RESERVES FOR MATURITY GUARANTEES UNDER UNIT-LINKED POLICIES

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Introduction

Mr Sidney Benjamin’s paper A Study of Maturity Guarantees under Equity-Linked Policies which, inter alia, discussed the level of reserves for these benefits was discussed in Staple Inn late in 1971 and a revised version was submitted to the 1976 Tokyo Congress (Trans. 20 International Congress of Actuaries, 1, 17). Generally the earlier paper and its conclusions were criticized, either on the basis of the Stock Exchange model used or on the high level of reserves produced, or both. It is clear, from the other papers which were presented at that first discussion and from the discussion itself, that a particular philosophical problem arose as to the approach to be adopted towards the determination of the reserve. This problem had arisen before. For example, in the discussion on a paper by Mr S. H. Turner (T.S.A. 21, 459), Mr Benjamin had said

I would, however, like to draw a very clear distinction between the author’s work and the problem of reserves. It was the latter problem which occupied my own work on the subject. The author has produced a premium rate, and ... he suggests that an appropriate reserve would be obtained by accumulating the premiums. My own feeling is that, if a basis is correct for premiums, it must ipso facto be too weak for valuation and vice versa—if it is correct for valuation, it is too strong for a commercial premium. Furthermore, a retrospective reserve is dangerous.

I do not agree that the accumulation of a commercial premium is a suitable actuarial reserve. A prospective reserve on a cautious basis should be set up, and the interest and profit return required on this capital should be incorporated in the commercial premium charged. My own arithmetic trebled the commercial premium chargeable when I inserted a profit return on the reserves into the calculation.

There are cases of companies issuing policies for one or two years and then ceasing issue. The result could be a lack of spread of maturity dates, and hence I think there is a very real problem of evaluating reserves at the commencement of this type of business.

In his reply Mr Turner took the view

Because of the nature of asset value guarantees and the different benefit forms in which such guarantees can be applied (e.g. benefits payable at death or maturity, annuity benefits, etc.) an ‘appropriate’ reserve for valuation will depend to a great extent on the actuary’s judgment in dealing with each particular situation. Therefore, regulations pertaining to reserves for asset value guarantees might include the following:

(1) That the reserve liability for asset value guarantees be established in accordance with actuarial procedures that recognize the nature of benefits provided and, to the extent applicable, the requirements of the standard valuation law.
(2) That a statement of valuation standard be filed setting forth the bases, methods, the assumptions used in determining the reserves for asset value guarantees. Subsequently, a company could indicate its continued compliance with such statement of valuation standard.

(3) That an opinion by a qualified actuary be filed stating that the reserves established for an asset value guarantee place a sound value on the liability with respect to such guarantees as of the valuation date.

As an example, the following valuation procedure for an asset value guarantee at maturity of an equity-based contract would appear to be consistent with the above and practical in its application. Valuation premiums would be determined for the guarantee using 1958 CSO mortality, 3½% interest, and such actuarial procedures for evaluating the risk as are deemed sufficient and appropriate by a qualified actuary. Average or composite net valuation premiums based on representative groupings would be acceptable. Such net valuation premiums would be accumulated in the general account of the company in a manner consistent with the reserve bases used. Additional contingency reserves could be maintained as deemed necessary or desirable by the actuary. Prospective gross premium valuations would be made after several years of experience became available and at periodic intervals thereafter (say, every five years) to test the adequacy of the reserves maintained.

Whilst many might approve of the principle underlying the approach advocated by Mr Turner it leaves open the determination of a consensus view as to how the actuary's discretion should be exercised. Further, within the context of valuation regulations prescribed by the Department of Trade it is open to question whether an approach such as this would be acceptable.

Another major difficulty to which attention was drawn in the course of the discussion on Mr Benjamin's 1971 paper was the level of seriousness of a situation which would be such that it would be unreasonable to require a company to hold reserves sufficient to cover it, and this problem also remains.

2. Following the discussion on Mr Benjamin's 1971 paper, a Working Party was set up to consider primarily the question of the statistical independence of stock market movements as a first step towards the development of an acceptable valuation basis. This Working Party reported as follows:

(a) Independence is not a meaningful concept, unless the alternative or alternatives are fully specified. Tests that are powerful for one alternative are weak for another.

(b) Statistical tests are essentially shades of probability, they cannot in general confirm or reject a hypothesis with certainty.

(c) The tests used in (Mr Benjamin's) paper were on the borderline of 'conventional' significance and, indeed, depended to a marked extent on how certain items on the boundary were treated in the analyses made.

(d) Similar tests applied to Australian data over a 79-year period suggested that there was significant non-randomness.

(e) The data examined for the U.K. was for a 50-year period. What is of interest in the problems under discussion is what happens in the next 10 years or so. Is an analysis of the 50 years wholly relevant to this problem, or only one dimension in the problem?

(f) The problem of reserving appears to be one of estimation, rather than hypothesis testing, together with some limits on the estimates.

(g) There are various methods of approach to the problem of estimation posed in this instance and we feel that these now need to be explored and evaluated. However, such an investigation moves over very directly into the second part of the exercise that (was) envisaged
in that it is necessary to examine alternative methods of using the models and setting up reserves.

(h) Any system of reserving depends upon some assumptions, e.g. in ordinary life assurance the rates of mortality, the fund earning capacity, the expenses. If these estimates are seriously wrong for reasons that are outside experience or expectation (e.g. there was a virulent plague or a complete stock market collapse) the system fails. The same is true of the present problem where we can only seek to derive prudent, but not absolute, measures of reserving.

(i) The provision of adequate reserves must also depend, from a risk of ruin point of view, as to whether these contracts are considered in isolation or in conjunction with other contracts with lesser guarantees. This should be spelt out in advance of work being done.

(j) We conclude that it is unrealistic to separate the basic issues in the way proposed and we recommend moving straight on to stage II.

3. The stage II referred to in § 2(j) was to form a further Working Party to make recommendations as to the proper level of reserves. A number of meetings were held but no report has been prepared, mainly due to problems in resolution of fundamental points. This note records the various conclusions I have reached as a result of the study by the Working Party and other events that have occurred since the Working Party was established. It is in no sense a report of the Working Party.

4. It is of interest to note certain papers which have been produced recently. The first four listed describe work done at the University of British Columbia. Copies are deposited in the Institute Library.

BOYLE, P. P., BRENNAN, M. J. & SCHWARTZ, E. S. Equilibrium Prices of death benefit guarantees and maturity guarantees under equity linked contracts and optimal investment strategies for the sellers of such contracts. A summary and commentary on this paper, prepared by Mr P. J. Nowell, is set out in the Appendix.

BOYLE, P. P. An approximation method to calculate the value of a maturity guarantee under a level premium equity based contract.

The paper develops an approximate method for calculating the premium required for a maturity guarantee for a level annual premium contract without using simulation and shows the results of the approximation compared reasonably for terms up to 15 years with those derived from simulation. The investment model assumes that the returns from an equity portfolio are log-normally distributed, returns in successive years being independent.

BOYLE, P. P. & SCHWARTZ, E. S. Equilibrium Prices of death benefit guarantees and maturity guarantees under equity linked contracts and optimal investment strategies for the sellers of such contracts.

The authors' synopsis is “The prices of death benefit guarantees and maturity benefit guarantees under equity linked contracts are obtained under conditions of market equilibrium using some recent results from the theory of finance. The model provides a theoretical basis for pricing these guarantees and some numerical results are given. In addition the model can be used to prescribe an optimal investment policy for the insurance company selling these contracts.
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The investment policy is optimal in the sense that it provides the insurance company with a hedge against the investment risk associated with the granting of these guarantees.

The analysis is largely based on single premium contracts. The model used is similar to that in the first paper listed above, and the conclusions as to investment strategy are subject to the same reservations as set out in the final paragraphs of the Appendix.

Brennan, M. J. & Schwartz, E. S. The pricing of equity linked life insurance policies with an asset value guarantee. (Journal of Financial Economics, 3, 3.)

The paper has the objectives of determining the expected value of an asset value guarantee under a linked policy and to recommend an appropriate investment strategy. The approach is similar to that in the other work and the conclusions as to investment strategy subject to the same reservations.

Wilkie, A. D. The Rate of interest as a Stochastic Process—Theory and Applications. (Trans. 20 International Congress of Actuaries, 1, 325.)

The object of the paper is to approach compound interest by assuming that the force of interest in successive time periods is a random variable and the author applies this approach to the consideration of the determination of reserves for maturity guarantees following a simulation process and using a log-normal model. He determines a number of results for the 'largest claim' in a group of policies, original term from 10 years upwards.

Scott, W. F. A Reserve Basis for Maturity Guarantees in Unit-Linked Life Assurance.

This paper was discussed at the Faculty on 17 January 1977.

A mathematical model is used, based on the conclusion that yearly growth rates, including reinvested net income, in equity prices may be represented approximately by the log-normal distribution, and are negatively correlated.

A probability of ruin $\gamma$ is chosen and $Y_\gamma$ found such that

$$\text{Prob (Total outcome} \leq Y_\gamma) = \gamma$$

The reserve needed is Guarantee (G) minus $Y_\gamma$ discounted in the usual way.

5. Following discussion, and having regard to the findings of the previous Working Party (§ 2 above), the second Working Party decided it was unlikely that it would be able to derive a model of stock market behaviour which would be satisfactory for extrapolation into the future and which would be generally acceptable as a basis for reserving. Some alternative approach was therefore sought. The approach which has been investigated is described below, but first it is helpful to set out a number of basic points which seem particularly relevant.

(a) Solvency of a life fund is not absolute. Whilst there is no reason why a fund offering linked contracts with maturity guarantees should be 'more solvent'
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than a fund consisting only of traditional business, its solvency ought not to be less. However, there is no measure of the acceptable standard of solvency of a traditional portfolio, and in the United Kingdom at any rate little attention has been paid to a ruin probability approach.

(b) There are of course circumstances when the solvency of even a traditional portfolio with a substantial proportion of with-profits business could be endangered, but an office whose portfolio consists almost entirely of unit-linked policies with maturity guarantees must be regarded as much more vulnerable.

(c) This type of vulnerability would be shared by an office whose portfolio consisted, for example, of group term assurances where the actuary would be concerned about the vulnerability to fluctuations and he might adopt a ruin probability approach but, here again, there has been little discussion and open debate.

(d) Perhaps the closest analogy to the problem is in the field of non-life insurance. Technical reserves may be regarded as calculated on an expected value basis but in addition the insurer is required to hold a solvency margin, laid down by the supervisory authority, and of an amount which is arbitrary, although developed on the basis of experience over the years. It is recognized that there is continuing debate as to exactly what the solvency margin is required to cover.

(e) In the United Kingdom no explicit solvency margin is required for life business, but in due course E.E.C. legislation is likely to stipulate one.

(f) The traditional approach to a life office valuation is tantamount to calculating a reserve policy by policy, and whilst one policy could not stand alone a large number of policies issued at the same time can be validly dealt with. They do not need to be part of a continuing business, although there are admitted problems with a closed fund. However, in the various notes and papers written about maturity guarantees it is stressed how important it is to ease the reserving problem by having a good spread of maturity dates. This is a new form of averaging, the acceptability of which has to be considered, but in any event it is necessary to determine a reserving basis which will be satisfactory for a portfolio of policies, all of which mature on the same date.

6. Whilst the expected value approach is appropriate (subject to an increase in the premium to provide an appropriate return on any capital required) as the basis for the calculation of the premium to be charged for the guarantee, it is not so for the calculation of the reserve. It is recognized that this may lead to a confusion of reserves and solvency margin but we do this anyway in that actuarial reserves normally contain an implicit margin. A consequence is that in general the prospective reserve will not equal the retrospective reserve, i.e. the reserve required cannot necessarily be built up out of the office premiums payable for the guarantee.
Reserve at Inception

7. The approach followed is to assume a trend line for the performance of the relevant index together with a spread about that line. To obtain the reserve at inception of a single contract, it is assumed that all purchases are made at the top of the range and all sales (i.e. maturity) at the bottom of the range. Specimen results are set out in § 8 below.

Mr R. J. Squires, in his paper Unit-linked Assurance: observations and propositions (J.I.A. 101, 1) also used a trend line approach—in § 17 and Appendix C of that paper—but he based his conclusions on expected values.

8. A simplified model to determine the reserve required at inception is set out below:

\[ I_t \text{ is index at time } t, \text{ allowing for reinvestment} \]

Assume \( I_{t+s} = (1+r)^s I_t \)

Maximum spread about the trend line is \( \pm k \) such that the index at time \((t+s)\) varies between \((1 \pm k) I_{t+s}\)

Term of policy \( n \)

Guarantee \( G \)

Maximum cost of guarantee at maturity is

\[
G - \frac{1-k}{1+k} \sum_{t=1}^{n} (1+r)^t = G - \frac{1-k}{1+k} s_n \text{ at rate } r\%
\]

If \( G = n \), i.e. the guarantee is a return of premium without interest, we have the following for the maximum cost of the guarantee at maturity (no figures being shown if this cost is negative):

<table>
<thead>
<tr>
<th>( n )</th>
<th>( k = 0.2 )</th>
<th>( k = 0.3 )</th>
<th>( k = 0.4 )</th>
<th>( k = 0.2 )</th>
<th>( k = 0.3 )</th>
<th>( k = 0.4 )</th>
<th>( k = 0.2 )</th>
<th>( k = 0.3 )</th>
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<td>--</td>
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<td>5.2897</td>
<td>--</td>
<td>2.9669</td>
<td>--</td>
<td>--</td>
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<tr>
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<td>--</td>
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<td>5.1203</td>
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<td>0.0489</td>
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</tr>
<tr>
<td>25</td>
<td>--</td>
<td>--</td>
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<td></td>
</tr>
<tr>
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<td>--</td>
<td>0.1025</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
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<td></td>
</tr>
</tbody>
</table>

The required reserve at inception is found by discounting the value of the maximum cost of the guarantee at maturity and deducting the present value of the office premium payable for the guarantee, reduced by the expense loadings in the premium. Thus if the maximum cost at maturity is \( M \) and the unloaded office premium is \( \pi \) the reserve at inception is:
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\[ \frac{M}{(1+i)^n} - \pi d_n \]

at rate \( i \), the valuation rate of interest.

