualty, Base Period: BP and Extended Agreement Period: EAP, all Scenarios)

Based on the above probabilistic (annual aggregate) risk profiles for high-excess property and casualty Oil & Petrochemicals industry insurance coverage ("Beta" risk maps), different criteria can be used to select optimal layers for insurance programs that an experienced underwriter might desire to offer. Overall, optimal excess layers selected for "Beta" are characterized by low frequency. In particular, ***from Swiss Re's risk management point of view, optimal layers for "Beta" property and casualty excess coverages are defined as follows***:

No annual loss should pierce the chosen property or casualty excess layer more frequently than once every four years (based both on the historical and scenario annual aggregate

loss distributions). This translates into a 75% confidence that annual aggregate losses for a given layer of "Beta" coverage will equal zero.[7]

Monte Carlo Simulation Output - 1000000 Trials
Distribution Below Attachment Point

	50.00	100.00	150.00	200.00	250.00	300.00	350.00	400.00	450.00	500.00
Sample Mean	187.32	255.08	293.22	319.36	339.08	354.81	367.82	378.90	388.52	397.00
Sample Std	88.52	132.64	164.27	189.89	211.87	231.33	248.90	265.02	279.99	293.99
%iles										
50.0%	179.35	241.36	272.70	293.01	307.15	315.20	316.56	316.56	316.56	316.56
66.7%	219.86	300.78	347.62	379.11	403.22	423.77	440.78	453.73	461.23	462.62
75.0%	243.15	336.79	392.76	432.60	462.46	487.64	510.66	531.34	549.86	564.90
80.0%	259.73	362.39	425.21	470.99	506.02	534.56	560.68	585.19	608.42	629.97
90.0%	305.22	433.32	515.07	576.81	628.37	672.29	709.44	741.45	771.58	801.22
96.0%	345.07	494.98	594.47	671.46	735.82	792.25	843.80	890.74	932.61	968.65
96.0%	356.73	513.61	618.32	700.21	768.89	828.89	883.94	935.30	982.69	1'024.34
97.0%	371.37	536.38	648.30	736.03	810.48	875.13	934.51	990.22	1'042.60	1'091.09
97.5%	379.96	550.68	666.67	758.43	835.80	904.01	965.74	1'024.10	1'078.70	1'131.19
98.0%	391.09	567.50	688.19	784.61	866.82	939.49	1'004.06	1'063.94	1'121.83	1'177.56
99.0%	423.03	618.55	754.66	863.29	958.10	1'042.37	1'118.74	1'188.56	1'254.69	1'316.97
99.9%	516.76	771.16	952.94	1'101.03	1'235.57	1'350.73	1'462.37	1'565.15	1'663.86	1'760.41

Monte Carlo Simulation Output - 1000000 Trials
Distribution Above Attachment Point

	50.00	100.00	150.00	200.00	250.00	300.00	350.00	400.00	450.00	500.00
Sample Mean	335.99	268.23	230.09	203.95	184.23	168.50	155.49	144.41	134.78	126.30
Sample Std	619.65	596.11	573.94	554.79	537.08	520.44	504.68	489.64	475.20	461.28
%iles										
50.0%	114.47	37.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
66.7%	233.71	137.38	72.84	19.01	0.00	0.00	0.00	0.00	0.00	0.00
[redacted]										
80.0%	435.72	324.29	246.96	183.56	126.65	73.47	22.43	0.00	0.00	0.00
90.0%	851.13	728.69	640.16	567.14	501.73	441.58	384.50	329.35	275.95	223.90
95.0%	1'543.79	1'416.69	1'324.26	1'245.44	1'175.59	1'110.98	1'048.58	989.96	932.69	877.22
96.0%	1'859.86	1'735.31	1'643.10	1'565.21	1'496.30	1'430.32	1'367.11	1'306.38	1'248.37	1'190.12
97.0%	2'360.71	2'235.44	2'143.00	2'061.86	1'990.18	1'923.32	1'859.13	1'798.99	1'741.24	1'682.69
97.5%	2'738.13	2'613.96	2'521.20	2'442.09	2'368.32	2'302.00	2'238.35	2'174.28	2'115.36	2'055.48
98.0%	2'950.00	2'900.00	2'850.00	2'800.00	2'750.00	2'700.00	2'650.00	2'600.00	2'550.00	2'500.00
99.0%	3'112.54	2'973.87	2'864.45	2'800.00	2'750.00	2'700.00	2'650.00	2'600.00	2'550.00	2'500.00
99.9%	4'416.39	4'243.44	4'096.63	3'969.85	3'842.47	3'729.47	3'616.93	3'510.64	3'404.12	3'296.89

Fig. 9a: Oil & Petrochemicals Industry Optimal Layer (Property, Base Period)

[7] This optimality criterion is mainly derived from Swiss Re's perception (based upon an extensive Oil & Petrochemicals industry analysis) of a *"Beta"* or *"catastrophic" event*. In the case of "Beta" programs with combined single limits/deductibles, lower percentiles and thus shorter contract maturities may be preferable from a marketing point of view.

