PROPOSALS FOR THE STATUTORY BASIS OF VALUATION OF THE LIABILITIES OF LONG-TERM INSURANCE BUSINESS

by


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"Life Assurance is an institution possessing such an important influence in the welfare and well-being of Society that it is most essential the principles upon which it is based and conducted should be such as to ensure its lasting stability and prosperity."

H. W. Manly (1868)¹

1. INTRODUCTION

1.1. Section 78 of the Insurance Companies Act 1974 makes provision for Regulations to be made for the valuation of assets and liabilities. Regulations relating to the valuation of assets, although not published at the time of writing, are expected to be laid before Parliament in the near future.

It is expected that the corresponding Regulations for the valuation of liabilities will be issued later in 1975.

1.2. The Department of Trade has consulted interested bodies, including the Faculty and the Institute, with regard to proposals for the content of the Valuation of Liabilities Regulations and with its permission part of the relevant Consultative Note is reproduced in Appendix I.

1.3. Discussions have so far taken place on the basis of the "six principles" (see paragraph 3.3.5). However, the period of rapid and quite unprecedented change through which we are passing has led to the need to reappraise these principles.

1.4. As a consequence, the Councils of the Faculty and Institute established a Working Party with the following terms of reference. "To consider the desirability and possibility of modification of the method of valuation embraced in the "six principles" (T.F.A., 33, p. 30 and J.I.A., 97, p. 161)² so that:

(a) for the general range of long-term life assurance contracts the value of the net liabilities can be compared with the
368 Statutory Basis of Valuation of the market value of the assets, even during a period of rapid change, to ensure a reasonable standard of adequacy (see J.I.A., 92, p. 76, paragraphs a, b, c) rather than a mere demonstration of solvency, and 

(b) statutory rules for such a valuation can be designed."

1.5. The membership of the Working Party comprised the Authors and Mr. G. E. Barrow, M.B.E., F.I.A., F.S.S. Unfortunately, due to indisposition, Mr. Barrow was prevented from participating in authorship of the paper.

1.6. The Authors wish to make it clear that although this paper results from the deliberations of the Working Party, the views expressed are those of the Authors.

2. OVERSEAS CONTROL SYSTEMS

2.1. Introduction

2.1.1. The United Kingdom is the only major life assurance market in the world where no statutory minimum basis for the valuation of liabilities exists. It has, therefore, for some time, been inevitable that a statutory basis would be imposed in this country.

2.1.2. Before bowing to the inevitable it is, however, appropriate to consider the systems that exist overseas. By analysing their strengths and weaknesses it is hoped that attention can be drawn to those elements of control that tend to stifle innovation and competition and on the other hand to those that ensure the fulfilment of the company's obligations to its policyholders.

2.2. Australia

2.2.1. The first life assurance policies written in Australia were issued by British companies. Subsequently, a local industry developed, largely based on the mutual principle. The local offices quickly extended their operations overseas, reaching the U.K. in the later part of the nineteenth century. The local market has close traditional ties with the U.K. and, therefore, its control system is of some relevance.

2.2.2. Australian legislation provides for a net premium valuation of liabilities at 3 1/2% interest with prescribed bases for mortality and Zillmer adjustment. A statement showing how values of investments are arrived at must be appended to the balance sheet together with a certificate that assets are in the aggregate fully of the value stated. There is no statutory definition of value.
2.2.3. Since, in general, the conduct of the business is on U.K. lines, with a heavy emphasis on traditional with profits, the statutory basis has tended to hold back surplus and restrict expansion. The problem was discussed by Ward⁵ who proposed a system of terminal bonuses to mitigate the problem.

2.2.4. Until recently market value was thought to be the maximum that would enable the asset value certificate to be given but the depreciation resulting from current levels of interest rates, coupled with the conservative liability valuation basis, would have imposed intolerable restrictions on the emergence of surplus. It now appears that redeemable securities may be taken at a value in excess of market value but not exceeding par.

2.2.5. Apart from fiscal measures designed to promote investment in Government Securities and close personal supervision by the Insurance Commissioner there are no specific controls on investments or premium rates.