For example, if \( n \) is 10, \( k \) is taken as 4 and \( \pi \) is 2½% of the premium, then the required reserve at inception is of the order of 20% or 25% of the total premiums payable.

It could be argued that in this case the premium \( \pi \) would be greater than the amount assumed because it would contain an allowance to service the capital required to provide the reserve at inception (see the quotation from Mr Benjamin in § 1 above) but premiums of this order appear to have been charged. Had \( \pi \) been at a more 'realistic' level the reserve at inception, for the example given, would be reduced to, say, 10% to 15% of the premiums payable.

9. Results of the order determined in the previous paragraph are comparable with those produced by Mr Benjamin in his 1971 paper and which were so criticized at that time. Mr Wilkie, however, in the paper mentioned in § 4 determined 'free reserves' required at a considerably higher level, whereas Dr Scott in his paper illustrated slightly lower figures.

10. It may be objected that this approach is unreasonably pessimistic and this leads to a consideration of the circumstances the reserve would be expected to cover—and would be expected not to cover.

The end of 1974 presented serious problems to life offices, but except for a few rather special examples, they survived. Consider a company which on 1 January 1965 had issued a block of 10-year annual premium linked policies with a maturity guarantee equal to a return of premiums paid. Using the de Zoete and Gorton index, net income reinvested, shows that the shortfall at maturity would have been in excess of 40% of the total premiums paid, i.e. the guaranteed maturity sum. Had this been the only business of the company how solvent would it have been—how solvent should it have been?

This may be regarded as an extreme example, but it adds support to the general approach suggested and supports a level of reserves for a single policy, or for a block of policies all due to mature on the same date and which for some reason have to be considered separately from other business, considerably higher than that generally advocated in the discussion on Mr Benjamin's 1971 paper.

Reserve between Inception and Maturity

11. Again consider a single contract. The trend line assumed and the associated spread can be regarded as a valuation basis. As the contract moves towards maturity, units will be purchased at the level of the index from time to time. To what extent should the level of the index at the valuation date influence the valuation? In considering the calculation of the reserve at inception the level of the index at the date of inception is not taken into account but is it realistic to adopt that approach as the contract nears maturity? Obviously to pay full regard to the value of the index at the valuation date could give rise to undue
fluctuations in the reserve from valuation to valuation, but equally obviously it cannot be ignored.

12. One approach is to consider at each valuation whether it is reasonable to regard the original valuation basis (i.e. trend line and spread) as still applicable. If it is, it would be appropriate to aim to produce the same sum at maturity as was implied by the reserve at inception. It could be objected to this that the reserve at inception was calculated on the extreme assumptions of purchases at the upper level of the spread about the trend line and sales at the lower. If, for example, in a 15-year contract the first 10 years' premiums had been applied at the upper level of the originally assumed valuation basis (as implied by the basis and the approach followed) then the original valuation basis would rightly be questioned. Whilst admitting this objection, I do not feel it invalidates the approach. If it is proper to regard the original valuation basis as being not borne out by events since inception, then it would be appropriate to aim to provide the same sum at maturity as would have been implied by the reserve at inception calculated on the revised valuation basis.

13. As an example of the application of § 12, using the notation of § 8 above, if the original valuation basis is $r = 7\frac{1}{2}\%$, $k = 0.4$, then the target sum at maturity is 35% of the guaranteed amount. If in the light of experience since inception it appeared that more realistic assumptions were $r = 5\%$, $k = 0.4$, then the revised target sum at maturity would become 43%, i.e. a valuation strain would arise.

14. Notwithstanding the above, near maturity—say within 5 years—the actuary would have to consider the range of possibilities for the future having regard to the number of units already purchased, the value of the index at the valuation date and the guaranteed sum. Prior to the issue of valuation regulations it would perhaps be sufficient if actuaries could agree that they would make a reasonably pessimistic assumption, value accordingly and publish what they have done. Even with regulations this could well still be necessary, since the regulations will presumably prescribe a minimum reserve on what will probably be a somewhat arbitrary basis. In any valuation which uses the current value of the index it is essential to remember that however low an index value may appear historically, it still has room to fall (unless it is too late)!

Reserve for a Portfolio with a spread of Maturity Dates

15. An analogous approach to that above for a single policy is to assume a valuation basis—trend line and spread—and to assume that in any one year purchases/sales are made at the top or bottom of the range in such a way as to maximize the reserve.

16. Calculations have been made on this assumption and examples are given below, where one policy for term $n$ is issued each year, the guaranteed sum is $n$, the growth rate is $r\%$ and the assumed spread about the trend line is as indicated. The shortfall shown for year 1 is for the year when the first policy matures and the subsequent years.
Reserves for Maturity Guarantees under Unit-Linked Policies

<table>
<thead>
<tr>
<th>n</th>
<th>r</th>
<th>Spread</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
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<tr>
<td>10</td>
<td>9</td>
<td>1·4-7</td>
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<td>1·3-7</td>
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<td>1·3-7</td>
<td>1·71</td>
<td>1·23</td>
<td>·71</td>
<td>·16</td>
<td>nil</td>
</tr>
</tbody>
</table>

Where nil is shown this is repeated for \( n \) years when the cycle is repeated.

Taking the first example \((n = 10, r = 9)\) and assuming that the unloaded office premium is 2½% of the premium and valuing at 5%, reserves of the following order would be required, counting from the year when the first policy is issued and the percentages shown relate to the guaranteed maturity amount:

- **Year 1**: 0·85 8·5%
- **Year 2**: 1·45 7·2%
- **Year 3**: 1·7 5·7%

Expressed as a percentage of the total of the guaranteed maturity amounts in force at the valuation date.

Thereafter the reserve would increase by interest, by the discounted value of the shortfall at maturity when the cycle repeats itself and by office premiums as paid, and reduce by the present value of future office premiums on each year’s new business.

17. Ultimately, with a stable portfolio and if the premium basis and the valuation basis are the same, the stable portfolio reserve will average out at the accumulated value of the premiums received on the business in force—say between 1% and 2% of the total guaranteed maturity amount for the business in force—but it is obvious that from time to time a reserve at this level would be inadequate to meet the calls upon it. In fact in the first example in §16 we would need to hold a reserve of the order of 3% of the total guaranteed sum as the first policies come to maturity in order to meet the calls which would be made in years 10, 11 and 12.

18. On the approach which has been adopted a spread of maturity dates significantly reduces the reserve which is required but in practice there are a number of constraints. It is necessary to adopt a conservative approach, financial options exist and there could be selective surrenders. At the least the minimum reserve should be the maximum of the reserves individually required for each year’s maturities. There will come a point (making the normal assumption of no future new business) where there is only one maturity date and the reserve must be adequate for that. Further, it would appear imprudent to release any part of the office premiums paid (to the extent they are calculated on an expected value basis) until the relevant policy has gone off the books.

Practice in other Countries

19. Unit-linked policies with maturity guarantees are available in other countries, and in Canada and France the supervisory authorities have laid down valuation bases for the guarantees, in both cases on what appear to be arbitrary lines.
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20. In Canada guidelines were prescribed in 1971. The maximum permitted guarantee is the total of gross premiums paid: minimum term 10 years. A separate maturity guarantee reserve is established for the guarantee.

'Valuation factors' are provided with which to calculate a special reserve for policies maturing in the 10 years following the valuation, which makes some allowance for the value of the guarantee. The aggregate of the maturity guarantee reserve and the market value of the investments to which the policies are linked must be at least equal to the special reserve.

The special reserve is based on the formula:

\[
\sum_{t=1}^{10} (G_t, V_t, \pi_t, \bar{a}_\eta)
\]

Where \( G_t \) is the guaranteed minimum amount for policies maturing in \( t \) years
\( \pi_t \) is the annual premiums payable under such policies
\( V_t \) and \( \bar{a}_\eta \) are special factors, e.g.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( V_t )</th>
<th>( \bar{a}_\eta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.2</td>
<td>0.6</td>
</tr>
<tr>
<td>5</td>
<td>1.105</td>
<td>5.278</td>
</tr>
<tr>
<td>10</td>
<td>0.845</td>
<td>10.178</td>
</tr>
</tbody>
</table>

The effect seems to be to home on to 120% of the guaranteed sum, but deducting in the calculation more than the 'normal' value of future premiums.

21. In France it is required to accumulate the office premiums payable for the guarantee, after deduction of expenses, as a special reserve. When the amount of this special reserve, together with the basic reserve calculated by reference to the value of units purchased and the reserve for participation in profits which arises from interest receipts in excess of those assumed in the formula for the premiums, exceeds by more than a stipulated percentage the reserve calculated by reference to whichever is the greater of (i) the current unit price and (ii) the minimum guaranteed unit price, then the amount of the excess is transferred to the reserve for participation in profits. This reserve is to be distributed to policyholders at maturity. The stipulated percentage varies between 10% and 25% according to the type of contract and the rate of interest assumed in the calculation of the premium.

The contracts provide a guarantee equal to the result of multiplying the number of units purchased at maturity by the original value of the units.

Conclusions

22. Working Paper 338 (§ 4 above) includes the sentence "It is clearly no longer sufficient to compute the expected value of these guarantees and to discount them back to the present." This must be accepted. Dr Scott (§ 4 above) takes the same view and develops a reserve based on the 'likely maximum cost'.

23. I believe that reserves higher—and in some cases very much higher—than those advocated in the discussion on the earlier paper by Mr Benjamin are appropriate. For a minimum to be specified by regulations it seems likely that only an arbitrary basis could apply, leaving the actuary free to require higher reserves if he consider them appropriate. If share prices had appreciated to such an extent that the reserves were generally agreed to be excessive, then some reduction in the required level could be negotiated. The level of reserve is necessarily subjective but for a single tranche of 10-year policies all maturing in the same year I would be unhappy to accept a target at maturity appreciably less than 30% of the guaranteed sum (for a guarantee limited to return without interest—higher guarantees require higher reserves broadly to the extent of the increase in the guarantee). For a stable portfolio of 10-year term policies a much lower percentage of the total guaranteed sum could be justified, say in the range 5% to 10%, but such reserve could be dangerously low if the portfolio were unbalanced. Perhaps the only possible approach is a percentage of the total guaranteed sum as a minimum, leaving it to the actuary to reserve more if this were justified. Factors which would be taken into account by the actuary in coming to his view are:

(a) Experience of the portfolio since inception.
(b) Current unit price (see § 11).
(c) The distribution of the portfolio by maturity dates.
(d) The original terms of the policies.
(e) The precise level of the guarantee, although in the first instance this might more appropriately be reflected in the minimum percentage required.

24. Reserves at the level suggested are higher than those generally adopted. If they are accepted, then some interim arrangement may be necessary.

25. Anyone considering this subject is forced to consider whether the uncertainty of the reserving problem is such as to preclude the issue of contracts with a guarantee above a certain level. I believe that it is, although the level of the capital required would in any event act as a strong deterrent.

26. These notes have been written with a view to stimulating a discussion on a problem which has not yet been resolved, and hopefully to help develop an agreed solution. I am indebted to many to whom I have talked on this subject over the years, but the views expressed are entirely my own.
Review of Equilibrium Prices of death benefit guarantees and maturity guarantees under equity-linked contracts and optional investment strategies for the sellers of such contracts. (See § 4.)

A maturity guarantee can be regarded as a 'put option' (option to sell at a fixed price) granted to the investor by the insurance company.

As such the value of the guarantee is given by

\[ P = e^{-rt} \int_0^T (E - S_T) \cdot p(S_T) \cdot dS_T \]

where \( P \) is the value of the option.

\( E \) is the guaranteed sum.

\( S_T \) is the value of the equity units ('reference portfolio') on maturity in \( T \) periods' time.

\( r \) is the risk free rate of return (discount rate).

\( p(S_T)dS_T \) is the probability that the value of the reference portfolio is \( S_T \) at time \( T \).

A.2. The problem therefore hinges on the prediction of \( S_T \). In other words a stock market model is required.

The paper suggests the basic stochastic process

\[ \frac{dS}{S} = Mdt + \sigma dz \]

where \( Mdt \) is the expected return over the period \( dt \) and \( \sigma^2 \) is the variance of this return. \( dz \) represents a probability distribution element which has mean zero and variance \( dt \), and follows a Gauss–Weiner process.

Integrating this with respect to \( t \) and using appropriate boundary conditions leads to the conclusion that

\[ P(S_0, T) = E e^{-rt} N(-d_2) - S_0 N(-d_1) \]

where \( N(d) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^d e^{-x^2/2} dx \) (the cumulative normal density function)

\[ \frac{\log S/E + (r + \frac{1}{2} \sigma^2)T}{\sigma \sqrt{T}} \]

\( d_1 = d_2 = d_1 - \sigma \sqrt{T} \)

(Perfect markets are assumed and hence it is possible to express \( M \) in terms of the other variables.)