Monte Carlo Simulation Output - 1000000 Trials
Distribution Below Attachment Point

	50.00	100.00	150.00	200.00	250.00	300.00	350.00	400.00	450.00	500.00
Sample Mean	119.35	158.04	181.38	198.41	211.93	223.20	232.88	241.40	249.00	255.89
Sample Std	68.69	101.49	126.72	148.24	167.43	184.98	201.26	216.53	230.97	244.72
%iles										
50.0%	112.50	144.07	162.40	167.65	167.65	167.65	167.65	167.65	167.65	167.65
66.7%	143.55	190.86	219.04	242.93	261.20	267.78	267.78	267.78	267.78	267.78
75.0%	162.06	219.43	254.24	283.03	309.61	333.30	350.00	356.71	356.71	356.71
80.0%	174.62	239.41	280.91	312.48	342.07	370.77	398.20	422.98	442.84	442.84
90.0%	211.78	295.84	354.48	402.01	439.91	474.51	508.82	542.89	577.58	612.32
95.0%	243.78	345.99	420.05	482.00	539.33	587.15	625.18	661.96	698.95	735.48
96.0%	252.90	360.93	440.27	505.52	566.47	621.89	666.05	704.89	742.26	779.82
97.0%	265.53	379.83	465.35	535.46	599.77	661.66	719.37	763.32	802.15	840.70
97.5%	272.48	391.55	480.31	553.90	620.67	685.14	746.89	800.00	843.61	883.39
98.0%	281.40	405.35	499.01	577.49	646.51	713.06	778.94	841.95	898.40	939.11
99.0%	308.19	447.69	554.08	646.43	728.68	802.30	874.09	944.73	1'015.69	1'084.74
99.9%	386.26	573.97	722.44	853.02	972.79	1'085.96	1'194.79	1'300.38	1'403.83	1'499.95

Monte Carlo Simulation Output - 1000000 Trials
Distribution Above Attachment Point

	50.00	100.00	150.00	200.00	250.00	300.00	350.00	400.00	450.00	500.00
Sample Mean	290.51	251.83	228.49	211.45	197.94	186.67	176.98	168.47	160.86	153.98
Sample Std	751.47	733.39	716.89	701.39	686.64	672.49	658.83	645.58	632.69	620.12
%iles										
50.0%	34.64	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
66.7%	111.88	46.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
[redacted]										
80.0%	271.99	193.74	133.28	79.11	27.65	0.00	0.00	0.00	0.00	0.00
90.0%	681.82	594.08	524.04	462.41	404.22	348.80	295.50	243.26	191.57	140.36
95.0%	1'542.57	1'450.22	1'374.72	1'307.51	1'244.40	1'184.24	1'127.45	1'072.70	1'018.00	963.29
96.0%	1'987.23	1'899.27	1'825.68	1'757.14	1'692.81	1'631.72	1'573.04	1'516.33	1'460.73	1'406.83
97.0%	2751.81	2660.33	2584.25	2515.28	2451.94	2390.74	2330.24	2271.79	2214.61	2159.40
97.5%	3370.88	3278.35	3200.45	3131.05	3066.95	3002.25	2940.08	2882.13	2827.69	2770.27
98.0%	3950.00	3900.00	3850.00	3800.00	3750.00	3700.00	3650.00	3600.00	3550.00	3500.00
99.0%	4001.37	3900.00	3850.00	3800.00	3750.00	3700.00	3650.00	3600.00	3550.00	3500.00
99.9%	5372.86	5234.51	5110.91	4989.90	4879.60	4774.73	4669.34	4563.09	4456.05	4351.46

Fig. 9b: Oil & Petrochemicals Industry Optimal Layer (Casualty, Base Period)

The following table characterizes the optimal three year excess layers (i.e., layers of property and casualty coverage where the probability of loss is low but where premium volume remains substantial) to be used by experienced Oil & Petrochemicals industry underwriters as a target range for "Beta" capacity:

