THE CONSIDERATION OF RISK IN PROJECT SELECTION

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INTRODUCTION

This paper is concerned with attempts that have been made to incorporate measures of risk in the selection of financial investments. The methodology that is developed here has been primarily aimed at the evaluation of capital investment projects, but the concepts and ideas are also relevant in the financial investment area. Implicit in all this discussion is the fact that when information is obtained in order to calculate an internal rate of return or a net present value, the figures put into the calculation are estimates rather than precise and exact quantities. In some instances the estimates may be felt to be very good, whilst in other cases it may be felt that there is a wide range of possible deviations. Hence the evaluation of any project or projects should, correctly, be described not by a single criterion but by a range of possible values, some of which are judged to be more likely than others. Having said this, there now comes the need to decide upon the method by which choice is to be exercised. Should it be by the choice of the highest average value of the selection, or by choosing some more conservative rule?

In this paper a distillation is given of previous work on this subject and extensions made of some of the procedures that we feel are the most promising. To do this some computer programs have had to be written as the computation by hand becomes so unwieldy. These programs are not, however, discussed in this paper. Extensive computation as such is only justified if it produces better results than would otherwise be obtained. Additionally, analyses along the lines proposed here do much to clarify the issues involved when assessing individual projects, or in choosing from amongst a set of alternatives.

I. RISK

The meaning of 'risk' in the context of capital investment projects is intuitively fairly clear, but to be more explicit we may quote Wagle's definition of the term as 'the potential for a project’s return to fail to achieve any given rate'. A distinction is often made between situations of risk and situations of uncertainty, the term uncertainty being employed when there is little or no information available on the relative likelihoods of each of the various possible different outcomes actually occurring. Such
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situations will not be considered in this paper, it being assumed that there will always be enough information available for subjective estimates of probabilities to be obtained—converting the uncertainty situation into one of quasi-risk.

In analysing the effect of risk on capital investment decisions it is useful to bear in mind the main sources of risk, of which Merrett and Sykes list five:

(i) Risk from an insufficient number of projects. The smaller the number of projects which are undertaken the more serious will be the consequences of any failures. Where the individual projects are very risky (e.g. drilling for oil) this form of risk may be considerable unless sufficient funds are available for a large number of projects to be financed.

(ii) Risk from inadequate knowledge. It may be impossible to forecast accurately some of the factors influencing the future profitability of an investment, (e.g. sales demand, the useful life of the investment, etc.).

(iii) Risk of bias. Some individuals may be unduly pessimistic or optimistic in their estimates. To a certain extent this kind of bias can be reduced by careful observation and feedback from the results of previous estimates of the individuals concerned.

(iv) Risk from external change. Changes in the economic environment (e.g. due to changes in government policy, in consumer tastes or to the introduction of revolutionary new products) may invalidate much of the usefulness of past experience.

(v) Risk from errors of analysis. Consideration of possible teething troubles, or of some necessary items of investment or cash advantages may have been overlooked. The methods of analysis used may have been too crude. (Hertz gives an example of this in which expected values have been used indiscriminately in the discounted cash flow calculations for a risky investment).

The issue of risk cannot be avoided by adopting a policy of conservatism and only investing in relatively ‘safe’ projects. A firm following this line of action merely exposes itself to risks of a different type and is likely to be rapidly left behind by its rivals. It is, therefore, important to have suitable methods of analysing risky investment proposals so that better decisions can be made about them.

Analysis along these lines is no substitute for sound intuitive judgment—its purpose is rather to ensure that all relevant factors have been taken into consideration and to combine a series of such judgments to give a sound overall picture which can be readily interpreted by the decision-maker(s).

Where risk is of little significance (e.g. if there is little uncertainty, or if a large number of small independent investments are undertaken) the conventional discounted cash flow methods can be used to give very
precise results. In these methods the decision is based on the expected value of the internal rate of return (IRR), or of the net present value (NPV) when the cash flows are discounted at some appropriate rate (the question of the choice of this rate lies outside the scope of this paper). When applied to risky situations, however, the apparent precision of these methods is an illusion, as the IRR and NPV may have quite large variances, which should be taken into account in some way.

Various methods have been suggested in the literature for bringing risk into the analysis.

(i) It may be left entirely to the informal judgment of the decision-maker(s).

(ii) Empirical adjustments may be made to some or all of the crucial variables. For example, the expected life of the investment may be reduced, sales estimates cut or a higher than usual discount rate applied.

(iii) Probabilities of different values of uncertain factors occurring may be used in the calculation of the expected value of the IRR or NPV.

(iv) Sensitivity analysis may be applied. This is often done by obtaining pessimistic, most likely and optimistic values of the key factors and computing the effect of various combinations of these.

None of these methods is really satisfactory. Only the last one tries to quantify the possible range of the IRR or NPV, and even this gives no information as to whether or not the pessimistic result is more likely than the optimistic one.

In analysing capital investment decisions under risk, it is really necessary (as has been pointed out by Hertz(3), Hillier(4) and others) to obtain an overall picture of the probability distribution of the IRR or NPV so that the chances of any particular level of return can be assessed. These decisions are essentially of a 'single shot' variety and it is more important to have even a vague idea of what the distribution is like than to know precisely its expected value.

Hertz(3) points out that for major capital proposals, managements usually make a significant investment in time and funds to pin-point information about each of the relevant factors, and comparatively little extra effort is required to obtain estimates of the necessary probability distributions. The less certainty there is in an expected value estimate, the more important it is to consider the possible variation in that estimate. The probability distribution approach aims to make the fullest possible use of whatever information can be obtained about such variations.

II. THE DISTRIBUTIONS OF IRR AND NPV

As in the conventional analysis of a capital project, it is necessary first of all to isolate a set of key factors or variables, on which the success of
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The following nine factors might form a typical set of this kind:

(i) Market size
(ii) Selling price
(iii) Market growth rate
(iv) Share of market
(v) Investment required
(vi) Residual value of investment
(vii) Operating costs
(viii) Fixed costs
(ix) Useful life of investment.

These key variables have then to be estimated. Some or all of them may be either probabilistic or uncertain in nature and information must then be obtained about their likely probability distributions. They are also likely to be interdependent and so estimates must be obtained which in some way quantify the correlation effects between them. Naturally the form in which these estimates are actually made will depend on the method it is proposed to use to transform the information into information about the probability distributions of the IRR or NPV. Consideration of the problems of obtaining the necessary data on the key factors is therefore given later in the paper, after the two main methods of analysis have been discussed. Basically, information on the required distribution can be obtained through either the simulation approach or the analytic approach.