It will be seen that this is in the general form suggested above. \( P \) then represents the value of a guarantee of \( E \) in \( T \) periods' time where an amount \( S_0 \) is to be invested in the stock market (assuming no dividends are paid). It represents the guarantee on a single premium contract.
A.3. Given the basic stock market model there are no theoretical problems involved in finding the value of more complex types of contracts (i.e. annual premiums). In practice, however, the handling of the algebra and differential equations becomes somewhat difficult. Simulation is however possible.

The model produces premium levels very much in line with the results derived by members of the sub-committee in their investigations.

A.4. Having issued the contract at the price suggested the paper goes on to describe an investment strategy which will insulate the insurance company from profit or loss on the contract.

For this purpose it is easiest to describe the contract as the promise of a fixed sum $E$ at maturity, and the option ('call option' or 'warrant') to buy the reference portfolio at maturity for an amount $E$. Thus the insurance company has sold a warrant valued at $W$ and a fixed interest stock of value $Ee^{-rT}$. The available markets are the equity market and the fixed interest market, and the insurance company can sell, but not buy, in the 'warrant' market. It has an amount $W + Ee^{-rT}$ to invest. Let the amounts invested be

- $a_1$ in the equity market
- $a_2$ in the warrant market
- $a_3$ in the fixed interest market.

Then the instantaneous return on the investment is given by

$$dy = a_1 \frac{dS}{S} + a_2 \frac{dW}{W} + a_3rdt$$

Overall the insurance company wishes to have a zero net investment, and hence in perfect markets will have a zero net return. Thus to immunize against movements in the stock market we require (with $dt = 0$)

$$a_1 \frac{dS}{S} + a_2 \frac{dW}{W} = 0$$

But the insurance company has invested $-W$ in the warrant market hence

$$a_2 = -W$$

and

$$a_1 = \frac{dW}{dS} \cdot S$$

= the amount to be invested in the equity market.

The total amount given to the insurance company is $W + Ee^{-rT}$; thus the amount to be invested in the fixed interest market is

$$W + Ee^{-rT} - \frac{dW}{dS} \cdot S$$
Now according to the model used
\[
\frac{dS}{S} = Mdt + \sigma dz
\]
and
\[
\frac{dW}{W} = \beta(S,T)dt + \gamma(S,T)dz
\]

The elimination of statistical risk involves the elimination of \(dy\). Hence we require

\[a_1 \sigma + a_2 \gamma = 0\]

or

\[
\frac{SdW}{dS}/W = \frac{\gamma(S,T)}{\sigma}
\]
\[
\frac{dW/dS}{W/S} = \frac{\gamma(S,T)}{\sigma}
\]

It will be seen from the last equation that by holding the assets in these proportions the statistical risk has been eliminated by holding 'warrant' liabilities and equity assets in the ratio of their volatilities. For instance if the 'warrant' is three times as volatile as the shares then if shares represent 30% of the assets and the 'warrant' 10% of the liability then no matter how much the market rises or falls the assets and liabilities will remain equal.

Unfortunately the 'volatility' \(\gamma\) is a function of both the share price and time to maturity. Continuous portfolio adjustment would therefore be required and any misspecification of the model would immediately leave the insurance company unmatched.

Summary and conclusion

A.5. The paper is based on the assumption that the stock market follows a particular type of stochastic process which can be handled reasonably easily from a theoretical point of view and apparently is likely to give a reasonable 'fit' to actual stock market behaviour.

Given this model the paper derives the necessary formulae to establish

(a) The net premiums which should be charged for the guarantee.
(b) A method of investment which will insulate the insurance company against profit or loss from the guarantee.

The model used is of a type which is much favoured in the explanation of stock market behaviour and the estimates made will therefore be much in line with the results derived by others who have used similar models for the same purpose.

The investment procedure depends on knowing the relative volatility of the 'warrant' element of the guarantee against the shares. If this relationship is known it is clear that the appropriate proportion of shares and fixed interest will match the liability. It also however depends on continuous cost less adjustment of these holdings since the volatility is dependent on \(S\), the share price and \(T\), the time to maturity.
Here practice and theory are irreconcilable. Even given costless transfer two practical matters will lead to a mismatched position. First the reaction time required and the 'size' of markets will make anything approaching instant adjustment impossible. Secondly if the market is misspecified by the model then the position will anyway be mismatched.

Most of the estimates and experience of modest maturity guarantees suggest that the probability of serious loss is remote. If in order to guard against this remote possibility an insurance company were deliberately not to invest in the units it is linked to, then it would run risks of significant losses if the stock market moved rapidly over a short period.

The paper relies heavily on existing studies of the behaviour of stock and option markets. The solution of differential equations is credited to someone other than the authors. The investment procedure is a perfectly reasonable theoretical concept but as a practical proposition it is one which contains risks greater than the risk which it is designed to eliminate.
Reserves for Maturity Guarantees under Unit-Linked Policies

ABSTRACT OF THE DISCUSSION

The author: In § 171 have lapsed—I think that is the right word—into an expected value approach; I do not in any sense advocate it. If it is followed, there could be times when there are not enough reserves to meet a call or to demonstrate solvency, both of which are a *sine qua non* for continued operations. Where then is the money to come from? Who will put it up in those circumstances? The assets must be there, and in evidence, not just hopefully available.

Mr P. Smith: It is now over five years since Mr Benjamin's paper was discussed by the Institute. That discussion was by far the stormiest I have ever attended; I remember enjoying it very much. Although the paper was criticized from many angles, it highlighted a very real problem which could not be solved using traditional techniques and suggested a novel, alternative approach. There is no doubt that Messrs Benjamin, Scott, Squires and Wilkie have moved us forward considerably since 1971; but there is also no doubt that we still have a long way to go. It is very surprising that we have taken so long to make so little progress but it is certain that we cannot afford to be here having a similar discussion in another 5 years' time.

At this stage I thought that it would be worthwhile to give my impression of the discussion on Dr Scott's paper to the Faculty in January 1977. The tone of the meeting was set by part of Dr Scott's introductory remarks which I will quote: "The danger I fear this evening is not that my efforts will be criticized but that no definite alternative proposals will be forthcoming and the present state of affairs, which may be regarded as unsatisfactory, will be permitted to continue."

I have divided the points made in discussion into three groups covering unit prices, the mathematical model and the results. Taking unit prices first, there was considerable discussion about their independence or non-independence. Although for me there is enough evidence, both visual and numerical, to support the non-independence theory I doubt whether anyone will ever win or lose the argument. One speaker enquired how movements could be independent when so much was spent on investment research, and wondered what would happen if there was a relationship which changed from time to time. Another speaker suggested that we should be conservative and assume independence if that produced the biggest reserves. Whilst I agree with the principle of being conservative we must be practical too and keep the probabilities in mind; no one would use a valuation basis which assumed that half their lives assured die tomorrow.

In preparing his paper, Dr Scott had found non-independence in the unit prices he was working on and had then adopted an artificial process to produce a modified independent model. Unfortunately, this satisfied neither those who believe that the movements are independent, who criticized his reasoning because he had gone too far, nor the people who think that the movements are non-independent, who also criticized his reasoning because he had not gone far enough. The discussion about the results included the relatively unimportant comparisons of monthly with annual premiums and of spot guarantees with continuous guarantees. There was also some expression of opinion as to whether 1974 was good or bad and, philosophically, whether it mattered.

It was clear that those applying probability theory directly to the maturity guarantee problem had made progress in the past year. They have refined their techniques and are beginning to consider the question of non-independence. There are, however, several important factors which require attention. At the top of my list would be the effect of a spread of maturity dates, which is vital for any office transacting this type of business. Next would come the effect of assets other than equities. In this respect it is interesting to note that Mr Benjamin's paper refers to 'Equity-Linked Policies' whereas the author refers to 'Unit-Linked'. Many people seem to ignore the difference between these terms when talking about maturity guarantees. In fact, over the last 5 years, the words have changed but the numbers have remained the same.

An alternative approach is the one first proposed by Mr Squires. His model assumed that prices follow a trend line but that there are variations about that line with suitable probabilities. The author's model is of this type. Both of these approaches have the advantage of simplicity.
and are easy to use. In fact the author's method is so easy to use that you can actually check his figures, something which I have not previously experienced with a simulation paper. I wonder, however, what is the probability of, for example, several prices being at the top of a range and only one at the bottom. Unfortunately, the only way I have of answering this question is to work back from the results produced by the other methods. Therefore I can see no alternative but to continue to make the basic attack on the problem by way of the direct probability models. I am sure, however, that methods such as the author's will be used at a later stage as a practical way of producing results when we understand their relationship with the direct models.

Considering next the problems of calculating the valuation liability for one policy, suppose that the shortfall at maturity for which we wish to reserve has been determined. My next step would differ from the methods published so far, in that I would not value the maturity guarantee separately from the other benefits: they are not issued separately so why value separately? Furthermore, double counting of liabilities may result from a separate valuation.

For example, consider a unit-linked whole life assurance with a guaranteed surrender value after 10 years. The benefits other than the guaranteed surrender value, would be valued by adding the unit liability, or surrender value, to the non-unit liability determined in a similar way to that described in Mr Smart's paper (J.I.A. 104, 125). For each year from the valuation date, the income less the outgo would be calculated; typically, this would involve premiums plus renewal management charges minus unit allocations, expenses and mortality costs. The non-unit liability would then be derived by determining the worst possible combination of the discounted values of these items. If the valuation basis involves a rate of renewal expense inflation, the worst possible result will probably arise from a series of positive values of income minus outgo for the first few years after the valuation date followed by a series of negative values which more than outweigh the positive ones. If the guaranteed surrender value benefit is then added, two important points arise. First, any negative values which arise after the tenth policy anniversary are unimportant if the guarantee takes effect: we, therefore, have alternative rather than additional liabilities and must take the greater. Secondly, we should not be looking at the guarantee premium less expenses in calculating the liability, but at the income less outgo: if in any year there is an excess of income over outgo, that excess can be accumulated to meet the deficit on surrender.

My method appears to offer a consistent approach to the valuation of a unit-linked contract and adopting it should result in lower levels of additional resources than are required if maturity guarantee benefits are considered separately from other benefits. The problems associated with liability calculations for blocks of business are the ones which have received the least attention so far, despite their importance; this is not surprising since the single policy problem has not yet been solved. For a real portfolio the calculation of liabilities would have to be a two-stage process if we are to achieve a consistent approach in the way described for a single policy. The first stage would be to determine a pattern of percentage shortfalls on maturity for each future year. These figures would then be passed to the second stage, which would consist of calculations on the single policy basis.

The determination of the pattern of shortfalls could be based on one of several methods. Using the author's approach it could be determined by searching through all possible combinations of unit prices to find the set which leads to the maximum value of the discounted deficits. This would take account of the real distribution of the portfolio and would guard against situations such as the apparent contradiction in §17 which the author has already mentioned. The residual problem is the worry that this two-stage process may not give the overall maximum liability due to the complexities of the interaction of the various benefits. Such a process does seem to provide a workable solution, however, although there is obviously more work to be done.

The question of the regulations which, I accept, are bound to be made, is the other major problem. The most important point is that the regulations should not be made until we have solved the problem.

A third working party should aim for a statement of a consistent approach to the valuation of
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maturity guarantees, covering the important variables such as the term of the contract; the nature of the guarantee (its level and whether it is a spot or a continuous guarantee); the investment linking (whether the assets are equity, property, gilts, cash or a managed fund); and finally the investment flexibility (whether the office is locked into a particular kind of investment or whether it has the freedom which could be vital in a falling market). Thought will also have to be given to the information to be shown in the Department of Trade Returns.

Such an approach is more likely to solve the problem satisfactorily than the minimum percentage idea, which is quite arbitrary. In particular, the only justifiable percentage is zero unless we are to reserve for the sake of it. What other figure could be justified for a single premium policy linked to deposits with the big banks which has a guaranteed surrender value of the amount invested less the initial management charge?

I hope the regulations will deal with all guarantees, not just maturity guarantees on unit-linked policies. Many offices issue traditional contracts with guaranteed surrender values, or guaranteed annuity rates, and I cannot see why they too should not be covered. It is precisely these benefits which have given rise to the difficulties of the recent past and there is need for more work in this area also.

Mr N. D. Freethy: When Mr Benjamin introduced the subject of reserves for Maturity Guarantees in 1971, I too had reservations about the assumption of randomness, or statistical independence, between the price of a given index, or units of a unit fund, at a particular point in time and at, say, the same point of time a year later. To judge from § 2 of tonight's paper the working party had similar thoughts because in conclusion (a) they reported that "independence is not a meaningful concept". There are almost insurmountable difficulties in attempting to build statistical theory on the basis of some form of dependence between one year and the next if only because of the difficulty of defining what such dependence might be in the first place. Furthermore, it is not certain that dependence would necessarily reduce the probable range of price variation between one year and the next as compared with the assumption of independence. For example, the price at a given time might, judged from some standard, be absurdly high, and the forces of natural reaction might make the price the following year equally absurdly low. The two prices are dependent upon each other in that the later one is an over-reaction to the earlier one, so that the difference between them is greater than might be expected from mere random fluctuation on the basis of independence between the prices in two successive years.

I would regard the example of dependence which I have just cited as untypical, however, and to the extent that the market price of a share reflects that of a year previously, I would expect the difference between the two would normally be less than the difference which would occur on the basis of randomness and independence. If this were so then I should expect that reserves for maturity guarantees would be lower than those towards which Mr Benjamin wished to lead us in 1971. And yet § 9 of the paper states that the results of the estimation process used produces results 'comparable' to those of Mr Benjamin in his 1971 paper.