Basic Scenario			Adjustment Scenario		
Property			*Property*		
BP	Opt. Attachment Point	300.00	BP	Opt. Attachment Point	600.00
EAP	Opt. Attachment Point	350.00	EAP	Opt. Attachment Point	800.00
Onshore			*Onshore*		
BP	Opt. Attachment Point	250.00	BP	Opt. Attachment Point	500.00
EAP	Opt. Attachment Point	290.00	EAP	Opt. Attachment Point	700.00
Offshore			*Offshore*		
BP	Opt. Attachment Point	90.00	BP	Opt. Attachment Point	180.00
EAP	Opt. Attachment Point	110.00	EAP	Opt. Attachment Point	240.00
Casualty			*Casualty*		
BP	Opt. Attachment Point	250.00	BP	Opt. Attachment Point	550.00
EAP	Opt. Attachment Point	300.00	EAP	Opt. Attachment Point	850.00

Fig. 10: Oil & Petrochemicals Industry Optimal Layers (Property and Casualty, Base Period: BP and Extended Agreement Period: EAP, all Scenarios)

3. Threat Scenarios

The "Beta" policy term is three years, with an option to extend the high-excess property and casualty coverage for another three years under the same conditions (assuming relative constancy of the underlying risk distribution and exposure base for a particular insured and industry). Oil & Petrochemicals industry "Beta" capacity is based on the notion of optimal layers of coverage which uses one year aggregate loss distributions for property and casualty claims. These parametric distributions can be estimated from corresponding loss information (i.e., Oil & Petrochemicals industry reference datasets) properly verified and adjusted by the experienced underwriter. In addition, in order to capture future risk dynamics, a sequence of standardized and adjusted loss scenarios should be developed for the initial three year "Beta" policy term (base period) from 1997 to 1999, in order to get a clearer picture of the sensitivity of the underlying layer optimization procedure to corresponding changes in risk exposure. Since the option to extend the "Beta" coverage is available at the inception of the initial three year contract term, additional scenarios for the extended agreement period from 2000 to 2002 should be developed by the experienced Oil & Petrochemicals industry underwriter in order to properly assess the impact of such a three year contract extension on "Beta"'s risk map (see Fig. 2 above). Five kinds of "Beta" ***threat scenarios*** following such a schedule are developed:

(i) ***adjustment scenarios*** showing the effects of an increase in the trending factor for both property and liability claims;

(2) ***frequency scenarios***[8] showing the effects of a higher claims frequency;
(3) ***severity scenarios*** showing the effects of a higher claims severity;
(4) ***batch scenarios*** showing the effects of claims series;
(5) ***MPL scenarios*** showing the effects of an extremely adverse maximum potential loss (MPL) estimate.

Bootstrapping[9] is the applied statistical/actuarial methodology. According to the experience of the "Beta" implementation team so far, under normal circumstances only an adjustment scenario (for property and casualty) has to be explicitly considered. The other scenarios just introduce additional parameter uncertainty into the original historical loss information and can therefore be replaced by a simulation approach to calculating aggregate loss distributions that allows for (e.g., normally distributed) parameter uncertainty. Recall that the "Beta" 3 year aggregate loss distribution for the Oil & Petrochemicals industry (see Fig. 8 above) was calculated with such a simulation approach under the assumption of at the 95th percentile 25% normally distributed[10] parameter uncertainty. Fig. 11 below shows the same aggregate loss distribution under the assumption of 0% parameter uncertainty:

	Basic Scenario		Adjustment Scenario	
	BP	EAP	BP	EAP
Sample Mean	201.00	244.72	443.43	678.85
Sample Std	172.67	189.52	255.86	311.63
%iles				
50.0%	200.00	200.00	406.85	650.79
66.7%	247.74	300.00	526.33	794.48
75.0%	300.00	359.67	600.00	875.43
80.0%	328.95	400.00	649.60	931.49
90.0%	428.96	500.00	793.67	1'096.37
95.0%	516.39	600.00	903.58	1'233.89
96.0%	556.32	620.29	943.50	1'277.85
97.0%	600.00	668.99	995.29	1'328.71
97.5%	600.00	699.94	1'014.39	1'362.25
98.0%	628.08	703.72	1'051.11	1'400.00
99.0%	700.00	800.00	1'151.11	1'514.38
99.9%	941.73	1'035.49	1'450.01	1'858.78

[8] Frequency scenarios play an important role when insureds require coverages below the optimal attachment point and also for examining the implications of "Beta" portfolio growth over time.