The simulation approach is described in the papers by Hertz(3) and by Hess and Quigley.(5) A random sample is taken from each of the relevant distributions to give a possible value for each of the key variables. Correlation effects between these variables are introduced by means of conditional probability distributions—the value obtained for one variable being used to determine which of several conditional distributions should be sampled to give the value of another variable. These sample values are used to calculate the cash flows which would then result over the various time periods, and the cash flows are either discounted at the appropriate rate to give the NPV of the project under the conditions simulated, or else used to give the IRR. (For the sake of completeness, Appendix I gives an efficient algorithm for finding the IRR of a series of cash flows—assuming, as is usually the case in practice, that a unique rate does exist.) The procedure is repeated taking new random samples until a sufficiently good picture of the overall criterion distribution (of the NPV or IRR) is obtained.

The method is fairly easy to apply, and quite general in that few simplifying assumptions have to be made in setting up the simulation model. However, it should be pointed out that the distributions obtained are only approximate and, in particular, little reliance should be put on their extreme points. The computation is best done by computer and although
The alternative analytic approach has been followed by Hillier(4,6) and Wagle(1). In essence it consists of estimating the means and variances of the key variables and certain correlations between them. Various known properties of statistical distributions are then used to derive the mean and variance of the project's NPV. It might be thought that this information would be insufficient for many purposes, but under certain conditions, which have been considered in detail by Hillier(7) and are summarized later in the paper, its distribution is approximately the Normal distribution specified completely by these parameters. More generally, the variance of the NPV can be very useful as some measure of the risk associated with a project.

One disadvantage of the analytic model is that, unlike the simulation model, it is difficult to adapt it to give information on the IRR of a project. If the NPV calculation is performed with a discount rate $i$, the equation

$$\Pr \{ \text{IRR} < i \} = \Pr \{ \text{NPV} < 0 \}$$

can be used to give a point of the cumulative distribution of the IRR where $\Pr \{ \}$ denotes the probability of the expression inside the bracket. However, this can only be done if the NPV distribution is assumed to be approximately normal, since otherwise the probability on the right-hand side of the equation is virtually impossible to evaluate. Further, the equation is not generally true (see Hillier(6)), but it is good enough for most practical purposes. Several points on the IRR's cumulative distribution must be obtained, which involves evaluating the NPV for a corresponding number of discount rates. In spite of these disadvantages an analytic model, suitably applied, may be expected to give a more accurate value of the variance of the NPV than would a comparable simulation model. If a fairly simple model is employed the use of a computer may not be necessary.

Whichever approach is used, by altering the distribution of an input factor it is possible to determine the effect of added or changed information (or of the lack of such information). This enables a sensitivity analysis to be performed on the criterion distribution allowing management to ascertain the sensitivity of the results to each or all of the input factors. It may turn out that fairly large changes in the distributions of some factors do not significantly affect the outcomes. For the most crucial factors it may be worth while to concentrate more effort on obtaining good estimates of their probability distributions. For example, the existing estimates may be improved and possibly lowered by the expenditure of additional engineering time. Again, management can sometimes suggest action to reduce the variability of some of the crucial factors and so reduce the overall level of risk. Thus it may be possible to reduce changes in sales volume and price by developing long-term customers. This would probably result in less risk at the expense of a reduced expected return.
Instead of estimating expected values, as for a conventional discounted cash flow analysis, we need to estimate the probability distributions of some or all of the key variables. If an analytic model is employed, then certain correlation coefficients must also be estimated. If simulation is used, then conditional probability distributions are estimated to give the correlation effects. Thus, if two variables are dependent, the range of one of them is split up into intervals and, assuming that its value lies in each of these intervals in turn, conditional probability distributions of the other variable are estimated.

Estimates are usually obtained by holding a series of meetings with management personnel at which each of the experts involved is probed and questioned until, it is hoped, a clear and consistent set of estimates is obtained. Some managers may have difficulty in expressing their judgments on a quantitative basis or in grasping the meaning of probability. Most of these difficulties can be overcome by careful explanation of what information is required and by the phrasing of suitable questions. What is important is that the experts themselves should have a clear idea of the range of results which might occur under different circumstances and, to this end, they must be encouraged to think deeply about the estimation problem.

In estimating a probability distribution, most estimators can fairly easily specify outer limits (a and b) and a modal value (m) which they consider most likely to occur. For an analytic model it may be possible to use these estimates, as for the well-known PERT technique used in network analysis, to give the mean and variance of the distribution. In this method it is assumed that the distribution is approximately of Beta form with six standard deviations between its outer limits. This assumption gives the estimate for the variance as

$$\sigma^2 = \frac{1}{4}(b-a)^2$$

and the estimate of the mean as

$$\mu = \frac{1}{2}(a + 4m + b).$$

If the assumptions required for this method are not felt to be sufficiently justified then estimates of \( \mu \) and \( \sigma^2 \) could be obtained from a direct estimate of the complete probability distribution.

There is relatively little material available in the literature on methods of deriving the complete form of a subjective probability distribution. The following notes attempt to summarize the more practical suggestions that have been made for accomplishing this important stage of the analysis.

**Method A**

If the estimator is capable of estimating probabilities directly then, having first obtained estimates of \( a, b \) and \( m \), attention may next be concentrated on a fairly small range of values containing \( m \). Once a probability
weighting has been assigned to this modal interval it should be quite easy
to obtain a graph of the complete probability density function (PDF) of
the distribution. Rough graphical drawings are made at first and these
are gradually improved by considering what probabilities (or areas of
the graph) should be assigned to various other intervals within the bounds
of the distribution.

Method B

This method, which will often be easier to implement, consists of
obtaining estimates of fractiles of the cumulative distributive function
(CDF) and graphing this before converting it into the PDF if this is
required. Morrison(8) gives a concise presentation of this method and its
virtues. The estimator might be asked the following questions:

1. At what level, $F_{50}$ do you feel there is a 50–50 chance that the
variable will fall below $F_{50}$?
2. Given that its value is below $F_{50}$, at what level $F_{25}$ do you feel
there is a 50–50 chance that it will be below $F_{25}$?
3. Given that its value is above $F_{50}$, at what level $F_{75}$ do you feel
there is a 50–50 chance that it will be above $F_{75}$?

