THE SOLVENCY OF LIFE ASSURANCE COMPANIES

[Submitted to the Faculty on 8th October 1984.]

1. INTRODUCTION

1.1. The Working Party's Brief

1.1.1. In October 1980 the Council of the Faculty of Actuaries set up a Working Party with the following brief:—

(1) To investigate the criteria by which the solvency of life assurance companies should be assessed and to determine the amount of the solvency margin which should be required in practice by supervisory authorities. The existing requirements of the E.E.C. Life Establishment Directive should be considered with a view to recommending any desirable alterations thereto to be made when those requirements are reviewed in due course by the E.E.C. Commission. In carrying out its work the Working Party should co-operate with Working Parties or Committees of other actuarial bodies.

(2) To report the result of their investigations to Council.

1.1.2. This Paper reports the result of the investigations of the Working Party. The names of its members are listed in Appendix 1.

1.2. Other Current Solvency Studies

1.2.1. The Institute of Actuaries set up a Working Party to study the subject of the solvency of general insurance companies. This Working Party, the work of which was contemporaneous with our own, has now produced a Paper which has been presented to the Institute of Actuaries (Daykin (1984)). The Department of Trade & Industry has been studying the subject of solvency extensively in connection with criteria for solvency required of life assurance companies within the U.K. The Groupe Consultatif des Associations d'Actuaires des Pays des Communautés Européennes has set up a Solvency Margins sub-committee, under the chairmanship of Mr. F. B. Corby, to study the subject of the solvency of both life and general insurance companies within the E.E.C. The Chairman of the Faculty Working Party is a member of this Solvency Margins
sub-committee and hence liaison with the work of that body and through it the work of the Institute of Actuaries Working Party has been assured. Mr. D. H. Loades of the Government Actuary's Department is a member of the Faculty Working Party, which has through his help become aware of thinking within the Department of Trade & Industry on the subject of solvency. The views expressed in this Paper are, however, the collective views of the Working Party.

1.2.2. The Working Party commenced its activities by studying a bibliography of material relating to solvency published in recent years in the journals of the Institute and Faculty of Actuaries and elsewhere: this bibliography is set out at the end of this Paper and a copy of the material concerned has been deposited in the Faculty library.

1.3. Outline of this Paper

1.3.1. Section 2 of this Paper, by way of an introduction to the solvency of life assurance companies, describes a number of the questions considered by the Working Party during its activities, with the views reached by them on these various matters. Section 3 comprises a description of the current E.E.C. solvency margins, with comments thereon. An attempt is made to describe and criticise the material which, as far as the Working Party has been able to discover, appears to have formed the basis for the current E.E.C. solvency margins. The final section of the Paper describes a fresh approach to the determination of solvency which has been explored by the Working Party. It also lists the conclusions reached by the Working Party.

2. GENERAL CONSIDERATIONS

2.1. In sections 2.2 to 2.5 a number of general questions are discussed, and section 2.6 suggests an approach to the investigation of solvency.

2.2. The Meaning of Solvency

2.2.1. Before investigating the criteria by which the solvency of a life assurance company should be assessed, it is necessary to describe what is meant by "solvency". The question is very much more elusive than might at first appear. It is plain that a life assurance company is insolvent if at any time it is unable to meet any claim which falls due under the terms of its contracts. This, however, is a retrospective definition of insolvency, whereas what we are seeking
is a prospective definition, and a means of assessing whether or not, in accordance with that definition, a life company is solvent.

2.2.2. It is perhaps helpful to begin by considering the criteria which would be applied by various parties in considering this question because a prospective or current policyholder, the company management, shareholders, and the supervisory authority (in the U.K., the Department of Trade & Industry) may well approach the question differently.

2.2.3. A prospective or current policyholder will be concerned to know whether or not the claim under his policy will be paid as and when it falls due. If the policy concerned participates in the profits of the company, that policyholder will also be interested to have an idea whether the profits to be paid by his assurer are likely to compare reasonably with those that he might have secured, in return for the same premiums, from a competing assurer.

2.2.4. The management of the company and through them the shareholders, if any, will be concerned to know not merely that the claims of current and immediately prospective policyholders can be met in the way that these policyholders wish, but also that the company is likely to be able to continue to operate properly and to take on new business providing satisfactory returns for future policyholders, and dividends for shareholders, throughout the foreseeable future.

2.2.5. As far as the Department of Trade & Industry is concerned, the concept of insolvency which has underlain its approach has been that to be found in Sections 222 and 223 of the Companies Act 1948. Section 222 provides that a company may be wound up by the court if (inter alia) the company is unable to pay its debts. Section 223 states that a company shall be deemed to be unable to pay its debts if (inter alia) it is proved to the satisfaction of the court that the company is unable to pay its debts, and in determining whether a company is unable to pay its debts, the court shall take into account the contingent and prospective liabilities of the company. In devising an approach on the basis on these sections, the supervisory authority will want to be assured, before it is satisfied that a company is solvent, that claims under policies in force and those likely to be written in the near future, can be met as and when they fall due but should not be concerned with the office’s ability to continue to write new business into the indefinite future. The supervisory authority may well be less concerned than the policyholder, when assessing the
solvency of a company, that the profits payable by way of bonus to existing and immediately prospective participating policyholders be fully competitive with those of other assurance companies operating within its territory. It will be very concerned, however, to ensure that the solvency margins set will be at a level which permits effective action to be taken before it is too late to ensure the adequate fulfilment of current and immediately prospective contracts.

2.2.6. The Working Party has concerned itself mainly with the requirements of the supervisory authority. If these requirements are met, then current and immediately prospective policyholders can be assured that contractual obligations are likely to be fulfilled, although perhaps the level of bonus payments may be less than such policyholders would hope for. The question of policyholders' expectations is discussed below. The requirements of management will no doubt go beyond those of the supervisory authority: they will indeed go beyond those of current and immediately prospective policyholders also. They will doubtless embrace the continued writing of a desired volume of new business into the indefinite future and will probably be investigated by the management of the office, using model office techniques, possibly incorporating a stochastic approach.

2.2.7. The Working Party has taken the view that solvency must be a matter of probability. Except in unreasonably hypothetical circumstances it is impossible to say that a life company is solvent in the sense that no circumstances could conceivably cause it to fail to meet the obligations which it has undertaken. To be in this position would require enough funds to cover the absurdly improbable situation that all policyholders died at once (for assured lives) or lived an unreasonably long time (for annuitants), together with the holding of copious reserves for expenses. Plainly this extreme situation, which may be described as absolute solvency, is quite impractical and hence any assurance company can be solvent only with a certain degree of probability. The probability of remaining solvent and the probability of ruin add up to unity. Determining whether a life company is solvent means therefore in practice fixing a probability of ruin and thereafter attempting to determine whether or not the situation of that company appears to be such that the probability of its ruin is lower than the selected probability.

2.2.8. The selection of a probability of ruin must to some extent be arbitrary, but even when an arbitrary value has been fixed, the estimation of the probability of ruin of a particular company at a
particular time must also involve estimates, to some extent subjective, of the probabilities of occurrence of various events which can lead to ruin. These events include, for example, the rate of escalation of expenses, future mortality rates and future investment returns.

2.2.9. We have, therefore, taken the view, for the purposes of this Paper, that a life assurance company should be regarded as solvent if it has assets sufficient to satisfy the supervisory authority that the probability of its being unable to meet its contractual liabilities under current and immediately prospective contracts (estimated on assumptions which are acceptable to the supervisory authority and to the actuarial profession as a whole) is less than a certain figure (the probability of ruin) prescribed by that supervisory authority. The purpose of setting solvency margins is to furnish sufficient warning to the supervisory authority to enable it to intervene if necessary in the affairs of the company so as to ensure that this can be achieved. The extent to which the supervisory authority should take into account the "reasonable expectations of policyholders" is a vexed question which is next discussed.

2.3. The Reasonable Expectations of Policyholders

2.3.1. The concept of "the reasonable expectations of policyholders" has appeared in Papers on this subject for some time. As far as the Working Party can trace, it appeared first in the Paper "A Solvency Standard for Life Assurance Business" (Skerman (1966)). In this paper, Skerman says: "Holders of with-profit policies have taken them out in the expectation that they will benefit from a share of profits from time to time. Although an office is not under a contractual obligation which can be quantified in relation to the benefits which its policyholders will derive from future profits, it would be unsatisfactory not to take some account of the policyholders' reasonable expectations when determining the value of the liabilities". The concept, although now embodied in the U.K. legislation, has never, however, as far as we are aware, been defined more precisely than that. There can be no doubt that policyholders can reasonably expect that contractual liabilities (including bonuses declared to date) be met. The area of doubt arises in the context of participating policies, with reference to the level of future profits that may reasonably be expected by policyholders.

2.3.2. It does not seem to us reasonable for policyholders to expect that the current level of bonuses declared by the company concerned should be maintained throughout the rest of the term of their policies, let alone increased. Nor, therefore, does it seem reasonable for
prospective policyholders to expect that illustrations given at the
time they effect policies should necessarily be fulfilled in practice. On
the other hand, it is scarcely reasonable to assume, either for current
or immediately prospective policyholders, that no bonus whatever
should be payable.

2.3.3. It may be thought a reasonable compromise that with-profit
policyholders could expect at least the level of bonus in future which
would be earned by the bonus loadings inherent in their premiums
were the expenses, investment and mortality assumptions under-
lying the non-profit premiums chargeable by the company concerned
to be experienced in future. This level of bonus might well for a
typical U.K. with-profit company be something like one half of
current bonus levels and we, therefore, suggest that reversionary
bonuses at this level could probably be considered a reasonable ex-
pectation for participating policyholders. The use of a conventional
net premium valuation implicitly reserves for levels of bonus not
dissimilar from those just described. We do not consider that any
level of terminal bonus should form part of policyholders' reasonable
expectations. It is probably unnecessary to seek any further refine-
ment of this figure, in view of the aim of the supervisory authority
as we see it, which is to have sufficient warning of the need to inter-
vene in a company's activities.

2.4. The Aims and Requirements of the Supervisory Authority

2.4.1. Supervisory authorities within the E.E.C. have a responsi-
bility to enforce the requirements of the existing E.E.C. Life Estab-
ishment Directive, including a requirement that the company be
able to meet the solvency margins set out in that Directive. The
on the co-ordination of laws, regulations and administrative pro-
visions relating to the taking up and pursuit of the business of direct
life assurance. The requirements of the Directive are described and
commented upon later in this Paper and we are concerned here
rather to consider, independently of that Directive, the matters which
a supervisory authority should take into account in determining how
to satisfy itself that a company is solvent in the sense described earlier
in this section.

2.4.2. In the first place, we take the view that the aim of the super-
visory authority will be to provide itself with information which
will enable remedial action to be taken in the case of a company
approaching insolvency with time enough to prevent that insolvency
occurring. The aim of the supervisory authority will, therefore, be to
devise regulations which will in the first instance give rise to a "warning signal" when the company is sufficiently far away from disaster to give an opportunity for the company concerned to put its own house in order. At this point the supervisory authority will take a close interest in the affairs of the company, and will require steps to be taken by the management of that company, supervised no doubt by officials from the authority, to ensure that insolvency is averted. The affairs of the company will be closely monitored by the supervisory authority and there should be regular investigations into the company's progress and the state of its solvency: at a certain point it may be necessary for the supervisory authority to deprive the management of the company of all further control of the company's affairs.

2.4.3. We are not concerned in this Paper with the detail of the steps which will be required once the "warning signal" has been sounded. Our concern is with the point at which the supervisory authority should first interest itself in detail in the company's affairs. In other words we are concerned with the point at which the "warning signal" should be sounded, not, in any detail, with what happens thereafter.

2.4.4. In practice, no doubt, action to be taken following the first indication that the company is approaching insolvency would depend upon the cause of the trouble. It may be necessary for fresh capital to be raised, for premium rates for new contracts to be revised if these were inadequate, or for contracts to be redesigned. It may be necessary for investments to be rearranged if these were unsuitable in either nature or term, for bonuses to be cut if this appears appropriate, for the level of new business to be more strictly controlled or even for new business to be stopped altogether until the company returns to financial health. Expenses may require more strict control, surrender values may have to be cut or other steps taken depending on the factors which were leading the company towards insolvency.

2.4.5. If these steps are not taken with sufficient firmness, or if they fail in their aim, it will be up to the supervisory authority to decide at what more serious level drastic action should be taken. Such action may involve preventing the company from writing new business or even petitioning the Courts to wind it up. In the U.K. the duty of a liquidator of a life insurance company is to continue the business with a view to transferring it to another company. Only
as a last resort are the assets distributed. A recent Paper (Barrow (1984)) considers these questions.

2.4.6. We have considered whether or not, in fixing the point at which the “warning signal” should be given, the supervisory authority should have regard to the future new business which the company may write, or whether on the other hand it should do no more than consider the company at the point of time when the information is furnished to the authority. We are of the view that an authority need not have regard—and indeed should not have regard—to possible new business which the company might write over the indefinite future, but should assume that new business will continue to be written over a short period in case this exacerbates the problem.

2.4.7. A suitable period might perhaps be one year. The reason for taking this view is that by the time this information has been furnished to the authority, which may well be up to six months after the time to which the information relates, and by the time the authority has had an opportunity to examine the information supplied and has reached the view that intervention of some kind is necessary, it may well be at least nine months after the time when the state of affairs revealed in the company’s returns pertains. Over that nine months no doubt the company will have been writing new business and it is proper to take this into account. The devising of a plan to avert impending insolvency will take time also and hence it seems reasonable to allow for say at least one year’s new business in considering how to fix the time at which the “warning signal” should be given.

2.4.8. There is of course a substantial difficulty in that the amount of new business written by a company can vary greatly from one year to the next and it may well be that if, for example, the problem of the company concerned resides in the writing of excessive new business on inadequate premium rates, then the management of that company may be pursuing quite indefensible new business acquisition policies and accelerating irresponsibly the amount of new business written. Applications for the authorisation of life insurance companies must include business plans which can be used to monitor the progress of the company. Also new companies can be required to provide quarterly accounts together with statements of new business and changes in premium rates. The supervisory authority can obtain similar information from established companies if grounds for doing so, as set out in the legislation, exist. Although, therefore, we think it right to take account of future new business over a limited
period, there seems no effective way of guarding against the possibility that grossly excessive new business might be written in that period except to lay upon a responsible actuary for the company concerned the duty of informing the supervisory authority should he feel that an irresponsible new business policy is being followed, and do this when he first becomes aware of what is happening. In the U.K., this duty is laid upon the Appointed Actuary to the company, by way of guidance notes issued by the professional actuarial bodies.

2.4.9. Supervisory authorities will wish to frame their requirements to guard against insolvency in such a way as to achieve equity amongst various types of offices. This laudable aim can probably not be achieved in practice with precision, but should certainly not be lost sight of. Broadly speaking, it is achieved if the regulations impact on companies of different sizes, with different portfolios of business, in such a way that the "warning signal" will be given for each of the companies concerned if it reaches the same probability of ruin. An allied point is the need for the supervisory authority to avoid excessive conservatism in framing regulations. If this is not done there may well be an unnecessary, and indefensible, effective deterrent to the setting up of fresh enterprises (the authorisation of which is already carefully controlled), or undue inhibition placed on the expansion of existing enterprises. There is plainly no absolute correct answer to the determination of that probability of ruin beyond which no company should be allowed to proceed unchecked, and judgement and pragmatism must be brought to bear in fixing this level.

2.4.10. The Working Party intended to attempt as far as possible to take account of conditions affecting life business in other E.E.C. countries, since plainly the problems faced by the supervisory authorities in different countries are far from uniform. They therefore sent a questionnaire to correspondents in the E.E.C. countries and thereby secured an indication of the salient features of life assurance in the countries concerned. The information derived is included at Appendix 2. The Working Party had hoped that it would be evident that information currently supplied to supervisory authorities in the various E.E.C. countries would enable those authorities to investigate independently the question of a particular company's solvency. Unfortunately it became clear that this was not the case. Appendix 2 is nonetheless of general interest and certainly highlights the different conditions prevailing in the countries concerned: these
differences are of particular significance in considering the appropriateness of uniform solvency margins throughout the E.E.C.

2.4.11. It may well be that a certificate from the Appointed Actuary (in the U.K.) and the establishment of a counterpart to the Appointed Actuary in other E.E.C. countries and reliance upon a certificate from him, in practice may be all that can be achieved. It has to be accepted that no regulations can be proof against rogues or fools. Reliance upon a certificate given by an Appointed Actuary who holds a special position of responsibility in relation to the company is therefore unavoidable, and is not necessarily less desirable or less effective than any other approach.

2.4.12. We would like to emphasise one further point with regard to the requirements of supervisory authorities. It will be seen later that we have examined a complex theoretical structure for determination of the probability that, at a given time, a particular company is solvent according to a particular assigned level for the probability of ruin. This approach involves computer projections of possible future out-turns for the office, on the basis of probability distributions for the variables having a bearing on the question of solvency (such as investment returns, expense levels etc.). Nonetheless, we do not believe that it is at present practicable for a supervisory authority to require of each of the companies under its surveillance that a similar complex approach be carried out by the company concerned. However, we would expect this to become practicable in the near future, as the requisite statistical and computer techniques become more generally known. We do not suggest that there are uniquely defensible probability distributions for the variables concerned.

2.4.13. In any event, we have taken the view that from the supervisory authority’s standpoint, what is required is a system that will give adequate warning of threatened disaster. It has to be appreciated that any such system will inevitably, however much care may be taken to avoid it, affect companies of different sizes with different portfolios of business somewhat differently. What is essential therefore is that the period of warning given by the system be adequately long. Our conclusion from all this is that a certain amount of rough justice must willy-nilly be accepted in practice, and that excessive refinement of calculation is out of place. Accordingly, we have pursued a theoretical investigation in the hope of determining a number of fairly simple rules which can be applied generally, for whilst these rules would inevitably bear somewhat unfairly on one
company vis-à-vis another, nevertheless a supervisory authority
should aim for simplicity as far as is reasonable and possible in
framing regulations designed to guard against insolvency. Great
care must be exercised in balancing the claims of simplicity and fair-
ness. We are obliged to admit that we have found no such rules,
 alas. The management of a particular company may well wish to
pursue more sophisticated investigations but these are out of place,
in our view, when considering the requirements of a supervisory
authority.

2.5. Causes of Insolvency

2.5.1. There appear to be four reasons which can lead a company
to insolvency.

(a) Mortality Risks
If mortality experience is adverse vis-à-vis that envisaged in
premium rates, insolvency can ensue. Under this heading we
must consider sudden changes in mortality levels such as might
result for example from substantial improvements in mortality
arising from medical advance or substantial deteriorations in
mortality due for instance to war or to disease. We must con-
sider also long term drifts resulting in major shifts in the mean
mortality experience extending over a protracted period, and
fluctuations about mean levels. Inadequate reassurance ar-
rangements can also lead to insolvency, as of course can the
failure of a reassurer.

(b) Investment Risks
(i) If the investment returns secured fall below those envis-
aged in the premium basis, there is danger of insolvency.
Once again, we must consider firstly fluctuations in a
company's portfolio arising because the investments
which it holds are atypical of the market as a whole,
secondly the risk associated with major changes in stock
market levels and thirdly variations in return from year
to year reflecting the experience of the market as a whole.
These risks may affect both capital and income.
(ii) We must consider also the danger of mismatching in-
vestments and liabilities. In the U.K. it is possible to
arrive, for some classes of business at any rate, at a
reasonably effective match of investments and liabil-
ities but this may not be possible in all E.E.C. countries.
There is plainly a risk of insolvency where assets and
liabilities are seriously mismatched and substantial free
reserves are not held.
Expenses
If expense levels exceed those envisaged when the premiums were set, there may be a risk of insolvency. This is particularly apparent in an inflationary climate. Excessive lapses giving rise to unrecovered initial expenses can also lead to insolvency.

Options and Guarantees
It is to be expected that, in general, options will be exercised in favour of policyholders and against the office: if experienced conditions differ from those envisaged when the options or guarantees were granted, there is a risk of insolvency.

2.5.2. It may well be that the matters considered above are not independent of one another: in particular, there may well be a correlation between investment returns and the escalation of expenses.