That this should be so is perhaps not surprising in that the theory expounded in §§ 7 and 8 assume that all shares are bought at the top of a conservatively widely chosen spread about the trend line and are sold at the bottom of that spread. I am not sure whether this is meant as some kind of jaundiced reference to the typical investor in such policies. What has to be guarded against is just the possibility that a substantial lump sum is required to meet the commitment on a day when the stock market (or whatever other area of investment outlet is applicable) is depressed. It applies with particular force when the company has sold a large block of policies all maturing and calling for the guarantee at or about the same time. What I find less than reasonable about the approach in § 8 is the assumption that whenever purchases are made the stock market (or other relevant index governing the unit price) is always at the top of the assumed spread about the trend line. This seems totally unreasonable, as purchases will be made regularly throughout the whole term of the contract and the assumption that the market is always at a top seems comparable to me to calculating a valuation reserve using the
assumption with regard to mortality that at some point in a policy's currency everyone will die from an epidemic!

I should like the estimation process based on the assumption that units will be bought at varying prices during the term of the policy, even if, to be duly conservative, we assume that the policy matures at a time when the unit price is low.

The general tenor of the paper is sufficiently alarmist to deter all but the most venturesome into offering maturity guarantees under equity-linked policies; but actuaries are noted for the reasonableness with which they can combine the setting-up of an adequate reserve with what is available to meet that reserve. Anyone who cannot invest so as to cover a return of premiums over as short a period as 10 years would generally be regarded by this profession as having performed pretty poorly and, if we contended that 30% of the sum assured was the right initial reserve for such a promise, most people would surely dismiss us as being theoretical rather than realistic. In the past we have normally arrived at reserves which we consider adequate, and which most outside observers would regard as reasonable, by taking into account all possible factors, not just those which are potentially most alarmist. Thus, in the current example, even if we were to accept that reserves of the size propounded by the author of this note are appropriate we must also accept that such reserves will only be required if the policy remains in force until maturity. Should the policy not run its full term the whole of the reserve built up will be released. In this area, therefore, the introduction of the probability of lapse is highly significant. A reserve of 30% of the guaranteed sum at maturity may be required on day one if it is known that the policy will survive but if, as is common these days, 40% of such policies lapse in the first year, then it is grossly excessive.

Perhaps the aim should be to set up a suitably conservative prospective reserve after 3 years when it is expected that perhaps 40% or 50% of policies may have lapsed, the actual percentage to be chosen conservatively by the actuary. If the size of the reserve after 3 years is then calculated on less stringent assumptions than buying at the top and selling at the bottom, the reserving problem reduces to more realistic proportions. One possibility which has been adopted in practice is to accumulate the premiums for the maturity guarantee retrospectively, not releasing the reserves held in respect of policies which lapse. A life office can clearly afford this as it is reserving on the basis of the amount actually charged for the maturity guarantee and is simply denying itself a contribution to profit by not releasing the reserve when this is no longer required for the specific policy. With this sort of arrangement we have a reserve after 3 years which is closer to that required for a basis slightly less stringent than that set out in § 8 of the paper, allowing for the possibility of lapse between years 3 and 10.

The following general conclusions may be drawn from the paper:

(1) Be cautious in offering a maturity guarantee of a return of premiums where the term of the contract is as short as 10 years.

(2) Be equally careful when selling a large tranche of policies all maturing at about the same time. One solution might be to have a quota of policies maturing in 10 years and when exhausted to offer a guarantee after 11 years, and so on.

(3) As far as practicable maturity guarantees should be restricted to markets where investments are fairly readily realizable and where there is a good record of price movements. This will enable the future to be predicted with some degree of confidence.

(4) The problem is less serious when the term of the policy is as long as 20 or 25 years. For these terms it would seem reasonable to accumulate retrospectively particularly if reserves in respect of lapses are not released but kept in the general fund for the maturity guarantee reserve.

(5) Regular prospective checks as in the model in § 8 of the paper modified so as to relate to slightly more reasonable assumptions, should be carried out to check that the overall amount reserved is still adequate.

Mr S. Benjamin: Our last meeting on this subject, in 1971, was closed and we had a confidential discussion; that paper and the discussion have never been published nor is the edited version prepared for the Tokyo Conference available yet.
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I referred then, as I shall now, to policies linked to outside unit trusts, rather than internal funds, where, although very difficult, the problem is different.

I should like to discuss the subject under several headings. The first one is the model: the one which I used at the time was to consider the de Zoete index history and to assume that it looked random. Although the working party thought otherwise I still disagree with them. The article "Random Walks and Investment Theory" by Peter D. Praetz (J.S.S. 20, 202) is a survey of the statistical literature on the subject. Most of the literature looks at the short-term effects, but it seems to be agreed that a random walk is a good representation of the market.

I found that there was a mean of +9.7% to annual changes and a standard deviation of 19%; it is a distribution with very fat tails. I examined the independence from year to year, applied some simple tests and decided that the evidence suggested that the model was suitable for calculating reserve values. It may not be random but I consider it adequate and I am interested to know why others are not prepared to accept that hypothesis now.

The philosophy which I used—and it was not easy to develop—started from the premise that expected values are not good enough. The second point was that for this situation the ruin probability was useful. In fact Mr Wilkie has described the philosophy in that paper as: 'bankruptcy' is 'no money to pay'; 'technical insolvency' is 'a probability of ruin (i.e. bankruptcy) of 1 in 50'. (That is when the supervisory authorities investigate.) 'Standard of adequacy'—and we all know what that means now—is obtained by ensuring at the outset of the policy that the reserve is derived in such a way that the probability of technical insolvency during the term is not greater than 1 in 50. This implies that the standard of adequacy (but not the standard for technical insolvency) will vary during the term. If a calculation on the in-force portfolio, which with a computer can be effected at any time, shows that the standard of adequacy is less than 1 in 50, no action is necessary. The reserve has been set up at the outset, and the fund is not technically insolvent. If the standard of adequacy is greater than 1 in 50 it is possible, but not mandatory of course, to release some surplus.

There are at least two desirable attributes of any method of reserving. The first is that the method should be cautious; the second is that the method should be coherent—that is it gives a reserving standard at the end of year \( t \) attaining the same reserving standard at the end of the year \( (t + 1) \) without injecting capital. Consider the alternatives put forward in the discussion and see if they are coherent by this definition. The suggestion in the paper is not even coherent within its own framework.

I considered 1 per annum for \( n \) years, with a guarantee of \( n \). I found that for a 10-year term the opening reserve was around 25% of the sum assured; for longer terms I found it did not decrease; algebraic analysis shows why.

Five to six years ago I published material which showed that for a portfolio with a mix of maturity dates the reserve does not decrease. Many speakers have asserted that you need a spread of maturity dates and that this lessens the reserves. But if the published work to which I have referred is correct, such a spread of reserves does not decrease the reserve. Certainly, no one has challenged my conclusions so far. Since then, Mr Wilkie has agreed in his work that the opening reserves, that I obtained for a 10-year term, were of the right order of magnitude and he has also shown that they do not decrease for longer terms.

All the arguments that the market is not random depend on the fact that there exists slight evidence of auto-correlation. For example, Dr Scott looked at the auto-correlation at 4 successive years. He ascertained the largest, which was at year 2, and applied a simple significance test. If the largest of 4 is taken I would not have thought that a straight significance test was really appropriate. Subsequently Mr Wilkie, in unpublished notes, has performed simulations incorporating such auto-correlation and discovered that it reduces my figures.

He also incorporated the de Zoete index from 1972 to 1976, which brought him back to my results. I further found, and Mr Wilkie has confirmed, that the worst market sequence in the simulation does not appear unfavourable; it does not show runs of steady decline, and looks a very reasonable sequence. The figures have to be very carefully considered in order to realize why the maturity guarantees were paid.

Let us consider what the working party might have done in the past 5 years. They could have
published the paper which I gave, and the discussion thereon, in a confidential form, thereby affording the speakers the chance to develop their arguments. They could also have checked my arithmetic. They could have done what Mr Wilkie has just done and included the autocorrelation, which we knew was there, in the market model. They could also have verified my results with an on-going portfolio rather than asserting that no work has been done. They could have commented on the philosophy of an expected, rather than a cautious, approach. (So far as I am aware most companies, at that time, were actually reserving on an expected approach.) They could have investigated and reported clearly on the methods of reserving actually used in the United Kingdom. They could have determined the extent of these guarantees and detailed the results, including reference to those companies unwilling to co-operate. They could have entered into discussion with colleagues in other countries to clarify their methods of reserve. The Canadian method is an *ad hoc* one, whilst the French method is unintelligible. They could have tried to ascertain why it is impossible to obtain reinsurance for this risk and they could have sought the views of many of the senior actuarial officers in the Institute.

Now let us look at some of the alternatives, such as the author's model. It uses a trend line, but does not name it. Apparently, the line is anchored to the first year of issue for that company; this is why most of these trend-line methods seem to be cautious, but are not. The paper suggests that the trend line can be adjusted. If this method is used, the trend line will soon need to be adjusted, and the consequence should be considered. As the trend line is moved the model becomes random and becomes more complicated.

The model specifies certain fairly narrow limits and suggests that the probability of going outside these limits is either zero or very small. My statistical training suggests that if you are given 50 numbers then the probability of exceeding the limits with the next number is either $1/51$ or $1/50^2$ depending on which approach you adopt. Presumably in order to use this model with a spread of terms and maturity dates across $n$ years, you search through the $n$ possible market sequences. One actuary has actually said that he could see the solution in practice by inspection of his own portfolio. But can he demonstrate how to use a changing portfolio and, in particular, can he demonstrate that the method is coherent: i.e. that the movement from one year to the next does not require an injection of capital? Furthermore, can he name his trend line? Is he currently above or below it?

Arguments are put forward that we do not use ruin probability as an approach to an orthodox portfolio. I do if there is a volume of temporary insurance business. I allow a ruin probability of around 1 in 1,000 on a fairly simple model of fluctuation.

*If there are mortality fluctuations and maturity guarantees, then the combined calculation must be carried out, by combining the two probability distributions; if it is difficult, do not issue them both. For ordinary non-profit endowment business I do not see the problem as probabilistic. The actuary can match with dated gilts and give management a rule of thumb to calculate a mismatching reserve at any point of time. The one which I use is to take a margin on equities of about one-third, and to take a margin on gilts allowing for about a 3 or 4% change in yield, and if the free capital is less than a mismatching reserve calculated on that basis at any point in time, it is necessary to rearrange your assets. I do not see it as a probability model at all, but rather as a decision and control model. It only becomes a probability model when the prices are outside control.*

We shall also hear of methods which require moving cash in and out as unit prices fluctuate; several suggestions are currently being made.

I refer you to the very last section of the paper, § A5. Presumably someone will have to show how fast the fund does not go downhill with buying and selling costs every day desperately trying to avoid both mismatching and influencing the price. Perhaps we should have another non-working party on the subject!

When doing research there is a duty to be cautious. The profession is faced with one model and method—that is to say the random walk approach—which has been written up, and other methods which have not been subjected to publication and which, from a brief description, do not sound coherent in the way that I have defined. The random model is apparently more
cautious and hence it seems to me to be the only one which is professionally justifiable in this situation. Unfortunately, it leads to very large reserves and it is sometimes condemned as over-cautious, for that reason, but that of course is nonsense; it is inverted logic.

When I submitted my previous paper in 1971 many people said that the model was excessively cautious. One of the main arguments which was stated time and time again was that there had never been a 10-year period which showed a loss. I went away and carried out simulations on the simple random model in order to calculate the probability of getting sequences of 51 years (as then) available with no period of 10 years showing a loss. The answer was 1 in 4. And then came 1974. Perhaps I should walk back to my seat in a random fashion.

Dr W. F. Scott, F.F.A.: To answer Mr Benjamin's criticism of my article, I did not, and indeed was careful not to, use the words 'significance test'. I also consider that there is evidence for cyclic movements in the stock market in addition to the correlation coefficients which he mentioned.

Despite any shortcomings my recent paper may have—and after two hours of discussion in the Faculty Hall, Edinburgh, I am still of the view that my paper, and in particular the table in § 7 thereof, provides the best way to solve the problem of the reserves for maturity guarantees—the only question which needs further serious consideration is that of withdrawals.

The supervisory authorities are apparently not prepared to permit an allowance for withdrawals. Therefore without necessarily conceding the principle they might be prepared to allow the level of reserve suggested in table 7 of my paper rather than the larger reserves of Mr Wilkie's and Mr Benjamin's methods, I do not think they will be prepared to accept the level of withdrawal suggested by Mr Freethy of 40% in the first year.

I am also of the view that all that is necessary in the recommendations is to establish sufficient reserves at inception to deter the irresponsible issue of guarantees.

Mr A. D. Wilkie: Expected value is not suitable as a way of calculating premiums. It is an elementary result of risk theory, which I think is universally valid, that, unless there is a contingency loading in the premium, the probability of running out of reserves, however large they are at the outset, is one. In the long run, the result is failure without contingency loadings. If the contingency loadings and contingency reserves are both adequate then the probability of running out of reserves is less than one and, by increasing both loadings and reserves sufficiently, the probability can be made as low as is desired.

We, as a profession, are in a muddle because our educational system has not been brought up to date in this respect during the last 100 years. We still teach life contingencies in a purely deterministic way, and if we continue to do so we shall be ridiculed by statisticians in every country. We need to revise substantially our teaching methods in this subject to get away from the assumption that expected values are applicable everywhere, and to introduce risk theory into the syllabus appropriately. The Faculty is no better than the Institute in this respect.