This gives estimates of the quartiles. As a check on his consistency the
estimator can next be asked:

- Do you feel that there is a 50–50 chance that the value will be
between $F_{25}$ and $F_{75}$?

If he is inconsistent then adjustments must be made until the inconsis-
tencies are resolved. Similar sets of questions can be asked to determine
other points of the CDF. When this has been satisfactorily plotted and
graphed the PDF can be obtained from it. At this stage there should again
be collaboration with the estimator so that possible inconsistencies are
avoided.

Method C

Perhaps the most simple method of obtaining a subjective PDF consists
of asking the estimator to distribute 100 points over his range of possible
values, so that the resulting ‘picture’ best reflects his judgment of the
uncertainty of the situation. Once this is done questions on the CDF
fractiles such as those given above may be used to check his consistency.

Some other less direct techniques have also been suggested. Most of
these suffer from being harder to implement and much harder to explain
to the manager (who is unlikely to take much trouble with a technique
he does not properly understand). One method suggested by Smith(8)
involves splitting up the range of possible values into equal intervals,
ranking these in order of probability and then ranking the differences
between the probabilities ranked next to each other. The probabilities
do not have to be estimated directly.
Another method, mentioned by Winkler,(9) asks the estimator how he would modify his estimate of the most likely value in the light of various hypothetical samples from the actual distribution (assuming independent repetitions were possible). Bayes's theorem is then used to deduce information about the shape of the estimator's subjective distribution. Unfortunately this method appears to underestimate the spread of the distribution because (see Phillips and Edwards(10)) people are conservative processors of information and do not correctly take into account the value of different numbers of observations.

Whatever method is used the spread of the distribution will be directly related to the degree of confidence that the estimator has in his estimate. It is important to try to avoid the tendency to spend less time on estimating the more uncertain variables.

It has been observed that estimators are usually reluctant to quantify correlation effects despite definite evidence of their existence. The most practical method of estimating correlation coefficients, when no relevant past data is available, is the following one suggested by Hillier.(7)

Suppose it is required to estimate the correlation coefficient $\rho$ between two variables $w_1$ and $w_2$ having estimated means $\mu_1$, $\mu_2$ and variances $\sigma_1^2$, $\sigma_2^2$ respectively. Then given that $w_1$ has the value $w$, the best (least squares) estimate for the expected value of $w_2$ is

$$\text{Est}(E(w_2 | w_1 = w)) = \mu_2 + \rho \frac{\sigma_2}{\sigma_1}(w - \mu_1)$$

Hence if an estimate can be obtained of the expected value of $w_2$ given that $w_1$ has the value $w$ ($\mu_1$), the equation can be used to estimate $\rho$.

It may be possible to use the PERT method described above to give the required expected value. Hillier suggests performing this estimation with $w_1$ equal to its upper and lower bounds in turn and averaging the two resulting values of $\rho$ to give the final estimate.

IV. AN ANALYTIC MODEL

The following analytic model, which is given in Wagle's paper,(11) is quite general and yet can be implemented without undue difficulty—the numerical calculations being performed, if necessary, on a desk machine.

Suppose $m$ sources of cash flows $S_1, \ldots, S_m$ have been identified for an investment. Let the random variable $Y_{tr}$ denote the cash flow in the $t$th year (or other convenient time period) from the $r$th source, $Sr$. Assume that $Y_{tr}$ has finite mean $\mu_{tr}$ and variance $\sigma_{tr}^2$.

Denote the net cash flow in the $t$th year by $X_t$ ,

where

$$X_t = \sum_{r=1}^{m} Y_{tr}$$

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Taking expectations and variances, we have:

\[ E[X_n] = \sum_{i=1}^{n} m_i (\text{say}) \]  
\[ \text{var}(X_n) = \sum_{i=1}^{n} \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} \text{cov}(X_i, X_j) \]  

where the symbol ‘\( E \)’ denotes ‘expected value of’, ‘\( \text{var} \)’ denotes ‘variance’ and ‘\( \text{cov} \)’ denotes ‘covariance’.

If the life of the project is \( n \) years, then discounting the cash flows over this period at the rate of 100\( i\% \) gives the NPV of the investment as

\[ P_n(i) = \sum_{t=0}^{n} \left( \frac{X_t}{(1+i)^t} \right) \]  

Taking expectations and variances of \( P_n(i) \), we have:

\[ E[P_n(i)] = \sum_{t=0}^{n} W_t(i) = W_t(i) \text{ (say)} \]  
\[ \text{var}(P_n(i)) = \sum_{t=0}^{n} \text{var}(X_t) + 2 \sum_{t=0}^{n} \sum_{j=0}^{n} \text{cov}(X_t, X_j) \left( \frac{1}{(1+i)^{t+j}} \right) = L_t(i) \text{ (say)} \]

Since

\[ \text{var}(P_n(i)) = E[P_n(i)^2] - E[P_n(i)]^2 \]

we also have:

\[ E[P_n(i)^2] = L_t(i) + W_t(i) = M_t(i) \text{ (say)} \]

In most cases the length, \( n \), of the project will not be known with certainty and thus will have a probability distribution which can be estimated. Suppose it is estimated that the project’s life will certainly be between \( N_1 \) and \( N_2 \) and that the probability of its being \( n \) years is \( p_n \).

Combining equations (5) and (7) over this distribution and writing \( P(i) \) for the NPV of the project at a discounting rate of 100\( i\% \) gives:

\[ E[P(i)] = \sum_{n=N_1}^{N_2} p_n E[P_n(i)] = \sum_{n=N_1}^{N_2} p_n W(n) \]  
\[ E[P(i)^2] = \sum_{n=N_1}^{N_2} p_n E[P_n(i)^2] = \sum_{n=N_1}^{N_2} p_n M(n) \]

We also have

\[ \text{var}(P(i)) = E[P(i)^2] - E[P(i)]^2 \]

Equations (8) and (9) complete the calculation of the mean and variance of the NPV distribution.
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This model requires estimates of the mean and variance of each of the random variables $Y_t$ and also of the covariances between them. The following procedure of Hillier[7] is suggested for simplifying the estimation of the covariances.

