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INTERNATIONAL ACTUARIAL NOTATION

The Sessional Meeting held on 26 April 1976 was devoted to a discussion on International Actuarial Notation. The twelfth edition of "ANZ-75", prepared by the Notation Sub-Committee of the Institute of Actuaries of Australia and New Zealand, and "Thoughts on the Harmonization of Some Proposals for a New International Actuarial Notation" (*Blätter*, **12**, 99) had been made available as a basis for discussion.

Mr J. D. U. Harsant, in opening the discussion, said that to a large extent the work which had been carried out on actuarial notation in linear form was summarized in the two proposals made available before the meeting. He would refer to those published in *Blätter*, as the 'continental' proposals, as there was no formal recognition of those proposals in the United Kingdom. Many of the recommendations would appear simple and obvious to all, but he knew from his own experience that many of the simplest things produced came paradoxically only after a great deal of work and that had been the case with those papers.

Mr Harsant gave a brief description of the proposals, indicating some of their salient features. His first example was the notation for an annuity as follows:

IAN $\quad {}^{\mathrm{T},i}_{m}\%a^{k}_{x\overline{n}|}$

Continental $\quad (k)(m:n)\ a\ (x)\ (i;\mathrm{T})$

ANZ-75 $\quad a(x,f=k,q=\mathrm{T},i=i,\#m+n,b=\#m)$

The fundamental differences between the two proposals would be seen at once. The 'continental' relied on the position of the parameters and the use of punctuation and was thus very similar to the IAN. ANZ-75 relied on 'key letters'; that was, each parameter was defined by a letter and given a numerical equivalent. He thought that the 'continental' was preferable; for although it was not so compact as the IAN it followed the same pattern, it did not introduce unnecessary items, and the alphabetic characters could be substituted by numerical values without ambiguity.

The approach to multiple life functions was very different in that the 'continental' approach was very general and could be used for a large number of lives, whilst the ANZ-75 approach, although it did not rule out functions on many lives, concentrated on a particular device which produced a notation which was very descriptive of the event. He gave examples to illustrate the differences of approach.

Continental

Status: exactly V out of W persons were dead

$$a(u3:x1,\ x2,\ \ldots\ x5:m)$$

was a temporary annuity paid while exactly 3 out of 5 lives were dead.

$$a(23\ u4:x1,\ x2,\ \ldots\ x5)$$

was an annuity paid while exactly 4 out of 5 lives were dead, ($x1$) having died second and ($x2$) third. That was a very general approach and could be extended to cover many particular situations:

Examples of assurance functions follow:

	IAN	*ANZ-75*	*Continental*
Last survivor whole-life assurance	$A_{\overline{xyz}}$	$A(x \,\&\, y \,\&\, z)$	$A'(x:x, y, z)$
Contingent assurance on death of x before y	A^{1}_{ny}	$A\,(x<y)$	$A'((r:x), y)$
Assurance on death of x after y and before z	$A^{2}_{\underset{1}{xyz}}$	$A\,(x>y \,\&\, <z)$	$A'((r:y, x), z)$

The difference in approach was very wide: the continental approach was general and in the proposals developed to a considerable extent, whilst the ANZ-75 proposal concentrated on functions in general use in a form whose meaning was immediately obvious. He preferred ANZ-75 for multiple life functions limited (in any IAN) to at most three lives.

That illustrated well one feature of a new notation which had been completely overlooked. Was the notation to cover every possible function which might be met or was the formal IAN to be restricted to a very limited group of functions in general use? His view was that actuarial practice was so varied that only the very basic functions admitted to an international notation and that they had suffered in the past from a lack of any clear-cut ideas of where an international notation could be of value and where it was not of value and local variations were appropriate.

Having described some of the most significant features of the proposed linear notations, it was worth asking the question: "Why change?" Three reasons were put forward: ease of typing and printing, the extension to other areas such as pension funds, and application to computer use. The second point had not been considered in depth previously and in no way added to the case for change. The application to computers was not relevant because the proposed adaptations from the linear forms were very little different from those which were needed from the current IAN.