2.3. Canada and the U.S.A.

2.3.1. The Canadian scene is also of considerable interest as there are a number of substantial companies operating in the U.K. whose world-wide operations are subject to the control of Canadian legislation.⁶ There is a statutory basis for computing the minimum reserve which specifies a maximum rate of interest of 3½% for assurances and 4% for annuities. A number of standard mortality tables are specified, distinguishing between ordinary, industrial and life annuity business. The Superintendent of Insurance has power to approve other mortality tables and higher rates of interest; the actuary requesting such treatment must justify his request. The net premium method is employed, sometimes modified to allow for initial expenses.

2.3.2. The terms and conditions of life assurance contracts are regulated by the laws of the individual provinces of Canada. In general, the practice is to guarantee surrender values. This is different from the U.S. situation where guaranteed surrender values are required by law. This feature has had considerable impact on North American systems for the control and valuation of assets and is well described by Noback⁷ whose description of the historical origins of many of the existing control practices is of considerable interest.

2.3.3. In general, throughout North America there is a very severe restriction on the proportion of life assurance funds that may be invested in equities. On the other hand, it is recognised that a
Statutory Basis of Valuation of the stringent market-value approach ignores the fact that life assurance companies are going concerns. Following the collapse of the New York security markets in 1907 the authorities moved away from the market-value concept and today amortised values are used for some categories of fixed-interest securities, there being differences in detail between Canada and the United States. In both countries provisions exist for the stabilisation of asset values by means of specific reserves.

2.3.4. One of the interesting features of Noback's book is the story of the origin of the use of the net premium system as a method of control in North America.

"In 1858, the first Massachusetts State Commissioner of Insurance, Elizur Wright, challenged the financial statements of companies that determined their policy reserve liabilities by using a gross premium method of valuation. He contended that the gross premium method was not a sound test of solvency and persuaded the legislature to adopt the net level premium method as the legal standard. . . .

"Wright wanted a uniform and stringent method of valuation. His strong stand led to a protracted dispute with the managers of the International Life Assurance Society of London, England. When this Society failed, the gross premium method of valuation was thoroughly discredited and Wright's position strengthened. His standard of valuation and calculation method were widely adopted."

2.4. Europe

2.4.1. One of the most rigid control systems applies in Germany where a 3% net premium valuation is prescribed. However, an explicit Zillmer of 3½% is permitted and in addition negative values can be regarded as assets but they are not admissible for the purpose of determining solvency. In general, assets are taken at the lower of cost and market value and premium bases are prescribed with virtually no scope for variation. There are published lists of approved investments for the technical reserves and a Trustee has to be appointed, any portfolio activity requiring his prior written consent. The Control Authority specifies maximum holdings of equities and properties, these being 15% and 25% of the total portfolio respectively. The overall effect of these controls is to leave very little scope for creativity and innovation.

2.4.2. In some respects a fairly similar situation prevails in Holland but unlike the German authorities the Dutch tend to exercise the discretions contained in their legislation. A net premium valuation is almost universally used, the basis being subject to scrutiny by the authorities. In general, equities and properties are taken, net of
Liabilities of Long-term Insurance Business

investment reserves, at the lower of cost and market value. Loans
and fixed-interest securities are normally valued at par. Although
premium rates are not controlled there is, in effect, a correlation
between premium and valuation bases largely induced by tax consi-
derations. The majority of offices are members of a tariff group
basing their premium rates on 4% interest. There are lists of
approved investments although gilts and near-gilts are accepted
automatically. Control is exercised through a unique system of first
private and then published warnings.

2.4.3. The situation in Switzerland is interesting as, although in
common with most other European countries a net premium basis
is prescribed for valuation of liabilities, specific regulations also exist
for valuation of assets. These provide for fixed-interest securities to
be valued at a rate of interest not less than ½% greater than that used
in the valuation of liabilities.

2.5. Conclusions

2.5.1. In most countries where a statutory valuation basis exists,
a mere test of solvency was discarded long ago as providing inadequate
protection for policyholders. The net premium valuation method
has, therefore, secured almost universal acceptance as a method of
control.