Estimate:

(i) The correlation coefficient $r_t$ between the cash flows from each source $X_t$ in successive time periods.

(ii) The correlation coefficient between $Y_t$ and $Y_{t+1}$ (assumed independent of $t$).

(iii) The correlation coefficient between the cash flows from $X_t$ and $X_s$ in the same time period.

Then for $t > 1$ the correlation coefficient between $Y_t$ and $Y_{t+s}$ is given by the product

$$r_t = (r_s)^{t-s}$$

This procedure requires the following reasonable assumptions:

(a) Cash flows from the same source are Markov dependent, i.e. a cash flow in year $(t-1)$ only influences the cash flow in year $(t+1)$ indirectly, by its influence on the cash flow in year $t$.

(b) The correlation between cash flows from the same source in adjacent time periods is independent of time.

(c) The cash flow from a particular source is independent of the cash flows from other sources in earlier time periods.

(d) The correlation coefficient between cash flows from different sources in the same time period is independent of time.

Some of the component cash flows will generally themselves be in the form of products of random variables. For instance, the cash flow from sales may be calculated as sales volume times price (or even as total market size times market share times price), and part of the production cost will be a variable cost times sales. We need, therefore, expressions for calculating the means and variances of, and covariances between, such products.

The general result for two random variables $U_1, U_2$ with means $\mu_1, \mu_2$ and dispersion matrix

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$$

is

$$E[U_1 U_2] = \mu_1 \mu_2 + \sigma_{12}$$

and

$$\text{var}[U_1 U_2] = \mu_1^2 \sigma_{22} + \mu_2^2 \sigma_{11} + 2 \mu_1 \mu_2 \sigma_{12} + 2 \mu_1 \sigma_{11} + 2 \mu_2 \sigma_{22} - \sigma_{11}^2$$
If \( U_1, U_2 \) are independent these expressions reduce to
\[
E_{U} = E[U_1 \mu_1^2(U_2 \mu_2)^2],
\]
and
\[
\text{var}(U_1 U_2) = \mu_1^2 \sigma_2^2 + \mu_2^2 \sigma_1^2 + \sigma_{12}.
\]

Generally, however, the \( E_{U} \) may be difficult to obtain and an approximation formula omitting these terms is often used, but this may not be sufficiently accurate.

More tractable results can be obtained if the factors involved have a joint multivariate Normal distribution. Consider random variables \( u_1, \ldots, u_k \) with a joint \( k \)-variate Normal distribution, with mean vector \( \mu = [\mu_j] \) and dispersion matrix \( \Sigma = [\sigma_{ij}] \). The characteristic function of this distribution is
\[
\phi(t) = E[\exp(itu)] = \exp(it\mu + \frac{1}{2} tu^t \Sigma t).
\]

Differentiating partially with respect to the components of \( t \) and evaluating at \( t = 0 \) gives
\[
E[U_1 \ldots U_k] = \phi_t(0) = \phi_t(0) = \phi_t(0).
\]

The following expressions can be obtained in this way for \( k = 2, 3, 4 \):
\[
E[U_1, U_2] = \mu_1 \mu_2 + \sigma_{12},
\]
\[
E[U_1, U_2] = \mu_1 \mu_2 + \mu_1 \sigma_{12} + \sigma_{12} \mu_2 + \sigma_{12} \mu_2 + \sigma_{12} \mu_2 + \sigma_{12} \mu_2 + \sigma_{12} \mu_2 + \sigma_{12} \mu_2 + \sigma_{12} \mu_2 + \sigma_{12} \mu_2 + \sigma_{12} \mu_2 + \sigma_{12} \mu_2 + \sigma_{12} \mu_2 + \sigma_{12} \mu_2 + \sigma_{12} \mu_2 + \sigma_{12} \mu_2 + \sigma_{12} \mu_2 + \sigma_{12} \mu_2 + \sigma_{12} \mu_2 + \sigma_{12} \mu_2 + \sigma_{12} \mu_2 + \sigma_{12} \mu_2 + \sigma_{12} \mu_2 + \sigma_{12} \mu_2.
\]

We see that these are all of the same general form—all permutations of the subscripts which give distinct terms being used with each \( \mu \) taking one subscript and each \( \sigma \) taking two. This result can be proved by mathematical induction, so \( E[U_1, \ldots, U_k] \) may be written down directly for any \( k \), and so may \( E[U_1^2, \ldots, U_k^2] \) by repeating some of the subscripts. No general mathematical expressions are given here because of the complex notation required.

The above result permits straightforward calculation of the variances and covariances as well as of the means, by using the formulae:
\[
\text{var}(Z) = E[Z^2] - (E[Z])^2
\]
and
\[
\text{cov}(Z_1, Z_2) = E[Z_1 Z_2] - E[Z_1] E[Z_2]
\]
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For example, if \( Z_1 = U_1 U_2 \) and \( Z_2 = U_1 U_3 \)

\[
\text{cov}[Z_1, Z_2] = E[U_1 U_2 U_1 U_3] - E[U_1 U_2]E[U_1 U_3]
\]

By changing the subscript 3 to a 2 the formula for the variance of \( Z_1 \) may be obtained:

\[
\text{var}[Z_1] = 2\mu_1\mu_2\mu_3 + \mu_1^2\mu_2 + \mu_1\mu_2^2 + \mu_1^2\mu_3 + \mu_1\mu_2\mu_3
\]

V. NORMALITY

It has already been noted that under certain conditions, discussed in detail by Hillier(7), the distribution of the NPV, \( P(i) \), is approximately Normal and, if this is so, the mean and variance are sufficient to give a very good picture of it. If the \( P(i) \) have nearly Normal distributions then the IRR distribution is usually also approximately Normal. \( P(i) \) is the sum of the random variables formed by discounting the cash flows for each year at the rate of \( 100i\% \). One of the most important problems of probability theory, the Central Limit Problem, is concerned with finding sufficient conditions for the distribution of such a sum of random variables to be asymptotically Normal so that, provided a sufficiently large number of random variables are included in the sum, it is approximately Normal. Many different sets of sufficient conditions for this Central Limit Theorem to hold have been and are being developed.

The following three results are those which are most likely to be of use in the context of capital project appraisal.

1. \( P(i) \) is Normal whenever the joint distribution of the net cash flows for each year \( X_1, \ldots, X_n \) is multivariate Normal.