There is disagreement concerning the right model for share price movements. The models proposed can be subdivided into two separate types, in one of which there is a centralizing tendency whilst in the other there is not. If the model has a centralizing tendency, or a trend line of some sort, then following a movement diverging from that line there is a tendency to return to it. In the other type of model, whether it is a pure random walk or exhibits autocorrelations or some sort of cyclic lag, wide divergence is not necessarily followed by return.

In Germany, in 1923, share prices multiplied by many powers of 10; there was no tendency for them to return to their previous levels afterwards. It is conceivable that there might be circumstances, while still retaining a capitalist economy, where share prices fell by factors of 10 and there need be no reason for them to revert to former levels. I do not accept the trend line as a tenable hypothesis; it is as ludicrous a way of predicting share prices as to quote immediate annuity values on the assumption that everybody lives for precisely three score years and ten.

The author's model is similar to the one which Mr Squires described in Dr Scott's paper; I would describe it as a malevolent deterministic model. One problem is that if you start with the
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de Zoete index in 1919 and follow a trend line of 7½%, taking the ratio of actual to trend, the result at each of the 58 year-ends in this period exceeds 1-4 eleven times and goes below 0-7 twice. The range is between 1-7 and 0-45. I think the author should have used those figures and I consider his reserves are ludicrously low, rather than too high.

Nobody, apart from Mr Benjamin some years ago, has satisfactorily studied the situation where there is a spread of business of varying dates and terms. Mr Benjamin has anticipated some of my comments, but I have this contribution: I have started simulating business growth using a model taking one policy per year, each issued for a 10-year term for a period of 50 years. Each sum assured is £1, each premium is £0.1; I used Dr Scott's auto-correlated model of the stock market. The starting reserve I had was £2.5— that is 25% of the maximum sum assured in force. I took guaranteed premiums equal to expected costs, plus a charge of 1% on the reserves to rent them, as it were. The offices were bankrupted 7 times within the 50 years. That is a preliminary figure, but I should like to do many more simulations and produce alternative answers in due course. I am sure that the 5% to 10% which the author quotes as a reasonable figure, is far too low.

Any reasonable statistical model can be used in a simulation, as Mr Benjamin mentioned, and I have re-done some of my own calculations using Dr Scott's auto-correlated models but using different bases. Dr Scott used the experience of the de Zoete index, and because there is auto-correlation he chose a lower standard deviation, roughly one-half of that experienced. Using that model, I find that the reserves again are much too low; they make too great an allowance for the auto-correlation. Using the actual auto-correlation, which is easily done for simulation, results which are equivalent to using about three-quarters of the standard deviation of the not auto-correlated model are obtained.

Mr M. J. Taylor: Some years ago when the subject of maturity guarantees was discussed at a Students' Society meeting a remark was made with which I then wholeheartedly agreed: were there to be a stock market crash of such size, and perhaps more important, such duration as seriously to embarrass life offices offering such guarantees, it would mark an economic catastrophe of such proportions that far more serious problems would have arisen than the loss of part of the savings of a number of investors in life offices. The implications seemed that we should not worry over-much about maturity guarantees, any more than we worry about the threat of nuclear war in our considerations on mortality. I no longer accept these implications.

When we examine the literature to which unit-linked guarantees have given rise, there appears to be one factor common to many of the mathematical models which are designed to reproduce stock market behaviour: namely, the assumption that the long-term trend of the underlying market or units is upwards. This assumption underlies the author’s analysis in § 8 of his paper. He demonstrates on his simple model that initial reserves of 10 to 15%, or perhaps even more, of total premiums, will be necessary. It does not take long to rework these initial reserves on the assumptions of zero growth, or indeed a modest negative growth, and figures of 50% or more soon start appearing.

I believe that there has been a historical trend line in the growth of the stock market indices and that there have been identifiable reasons for this. Without preaching economic gloom it does not require much imagination to admit to the possibility of negative growth. Historically, the growth of the economy of the western world has been based on a number of factors: cheap raw materials from primary producing countries, fairly cheap workforce and the technological skill to manufacture and export goods. The prospects for the future could be vastly different. The primary producing countries seem determined to take a larger share in the profits of manufacture by charging higher prices for their raw materials; the increase in oil prices over recent years is but one example. The workforce is no longer as cheap as it was. It is merely a matter of time before what has hitherto been referred to as the backward countries develop their own technologies to compete against ours. Japan is a prime example of this having occurred.

I do not claim the above to be a profound theory of economic decline. I merely submit that a number of the factors which have in the past given rise to a positive growth in the economy
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and in the stock market have altered radically. The possibility of zero or even negative growth in equity prices is in my judgment too strong to be pushed aside but should be taken into consideration in formulating a reserving basis for maturity guarantees under equity-linked policies.

Companies should think very hard indeed before issuing policies under which the liabilities cannot be, at least notionally, matched by appropriate assets. There is no way in which maturity guarantees on a unit-linked policy can be matched where the underlying unit fund largely comprises equity or property investments.

Secondly, where maturity guarantees are required, the investment mix of the units should be such that there is sufficient cash or suitably dated fixed-interest investments to underpin a positive growth rate in the units, and in consequence to reduce the effect of market variations in the price of the units. A company whose business consists mainly of equity—and I stress equity—unit-linked policies with maturity guarantees I think has a difficult problem to resolve in determining its reserving basis. It is a problem which, if we are to adopt the principle of less likely maximum cost mentioned in the author's paper, could conceivably be greater than has hitherto been realized.

I hope that the answer could be found by varying the investment mix of the units rather than by requiring such levels of reserve as might be embarrassing both to offices and the industry. There are problems in this approach, especially where the linking vehicle is an authorized unit trust. Even when the life office management is in a position to influence the investment policy of the trust managers, it could be difficult for the life office to influence the outside unit holders who are presumably intending to invest in something other than sizeable holdings of fixed interest stocks.

Mr J. C. Fagan: In his 1971 paper, Mr Benjamin used an example of tossing a coin in order to illustrate the difficulties involved in trying to set aside adequate reserves for maturity guarantees. I would like to use a similar, but simpler, example which I hope, will point the way to a possible solution.

Suppose I meet a gambler who wants to bet £50 with me that the toss of a coin will turn up heads. I do not have to be an expert in risk theory to know that I need to have £30 in my pocket before I accept the bet. An alternative is to find another gambler who will bet £30 with me that the same toss of a coin will result in tails. In this case I can accept both bets without even a penny in my pocket. On the subject of maturity guarantees we have spent too much time worrying about the gambler who bets on heads turning up without looking for the gambler who is prepared to back tails. We have that second gambler. If an office holds less units than are credited to policyholders it stands to lose if units increase in value, but stands to gain if units decrease in value. The surplus movements are in exactly the opposite direction to those arising from changes in maturity guarantee reserves.

A satisfactory state of immunization can be achieved by using this technique, in which event it is sufficient to base valuation reserves on the expected cost of the guarantee only. A margin must be retained to allow for delays in changing the immunized position and for dealing costs, but this margin can be implicit in the form of conservative assumptions regarding the future earned rate of interest, the model for assumed price behaviour, and so on.

If it so wishes, an office need not immunize; in this event it is in an analogous position to an office transacting traditional business and which invests substantially in equities. In fact there is not much difference between an office which transacts traditional business and one which transacts unit-linked policies with guarantees. I conclude that the problems in this area are not nearly as great as some may have us believe. Satisfactory valuation regulations can be devised on these lines which will not result in excessive financial strains being placed on the offices transacting this type of business.

I prepared the foregoing remarks on the assumption that there would be some support for this point of view at this meeting, but so far there is little indication of this. Mr Benjamin has dismissed the argument; but I should like to refer to his own contribution in which he defined a coherent reserving basis as one in which, starting with a certain reserving basis at the end of
year $t$ it was possible to maintain the same basis at the end of year $t+1$ without raising additional capital from the shareholders. I would contend that Mr Benjamin’s basis as set out in the 1971 paper does not fulfill the condition which he now asserts. If the independence of successive price movements is accepted—and I agree with him on this point—and if there is a 2% probability of ruin in the reserving basis at the end of year $t$ and if at the end of year $t+1$ the price has gone down, where under the Benjamin system is the extra capital for those additional reserves which will definitely be necessary at that time to come from? The only automatic method of raising more capital when prices have fallen is by under-investing in units.

Mr J. P. Ryan: There seem to be two approaches: the straightforward random walk of Mr Benjamin and the trend line approach of the author and Mr Squires. The numbers that arise are not as far apart as might be expected, the reason being that the author’s assumptions of insolvency are much greater than those of the random walkers.

There is, however, a philosophical danger in the random walk model. First of all the fact of independence from one year to another does not imply that there is no cycle at all. If it is assumed that there are whole series of economic cycles over 50 years but that the amplitude and length of those cycles vary in a random manner over a number of years, then simply to correlate 1 year with the next or even up to the next 4 years as Dr Scott has done, is not going to prove the independence of share prices. A more satisfactory model, although obviously rather more complicated, is a sine or other wave type of model with fluctuating amplitude and length of cycle.

The random walk model raises problems when one compares this method with that of a conventional non-profit endowment assurance. The conservative assumption is that the share or unit price is going to fall each year, and over a period of years, the implication is a serious economic situation in the country. There will be fears of substantial cuts in dividends, bankruptcies or something similar. On these assumptions the same problem is going to apply to companies issuing non-profit endowments backed by unsecured loan stocks. Therefore, on accepting Mr Benjamin’s approach, the statutory interest margin is much too low.

While the model is unsatisfactory in many ways, the author’s approach provides a practical compromise for considering reserves. The model itself is not conservative but the assumption of dealing spread between the upper and lower trend lines is relatively conservative, although the 30% fluctuation is too low as was borne out by the events of 1974. The other advantage of the trend-line approach relates to formulating rules for a statutory valuation basis. It is relatively straightforward to use a method of adaptive forecasting to provide a simple rule for altering the reserve from year to year. Kendall (Time Series, pp. 118–22) gives an account of the method.

Several papers have referred to the investment of maturity guarantees reserves. They have all suggested a matching dated gilt as being appropriate, but the price of the maturity guarantee investment should preferably move in the opposite direction to the unit price. It is by no means obvious to me that a dated gilt is going to move in this way, indeed the reverse is more probable. The ideal solution would be to ask some reputable institution for a 10-year option on the market; perhaps the development of option markets in North America will make this a future possibility. Mr Taylor made the point that there is no way of matching these guarantees; however, consider a straightforward convertible issue. The stock market values convertibles on an expected basis together with Mr Wilkie’s contingency margins. These are attractive investments from the company’s point of view in that, provided it is monitoring its cash flow either for investment, or for repayment of a convertible loan stock, it will almost certainly wish to defer any investment plans it has at a time of extreme economic depression, and therefore, it will be in a position to redeem the loan stock.

Mr E. A. Johnston: The discussion this evening, and at the Faculty in January, has shown that there is no general agreement on the mechanical models which should be used for assessing these reserves. I am certainly not going to venture any opinion on which mathematical model is right, but there does have to be a practical answer to this question because companies do
have to set up reserves of some size or other. I thought I saw the glimmerings of a first approximation in § 9 of the paper, which suggests that the results obtained by at least two of the different methods were not all that far apart in practice. Indeed a great many of the proposals put forward produce reserves well above the expected value of the liability, and, particularly for the shorter-term policies, far greater than any premium loading which could realistically be charged to policyholders. If we are really getting some general agreement in the profession on those propositions, that is something which we in the Government Actuary's Department will have to take seriously, and it may be helpful if I outline our current thinking on the matter, which is going to look rather simplistic beside a lot of what has been said this evening.

First, reading on through the paper, I found in § 23 a figure of 30% as the target reserve at maturity for the 10-year policy. Some other figures which we have looked at suggested that the target reserve required at maturity would vary between 10 and 50%, so 30% seems a reasonable first approximation. But the initial reserve needed to achieve such a target will be very substantial. The many other aspects to be considered, such as overlapping tranches of business, the treatment of longer term policies, and the question of mid-term reserves when a longer policy has become shorter term, seem to be quite different, according to which mathematical model is adopted. The possible use of withdrawal rates in the valuation is something which has not been done hitherto in statutory valuations, and the question of how maturity guarantees fit into the valuation of unit-linked policies must be considered as a whole.

When all these factors have been considered, the main conclusion is that heavy reserves are necessary.

Mr G. E. Barrow: We can consider the reserves as consisting of two parts: the hard core which have been accumulated by the expected premiums, and the contingency margin over and above that, which we all regard as essential but on which we differ as to size. What are the practical consequences? The contingency element will need to be set at such a level as to discourage new offices from writing this type of business unless they have adequate resources. Also, in calculating the contingency reserve, there is room for the injection of some lapse rates without in any way introducing the concept into the normal reserving basis. It is clear that the contingency reserve is going to be uncomfortably high even for offices which are currently writing this type of business. If the contingency reserves are set at such a level as to seriously embarrass such offices, the result is a considerable practical problem. Yet we cannot ignore the fact that contracts are backed by the Policyholders Protection Board and in equity to other offices there must be some reasonable contingency reserve.

Mr P. A. C. Seymour: Seven years ago I wrote a paper for the Students' Society about Unit-Linked Assurance and Capital Gains Tax using the de Zoete index in the historical order in which the events occurred. In other words, I performed only one simulation, based on what had actually happened in the past. I was looking principally at the problem of what to deduct for capital gains tax but as a by-product, I produced some data on the cost of capital guarantees. At that time it was just about zero, even for 10-year policies.