2.5.2. The greatest weakness of many foreign control systems is
their failure to link the valuation of assets to the valuation of
liabilities. Asset values have now fallen to a similar extent to the
1929 crash but at twice the pace. There is no doubt that throughout
the world many offices may have great difficulty in demonstrating
an adequate solvency margin on the basis currently laid down by
the authorities.

3. DEVELOPMENTS IN THE U.K. OVER THE
PAST 30 YEARS

3.1. The Development of Life Office Practice

3.1.1. A control system should not unnecessarily repress com-
mercial initiative and so must take cognisance of the traditions and
structure of the market it seeks to govern. Therefore, at this point,
it is helpful to consider the development of life assurance in the U.K.
during the last 30 years and related actuarial practice.

3.1.2. At the end of the Second World War only a handful of
British offices had been established for less than 50 years and over
one-half had been established for more than 100 years. The Canadian
and Australian offices were beginning to obtain a sizeable share of the market. The life departments of (mainly proprietary) composite offices were beginning to expand.

3.1.3. The liabilities of offices at that time broadly consisted of a significant proportion of predominantly with-profit endowment assurances, whole-life assurances and annuities. Group deferred annuity business was in its infancy. With-profit premium rates contained fairly large bonus loadings, to support reversionary bonuses at the then current rates of about $1\frac{1}{2}\%$. To obtain the higher yield then prevailing, it was considered appropriate for offices to invest what has since come to be called the "estate" in equities, although not much of a move had been made in this direction.

3.1.4. By 1945, within the working lifetime of the older actuaries, all four elements of the actuary's "MITE", that is mortality, interest, taxation and expenses, had given cause for concern. The lean war years had been financed on high rates of tax and low rates of interest, the dangers of which had been officially recognised in 1940 by the limitation of the life office tax rate to $37\frac{1}{2}\%$. The war also involved the risk of heavy mortality costs, as, in general, offices did not include a war risk exclusion clause in their policies. The actuaries and the Board of Trade, as supervisory authority, therefore had reason for some self-congratulation; the insurance companies had weathered the storm, and policyholders had been fairly treated under the supervisory system of "freedom with disclosure".

3.1.5. In the years immediately following the war the Government policy of low-interest borrowing continued, culminating in the issue of Treasury 2\% (1975) at par between October 1946 and January 1947. By the end of 1947 the price had fallen to 82, and apart from a brief period in 1948, has not touched 80 since. In these conditions it seemed clear that closer attention should be paid to the matching of assets to liabilities. Two papers were presented in 1952; by Haynes and Kirton to the Faculty$^9$ and by Redington to the Institute$^{10}$. Although written independently the papers show a close similarity of approach; Haynes and Kirton described "paid-up immunisation" and Redington "full immunisation". Virtually the whole of Redington's paper is apposite to this subject and in particular he stressed three ways in which the solvency of the fund could be jeopardised—failure to immunise, unrestricted war risk cover, and onerous policy options, particularly guaranteed surrender values.

3.1.6. During the 1950's the cult of the equity developed. This led to consideration of the problem of distribution of surplus arising from
investment profits. In 1953 Bayley and Perks addressed themselves
to the subject in their paper to the Institute on "A consistent system
of investment and bonus distribution for a life office". The prices
of ordinary shares advanced and the reverse yield gap first appeared
in August 1959. The first terminal bonus systems made their
appearance and also the unit trust movement began to seek new
avenues for marketing its products, one result being the introduction
of linked life assurance.

3.1.7. The established life offices viewed this last development with
misgiving, feeling that it might endanger tax relief on life insurance
premiums. In the event the Inland Revenue decided that no distinc-
tion should be drawn between unit-linked and traditional life
assurance. In retrospect this decision can be seen to have had a
critical effect on subsequent development of life assurance in this
country. Many of the unit trust companies formed their own life
companies and, not to be outdone, the older offices produced their
own unit-linked policies. A further development was the linking of
benefits to property values as well as traditional equity-based unit
trusts. More recently the managed-fund concept has emerged,
turning the wheel full circle back towards the mixed portfolio of the
traditional offices.