[9] For further details, see *An Introduction to the Bootstrap*, B. Efron and R. J. Tibshirani, Chapman & Hall 1993.

[10] For example, consider the shape parameter ξ of the property GPD in the basic scenario, base period (see Fig. 5 above): We assume then that $\xi \equiv \xi(\omega)$ is a normally distributed random variable with mean $m = 0.869$ such that

$$P(0.75m \le \xi \le 1.25m) \ge 0.95 .$$

The same assumption is made for the frequency (Poisson) parameter λ and the other severity (GPD) parameters μ and σ

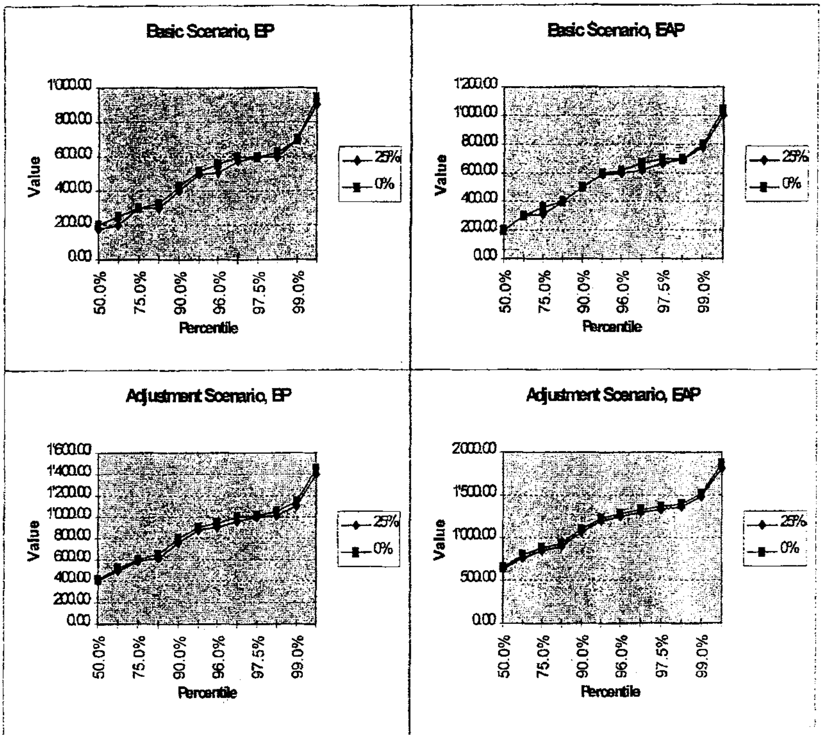


Fig. 11: Oil & Petrochemicals Industry "Beta" Loss Portfolio (3 Year Aggregate Loss Distribution, Property and Casualty, Base Period: BP and Extended Agreement
Period: EAP, all Scenarios, 0% parameter uncertainty)

4. Pricing/Rating

"Beta" Base Coverage. We recommend a simple, practical ***actuarial pricing principle*** (slightly modified because of the large "Beta" limits and the resulting potential risk exposure in a "particularly bad" three year coverage period) for rating "Beta" coverages:

$$P = \max\left\{\frac{mL}{n}, E[X] + k\sigma[X]\right\}$$

P: Premium

m: Number of Full - Limit Losses to be Considered

L: "Beta" Limit

n : Payback Horizon (Years)

E[X]: Expected Loss Burden in the "Beta" Layer

σ[X]: Standard Deviation of this Loss Burden.

Note that individual loss information and corresponding industry loss data (i.e., the Oil & Petrochemicals industry reference datasets) can be taken into consideration to calculate a weighted (***credibility***[11]) premium:

$$E[X] = \alpha E[X_i] + (1-\alpha)E[X_R]$$
$$\sigma[X] = \alpha\sigma[X_i] + (1-\alpha)\sigma[X_R]$$
$$0 \le \alpha \le 1$$

i: Individual Loss Burden

R: Reference Loss Burden (Industry Data).