My mistake was to mention the subject at all. It was my first encounter with the redoubtable Mr Benjamin, armed with his risk theory and probability of ruin. How naive I was then; I had been brought up on the traditional British approach of calculating premiums using formulae based on expected values. The Scandinavians with their modern risk theories were an unheard-of mystery. Perhaps many of us are still rather daunted by such high level statistics. Nor do we fully understand the arguments about what is a suitable model for the stock market. Nevertheless, the warnings of the statisticians and the events of 1974 will have convinced everyone of the real dangers involved. It is clear that the volatility of stock market prices makes it necessary to hold reserves of much greater strength than merely an accumulation of past premiums calculated on an expected cost basis. Some form of prospective testing is required. If the statisticians cannot agree, how are we ordinary mortals to decide on an appropriate basis? Let us remember, it should be practicable enough for general use. Probably we will tend
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to sympathize with the simple trend line approach first propounded by Mr Squires, and echoed in the notes now before us.

I find it intuitively correct that the reserves required for a single tranche of 10-year policies should be substantially higher than for 30-year policies. The table in § 8 exhibits this feature, but some other models do not. Similarly, it seems right that a good spread of maturity dates should also reduce the reserves required. Again, the table in § 16 demonstrates this feature. Mr Benjamin's original simulation work in 1971 did not, but while I could broadly reproduce his simulation results for single generations, I could not when successive generations were merged. He considered his results surprising, so I wonder if his simulation model was working correctly. It seems that with a satisfactory spread of maturity dates for policies with say 30-year guarantees, the cost of reserving appropriately may be very modest, particularly if an allowance is made for withdrawals before maturity. Thus, unlike the author in § 25, I do not consider "the uncertainty of the reserving problem is such as to preclude the issue of contracts with a guarantee above a certain level", unless 'level' in this context somehow involves term also.

The question of maturity guarantees should not be considered in isolation. There are other risks at least as important, and some of them may be mutually exclusive: one obvious example is capital gains tax. If unit prices rise, there is less need for a reserve for guarantees but more will be required for gains tax. Furthermore, an incorrect level of deduction for gains tax, from policies linked to separate authorized unit trusts, could be more damaging than maturity guarantees. If a deduction of two-thirds the full rate is made on the assumption tax will be deferred, and the fund closes more rapidly than expected, it will be impossible to correct the position by adjusting the deduction from later policies. Another example of compensating effects is in 'Type B' policies where the company retains the income from the units. Assuming that dividend levels are maintained, if the unit price falls the yield will increase. The extra income accumulating to the company will help to offset the cost of the maturity guarantee.

A further important point to consider is the possibility of immunizing the risk. The statisticians' models assume that the investment policy of the office is to buy units to the full extent of the allocations. Instead of worrying about the consequences of such a policy, it might be better to research whether this was the best policy to adopt. I have already mentioned the complementary nature of maturity guarantees and capital gains tax. This points the way to Mr Fagan's immunization technique. A suitable investment to hold for maturity guarantees is something which goes up when units go down—for example 'negative' units. One way of holding such an investment is intentionally to underpurchase units against allocations. Mr Fagan's algebra, which is similar to Mr Redington's, turns this concept into mathematical formulae. Whether it works or not, and if so, whether it poses more new problems than it solves, I am not yet sure. But it is preferable to consider such an approach before being frightened away from guarantees altogether. Imagine a risk-of-ruin approach applied to the issue of non-profit endowment business, with a stochastic model for the future rate of interest. In that event the mathematicians would be calling for far stronger reserves than we currently adopt. Why are they not doing so? Presumably because it is possible to immunize, so it is only a question of deciding on an appropriate mismatching reserve. Offices will only mismatch if they can well afford it. Admittedly, the maturity guarantee immunization theory implies constant switching but so does Redington's which is still accepted as solving the ruin problem for conventional non-profit endowment assurances.

Whilst I accept that my historical analysis of the costs of such guarantees, made 7 years ago, was over-simplified, nevertheless I feel that we are now in danger of over-reacting in the opposite direction, and so getting the whole matter out of perspective.

Mr R. J. Squires: If a model is postulated and surprising answers are obtained, one of two conclusions may be drawn: either that the result is significant or that the model does not represent reality.

My model has been described as being a trend line approach, which I accept; but there was the additional assumption of a regular cycle. That is very important and influences the answers considerably. My model is adequate for calculating what might be called a pure risk premium,
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or for deriving reserves for guaranteed death benefits, but I accept that it may be too weak for maturity guarantees, and I would be quite happy to accept the author's model.

The note postulates a set of future prices and the task then is to find that particular set—that particular hypothesis—which gives the least favourable result over all. The great advantage of the procedure is that it can be done and it can be demonstrated to have been done within the limits of the hypothesis. I would totally reject any approach, however, that applied the procedure to a single generation of policies and derived some figure whether it be 10% or 30%, to be applied to every generation, the total reserve being the sum of the results. That approach implies a set of mutually contradictory hypotheses. To use the author's method, choose a central growth rate to fix a trend line and then choose a spread: two very difficult judgments to make. Unfortunately, they are very critical to the answer. Eventually the profession must come to some sort of consensus or even accept regulation.

The third factor, however, is to choose the central point from which to project the trend line in the future and in my paper I suggested projecting by exponential smoothing of prices. In effect, the use of this method entails the choice of a new base at each valuation as the smooth price will have moved. Of course the more accurately and satisfactorily the smoothing is performed the less the base will change, and that has to be one of the keys. If we can find an acceptable method of determining where the trend line will be seen to have been in retrospect, then the fluctuations in reserve are considerably less. In the meantime, I suggest that as a practical device a system should be used which I would describe as "jump up but creep down"—in other words, when the basis gives a higher reserve than the last valuation, the additional amount is put up immediately; but when it gives a lower result release only 10 to 25% of the difference.

Mr P. J. Turvey: I would like to broaden the scope of the discussion to include other types of guarantees. The author refers to maturity guarantees, and by implication excludes minimum death benefit guarantees; there is also the question of surrender value guarantees under unit-linked contracts, and I propose to adopt the terminology performance guarantees for all risks contingent upon the performance of a unit-linked fund at the date of claim.

For an annual premium contract, the major option is the guaranteed sum on maturity, which for some types of contract is an open-ended guarantee more akin to a surrender value guarantee. For certain annual premium contracts with a guaranteed minimum sum assured on death, the premium payable includes a charge for the projected cost of cover. There is therefore an implicit guarantee, equal to the excess (if any) of the sum at risk actually payable over the amount projected and reserved for, with a claim probability equal to the probability of death. In many cases, especially at early durations, this will be minimal because the size of the actual accumulated units will be small relative to guaranteed sum assured, but we should be aware of the existence of the risk and also in a position to evaluate it. Furthermore, it is quite likely that these guarantees, measured across a large and mature portfolio, would be significant.

For single premium contracts where the guarantee is a multiple of the original single premium, the death guarantee can be calculated in a similar way, being the excess of the guaranteed sum over the units allocated and the death strain supported by the loadings and reserves, with a probability of claim equal to the probability of death. The value of the guarantee can be significant and, in a medium-sized portfolio, can accumulate to a substantial sum. In contrast, where the sum assured on death is a function of the surrender value at that time there is no guarantee.

A further question which has had little discussion so far is that of the nature of the unit fund. Some funds—for example a specialized unit trust, or a geared fund—are more volatile than the FT-Actuaries Index, and these call for greater caution. At the other extreme it has been suggested that internal funds with no commitment to a specific investment policy provide a safe link for performance guarantees because investment policy can in extremis be arranged with a view to minimizing guarantee payments. Apart from the ethics of such a policy, and the problem of ensuring that the actuary will, at all times in the future, be able to exercise the
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necessary degree of control, there is a fundamental technical problem. It is impossible to immunize a linked portfolio in the same way as for a conventional portfolio of business; I exclude the concept which Mr Fagan brought out. Without going into detail, the mis-matching occurs because the operation of the unit pricing mechanism forces the change in asset values arising from a change in interest rates to be shared in the wrong way between groups of policies; this point was dealt with in greater detail by Mr Wales during discussions at the recent Tokyo Congress.

I agree with the speakers who suggested that a model of future stock exchange performance with useful predictive value is unattainable. The profession has examined past performance with great care and has had some difficulty in reaching agreement. No wonder: in the period under review there have been two world wars, a world-wide recession, and an oil crisis, not to mention raging inflation and other factors of substantial magnitude in the economic sphere. How do you draw a trend line through world wars? When is the next one due? What is the trend line in the United Kingdom political scene? As for extrapolation of these trend lines into the future, I would offer the analogy—which is not original—of predicting the next bend in the road from the experience of the last two miles. I have come to the conclusion that performance guarantees represent a risk that we have underrated in the past, and which should only be offered in restricted cases. What, then, should we do about existing contracts?

First and foremost, we should ensure that any office which relies upon our advice should only issue contracts carrying performance guarantees when there is adequate capital backing, and that we should particularly beware of creating portfolios consisting solely of performance guarantee business. I hope we shall so do voluntarily before we are compelled by new regulations from the Department of Trade. If common agreement cannot be obtained—and I am not aware of any mechanism within the Institute for reaching such agreement—I would welcome regulations that call for such a stringent valuation basis that offices will be deterred from offering guarantees unless the above conditions are satisfied.

Mr A. F. Wilson: I have tried approaching the problem through models, starting with re-invested indices, in the style of Messrs Benjamin and Wilkie and, perhaps not surprisingly, have reached similar conclusions. However, I have never been happy with the random walk approach which they adopt, and I consider it inapplicable to the present problem of establishing adequate reserves. To simulate the stock market, it is important to start with the basics. These are the level of dividend income from time to time and the immediate yield which the market is seeking. Dividend income may well exhibit the properties of a random walk and auto-correlation, but that immediate yield does not. In any event, what meaning can there be in extreme values of a random distribution of a reinvested index which uses the same future expected rate of growth, including reinvested income, whether the net yield implied is 2% or 20%.

For the purpose of reserving, I prefer the deterministic approach, as embodied in §§ 15 and 16 of the author's notes. However, I would make one change, and that is to express the parameters in terms of yield rather than capital values. Thus, the strength of any reserve held by a company can be considered as an answer to the following question: given the future rate of growth of dividend income, and the yield on the index during the investment period, what is the yield implied by the maximum fall in the market which could be sustained, should this happen at the worst possible time? I would also make one slight modification to the method in that I do not believe that the market can stay at very low yields indefinitely. In consequence, I would assume that the yield on the index was at its minimum value for 5 years only; prior to that, I would assume that it had taken the average value.

The advantages of this approach are firstly, that the method is readily checked by another actuary and the relative strengths of two offices can be easily compared; secondly, that the question—and the answer—can be readily understood by lay observers and thirdly, that direct allowance can be made for how the market—and, indeed, a particular unit trust—has performed in the past. (I am considering here contracts linked to equities in authorized unit trusts.) Fourthly, that the reserves are stable and do not fluctuate violently as does the immediate yield or the index, and fifthly that the reserves increase gradually while the market is
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high and decrease slightly when it remains low. Some actuaries suggest that the higher the market, the lower the reserves required. To the extent that the higher market is a reflection of dividend increases, this is true; and the method allows for it. However, to the extent that it is not, the reverse is the case. The propensity of this market to fall to a given yield basis, is not less just because the present yield is low. Rather, the cost of a fall is greater, due to the investments made while the market is high. Sixthly, that the parameters can easily be changed to allow, for example, for the different type of unit link, or the control the office has over the investments being made. Lastly, that the standard deviation of the rate of dividend increases in recent years has been small and a trend line which assumes a reasonably cautious rate of growth is, in the long run, over cautious.

Over the past 15 years dividends have increased at 5½% per annum, with a standard deviation of less than 4%. A growth rate of 4½% per annum gives a value more than two standard deviations below the expected value using 5½% increases after just 10 years. The parameters I would then choose in this type of approach would be: increases in dividends, 4½% per annum; mean yield on index, 3½% per annum—and that is slightly lower than it has been over the last few years; minimum net yield on the index, 2%—it has never been below that; maximum net yield on the index 6%—it has only been above that for 4 months. With the format of the table in §16 this shows the following figures: n = 10; r = 8; spread = 1.73-58; Year 1 = 2.67; Year 2 = 2.15; Year 3 = 1.60; Year 4 = 1.01; Years = .39.

The total for the 10 years is 7.82. If the initial position of any policy was considered the discounted value would be 7.82% of the sum assured which I would discount at a medium rate of interest rather than a low one.

Is this reserving basis sufficiently stringent? I consider that it is. The rate of growth of dividends has already been described as cautious, and so is the mean yield. I do not believe in taking unduly cautious values in all the parameters: if such a course is taken it becomes impossible to tell just how pessimistic a view is being taken. In my method the caution is based on the maximum yield assured.