3.1.8. Another aspect of the "marketing revolution" was the
development of single-premium contracts which enable the investor
to enjoy the roll-up of his investment on a favourable tax basis.
These products, remote from the mainstream of traditional life
assurance, were in direct competition with building societies and other
traditional avenues for lump-sum investments.

3.1.9. Although these developments have stimulated the life assu-
rance industry into some much needed changes, they have had two
very damaging consequences. The deliberate exploitation of artificial
life assurance contracts to secure tax reliefs resulted in the imposition
of the "qualifying policy" rules in the Finance Act 1968. This
justified the misgiving referred to in paragraph 3.1.7 and has had the
inhibiting consequences that the self-policing rules, operated until
then by the traditional offices, had been designed to avoid.

3.1.10. The other consequence has been the recent insolvencies
of some offices who, with inadequate resources, had chosen to
disregard basic principles that enabled the traditional offices to
weather earlier storms.

3.1.11. In the traditional life assurance field, the actuaries,
stimulated by the demand for a share in unrealised investment gains
and by the competition from unit-linked business, continued to develop the terminal bonus system in the early 1960’s. Many different philosophies have emerged, ranging from the relatively stable terminal bonus, which treats the bonus as a means of distributing surplus locked up in the valuation basis, to the much more unstable bonuses, associated with giving full effect to market conditions and the resulting fluctuations in unrealised investment appreciation.

3.2. Legislative Developments

3.2.1. In 1958 a new Insurance Companies Act was passed, largely as a consolidation measure, but this was followed in the mid-1960’s by a crop of failures of cut-price motor insurers. The Companies Act 1967 was, therefore, used as a means of introducing considerable “amendments to the law relating to Insurance Companies”. Resulting from these changes new Regulations under the 1958 Act governing statutory returns were made in 1968, but it was subsequently realised that these Regulations were not sufficient to provide early warning of such insolvencies when the Vehicle and General Group collapsed in March 1971, following losses on its motor account.

3.2.2. The report of the Tribunal of Enquiry\(^\text{12}\) into the V & G collapse criticised certain senior Civil Servants for not intervening more quickly. At the time, the prevailing view was that a Civil Servant would be at risk if he intervened without being able to show due grounds, because the intervention might in itself precipitate a collapse. The Tribunal report in February 1972 resulted in a shift in public sentiment towards further protection of policyholders, particularly the life policyholders in composite companies and in subsidiaries of conglomerates.

3.2.3. In February 1971 the Hilary Scott Committee was commissioned to study linked life assurance and its report\(^\text{13}\) was presented in April 1973. Some of the suggestions of the report together with a number of recommendations from the industry were included in the Insurance Companies Amendment Act which was passed in July of the same year and which gave the Secretary of State greatly enhanced powers.

3.2.4. The new Act included power to make Regulations covering a wide variety of subjects including valuation of assets and liabilities. A number of Regulations have already been issued and added impetus has been given by the recent failures referred to in paragraph 3.1.10.
3.2.5. With insurance business being controlled by three separate Acts of Parliament, there was a clear need for consolidation. This has recently been achieved by passage of the Insurance Companies Act 1974.

3.3. The Influence of Europe

3.3.1. Before leaving this historical survey, it is necessary to consider how the strengthening of links with Europe has influenced legislative developments.

3.3.2. Continental life assurance practice differs fundamentally from that of Britain. As discussed in section 2.4 the authorities frequently control the bases used for premiums, valuation and surrender values, although the rules vary widely from one country to the next.

3.3.3. With the establishment of the E.E.C in 1957, it became apparent that the question of harmonisation would require detailed study. Work was undertaken by the O.E.C.D. Insurance Committee eventually leading to publication of the Buol Report. At the same time European insurers carried out their own studies through the Life Working Group of the Comité Européen des Assurances (C.E.A.), which, of course, included representatives from non-E.E.C. countries.

3.3.4. Despite the breakdown of Britain’s first attempt to join the E.E.C. in the early 1960’s, British actuaries were closely involved because they realised that if Britain did eventually enter the E.E.C., the older system of “freedom with disclosure” would not be entirely acceptable to other members. The British were anxious to avoid the need to control premium bases and investments as this would not have been compatible with the with-profits system that formed such a vital element of the U.K. market.