2.5.3. We have emphasised the need to consider the possibility, in the case of the first three causes of insolvency listed above, of sudden shifts in the level of the factor concerned, of gradual drift in the mean values appropriate to the factor, and of stochastic fluctuations about the mean at any time. It is likely, in our view, that in the case of most large old-established companies, stochastic fluctuations in mortality are probably of little significance compared with the possibility of long term drift or of sudden change to a new level. We are accustomed to the notion that there is a mean level of mortality and stochastic fluctuations about this mean level, with the possibility of gradual changes in that mean level. The experience of recent years, and indeed of the more distant past, makes it harder to accept that it is reasonable to adopt a deterministic approach for investment return or inflation rates in financial projections. Nevertheless, premium bases have to be fixed on the assumption of certain levels of investment return and of expenses, and reserves have to be calculated at a valuation of liabilities derived by assigning figures to these parameters. The environment within which companies operate is far from docile, however, and this emphasises the need for caution in fixing premium rates, in the design of contracts, and in setting the basis for computing mathematical reserves from time to time.

2.6. An Approach to the Investigation of Solvency
2.6.1. Recent work, and modern computer techniques, permit the formation of multivariate time series models which make possible the examination of a wide number of possible future scenarios for the environment within which life offices will have to operate. Thus
for example, given a postulated statistical model for a price index, it is possible to generate a large number of "possible futures" for that price index year by year into the future. Careful statistical analysis of a series of past values of prices allows one to postulate a model and the values of the parameters of that model which are consistent with the series of past observed values. A similar approach can be followed with share dividends, share dividend yields, fixed interest yields and other series. Using this approach, together with a deterministic approach for mortality, and a particular portfolio of assets and liabilities, it is possible to examine as many conceivable future out-turns for a particular office as may be desired. In this way, the assets required to be held by the office at the outset of the investigation in order that it may meet its liabilities as and when they fall due can be determined. By examining a large number of possible out-turns, the minimum figure for the value of assets to be held at the outset in order to achieve solvency with a particular preselected degree of probability can be determined also. Hence it becomes possible to examine the appropriateness of the E.E.C. solvency margins for a range of possible portfolios of liabilities and assets.

2.6.2. A detailed description of this approach will be found in a later section of the Paper. Examples of results obtained are also to be found later. It is appropriate here, however, to draw attention to some of the limitations and, of course, advantages of this approach. It may first be argued that any attempt to fix future values of, for example, the rate of inflation as it will no doubt affect expenses in future, based on the past history of inflation, suffers from an inherent credibility gap. Indeed, recent work (Wilkie (1981)) has demonstrated the difficulties of finding an appropriate model for inflation rates which is durable over long periods of time. Furthermore, although there must always exist the possibility of major influences affecting the levels of relevant variables in the future and which have not affected them in the past, it is surely impossible to foresee or quantify these influences or their magnitudes. We recognise that the degree of confidence one can have in a postulated probability distribution for future rates of inflation (or gilt edged yields, or share prices etc.) is less than one would have for probability distributions in other circumstances, for example, in the physical sciences, but nevertheless we believe that this approach is the best that can be devised.

2.6.3. Against this, however, one can fairly point out that the approach avoids the setting of wholly subjective "prudent" or "best
estimate reserve bases, and any attempts to devise margins around these. Whilst the approach cannot command one's full confidence, it is difficult to think of any sounder approach to the problem. The question of solvency is in essence a matter of how events will in fact turn out in the future. We cannot know this with full confidence and any approach must therefore have some level of doubt attached to it. An approach firmly rooted in the past is more persuasive than one proceeding from subjective judgement alone.

2.6.4. It must be admitted that the results illustrated later in the Paper may well be thought to demonstrate all too clearly the inappropriateness of any particular simple formula for determining the amount of solvency reserve. This reserve will depend on the portfolio of assets and the nature of the portfolio of liabilities in such a way as to illustrate the dangers of reliance on any particular formula. However, the question of a solvency margin has to be faced, and it may well be that in future companies should be required to demonstrate, by means of an approach similar to that which we have adopted, the degree of solvency exhibited by their operations.

3. CURRENT E.E.C. SOLVENCY LEGISLATION


3.1.1. The E.E.C. Life Directive refers to the requirement for a solvency margin in Article 18 by stating that "Each Member State shall require of every undertaking whose head office is situated in its territory an adequate solvency margin in respect of its entire business". Article 18 then proceeds to outline what items may be used to constitute a solvency margin, these being, in general terms, the amounts corresponding to share capital, free reserves and profits both earned but not distributed and also to be earned in the future. This last amount considered as permissible in constituting a solvency margin has itself been the subject of much discussion and we do not intend to comment further on the correctness or otherwise of admitting any portion of future profits as an element of a solvency margin. Article 18 also places an upper limit of 3½% of the capital sum at risk on any Zillmer adjustment employed and permits the admission of "hidden reserves not of an exceptional nature" as part of the solvency margin. If the basis for valuing liabilities is stronger than that prescribed by the appropriate supervisory authority, it is not permissible to use any part of the excess towards the solvency margin.

3.1.2. Article 19 sets down the arithmetic bases to be employed in assessing the minimum solvency margin for non-linked assurance
and annuity funds and they are 4% of mathematical reserves for business written gross of reinsurance plus 0.3% of the capital sum at risk. The maximum allowance for reinsurance is 15% of mathematical reserves and 50% of capital sum at risk. Some reduction in the 0.3% margin is given for short term temporary assurances. Supplementary insurances and accidental death benefits require a solvency margin as for non-life insurance—that is 18% of premium revenue up to 10,000,000 units of account (ECU) with a reduction to 16% for any premium revenue in excess of that figure.

3.1.3. Article 20 requires the establishment of a guarantee fund equal to one-third of the minimum solvency margin as determined by Article 19. At least 50% of this fund shall consist of share capital, free reserves and unallocated profits carried forward, with a minimum of 800,000 ECUs.

3.1.4. The above is intended to be a summary of the provisions of the E.E.C. Life Directive and is, therefore, because of the requirement of brevity, a précis of the wording contained therein.

3.2. Solvency Requirements as Incorporated into U.K. Legislation

3.2.1. The E.E.C. requirements are now incorporated into the U.K. legislation through the consolidated Insurance Companies Act 1982. For direct writing offices or for offices transacting both direct and reinsurance business, the requirements are the same as those embodied in the Directive. However, the U.K. Government has decided to extend the solvency requirements to pure reinsurers who are specifically excluded from the Directive, but has modified the margins applicable. Pure reinsurers are allowed to reduce the margin of 4% of mathematical reserves by 50% for business given off whereas for other offices the reduction has a maximum value of 15%. In respect of the second margin, if it is risk premium business, the factor to be applied is 0.1% in place of 0.3%. In addition the supervisory authority is prepared to use its discretionary powers to permit a pure reinsurance company to use the factor of 0.1% for all its assurance contracts.

3.3. Fundamental Flaws

3.3.1. We consider that there are fundamental flaws in the solvency requirements prescribed by the Directive stemming first from paragraph 1 of Article 17 and second from Article 19 (see 3.1. above). Article 17 paragraph 1 states:

"Each Member State in whose territory activities are carried on by an undertaking shall require the undertaking to establish sufficient technical reserves, including mathematical reserves."
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"The amount of the technical reserves, including mathematical reserves, shall be determined according to the rules fixed by the Member State or, in the absence of such rules, according to the established practices in such State."

3.3.2. We have been unable to locate definitions of "technical reserves" and "mathematical reserves" universally applicable throughout the Community.

3.3.3. The Directive, not unreasonably, allows each Member State to determine its own basis for the calculation of liabilities. However, these bases should in equity lead to consistently calculated reserves aimed at satisfying the criterion contained in the Buol report (Buol et al. (1971)) that technical reserves (which include mathematical reserves) must be "equivalent to the most accurate estimate possible of the amount which the concern must possess at any time in order to be able to pay claims arising."

3.3.4. Notwithstanding this constraint necessary to secure equity among Member States, the supervisory authorities have not all acknowledged that this is their goal in setting reserve bases, and there is no provision to take account of the relative strengths and weaknesses in the normal bases prescribed by Member States.

3.3.5. As regards the flaw stemming from Article 19, the Working Party took the view that, in principle, there is no objection to the imposition of an explicit solvency margin on top of the reserves calculated on a "best estimate" basis provided that the explicit solvency margin is properly calculated. However, as set out below, the Working Party has been unable to satisfy itself that this is the case.

3.3.6. It is worth reiterating that preceding paragraphs seek to make two points. Firstly, as is discussed in the rest of this section of the paper, the theoretical basis of the prescribed solvency margin factors has not yet been found. Secondly, even if the margins had been proved to be of the correct size, it is unsound to apply the margins to reserves, the basis of calculation of which is not uniform amongst Member States, with the result that the strengths of the emerging reserves may well vary considerably.

3.3.7. This failing could be of substantial importance once regulations are fixed relating to the freedom of provision of services within Member States. There could be considerable divergence in the strengths of reserves required in various States. Hence, unless these bases are co-ordinated, the amount of solvency margin required for
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any given block of business could vary depending on whether it was transacted by a concern having its Head Office in a given state or by a concern based elsewhere in the Member States.

3.4. The Campagne Report

3.4.1. One of the two most important parts of the solvency margin required by the Directive is the 4% fraction of "mathematical reserves". This 4% figure has existed without alteration since the publication of the second Campagne Report (Campagne (1961)) under the auspices of the O.E.E.C. as it then was. This second report by Professor Campagne expanded on an earlier report published some four years previously which drew upon the studies of ten Dutch insurance companies over the years 1926 to 1945.

3.4.2. Although further investigations were performed in the second Campagne Report, both in relation to more up-to-date data and also data from other countries, it was not thought necessary to change the basic 4% figure even in the light of the widely differing results produced from country to country. This second amount of data included both life and general insurance companies and was examined separately for each class. The Working Party thought that the approach used by Professor Campagne had much to commend it but that, because it was based on data which were so out-of-date and which were admitted not to be independent, there must be serious doubts as to the validity of the conclusions drawn.

3.4.3. The general approach used by Professor Campagne was to compare the profits or losses as stated in the profit and loss account (adjusted for any amortization thereof or any transfer from other available reserves) with the mean value over the financial year of the reserves. The resulting ratios were then grouped together in a frequency curve representing calculations over one year. From this curve other frequency curves were constructed showing the frequency distributions for profits and losses as percentages of reserves for periods of two, three, five and ten years. The frequency curves thus obtained were then used to determine the necessary level of free assets expressed as a percentage of the reserve, so that the free assets were large enough to ensure that the chance of a loss exceeding these assets in any particular period of years was kept below a given figure. The 4% figure was brought out in analysis and represents a 5% probability that accumulated losses over a period of three years would exceed the free reserves available. To reduce this 5% probability to 1% would involve a doubling of the 4% figure to 8% of reserves.
3.5. Criticisms of the Campagne Report

3.5.1. As mentioned in the previous paragraph the Working Party are not convinced of the relevance of Professor Campagne’s work to the present time for the reasons given below.

3.5.2. In the first place, the data were gleaned from a period which is between thirty-eight and fifty-eight years ago; the economic conditions ruling during that period were considerably different from those being experienced at present even allowing for the fact that we are currently undergoing a period of economic depression. As an illustration of differing economic standards, Campagne evidently felt that a period of falling prices was to be accepted and that overheads would be adjusted accordingly. The Working Party thought this would be unlikely in current circumstances. They were more concerned that no reference was made to the financing requirements of writing large amounts of new business whether due to inflation or by expansion of market share.

3.5.3. Campagne admitted to the weakness of his analysis relating only to Dutch companies, in stating “the purchasing power of the currency did not decline to the same extent as in other countries”.

3.5.4. Although the approach used by Campagne, by definition, includes fluctuations in profits occasioned by variations in mortality, he did not think it necessary to reserve explicitly for catastrophe, preferring to state: “in the event of a nationwide catastrophe, the Authorities would undoubtedly come to the assistance of all concerns which are vitally necessary to the country’s economy.” This remark is all the more surprising when it is recalled that the period of investigation encompassed the Second World War. Further comment on this aspect is made in our observations on the Buol Report with particular reference to the other major part of the solvency margin.

3.5.5. The Working Party felt, however, that their main criticism must be that Professor Campagne’s research is based on data which are likely not to be independent. This conclusion is based on the fact that the ten Dutch insurance companies studied were all similar and long established. In fact, Campagne himself studied the independence of his data and concluded, in relation to the probability of losses or profits in any given year being affected by the previous year’s results: “the probability of another loss following a loss is generally smaller than it would be if there had been a profit in the previous year, and vice versa. But this influence is of only secondary
importance. There seemed to be more likelihood of a profit when the previous (several) years had shown losses. Where a Company is otherwise sound, there is therefore a normal tendency to earn profits after several years of losses."

3.5.6. There are other imponderables which the Working Party has been unsuccessful in investigating. In the Campagne Report itself discussion covers the full range of investments currently available; it also refers to virtually all methods of distributing surplus to with-profit policyholders from cash payments to reversionary and terminal bonuses. However, we have been unable to discover the investments actually available to the Companies examined or the bonus policies they used and, for example, cash bonus offices could produce substantially different analyses from reversionary bonus offices. Likewise, we have no details of their product mix.

3.5.7. It is the opinion of the Working Party that the statistical evidence being used to generate a global solvency margin is insufficient and unsuitably based.

3.5.8. Before leaving the Campagne Report, two items which have been changed in the Directive should be recorded. First, the Zillmer adjustment percentage of 2% of sum assured recommended by Campagne has been raised to 3½% in the Directive but we were not able to trace the reasoning behind this move, although it was felt that it was not a reflection of the increase in the overall rate of new business acquisition expenses. Second, as far as we are aware, unit-linked business was very new at the time of the Campagne Reports and the 4% figure of reserves has since been relaxed, if the company bears no investment risk, to 1% where the term of the contract exceeds five years and the allocation to cover management expenses set out in the contract is fixed for a period exceeding five years and to nil otherwise. This latter provision is of great importance to Pension Managed Fund Companies.

3.6. The Buol Report

3.6.1. This report was submitted in 1969 under the aegis of the OECD Insurance Committee and represents the deliberations of a Working Party, under the chairmanship of Mr. Buol, at that time a member of the Swiss Insurance Supervisory Service. The Working Party included, for all or part of its studies, members of the Insurance Supervisory Services of all the then E.E.C. Member States (excluding Luxembourg) together with Austria, Denmark, Finland, Japan, Spain, Sweden, Switzerland, Turkey and the United Kingdom
and thus, it may be construed, had a wider than merely E.E.C. view
on the subject of “Financial Guarantees required from Life Assurance
Concerns”.

3.6.2. The rationale behind the main conclusion of this Report
was that a solvency margin formula could be provided by evaluating
the difference in reserves required on “unstrengthened” and
“strengthened” interest rate bases. In order to do this it is neces-
sary to define both these terms and an “unstrengthened” rate was
considered to be the retrospective average of the yield on the assurer’s
assets over a sufficiently long period (say 20 years), reducing this
figure by 10% or some other formula (but in no circumstances
should the unstrengthened rate exceed 90% of the current relevant
yield.) Having defined this term, “strengthened” was taken to
mean a figure of 80% of it although a figure of 85% was considered
as an alternative by the Buol Working Party “should the European
Communities opt for a system leading to a markedly lower level of
solvency than that resulting from the measure proposed in this
report”.

3.6.3. Expressed in algebraic form, for an endowment assurance,
the total margin, $M^t$ is made up of two parts $M^r$ and $M^p$ as follows,
where $A, a$ and $\pi$ are on the strengthened basis and $A', a'$ and $\pi'$
are on the unstrengthened basis:

$$M^t = (A_{x+t} + t \cdot n - q) - (A'_{x+t} + t \cdot n - q)$$
$$= (A_{x+t} + t \cdot n - q) - (A'_{x+t} + t \cdot n - q)$$
$$+ (\pi - \pi') a_{x+t} + t \cdot n - q$$
$$= (V_{x:m} - t V'_{x:m}) + (\pi - \pi') a_{x+t} + t \cdot n - q$$

where $M^r$ is the difference in the policy reserve on the strengthened
and unstrengthened bases and $M^p$ is the present value on the streng-
thened basis of the difference in net premiums on the two bases.

3.6.4. These two elements of $M^t$ were then studied separately in
order to establish a working rule and it was found that, not surpris-
ingly, “the very different forms of benefit can show essentially dif-
ferent curves representing their implicit safety margins”. The
research brought out a figure of 0·6% × outstanding term × reserve
for $M^r$, which for the average portfolio Buol considered became 9%
of unstrengthened reserves. Buol also found a figure of 6% of sum
assured at risk (defined as claim sum assured less unstrengthened
reserves) for $M^p$. 
3.6.5. In looking at the Buol Report it first has to be mentioned that as it is recommending an approach based on strengthened and unstrengthened reserves, the concept of a "probability of ruin" is not explicitly considered. Furthermore, the interest rates on which it operates are $2\frac{1}{2}\%$ and $3\frac{1}{2}\%$. We understand that $3\frac{1}{2}\%$ was a common interest rate in other European countries for premium rates at that time. Finally, the Working Party thought that, although the Buol Report contains much excellent work, nevertheless in trying to find a simple operating rule (which would not now be required for computer held records) there had been undue simplification. The Buol Report considers a "classic portfolio" of business standardised by class, term and duration and this simplification may obscure many of the dangerous situations which can arise in other portfolios.

3.6.6. It may be that this Report forms the rationale for the double-barrelled approach to an explicit solvency margin adopted in the E.E.C. Life Directive, i.e. a term dependent on reserves plus a term dependent on sum assured at risk. We attempted, without success, to reconcile the factors of 4\% and 0.3\% which seem to have replaced the figures of 9\% and 6\% in this Report. Our research has not brought to light any other document recommending this approach although the Buol Report appeared in 1969 and we know that the 0.3\% was already in the draft E.E.C. Life Directive by 1968.

3.7. Criticism of the Buol Report

3.7.1. The Working Party believes that prescribed solvency margins should ensure that the probability of ruin of an insurance enterprise is no greater than an agreed figure, and should bear as equitably as possible on all the enterprises to which they are applied. However, the approach developed in the Buol Report bears no direct relationship to the avoidance of ruin. Moreover, it is not clear that, in practice, the unstrengthened rates of interest, being related to yields obtained, will result in equivalent strengths of reserves from one company (or country) to another. Furthermore, the definition of the "unstrengthened" valuation rate of interest itself contains an implicit solvency margin by virtue of the overriding stipulation that, at most, 90\% of current yield should be permitted. To superimpose a further explicit margin of 20\% of the valuation rate of interest seems to be very much a belt and braces approach.

3.7.2. There appears to be no justification for the figure of 80\% to apply to the "unstrengthened" rate of interest apart from the
fact that it should be expressed in a percentage form thus having more impact when interest rates are high. As a result, the figures brought out represent the difference between two net premium reserves and are only as relevant to a solvency margin as is the difference in interest rates used.

3.7.3. Finally, the Buol Report was prepared at a time of relative stability in all the major financial markets of the world and we again question if an approach based purely on conditions 30 years ago is appropriate to the conditions currently facing certain, if not all, Member States of the E.E.C. As the most fundamental support for this argument, as well as being a meaningful observation in its own right, we quote from the Buol Report—"the application of the schematic formula 5 (Mr = 9% reserves) should not be considered unless it is certain that one is dealing with a normal classic portfolio, for in all other cases the results are likely to be completely unacceptable or even absurd". We feel that the normal classic portfolio of the era of the Buol Report has little or no relevance to present day conditions for some companies in some Member States of the E.E.C.

3.8. The Final Compromise

3.8.1. As we have illustrated in the foregoing, we believe that the solvency margin content of the E.E.C. Life Directive has its roots in the Buol Report because of the form in which it is expressed, i.e. a margin of reserves plus a margin of sum assured at risk. Despite the intensive research efforts of the Working Party the reason for the dilution of the factors produced by Buol to those contained in the Life Directive remains a mystery. We suppose that the reserve dependent factor was reduced to 4% because of the already available research of Professor Campagne. If this was so, then why should there be a factor dependent on sums assured at risk because this would have already been taken into account, as a consequence of the methodology used in Campagne's results? And why should it have been set at 0.3%? The net result of several months' investigations in many areas is most unsatisfactory and is summarised by the reply of the Supervisory Authority of one of the original E.E.C. Member States to our enquiries on the point. The reply said: "The rules are purely set through negotiations and are a compromise reflecting each Member State's positions and interest".