Why did ease of typing justify a change from the existing notation? It had been used for many years, was understood by all and formed the basis for the current literature, substitution of numerical values for alphabetic characters brought no confusion and the notation was compact and each function could be understood at a glance.

His view was that as far as the examples he had given earlier went there would not be a great deal of loss in changing from the existing form of notation to the continental proposals for the single life functions and to the ANZ-75 proposals for multiple life functions, subject to adaptation as appropriate.

The profession was only just beginning to use computers properly. Much of the actuarial textbooks could be written as computer programs, thus leaving to the computer all the routine decisions. He questioned, therefore, whether in future many of the present functions would continue to be used. Certainly some of them would be implied inside computer programs, but for the purpose of writing those programs those functions would have to be specified by their formulae and not by their symbols. The stage was rapidly being reached where actuarial work on computers was a form of matrix arithmetic. He wanted to answer the question already touched upon: "Which functions were included in the notation and which were to be omitted?" Only the basic functions should be included, those for everyday use. Complicated ones which were either used infrequently or only by one or two offices or were subject to change should not form part of an IAN.

It was relevant to consider what was going to happen at the International Actuarial Congress at Tokyo, where undoubtedly the two proposed forms of notation would be debated with a view to replacing the existing IAN. The IAA had published a questionnaire to all national actuarial bodies seeking their views on the future of actuarial notation. He could sympathize with the reasons for changing the notation and it would be foolish to deny that the proposals, modified in the way he had indicated, were reasonable. No one wanted anything to be changed simply for the sake of making a change and he had very considerable reservations as to whether any forms of notation, either the current IAN or one of those proposed would in fact prove suitable in the long term having regard to all the changes which he believed would occur with the increasing use of computers.

Professor Dr Engelfriet (a visitor) said that the members of the small committee which published some thoughts on harmonization of some of the proposals for a new actuarial notation had felt very honoured by the fact that the Council of the Institute had chosen their paper as one of the two papers to be discussed. He emphasized the fact that it was the product of a closely co-operating team. It might be that some took the initiative of writing thoughts more than others. The extensive discussions in their meetings were essential in the process of arriving at a result which at least in their eyes was reasonably coherent.

What sense was there in taking so much trouble to analyse the possibilities of a new notation? It was known that a change in the present notation would meet great opposition, as had already been heard, from many actuaries. Some answers by the actuarial societies to the questionnaire had not given an indication of overwhelming enthusiasm for linearization. That lack of enthusiasm would not be stimulated by the fact that different proposals—perhaps even antagonistic ones—were submitted for the judgment of the actuarial world.

His answer to the question "Why so much trouble?" was "Curiosity"; curiosity about the feasibility of linearization, from which followed deep

probing into the problem. He was in favour of linearization and would answer the opener, not for the use of computers, not for typesetting for papers or for textbooks, but especially for typing. Lectures and exercises could be adapted much more frequently. Linearization would lighten those tasks and he was in favour of linearization in that sense only.

Another question was the purpose of developing a coherent and more or less generalized notation. It was not only for giving the actuary a tool in his daily, practical work. The present notation gave much more than was immediately needed in practice, and not without reason. There was a difference between the needs of daily work and those of tuition and training of actuarial students. To give them an insight into a more simple formula and make memorizing more easy, it was often preferable to give the proof of a generalized formula from which all the simple specific formulae could be derived. That was a method which had been extremely useful in all branches of science, and in his experience it worked well in actuarial theory. The thoughts on harmonization were related to papers of Boehm and others on one side, and from Kool and himself on the other. From the Boehm paper was derived, in a somewhat altered form, the basic idea of the so-called structured parameter list which had been demonstrated by the opener, giving a linearization as close as possible to the two-dimensional parameter location in the current notation. From Kool and his paper was derived the idea of consequently handling the concept of status of an insurance as a basic principle, payment of an annuity as soon as and as long as an insurance was in a given status, payment of the sum assured in case of transition from one status into another or on entering into a given status or exit from a status.

Consequent handling of this principle gave the possibility of covering with the same technique insurances from different fields: single and multi-life annuities and life assurances; disability or sickness insurance, distinguishing total from partial disability. It proved to be very useful to derive general formulae for a generalized insurance which might be in one of a given number of distinct statuses and which might be further defined by a matrix indicating which transitions from one status into another were possible and which were not.