3.3.5. In 1966 Skerman published his five principles for a solvency standard in J.I.A. These were adopted by the C.E.A. Working Group, with the addition of a sixth principle, namely that the reserves must cover any guaranteed withdrawal benefits. The six principles were then put forward as suitable for inclusion in a “life directive”.

3.3.6. Later in the year a European viewpoint was put by Ammeter in two notes in J.I.A. Ammeter commented that Skerman’s proposals to limit the valuation interest rate to the current yield on the fund (less a margin) would not be regarded on the Continent as providing adequate protection; the limit should rather be the rate
376  Statutory Basis of Valuation of the
of interest laid down by the authorities for premiums, which would
have regard to cautious long-term considerations.

3.3.7. The proposed Valuation of Liabilities Regulations are based
firmly on the "six principles", and since they have secured wide-
spread acceptance by European insurers any major change in the
proposals would have repercussions outside the U.K.

4. VALUATION OF ASSETS

4.1. In the Consultative Note relating to the proposed Valuation
Regulations the Department of Trade stated, "It is essential for
assets and liabilities to be valued on consistent bases and Rule 2 of
the proposed Rules for valuing long-term business liabilities provides
the link." This is a fundamental principle with which it is hard to
disagree. It follows that before considering the proposals for valua-
tion of liabilities it is essential to consider the proposals for Valuation
of Assets Regulations.

4.2. Although not published at the time of writing it is known that
the Regulations will cover both general and long-term business. For
long-term business market values will be used for quoted securities
and real property. Subsidiary companies will be valued on a "look-
through" basis and the break-up concept will, in general, apply to
other assets e.g. motor cars and office equipment are valued at 50%
of cost in year of purchase and nil thereafter.

4.3. Consideration of the detail of the asset value Regulations is
outside the scope of this paper, but it should be noted that the full
value of an asset as determined may not in certain circumstances be
admissible and some classes of asset will be wholly inadmissible.

5. VALUATION OF LIABILITIES

"A particular valuation basis may be desirable for many reasons,
but it must be a servant of realities for it cannot be their master."
F. M. Redington

5.1. The Valuation Method

5.1.1. The valuation method suggested in the Consultative Note
is the net premium system. The choice of this method follows
inevitably from the events already outlined.

5.1.2. The net premium method has been subjected to considerable
criticism as a basis for statutory controls. The critics usually argue
that the only valid method for testing solvency is a gross premium
Whilst this may be true it must be remembered that the authorities are not seeking a mere demonstration of solvency but wish to establish a standard of good conduct, i.e. achievement of the reasonable standard of adequacy referred to in the Working Party’s terms of reference. The desire of the authorities to insist on a standard of adequacy seems understandable and reasonable. After all, one of the main purposes of British insurance legislation is to prevent insolvencies. Thus, failure to reach the required standard of adequacy would not, in itself, mean that the company was insolvent but would act as a warning signal to enable remedial action to be taken. On the other hand, if the authorities employ a simple solvency test this will not prevent failures, as in any case where the test is failed, by definition it is too late for remedial action to be taken.

5.1.3. Another objection to the net premium system is that the provisions for future bonuses and future expenses are implicit, that is not specifically defined. On the other hand, the net premium system has great attractions as a measure of good conduct in that it does not capitalise profits arising from future premium payments and thus could be said “to take some account of policyholders’ reasonable expectations.” Indeed, it can be argued that it is not the function of a control system to seek to make specific provision for future bonuses and that the mere act of preventing a company from taking credit for future bonus loadings is quite adequate. The alternative to this approach is to make provision for explicit solvency margins in a gross premium valuation. The dangers of such measures, particularly when applied as percentage increments to calculated reserves, are well illustrated by Ammeter.