2. If the discounted cash flows \( \{ X_i \} \) form a uniformly bounded non-degenerate sequence of mutually independent random variables, then the Central Limit Theorem holds for \( P(i) \) and its distribution is approximately Normal provided \( n \) is sufficiently large. The assumption of independence which has to be made here is unfortunately very restrictive.

3. If the discounted cash flows \( \{ X_i \} \) are \( q \)-dependent (i.e. if cash flows more than \( q \) years apart are independent) then under a condition, which requires a certain degree of uniformity and constancy in the pattern of variances and covariances of these cash flows, the Hoeffding and Robbins Theorem applies and \( P(i) \) is again asymptotically normal.
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In some cases where insufficient conditions are satisfied for the NPV distributions of individual projects to be normal, it may still be possible to conclude that the distribution of the NPV of a collection of several projects is approximately normal. Depending on the number of projects which can be undertaken at once it will often be possible to apply a form of the Central Limit Theorem to the net cash flow in each year resulting from a whole collection of projections. If these cash flows have Normal distributions, then their NPV, which is a weighted sum of them, will also be approximately Normal.

One difficulty which arises in applying these results is that of knowing how many random variables must be present in a finite sum before its distribution becomes reasonably close to Normal. Even when the Central Limit Theorem applies there is no guarantee that the rate of convergence to Normality is sufficiently rapid for the sum yielding $P(i)$ to be nearly Normally distributed. If the random variables are completely independent it is known that a number of them as small as four or five is sufficient to give a reasonable approximation to Normality. Less is known about the case of dependent random variables although it seems evident that a larger number will usually be required, depending on the degree and type of dependence. An additional factor is the distributions of the individual random variables—the closer these are to being Normal the more confidence we may have in their sum being nearly Normal.

Lastly, where there is doubt as to the validity of assuming that the distributions are approximately Normal, it can if necessary be resolved, one way or the other, by applying the simulation approach for a few typical distributions.

VI. SEVERAL PROJECTS

So far this paper has been mainly concerned with the analysis of individual projects. We now turn our attention to the problem of obtaining the criterion distributions for portfolios consisting of a number of separate projects. Conceptually there is no reason why such portfolios should not be treated in the same way as individual projects, and the methods described above then used to obtain these distributions. In practice, however, some less cumbersome procedure is needed because, with only a moderate number of individual projects, the number of possible portfolios which have to be considered becomes unmanageable.

As explained above, where a number of projects are undertaken at the same time, there is a strong possibility that the criterion distribution of the portfolio is approximately Normal, for the Central Limit Theorem can usually be applied to the distribution of the net cash flows for each year. The most practical method is therefore to set up a simple analytic model which can be used to give the mean and variance of the criterion distribution for any portfolio under consideration. Even if the distributions are not Normal, it is very useful to be able to calculate their means.
and variances efficiently. Any method of doing this must take account of the correlations between the results of different projects, as it will only occasionally be reasonable to assume that they are independent. When two projects are combined together in a portfolio their NPVs, calculated at the same discounting rate, are added together. It might be expected that their combined IRR is the weighted average of the two IRRs, with the weights proportional to the initial outlays required, but this is not generally the case. The rate of return of the combined projects would have to be recalculated from the combined cash flows for each year, unless the cash flows take an especially simple form (e.g. are constant in each year for both projects). This means that such a simplified model is often not possible for the IRR of a portfolio of projects, particularly where these are projects of capital investment rather than stock exchange investment. The following model, based on work by Markowitz, is therefore presented in terms of NPV.

Suppose there are \( m \) projects under consideration with NPVs \( p_i (i = 1, \ldots, m) \) and that a relatively small number \( n \) of factors or indices \( v_j (j = 1, \ldots, n) \), which are expected to influence the degree of their success or failure, have been identified. The \( p_i \) and the \( v_j \) are random variables. As long as the number of indices \( v_j \) is not too large it should be possible to estimate their covariance matrix \( C_v \), either statistically or else by subjective estimates as described earlier.

Assume relationships of the form

\[
p_i = a_i + \sum_{j=1}^{n} b_{ij} v_j + u_i \quad (i = 1, \ldots, m)
\]

where the \( a_i \) and \( b_{ij} \) are constants and the \( u_i \) are random deviations with zero means. The constants \( b_{ij} \) may be estimated for each \( i \) and \( j \) by examining the sensitivity of \( p_i \) to a change in the value of \( v_j \). The \( a_i \) do not have to be estimated.

We first obtain the covariance matrix \( C \) of the \( p_i \). Writing (1) in the obvious matrix notation gives

\[
\begin{align*}
p & = a + B v + u \\
E(p) & = a + B E(v)
\end{align*}
\]

Hence, writing \( P = p - E(p) \) and \( V = v - E(v) \)

\[
P = R(V - E(v)) + u = RV + u
\]

Thus

\[
C = E(P^T P) = E(B V^T V' B + B V u' + u V' B + uu') = BC_v B^T + D
\]

since \( E(V V') = C_v \), where \( D \) is a diagonal matrix with non-negative elements. Generally \( D \) is not known directly, but the diagonal terms of \( C_v \)
var (p_i) will be known, and the equation used to give the off-diagonal ones.

It is, however, necessary to check that
\[ \text{var} (p_i) \leq (B' C B)'_{ii} \quad \text{for each} \; i, \]
or else equation (2) is obviously inconsistent.

For any vector \( x \) the mean of \( p' x \) is
\[ \mu = E(p') x \]
and its variance is
\[ \sigma^2 = x' C x. \]

These are the mean and variance of the NPV of the portfolios of projects defined by the vector \( x \), where \( x_i = 1 \) if the \( i \)th project is included, whilst otherwise it is zero. Where different levels of investment are possible with linear returns then \( x_i \) can be varied continuously and is proportional to the amount invested in the \( i \)th project.

If all cash flows are approximately constant from year to year then the model is easily adapted to give the mean and variance of the IRR of a portfolio. It is merely necessary to work throughout in terms of IRR instead of NPV and to define \( x_i \) as the proportion of the total current outlay which is invested in the \( i \)th project.