3.8.2. The Working Party have, therefore, been driven to the conclusion that no statistical basis underlay the 0.3% factor in the E.E.C. Solvency Margin and that the basis underlying the other 4% factor is, at best, dubious.
4. APPROACH FOR ESTABLISHING A SOLVENCY MARGIN FOR A GIVEN PROBABILITY OF RUIN

4.1. The Stochastic Approach

4.1.1. As mentioned in Section 2 of this Paper the Working Party has concerned itself mainly with the requirements of the supervisory authority and took the view that, in order to satisfy the supervisory authority that it was solvent, a company would have, in some way, to demonstrate that the probability of its being unable to meet its contractual liabilities under current and immediately prospective contracts (on assumptions acceptable to the supervisory authority and to the actuarial profession as a whole) was less than a probability of ruin prescribed by that supervisory authority. But how could the probability of ruin for a particular company be determined in a particular set of circumstances? We decided that the best approach would be to use a stochastic model office technique to simulate a wide number of possible future scenarios so that we could analyse the results in the hope that we could arrive at a relatively simple set of rules by which the probability of ruin of a particular office could be estimated, allowing for all the features of that office as regards the type of business it had written, the premium rates it had charged and the assets it held.

4.2. Allowance for Mortality

4.2.1. In Section 2.5 we listed four areas which affected the solvency of a company. Dealing first with the question of mortality, the Working Party decided that the solvency margin for this item could be dealt with separately from the other items and in this way one less factor would have to be taken account of in the stochastic model. A separate investigation was therefore carried out into the effect on solvency of the various factors referred to in (a) of that section—a substantial improvement in mortality arising from medical advance, a substantial deterioration due, for example, to war or to disease, the effects of long term drifts resulting in major shifts in the mean mortality experienced over a protracted period and fluctuations about mean levels. Details of this investigation are given in full in Appendix 3, from which it will be seen that we concluded that in rough terms it would not be unreasonable to cover the possible adverse variations in mortality rates by a solvency margin of 1.3% of the amount at risk under assurance contracts and 1.6% of the reserves for annuity contracts. These figures are approximate. In the case of certain portfolios of liabilities reduced margins may be acceptable.
4.3. Allowance for Options and Guarantees

4.3.1. We considered carefully what allowance could be made for options and guarantees but concluded that, as there was such a variety available under contracts issued in the U.K. market, it would be impractical to provide for them in our model. The guarantee which has recently been of particular concern to actuaries in this country, the maturity guarantees given under some unit-linked contracts, has been dealt with adequately elsewhere (Benjamin et al. (1980)). There are other guarantees which can lead, and indeed have led, to offices becoming insolvent—for example the guaranteed surrender values given under single premium deferred annuities particularly where the assets are stock exchange securities and not deposits. Similarly guaranteed surrender values are in effect given under the "open ended" type of assurance contract where the contract can be generally cashed in for a value equal to the sum assured and accrued bonuses scaled down in the proportion of the premiums actually paid to the total premiums payable under the contract.

4.4. Allowance for Investment Risks and for Expenses

4.4.1. We intended, therefore, that our approach should deal only with the effect on the solvency of an office of the two factors mentioned in Section 2.5—investment risks and expenses. Our aim was to construct a model which would enable us to deduce what was the likely value of the assets needed by an office at a particular time to enable it to meet all its future liabilities without recourse to further capital if the chance of its failing to do this was to be no more than the prescribed probability of ruin. In the event time constraints and the results obtained from the investigation of investment risks led us not to pursue investigation into expense risk.

4.5. Use of Time Series Analysis

4.5.1. In order to carry out the necessary projections required for the model we had to make assumptions about possible future movements in the equity and fixed interest markets and in the rate of inflation. The Maturity Guarantees Working Party had been concerned only with future movements in equity share prices so that further investigation was needed into a suitable stock market model. To this end historical information was supplied to Messrs. Gwilym Jenkins and Partners relating to rates of inflation, rates of dividend growth, dividend yields and short term and long term interest rates, and the Standard Life (to which the Working Party is much indebted) commissioned them to carry out a multi-variate time series analysis. The result of this analysis was a set of inter-related formu-
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lae which can be used to produce possible values into the future of the rate of inflation, equity share prices, equity share yields, along with the yields of short term and long term fixed interest securities. From this the Working Party derived a suitable stochastic model for these investment variables. This model will be fully discussed in a forthcoming Paper to the Faculty by Professor Wilkie and is outlined in Appendix 4.

4.6. Construction of Model for Use in Projections

4.6.1. We mentioned earlier that we considered that a life assurance company should be regarded as solvent if it had assets sufficient to satisfy the supervisory authority that the probability of its being unable to meet its contractual liabilities under current and immediately prospective contracts was less than a certain figure (the probability of ruin) prescribed by that supervisory authority. We suggested that a statistical model could be used to generate a number of “possible futures” in order to determine the minimum figure for the value of the assets that had to be held by the office in respect of a particular portfolio of liabilities so that if it did not have recourse to further capital the probability of ruin did not exceed the prescribed figure. In practice this is not the approach adopted at present under the E.E.C. Life Directive, and we considered that what we should do inter alia was to compare the value of assets required for solvency (the solvency reserve) with the reserve on the U.K. statutory minimum basis to see how the difference between these two reserves compared with the solvency margin required by the E.E.C. Life Directive.

4.6.2. We used an approach which allowed us to determine for a particular portfolio of non-profit business the solvency reserve $(SR_t)$ that would be required at each duration $(t)$ for any particular simulation $(s)$ of future investment conditions. Each such simulation was produced using the investment model mentioned in 4.5.1. If future events followed that simulation exactly then a solvency reserve of $SR_t$ at duration $t$ would guarantee solvency (assuming that the assumptions made about mortality and expenses in the premium formula were correct). Of course the set of future investment conditions produced by this simulation $s$ is only one of an infinite number of possible sets of conditions, and we, therefore, carried out a number of simulations—1000 for each portfolio. We examined the 10th and the 50th highest value of $SR_t$, $1 \leq s \leq 1000$, for each value of $t$, to show the size of the solvency reserve that would be required if the probability of ruin was to be 1% or 5%. The mean solvency reserve
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for duration $t$ was then calculated, as was its standard deviation. At
the same time we determined for each simulation the statutory
minimum reserve ($\text{SMR}_t^s$) that would be required at each duration
(excluding the solvency margin), and calculated the margin ($\text{M}_t^s$)
that would be required over this statutory minimum reserve to pro-
vide the solvency reserve, i.e. $(\text{SR}_t^s/\text{SMR}_t^s - 1)$, expressing this as a
percentage. Again the mean value of $\text{M}_t^s$ was calculated over the
1000 simulations, along with the 10th and the 50th highest values
and its standard deviation. These calculations did not require us to
calculate the retrospective reserves from premiums paid (except as
mentioned in 4.6.7 to enable us to determine the bonus to be declared
under a with profits policy).

4.6.3. The method we used to determine $\text{SR}_t^s$ and $\text{SMR}_t^s$ for a non-
profit policy was as follows. The premiums expected to be received
and the claims to be met were calculated each year for the particular
contract being considered. Appendix 5 gives the actuarial basis used
in this calculation. An accumulation of premiums less claims,
allowing for dividends and interest payments received as well as
appreciation according to the particular investment conditions, was
carried out from the present time until the last claim was met, for
each simulation $s$. To simplify the calculations it was assumed that
all claims were paid at the end of the year, and that dividend and
interest payments were also received at that time.

4.6.4. If the assets were fixed interest securities calculations were
carried out at the start of each year to determine whether or not the
business in force could be immunised by investing in irredeemables.
If longer-dated securities were required investment was assumed to
be in irredeemables, otherwise in the dated securities required for
immunisation. A flat yield curve was used and it was assumed that
all stocks would be purchased at the beginning of a year at par.

4.6.5. The next step was to work backwards from the time the
last claim would be met, to determine for each year-end the solvency
reserves ($\text{SR}_t^s$) that would be required, according to the particular
set of investment conditions produced by the simulation $s$, to meet
all future liabilities, allowing for any appreciation/depreciation for
the year in question.

4.6.6. The statutory minimum reserves $\text{SMR}_t^s$ were calculated at
each duration and for each simulation, using a valuation rate of
interest of $92\frac{1}{2}\%$ of the net current yield on the particular type of
assets assumed, restricting the net premium to 95% of the office
premium as prudent provision for future expenses. The margin required over this statutory minimum reserve, $M^s_t$, was calculated as described earlier.

4.6.7. If the contracts were with profits then, before determining the amount of the solvency reserves, the bonuses that would be declared for these policies for each simulation had to be determined. Two different approaches were adopted:

1. The bonuses $B^s_t$ to be declared at duration $t$ for simulation $s$ were determined by comparing the retrospective accumulation of premiums less claims (ignoring capital appreciation/depreciation), with a net premium valuation on a 3% net interest basis. If the former exceeded the latter then the difference was used to provide one year’s bonus; if not, then no bonus was declared for that year. The average value of $B^s_t$ over the 1000 simulations was calculated at each duration, as well as the standard deviation.

2. This was an attempt at a more realistic basis, closer to what might possibly be adopted in practice to determine an office’s bonus policy. The values of assets and liabilities to be used for valuation were derived as follows:

Where investment was in fixed interest securities the net current income and any gross redemption payments (at par) were valued using the five-year average of the net fixed interest yield. In valuing liabilities this same average rate was used.

Where investment was in equities a notional value of the assets was calculated by valuing the actual dividend income at the valuation date using the average over the previous five years of the dividend yields. In valuing the liabilities the valuation rate was taken as the five-year average of the net dividend yield plus the difference between the five-year average of the gross fixed interest yield and the five-year average of the gross dividend yield. This adjustment was made to produce a valuation rate for liabilities consistent with the growth rate apparent in the investment experience. A bonus reserve valuation was carried out, the bonus rate to be declared at the valuation date being the rate which would result in the value of the liabilities being equal to the notional value of the assets if allowance was made for this rate to be declared in all future years.
4.6.8. Having calculated the bonus to be declared each year on the particular basis adopted, it was then possible to work backwards from the date of payment of the last claim, as was done for non-profit contracts in 4.6.5, to determine what reserves had to be held each year, including an allowance for future bonus payments.

4.6.9. The actuarial bases set out in Appendix 5 cover the premium basis for the cohorts of policies which have been investigated with the results shown in Appendix 6. Whilst the Working Party are confident that the investment model used is suitable for producing the projections illustrated in Appendix 6—it is certainly the best model they have been able to produce—they hope that discussion of this Paper may focus predominantly on the ideas developed earlier in the Paper and on the main conclusions to be drawn from the illustrations produced, rather than on the details of the investment model. This point is made because, whilst it is a truism that no model, no matter how carefully produced, can be unarguably a valid representation of what the future holds in store, it is believed by the Working Party that small changes in the structure of the investment model used are unlikely to vitiate the conclusions drawn. Discussion on the appropriateness of the use of any stochastic model whatever in investigating solvency is, of course, an entirely different matter, but for reasons given earlier the Working Party believe it is valid and, despite its complexity, is a necessary tool in discussion of life office solvency, just as it is a necessary tool in the establishment of reserves to be held against guaranteed maturity values for investment linked policies.

4.7. Illustrative Projections

4.7.1. Earlier sections of this paper have described general considerations relating to solvency, particularly from the standpoint of the supervisory authority, and have examined current E.E.C. solvency legislation, including an only partially successful attempt to ascertain its origins. This fourth section of the paper puts forward the Working Party’s contention that a stochastic approach should be used to establish, for a particular enterprise at a particular time, the solvency margin required for a given probability of ruin. The Working Party has, however, not found it possible within the confines of the resources at its disposal to produce an extensive series of illustrations dealing with a variety of offices at different stages of development and with widely different portfolios of assets and liabilities. Instead, we have chosen to illustrate a number of simple positions, some of which, no doubt, will be thought somewhat extreme.
This has been done to focus attention on the importance of suitable investment selection and on the importance of bonus distribution policy and of future bonus assumptions when considering the solvency of any particular life office. The Working Party believe that the importance of these matters is such as to make any simple margin of solvency universally applicable to all offices, regardless of their portfolios of assets and liabilities, no more than a chimera.

4.7.2. Appendix 6 contains 11 tables of statistics. The first three tables, A.1 to A.3, illustrate the progress of portfolios of non-profit policies over 1000 simulations. They deal with a single generation of twenty-year policies all issued together. Statistics are shown on the assumption of 100% fixed interest investment, 100% equity investment, and a 55%/50% mixture of the two by market value at any time.

4.7.3. The next four tables, B.0 to B.3, illustrate the progress of similar portfolios of with profits policies. Here, the reversionary bonuses have been calculated to exhaust the surplus emerging each year on the basis of a book valuation of assets set against a 3% net premium valuation as set out in 4.6.7 (1). In very general terms, such a valuation approach will represent the results which might be exhibited by an office which chose to reserve for something approximating to one half or perhaps rather more of its current rate of bonus. It will be recalled that it was suggested earlier that this may reasonably be thought to correspond to a policyholder's reasonable expectation, though it must be emphasised that the Working Party regards these figures as illustrative of such a view rather than endorsing that view! Tables B.1 to B.3 correspond to the equivalent non-profit tables in A.1 to A.3, whilst table B.0 gives some information about the bonus rates themselves.

4.7.4. Finally the last four tables, C.0 to C.3 are similar to tables B.0 to B.3 but assume that reversionary bonuses are calculated to exhaust the surplus emerging on the basis of a notional valuation of assets set against a corresponding valuation of liabilities, as described in 4.6.7 (2) above.

4.7.5. All of the tables, other of course than those dealing with the bonus rates themselves, are set out in a similar form. First, they show the mean, standard deviation, 95% level and 99% level of the solvency reserve, calculated as described in 4.6.2 to 4.6.5. It is perhaps worth recalling here that the 99% level solvency reserve is simply the 990th highest such figure out of the 1000 simulations made.
The next set of four columns (columns 5 to 8) shows the mean, standard deviation, 95% level and 99% level of the percentage margin required over the market value of the assets held. Once again, it may be worth recalling that these margins have been worked out for each simulation (the market value of assets being derived by a retrospective accumulation) and the mean, standard deviation, 95% level and 99% level of the margins have been thereafter computed. The next set of four columns (columns 9 to 12) has been similarly computed where the statutory minimum reserves have been calculated as described in 4.6.6 et seq. Finally, the last four columns set out similar statistics for the percentage margin required over a gross premium, or bonus reserve valuation.

4.7.6. We comment first on the results shown in table A.1. It will be recalled that the investment policy chosen has been such as to match the liabilities by fixed interest securities at suitable term wherever possible. It is at once evident that the statutory reserve basis is the strongest of the bases used for the first ten years or so of the projection. (The absence of figures for the first nine years in the case of the gross premium reserve valuation indicates that some of the reserves concerned are negative.) It is evident that at the 99% level, very substantial additional reserves are required over the market value of assets throughout. This may be seen as confirming the well known fact that capital is required before undertaking a risk-taking business! The most interesting feature is, however, that at the 99% level, negative margins over the statutory reserves are required throughout, indicating the strength of the statutory reserve basis and suggesting that no solvency margin (at any rate at the 99% level) is required over and above the statutory reserve in this case.

4.7.7. Turning now to table A.2 where a single generation of non-profit policies has been (somewhat perversely) invested 100% in equities, it is at once evident that in every case, including the statutory reserve case, very substantial margins are required at the 99% level. A 4% solvency margin would be of no use here. This table confirms at once that any company writing non-profit policies and investing only in equities (and having no substantial estate) would be at severe risk of insolvency. This will come as no surprise, but the table dramatically illustrates the extent of the risk taken where an inappropriate investment policy is chosen.

4.7.8. Table A.3 supposes that a mixed investment policy is followed for non-profit policies. As might be expected, the results,
The Solvency of Life Assurance Companies 281

whilst still demonstrating total inadequacy of a 4% solvency margin, are less alarming than in the case of table A.2.

4.7.9. The next set of tables, B.0 to B.3, relate to with profits policies where the reversionary bonuses have been calculated to exhaust the surplus emerging each year on the basis of a book valuation of assets set against a 3% net premium liability valuation. Table B.0 shows each year for twenty years the reversionary bonus rates which would be declared under various investment policies, including both the mean and standard deviation for the bonus concerned. Table B.1 shows the very high margins required over the statutory reserve basis at the 99% level where investment is in fixed interest securities. Table B.2 gives similar figures where investment is in equities, and the margins required are even more alarming. In virtually no case would the E.E.C. margin of 4% be of much comfort. Table B.3 indicates the effect of a mixed investment policy.

4.7.10. Turning now to tables C.0 to C.3, where a somewhat more realistic bonus distribution policy is envisaged, we see that the relatively low bonus rates emerging where fixed interest investment is undertaken produce a satisfactory out-turn vis-à-vis market values of assets, but the weakness of the statutory reserve basis is again apparent. The bonus rates illustrated in C.0 for equity or mixed investment policies presage the results shown in tables C.2 and C.3 illustrating the prime importance of bonus distribution policy in ensuring solvency.

4.7.11. The reason for these disquieting results lies not so much in the bonus distribution policy actually followed, but more particularly in the assumption that that policy will be rigidly adhered to regardless of future conditions. Plainly, if no future bonus were reserved for, the results emerging would, in general, be more palatable than the corresponding figures for non-profit policies. To put matters another way, the model effectively demonstrates that, in far more “possible futures” than a 4% solvency margin would cater for, future investment out-turn will be such as to make it impossible to continue to follow the bonus distribution policy assumed. But, of course, in the event of adverse circumstances, an office would modify its bonus distribution policy and unless previous bonus declarations had been made at so high a level as to make it impossible for the office concerned to recover from future investment adversity, the office would in fact avoid insolvency. It would seem therefore that even the relatively modest expectations which the Working Party at first thought that a policyholder might reasonably entertain
are too exacting to form part of the basis for a test of solvency: it may be that in testing solvency, as distinct from future viability as an ongoing concern, no level of future bonus should be required to be considered.

4.7.12. We would have liked to produce results illustrating the position of an office issuing cohorts of policies year by year. The problem is that in practice the terms on which future policies are issued depend on conditions at the time. The practical problems involved in producing sufficient such projections have proved to be too great for the resources available to us. Some tentative experiments we have performed do, however, confirm our intuitive expectation that there is a stabilising effect associated with the continuing issue of cohorts of policies on reasonable terms.

4.7.13. It will be observed that we have not pursued investigation of the effects of future conditions on experienced expenses. The Working Party felt that the results produced from the investigations described for future investment conditions warranted presentation to the Faculty and decided not to pursue the problems associated with variations in experienced expenses.

4.8. Conclusions

At the time this paper is presented to the Faculty, the Working Party will have been in existence for four years and thought it as well to present such conclusions as it has reached. We have not found it possible to fulfil our brief in the terms in which it was set out. We mean by this that we have not found it possible to determine the amount of the solvency margin which should be required in practice by supervisory authorities. It is our hope however that some of the conclusions we have reached may be of use in future work on this important topic. We would state our main conclusions as follows:

(1) Solvency is a matter of probability and not, except in unrealistic and highly improbable circumstances, a matter of certainty. We think it reasonable to say that a life company should be regarded as solvent if it has assets sufficient to satisfy the supervisory authority that the probability of it being unable to meet its contractual liabilities under current and immediately prospective contracts (estimated on assumptions which are acceptable to the supervisory authority and to the actuarial profession as a whole) is less than a certain figure (the probability of ruin) prescribed by that supervisory authority.
(2) Whilst many bodies have an interest in solvency, the purpose of setting public solvency margins is to furnish sufficient warning to the supervisory authority to enable it to intervene if necessary in the affairs of the company so as to ensure that the company does not, in fact, fail to meet its contractual liabilities.

(3) The supervisory authority will, therefore, require solvency margins to be set at such a level that they have adequate time to fulfil this function.

(4) It is necessary for the supervisory authorities, therefore, to require some account to be taken of future new business. In practice, because of some of our later conclusions, this point may be more theoretical than real and in any event it is difficult, if not impossible, to guard totally against the possibility that unrestrained new business expansion over a short period may jeopardise the solvency of a company. In practice, it may be necessary to rely upon the Appointed Actuary to alert the supervisory authority if he believes that dangerous risks are being taken, whether in the new business expansion area or in any other areas which can imperil solvency.

(5) Solvency margins should be fixed in such a way as to achieve broad equity amongst various types of offices.

(6) There appear to be four main reasons which can lead a company to insolvency: mortality risks, investment risks, expenses and options or guarantees.