For single life functions the concept of status was handled implicitly by the current notation in the same way. For multi-life annuities status was defined by the given or minimum number of survivors and, possibly, by a description of the series of events to take place before the given number was attained. It was different for life assurances in the current notation. There the notation gave a description of the series of events which had to take place, starting from the initial status where all insured lives were alive, up to the last event, decisive for the moment of payment, which event was marked in a special way. The curious fact was that such a series of events defined a status, the status into which the insurance entered after the last event. One could define, for instance, an annuity to be paid from the moment the insurance entered into that status. The simplest example was the assurance of a sum to be paid if x died before y and the corres-

ponding annuity, for instance, the widow's pension. The implicitly defined status in the assurance was coded, however, in a different way from the coding of the status for the annuity.

In their paper published in *Blätter* Professor Engelfriet and his fellow authors had avoided the difference in coding of one and the same status, not because it was ambiguous, which it was not, but because it was simpler to code the implicitly defined and explicitly defined status in one and the same way and it offered the possibility of generalizing and coding in a simple way for a broad group of insurances the well-known relation:

$$A_x = 1 - d\ddot{a}_x$$

which could be proved easily for the most generalized insurance and then held for all special cases.

The description of status had been given as much as possible in conformity with the system applied in the current notation for annuities with a slight extension. Some tests with actuarial students had shown that the notation could be easily memorized.

The ANZ-75 approach was exactly the opposite of that of his committee. Status was defined implicitly by a preceding series of events (which might be empty or consist of one event) and a subsequent event, the preceding series (unless empty) were indicated by $b = \ldots$ That led to the following consequences which could be demonstrated with a widow's pension. If the annuity was to be paid to y after the death of x the preceding event was coded by $b = x$. The subsequent event was coded by simply putting y in the first place. But in the corresponding survivor assurance, the preceding event (x died before y) had to be coded in a different way and also the subsequent event (y died) as second. That had to be coded differently.

He admired the ingenious way that the formulae had been worked out in many special cases, but the basic concept was open to criticism. Another objection was that all multi-life probabilities could be based on the application of two basic formulae, one giving the probability that out of an initial group of insured lives all died within a given period in a fully specified order, and the other that all the lives would survive after a period.

His committee's system made it easy to see how the formulae for more complicated probabilities could be derived from the basic ones. He had not found such a systematic approach in ANZ-75, but that was no proof that it might not be possible there. Other differences might be purely questions of personal preference.

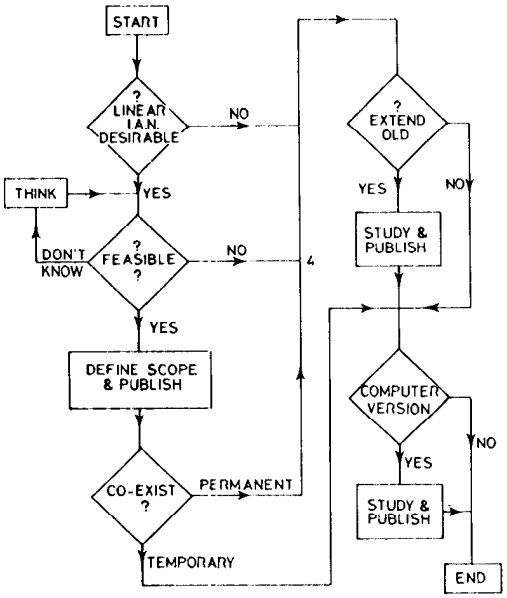
The opener made a proposal of combining some techniques of the multi-life functions of ANZ-75 with the single life techniques of the 'continental' report, but that was very difficult. At a meeting of the Actuarial Society in Germany Dr Sachs had made the suggestion that the IAN should be restricted to functions on one life only, and the notation for other functions should be explained at the beginning of each paper. The difficulty was—and it was the

same for the opener's proposal—that the choice of a certain notation for multi-life functions defined to some extent the notation for single life functions, since duration may be handled as a 'life'.