Provided that \( C \) is correctly estimated and turns out to be positive semi-definite (i.e. it satisfies \( y' C y \geq 0 \) for all \( y \)) then for any \( x \),
\[ x' C x = x' B' C B' x + x' D x = (B' x)' C (B' x) + x' D x \geq 0. \]

Thus \( C \) is also positive semi-definite, and the absurdity of estimating a negative value for \( \sigma^2 \) cannot arise.

In the above model no account has been taken of the various kinds of restriction on which sub-sets of the set of proposed investments can feasibly be undertaken. For example, some investments may be mutually exclusive, such as when they consist of the same project delayed to different points in time, or more generally when they involve different methods of satisfying the same demand, so that no more than one of them can be approved. There is no difficulty in handling sets of mutually exclusive projects, for when combinations of projects are analysed it is easy to ensure that no more than one project is allowed from each such set. The meaningless covariances between mutually exclusive projects which could be obtained from the model are never used. The feasibility of some investments may depend on the approval of others. Again this should cause no difficulty and can often be conveniently handled by combining such projects with the projects they depend on and grouping all their feasible combinations together into a set of mutually exclusive projects.

Other forms of constraint must also be included in the analysis. The most important of these will usually be a budget constraint, either as a restriction on the capital outlay in one particular time period or as a 

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series of such restrictions. There may also be restrictions on other resources, such as labour, machines and materials. Depending on the degree of risk involved these restrictions can often be formulated as linear constraints which are fairly easily dealt with.

In addition it may sometimes be desirable to impose other financial restrictions to take account of the structure of the investments which are already being undertaken, and to ensure that the published financial results will be satisfactory over a specific future period of years. Chambers(12) describes this aspect of capital budgeting in his paper. The example he gives includes the following four constraints on published results:

1. Published profits are required to increase by at least some fixed percentage in each year.
2. The ratio of current assets to current liabilities, as reported on the balance sheet, is kept at least at a specified value.
3. Dividends must not fall below a specified value.
4. The return on gross assets is not allowed to fall below some fixed value.

When risk can be disregarded these constraints take a linear form. Generally, however, they will be non-linear and more difficult to manipulate.

A numerical example illustrating the incorporation of risk analysis into the selection of a portfolio of capital investment projects is given in Appendix II.

VII. DECISION CRITERIA

So far this paper has only shown how a mathematical model of a capital budgeting situation may be set up. Before such a model can be of any use, two basic issues must be decided. The first of these is the question of the criterion on which the investment decision is to be made. Given the NPV (or IRR) distributions of two portfolios of projects, which of them is to be preferred?

The classical utility theory approach consists of defining somehow a measure of the utility to the firm of each possible different level of return, and then maximizing the expected value of the firm's utility. (For a discussion of utility see, for example, Luce and Raiffa(13)) This approach, however, for all its theoretical appeal, is of little or no practical use for two reasons. First of all, it is usually impossible to obtain meaningful estimates of the utility of the different possible outcomes. Secondly, even if these are obtained and the portfolio with maximum expected utility is successfully derived, this presents management with a fait accompli which may be difficult to justify and which they will not unreasonably be reluctant to accept.
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A more practical approach to the selection problem is that set out by Markowitz. He first recognizes high return and low risk as the two most fundamental objectives. These objectives will often conflict—the portfolio with the highest 'likely return' is unlikely to be the one which offers the least 'uncertainty of return'. If two feasible portfolios A and B are such that A has both a greater 'likely return' and a smaller 'uncertainty of return' than B, then A will always be preferred to B which may be eliminated as 'inefficient'. The Markowitz approach consists of eliminating all such 'inefficient' portfolios, or rather, of obtaining the 'efficient set' of portfolios which give the greatest possible 'likely return' for any acceptable level of 'uncertainty of return'. In doing this the mean \( \mu \) of the criterion distribution may be used as a measure of 'likely return' and its variance \( \sigma^2 \) as a measure of 'uncertainty of return'. At this stage it is left to the decision-maker(s) to make a value judgment between the different 'efficient' portfolios, trading off the level of probable return against the level of risk incurred. If this method is followed the 'efficient set' may be restricted by insisting on some minimum value of \( \mu \). It may be possible to restrict it further by imposing a lower limit on the expected floor \( L \) of the return, where

\[
L = \mu - k\sigma
\]

and \( k \) is a constant chosen so that the return is very unlikely to fall below this floor. Baumol has suggested using \( L \) instead of \( \sigma^2 \) as a measure of risk, but objections can be raised against this (see Russell and Smith). In some ways the 'efficient set' approach begs the question of criteria, but is of great practical value in reducing to a comparative handful the number of portfolios which have to be considered by the decision-maker(s).

In practice the decision boils down to selecting the portfolio with the highest 'likely return' subject to the risks not being too great. If sufficiently harsh probability constraints are imposed on the worst acceptable future financial results then it may be felt that no other consideration of risk is necessary. In this case it is sufficient to maximize the expected value \( \mu \) subject to all the various constraints.

VIII. TECHNIQUES

The second issue which must be decided before an investment model can be used is the mathematical technique to employ in obtaining from the model either the 'best' investment portfolio, or the set of 'efficient' portfolios. A review of the ones likely to be the most useful in practice is given below. To a large extent the structure of the model used and the number and kind of the projects themselves will determine which technique will be the most satisfactory to use. Whatever the choice of technique, it will usually be worth while to use simulation afterwards to give further insight into the likely results of adopting the portfolio(s) obtained.
Mathematical programming

The techniques of mathematical programming are particularly appropriate when the levels of investment $x_i$ can be varied continuously with linear returns. This is typically the case with stock exchange investment. Solving the problem

$$\text{Minimize } \mathbf{c}^T \mathbf{x} \text{ subject to}$$

for different values of $\mu$ gives portfolios belonging to the 'efficient set'. There will normally be some additional constraints restricting which portfolios can be adopted, but provided that these are linear the problem is one of quadratic programming and a number of algorithms have been written and some computer codes are available for its solution. The methods of Markowitz(11) and of Wolfe(16) were developed with this application to portfolio selection specifically in mind. Sharpe(17) has produced a method which efficiently solves the simpler case in which covariances between the different investments are obtained by relating their returns to a single index instead of to several. Maximizing directly the expected return may not be convenient in the continuous case, for although the return itself is linear, the constraints imposed will almost certainly not be, and resort must be made to the more complicated techniques of non-linear programming. The effectiveness of such techniques is largely dependent on the nature of the non-linearities involved.