5.1.4. Another criticism of the net premium method arises from the effects of a change in the mortality basis. Adoption of a lighter mortality table for assurances does not necessarily result in a reduction in reserves as would occur in a gross premium valuation. The precise effects are dependent upon the shape of the mortality curve and the results are, therefore, to some extent unpredictable. This feature was discussed as long ago as 1868 by Manly who provided many interesting examples. However, in current conditions, it is felt that this objection should not be accorded undue weight. Although the phenomenon exists its effects are relatively trivial when compared with the effects of the large changes in interest rates which have now come to be regarded as normal.

5.1.5. The greatest objection to the net premium system is that it is not possible to secure complete consistency between the valuation
of assets and the valuation of liabilities using the conventional valuation formula. If the market rate of interest increases this is manifested by a fall in asset values. Use of a corresponding rate of interest in a net premium valuation does not necessarily give rise to a proportionate decrease in the calculated liability. The reason is that as the valuation rate is increased the net premium valued is decreased.

5.1.6. Current yields are at historically high levels, with correspondingly low asset values. Fears have been expressed that for the reasons given in 5.1.5. the net premium method is likely to prove unduly stringent in today's conditions. Various modifications have, therefore, been suggested and these are developed in the following section.

5.2. Modifications to the Net Premium Valuation Formula

5.2.1. The formula generally used in the net premium valuation of a non-profit endowment is:

\[ S \cdot t \cdot V_{x+n} = S \cdot A_{x+t; n-t} - S \cdot P_{x+n} \cdot d_{x+t; n-t} \]

where \( S \) is the sum assured
\( t \) is the duration at valuation date
\( x \) is the age at entry
\( n \) is the term of the policy at entry.

The formula assumes that the sum assured is paid at the end of the policy year in which death occurs, that premiums are payable yearly in advance and that the policy is valued immediately before the \((t+1)\)th premium falls due.

This equation of value can be rewritten in the form:

\[ V_1 \cdot A_{x+t; n-t} + \left( \frac{i}{1+i} \cdot V_1 + S \cdot P_{x+n} \right) d_{x+t; n-t} = S \cdot A_{x+t; n-t} \]  \( (1) \)

where \( V_1 \) represents the policy value for a sum assured of \( S \). This expresses the fact that the future income required to provide the sum assured on death or maturity can be regarded as the annual premium plus the interest income from \( V_1 \) both accumulated at further interest, with \( V_1 \) itself being returned at the expiry of the policy.

5.2.2. This method of valuing the liability can be criticised on the grounds that being based on a single rate of interest it takes no account of the fact that the assets representing the liability are invested to earn interest at a rate currently ascertainable, while future income will fall to be invested at rates which are not known but which it is prudent to assume will average out at less than that
currently being earned. Let the first of these, the current return on
assets, be denoted by \( g \) and the best estimate of the second, the
expected return on future investments, by \( i \). Strictly speaking \( i \)
should vary with term to run but for simplicity an average rate will
be assumed applicable to all terms.

5.2.3. One possible modification of (1) which would allow for the
different rates of return would be as follows:

\[
V_2 \cdot A_{x+t: n-t}^t + \left( \frac{g}{1+i} \cdot V_2 + S \cdot P_{x:n}^t \right) \ddot{a}_{x+t: n-t}^t = S \cdot A_{x+t: n-t}^t
\]

This assumes that all future income including the income from \( V_2 \),
namely \( gV_2 \), is reinvested at \( i \). The equation of value reduces to:

\[
V_2 = V_1 \div \left( 1 + \frac{g - i}{1+i} \cdot \ddot{a}_{x+t: n-t}^t \right)
\]

It is self-evident that if \( g > i \), then \( V_1 > V_2 \).

5.2.4. Another possible modification takes the form:

\[
V_3 \cdot A_{x+t: n-t}^t + \frac{g}{1+g} \cdot V_3 \cdot A_{x+t: n-t}^t + S \cdot P_{x:n}^t \cdot \ddot{a}_{x+t: n-t}^t = S \cdot A_{x+t: n-t}^t
\]

Here the assumption is that the income from \( V_3 \) can be applied as the
annual premium for a new policy for the balance of the term with
an office which bases its premiums on \( g \), i.e. that the income from \( V_3 \)
can be invested at \( g \). This equation reduces to:

\[
V_3 = V_1 \cdot \frac{A_{x+t: n-t}^t}{A_{x+t: n-t}^t}
\]

which is the paid-up sum assured calculated at \( i \) and discounted
at \( g \). It is clear from the development of the respective equations
of value that when \( g > i \), \( V_2 \) is always greater than \( V_3 \).