If Dr Sachs's suggestion were to be followed he would be for total freedom. It would always mean that only single life functions would occur in whatever notation in any paper, with the exception perhaps of Chinese characters. If a reform of the IAN were agreed upon, then it would be impossible to make a choice from the different proposals. Had complete freedom regarding notation to be rejected?

Mr P. J. Turvey wanted to rephrase the reasons for the discussion as he saw them: first, to inform the members of the Institute of the stage of play; second, to provide a forum for the expression of opinions; third, and by no means least, to brief those attending the meeting in Tokyo.

Members at the meeting had before them a copy of the questionnaire issued by the International Actuarial Association. A consideration of the flow-chart shown might help to highlight the points involved.



The first box of the flow-chart after the start was the question of whether a linear version of the actuarial notation was desirable. As the opener had pointed out, the arguments in favour of a linear version included the ease of typesetting, the ease of conversion to 'computerese' and ease of extension. Arguments against

were that the existing notation was adequate and familiar, that the typesetting difficulties described were in any case exaggerated and that the problems of conversion would be too severe.

The second decision box questioned whether it was possible to construct a linear version of the notation which the profession would find acceptable. The pessimistic answer was 'No' on the grounds that the efforts made so far were discouraging and that there was no solution in sight. The optimists answered 'Yes' on the grounds that substantial progress had been made in the past 10 years and they could see the end in sight. There was also a 'Don't know' route, where the logical conclusion was to refer the matter back to the experts for further study. To help them make up their minds in that case experimentation with possible new notations in papers about subjects other than notation might be allowed, but he felt that is would only lead to confusion. However, the 'Think' box was an important option.

If a linear notation was both desirable and feasible, the path continued down the left-hand line of the flow-chart. If a linear notation was rejected the path went to the right and whether or not the existing notation needed to be extended to cover areas at present not covered, in particular pension funds had to be considered, that decision was very much a matter of taste.

Following the left-hand route, if a linear notation was to be adopted, then the areas to be included had to be defined. The flow-chart was simplified: it would be more logical to define the areas of extension before publishing the complete notation; but in practice it would probably be desirable to publish first only those areas of the new notation at present covered, and then later a second edition defining the extensions.

How long ought the existing notation be retained? The principal options were either permanent co-existence or a deliberate phasing out. The case for permanent retention was that that might make a consensus possible; if the new linear notation did prove to be more practical, then it would quickly exert its superiority. Transitional arrangements of some kind would be necessary if there were a deliberate phasing out, and students would continue to need to be familiar with the old notation for some time. If permanent co-existence was decided then the question of whether to extend the existing notation had to be considered. Otherwise the path went straight to the last decision box on the flow-chart, which was common to all lines of action, whether or not to develop a computer-compatible version of the notation. The use of that notation would be to establish a standard self-documenting method of communicating not only with the computer, but also with other computer users. Such a notation could be used not only in actual computer programs, but also in conversational mode on computer terminals which were seen more and more in actuarial offices.

The computer-compatible version of the notation should be as close as possible to the published version, and obtainable from it by some simple algorithm. The 'No' line would be taken by those who were prepared to wait for a

major increase in computing power, and by those who thought that actuarial computing was too far committed to existing *ad hoc* usages. He suggested that every decision box needed to be considered, even if it did not appear on the preferred path.

Currently there were still many actuaries who had no access to computers: of those with access, the vast majority were restricted to the FORTRAN-type languages, and only a small minority had APL or other highly sophisticated tools. Therefore, in planning links with the computer the level of technology likely to be available to the majority of the profession during the rest of the century should be considered. He predicted that the majority would still be limited to the FORTRAN-type languages, even though some sophisticated technology might be available to the lucky few. When he had referred earlier to a computer-compatible notation, he had in mind unique function names and fixed-length parameter strings.

Turning to the main question of whether to adopt a linear notation, the speaker said he was a 'Don't know' at present, because he did not think that either of the notations considered by the meeting was acceptable. He agreed with the opener that a solution could well be found in a merger between the continental and Australian approaches: but he differed from him on two points.