(7) The E.E.C. Life Directive solvency requirements do not appear to be based on sound theoretical analysis applicable to current conditions.

(8) We believe that the approach which must be adopted for establishing a solvency margin (once an acceptable probability of ruin has been prescribed) is of necessity a stochastic approach. This will probably entail the fixing by the supervisory authority, in consultation with the actuarial profession, of acceptable stochastic models for future investment variables, and possibly other variables also. Each company will be required to produce a simplified model of its own assets and liabilities, and carry out future projections using this stochastic model. We recognise that, even if this is accepted as the way forward, much work requires to be done before it can in practice be implemented. We do not, however, believe that the difficulty of this work should serve as a pretext for ignoring its validity.
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(9) Primary in determining the solvency of a company will be the propriety or otherwise of the matching of its assets and liabilities by nature and by term. This conclusion is scarcely new, but has been dramatically illustrated once again by the projections we have carried out.

(10) Despite the doubts attaching to its origins, the 4% solvency margin may not be inappropriate for a matched portfolio of non-profit liabilities and fixed interest assets, when taken in conjunction with the minimum statutory reserves prescribed in the U.K.

(11) The requirement that any substantial level of future bonus be reserved for over the future lifetime of existing with profits contracts is likely to mean that the 4% solvency margin on top of statutory reserves is inadequate.

(12) We think it unlikely, in view of our earlier conclusions, that any simple solvency margin, expressed as a percentage of reserves (even reserves calculated on a statutory minimum basis) will be adequate for all companies regardless of the nature of their assets or liabilities.

(13) It is important that offices be prepared to reduce reversionary bonuses should investment conditions warrant it. Greater emphasis should perhaps be placed on the use of terminal bonus.

BIBLIOGRAPHY


The Solvency of Life Assurance Companies

APPENDIX 1

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I. C. Lumsden, M.A., F.F.A.
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APPENDIX 2

LIFE ASSURANCE PRACTICE IN THE E.E.C.

1. A good deal of written material is available describing the conditions under which life assurance is transacted in the member countries of the E.E.C. In particular, mention might be made of the comprehensive report on insurance supervision published under the auspices of the O.E.C.D. in 1963, of the many reports published by the Life Working Group of the Comité Européene d'Assurances (C.E.A.), and of a number of individual papers only some of which are referred to in the short bibliography at the end of this Appendix.

2. Excellent although these published works are, some of them are of considerable age and they do not in any event provide a systematic review of those aspects of life assurance practice germane to the question of solvency. In order to build up a general picture of current practice, therefore, we circulated to our correspondent representatives of the actuarial professions in each country a simple questionnaire the answers to which are summarised in Tables 1, 2 and 3. Naturally no such questionnaire can hope to impart a detailed knowledge of the customs and practice of the individual countries concerned but our hope is that we have been able to highlight the main aspects relevant to our work.

3. Table 1 shows briefly the different types of business written in each country. The usual forms of whole life, endowment and term assurance policy are written in all countries, as are deferred and immediate annuities. In some countries, notably France and Germany, the "fixed-term" assurance whereby a sum is provided at maturity irrespective of survival but premiums cease on death, is also common. Group pensions business is transacted in all countries, usually on a deferred annuity basis. Throughout the Community insurance is written on a with-profits basis, usually on the basis of cash bonuses but sometimes with reversionary bonuses, whilst in some countries non-profit policies are also written. Unit-linked business is increasing in popularity throughout the Community. In most countries the insurance contract can incorporate guaranteed insurability options and options to alter the policy on guaranteed terms, whilst guaranteed surrender and paid-up values are common and, on occasion, a legislative requirement.
4. Until a few years ago, it was common practice in many countries of the community to allow composite concerns to write both long-term life assurance business, and short-term business, within a single account in such a way that the losses of one branch could be borne by the other. Partly as a result of the First Council Directive, legislation has now been or is being passed in all countries to prohibit the licensing of new composite concerns, although those in existence already will, of course, continue in their present form.

5. Just as Table 1 shows the spreads of business written by insurers in each Community country to be similar, Table 2 shows that the ranges of available investments are similar also. Although the proportions of total funds invested in different types of security differ from country to country, the availability of common stocks in particular being restricted, in all countries each of the five main types of investment is available. The normal methods of valuation of assets for solvency purposes differ widely, however, with market value (or the lower of market value and cost) being common for equity stocks but a valuation at cost being common for fixed-interest stocks. In most countries, as in the United Kingdom, legislation restricts the proportion of total assets which may be held in a single investment whilst in some countries there is a maximum percentage which may be held in equity-type assets, and in others a minimum proportion which must be held in government-guaranteed stock.

6. If Tables 1 and 2 suggest that the general conditions under which insurance business is transacted in the E.E.C. countries are similar, Table 3 shows that the legislative framework of supervision differs widely from country to country and certainly between the United Kingdom and the majority of others. In all countries a government agency is specifically charged with the supervision of insurance, and in many that agency either must or in practice does consult with actuarial and other bodies before recommending legislation. The minimum capital requirements for transacting business, and of course the solvency margin requirements, either are or will be uniform as a result of the implementation of the First Council Directive. Most supervisory authorities require yearly (or occasionally quarterly) returns to be made, and there is generally a requirement for certificates of satisfaction by an employed or independent actuary, and by an independent accountant.

7. Unlike the United Kingdom, however, most countries lay down strictly the minimum liability valuation basis and the premium rate tables in use must be approved by the supervisory authority before
### Table 1

**Life Assurance Contracts in the E.E.C. Countries**

<table>
<thead>
<tr>
<th>Types of Life Business</th>
<th>Belgium</th>
<th>Denmark</th>
<th>France</th>
<th>Germany</th>
<th>Greece</th>
<th>Holland</th>
<th>Ireland</th>
<th>Italy</th>
<th>Luxembourg</th>
<th>United Kingdom</th>
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<tbody>
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<td>Yes</td>
<td>Yes</td>
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<td>Yes</td>
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<tr>
<td>Term Assurance</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<td>Yes</td>
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<tr>
<td>Endowment Assurance</td>
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<td>Yes</td>
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<td>Yes</td>
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<tr>
<td>Fixed-Term Assurance</td>
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<td>Deferred Annuity</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<td>Yes</td>
<td>Small</td>
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<td>Tontine</td>
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#### 1.2 Guaranteed Options Commonly Offered:

<table>
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<tr>
<th>Guaranteed Options</th>
<th>Belgium</th>
<th>Denmark</th>
<th>France</th>
<th>Germany</th>
<th>Greece</th>
<th>Holland</th>
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#### 1.3 Approximate Proportion of Total Premium Income Derived from:

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<td>Non-Profit Business</td>
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<td>30%</td>
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<td>15%</td>
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<td>--</td>
<td>50%</td>
<td>70%</td>
<td>Nil</td>
<td>30%</td>
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<tr>
<td>With Profit Business</td>
<td>85%</td>
<td>70%</td>
<td>75%</td>
<td>100%</td>
<td>100%</td>
<td>--</td>
<td>10%</td>
<td>15%</td>
<td>85%</td>
<td>55%</td>
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</tbody>
</table>
### Table 2

**Investments of Life Assurance Concerns in the E.E.C.**

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<th>Belgium</th>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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</tbody>
</table>

#### 2.1 Types of Investment Available:

1. **Government Fixed Interest**
   - Belgium: Yes
   - Denmark: Yes
   - France: Yes
   - Germany: Yes
   - Greece: Yes
   - Holland: Yes
   - Ireland: Yes
   - Italy: Yes
   - Luxembourg: Yes
   - Kingdom: Yes

2. **Other Fixed Interest**
   - Belgium: Yes
   - Denmark: Yes
   - France: Yes
   - Germany: Yes
   - Greece: Yes
   - Holland: Yes
   - Ireland: Yes
   - Italy: Yes
   - Luxembourg: Yes
   - Kingdom: Yes

3. **Common Stocks**
   - Belgium: Yes
   - Denmark: Yes
   - France: Yes
   - Germany: Yes
   - Greece: Yes
   - Holland: Yes
   - Ireland: Yes
   - Italy: Yes
   - Luxembourg: Yes
   - Kingdom: Yes

4. **Real Property**
   - Belgium: Yes
   - Denmark: Yes
   - France: Yes
   - Germany: Yes
   - Greece: Yes
   - Holland: Yes
   - Ireland: Yes
   - Italy: Yes
   - Luxembourg: Yes
   - Kingdom: Yes

5. **Mortgages**
   - Belgium: Yes
   - Denmark: Yes
   - France: Yes
   - Germany: Yes
   - Greece: Yes
   - Holland: Yes
   - Ireland: Yes
   - Italy: Yes
   - Luxembourg: Yes
   - Kingdom: Yes

#### 2.2 Constraints on Investment:

1. **Minimum proportion in Government Securities**
   - 15%

2. **Maximum proportion in other categories of security**
   - **stocks**: 25%
   - **stocks**: 20%
   - **stocks**: 33%
   - **stocks**: 20%
   - **overseas**: 50%
   - **realt**: 65%
   - **property**: 40%
   - **loans**: 25%
   - **mortgages**: 35%
   - **property**: 40%

3. **Maximum proportion in any one Stock**
   - 5%
   - None
   - None
   - None
   - None
   - None
   - 5%
   - Usually 24%

#### 2.3 Basis for Valuation of Assets:

1. **Fixed Interest**
   - Market Value
   - Cost
   - Book Value
   - Cost
   - Book Value
   - Cost
   - Book Value
   - Market Value
   - Cost
   - Market Value

2. **Common Stocks**
   - Market Value
   - Market
   - Book Value
   - Cost
   - Market
   - Cost
   - Market
   - Market
   - Market
   - Market

3. **Real Property**
   - Market Value
   - Market
   - Cost
   - Book Value
   - Cost
   - Market
   - Market
   - Market
   - Market
   - Market

**Note:** Book Value = Lower of Cost and Market Value
<table>
<thead>
<tr>
<th>Basis for Valuation of liabilities</th>
<th>Belgium</th>
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<th>France</th>
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<tr>
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<td>Specified</td>
<td>Specified</td>
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<td>Specified</td>
<td>Regulated</td>
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<td>3.2 Premium Rates</td>
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<td>Advised</td>
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<td>3.3 Frequency of Returns</td>
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<td>Yearly</td>
<td>Yearly</td>
<td>Yearly</td>
<td>Yearly</td>
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<td></td>
</tr>
<tr>
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<td>Actuary</td>
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<tr>
<td>3.5 Independent Auditors</td>
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<tr>
<td>3.6 Deposited Assets</td>
<td>Technical Reserves</td>
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<td>Mathematical Reserve</td>
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<td>Technical Reserves</td>
<td>None</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3
Supervision of Life Assurance Concerns in the E.E.C.
implementation. These constraints, combined perhaps with a restricted availability of investment in other than fixed-interest and mortgage-type assets, must in our view imply a potentially less volatile atmosphere for the conduct of business than applies here in the United Kingdom and suggest that, whilst there can be no doubt of the need in all countries for a margin to be held by insurers to cover the aleatory risks of mortality, investment and management error, the need for such margins is probably at least as great here in the United Kingdom as anywhere in the Community.

BIBLIOGRAPHY


APPENDIX 3

THE EFFECTS OF MORTALITY VARIATIONS ON SOLVENCY

1. Provided that life offices employ mortality tables appropriate to the type of business being transacted, and to their own experience, the main sources of risk from mortality variations are as follows:

(a) random fluctuations;
(b) making an inadequate allowance for future improvements (in annuity and pensions business), or (possibly) an inadequate allowance for deterioration (in the mortality of assured lives);
(c) a very large temporary increase in mortality rates.

2. We shall examine each of these possible sources of risk in turn. Mortality risks for an individual office may be reduced by an appropriate reinsurance policy (provided that the solvency of the reinsurer may be relied upon).

(a) The costs and reserves associated with a given type of policy are, in principle, random variables (the values normally used by life offices being their means). Given that interest and expense assumptions are exactly fulfilled, the means and standard deviations of the reserves for the various policies may be calculated as in the note published by the Institute and Faculty of Actuaries: “A Statistical Approach to Life Contingencies” or Pollard and Pollard (1969). The practical effects, however, of random variations are unlikely to be large unless the office has very few policies, or some of them are much larger than the rest. As mentioned above, reinsurance arrangements can substantially reduce the risks from this source.

(b) An allowance for future mortality improvements is normally made when valuing annuity and pensions business, but it may turn out that this allowance is not large enough. To obtain estimates of the extra reserves which might be required, we refer to Addendum 1 of Archer, B. S., et al. (1978). The following reserves were needed for the two model funds under the PA(90) tables and “optimistic” assumptions about future mortality trends respectively.

<table>
<thead>
<tr>
<th></th>
<th>PA(90)</th>
<th>“Optimistic”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model Fund No. 1</td>
<td>100.80</td>
<td>102.44</td>
</tr>
<tr>
<td>Model Fund No. 2</td>
<td>100.67</td>
<td>103.44</td>
</tr>
</tbody>
</table>

The extra reserves needed under the “optimistic” assumptions are about 2.2% of those under the standard (in this
example, PA(90)) basis. An appropriate solvency margin for this source of risk is, therefore, about 2% of $V_A$, where $V_A$ represents the reserves held for annuity and pensions business. The extra reserves may, however, be built up slowly over a long period.

(c) The most probable causes of an extremely large temporary increase in mortality rates are epidemics and wars, the effects of which we consider in turn.

3. By far the most serious epidemic to strike the U.K. since the cholera epidemic of 1849 was the influenza epidemic of 1918-19 which is fully described in the supplement to the 81st Annual Report of the Registrar General. It affected both sexes and all social classes more or less equally, and struck in three waves between 23rd June 1918 and 10th May 1919. In Table A.1 we study the influenza mortality of females, the males' data being more difficult to analyse because of war conditions.

4. It may be seen that the excess central death rate (or force of mortality, approximately) was, to a first approximation, constant at about 0.4%. According to Scott, W. F. (1984) the mortality loss in one year associated with an epidemic on this scale is approximately equal to

$$
\frac{0.004}{2} \left\{ (S_0 - V_0) + (S_1 - V_1) \right\}
$$

where $S_0, V_0$ denote the sums assured and reserves at the present time, and $S_1, V_1$ denote these quantities at the end of the year. For practical purposes we may assume $S_1 - V_1 = S_0 - V_0$, giving a mortality loss of approximately

$$
0.004 (S_0 - V_0).
$$

5. Naturally there would be a release of reserves in respect of annuity and pension business, so the mortality loss may be written as

$$
0.004 (S_L - V_L) + 0.004 (S_A - V_A)
$$

where the suffices "L" and "A" refer to life business and annuity and pensions business respectively; (since $S_A < V_A$, the second term is a profit).

6. We now consider the effect of wars on life office solvency. Although it is possible that a nuclear war could wipe out most of the population, such an eventuality is so catastrophic that life office stability would then be of little consequence. It is, however, conceivable that war casualties might be similar to those of recent
### Table A.1

*England and Wales, females; 1 April 1918 to 31 March 1919*

<table>
<thead>
<tr>
<th>Age-group (age last birthday)</th>
<th>Central exposed to risk</th>
<th>Influenza deaths</th>
<th>Influenza death rate</th>
<th>Adjusted influenza death rate</th>
<th>&quot;Normal&quot; influenza death rate</th>
<th>Excess influenza death rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3) = (2)/(1)</td>
<td>(4) = (3) × 1.25</td>
<td>(5)</td>
<td>(6) = (4) − (5)</td>
</tr>
<tr>
<td>0-4</td>
<td>1,644,197</td>
<td>8,885</td>
<td>0.0054</td>
<td>0.0067</td>
<td>0.000098</td>
<td>0.00066</td>
</tr>
<tr>
<td>5-9</td>
<td>1,913,755</td>
<td>4,041</td>
<td>0.0021</td>
<td>0.0026</td>
<td>0.000023</td>
<td>0.00026</td>
</tr>
<tr>
<td>10-14</td>
<td>1,880,949</td>
<td>3,056</td>
<td>0.0016</td>
<td>0.0020</td>
<td>0.000025</td>
<td>0.00020</td>
</tr>
<tr>
<td>15-19</td>
<td>1,780,117</td>
<td>5,110</td>
<td>0.0029</td>
<td>0.0036</td>
<td>0.000039</td>
<td>0.00036</td>
</tr>
<tr>
<td>20-24</td>
<td>1,723,332</td>
<td>7,555</td>
<td>0.0044</td>
<td>0.0055</td>
<td>0.000048</td>
<td>0.00055</td>
</tr>
<tr>
<td>25-29</td>
<td>1,712,714</td>
<td>10,709</td>
<td>0.0063</td>
<td>0.0079</td>
<td>0.000053</td>
<td>0.00078</td>
</tr>
<tr>
<td>30-34</td>
<td>1,612,970</td>
<td>8,714</td>
<td>0.0054</td>
<td>0.0067</td>
<td>0.000057</td>
<td>0.00066</td>
</tr>
<tr>
<td>35-39</td>
<td>1,486,978</td>
<td>5,243</td>
<td>0.0035</td>
<td>0.0044</td>
<td>0.000094</td>
<td>0.00043</td>
</tr>
<tr>
<td>40-44</td>
<td>1,300,985</td>
<td>3,702</td>
<td>0.0028</td>
<td>0.0035</td>
<td>0.000115</td>
<td>0.00034</td>
</tr>
<tr>
<td>45-49</td>
<td>1,113,574</td>
<td>3,473</td>
<td>0.0031</td>
<td>0.0039</td>
<td>0.000174</td>
<td>0.00037</td>
</tr>
<tr>
<td>50-54</td>
<td>944,639</td>
<td>3,347</td>
<td>0.0035</td>
<td>0.0044</td>
<td>0.000221</td>
<td>0.00042</td>
</tr>
<tr>
<td>55-59</td>
<td>760,518</td>
<td>2,542</td>
<td>0.0033</td>
<td>0.0041</td>
<td>0.000406</td>
<td>0.00042</td>
</tr>
<tr>
<td>60-64</td>
<td>598,772</td>
<td>2,626</td>
<td>0.0042</td>
<td>0.0052</td>
<td>0.000680</td>
<td>0.00052</td>
</tr>
<tr>
<td>65-69</td>
<td>474,469</td>
<td>2,630</td>
<td>0.0055</td>
<td>0.0060</td>
<td>0.001044</td>
<td>0.00059</td>
</tr>
<tr>
<td>70-74</td>
<td>354,814</td>
<td>2,066</td>
<td>0.0058</td>
<td>0.0072</td>
<td>0.001943</td>
<td>0.00053</td>
</tr>
<tr>
<td>75-79</td>
<td>208,270</td>
<td>1,361</td>
<td>0.0065</td>
<td>0.0081</td>
<td>0.003141</td>
<td>0.00050</td>
</tr>
<tr>
<td>80-84</td>
<td>94,893</td>
<td>663</td>
<td>0.0072</td>
<td>0.0090</td>
<td>0.000529</td>
<td>0.00037</td>
</tr>
<tr>
<td>85 and over</td>
<td>42,522</td>
<td>370</td>
<td>0.0087</td>
<td>0.0109</td>
<td>0.0008421</td>
<td>0.00025</td>
</tr>
<tr>
<td><strong>ALL AGES</strong></td>
<td><strong>75,964</strong></td>
<td></td>
<td></td>
<td><strong>0.0047</strong></td>
<td><strong>(average)</strong></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

1. Column (1) is the average of the mid-year populations in 1918 and 1919 (see 81st and 82nd Annual Reports of the Registrar General).
2. Column (2) is from table 2 of the supplement to the 81st Annual Report of the Registrar General.
3. Column (4) allows approximately for deaths due to the epidemic but not recorded as being due to influenza (see the supplement to the 81st Annual Report of the Registrar General).
4. Column (5) refers to 1916 which was about average for the years around (but excluding) 1918-19 (see 79th Annual Report of the Registrar General).
The Solvency of Life Assurance Companies

history. So far as the U.K. is concerned, the worst casualties occurred in 1914-18; statistics relating to the mortality experience of the male policyholders of a large life office before and during this war were published in Burn (1919), from which we have taken Graph A.1.