Where each project must either be accepted at a fixed level of investment or rejected completely, the problem is one of discrete optimization with each corresponding $x_i$ restricted to take the value 0 or 1. Capital investment projects are almost always dealt with in this way, and where several different levels of investment are possible these are considered as separate mutually exclusive projects. This time the problem can be formulated as one of integer quadratic programming, but as even most integer linear programming algorithms are inefficient, this approach seems rather fruitless. Weingartner(18) has suggested dropping the integral requirement and using mathematical programming to obtain a solution with $0 \leq x_i \leq 1$. This can be used to give near optimal solutions to the discrete problem, for although some of the $x_i$ will usually turn out to be fractional the number of these is limited by the number of other constraints employed. However, one of the following three methods of dealing with discrete variables will usually be more satisfactory. Where there is a mixture of discrete and continuous variables it may be best to use some combination of methods.

Dynamic programming

The technique of dynamic programming has often been suggested as a means of calculating which portfolio of investment projects will be optimal.
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This approach, however, is of limited application and even where it can in theory be applied, an excessive amount of computation may be involved.

The simplest form of capital budgeting problem, the single period Lorie-Savage problem, is that in which there is a single budget constraint and the returns from the different projects are not subject to uncertainty. This is an example of the 'knapsack' or 'flyaway kit' problem which is easily solved by dynamic programming. Any additional constraints, however, make dynamic programming harder to apply, and so when uncertainty is introduced the problem is increased by having an extra constraint on the total variance. One method of handling this constraint is to introduce a Lagrange multiplier, and to maximize \( \mu - \lambda \sigma^2 \) for different values of \( \lambda \), which may be interpreted as a measure of the cost of the variance or risk associated with a portfolio. A drawback to this use of a Lagrange multiplier, in a situation in which the other variables are discrete, is that there may be gaps in the set of solutions produced, making it impossible to obtain some numbers of the 'efficient set' by this technique.

The method of dynamic programming is a sequential one and depends for its validity on the following principle of optimality:

\[ \text{'The best portfolio which can be selected from the first } (k + 1) \text{ projects, using up to some fixed amount of capital, includes the best portfolio which can be selected from the first } k \text{ projects for some level of available capital.'} \]

This principle holds when the individual projects are independent but, unfortunately, it seems that otherwise, unless there are severe restrictions on the correlations between projects, it will not hold and some other technique must be used.

Exhaustive examination

The most naive way of handling the discrete case is to generate all the possible combinations of projects and perform the optimization by making direct comparisons between these different possible portfolios. As long as the number of projects involved is fairly small, then this procedure is quite satisfactory on a computer. The portfolio can be efficiently generated by changing from one portfolio to the next by just adding or omitting a single project, on the principle of the binary Gray code often used in electronic counting devices. Once a portfolio has been generated it is tested to see if it satisfies the various constraints and, if it does, it is compared with the list of 'best' portfolios encountered so far, which is updated if necessary. Continuing until all possible portfolios have been considered, the set of 'efficient' projects, or a set of those portfolios with the highest \( \mu \) or which maximize some other criterion, is obtained. Using this method, the number and kind of constraints is unimportant but the size of computation increases very rapidly with the number of projects, as there are \( 2^n \) ways of combining \( n \) projects.
Branch and bound is another combinatorial technique, but one which offers a means of reducing the number of portfolios which need to be considered. Suppose we are trying to obtain the portfolio which has the maximum value of $\mu$. By making use of the key constraints, such as the amount of capital available, an upper bound for $\mu$ can be calculated. Next, the effect of the alternative decisions to include or not to include the first project is considered and gives two partial solutions or nodes, for each of which new bounds can be obtained. The process is continued by branching out again from one of these, to give two more nodes, which correspond to the two possible decisions on the next project and bounds are again calculated for these. A node for which decisions have been made on all of the projects is a possible solution to the problem, so provided it satisfies all of the constraints it is compared with, and may replace, the current best solution. All nodes with upper bounds less than the value of such a feasible solution can be discarded. The procedure is continued until all the nodes have been eliminated in this way and the best solution remains unchallenged.

It is possible to produce many variations of this branch and bound technique. By only eliminating nodes with upper bounds on $\mu$ which fall below some pre-assigned level, all portfolios above this level are generated and their 'efficient set' may be obtained. Other approaches to the problem of obtaining efficient sets by branch and bound are hampered by the difficulty of obtaining reasonable lower bounds on the variances of the different nodes.

**APPENDIX I**

**RATE OF RETURN CALCULATION**

(a) Basic equation

Given $N$ successive cash flows $a_1, \ldots, a_N$ the rate of return $r\%$ is given by solving:

$$a_1 \left( \frac{1 + x}{100} \right) + \cdots + a_N \left( \frac{x^N}{100} \right) = 0$$

Putting $x = \sqrt[1 + r]{\frac{r}{100}}$, where for $r > 0$, $x$ is in the range $0 < x < 1$, whilst for $-100 \leq r \leq 0$, $x$ is in the range $1 \leq x \leq \infty$ gives

$$r = 100 \left( \frac{1 - x}{x} \right)$$

(1)
This implies we can solve
\[ f(x) = c_1 + c_2 x + c_3 x^2 + \ldots + c_n x^{n-1} = 0 \]
and use equation (1). If \( r \) is very close to zero this may not be very accurate.

(b) Method of solution
(i) The equation \( f(x) = 0 \) may be solved by Newton's iterative method; i.e.,
\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \]
The convergence of this method is very rapid (second order) which makes the calculation of \( f(x) \) and \( f'(x) \) worthwhile.