Mr F. R. Wales: The task of closing tonight's discussion is a particularly difficult one. The concept of a maturity guarantee for unit-linked policies is simple and superficially attractive. In the early stages of development of unit-linked life assurance, the provision of such a guarantee seemed a logical development. I, like many other actuaries who examined the problems of product design 10 or more years ago, postulated the inclusion of such guarantees at modest rates of premium, reserving on an experience basis. The methods used were those that had stood the test of time when applied to other forms of benefit. With hindsight it is clear that much of our work was wrong and could have led to potential disaster. Indeed, I must confess to making the remarks at the Students' Society to which Mr Taylor referred. I agree therefore with the author and the opener that we all owe a great debt to the proponents of the stochastic methods, particularly Messrs Benjamin, Scott and Wilkie, who have demonstrated mathematically the magnitude of the risk to which companies can be exposed. However, it is one thing to postulate a mathematical model of the stock market in an attempt to determine the extent of the risk exposure, but quite another to claim that it is possible to simulate future stock market price movements. A number of speakers referred to this problem; but like the first working party I am totally convinced that it is a fruitless exercise to attempt to find a satisfactory model of stock market behaviour. As I said during a discussion of Mr Benjamin's paper in Tokyo (Trans. 20 Int. Congress. of Actuaries, 1, 17), stock markets operate in a constantly evolving environment. That means that the rules are always changing and thus the appropriate models must always be changing, as was really implied by the investment expert referred to by the opener. Indeed, Mr Taylor also implied this.

Now, let us put the problem into perspective. Messrs Ryan, Barrow and Seymour all referred to conventional business and this is most important. Mismatching under conventional policies can be just as serious as under maturity guarantees. We must avoid becoming obsessed with one particular actuarial problem to the exclusion of others. In fact, we have new light brought on the subject tonight by Mr Fagan, who very appropriately followed Mr Taylor
and answered his remarks by suggesting an immunization approach, this being supported by Mr Seymour. It is obviously something which is well worth considering and further work could be of great value. But there is a problem that so far as internal funds are concerned very often the policy conditions compel the company to fully invest or over invest thus preventing the use of negative units. However, if I understood Mr Ryan correctly, I am not sure that his approach would work unless the convertible stocks, which he suggests, are held outside the unit account. Otherwise, although he may match his maturity dates, the assets share of any policy includes a share in every convertible, not just the ones which are appropriate to the relevant maturity date.

We had a typically eloquent contribution from Mr Benjamin who claimed that his results have so far not really been disputed; but I was present at the earlier meeting and there was a very strong body of opinion which disagreed with many of his findings and indeed with much of his statistical work. I do not agree with him that if the statistical results are suspect, they are still suitable for a reserving basis; that is a rather dangerous approach. We have also had comments on his and Mr Wilkie's views that the immunization approach cannot be followed because of dealing costs; but, as Mr Seymour says, we can apply the same arguments to Redington's theory. Mr Benjamin has asked us to reach a consensus on the subject under discussion; obviously he feels affronted that the profession has so far failed to reach agreement. However, I consider there is always room for honest differences of opinion. Unlike the opener, I hope that in 5 years' time we will still have something to discuss on this subject.

Appropriate levels of reserving must vary dramatically from office to office, depending upon the nature of the total portfolio of the business. Nevertheless, I think it is fair to conclude from discussion that there is a general agreement among the profession that, by itself, a reserve determined on an expected value basis does not provide adequate protection against extreme fluctuations in security values. It is also fair to conclude that there is a general acceptance that for a single tranche of business maturing at a fixed date the reserve required for the maturity guarantee could be to the order of 30% or more, as suggested by the author and Mr Johnston, if this reserve is considered in isolation. I say 'in isolation' advisedly, for as the opener and Mr Seymour pointed out, reserves for maturity guarantees are only one valuation problem among many. Provision of death benefits, future expense levels, capital gains tax deductions, and so on, are also of vital importance to offices with unit-linked business, and factors that are adverse for one benefit may have the opposite effect on other benefits.

On the other hand, the opener's comments on a matching valuation approach are particularly important. In fact, it is interesting that one of the earliest proponents of this method was Mr Benjamin, who introduced a very interesting paper (J.I.A. 85, 373) on it. It is perhaps a pity he has not developed his work along these lines. Construction of an appropriate mathematical model to allow for the interactive effects of all relevant parameters in the valuation of unit-linked business presents horrific problems. The opener's approach has a virtue of practicability. I cannot, however, accept Mr Benjamin's view that we must not have a complex portfolio if we cannot construct a model to reproduce it. Turning to Mr Johnston's contribution, I hope that he will be taking a 30% reserving level as a form of warning signal. If an appointed actuary wishes to reserve at the lower level then he should be required to demonstrate his method fully in order that the Department of Trade can be satisfied that the reserves set up are indeed adequate. If I have interpreted him aright, I fully support this approach. I feel that often a lower level of reserving will indeed be appropriate. Mr Barrow suggests that the appropriate method from which to derive a reserve is to produce individual reserves on a per policy basis, using an expected value approach. In addition, the company should hold a contingency reserve to cover fluctuations from expected value; I fully support this. The adequacy of this fluctuation reserve can be tested using simulation techniques but not determined in this way. For example, no two simulations will ever give the same result: it all depends on the random number generation method.

In answer to one of Mr Johnston's questions, I think, like Mr Freethy, that we should accept that it is appropriate to take some account of withdrawals. These have a very significant effect on the cost of guarantees. An approach which tests for a 1% probability of ruin and
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ignores withdrawals is in effect demonstrating an infinitesimal probability of ruin, bearing in mind the multiplication rule for probabilities which seems so far to have been ignored in this discussion. For a similar reason, the company's contingency reserve should be considered in aggregate. The adequacy of each element of the reserve can be tested at a fairly high probability of ruin, say 5%, on the basis that the probability of catastrophes occurring simultaneously is the product of the individual probabilities. A further test at a low probability of ruin should then check that the overall reserve exceeds the maximum potential loss from any one individual risk.

The President (Mr C. M. O'Brien): In one sense the discussion has been disappointing because who, if not members of this profession, should be able to determine the right answer to the problem confronting us? On the other hand, it is perhaps hardly surprising that we have not found a complete solution. It took us a very long time recently to discover that there was no simple or indeed single way to frame regulations which would test traditional business, something with which we have been concerned for more than 200 years. It is perhaps not surprising that it is taking time to crystallize views and theories about unit-linked contracts which are relatively new.

It is equally true that the resources which the Institute can command for this sort of work, in terms of manpower in particular, are somewhat limited. Two things are clear: first, from the discussion tonight it would seem that we have not reached the stage where theory is so proven that it commands general agreement; and, secondly, the total problem of reserving for these contracts needs to be dealt with. I propose therefore to encourage Mr Wales, and the joint working party with the Faculty, to press forward as quickly as possible.

In the meantime, it is clear that if an office has only a limited volume and spread of this type of business—and some would question that last requirement—which, by definition, is the position of an office which is considering starting to write such business with guarantees, its actuary needs to exercise considerable caution. It could be right, initially to take the most stringent of the views expressed here and only to relax from that standard when the volume—more particularly the spread—of maturities is achieved. Alternatively, do not write guarantees at all. Even if we wanted as a profession to suggest that, I doubt if we could, having regard to things like the Restrictive Practices Act. Nevertheless, that is one clear solution to the difficulty if there is no real necessity to write a maturity guarantee into the contract.

The author: The opener, Mr Seymour and Mr Wales suggested that we should look at contracts as a whole. I agree, but only up to the limits of practicality; we must not use the same reserve twice to cover two different risks. It may be that those two risks are in fact mutually exclusive; but then again it may not, so caution is necessary. The regulations are not yet made and the opener suggested that we should resist control until the profession is agreed. There may be insufficient time so speed is important. As to information shown in the Department of Trade returns, Mr Benjamin commented that we ought to know what is the total liability under maturity guarantees, but this information seems to be remarkably difficult to obtain.

Various speakers referred to different types of guarantees. These will have to be dealt with in the regulations, but it is a matter of getting things in perspective. The traditional office might have many guarantees but the total loss that it could possibly experience on all of them would be minimal in relation to the total reserves. A unit-linked office with a lot of business of this type with guarantees on its books could be in an entirely different position.

Mr Freethy made me most unhappy. I felt he was proceeding too much on hope and did not seem to be displaying the right degree of pessimism in his approach. Mr Benjamin, not for the first time, displayed a certain messianic quality in what he said. I realize that he feels very strongly and I know some of the reasons; he has come to a view which is different from that of others, and he has failed to convince them of the suitability of the model which he has chosen. I agree with Mr Wilkie's comments on expected values and I also agree with his remarks about risk theory. I am happy to accept the criticism of a trend line, which I do not regard as a model of stock market behaviour at all; but, again, in the absence of an agreed model, we still have to try to answer the question posed by the uncertainty of future events. I agree, too,
with Mr Wilkie in his criticism of the possibility of an investment solution to this; Mr Taylor was very pessimistic, I think rightly so. There are a number of situations which could develop where it would be impossible to meet the liabilities. I am led to the conclusion to which Mr Turvey was tending, that possibly we ought not to issue unit-linked contracts with maturity guarantees.

I am not sure what Mr Fagan was proposing; I think it is a little bit too simple to be true. In his gambling example, he introduced three people, whereas we are discussing the company and the insured. I am not quite sure where the third player is; perhaps if I have the opportunity of reading his paper it might become clear to me. Mr Ryan and Mr Wilson echoed my own doubts as to the random walk model and the limitation of mathematical models for this purpose. Mr Squires made one remark which worried me a little; he said that a shortfall of reserves would need to be made up. Where would the money come from? That is the least part of the problem. Mr Johnston needs a practical solution. He thought that we were making progress; strangely enough, as a result of this discussion, so do I. Finally, Mr Barrow, below the acerbity sensed consensus; I hope he is right.

WRITTEN CONTRIBUTIONS

MR G. E. BARROW: I had to cut out two points from my oral contribution because of the pressure of time.

First, as a matter of general semantics, there is a danger in the use of the adjective 'actuarial' with insufficient precision. We are accustomed to submitting reports prepared in accordance with received actuarial principles. Such reports are commonly designated 'actuarial reports'. There are, however, occasions when, as actuaries, we are asked to report on some subject where there is no such received body of opinion, such, as in connexion with non-life insurance. In submitting such a report most of us would state our premises, develop our argument and state our conclusions with suitable qualifications. Such a report would properly be described as an 'actuary's report', but I think there is a danger in describing it as an 'actuarial report', though grammatically the adjective is correctly derived. There is essentially an imprecision of terminology. My specific point is that there is a danger of designating contingency reserves which have been developed from models by the description 'actuarial'.

My second point is in connexion with the use of models. On reading Mr Corby's notes I thought that the use of models had been put into its proper perspective. After listening to the discussion I am no longer sure that this is so and I would like to venture upon an analogy.

In some parts of the Pyrenees torrents issue from the mountain side having a curiously cyclical rate of flow. It is fairly easy to deduce that this must be caused by the operation of some form of natural syphon, a system of caverns and connecting passages within the mountain. It would be possible by careful observation to develop a hypothetical model of that system which would counterfeit with a remarkable degree of accuracy the observed characteristics of the natural phenomenon. That model could be a most powerful tool; it could provide a great deal of insight into the natural process; it might even give advance warning of potential dangers. But ultimately the model is only a hypothesis and danger arises when, for those who have devised it, the model becomes the reality and they fail to recognize its limitations. Such, for example, in my analogy would be the possibility that, in certain extreme circumstances, another stream might overflow into the system one was studying causing a total change of characteristics.

Before we can place our reliance on analogues I think we must be sure that they are analogues of closed, not open, systems and that there is no possibility of some external influence affecting the validity of the model. We do not know enough of the circumstances which affect stock market prices to be certain that it is a closed system; hence we cannot have sufficient confidence in any model of the pricing mechanism of the Stock Exchange to justify determining our reserves by reference to the model alone. However, I do recognize the value of models in calculating the expected values of maturity guarantees and hence in the derivation of equitable...
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premiums, in demonstrating the possible cost of meeting such guarantees, and in revealing latent and unsuspected instability in models based on assumptions which seem in no way improbable.

Professor R. E. Beard: This note updates some work I did at the time of the meeting in December 1971. My remarks are a précis of this longer study to which reference should be made for justification of the comments in the following.

The central feature underlying these contracts is the basic time series of movements in a reinvested index. Before developing any calculations it is essential to obtain as much information as possible of the structure of the series and it is in this direction that much of my earlier work was slanted. The most important feature of the series is that there are important correlations between successive observations. The normal analysis in such circumstances is the calculation of a correlogram and to examine the behaviour to try and find some indication of the genesis of the variations. In this case my calculations suggested that the lag correlations exhibited a marked cyclical behaviour, without damping. The period was about 4½ years, the sort of value found in economic time series.

Both the period and amplitude of the cyclic component would be random variables but the existence of a cyclic component would invalidate any calculations which ignored its presence. It may be reasonable to assume that the spread of business in time and original term would mean that calculations could be based on the overall mean, with allowance for the spread, but this overlooks the fact that the contract term would not be an integral number of cycles and there could be serious option situations.

This means that the underlying model requires a triple tabulation, i.e. date of entry, original term and date of valuation. At each future date there is an expected index level determined by the cyclic component, plus of course trend and other components, and the maturity strain has to be measured against this model value. If the spread of values around the expected index is known then the probable cost can be found.

It appears that the series is not homogeneous over the period from 1919. Whether it is possible to identify the reasons for the changes in structure which appear to have occurred around 1930 and 1950 is speculative, but I hope I have written enough to show that much more needs to be done on this problem. The qualitative consequences of a cyclic component can be considered as a problem in its own right but it is hard to escape the conclusion that a triple tabulation will be needed.

The first question I set myself in 1971 was to find a theoretical distribution of maturity values based on the assumption that the series of annual changes, including reinvestment, were random and independent. The results of this, which have been utilized by Messrs Wilkie and Scott in their respective papers, showed that it was practicable to derive the moments and also that the results were consistent with the simulation calculations made by Mr Benjamin. The next experiment was to calculate the single premium maturity values from the theoretical model and compare these with values derived from the annual series. It soon became clear that the observed means for terms greater than 4 years were progressively less than the theoretical as the term increased but a more important observation was that the observed standard deviations were also less than the theoretical values, the value for term 10 years being about one-half of the calculated figure.