7. The "excess" mortality rates of 1917 (the worst year for British casualties) may be calculated by subtracting those of the office's 1913 experience (which corresponded well with that of
The Solvency of Life Assurance Companies

The English Life Table No. 8—Males from the 1917 experience. The results are as follows (taking the averages over quinary age groups):

<table>
<thead>
<tr>
<th>Age</th>
<th>&quot;Excess&quot; rate of mortality, $q'_x - q_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>0.012</td>
</tr>
<tr>
<td>22</td>
<td>0.039</td>
</tr>
<tr>
<td>27</td>
<td>0.027</td>
</tr>
<tr>
<td>32</td>
<td>0.018</td>
</tr>
<tr>
<td>37</td>
<td>0.011</td>
</tr>
<tr>
<td>42</td>
<td>0.003</td>
</tr>
<tr>
<td>All other ages</td>
<td>0</td>
</tr>
</tbody>
</table>

The effect of "excess" mortality of this magnitude upon a present-day life office may be estimated by means of the following schedule (Table A.2), which refers to a hypothetical medium-sized office's life business.

**Table A.2**

<table>
<thead>
<tr>
<th>Age last birthday</th>
<th>Sums assured (including Reserves)</th>
<th>Death strain at risk</th>
<th>&quot;Excess&quot; force of mortality strain</th>
<th>Extra death strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>$S(x)$</td>
<td>$V(x)$</td>
<td>$S(x) - V(x)$</td>
<td>$\mu'<em>x + \mu</em>{x+1}$</td>
</tr>
<tr>
<td>17</td>
<td>16,000,000</td>
<td>160,000</td>
<td>15,840,000</td>
<td>0.012</td>
</tr>
<tr>
<td>22</td>
<td>86,000,000</td>
<td>1,290,000</td>
<td>84,710,000</td>
<td>0.039</td>
</tr>
<tr>
<td>27</td>
<td>145,000,000</td>
<td>4,490,000</td>
<td>140,510,000</td>
<td>0.027</td>
</tr>
<tr>
<td>32</td>
<td>150,000,000</td>
<td>5,700,000</td>
<td>144,300,000</td>
<td>0.018</td>
</tr>
<tr>
<td>37</td>
<td>130,000,000</td>
<td>6,760,000</td>
<td>123,240,000</td>
<td>0.011</td>
</tr>
<tr>
<td>42</td>
<td>130,000,000</td>
<td>9,490,000</td>
<td>120,510,000</td>
<td>0.003</td>
</tr>
<tr>
<td>All other ages</td>
<td>543,000,000</td>
<td>104,110,000</td>
<td>438,890,000</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>1,200,000,000</td>
<td>132,000,000</td>
<td>1,068,000,000</td>
<td>11,602,110</td>
</tr>
</tbody>
</table>

$x \times 0.85$ to allow for males only

$= 9,861,703$

**Notes:**

(1) Columns (1) and (2) were estimated from valuation returns and other data supplied by several offices. It is assumed that 15% of business refers to female lives (whose ages have been rated down) and an allowance for this is made at the end of the calculations.

(2) In column (4) we use the approximation $q'_x - q_x \approx \mu'_x + \mu_{x+1} - \mu_{x+2}$.

(3) The extra death strain in column (5) is computed by formula 5.2 from Scott (1984) with $T = 1$ and $S(x) - V(x) = S(x) - V(x)/T$, which depends on a replacement model. If war were declared, offices might stop issuing policies to young men (or issue policies with a "war clause") so the actual excess costs would probably be rather less than those given above (but not by much).
8. It is seen that war deaths would, in the above circumstances, cost about £9.8 million, representing about 0.9% of $S_L - V_L$, the death strain at risk in respect of life business. The effect on annuity and pensions business of war casualties is small (in the above circumstances).

In the event of very severe epidemics or war casualties, with profits bonuses might be reduced, as happened during and after the 1914-18 and 1939-45 wars. If policies already contained a war clause, this could be invoked to limit claims from war deaths.

9. Conclusions

The main dangers to the solvency of life offices arising from variations in mortality rates are:

(i) war deaths, and, to a lesser extent, epidemics;
(ii) mortality improvements exceeding the allowance made for them in valuing annuity and pensions business;
(iii) a lack of adequate reinsurance arrangements, especially among smaller offices.

Defining $S_L$, $V_L$ and $S_A$, $V_A$ as the death benefits and reserves for life business and annuity business respectively, we obtain as an estimated solvency margin for risks (i) the expression

$$
0.009(S_L - V_L) + 0.004(S_L - V_L) + 0.004(S_A - V_A) = 0.013(S_L - V_L) + 0.004(S_A - V_A).
$$

The margin for risk (ii) is estimated at about 2% of $V_A$, so we obtain a total solvency margin, in respect of mortality risks, of

$$
1.3\%(S_L - V_L) + 0.4\%(S_A - V_A) + 2\%V_A. \quad (*)
$$

for a typical life office.

An office with a large with profits element in its business may be less at risk because it may reduce its future bonuses: but if future bonus loadings in any case form part of the solvency margin, it is no better off than any other office with the same solvency margin.

If many of the office's life policies have less than a year to run, it may reduce the first term of the above solvency margin (*) pro rata (unless the policyholder has the right to effect a new policy).
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81st Annual Report of the Registrar-General. H.M.S.O.


82nd Annual Report of the Registrar-General. H.M.S.O.
APPENDIX 4

1. This Appendix outlines the stochastic investment model used to simulate the projections whose results are given in Appendix 5. The model will be more fully discussed and its derivation explained in a forthcoming paper to the Faculty by Professor A. D. Wilkie. The model outlined here is described in that paper as the Reduced Standard Version, and the parameters given are those considered appropriate to that version of the model.

2. The four series of variables involved in the model are:

- \( Q(t) \) The Retail Prices Index;
- \( D(t) \) An index of share dividends;
- \( Y(t) \) The dividend yield on these same share indices, that is, the dividend index at the specified date divided by the price index at that date.
- \( C(t) \) The yield on 2.5\% Consols, which is taken as a measure of the general level of fixed interest yields in the market.

3. The model for the Retail Prices Index series, \( Q(t) \), is

\[
\nabla \ln Q(t) = QMU + QA(\nabla \ln Q(t-1) - QMU) + QSD.QZ(t),
\]

where the backwards difference operator \( \nabla \) is defined by

\[
\nabla X(t) = X(t) - X(t-1).
\]

and \( QZ(t) \) is a sequence of independent identically distributed unit normal variates, or "white noise".

4. This model says that the annual rate of inflation follows a first order autoregressive process, with a fixed mean \( QMU \), and a parameter \( QA \) such that the expected rate of inflation each year is equal to the mean plus \( QA \) times last year's deviation from the mean. Appropriate values for the parameters are:

\[
QMU = 0.05, \ QA = 0.6, \ QSD = 0.05.
\]

5. The share yield, \( Y(t) \), depends both on the current level of inflation and on previous values of itself and on a white noise series. The model is:

\[
\ln Y(t) = YW. \nabla \ln Q(t) + YN(t),
\]

where \( YN(t) = \ln YMU + YA(YN(t-1) - \ln YMU) + YE(t), \)
\( YE(t) = YSD.YZ(t), \)
and $YZ(t)$ is a sequence of independent identically distributed unit normal variates.

6. This model says that the natural logarithm of the yield consists of two parts: the first is directly dependent on the current rate of inflation (a high rate of inflation implying a high share yield and vice versa), and the second, $YN(t)$, follows a first order autoregressive model, similar to that of the rate of inflation itself. Appropriate values for the parameters are:

   $YMU = 0.04, YA = 0.6, YW = 1.35, YSD = 0.175.$

7. The index of share dividends, $D(t)$, depends on inflation, both with an exponentially lagged effect and an additional direct effect, and on the residual, $YE(t)$, from the yield model, plus a white noise series. The parameters are such that a given percentage increase in the Retail Prices Index ultimately results in the same percentage increase in the dividend index, so the model has unit gain. The model is:

   $$\nabla \ln D(t) = DW \left( \frac{DD}{1 - (1 - DD)B} \right) \nabla \ln Q(t) + DX \nabla \ln Q(t) + DMU + DY \cdot YE(t-1) + DE(t)$$

   where the backwards step operator $B$ is defined by

   $BX(t) = X(t-1),$

   $DE(t) = DSD \cdot DZ(t),$

   and $DZ(t)$ is a sequence of independent identically distributed unit normal variates.

8. The term in parentheses above involving $DD$ represents an infinite series of lag effects, with exponentially declining coefficients:

   $DD,$

   $DD (1 - DD),$

   $DD (1 - DD)^2,$ etc.

   The sum of these coefficients is unity, so this part of the formula represents the lagged effect of inflation, with unit gain. This means that, if retail prices rise by $1\%$, this term will also eventually rise by $1\%$. We can alternatively describe it as the "carried forward" effect of inflation, $DM(t)$, where

   $$DM(t) = DD \cdot \nabla \ln Q(t) + (1 - DD) \cdot DM(t-1),$$

   from which we see that the amount that enters the dividend model each year is $DD$ times the current inflation rate, plus $(1 - DD)$ times
the amount brought forward from the previous year, and that this total is then carried forward to the next year.

9. Appropriate values for the parameters are:

\[
\begin{align*}
DW &= 0.8, \quad DD = 0.2, \quad DX = 0.2, \quad DMU = 0.0, \quad DY = -0.3, \\
DSD &= 0.1.
\end{align*}
\]

10. The model makes the dividend index appear to depend on the residual of the share yield. In fact share prices to some extent correctly anticipate changes in dividends. For example, an unusual rise in dividends may be correctly forecast by investment analysts, so that share prices take account of this and so rise. The yield is calculated on the previous year’s dividend, and so falls. Although this is the causal sequence, it is convenient in the model to reflect the temporal sequence, so that an unexpected fall in yields results in an upwards change in the dividend index in the following period.

11. Although the parameter DMU is set to zero, it is retained in the model, since one may wish to investigate the results of assuming a small positive or negative value for it, implying a positive or negative long-term change in real dividends.

12. The Consols yield is assumed to consist of a real part, \( CN(t) \), plus an allowance for expected future inflation. The latter is based on the actual values of present and past inflation. The real part is defined by a first order autoregressive model, with a residual white noise series. The model is:

\[
C(t) = CW \left( \frac{CD}{1-(1-CD)B} \right) \nabla \ln Q(t) + CN(t),
\]

where \( \ln CN(t) = \ln CMU + CA(\ln CN(t-1) - \ln CMU) + CSD.CZ(t) \),

and \( CZ(t) \) is a sequence of independent identically distributed unit normal variates.

13. The term in parentheses in CD has a similar form to the DD term in the dividend model, though the parameter value is different. It represents the current value of expected future inflation as an exponentially weighted moving average of past rates of inflation.

14. Appropriate values for the parameters are:

\[
CW = 1.0, \quad CD = 0.05, \quad CMU = 0.035, \quad CA = 0.91, \quad CSD = 0.165.
\]

The value of CW is 1.0, and it might appear that this term could be omitted; however, it may be of interest to investigate variations in this parameter.
15. This model says that the influence of inflation on the Consols yield is reflected by using as expected inflation an exponentially weighted moving average of past inflation, with a parameter of 0·05. The real rate of return has a mean of 3·5%, and follows a first order autoregressive series with a parameter of 0·91, so that it tends back towards its mean rather slowly.

16. It will be seen that the complete model is wholly self-contained. The only inputs are the four separate white noise series, and no exogenous variables are included.

17. The method of simulation that is appropriate for this model is similar to that used by the Maturity Guarantees Working Party. On the basis of a starting position at time \( t = 0 \), one can generate values for the four series, \( Q(t) \), \( Y(t) \), \( D(t) \) and \( C(t) \), for \( t = 1 \) to \( N \), where \( N \) is for example 100. It is necessary to simulate independent unit normal pseudo-random variables for each of the white noise series, \( QZ \), \( YZ \), \( DZ \) and \( CZ \), using for example Marsaglia's Polar method, as described in Appendix E of the MGWP Report.

18. It is necessary to choose certain initial values to represent the present state, and to start the indices. One can set \( Q(0) \) arbitrarily as 1. The model for the Retail Prices Index requires us to postulate a value for \( \nabla \ln Q(0) \), the rate of inflation "last year", i.e. in the year just preceding the beginning of the simulation period. This is denoted by \( QI \). A neutral value for \( QI \) is \( QMU \), the average force of inflation, taken as 0·05.

19. The model for yields requires us to choose a value for the share yield at the start of the simulation period. This is \( Y(0) \) or \( YI \). A neutral value for this is given by \( YMU \cdot \exp(YW \cdot QMU) \), which equals 4·27932%. The model for yields requires also a value for \( \nabla \ln Q(0) \), which has already been given by \( QI \).

20. To start the dividend series one needs to choose an arbitrary value for \( D(0) \). It is of no importance whether one uses a value of 1, or a value equal to \( Y(0) \), which would then imply a starting share price, \( P(0) \), of 1; either may be used. One then needs to choose a value for the carried forward exponentially lagged effect of inflation, viz:

\[
DM(0) = \left( \frac{DD}{1-(1-DD)B} \right) \nabla \ln Q(0),
\]

which is denoted as \( DM \). The neutral value for this is also \( QMU \), which equals 0·05. One also needs a value of \( \nabla \ln Q(0) \), given as
One then needs a value for YEI = YE(0), the random residual that took the share yield to its present level. This could either be stated explicitly, or be calculated given also values for \(Y(-1)\) and \(\nabla \ln Q(-1)\). The neutral value is zero.

21. The starting values required for the Consols yield series include a carry forward from past inflation, similar to that required for dividends, though based on a different parameter, viz:

\[
\left( \frac{CD}{1-(1-CD)B} \right) \nabla \ln Q(0),
\]

which is denoted CM. The neutral value for this is QMU, again 0.05. One also needs to select a value for the starting Consols yield, C(0). The neutral value for this is QMU + CMU, which equals 8.5%. The model for C(t) would allow the possibility of negative values of the yield if inflation were negative for long enough. To avoid these occurring a minimum value for C(t) of CMIN, set equal to 0.5%, is postulated.

22. From the values for the basic series one can calculate the share price, P(t), which is easily derived from the formula:

\[
P(t) = D(t)/Y(t).
\]

23. Neutral values for the starting parameters were chosen, and the model was then run for five years before reaching "now". This produces some scatter of actual starting positions. The values of all the parameters used are listed in Table 1.
### Table 1

Values of Parameters in Reduced Standard Basis

#### Inflation

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<td>QA</td>
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<tr>
<td>QSD</td>
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<tr>
<td>QI</td>
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#### Share Yield

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#### Share Dividend

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<td>DEI</td>
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#### Consols Yield

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<td>CSD</td>
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<td>CI%</td>
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APPENDIX 5

RESULTS FROM THE MODEL

1. The investment model used was that described in Appendix 4 using the Reduced Standard Basis. This produces over a 20-year period mean rates of money return on equity shares of 9.81% p.a. and on fixed interest securities of 8.77% p.a., in both cases before tax. In the calculations it was assumed that tax would be paid at 30% on equity dividends and at 37½% on fixed interest income, with no tax payable on capital gains in view of the indexing of capital gains.

2. The 20-year non-profit endowment assurance used in the model was one for £10,000 and was calculated for a male aged 35 on entry, using A67/70 ultimate mortality at 5% net interest and ignoring expenses, producing a premium of just under £300 p.a. The premium for the with profits policy was calculated on a similar basis but with a bonus loading of 3% p.a. compound, producing a premium of about £530 p.a.
### Table A.1

**A Single Generation of Without Profits Policies Invested 100% in Fixed Interest Securities**

<table>
<thead>
<tr>
<th>Duration in force</th>
<th>Solvency Reserve</th>
<th>Percentage Margin Required over Market Value of Assets</th>
<th>Percentage Margin Required over Statutory Reserve</th>
<th>Percentage Margin Required over Gross Premium Reserve</th>
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### TABLE A.2

*A Single Generation of Without Profits Policies In vested 100% in Equities*

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<th>Duration in force</th>
<th>Solvency Reserve</th>
<th>Percentage Margin Required over Market Value of Assets</th>
<th>Percentage Margin Required over Statutory Reserve</th>
<th>Percentage Margin Required over Gross Premium Reserve</th>
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<td>Mean deviation level</td>
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### Table A.3

**A Single Generation of Without Profits Policies**

Invested 50% in Fixed Interest Securities and 50% in Equities

<table>
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<th>Duration in force</th>
<th>Solvency Reserve</th>
<th>Percentage Margin Required over Market Value of Assets</th>
<th>Percentage Margin Required over Statutory Reserve</th>
<th>Percentage Margin Required over Gross Premium Reserve</th>
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<td>Standard 95% level</td>
<td>Mean deviation</td>
<td>Standard 95% level</td>
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### Table B.9

**A Single Generation of With Profits Policies**  
*Reversionary Bonus Rates—Based on Book Values*

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<th>Duration in force</th>
<th>100% Fixed Interest Investment</th>
<th>100% Equity Investment</th>
<th>50% Fixed Interest/Equity Investment</th>
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<td>Bonus % Standard deviation</td>
<td>Bonus % Standard deviation</td>
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<tr>
<td>Duration in force</td>
<td>Solvency Reserve</td>
<td>Percentage Margin Required over Market Value of Assets</td>
<td>Percentage Margin Required over Statutory Reserve</td>
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**A Single Generation of With Profits Policies**

*Invested 100% in Equities*

*Bonuses Based on Book Values*

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### Table B.3

A Single Generation of With Profits Policies
Invested 50% in Fixed Interest Securities and 50% in Equities
Bonuses Based on Book Values

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## Table C.0

**A Single Generation of With Profits Policies**

*Reversionary Bonus Rates—Based on Notional Values*

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### Table C.3

**A Single Generation of With Profits Policies**  
*Invested 50% in Fixed Interest Securities and 50% in Equities*  
*Bonuses Based on Notional Values*

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DISCUSSION

Mr. A. P. Limb, introducing the paper, said:—Mr. President, Ladies and Gentlemen, it is an honour and a privilege to introduce this paper to you tonight. I feel the responsibility of doing so all the more since I speak not merely on my own behalf but on behalf of all nine members of the Working Party who have been concerned for nearly four years with the brief set out in the first paragraph of the paper and with whom it has been my pleasure to work.

Mr. Micawber was able to say as we all recall, "Annual income £20, annual expenditure £19.19.6d. — result happiness. Annual income £20, annual expenditure £20.0.6d. — result misery."

Would that the problem were so simple where life assurance companies are concerned. It is a far from simple problem. As early Actuaries observed, the annual income and outgo of a life assurance company viewed over a short period are neither here nor there as regards that company's solvency. What matters is annual income and outgo over the whole life of the company until the last policyholder has been paid off.

We are concerned therefore with looking into the future as best we can, in an attempt to foresee the range of possibilities which may eventuate both as to income and as to outgo, but the future is a bourne to which no traveller has yet ventured, nor can he. Hence the major part of the intractability of the subject.

Actuaries are sometimes defined as those who forecast what the accountant will later record. The aphorism is neat but conceited. Traditionally, Actuaries have evaluated reserves for future liabilities on the basis of assumptions as to the values which relevant variables will assume in the future and they have done so for simplicity on a deterministic basis. That is to say, they have assumed for example that at a particular time in the future, the rate of return on investments will have a particular value. Quite frequently they have assumed and still do in evaluating reserves, a constant value over the future. Naturally they have attempted to be reasonably conservative in fixing such a value and the values of other relevant parameters, but our recent experience has made it painfully clear to us all that we scarcely know what the rate of interest will be a year hence let alone twenty years or more. But the solvency of a life company may well be severely affected by wide fluctuations in the rates of inflation, investment return and mortality.

We cannot then pretend that a deterministic approach gives us very much useful information on the topic. From this start point flows the Working Party's conviction that future investigations of solvency must be on a stochastic basis. That is to say they must explore a range of possible futures which might arise if mortality, investment return and inflation as it affects expenses, exhibit a fluctuating pattern in future. That is what we have tried to do, but we would be the first to admit that we have by no means explored the whole of the terrain or examined all the problems which arise in pursuing such an approach. Those problems can really be divided into two areas.

The first is that of deciding on the range of fluctuation which will be examined for each of the relevant variables which in practical terms must mean fixing on some probability distribution for the value of each at a given future time. The profession is already used to such an approach introduced recently by the work of the Maturity Guarantees Working Party considering the problem of fixing reserves for maturity guarantees under unit-linked policies and in the work of Professor Wilkie when investigating future inflation rates. It would not I think be true to say that the approach has gained universal support. Indeed it has inspired some clamant opposition. I will return to the question in a moment.
The second main problem associated with the approach is the sheer practical difficulty in applying it. I am the first to admit that the problem is massive, just as it is in the application of a similar approach to the determination of reserves for the guarantees just mentioned. I do not pretend that the Working Party has addressed itself to this practical problem in any great detail. I hope that others will tackle the question more thoroughly than we have been able to do.