5.2.5. A further suggested formula is derived from:

\[
V_4 \cdot A_{x+t: n-t}^t + \left( \frac{g}{1+g} \cdot V_4 + S \cdot P_{x:n}^t \right) \ddot{a}_{x+t: n-t}^t = S \cdot A_{x+t: n-t}^t
\]

which assumes that the premiums and the interest on \( V_4 \) can both
be invested at \( g \). Therefore \( V_4 \) clearly gives the lowest values of all
the formulae considered so far. The equation reduces to:

\[
V_4 = S \cdot A_{x+t: n-t}^t - S \cdot P_{x:n}^t \cdot \ddot{a}_{x+t: n-t}^t
\]

which is the normal net premium reserve, \( V_1 \), calculated at \( g \), but
using a net premium calculated at \( i \).
5.2.6. The successive adjustments to the equation of value show that when $g > i$:

$$V_1^g > V_2 > V_3 > V_4$$

As a general rule $V_1^g > V_2$, since a paid-up sum assured is usually greater if calculated at a higher rate of interest. Sometimes $V_2 > V_1^g$ and sometimes the opposite applies. In order to investigate the behaviour of the four possible methods, various calculations have been made. The results are discussed in the next section.

5.3. **An Examination of Some Numerical Results**

5.3.1. The effect of changes in interest rates on the value of assets of various terms is shown in Table A of Appendix II. It is assumed that in each case the asset is a fixed interest stock with a coupon of 4.8% gross (3% net). The net rate of interest shown in the table represents the net redemption yield and not the gross redemption yield net of tax. (See 5.4.8).

5.3.2. Table B of Appendix II shows the effect of changes in the net interest rate on the unadjusted net premium reserves ($V_1$) for various different typical policies. Non-profit contracts have been considered in order to simplify the illustrations. By setting out the two tables separately, no attempt has been made to hypothecate a particular term of investment to a policy of a particular outstanding term, or indeed a particular valuation rate to a particular current yield on assets ($g$).

5.3.3. The chief criticism of the net premium method (see paragraph 5.1.5) is that it is not sufficiently sensitive to changes in the rate of interest and hence is too strong at a time of rising interest rates in the context of what would normally be regarded as a reasonably matched asset situation. In particular, this objection is raised in circumstances when full immunisation can be assumed and hence valuation is made at rate $g$. It is also open to the converse criticism that it does not react sufficiently to a reduction in the rate of interest. An examination of Tables A and B in Appendix II shows that, contrary to expectations, the net premium reserve ($V_1^g$) is far from being too stringent at high interest rates and the longer outstanding terms. For example, consider the case of the policy value $5V_{30}$. If it is assumed that the liability is matched by a holding of an irredeemable stock, a change in the interest rate from 3% to 4% net gives rise to an asset value of 750 and a liability of 817, a net deficit of 67. However, as the interest rate rises the deficit reduces until it is totally eliminated when the interest rate reaches 8% net. Subsequent
increases in the interest rate give rise to a profit. The origin of the phenomenon is that, as interest rates rise, the mean term of the asset-income (using Redington's terminology) reduces faster than the mean term of the liability-outgo. At very high rates of interest it is only possible to immunise policies of relatively short outstanding terms.

5.3.4. The chief problem therefore appears to be with offices whose business is generally of a short term to run, where the net premium method becomes progressively more stringent as interest rates rise. Conversely, offices with longer term portfolios may be lulled into a false sense of security by mis-matching profits when interest rates rise to a high level, and find themselves in difficulties if interest rates subsequently fall.