The interpretation of these differences was that the annual movements were not independent. Accordingly a correlogram was calculated giving the correlations at successive lags and from these values the theoretical formulae for the expected means and standard deviations were derived and the resulting values reproduced the actual values, confirming quantitatively the lack of independence. It was not practicable to derive the higher moments of the maturity distribution allowing for the correlations but approximate values of the skewness and kurtosis derived from the data suggested that the distribution could well be a limited range Beta distribution with a moderate skewness. This was tested against the simulation values derived by Meldrum in 1971 and a satisfactory agreement was found. One very important feature was
that for a 10-year term the maturity value was never below unity. Intuitively this threw doubts on the adequacy of this model.

The next stage was to examine the correlogram to see if there were any clues to the structure of the series. Although the series consisted of 51 terms, only the coefficients were calculated up to a lag of 34 years because the shorter lags seemed to indicate a cyclic behaviour. The whole series continued to show cyclic behaviour with no indication of damping, the cycle having a period of about 4½ years the interpretation of this feature being that the original series contained a cyclic component with this period. It will be remembered that these experiments were made on the series up to 1970.

The original series and the series derived from successive products were then subjected to ‘classical’ harmonic analysis techniques to find more about the cyclic behaviour. There were repeated indications of cycles of about 4½ and 9 years. However, an attempt to fit the whole series with periodic terms was disappointing as the reduction in variance was only about 25%. Accordingly a systematic study of the series was made in the light of these findings. This led to the suggestion that there were three basic subsections. The first was from 1919 to 1929 which could be interpreted as a damped harmonic with a term of about 2 years. The second from 1930 to about 1948 with a damped harmonic of period about 9 years, but with a ripple of term 4½ years; the final period showed a cycle of about 4½ years; there was some evidence that the means of the three periods were different.

Since these studies were made there has been the remarkable fluctuations in the index with the low in 1975. Perhaps the most important observation is that the series has continued the cyclic behaviour with a period of about 4½ years. A re-examination in the light of this recent behaviour suggests that the third period really consists of two sub-periods from 1953 to 1960 and from 1961 to 1977. (The original second period could reasonably be extended to 1952). The first of these sub-periods could be regarded as an undamped cycle of 4½ years and the second a cycle of similar length but with an increasing amplitude, culminating in the 1975 low and 1976 high. One other relevant observation was that the mean could be regarded as having a long-term cyclic period of about 36 years. Calculation of the lag correlation showed a very high value at 35 years and a slightly lower value at 36 years.

Although it would be desirable to recalculate the correlogram based on the additional data it would appear unlikely that the above observations would be materially changed. Of more importance would be the calculation of the spectrogram, and the modern technique for identifying behaviour.

The evidence from these studies is that the basic series can probably be regarded as made up of a mean value with a long cycle of about 36 years with fluctuations based on a cycle of about 4½ years and a random component. The 4½-year cycle seems very persistent but the amplitude changes from time to time, roughly at the ends of the sub-series. From 1929 the amplitude decreased but from about 1961 the amplitude increased.

If we are to try and use this information as a base for premium calculation and for valuation purposes some attempt must be made to find a rational basis for the features found. The existence of cyclic behaviour has long been known in economic series and can be derived from the difference equations which can be used to define the dynamics of the systems. It would be surprising if such cycles did not emerge in financial series, particularly when attempts are made to manage the economy. It is possible that the basic underlying dynamics generate the cycle of 4½ years but the ‘corrective’ adjustments made from time to time, particularly as they may be made against differing political backgrounds lead to discontinuous changes, some of which lead to converging and some to diverging amplitudes.

This reasoning suggests that a possible model would be to assume that there is an underlying cycle, which will, of course, be a deterministic series tied to calendar time, but with the amplitude following some form of auto-regressive series based upon random ‘shocks’. If this could be shown to be a reasonable representation of past history it would then be possible to build a probabilistic model for projection purposes.

This would then lead to a model in which we would project the deterministic components, namely the trend in the mean and the cyclic component with the amplitude of the latter
defined by a probability distribution. This might well be equivalent to the deterministic components plus a random element with a defined distribution.

All of which suggests that the first stage in resolving the problem is to consider the problems of premium determination and valuation on the assumption that the underlying model is based on a cyclic series.

For simplicity we may first restrict discussion to the single premium case and will assume that the mean is constant. The maturity proceeds will then be the product of the \( n \)th power of the mean, where \( n \) is the term and the cyclic component. If \( n \) coincides with an integral number of complete cycles the multiplier is unity. If an incomplete number of cycles is involved the expected value will be greater or less than \( mn^n \), the value depending on where in the cycle the case entered. With a random distribution of cases in time the deviations would balance out, but it must be noted that the flow of business will probably not be independent of the state of the market so that an option situation may exist which should be allowed for.

So far as the guarantee is concerned the cost should be measured against the expected value (including the cyclic component) with a suitable multiple of the standard deviation of the distribution of the random errors. Mr Benjamin assumed all variation was random; if the variations are separated into cyclic and random components the overall spread would appear to be larger but it will be appreciated that this ignores the spread in time from the cyclic component. Mr Meldrum ignored the cyclic component and the resulting maturity spread, in effect, ignores the time relationship. Dr Scott has compromised by selecting a variance between Benjamin and Meldrum and introduced a ‘maximum’ measuring point which can be regarded as a sort of compromise which does not respect the time variation from the cycle.

If the cyclic component is ignored it appears extremely unlikely that there is an appreciable expected cost for the maturity guarantee but this assumes that the expectation is taken over time and term so that the cyclical effect balances out. In so far as premium rates should in theory be a function of the calendar date of entry this assumption can be called in question.

There remains the loading which is needed to service the capital required to finance fluctuations. It will be assumed that the expected values of the relevant index have been calculated at all future dates and will consist of the ‘deterministic’ components of the trended mean and the cyclic components based on an average amplitude but with a fixed phase. The amplitude will probably be relatively large, i.e. 20% or 30%. There will also be a distribution of random components about a mean of zero with a known standard deviation but with a distribution which may be skew. Thus at any future date we will have an index with a probability distribution for the spread.

The portfolio of contracts can be classified according to the date of maturity but the guarantee situation will depend on the actual date of entry since the relative proceeds will depend on the position within the cycle, or more precisely, on the relation between the original term and the phase of the cyclic component. If we ignore the complication that the behaviour of the series between the date of entry and valuation will differ from the model then the amount of the guarantee (and its probability distribution) can be found for the whole portfolio. This will be a series of amounts at various future dates and the sum required to generate this series, which will include the margin to cover random fluctuations, can be found by some form of discount procedure. In this latter regard, since the series of payments is known a portfolio of redeemable securities, suitably matched, it will provide a basis which reflects the market situation and opinion of future interest movements and will avoid the problem of selection of a discount rate.

The dual tabulation implied by the foregoing provides an immediate view of the points of time when maturity strains might arise. This is a very necessary piece of information because the flow of business and its constitution as regards term will have fluctuated from time to time and it would not be satisfactory to assume uniformity because of the incidence of the options referred to earlier.

The simplification referred to earlier in regard to the performance of the index between date of entry and valuation date must be dealt with. At the valuation date the position of the index in the cycle will have to be defined. It would seem unlikely in terms of the overall situation that
very high precision is needed in placing the precise position in the cycle but it would be desirable to eliminate as much subjective judgment as possible so that extraneous fluctuations are not introduced. Once the position in the cycle has been fixed the maturity proceeds are determined and the guarantee strains can be found if the guarantee amounts are tabulated according to the maturity dates.

Unfortunately it is impossible to know if there are practical snags. The numerical analysis has been based on the performance of a special index and it would be necessary to analyse other practical series to see that they conformed to a similar pattern. It would be expected that there would be similar features but with some trend features reflecting the particular pattern of the various portfolios. The statistical evidence from comparative studies of different trusts suggests that portfolio selection may not be as effective as the objects would imply so that the underlying cyclic features will be in evidence.

The foregoing considerations have been largely based on single premium contracts. The extension to annual contracts is straightforward if complicated since the theoretical maturity proceeds can be found. Once the expected pattern of prices has been set future proceeds can be calculated and the maturity strains found provided the guarantee amounts at maturity are tabulated.

Whether the development of a whole technique based on the assumption of a cyclic component is unnecessarily complicated is a question that might be settled by some simulation studies of actual distributions of business. The practical difficulty arising from the introduction of a third variable into valuation methods is probably not relevant now that computers are available to process the data. Our attitudes are still conditioned by the conditions that when data processing was by hand it was inconvenient and expensive to work with more than two basic variables. The essential point with unit-linked business is that the guarantee amount is dependent on the calendar date as well as date of entry and original term. The cost depends on the assumptions made regarding the behaviour of stock exchange prices in the future. The problem is to prescribe a pattern which is consistent with assumptions regarding the continuation of a mixed economy. Past experience indicates that a distinguishing characteristic is the auto-regressive nature of economic series and it does not seem unreasonable to assume this will continue.

Mr P. D. Smith: Although valuation of maturity guarantees by expected values is unrealistic, the alternative methods seem to cause far more problems than they solve. In determining any quantity like 'Likely Minimum Investment Proceeds' the assumed value of the standard deviation of yearly price movements is absolutely crucial. For illustrative purposes Dr Scott used a value of 1 in his paper to the Faculty. With dividends reinvested net of tax at 37.5% the de Zoete equity index has exhibited a value of ·35 over the last 30 years. If the last 59 years are taken this only declines to ·28. If the 1970s with the slump and recovery are excluded one has the value of ·19 used by Mr Benjamin 5 years ago. However ignoring the most recent past because it is inconvenient would destroy any claim of realism. This is especially true since the recent past is a demonstration of precisely the performance we are trying to model. Dr Scott used a deliberately low standard deviation to compensate for his model assuming independent year to year price movements. He did not however justify his choice of so crucial a parameter, or demonstrate the effects of reasonable variations in it. Before any attention is paid to the numerical results of a complex model the assumptions should be critically examined, and variations in them tested.

Returning to the value of ·35, the Scott model now requires reserving against the possibility of investing ·9 for 10 years, and ending up with proceeds of only 2·2. Required initial reserves are now about five times annual premium for a 10-year contract.

In practice a managed fund is not 100% equities. An imaginary fund has been created which maintains a 50/50 mixture of de Zoete equities and cash. Net income is reinvested. This enables the initial reserves to be reduced by just over one-third. The reduction is remarkably independent of term of contract. A managed fund probably lies between the two. The problem is how to choose a single value from a wide range.
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The Scott/Benjamin random walk approach assumes no centralizing tendency after abnormal price fluctuations. The other extreme is to take independent lognormal fluctuations from an exponential trend line. Deriving parameters from the 50/50 fund mentioned above initial reserves can be cut by about 50% for a 10-year contract. For longer terms far larger reductions are possible. The point is that when the random walk and trend line methods are applied to the same model contract, using the same ruin probability of ·5%, and with parameters derived unadjusted from the performance of the same model fund, the results differ. Even at the closest there was a factor of 2 between comparable answers. The problem again is which answer to choose from the range.

The various papers written on maturity guarantees are good as starting points for discussion. It seems, however, that the more complex models become, the less reliable they are for practical use. Any further complexity will probably not be worthwhile.

No method, yet comes close to giving a valuation formula which is both theoretically devised and practically acceptable. There is a wide range of plausible answers. Choice of a single value is less scientific than arbitrary.

The author: As I said in my opening remarks, any form of extrapolation without a basic understanding of the underlying processes is dangerous. This is a fundamental problem which many of us face in respect of Mr Benjamin's proposals. He has not managed to convince me—nor clearly others also—that his model is suitable to apply to stock market behaviour in the future and it is not an acceptable method of argument to contend, as he appears to do, that until he has been convinced that his ideas are wrong or that some other approach is better his conclusions must be accepted by all.

The President rightly reminded us in the course of his remarks how difficult we are finding it to establish valuation regulations for traditional business with which we have been dealing for 200 years. In the light of this our problems with maturity guarantees are more understandable. The writing of these guarantees must be regarded as more akin to non-life business and here, despite the theoretical work done in ASTIN and elsewhere, the required solvency margin is determined on a more or less arbitrary basis—based on experience—and the attitude of companies writing non-life business clearly indicates that for continuing operations they intend to hold free reserves in excess—perhaps considerably in excess—of the statutory minimum. This shows a properly cautious view in a state of uncertainty and it is clearly incumbent on actuaries in reserving for maturity guarantees to adopt a similarly cautious approach.

Such an approach did not appear to find favour at the 1971 discussion but from the remarks in the present discussion it is clear that it is now shared by the majority. That is a step forward and, together with the rejection of the expected value approach to reserving, points the way to further progress. What is now needed is further research of the 'pure' variety, that is without the constraint of trying to find an answer to apply almost immediately within the context of the forthcoming valuation regulations. For the latter, there seems little alternative in the short run to an arbitrary basis based on premiums unless the Department of Trade were to agree to an approach taking into account the factors suggested at the end of § 23 subject of course to full disclosure of the methods used and the assumptions made. Some actuaries have tried to handle the problem of reserving by a method which is very similar to emerging cost methods and it may be that this is worth further development. At the very least such a method would demonstrate clearly the consequences of different assumptions as to prices from year to year even if at the same time they earned the rebuke of malevolent determinism from Mr Wilkie.