I said I would return to the first problem, that of quantifying the probability distributions to be used for the relevant variables. There are some intellectual difficulties in supposing that any probability distribution can be an accurate description of any one of these variables. I admit that. Nevertheless, we know that the values of the variables fluctuate over time and if we can find a practical description of their behaviour which is consonant with past history and does not offend our good sense there seems to me to be no reason why we should not use it whilst being aware of its limitations. It is a better approach than any other. I hope that our discussion this evening in so far as it may deal with this point will concentrate on the principle of using any stochastic approach at all rather than on detailed examination of the particular probability distributions we have used. I say this for two reasons. Firstly, a forthcoming paper by Professor Wilkie will describe the particular probability distributions we have used in some detail, and comment on the reasons for their adoption. It seems to me that discussion of that paper is the best place then for a detailed examination of the particular probability distributions we have used. Second, I am convinced that even if we had used other probability distributions which were consonant with past experience as far as good sense will allow, our results would not have been significantly different.

Of course, if the Faculty are disposed to reject the stochastic approach entirely that is another matter, and the Working Party would welcome views on the point. Speaking for ourselves however we believe that a stochastic approach must be the technique of the future and Actuaries must wean themselves from undue attachment to the deterministic simplism of the past. We now have the techniques and the tools and we must use them.

Mr. D. O. Forfar, opening the discussion, said:—I think it was the famous physicist Sir Arthur Eddington who when he was seeking to find a suitable simile to describe something in the real world as near to certainty as it was possible to be chose for his example the solvency of a well run Life Assurance company.

The paper before us this evening has addressed itself to the consideration of how to set a standard which will continue to ensure the security and solvency of Life Assurance companies in order that Eddington's words may continue to be apt in the real world of today. Our membership of the E.E.C. and the enactment of the E.E.C. Life Directive means that the Working Party has been considering the problem of solvency on a larger canvas than it has perhaps been considered before. We now share with our E.E.C. neighbours a set of regulations designed to ensure, in so far as any regulations can, the solvency of Life Assurance operations which is so vital to the health and prosperity of our industry. The Working Party has been set the difficult task of investigating the criteria by which the solvency of Life Assurance companies should be assessed and recommending any desirable alterations to the E.E.C. Life Establishment Directive.

The Working Party have set out clearly the type of stochastic approach which they are recommending but what is perhaps not quite so clear is the extent to which they feel the E.E.C. Directive should now be modified to take account of their new approach.

Section 3 of the paper provides an interesting historical background to the current solvency margins contained in the E.E.C. Directive. The Working Party
reassures us at first by acknowledging that the approach adopted by the Campagne Report has much to commend it but then shakes us out of any complacency by pointing out that the data on which the 4% margin was based was determined from data from only one E.E.C. country and in respect of the years 1926 to 1945. I rather share the same feeling of dissatisfaction of the Working Party with this theoretical basis.

The Working Party then turned to the Boul Report. The Boul solvency margin is 9% of unstrengthened reserves and 6% of the sum assured at risk — these figures being substantially higher than those in the Campagne Report. The Working Party again point out that the justification for these figures is rather weak, depending as it does on a rather arbitrary figure of 80% in order to derive the strengthened rate of interest from the unstrengthened.

The Working Party also points to a fundamental weakness, namely that current E.E.C. regulations take no account of the strength of the statutory valuation bases in the different countries. I agree therefore with the authors that it is unsatisfactory to find that in the current E.E.C. regulations we have ended up with a solvency margin which takes little account of statutory bases and which is a rather curious mixture of Campagne and Boul, and while the origin of the 4% margin can be clearly traced to the Campagne Report, there is apparently no clear justification for the margin related to the sum at risk, namely 0·3%.

The problem of solvency is one of long standing for the actuarial profession and has been discussed not only in papers dealing explicitly with solvency itself but also in papers dealing with valuation methods, particularly the valuation methods which should be adopted for statutory valuations. The idea of probability of ruin has been mentioned before in our literature, for example in the 1976 paper on valuation by Stalker and Hazel (T.F.A. Vol. 35), but no one has been able until now to quantify the reserves required for a given probability of ruin. Tonight’s paper has given us a method of doing this, and must be regarded as a milestone in the advance of actuarial science. It gives a theoretical method of testing the strength of any valuation basis. The point is clearly made in tonight's paper, and brought out particularly clearly in the simulations of non-profit policies which appear in Table A1 in Appendix 6, that the need for a solvency margin in addition to the statutory valuation very much depends on the investment policy pursued and on the strength of the underlying basis itself.

Table A1 answers the question — what additional reserves are needed for a non-profit policy over and above the statutory valuation in order to ensure solvency to a high degree of probability if the office follows a policy of immunisation? I emphasise the latter point for it seems to me very important. The figures given in columns (11) and (12) of Table A1 show that no additional reserves need to be held over and above the statutory basis, which is strong enough by itself. If however, the office was so rash as to invest 100% in equities then the figures in columns (11) and (12) of Table A2 show not surprisingly that large additional reserves need to be held over and above the statutory valuations. Indeed these additional reserves amount to a multiple of the statutory reserves.

This to my mind gives a vivid demonstration of the well-known connection between the strength of the reserves which need to be held and the investment policy pursued by the office. If an office pursues an inappropriate investment policy then, unless large solvency margins are held, solvency cannot be assured with any degree of certainty at all.

If however a sensible investment policy is pursued, where assets are selected according to their appropriateness in relation to the liabilities, the solvency margins can be reduced radically.

This seems to me to illustrate that either the solvency reserves have to be related to the asset distribution, or if the reserves are not so related a statement must be made about the suitability of assets to liabilities. In the U.K. we have an
element of both these approaches. The statutory valuation basis does make some allowance for the equity backing of a contract by relating the valuation yield to the yield on the portfolio based on market values. It does not therefore recognise that valuation yields need to be lower and reserves need to be stronger if investment risks are higher. Furthermore, in the statutory returns to the Department of Trade and Industry, the Appointed Actuary is asked to sign a statement that he has taken due account of the relationship between the nature and term of the assets and the nature and term of the liabilities. There is no better illustration of how important this is than the figures which emerge in columns (11) and (12) in Tables A1, A2 and A3. It does to my mind call for greater prominence to be given to the statement by the Appointed Actuary that assets are appropriate to the liabilities. In paragraph 2.4.8 the authors again emphasise the role of the Appointed Actuary, by saying that the only way they see of guarding against the writing of excessive new business is to place the duty, as is currently done in the U.K., on the Appointed Actuary to inform the authorities if an irresponsible new business policy is being followed.

Again in paragraph 2.4.10 the authors say that the information currently supplied to the supervisory authorities in the various E.E.C. countries is not enough to enable these authorities to investigate solvency. They suggest that a certificate from the Appointed Actuary in the U.K. and the establishment of a counterpart to the Appointed Actuary in other E.E.C. countries and reliance upon a certificate from him is all that can be achieved.

The Working Party does not however state explicitly the conclusion to which these steps appear to be leading namely the desirability that the concept of the Appointed Actuary be established across the whole of the E.E.C. This may play a greater part in ensuring the solvency of companies than many other measures that might be taken. It would presumably mean that it would be necessary to establish a legally recognised definition of an Actuary in countries where this legal recognition does not exist, but I think that this is something that would be welcomed by actuarial bodies in the E.E.C. I wonder whether the Working Party feel sufficiently strongly about this to put it forward as a definite recommendation?

In Section 2 the Working Party makes it clear that in regard to solvency it has concerned itself mainly with the requirements of the supervisory authority. The setting of solvency margins is intended to act as a warning signal, to direct the attention of the authorities to cases which may need to be investigated further. There is I feel, some onus on the supervisory authority to indicate the circumstances in which they wish the warning signal to be given. It is clear that if a company is unable to meet its contractual prospective liabilities, the warning signal should be sounded at the earliest possible time.

In respect of a portfolio of with-profit business the situation is not so clear and beset with the thorny problem of the reasonable expectation of the policyholder. Clearly it is far too late if a company only comes to the attention of the authorities when it is in danger of not being able to meet the contractual liabilities, far less be able to pay a rate of future bonus.

If a policyholder has in his own mind the expectation of a certain level of reversionary bonus and this level of reversionary bonus is reduced because of a fall in interest rates, then the policyholder cannot reasonably complain that his expectations have not been met. His expectations must be tempered by the actual course of interest rates and investment returns in future years.

As it is stated on many future illustrations for with-profit policies: the level of future bonus depends on profits still to be earned and cannot be guaranteed. If however, future bonus expectations are seriously impaired because of poor investment decisions or runaway growth in expenses or substantial outflow of money on overgenerous surrender terms or by inequitable distribution of profits,
then the policyholder can legitimately complain and the supervisory authority will wish some early warning that all is not well.

In paragraph 2.2.5 it is stated that the supervisory authority will be less concerned than the policyholder that the level of future bonus will be fully competitive with other companies. I feel this should be emphasised rather more strongly. I feel that the supervisory authority is substantially less concerned than the policyholder that the results will be fully competitive in this sense. In respect of the actual results achieved on with-profit business the supervisory authorities will expect a distribution of actual results across the offices, and will only wish to investigate when actual returns appear to be falling very far short of the average. The purchaser of any product, insurance policy or otherwise, where the returns are geared to profits earned, cannot expect a uniformity of performance across all offices, in the same way an investor in the stockmarket does not expect all enterprises, even if operating in the same area, to be equally profitable. Hence, I feel there is some onus on the supervisory authority to give some guidance on the point at which it wishes the warning signal to sound.

To my mind a net premium valuation method which values the sum assured and vested bonuses only, together with a premium excluding any bonus loading, seems to make adequate allowance for the expectations of with-profit policyholders. As pointed out in the paper this is broadly equivalent to reserving for the rate inherent in with-profit rates on the assumptions inherent in the non-profit rates—it is also about half the rates currently being declared. This is just what is incorporated in current U.K. regulations and seems to me to strike just about the right balance, but I err in the direction of thinking that we are being over-assiduous in our efforts to protect the reasonable expectation of policyholders.

In trying to gauge the strength therefore of the net premium valuation for with-profit business, an important assumption concerns how the assets are invested. We have already seen in Tables A1, A2 and A3 how the solvency reserves required are dependent on the investment policy assumed. Equally in the case of with-profit policyholders we need to pay attention to the nature and term of the liabilities and it would seem important to know the degree of immunisation assumed in B1 and C1. Indeed, if a measure of immunisation is assumed it is not altogether clear why the statutory basis should not be as strong for with-profit business as it clearly is for non-profit business with the appropriate matching of assets and liabilities. If, however, it is proved that the net premium method is less strong for with-profit business, than it clearly is for non-profit, I wonder if we should not err in the direction of reducing our concept of reasonable bonus expectations rather than strengthening valuation bases.

I would now like to come to the matter of determining how regulations are to ensure that unacceptable investment risks are not being run by an office. Conclusion (8) in the paper states the method recommended by the Working Party, namely

"Each company will be required to produce a simplified model of its own assets and liabilities and carry out future projections using a stochastic model."

This method ensures that the solvency reserves increase sharply according to the degree of investment risk of the assets in relation to the liabilities. In this way an office, if it is to show solvency will be encouraged to invest in an appropriate way. A statement by the Appointed Actuary regarding the matching of assets and liabilities might not on the face of it be needed. However if each office were to make a stochastic projection it would seem to have to state its investment policy particularly with regard to matching, otherwise the appropriate margin could not be determined. This is perhaps not so different from the statement from the Appointed Actuary which I referred to earlier.

I am not sure as to the extent to which the Working Party feels the E.E.C. Directive could or should be modified to take account of their new approach. I
would rather suspect that they would agree that it is rather over-ambitious to suggest that a stochastic approach should be incorporated in the regulations. It is perhaps rather too early to gain acceptance of such a major change of approach. I think it would take a fairly long time to convince the authorities throughout the E.E.C. that this was a practical alternative. A lot more work would be required and a trial run, probably in the U.K., would seem a prerequisite. It would no doubt take a substantial time to iron out all the practical difficulties involved.

What I think could perhaps be done is to use the power of the new techniques to give an objective assessment of the strength in terms of probability of ruin of a particular valuation basis associated with a particular investment policy. It might therefore be possible in any revised E.E.C. Directive to give the supervisory authorities in each country power to modify the solvency requirements according to the strength of the statutory valuation basis. If it can be demonstrated using the techniques before us tonight that a particular valuation basis and an associated investment policy give a probability of ruin below a stated level without the need for additional margins then should the E.E.C. authorities not be prepared to accept the valuation basis and the statement from the Appointed Actuary without the need for further margins? I should be interested in the Working Party's view on this.

I have not specifically covered the matter of risks due to mortality, expense inflation or options. I leave this to other speakers, but I would just like to make the point that in respect of mortality risk it surely depends on the rate of mortality assumed in the valuation basis — so again we cannot get away from the need to relate solvency margins to the valuation basis.

I should like to finish by thanking the Working Party very much for bringing before us tonight the very considerable amount of work that they have done in tackling the problem of solvency and giving us this opportunity to discuss a completely new approach to the determination of solvency reserves.

Mr. A. J. Wise:—Making simulations is like throwing darts at a wall. We can measure the height at which a dart lands in the wall and use that to represent the outcome of the fund. The higher the dart the bigger the fund. If we take a single generation of endowment assurance policies we need a certain sum in hand at the end of the day to cover the endowments for the survivors of that generation. That required amount can be marked as a horizontal line on the wall. If a dart lands above the line we are solvent; if it lands below the line we are ruined.

If I stand here with a dart in my hand, about to throw it, I need to know whether it will land above the line or not. This will be my test of solvency as of now, before the event unfolds. To be a reliable test it must be based upon a good mathematical model of dart-throwing. The calculations will use a statistical model of the action, so I must measure solvency in terms of probability, according to my present calculations, of the dart landing above the line. This is my understanding of the approach adopted by the Working Party and I support it.

Naturally there remain many questions to be answered. How realistic is the model which is used to represent the flight of the dart — that is to say the investment model? The results will be somewhat sensitive to the numerical parameters of the investment model, but are they also sensitive to its general structure? These questions should be investigated. But my chief interest in this paper lies in the technique which is proposed for the determination of solvency margins, namely the repeated use of simulation.

I have adopted a different approach to handling statistical models, as described in the paper on matching assets to liabilities which I presented to the Institute last March. The basic idea behind that paper was that in principle the
variation of outcome of a statistical model can be formulated explicitly and calculated directly, without simulation.

Now, instead of throwing hundreds of darts at the wall, with the direct method to which I refer we are casting a spotlight. We can tell at a glance whether the given spotlight produces a diffuse or narrow beam and how high it strikes the wall. If the beam of light strikes well above the break-even line we are assured of solvency. If it does not, the spotlight will have to be raised and simple observation will show by how much.

Whether or not the degree of variation of outcome is computed directly in this way or estimated by simulation, it is interesting to consider what will happen when offices do come to make such calculations as a matter of routine. They will start to investigate the effects of holding alternative investment portfolios, and they may come to find that by switching they can arrive at a position with a lower solvency reserve requirement. If such a switch can be made without unacceptable loss of prospective investment return it may well be worth taking.

This brings me to the central conclusion drawn in my own paper, namely that for a given portfolio of liabilities there is a unique portfolio of investments which minimises the variation of outcome about the break-even position. This is true even when inflation effects are brought into both the assets and the liabilities. Moreover this unique portfolio — which I call the matching portfolio — can be found directly for any specified liabilities.

In terms of the spotlight, what we are now talking about is aiming the beam at the break-even line and focussing it down so that the spread of illumination about the line is as small as possible. If you wish to be assured of solvency, just raise the spotlight a little and because it is focussed you can be as confident as you wish of being above the line. This is surely a more powerful technique than dart throwing.

In practice an office may not wish to invest in the matching portfolio. The highly focussed spotlight corresponding to the matching portfolio may be outclassed by a more diffuse beam which strikes higher up the wall. But this is a matter of particular circumstance. When contemplating solvency in the general sense, as for statutory requirements, it seems natural to consider the matching portfolio. For a given investment model and specified liabilities of any kind the matching portfolio is unique and can in principle be computed directly, without simulation. For a given office portfolio the solvency reserves required would then have two constituents: that solvency reserve which would be required if the office was investing in the matching portfolio, and the additional mis-matching reserve required because the actual portfolio is not so invested.

Incidentally, the matching technique described in my paper can answer one question which I think is not directly answered by the simulation method as presented. That is, given the prices of investments in today's market, what solvency reserves do I need for today's particular market conditions?

I have tried one or two matching calculations for a non-profit endowment assurance, and I found that the matching portfolio consisted simply of stock redeemable precisely at the end of the policy term. This portfolio does not immunise the liabilities, but is there any reason why it should do?

The practical success of an immunisation strategy in securing the outcome relies on the re-immunisation of assets whenever conditions change. Such a strategy might well be the best in a practical environment, but it is not clear to me that for the purposes of assessing solvency we should have regard to the likelihood of an office being able to take advantage of investment conditions in the future so as to keep itself in an immunised position. If one has confidence in the underlying model, is it not preferable to identify a minimum risk portfolio for solvency purposes on the assumption of no future investment activity except as required for new money?
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If the answer to this question is yes, or if only perhaps, then I submit for consideration the method described in my paper as a way of giving direct answers to probability questions of solvency for any given market conditions.

Mr. A. H. Godson:—I would like to congratulate the Working Party on travelling at least some fair distance along the road towards establishing an effective method measuring the solvency of life assurance companies. Before coming to my main comments I would like to draw attention to two minor points.

The first is in paragraph 2.3.3. where it is suggested that the reasonable expectations of policyholders would be a level of bonus secured by the policy loadings inherent in their premiums were the expenses, investment and mortality assumptions underlying the non-profit premiums chargeable by the company concerned, to be experienced in future. It seems to me more appropriate to base these expected bonus levels on the actual rates of interest and expense levels that are experienced rather than those inherent in the non-profit premium basis.

My second minor point concerns the conclusions in paragraph 4.8 where I thought it might be reasonable to add to the possible risks of insolvency such items as the system of unit pricing and the method of allowing for C.G.T. liability in the case of a unit linked office. These are very small details on their own but they perhaps indicate that the Working Party has concentrated on conventional business rather than unit-linked. Most of the problems of unit-linked companies are, however, related to expenses which have not been considered in any detail in this investigation.

Turning now to the Tables in Appendix 6, I am not sure how much they tell us. It seems to me that the really important figures in Table A1 are the first figures in columns 3 and 4. They tell us the amount that the office ought to hold at the outset if it is to have a 95% or 99% chance of meeting its contract without recourse to further capital.

At later durations of a policy I do not think we are interested in the mean or standard deviation of the solvency reserve according to a thousand possible situations. The investment conditions at these later durations will be known when we get there, and what we then want is another thousand simulations to tell us the mean solvency reserve, its standard deviation and the amount required for 95% and 99% levels of success, given the investment conditions at that time. I appreciate that this would have meant twenty thousand instead of one thousand simulations for each group of policies.

I also wondered about the effect of averaging. For example, I would not consider a 5% or even a 1% chance of an entire office being unable to meet its claims as acceptable. But it may turn out that the reserves required for a mixture of business of different types and durations, assuming immunisation, would not be prohibitive even if aimed at a probability of ruin of only a fraction of 1%. This may become clear after further investigation.

On the question of matching, in Mr. Wise's sense, and immunising, I find myself in some difficulty. If solvency margins are to be determined according to the degree to which an office's investments differ from a best matched portfolio, we shall have the position that an office which immunises may have to hold higher reserves than one which holds a better matched portfolio. The question is which position carries the greater risk. The alternatives cannot be measured against each other directly because they are a different sort of risk. The matched position assumes that investments are held until their redemption proceeds are required to meet claims. The immunised position requires an active investment policy, investments being switched so as to stay immunised. On Mr. Wise's analogy we are allowing the dart player's teammates to blow on the dart as it passes in the hope of increasing its accuracy but with the risk of blowing it further of course. I do not pretend to know the solution to this, but I think it is a question which we
must examine. We have to make up our minds whether we are matchers or immunisers. Meanwhile I propose to think very hard about what I mean when I next answer Question 5(1) (a) of the 4th Schedule of the 1983 regulations.