First, pensions should be at least retained if not extended. There would be a need for a notation to document programs, to communicate ideas, to teach students, to deal with exceptions and to develop theoretical treatments. He was horrified by the thought that actuaries of the future might adopt a 'black box' approach to pension fund valuation and costing.

Second, he would not limit the notation to an arbitrary number of lives such as three, provided that could be done without detriment to the average working actuary: the needs of the practitioner should not be overlooked in order to satisfy the theorists. The Australian concept was of complete generality, even if somewhat cumbersome. The theoretical notation should be free to deal with complex multi-life statuses, and it might even be to the ultimate advantage of the practising actuary with his single-life assurances.

Mr G. B. Hey realized that what the various authors were saying was: "If you want a linear notation this is what you are going to have to put up with." He made a confident prediction about linear notations: in 10 years' time they would be forgotten about, and would not be missed.

The most important purpose of a notation was to convey information quickly and clearly; the present notation did that within its limitations. He agreed that it could usefully be extended at least to pension funds. But all the proposals before the meeting failed lamentably. To convey information, the following was clear:

$$a^2 + b^2$$

but if they made it linear it was not as good:

$$a**2+b**2$$

and if they moved it one further, it was worst still:

a squared plus *b* squared

The present proposals were hopelessly complicated with little hope of simplification and were simply not needed. They did not represent the best way to talk to a computer, although they might have done some 10 years previously. All that the computer was asked to do was to calculate whether at each of a series of intervals a payment was to be made to not. That was determined by a simple decision table, for example:

x dead	yes	yes	yes	no
y dead	no	yes	either	either
z dead	no	no	yes	either
ACTION	—	PAY	—	—

Those decision tables were easily converted into COBOL or, if necessary, any other high-level language by an existing pre-compiler. The prospect of converting even the limited range in *Blätter* to machine-independent computer form was quite daunting.

In many fields, especially pension funds, it was the amount of the payment or the condition under which it was paid that was often the hardest to calculate not the probability that it would need to be made. The proposals before the meeting, complicated as they were, only partly coped with the probability statement and went virtually nowhere to deal with conditional payments (as in ill-health pensions), variable amounts on fluctuating discount rates, and so on. The proper way to do this was set out in an earlier paper (*J.I.A.* **101**, 299).

The computer could code the English, it could recognize the word ASSURANCE or ASSUR just as easily as A or AS, and it would be easy to produce a compiler to convert statements such as the following into code, and much simpler and safer for the actuary.

benefits	ASSURANCE	ANNUITY
payable	DEATH OF 30, A49-52	30 ALIVE, a(55)
within (periods)	30	420†
if	40 LIVING, a(55)	65 DEAD, ELT 12
and	37 DEAD, ELT 12	‡
frequency per year	1	12
in advance	No	Yes
complete	No	No
interest	·11	·14

† That is deferred from age 65 (420 = (65−30) × 12).

‡ Could be "still married to 27" on a suitable table for age and marital status.

There was no need for linear expressions to help with computers and effort would be better spent in learning to use the machines properly.

So far as printing was concerned, the case had been grossly overstated. Most communication was in manuscript between students, tutors and examiners, or in office instructions and forms. Printing in type was trivial by comparison and modern typesetting methods were rapidly overcoming any real difficulty there.

Mr G. T. Humphrey said that the two proposed notations under consideration were primarily conversions to linear form of the existing notation. That notation was developed to meet the needs of late nineteenth-century life offices, and both the European and Australian groups had tried to cover all functions that could currently be expressed, together with a few obscure multi-life functions which the present notation did not express. Those were not the problems which a new notation should be designed to cover. On consideration of the relative amounts of time which the profession as a whole spent on problems involving variable benefits, and problems involving more than two lives, it would be seen that much of the two papers under discussion was devoted to functions of limited interest. A new notation which only covered life office and compound interest functions would not only be of little use, it would be positively damaging to later attempts to get international agreement on a more comprehensive notation.

The ideas underlying the Australian notation could be extended to produce a more comprehensive notation. However, he suggested a new basic approach, to move certain ideas from the pension fund section of the paper into a more central position.