The "Beta" implementation team's experience has so far been that usually there is not enough individual loss information available in order to be able to apply the above actuarial rating approach. As an alternative therefore, a ***Value Proposition pricing formula*** on the basis of the ***risk-adjusted capital***[12] ***(RAC)*** necessary to support "Beta" in the Oil & Petrochemicals industry can be used:

$$P = \max\left\{\frac{mL}{n}, E[X] + rRAC[X]\right\}$$

RAC[X]: Risk - Adjusted Capital for the "Beta" Layer

r: Rate of Return on Risk - Adjusted Capital

$$RAC[X] = \alpha RAC[X_i] + (1-\alpha)RAC[X_R]$$
$$= \left(\alpha\frac{\sigma[X_i]^2}{\sigma[X_R]^2} + (1-\alpha)\right)RAC[X_R]$$

Whether the traditional actuarial pricing principles (variance or standard deviation principle) or the above RAC-based Value Proposition formula is applied by the Oil & Petrochemicals industry underwriter does not seem to be of too much importance. Much more important is the question of determining the expected losses in the "Beta" layers properly; i.e., of choosing the right (***stable parameters***, see Fig. 4 above) frequency and severity distribution functions.

Futures and Options. To begin, consider the "Beta" option for a property and liability coverage in three years time. Suppose the strike (or exercise) price of this option is Q and the

[11] For more details, see *Non-Life Insurance Mathematics*, Erwin Straub, Springer 1988 and *Mathematical Methods in Risk Theory*, Hans Bühlmann, Springer 1970. Note that we use the standard deviation principle merely for reasons of practicality: the numbers involved otherwise get too big.

[12] Note that the calculation of the ***risk-adjusted capital (RAC)*** necessary to support "Beta" in the Oil & Petrochemicals industry is a very intricate process which has to take the risk landscape of the entire Swiss Re portfolio into consideration and cannot, of course, be disclosed here. We found that by using the pragmatic formula: $RAC[X_R]$ ***equals 2 times the 99th percentile of the "Beta" aggregate loss distribution (see Fig. 11 above) minus USD 420M (corresponding premium estimate)***, we get a tolerable (conservative) approximation of the true value for $RAC[X_R]$. For more details, see the appendix: p. 24-3.

price of the "Beta" coverage in three years time is P_3 (random variable). Then the option premium P (on an industry basis, forgetting about various technicalities[13] here) is

$$P = \frac{1}{(1+r)^3} E\left[\max\{P_3 - Q, 0\}\right]$$
$$= \frac{1}{(1+r)^3} \sum_{i=0}^{5} \alpha_i \max\{P_3^i - Q, 0\}$$

where r is an appropriate discount rate, the index i ranges over the respective industry scenarios (0 = Basic Scenario, 1 = Adjustment Scenario, 2 = Frequency Scenario, 3 = Severity Scenario, 4 = Batch Scenario and 5 = MPL Scenario), α_i is the weight (probability) given to scenario i and P_3^i is the future price of the underlying property and liability coverage in three years time under scenario i.

An Example. Consider the following "Beta" rating problem:

(I) "Beta" Option on the Coverage
USD 100M p.o. 550M xs 250M (onshore property)
USD 100M p.o. 525M xs 250M (offshore property)
USD 100M xs 350M (liability)

The corresponding combined ***present ("Beta" base period) three year coverage*** is then

(a) USD 100M p.o. 550M xs 250M (onshore property)
USD 100M p.o. 525M xs 250M (offshore property)
USD 100M xs 350M (liability)

Premium (3 Year Agg., Prop. & Liab.)$_0$ = 1,989,600 + k_0 * 13,390,831 USD
Premium (3 Year Agg., Prop. & Liab.)$_1$ = 4,633,257 + k_1 * 20,381,035 USD

whereas the corresponding combined ***future ("Beta" extended agreement period) three year coverage*** is[14]

(b) USD 100M p.o. 550M xs 250M (onshore property)
USD 100M p.o. 525M xs 250M (offshore property)
USD 100M xs 350M (liability)

Premium (3 Year Agg., Prop. & Liab.)$_0$ = 2,483,543 + k_0 * 15,007,547 USD
Premium (3 Year Agg., Prop. & Liab.)$_1$ = 6,997,943 + k_1 * 25,095,997 USD.

---

[13] The premium payable for the option to extend the "Beta" coverage is $P = E\left[\xi_3 \max(P_3 - Q, 0)\right]$, where ξ_3 is an ***appropriate discount factor consistent with the originally given probability measure.*** For more details, see the paper *Theoretical Considerations for the Pricing of β-Insurance* by Freddy Delbaen and Uwe Schmock, ETH Zurich.

[14] In principle, attachment points and limits could be different. Note that the ***optimal attachment points move*** according to Fig. 10. The rating parameters are on an ***industry-average basis***, i.e., 50 "Beta" Oil & Petrochemicals industry target clients are assumed.