Mr. G. G. Newton:—I am grateful for the opportunity to take part in tonight's discussion. The Working Party has really produced two papers it seems to me. The first is a general canter over the territory that surrounds the E.E.C. Life Directive solvency margins, to which their terms of reference direct them. The second, and I believe by far the more important, reports the results of the research which they have undertaken opening up fundamental questions about the reserving standards that should be required of life companies.

For reasons which I hope to make plain, I am not sure that there is very much connection between the two parts of the paper. If I concentrate my comments almost exclusively on the first part (the E.E.C. solvency margins and their origins), where I believe the Working Party has in fact strayed into error in places and has misread the evidence, which I feel should not go uncorrected, this should in no way be interpreted as detracting from the debt that I feel we owe the Working Party for the research they have carried out.

I think it is necessary, if we wish to consider the E.E.C. solvency margins, to go back in history a bit and ask why in the 1950's and 1960's this interest in solvency margins suddenly arose. It was not by chance that almost simultaneously the E.E.C., C.E.A. and O.E.C.D. all set up working parties to study this subject. Why should these organisations suddenly take a detailed interest in the solvency of life assurance companies? As far as I know there was no spate of insolvencies at that time to cause concern, and, even if there had been, why should the O.E.C.D., a body concerned with economic matters and trade, become interested?

The reason I suspect is that there was a general desire at that time to facilitate insurance operations across frontiers. All the countries concerned exercised detailed supervision over their own companies but were reluctant to admit companies with head offices in other countries to their national markets, ostensibly because they were suspicious about their solvency. Although foreign companies would be required to set up technical reserves in accordance with national laws in respect of the business written in each country, nevertheless, it would create problems if the rest of the operation were to become insolvent, even if (as unlike in the U.K.) there was a law that would require the assets corresponding to the liabilities in that country to be reserved for those policyholders rather than treating the company's operation as a whole. So it came to be recognised that before the establishment of branches across frontiers could take place on any scale in life assurance some sort of internationally recognised standard of soundness of companies would be required and it was to that end that all this work was put in hand.

The solvency margin was thus in some ways the buffer that the supervisor in one country felt was necessary as protection against inadequate supervision elsewhere and inevitably this would have to be set up arbitrarily, by international agreement, and would be something required over and above the prudent technical reserves which each supervisor would still require for the companies operating in his own country. And those reserves are by no means expected values as I think something in the paper could be interpreted as suggesting, but, as they should be, are very prudent reserves which the supervisor hopes will give a very high probability of ensuring that a company's liabilities can be met.

It was envisaged that there would be two standards. Companies that only wished to operate in their national market would only have to meet the reserving standards under national legislation but companies that wished to operate across frontiers would in addition have to possess the cushion of free assets representing
a solvency margin. However, this system of dual standards was not acceptable to the E.E.C. where the aim is a single market covering the whole of the Community.

There are additional reasons, in a purely national context, for requiring a solvency margin. In the first instance there is the need for a trip-wire, to ring a warning bell, to which the paper draws attention. Also the experience is that companies often get into difficulties not through extreme stochastic variation in, say, interest or mortality but through what has been called the management risk — failure to control expenses, errors in policy drafting resulting in unexpected liabilities and so on. These management risks clearly point to the value of a solvency margin, which must inevitably be arbitrary because the nature of the risks cannot be quantified, in addition to technical reserves which contain all the margins to ensure a very high probability of proving adequate against stochastic variation in the underlying variables.

In addition, of course, there is always the possibility, even with models produced by the Working Party, that the model might be wrong, and that all the calculations showing a 999 out of 1000 probability of not becoming insolvent are in fact flawed because conditions change, the future does not reproduce the past and the model is found to be faulty.

For all these reasons I think solvency margins have come to stay but it will be clear that at least in the international context, like anything that has to be agreed between different states, the level of the solvency margin is a matter for political negotiation and I believe it is a weakness of the Working Party’s report that nowhere do they draw attention to these political realities.

In negotiating the level of the solvency margin the supervisor in each country is in something of a dilemma. On the one hand he is looking for a substantial solvency margin to protect his policyholders against shortcomings in supervision elsewhere. On the other hand, all supervisors I am sure feel that their system is broadly satisfactory and that the main core of their market, represented by their major companies, is as sound as anybody could wish and that these companies should not be put in the position of being shown not to measure up to the agreed solvency standard and have to raise extra capital. These two considerations will obviously conflict but in practice the impact on companies will carry at least as much weight as any theoretical considerations.

This is illustrated by the development of the E.E.C. non-life solvency margins where the history has been documented in a paper by my colleague Mr. C. D. Daykin for the forthcoming International Congress. Development of the work of Professor Campagne produced a figure of 24% of premiums, no doubt again open to the criticisms and qualifications which the Working Party make in relation to the corresponding life margin, but that was not the solvency margin actually adopted. Tests showed that about half the companies considered would not be solvent on this standard (if applied to gross premiums) and after further research and negotiations a more acceptable standard of 18% of premiums was adopted. I do not mention this to ridicule the whole operation, but just to bring out the practical realities in agreeing on solvency margins in, for example, E.E.C. directives.

One thing that does emerge in these negotiations is that there is a deep suspicion that other supervisors may be permitting their companies to avoid the agreed solvency margins, particularly where the system of supervision is totally different from that with which the supervisor in question is familiar. This is a major problem in negotiating within the E.E.C. My predecessor spent five years trying to convince the other E.E.C. supervisors, mostly non-actuaries, of the reality of the 10% interest margin under the original C.N. 10 proposals and that the unstrengthened interest basis really did represent a concrete standard and was not something quite arbitrary which each company selected to suit the resources it had available. Although he went over the ground again and again the
only result at the end of the day was that the more we tried to explain the U.K. system, with the discretion of the actuary and so on, the more we sowed doubt about whether there was any effective supervision in the U.K. at all. I hereby give notice that I have no intention of going through a similar fruitless exercise with proposals on the lines put forward by the Working Party.

May I now turn to the origins of the E.E.C. margins. This may seem just a question of academic interest, but as I have already indicated, I believe that the Working Party have not got it right, and I do not think that the statements in this area of the paper should be left uncorrected.

There is no doubt that the E.E.C. was greatly influenced by the work of Professor Campagne in regard to both the Non-Life and the Life Directive solvency margins and that the origin of the 4% of mathematical reserves does go back to the Campagne Report: the only mystery is about the 0.3% of the sum at risk factor.

The E.E.C. were firmly wedded to the idea that the solvency margin should be required in the first instance in explicit form, though of course there are the provisions for counting implicit items within certain limits towards meeting that solvency margin. They firmly rejected the approach put forward by the U.K. and codified in Skerman's Six Principles involving implicit margins.

The Buol Committee was set up under O.E.C.D. auspices. It was a technical sub-committee, which explains why though the E.E.C. countries had representatives on the committee and participated in its work, their governments were not committed to the recommendations. It is interesting to note that the committee consisted of a mixture of government representatives and representatives of the industry. I find their report a very impressive piece of work to which I do not think the authors of the report we are discussing tonight have done full justice.

The Buol Report, while showing how all the different systems could be brought within the suggested framework, came down firmly on the side of implicit margins. The starting point was an unstrengthened interest basis which was intended to be a prudent estimate of future experience; the consideration of the experience over the previous twenty years was solely to that end. It was intended to constitute a basis that would be perfectly adequate for national reserving standards and, it goes without saying on the Continent, would also be the premium basis (the two are almost invariably synonymous). The 20% margin was intended to give a contingency margin over and above the margin of prudence already inherent in the unstrengthened rate. The Working Party's criticism of this feature I believe shows a misunderstanding of what was intended. The idea of a contingency margin in addition to a margin in the assumed rate of interest itself really corresponds to the same philosophy as the 7½% interest margin in our own regulations. This margin is over and above all the specific allowance for risk and so on that the actuary is required to make.

The Buol Report came out firmly, as I have said, for implicit margins and was unacceptable for that reason to the E.E.C. who were wedded to explicit margins. In disagreeing with the Working Party's statement in Section 3.8.1 that the origins of the Life Directive Solvency margin have their roots in the Buol Report, the first point I would make is that this would be highly improbable because it was the wrong "church". The second reason is that the Working Party base their statement on the shape of the margins; in both instances you have a proportion of the reserve and a proportion of the sum at risk, but this last factor was quite a different animal in the two cases. There is really no doubt from internal evidence in the Life Directive that the 0.3% of the sum at risk is basically intended for mortality. The Buol 6% (as the Working Party quote it) of the sum at risk is the equivalent of the present value of the difference between two net premiums. In suggesting an interest margin of 1% (or whatever the 20% formula gave) it was
the full interest margin that Buol intended, not the difference between the net premium reserves on the two bases which of course is reduced by the fact that on the stronger basis a higher net premium is valued. He wanted to use the lower rate of interest, but the same net premium. Therefore he got a second factor which was just the difference between the net premiums multiplied by an annuity value. That is what his $6\%$ of the sum at risk represents and it has nothing to do with mortality.

Finally, I do not really think that $0.3\%$ and $6\%$ are in the same league and this also suggests that we are not talking about the same thing. There would thus seem to me to be sufficient evidence — I am sorry to bore you with all this history but I think we ought to get the record right — that the Buol Report was not the origin of the E.E.C. margins as the authors maintain.

In the last couple of days, in preparing myself to come here tonight, I have found a working paper of the E.E.C. Committee on the Life Directive dated early in 1968 which, let me hasten to say, does not give the origins of the $0.3\%$ margin but it does set out the philosophy of what they were trying to do and analyses the sources of possible variation against which a solvency margin was required. It is clear that what became the $4\%$ was essentially intended against the risk of loss or depreciation in value of the assets, and for that naturally we want a proportion of the reserves. A proportion of the sum at risk was required to deal with mortality fluctuations and in addition they wanted a solvency margin against variations in expenses. In the premium formula the allowance for the continuing administrative expenses is normally a percentage of the sum assured. The latter can be expressed as $V + (S - V)$ so the required margin can then be expressed as a percentage of the reserve and a percentage of the sum at risk. That I think we must regard for the moment at any rate, as the available evidence about the origin of the shape of the E.E.C. solvency margins.

As to how they got the $0.3\%$, which seems very high compared with most temporary assurance premiums, I can only hazard my own guess. Whenever the solvency margin for pure risk contracts comes up in the E.E.C. context thoughts tend to turn to the Non-Life Directive formula of $18\%$ of premiums. Although of course $18\%$ of a typical term assurance premium would not give $0.3\%$ or even $0.2\%$, I would remind you of the survey done in the late 1970's which compared the cost of term assurance in different E.E.C. countries and showed the premiums in most of the Continental countries to be 3 times or more the the rates charged in this country. I suspect that roughly at least $0.2\%$ of the $0.3\%$ could represent about $18\%$ of a typical Continental term assurance premium at the time with the balance representing part of the solvency margin for the expenses risk. But I must emphasise that this is speculation and no doubt some further papers will come to light to prove me wrong.

Let me return to the main theme: what changes can we expect when the Life Directive comes to be renegotiated? My guess is that very few changes will be made in the general structure, which was negotiated with so much difficulty over six years or more. If a number of countries agree that the margins pinch excessively on particular classes of policy or types of company some modification could be achieved, but I think from my experience it would have to be more than one country. Equally of course if attention is drawn to gaps, particularly if there have been insolvencies on account of some factor which was not adequately covered by the solvency margin, then there will be attempts to strengthen the margin. But drastic changes are not to be expected.

Does this mean that the actuarial profession should not press on with work on solvency margins? Even if international agreement is unlikely the answer must be definitely "no". We must continue with research and the process of education, but the need for education starts a long way further back than with stochastic methods. The concept that assets and liabilities should not be treated as entirely
separate entities has barely permeated thinking in most of the Continental
countries or indeed in the United States and elsewhere.

There are some signs that this is beginning to get through but inevitably, as the
opener said, this process of education is very much a long term matter and we
should not expect quick changes in what is internationally negotiated. In the
meantime, personally I would wish that the Working Party continue its research
into what seems to me is essentially reserving standards. The distinction between
technical reserves and solvency margins in this context is totally arbitrary; it
may be of interest to the tax authorities but it is of little interest otherwise. Even
though I doubt the practicality of adopting the sort of standards which the
Working Party are suggesting, I believe the results of the type of research they
have undertaken will inevitably determine the reserving standards that, even
though expressed in simpler terms, will gradually be adopted.

Mr. A. K. Gupta:—Many of you here tonight work for companies with large
estates and substantial portfolios of with profits business. Therefore, you will
have no problem in meeting the E.E.C. Solvency Margins.

Many actuaries who work for companies selling mainly unit linked business do
not have this luxury. I would like to consider that effect of the imposition of
E.E.C. Solvency Margins on such companies. Consider the simplified example of
a unit linked company selling two regular premium contracts and one single pre-
mium contract. The regular premium contracts are a ten year savings plan with
the minimum qualifying sum assured and a flexible whole life policy in which the
policyholder can choose a sum assured between two specified levels and the single
premium contract is a unit linked bond.

New contracts will be designed so that the margin required on the reserves will
be zero per cent. Existing contracts will either require a 1% or a 4% margin. The
4% margin if, say a maturity guarantee exists, although the D.T.I. have stated
that they will permit maturity guarantee reserves to offset a proportion of the
solvency margins on such policies.

The margin of 3 per mill will be required on the sums at risk on all policies but
these are likely to be small for the ten year savings plan and the single premium
unit linked plan. However, consider the flexible whole life plan. These policies are
sometimes sold with very high sums assured and are often nothing more than a
ten year term assurance. Consequently, the 3 per mill solvency margin can be
very high. Furthermore the loss of L.A.P.R. has reduced the competitiveness of
life assurance savings plans. Many companies, therefore, are now selling a higher
proportion of protection plans than savings plans resulting in an increasing
solvency margin requirement. Besides this direct effect the solvency margins
have a further indirect effect.

Whole life commissions and related expense allowances are incurred by the sale
of these policies and very little investment income is generated, with the result
that the office soon moves to a heavily gross tax position. When this is reflected in
the pricing basis the products either become highly uncompetitive or decrease in
profitability, or even become unprofitable.

The classic solution to this problem is to sell income bonds which generate
investment income which would then move the office back into a net position. However, if a 4% solvency margin is required on income bonds this may not be an
effective solution in the future. The office can try to sell a large amount of single
premium unit linked bonds; however, these contracts tend to produce as much
unrealised gains as investment income and this can lead to a conflict where it is in
the life office's interests to realise capital gains but not in the policyholder's.

What can be done about this? You could argue that the cost of solvency
margins should not be incorporated into the pricing basis. I strongly disagree
with this view, but I may be in the minority, since so far I have not seen a
wholesale rise in the term assurance rates to reflect the cost of solvency margins.
This excellent paper has quite correctly pointed out the arbitrary nature of the
solvency margins and the fact that any simple margin is unlikely to be adequate
for all companies.
The only true protection against insolvency is proper financial control and
planning and adequate reserves.
Regulation 56 on the elimination of future valuation strain has now made the
need to hold adequate reserves a statutory requirement.
There is another unfortunate consequence of the legislation enacting E.E.C.solvency margins. This legislation should be included in the exam syllabus
if it has not already been included.
But the more legislation included in the syllabus the less room is left for
introducing the modern techniques of life office management such as profit
testing and office modelling. I believe these are the future skills of the actuarial
profession. I agree with the Chairman of the Working Party in that stochastic
models are the way ahead. However, until students get to grips with simple
deterministic models it is difficult to see how life offices are going to get to grips
with more complex stochastic models.
So far, both the Institute and the Faculty have failed to include these in their
exam syllabus. Because of the greater proportion of Scottish Actuaries working
in life assurance maybe the Faculty should take the lead in the introduction of
these techniques.

Mr. C. S. S. Lyon:—Perhaps it would be an overstatement to paraphrase the
paper by saying it contains all we want to know about solvency margins but
never dared to ask. But I am quite sure, as other speakers have said, that the
Working Party has done us a service by examining the solvency margins that
would be needed to achieve or to limit the probability of ruin to a particular level
albeit in simplistic situations. And even if it is an inconvenient truth the Working
Party has demonstrated that there are situations in which solvency margins in
some form are needed. I found myself in broad agreement with almost all the
conclusions of the Working Party's Report, with the exception of number 11
where it is implied that there is a requirement for a reserve to be held for the
bonuses that may be declared over the future lifetime of with-profit contracts.
Future bonuses are normally implicitly reserved for through the net premium
valuation basis, not by explicitly reserving for the continuation of current bonus
rates.
As Chairman of the Joint Committee on Financial Standards for Long Term
Business, I am concerned to answer the question where do we go from here? What
lessons do we draw from this report? And I have identified five in the course of
tonight's discussion. The first, from Mr. Limb, is that actuaries must learn to
wean themselves from the deterministic simplicities of the past. Redington would
have applauded this. It was one of the last messages he wanted to give to the
profession. I do not know if he would have favoured the Working Party's route
through simulation or Andrew Wise's through the use of a spotlight, but it
doesn't matter for the purpose of this discussion.
Then the opener, I think it was, said that we should experiment with a
stochastic evaluation of the strength of the valuation basis given the asset
portfolio. The point that the solvency margin needs to vary with the nature of the
asset portfolio is just as important for general insurance as it is for long-term
business, as indeed has been pointed out in previous papers.
My third point is that we would take a long time, and a long time in European
politics is an eternity, to convince the European Commission of the need for a
change to a stochastic process. That was another point the opener made and with
which I have to agree. It does seem to me that solvency margins have come to stay whether we like it or not. The important thing is to try to make sure that they interact sensibly with the valuation of the liabilities. That will take time and will involve research of the kind that the Working Party have done for us.

Fourthly, and again I am picking up a point that the opener made, we need to establish the “Appointed Actuary” principle throughout the E.E.C. This is desirable and it is something that the Groupe Consultatif would like to achieve, but I do suggest in the light of the way in which our life assurance market is behaving, that perhaps the Appointed Actuary is needed here most of all because of the freedom of contract design and pricing that exists.

This brings me to my fifth point, that is the need to strengthen the role of the Appointed Actuary in this country. It arises, firstly, through what Mr. Gupta had to say on the difficulties that small companies may have in facing the implications of solvency margins; and, secondly, from some of the things that are said in the paper, about the expectations of policyholders in relation to bonuses.

The last of the conclusions of the Working Party, perhaps appropriately numbered 13, is that it is important that an office be prepared to reduce reversionary bonuses should investment conditions warrant it. I had some quarrel with the Working Party early on when they said that terminal bonus should not form any part of a policyholder’s expectations. I think that is an over-simplification, given the role that terminal bonuses have come to play in the years since they were introduced. In a sense they are recompensing an out-going policyholder for the capital value of future deferred yield on equity-type investments. To that extent he is perhaps entitled to think that terminal bonus is part of an expectation. But I have a concern, as does the Financial Standards committee, over the way in which we fuel these expectations, in other words over the way in which life companies illustrate their quotations. This is not only in the context of bonuses, but also illustrations on unit-linked contracts, and perhaps it comes home most acutely at the present time in the illustrations that are being given for section 32 buy-out annuities to people leaving pension schemes, where I fear we are not giving the customer a comparison of like with like. We may be illustrating a quotation on the basis of current rates of bonus both normal and terminal, or by projecting the past performance of unit-linked contracts, and we may be comparing the result with the preserved pension that the customer could have left behind in his former employer’s pension scheme. Do we enquire whether that preserved pension would have been subject to discretionary revaluation (or even something more than discretionary revaluation) by that employer?

We are, I believe, in very grave danger of bringing ourselves into disrepute, unless we can discipline the way in which we sell contracts, particularly where somebody has a choice of savings media, and we really owe it to ourselves to enable the customer to come to a proper conclusion on what he ought to do. In this the actuarial profession must surely have a role, as does the insurance industry. I know that the industry is as concerned about it as we are, but I thought I would place my concern on the record in the Faculty Hall in the context of this important and very interesting paper.

Mr. D. G. Robinson: Mr. Lyon has unfortunately just “stolen” some of what I had planned to say, and I fear he may also have said it more eloquently than I am about to do.

My comments also concern the reasonable expectations of policyholders. In section 2.3.2 of the paper the authors say that when determining the value of the liabilities, “it does not seem reasonable for prospective policyholders to expect that illustrations given at the time they effect policies should necessarily be fulfilled in practice”.