Any assurance-type function could be expressed in the following form:

$$\int V(t) \cdot {}_tp_x \cdot \mu_{x+t} p(t) B(t) \cdot dt$$

where $V(t)$ was a discount function, μ^j_{x+t} was a force of exit from cause j, $p(t)$ was a probability of receiving the benefit, and $B(t)$ was the amount or value of the benefit. Annuity-type functions were similar, excluding the μ term. $p(t)$ could be the probability of survival of a different life; it could be a proportion married; it could be a sickness rate. The important point was that large classes of actuarial functions could be defined in that way. Looking at an actual function:

$$A^{3:4}_{\substack{xyzw \\ 21}}$$

could be written in an obvious modification of the Australian notation as:

$$A(y, p = (w, x, \sim y), B = A(x+t)).$$

That approach had two important advantages. It demonstrated the calculation method, the evaluation of a function was either done manually by approximate integration, or on a computer by an emerging cost program, both methods being of course the same, emerging costs were merely approximate integrations with annual intervals. Either way the calculation would follow the formula.

Also, the approach led naturally to a lot of useful functions that could not at present be expressed. Considering a widow's annuity, in the Australian proposals an annuity payable on the life of y beginning at the death of x was straightforward. An alternative way of expressing it was also in the Australian

notation: an assurance payable on the death of x and the benefit was an annuity to y at that time and multiplied by the probability that y survived until the death. It could be generalized so that, for example, $a(y+t)$ could be select from the date of death of x; it could contain a remarriage rate; it could escalate from the date of death and the initial amount of the annuity could be some function of t which might be a salary-based function. The point was that all of those could now be described by altering the B = clause. Similarly, where a proportion married rather than an existing wife was required, it could be written:

$$\mathrm{A}(x, p = h(x+t), \mathrm{B} = a(y+t)).$$

In general where a continuing benefit started on a change of status, for any deferred or reversionary benefit, the change of status often involved a discontinuity in some other variable. Examples were: selection, underlying mortality, possible causes of decrement, interest and escalation. A typical example was the family income policy. To use the expression:

$$\bar{a}_{\overline{n}|} - \bar{a}_{[x]:\overline{n}|}$$

did not help because a different interest rate was likely before and after death. The reversionary benefit might be defined for an individual retirement annuity, but for the wife assured life mortality would probably be used; once she became a widow then annuity mortality would be used.

Mr D. E. Purchase wondered whether actuaries really appreciated what was at stake. The designers of new notations were naturally enthusiastic for their changes. Those who were not so enthusiastic might not realize what was going on until it was too late. He was not speaking entirely from the outside because he also had designed a notation. He was sufficiently proud to think it was better than any other, but they would be relieved to hear that he was not going to publish it, or present it to them in any way, or suggest it should replace the existing IAN.

The opener had proposed three main questions, one of which related to extensions to other fields. The basic question was "Should the international notation be changed to a linear form?" He felt that the Institute should not support any such change. Leaving aside the computer aspects, the advantages of the change seemed to have been reduced to two, or perhaps three, following the discussion: economy in typesetting, comprehensibility in speech, and a theoretical soundness which improved development work. He did not really believe that comprehensibility in speech was obtainable for other than the simplest functions. Unless the functions were written down, in whatever notation they might be, they could not be remembered. The ease of typesetting seemed to be an advantage which was grossly exaggerated. There was a trend to consider the convenience of the author, one person, instead of that of the readers, many people. It was perhaps no coincidence that that trend was most marked when computer output was designed. Economy should always be borne in mind, but the more widespread

use of photographic reproduction and actuarial expressions written in manuscript was the commonsense and cheap solution.

On the other hand, there were many disadvantages of changing from the International notation. The main one was the hidden investment in the existing notation, as represented in the knowledge of present actuaries and in a large body of literature which would still be referred to for a long time to come. A new formal IAN would need to be changed completely, although that was not necessary for practical use. It would, however, be unsatisfactory to have a formal notation that could not be extended to the complicated multiple-life functions that some of the speakers had delighted in presenting to the meeting. Also none of the proposals which had been put before the meeting were directly usable on computers without modification, and there was a danger that they would finish up with three notations instead of two.