With an ***exercise price*** of ***Q = 4,477,756 USD*** (i.e., the trended ***current price*** of ***USD 3,980,709*** of the "Beta" coverage), a discount rate of r = 4% and scenario weights of 95% (basic scenario) and 5% (adjustment scenario), respectively, the "Beta" ***option premium*** is then ***P = 528,144 USD*** (note that the ***futures price*** $E[P_3]$ of the above "Beta" coverage is ***USD 4,508,853***).

In the above calculations we have used the ***Value Proposition principle***

$$k_i = \frac{r_R RAC[X_R|i]}{\sigma[X_R|i]^2}\sigma[X|i] \quad \left[\begin{array}{l} \text{of course, in general: } k_i\sigma[X|i] \geq r_R RAC[X|i] \\ \qquad\qquad\qquad\qquad = r_R RAC[X_R|i]\dfrac{\sigma[X|i]^2}{\sigma[X_R|i]^2} \end{array}\right]$$

with e.g., $RAC[X_R|0] = 980{,}000{,}000\text{ USD}$, $\sigma[X_R|0] = 172{,}670{,}000\text{ USD}$ and a RORAC $r_R = 10\%$ (p.a.)[15].

5. Value Proposition

The "Beta" ***standard coverage***

(II) USD 200M xs 300M (property)
USD 100M xs 200M (liability)

with ***current ("Beta" base period) premiums***

Premium (3 Year Agg., Prop. & Liab., Ind.)$_0$ =	201,000,000 + k_R^0 * 172,670,000 USD
Premium (3 Year Agg., Prop. & Liab., Ind.)$_1$ =	443,430,000 + k_R^1 * 255,860,000 USD

and ***future ("Beta" extended agreement period) premiums***

Premium (3 Year Agg., Prop. & Liab., Ind.)$_0$ =	244,720,000 + k_R^0 * 189,520,000 USD
Premium (3 Year Agg., Prop. & Liab., Ind.)$_1$ =	678,850,000 + k_R^1 * 311,630,000 USD

implements Swiss Re's Value Proposition for Fortune 500 clients in the Oil & Petrochemicals industry: the associated "Beta" risk maps (see Fig. 9 above) indicate the ***optimal self-insured retentions (SIRs, = optimal "Beta" attachment points)*** for such corporates. Of course, an insured's needs for high-excess coverages that are different from the above standard "Beta" coverage can easily be accomodated within Swiss Re's "Beta" program.

If we now similarly to above define

[15] See the appendix for details: p. 24-1, 24-2 and 24-3.

$$k_R^i = \frac{r_R \mathrm{RAC}[X_R|i]}{\sigma[X_R|i]} \quad \left[\text{of course, in general: } k_R^i \sigma[X_R|i] \geq r_R \mathrm{RAC}[X_R|i]\right]$$

on an industry basis, then

$$k_R = \max\{..,k_R^i,..\}$$

is a ***RORAC-equivalent actuarial loading factor*** which allows the easy calculation of a ***standard "Beta" premium on an industry basis***

$$P_R^i \equiv P[X_R|i] = E[X_R|i] + k_R \sigma[X_R|i].$$

The ***individual "Beta" premium*** is then

$$P_i \equiv P[X|i] = \frac{E[X|i]}{E[X_R|i]} P[X_R|i]$$

and

$$\frac{\sigma[X|i]^2}{E[X|i]} \leq \frac{k_R}{r_R \mathrm{RAC}[X_R|i]} \frac{\sigma[X_R|i]^3}{E[X_R|i]}$$

$$\left[\text{derived from: } k_R \sigma[X_R|i] \frac{E[X|i]}{E[X_R|i]} \geq r_R \mathrm{RAC}[X_R|i] \frac{\sigma[X|i]^2}{\sigma[X_R|i]^2}\right]$$

is a straightforward ***test for the acceptability of any new "Beta" client and coverage***[16]. Of course, futures and options are subsequently rated as above[17]: with a projected RORAC $r_R = 10\%$ (p.a.), the RORAC-equivalent actuarial loading factor is $k_R = 2.5114$ (3 years), the "Beta" coverage

(I) USD 100M p.o. 550M xs 250M (onshore property)
USD 100M p.o. 525M xs 250M (offshore property)
USD 100M xs 350M (liability)

is acceptable under all considered scenarios, its ***current price*** is ***USD 6,535,286***, ***its futures price USD 6,846,541***, and with a ***strike*** of ***USD 7,351,307*** (i.e., the trended current price of this coverage), the ***option price*** is ***USD 342,903***.