Well I think it depends. It depends in my view on how reasonable a particular
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illustration given by a Life Office was at the time the policy was effected. Now if one looks at the returns which are implicit in for example some 10 year illustrations being made today, one finds that returns of as much as 15% net of tax are required if the projections including terminal bonus are to be achieved. 15% net of tax! These projections are admittedly hedged with qualifications saying for example that future bonuses depend on profits yet to be earned and cannot be guaranteed, but these tend to be in smaller print and more prominently will often appear a sales message proclaiming the excellence of the company’s investment team. The overall impression which tends to be given is that prospective policyholders can look forward with some confidence to the illustrations being achieved, the probability of a shortfall being small. Is it reasonable for an office to give this impression? Not in my view, unless the office has significant free reserves which it proposes to use to supplement any future shortfall in maturity proceeds. I believe that in judging what a policyholder’s reasonable expectations are, some account must be taken of what he has been led to expect by the new business illustration even if we might judge the illustration to have been over optimistic and thus unrealistic. The suggestion in section 2.3.3 that a reasonable expectation would be for reversionary bonuses to be at one-half of current levels in my view considerably understates what is reasonable.

There are also implications here for the appointed actuary’s reserving bases. It seems to me that the appointed actuary to an office which illustrates lavishly would be wise to hold stronger reserves for participating business than he might otherwise do. In effect he ought to make an allowance for the fact that the premium rates on which business is being and has been transacted are inadequate in relation to the prospective benefits. This isn’t just common sense — it is something that he, as an appointed actuary, is required to do by the profession — namely to take all reasonable steps to ensure the satisfactory financial condition of his office. If his premium rates are inadequate and his reserving bases do not take this into account he is failing in this duty.

The real answer of course is not for the appointed actuary to strengthen reserves on account of lavish illustrations, but for the illustrations to be made more realistic.

In conclusion I would like to congratulate the authors for their interesting and very readable paper.

Professor A. D. Wilkie: — There are some small points that I should like to comment on that have arisen in the course of the discussion. They relate to the tables at the end. Consider Table A1 and Table A2. Mr. Godson said that the most important line in it was the first one and the most important figures in his view were in columns 3 and 4. I think also the figure in column 1 should not be ignored. What we are told in Table A1 is that before this business is written at all the average reserve that is needed is £168 but at the 99% level one would need £800 initial reserve. On the other hand if the investment was in equities we would need, in order to write the business at all, a solvency reserve of £6000. But the average initial reserve needed is £601. By implication, if there is investment in equities on average this business is going to produce a profit of £600 and if it is invested in fixed-interest stock on average it will produce a profit of £168. So it may be perfectly sensible of the office to set up an additional reserve of £6000 and get a very comfortable average profit. Even on average it is going to get £6600 back and assuming symmetry (which is not necessarily right) it might get not only its original £6000 back but an extra £6001 whereas in the first case it might get at the 1% best chance only £968 back.

So I don’t think it is obvious that we should be going for the safest possible portfolio in all possible circumstances. As Mr. Wise said, it is sensible to know what that portfolio is, but, as Mr. Gupta said, one should take into account profit
testing as well. I think that this approach is a useful way for actuaries also to look at profit testing.

Mr. Godson also suggested that in order to see the whole thing properly one would like to have seen 20,000 simulations, so as to see the progress of the fund throughout the 20 years. I do not think that is the right way of putting the two figures together. I think you need to take 1000 to the power of 20. After ten years for example you actually have had 1000 different first ten years and you have got to choose one of those on which to condition for the future. The Maturity Guarantees Working Party did this calculation on very similar lines but ran for a number of years on a particular basis and said: let's assume that we know exactly what has happened for the first ten years and then let's see what happens to the remaining ten years in a random way. I think it is interesting to do exercises like this but it does produce mountains of computer paper and the Editor of the Transactions would not really allow that number of pages to be printed.

Mr. R. P. Bews, closing the discussion, said:—"What is truth?" asked Pontius Pilate, but despite being in illustrious company at the time he did not receive a reply. In posing the question "What is solvency?" the Council has been more fortunate and there can be no grounds for complaint concerning the quality of the Working Party's response.

The paper which they have presented to us tonight represents the fruits of four years' investigation into the nature of life office solvency. Clearly the preparation of this paper has involved an immense amount of work and probably represents only a fraction of the effort which has gone into the investigation as a whole. We are all in the authors' debt not only for the work they have done but also for the light they have cast on the problem they have undertaken to examine.

It is probably fair to say that before reading this paper many of us would have said that an office could be considered solvent if its liabilities did not exceed its assets. Of course, this begs the question as to precisely what values should be placed on the assets and liabilities, but there are generally agreed methods for arriving at these. The application of these techniques would involve the exercise of judgement, which is essentially a subjective process, and although the investigation would include tests on several different bases the approach was basically a deterministic one. From the tenor of tonight's discussion it is clear that this attitude will be more difficult to sustain in future.

The authors have, I think, made their case that the determination of solvency is a question of probability. They have put forward a solvency criterion based on a probabilistic approach and have indicated practical techniques which could be employed in determining a critical value. The criterion is the probability of ruin, but although two values of this probability, namely 5% and 1%, are used for illustrative purposes in the paper, the authors resist the temptation to choose which of these figures, if any, would be the proper one to use. In this I think they are wise. The development of the theory has obviously some way to go and it would clearly be premature at this stage to fix a value which eventually may prove too restrictive.

The tables at the end of the paper suggest that even a 5% probability of ruin — which does not on the face of it seem unduly conservative — could in some cases require an office to hold substantial reserves over and above those on the statutory valuation basis. Clearly it would not be practical to set up a solvency basis which could not be met by the majority of well-established offices and any basis which failed this test would have to be called into question. This is a similar point to that made by Mr. Newton when he talked about the practicality of solvency tests.

However, I am running ahead of myself, since I have not yet considered what the authors understand by the term solvency. To paraphrase their unavoidably
lengthy definition, a life office may be considered solvent if the statutory authority is satisfied that the probability of the office’s being unable to meet its contractual liabilities, both existing and imminent, is less than a specified probability of ruin. I note in passing that they do not say what value should be placed on the assets. Perhaps they feel that the existence of asset valuation regulations renders comment on this point superfluous.

Now this definition has been framed bearing in mind the supervisory authority’s power to intervene in order to prevent any office from following a course which would lead to failure to meet its contractual liabilities. By the time a U.K. office has submitted its statutory returns to the D.T.I. they can be up to six months out of date, so in the case of the U.K. — and, no doubt, other countries as well — if the supervisory authority is to take timeous and effective action it will need to consider the effect on the office’s solvency of the volume and nature of the business written since the end of the period to which the returns relate and any other factors such as changes in asset mix which may also be relevant. This is almost certainly a counsel of perfection since it is difficult to conceive of a practical system which would achieve this in all possible cases. As the authors point out it will still be necessary for the appointed actuary to act as watchdog and in the final resort to report to the statutory authority any practices which in his opinion seem likely to endanger the solvency of the office.

How far do the existing solvency regulations satisfy the requirements set out in the definition? It is disturbing to learn that despite extensive research the authors have failed to establish a scientific basis for the solvency criteria set out in the E.E.C. Life Directive and now enshrined in U.K. legislation.

This section of the paper serves to confirm the worst fears of those of us who suspected that the E.E.C. criteria had been chosen to some extent arbitrarily for want of anything better. The suggestion in section 3.8.1 that they represent a political compromise is not calculated to inspire us to hold in higher regard those responsible for framing this section of the Directive.

Regardless of how these criteria have been derived, are they adequate for our purpose? The answer to this question requires the establishment of proper solvency criteria and this brings us to the heart of the paper in which the Working Party set out to evolve a solvency test based on stochastic principles. They acknowledge that this derives from the work of the Maturity Guarantees Working Party who, in a paper presented in this hall almost four years ago to the day, employed these techniques to develop an investment model derived from sets of time series representing equity dividends and yields which were then combined to produce share prices. This was considered sufficiently novel at the time to justify setting up a crash course for those members of the profession who were interested in acquiring an understanding of the basic theory. It says something for the progress made in this field that a similar educative process has not been thought necessary as a preliminary to this discussion, though it might be something to be considered elsewhere.

The techniques pioneered by the Maturity Guarantees Working Party were generally hailed as a significant improvement on previous work in this area. One charge levelled against their model was that it contained no allowance for the effect of inflation and I am glad to see that tonight’s authors have supplied this deficiency. They have expressed the hope that the details of their model should not come under discussion at this stage since it will form the subject of the future paper. The speakers tonight have respected their wishes in this respect and I shall do so too, though it does seem to be asking a great deal of us to consider the results of applying a model of whose nature we have only been given a bare description. Mind you, this bare description runs to six pages in the paper.

As it stands the usefulness of the model is limited by the fact that it copes with only one of the four elements which the authors identify as causative factors in
insolvency. No doubt more work will be done on the model to extend it into these areas though the authors seem disinclined to treat one of the factors, namely mortality, by the stochastic method. They do not deny that mortality is subject to random fluctuations but justify treating it in a deterministic fashion on the grounds that the stochastic element in mortality is less significant than that in the investment market, for example. I would have thought that if there is a stochastic process operating then a probabilistic method is required regardless of the degree of significance involved. The deterministic test adopted by the authors for mortality is a bit like testing the reserves for the investment risk by assuming that the equity portfolio were suddenly to become worthless overnight. My intuitive reaction is to regard a test of this degree as very stringent indeed but it would be preferable to have this impression confirmed by the application of stochastic principles.

The model at this stage then is purely an investment one which for illustrative purposes has been applied to simulate the progress of a twenty year endowment assurance, both on a with and a without profits basis. These examples have been the subject of much comment tonight and no doubt they will continue to be discussed for some time to come. As the authors point out, they illustrate the effect which varying the investment mix, bonus distribution and future bonus assumptions can have on an office's solvency requirements. It is possible to read them as showing that, in conjunction with the statutory reserves, the statutory solvency margin, despite its arbitrary origins, may well be suitable for a non-profit portfolio matched to fixed interest securities. However, it is less likely to be sufficient in the case of with-profit policies unless virtually no future bonus is reserved for.

This must cast some doubt on the appropriateness of using the 4% margin for all offices since it seems desirable for the purpose of the authors' solvency test to reserve for a minimum bonus at the rate loaded for in the premium basis. After all, the purpose of the test is to alert the statutory authority whenever an office's ability to meet its contractual liabilities is under threat and it seems to me that any office which is forced to cut its reversionary bonus rate to below the level loaded for, on the basis of the rates of interest, mortality, etc. applicable at that time, has reached the point where the statutory authority ought to intervene. If this is accepted then it follows that the rate of bonus loaded for should be the minimum reserved for in the solvency test. In this respect I arrive at the same conclusion as Mr. Godson, though from a different direction. As a consequence I find it hard to disagree with the authors' conclusions that no simple test based on a percentage of reserves is likely to be appropriate for all offices.

In conclusion I would like to thank the Working Party for the valuable insight they have given us into an important aspect of our work. If I have seemed to harp on the fact that certain features remain to be dealt with this is not because I feel the paper is a case of Hamlet without the Prince. On the contrary, the Prince is very much in evidence; but it is as though we have been given Acts I and III of the play and have been told that Act II will be placed before us shortly. May I express the hope that Acts IV and V will follow in due course because I am anxious to see the rest of the play?

Mr. E. S. Robertson, replying to the discussion, said:—I found the discussion this evening very interesting indeed and I am sure that all the members of the Working Party will be very pleased at your reception of the report. I would just like to comment on a few points which have been raised by contributors and we shall be replying in the usual way in due course when we have had an opportunity of studying the transcript of the discussion.

Now in the report we did recommend the stochastic approach to solvency and as Mr. Limb said in his opening remarks we welcome views on this method. We
were aware that there would be criticism from some quarters of this particular approach rather than a deterministic one. But I have really been mildly surprised by its acceptance by those who spoke tonight. Of those who discussed the stochastic approach in opposition to a deterministic one according to my reckoning we have five for the stochastic and one against, not counting Professor Wilkie. I said in this Hall in closing the discussion on the report of the Maturity Guarantees Working Party that there was no doubt in my mind that a stochastic approach was the only one to use to calculate the reserves for maturity guarantees and I am certainly of the same opinion here.

Reasonable expectations — this was commented on by several speakers. In the main I would think speakers suggested that some allowance should be made for the reasonable expectations of policyholders in the solvency test. We of course started off on this basis and later on as we came through our calculations we did wonder whether it should be proper in testing for solvency that an allowance should be made for future bonus, and we mentioned this point in 4.7.11. In fact the conclusion 11 to which Mr. Lyon referred was unhappily worded. We were just suggesting that if the supervisory authority required a level of future bonus then the 4% solvency margin was inadequate, not that there was presently a requirement that allowance should be made for future bonus. In fact it is interesting that Messrs Barrow and Ferguson in a paper to be presented to the 22nd Congress later this month suggest that a regulatory authority should concern itself only with the policyholders' contractual expectations as far as solvency is concerned although it might be reasonable, they suggest, for an office to demonstrate to policyholders as a separate matter that its business is being conducted in such a way as to fulfil the reasonable expectations of policyholders in general.

Personally, despite the qualification on new business illustrations to which Mr. Lyon and Mr. Robinson referred, I am certain that prospective policyholders' expectations are inflated beyond what they should be and even though these are equitable expectations, to use Messrs Barrow and Ferguson's expression, rather than contractual ones the industry is storing up trouble for itself. The sooner an agreement can be reached on the form more realistic new business illustrations should take the better.

On the question of the E.E.C. solvency margins we were asked whether we thought these should be amended. We had of course originally intended to take account of the situation in the E.E.C. countries set out in Appendix 2 but we did not in the event. I think it is too early yet to say what should be done to the E.E.C. solvency margins: certainly a lot more work needs to be done on the general question of solvency. Mr. Newton mentioned the question of the political considerations. It may well be politically difficult to reach E.E.C. agreement but as a profession this should not deter us from examining the position and I think he has agreed with that point.

Regarding the history of the E.E.C. Life Directive I am sorry if we did not have the full story but it is not from want of trying. We did make a very exhaustive search to try to trace all the paper that might have a bearing on this matter but I am afraid we did not locate the paper to which Mr. Newton referred.

And finally if I could just say something about our last conclusion, that was conclusion 13 in Paragraph 4.8. We said there that it was important that offices be prepared to reduce their reversionary bonuses should investment conditions warrant it. Unfortunately marketing pressures are such that offices given a free hand are more likely to reduce terminal bonuses rather than reversionary bonuses and a move to lower reversionary bonuses may have to come from a need to maintain satisfactory solvency reserves. However, more work does have to be done on this subject and we are glad that we were given the chance to make some contribution to this important topic.
Mr. R. S. Skerman wrote:—The Report makes a valuable contribution to the debate on statutory solvency standards. As it points out, the present standards used in the U.K., which embody the requirements of the E.E.C. Life Establishment Directive, are open to significant criticism.

In my view the paper points to a valuable means by which an office can assess its financial strength having regard to its assessment of the reasonable expectations of its policyholders. I am doubtful, however, whether it indicates the best method of improving statutory requirements in present circumstances.

The basic problem is, in my view, raised by the last sentence of para. 2.2.9 which says that the extent to which the supervisory authority should take into account the reasonable expectations of policyholders is a vexed question. I do not think that the Report describes the present position fully because it makes no reference to the requirement in Regulation 54 that the valuation "shall be made on actuarial principles and shall make proper provision for all liabilities on prudent assumptions in regard to the relevant factors". The figures in the Appendix take no account of this requirement, being based on the minimum statutory reserve.

An appointed actuary making a valuation on actuarial principles as required by Regulation 54 should have regard to the need for equity to with profit policyholders and to the fact that the supervisory authority has power to intervene to protect policyholders against the risk that the office will be unable to fulfil their reasonable expectations. Thus, in summary, the statute defines a minimum basis which, because of the use of the net premium method, takes some account of the expectations of with profit policyholders by preventing future bonus loadings being capitalised into current surplus. Over and above this the appointed actuary has the responsibility of making a valuation on actuarial principles. This responsibility of the appointed actuary undermines the assertion that the basis does not bear equitably on different offices.

Is it desirable or practicable to move away from this basic approach in present circumstances? To proceed in the direction envisaged in the Report would mean quantifying all or part of what is now left in the hands of the appointed actuary. Unless and until offices are seen to be treating with profits policyholders unfairly or disregarding their expectations I do not think such a move is desirable. As regards practicability, I doubt whether reasonable expectations are capable of definition in a statute in a way that would bear equitably on different offices. They depend not only on the rates of bonus used in illustrations but on the manner in which the illustrations are presented. It does not seem satisfactory, as is proposed in para. 2.3.3, to exclude terminal bonuses unless the statute defines the relationship between terminal and reversionary bonuses. Moreover illustrations of non-guaranteed benefits such as surrender values create expectations.

Even if the problem of reasonable expectations could be solved there would be formidable difficulties in defining in a statute the various elements entering into a basis on the lines envisaged in the Report. They would have to take into account options in both assets and liabilities and be applicable not only to U.K. conditions but also to conditions in other countries where assets and liabilities existed.

Thus I think it that would be better and more realistic to seek to improve the bases used to give effect to the present basic approach than to change the approach as the Report envisages. This would avoid reducing the freedom of the appointed actuary to use his judgement in ensuring equity in bonus distributions and in providing for reasonable expectations.

The Working Party subsequently wrote:— We have now had an opportunity to consider at leisure the comments made by the contributors to the discussion, and
would like to thank them for the very thorough and helpful observations which were made.

A number of speakers drew attention to the fact that, although we had exposed what seemed to us to be shortcomings in the solvency requirements presently operative, we had been less explicit about the changes required in these solvency requirements. The criticism is a fair one, but we feel that in the long run a radically different approach to the determination of solvency is required. Our main theme is that no simple set of rules is likely to be appropriate to all companies, but rather that the mix of their business, and the investment policies followed, require some form of stochastic approach to underlie any regulations which may in the future be made to replace the existing rules. On our own admission, more work — in fact, considerably more work — would be required before a workmanlike solution to this intractable problem can be found. We have advocated a stochastic approach although we fully recognise the great difficulties involved in embodying such an approach in legislation.

We are asked specifically by Mr. Forfar whether or not we would recommend the appointment of an Appointed Actuary in other E.E.C. countries. Our view is that this would indeed be a desirable development, not merely in connection with the establishment of solvency but in other areas as well. We recognise that much water will have to pass under the bridge before this concept would be accepted within the E.E.C. generally.

A number of speakers, including Mr. Forfar and Mr. Newton, drew attention to the enormous practical difficulties which would be encountered in securing understanding and acceptance of the concept of a stochastic approach within the E.E.C. We fully accept that this is so, but do not believe for an instant that the profession in the U.K. should abandon its pursuit of what it believes to be the appropriate method of coping with the problem of solvency because of the difficulty of expressing it in legislation either in the U.K. or the E.E.C. So to do would be to follow a counsel of despair.

Mr. Wise prefers an analytical method (his spotlight) to our use of simulation methods (dart-throwing). We agree that analytical methods are preferable where it is in fact possible to apply them. However, simulation methods can be used more quickly and in many more circumstances than analytical ones, and the particular problem we were trying to solve did not appear to be tractable. However, analytical and simulation methods are just alternative ways of getting the same result.

Mr. Godson distinguishes between "immunisation" in Redington's sense and "matching" in Wise's sense. Yet another very similar concept would be that of the portfolio of assets that requires the lowest solvency margin in our sense. This seems to be an area where further investigation and discussion of concepts would be useful.

Mr. Gupta points out the fact that ought by now to be obvious to Actuaries, but is not much emphasised in our traditional textbooks, that a life office needs capital. We agree that the cost of solvency margins should be incorporated into the premium basis. We did not turn our attention to this, and we do not mean that the premium should be sufficient to set up the whole of the solvency reserve required. However, the premium basis should take into account the cost of "renting" the capital required for the solvency margin.

Mr. Bews notes in passing that we do not say what value should be placed on the assets. We think that here he has misunderstood our method. We have attempted to assess the solvency of a portfolio in terms of the actual assets held, without reference to their market values or any other present values. Market values give an indication of the price at which one might exchange one set of assets for another set, and are therefore relevant if this is under consideration. They also indicate the prices at which new money can be invested or assets sold at
some future date. But our method takes account of the assets and liabilities themselves, and not their present values.

Finally, we would like to wish every good fortune to those who may take up the arduous task of continuing the work of putting flesh on the skeleton we have suggested!