It was possible that they were considering change in an attempt to solve a relatively short-term problem. If a change had to be made to one of the two papers that had been presented to the meeting he would support the ANZ-75 proposals which he found more readily understandable than the continental ones which had subtlety hidden in punctuation and order of parameters. He regretted that they could (or should) not have a linear notation, because in regard to functions on more than one life he found the ANZ proposals easier to understand than the existing IAN.

The stage had been reached where there was sufficient experience so that all did not need to set to work to devise their own notation, for lack of anything else. He saw the need as being for an interim programming notation which could be used by that substantial minority of actuaries working in the field. It should not be given full international status because of the likely development of computer techniques. The advantage was that there could be experiment, refinement and expansion and new versions could be published frequently. The constraints to which such a notation should conform were well set out in a paper by Mr Turvey, but there was one constraint which he wanted to mention: any such notation should be capable of use with the generally available high level languages rather than needing a special preprocessing technique which only a few people would have available.

Mr G. C. Taylor said he was a member of the Australian and New Zealand sub-committee and was delighted to have heard ANZ-75 subjected to such a searching discussion. There had been a certain amount of criticism of both the papers which had been considered at the meeting, but there had not really been any critical approach to the existing notation. In order to obtain a balanced viewpoint it was necessary to do that also.

He hoped that in criticizing the IAN he would not seem to be extremist; the fact was that he was not fanatical about the adoption of the ANZ-75 notation or even fanatical about the adoption of the linear notation. However, there were advantages from both of those. The first disadvantage of IAN was the

placement of parameters in a function which was unconventional mathematically. Unconventional symbols occurred in function expressions; there were unconventional forms of a continuous argument, that was appearing as a subscript rather than in parentheses. Certain functions could not be represented in a descriptive way but had to be represented by a formula ("ANZ-75", 6). The notation was not computer-orientated; it was inadequate and there were ambiguities. IAN did not make provision for inclusion of interest or mortality specifications. Finally, it was confusing in certain cases; for example, P representing a premium. P_x meant an assurance payable on the death of x with premiums payable up to the death of x. It would be appreciated that he had made two statements about one symbol; that highlighted the fact that in order to specify a premium it was necessary to specify both the conditions under which benefits were paid and the conditions under which premiums were paid.

For more complicated functions, such as contingent assurances, the position was no longer clear. P_{xyz} with some placement of ones and twos was not clear—it was not defined in the IAN. Some of the points to which he had referred were not actually wrong, but ambiguities, inadequacies and confusions should be removed, whether by linearization or modification of existing notation in some other way. So long as there was some considerable case for making alterations to the existing notation, deviations from common practice also added weight to that argument. There was nothing actually wrong with using A_x where x was allowed to be a continuous argument. It just happened to be counter to the common practice of representing a real, continuous argument in parentheses.

In considering a linear notation it was necessary to make a choice between a 'structured parameter list' and key letters. A 'structured parameter list' meant that for a given function name the parameters were defined by being written in particular places in a list. The alternative was the use of key letters, where each number was preceded by a key letter defining its meaning. The order in which the parameters of the function appeared was inconsequent, and key letters therefore made the linear form clearer. They also facilitated computer usage because it would be far easier to write a preprocessor which would develop a key letter expression into a computer-usable expression that it would be to write a preprocessor to process a structured parameter list.

It was interesting to note that ANZ-75 underwent considerable alterations in the course of its development, as the members of the sub-committee found that they were being forced to look further and further into the syntax of their statements. They were logically forced closer to what would be more acceptable by a syntax-sensitive device such as a preprocessor.

Mr A. D. Wilkie, in closing the discussion, thought it was a pity that there was not sufficient time available to do full justice to the topic under discussion. Over the previous 8 years they had had many hours of discussion on the subject in Mr Turvey's perpetual loop. In the diagram Mr Turvey had shown there was

a situation where if the feasibility of something was unknown then the action was think and go back, and this could continue indefinitely.