(ii) The quantities \( f(x), f'(x) \) may be calculated by Horner's method
\[ f(x) = (\ldots (a_n x + a_{n-1}) x + \ldots + a_1) x + a_0 \]
Calculate
\[ \begin{align*}
\hat{b}_n &= a_n \\
\hat{b}_n &= \hat{b}_n + a_{n-1} \quad (n = N - 1, N - 2, \ldots, 1) \\
\hat{b}_1 &= f(x) \\
\end{align*} \]
Then
\[ b_1 = f(x) \]
Calculate
\[ \begin{align*}
\hat{c}_n &= b_n \\
\hat{c}_n &= \hat{c}_n + a_n \quad (n = N - 1, N - 2, 3, 2) \\
\end{align*} \]
Then
\[ c_1 = a_n x^{n-1} + a_{n-1} x^{n-2} + a_{n-2} x^{n-3} + \ldots + a_1 x + a_0 \]
(c) Algorithm
Set \( X_{\text{NEW}} = 1.0 \)
Repeatedly calculate
\[ \begin{pmatrix}
X_{\text{OLD}} = X_{\text{NEW}} \\
\hat{b}_1 = b_1 \\
\hat{c}_1 = c_1 \\
\end{pmatrix} \]
until
\[ X_{\text{NEW}} = X_{\text{OLD}} - \hat{b}_1 / \hat{c}_1 \]
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ABS(\(X_{\text{new}} - X_{\text{old}}\)) < SMALL

Then, from (3), calculate

\[ R = 100 \left( \frac{1}{X_{\text{new}}} - 1.0 \right) \]

as the required percentage return.

APPENDIX II

A NUMERICAL EXAMPLE

The following numerical example illustrates the incorporation of risk analysis into the selection of a portfolio of capital investment projects. A budget of £50,000 is available for investment in twelve such projects, each requiring an initial capital outlay of between £5,000 and £17,000. After deciding to make the NPV calculated at a discount rate of 8% the basis of the analysis, each project is considered in turn. The means and standard deviations of their NPV distributions are given in Table I (a).

<table>
<thead>
<tr>
<th>Project</th>
<th>Capital required (£000s)</th>
<th>Mean of NPV (£000s)</th>
<th>S.D. of NPV (£000s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>5.46</td>
<td>0.63</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>7.84</td>
<td>0.85</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>7.56</td>
<td>0.73</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>7.80</td>
<td>0.80</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>9.08</td>
<td>0.75</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>1.99</td>
<td>0.71</td>
</tr>
<tr>
<td>7</td>
<td>12</td>
<td>1.20</td>
<td>1.00</td>
</tr>
<tr>
<td>8</td>
<td>13</td>
<td>1.30</td>
<td>1.00</td>
</tr>
<tr>
<td>9</td>
<td>13</td>
<td>1.00</td>
<td>0.90</td>
</tr>
<tr>
<td>10</td>
<td>14</td>
<td>1.24</td>
<td>1.00</td>
</tr>
<tr>
<td>11</td>
<td>16</td>
<td>1.28</td>
<td>1.00</td>
</tr>
<tr>
<td>12</td>
<td>17</td>
<td>1.57</td>
<td>0.90</td>
</tr>
</tbody>
</table>

The covariances between the NPVs of different projects are calculated next. In this example each project was assumed to be correlated with just one of the other of two indices along the lines discussed in section VI. This made the calculations from the 2 x 2 covariance matrix of these indices particularly simple. Restricting attention to those portfolios with an expected NPV of at least £4,000, the 'efficient set' shown in Table I (b) was obtained.
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### Table I (b)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Projects included</th>
<th>Capital required (£000s)</th>
<th>Mean of NPV (£000s)</th>
<th>S.D. of NPV (£000s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1, 2, 3, 4, 5, 7</td>
<td>50</td>
<td>5.14</td>
<td>2.64</td>
</tr>
<tr>
<td>B</td>
<td>2, 4, 5, 6, 8</td>
<td>50</td>
<td>5.12</td>
<td>2.43</td>
</tr>
<tr>
<td>C</td>
<td>2, 3, 4, 8, 10</td>
<td>50</td>
<td>5.02</td>
<td>2.42</td>
</tr>
<tr>
<td>D</td>
<td>1, 2, 4, 8, 10</td>
<td>48</td>
<td>4.92</td>
<td>2.34</td>
</tr>
<tr>
<td>E</td>
<td>1, 4, 5, 8, 9</td>
<td>50</td>
<td>4.94</td>
<td>2.09</td>
</tr>
<tr>
<td>F</td>
<td>1, 3, 4, 8, 9</td>
<td>47</td>
<td>4.40</td>
<td>2.08</td>
</tr>
<tr>
<td>G</td>
<td>4, 5, 8, 9</td>
<td>45</td>
<td>4.38</td>
<td>1.99</td>
</tr>
</tbody>
</table>

The number of portfolios under consideration has now been reduced from 4,096 to 7. The final choice reflects the firm’s attitude to risk and must be left to the decision-maker(s). Their decision can be made easier by presenting the results of the analysis in a more readily comprehensible form. If the distributions are nearly Normal, approximate values can be obtained directly for the probabilities of different levels of NPV. Table I (c) gives these values for each of the ‘efficient’ portfolios. Alternatively it may be desirable to use simulation to give a fuller picture of these distributions, and possibly to give similar figures for the possible levels of IRR.

### Table I (c). Probabilities of NPV falling below various levels

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Pr(NPV &lt;0)</th>
<th>Pr(NPV &lt;£1,000)</th>
<th>Pr(NPV &lt;£2,000)</th>
<th>Pr(NPV &lt;£3,000)</th>
<th>Pr(NPV &lt;£4,000)</th>
<th>Pr(NPV &lt;£5,000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.026</td>
<td>0.058</td>
<td>0.117</td>
<td>0.212</td>
<td>0.333</td>
<td>0.479</td>
</tr>
<tr>
<td>B</td>
<td>0.018</td>
<td>0.045</td>
<td>0.100</td>
<td>0.192</td>
<td>0.323</td>
<td>0.488</td>
</tr>
<tr>
<td>C</td>
<td>0.019</td>
<td>0.048</td>
<td>0.106</td>
<td>0.202</td>
<td>0.337</td>
<td>0.497</td>
</tr>
<tr>
<td>D</td>
<td>0.018</td>
<td>0.047</td>
<td>0.106</td>
<td>0.202</td>
<td>0.337</td>
<td>0.497</td>
</tr>
<tr>
<td>E</td>
<td>0.010</td>
<td>0.035</td>
<td>0.087</td>
<td>0.189</td>
<td>0.344</td>
<td>0.530</td>
</tr>
<tr>
<td>F</td>
<td>0.017</td>
<td>0.055</td>
<td>0.125</td>
<td>0.251</td>
<td>0.424</td>
<td>0.613</td>
</tr>
<tr>
<td>G</td>
<td>0.014</td>
<td>0.045</td>
<td>0.115</td>
<td>0.244</td>
<td>0.424</td>
<td>0.622</td>
</tr>
</tbody>
</table>

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