6. "Beta" Options: Some Concluding Remarks

Apart from the already existing option to extend, the following "Beta" options are conceivable candidates for future enhancements to the program:

---

[16] Recall that very often there is ***no individual historical loss information*** $X|i$ and therefore $E[X|i]$ and $\sigma[X|i]$ are a priori unknown. On the other hand, usually there is ***very good industry loss information*** $X_R|i$ ***(Oil & Petrochemicals industry reference datasets)*** and consequently we have reliable estimates for $\mathrm{RAC}[X_R|i]$, $E[X_R|i]$ and $\sigma[X_R|i]$ - and therefore P_R^i. ***Classical actuarial (exposure, increased limits factors: ILF, etc.) rating techniques*** as well as discussions with the insured (and on-site inspections if necessary) however provide a reliable estimate for $E[X|i]$ and therefore P_i. ***The "Beta" rating approach presented here is therefore an attempt to make the best possible use of all available information about an insured by combining modern extreme value theory and classical actuarial techniques.***

[17] See the appendix for details: p. 24-3, 24-4 and 24-5.

Option for a Single Reinstatement[18].

- Conditional on the excess of loss experience.
- Separate reinstatements of up to USD 200M and USD 100M, respectively, for property and liability.

Start: Beginning of the "Beta" contract.
Maturity: End of the first coverage period (3 years).

Pricing/Rating Principle:

- Let $X_T = X_T^P + X_T^L$ be the total (property and liability) reinstatement necessary after a first loss piercing the "Beta" base coverage.
- Then the premium payable for the reinstatement option is $P = E[\xi_T X_T]$, where ξ_T is an appropriate discount factor consistent with the originally given probability measure.
- Open questions are of course the determination of (distributions of) $\xi_T, \xi_T X_T, T$.

Option to Refill.

The insurance industry struggles with the issue of liability claims that surface many years after a drug, chemical or product is introduced into the market. One approach to address ***latent liability exposures*** is to understand the potential financial impact of the growth of latent liability reserves and to price coverage accordingly (see for instance the Swiss Re publication: "Late Claims Reserves in Reinsurance"). "Beta" may incorporate a ***mechanism that reduces the amount of claims settlement based on the lapse of time from the first occurrence of exposure to the emergence of injury*** (from a technical perspective, the amount of claim settlement reduction should be based on the estimate of reserve growth). This feature of the "Beta" design is intended to limit the impact of latent liability claims on the performance of the Swiss Re "Beta" portfolio. The contractual mechanism operates as follows:

(1) The "Beta" liability contract follows an occurrence first reported mechanism similar to that used by the excess liability insurers in Bermuda (e.g., ACE and XL). Consequently, exposures that first commenced prior to the retroactive date agreed by the insurer in the "Beta" policy an insured of "Beta" is not covered even if reported while an insured of "Beta".

(2) The amount of loss payable to an insured is based on the limits provided for the three year policy term in which a claim is reported (no stacking of limits). If, in other words, a claim is reported after the policy period, the insured has no coverage (unless the insured purchased a discovery period policy).

(3) Assuming that the exposure first commenced after the company became an insured of "Beta", ***a schedule reduces the amount of claim settlement based upon the number of three year policy terms that had elapsed from the time of first occurrence to the time of reporting***

[18] Furthermore, options for the n^{th} consecutive reinstatement could be incorporated into future "Beta" program designs.

(***sliding-scale***, see Fig. 12 below) if the cover is still in operation (either via discovery period or via active policies as a result of renewals).

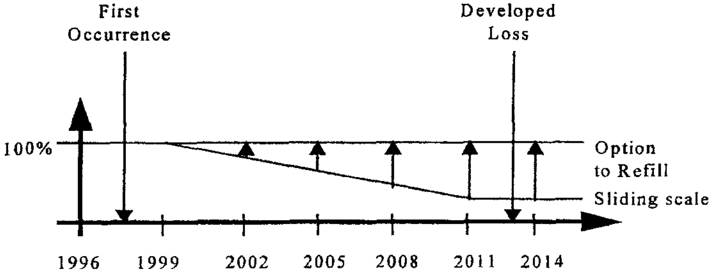


Fig. 12: "Beta" Latent Liability Provision (Sliding Scale) and Options to Refill

It is important to note that the date of first occurrence is the first date that any court opinion decides an injury had (or would have) occurred. From a practical perspective, this is the first date that a drug or product was introduced into the market. The use of existing or developing legal precedent should reduce associated controversy. The "Beta" latent liability provision is likely to cause insureds to accelerate claim notifications (balanced against the impact of premium increases: while "Beta" is for a three year term, a provision is included in the "Beta" contract to increase premiums for change in exposure base and a change in exposure base should include changes in claim notification patterns for liability exposures). The option to refill allows a "Beta" insured to recover the full loss amounts of long latent liabilities in future contract periods; the associated payments depend on the sliding-scale mechanism outlined above (see Fig. 12).