Mr Hey referred to a paper which had been presented a couple of years previously (*J.I.A.* **101**, 299), in which he had had to print the equation:

$$\frac{l_{(x+t)}-l_{(x+t+\delta t)}}{l_{(x)}} = -\frac{l'(x+t)\delta t}{l(x)}, \text{ in linear form,}$$

for typographical convenience because the editor had wanted it that way. When actually printing, it was so much easier to use parentheses rather than subscripts. There was nothing wrong with that in the present notation. If he produced a paper entirely with parentheses rather than subscripts, he hoped that the editor would not say, "That is not the correct international notation", but would accept it as valid.

All notations were hopelessly cumbersome, including the present one, so as to be at times unintelligible. But Mr Humphrey, Mr Taylor and the paper by Professor Dr Engelfriet and others had made the point that notation was not simply to express concepts; for that words did just as well. The point about notation was to be able to manipulate it, as for instance in the definition:

$${}_{n_1|n_2}\bar{a}_* = \int_{n_1}^{n_1+n_2} v^t \cdot {}_t p_* \, dt,$$

whatever the value of *. Once everything inside the integral was defined everything else was automatically defined. The European version was attempting to do that. The present international notation did it to some extent, but became muddled as soon as it used subscripts and superscripts because there was no way within that notation of manipulating such conditional statements satisfactorily. The notation suggested by Mr Taylor which had been summarized within the European paper, did it very satisfactorily for joint life and last survivor functions:

$$a(x \vee y) = a(x)+a(y)-a(x \wedge \delta^y)$$

Anybody who had worked with probability theory would be able to recognize the truth of that statement. It did not need any actuarial proof because it was a fairly regular result from ordinary probability theory about combinations of events. Mr Taylor's part of that notation was immediately intelligible if one understood 'ands', 'ors' and 'nots' to begin with. It seemed to be necessary for a notation to attempt such manipulation. He did not think any of the present proposals did that sufficiently clearly and well.

If for pedagogic purposes it was desirable and convenient to explain new functions, say to explain pension fund functions using new methods of calculation, then the tutor should be free to use his own notation defined at the start. That would be the simplest method of dealing with the whole problem rather than by trying to construct in a vacuum a notation which might possibly fit

all circumstances, but which really none of the authors, except perhaps Professor Engelfriet, had really tried out hard enough in practical use for teaching purposes. That was what it was all about; real office work did not need a notation at all.

The President (Mr G. V. Bayley) said that the subject of international notation was on the agenda for the forthcoming Congress in Tokyo. It was important that the views of the United Kingdom actuaries should be formulated in good time and then expressed adequately in Tokyo. The discussion had been very useful to prepare themselves and see that the U.K. views did not go by default.

The joint working party now had to complete the answers to the questionnaire and they would want to take note of views which members had expressed in the discussion. He said that the formal item on the agenda was for him to propose a vote of thanks to Mr Harsant for preparing his note and for his opening remarks, and to Mr Wilkie for closing the discussion. In reality, they had to thank them both not only for doing those things so very well, but for all that they had contributed to the important subject and he coupled those thanks with the names especially of Mr Taylor and Professor Dr Engelfriet, the visiting co-authors, and the members of the joint working party.

Mr. A. Neill wrote subsequently: The main point I wish to make does not concern the technical details of notation where my position is perhaps that of being sceptical of the advantages of change but willing to be convinced provided each notation is readily comprehensible by an actuary more accustomed to dealing with the other. What concerns me is the lack of a standard notation even within this country on such matters as pension funds. I suggest you may wish to look at the three different definitions of ${}^{s}D_{x}$. in our standard text books: Lee on *Introduction to Pension Funds* and Hooker and Longley-Cook on *Life and Other Contingencies* and the stencilled note also in the course of reading.

I feel that even if no alternative notations are introduced we must ensure that within the UK we do make an effort to have a standard notation. I do not consider that it is satisfactory to say that there is no International notation and therefore each function is defined as it is used; we could and should have a British Standard even if only for teaching purposes. I would go even further and say that our terminology is standardised so that for example if we use such terms as 'discontinuance funding' of a pension fund all these concerned know exactly what we mean.

In my view it is important that we should try and standardise at least within the UK on matters of notation and terminology and my suggestion is that the Councils of the Institute and Faculty set up a permanent joint committee which will keep notation and terminology under continuous review.