Option Price: Expectation of the discounted amount of all refillments.
Start: The end of the "Beta" contract period.
Maturity: One forward "Beta" contract period.

Option for Retroactive Coverage.

- Enables the client to buy "Beta" coverage for liability claims whose "first occurrence" date lies several years before contract start (see Fig. 13 below).

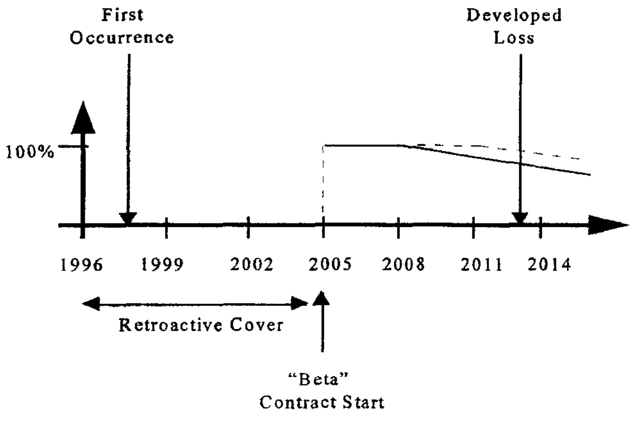


Fig. 13: "Beta" Option for Retroactive Coverage

7. Appendix: Calculation Sheets

- The Current and Future Price of a "Beta" Coverage 24-1

For both the "Beta" base period and the "Beta" extended agreement period and all relevant scenarios, the price of the combined onshore / offshore property and general liability coverage considered in the example on p. 19 is calculated by using the actuarial standard deviation rating principle and the Value Proposition principle (sheet 24-3 gives the corresponding coverage period and scenario dependent RAC figures) to determine the associated (coverage period and scenario dependent) loading factors. The present and future "Beta" coverage prices (on an industry-average basis) are then derived by taking expectations (weighted sums of the scenario dependent actuarial coverage rates).

- Pricing of the Option to Extend 24-2

The option the extend the example "Beta" coverage on p. 19 is calculated by using the valuation formula on p. 18.

- RAC and RORAC equivalent k 24-3

Applying the pragmatic formula on p. 18 (footnote 9) to a "Beta" target portfolio of 50 clients in the Oil & Petrochemicals industry (i.e., the reference datasets under consideration in this paper), risk-adjusted capital (RAC) as well as the corresponding industry means and standard deviations are calculated for the "Beta" standard coverage in the Oil & Petrochemicals industry (i.e., USD 200M xs 300M property and USD 100M xs 200M liability) for both the "Beta" base period and the "Beta" extended agreement period and all relevant scenarios. Then, using the Value Proposition principle (see p. 20), a RORAC-equivalent actuarial loading factor is determined that allows the easy calculation of a standard "Beta" premium on an industry basis.

- The Current and Future Price of a "Beta" Coverage (Value Proposition) 24-4

For both the "Beta" base period and the "Beta" extended agreement period and all relevant scenarios, the price of the combined onshore / offshore property and general liability coverage considered in the example on p. 19 is calculated by using the Value Proposition rating approach on p. 20-21 (sheet 24-3 gives the corresponding coverage period and scenario dependent RAC figures and RORAC-equivalent actuarial loading factor). The present and future "Beta" coverage prices (on an industry-average basis) are then derived by taking expectations (weighted sums of the scenario dependent actuarial coverage rates).

- Pricing of the Option to Extend (Value Proposition) 24-5

The option the extend the example "Beta" coverage on p. 19 is calculated by using the valuation formula on p. 18.

8. References

[1] P. Embrechts, C. Klüppelberg and T. Mikosch, *Modelling Extremal Events for Insurance and Finance*, Applications of Mathematics, Stochastic Modelling and Applied Probability 33, Springer 1997

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[4] A. J. McNeil and T. Saladin, *The Peaks-Over-Thresholds Method for Estimating High Quantiles of Loss Distributions*, Proceedings 28th ASTIN Meeting Cairns 1997