5.3.5. Before examining the proposed modifications to the net premium method, it is worth remarking on the effect of the requirement to have a margin of 10% in the interest rate, with a minimum of ½% net. Table C of Appendix II shows the percentage increase in the reserve, which the interest margin would produce. Generally speaking, the higher the interest rate and the longer the term to run, the greater will be the implicit margin in the reserves. The effect will tend to offset the mis-matching profits in paragraph 5.3.4.

5.3.6. For a mixed portfolio of policies of short, medium and long outstanding terms, the strains and releases introduced by use of the net premium method may well roughly cancel out. Nevertheless, it would be preferable to find a valuation formula which would fit the "matched" asset values across the whole spectrum of interest rates and outstanding terms, if it exists.

5.3.7. Examples of the first modification discussed (V2, paragraph 5.2.3), are set out in Appendix III in tables based on different assumptions for the long-term interest rate i. The thinking behind this modification differs from that adopted hitherto, since the assumption is that the valuation should be made at the long-term interest rate, but allowance made for the higher yield on existing reserves. In other words, the method veers towards the assumption of paid-up policy matching rather than full immunisation.

5.3.8. It is interesting to compare the results shown in Appendix III with V1. A line has been drawn in each of the three tables, to the left of which, V2 ≤ V1 and to the right of which, V2 > V1. It can be seen that for low values of i, V2 exhibits a greater degree of sensitivity to changes in g in those areas where V1 is particularly insensitive.
On the other hand, \( V_2 \) provides stronger reserves in those areas where \( V_1^g \) is weak. The reason for this is that in the first case the use of a net premium calculated at rate \( i \) is the dominant effect, whereas for the longer terms to run, the restriction of the reinvestment assumption is more important. As \( i \) increases the use of a net premium calculated at rate \( i \) becomes less significant and for \( g > i = 10\% \), \( V_2 \) is always \( > V_1^g \).

5.3.9. The findings discussed in the foregoing paragraph lead to the paradoxical conclusion that a strengthening of the reinvestment assumption can result in a lower reserve. This is due to the corresponding increase in the net premium valued and thus it is clear that if this method is adopted it is particularly important that a limit should be placed upon the premium valued, i.e. the net premium should not exceed the office premium less a loading for future expenses.

5.3.10. The next modification to consider is the formula \( V_3 \) which, as mentioned previously, in effect represents the paid-up sum assured calculated at rate \( i \), multiplied by a discount factor at rate \( g \). One obvious effect is that low rates of \( i \) will generally give lower paid-up sums assured and hence lower reserves. Table A of Appendix IV gives values of \( V_3 \) for \( i = 3\% \). It will be seen immediately that in all cases the reserves appear far too weak.

5.3.11. Although not tabulated in Appendix IV, values of \( V_3 \) have been calculated for \( i = 6\% \) and \( 10\% \) and in both cases the low values obtained indicate that the reserves are highly sensitive to the difference between \( i \) and \( g \). It is quite clear that for the formula to give satisfactory results the relationship between \( i \) and \( g \) must be tightly constrained. Experiments have been carried out in an attempt to evolve a satisfactory relationship and the results are given in Tables B and C of Appendix IV. In the first case \( i \) was defined as \( (g - 0.0025) \) and in the second case as \( 0.9g \).

5.3.12. It will be seen from Tables B and C that both definitions of \( i \) have the effect of producing values of \( V_3 \) slightly lower than \( V_1^g \) at low rates of interest but as \( g \) increases the difference narrows so that \( V_1^g \) approximately = \( V_3 \) at high rates of interest. The relationship \( i = 0.9g \) gives results that are more in accord with the movement of asset values but use of the formula in practice is easier to justify on the grounds of expediency than theoretical considerations.

5.3.13. The final formula developed in section 5.2 (\( V_4 \), paragraph 5.2.5) involves an adjustment to the net premium valued. For a
given value of \( i \), changes in \( g \) exhibit similar characteristics to a gross premium valuation since the premium valued does not vary with the rate of interest. Since the immunisation theory depends upon a gross premium valuation method it might be thought that this modification is most likely to achieve proportionate changes in liabilities and suitably matching assets. Table A of Appendix V gives values of \( V_4 \) assuming \( i = 3\% \). The weakness of the method is clearly exposed by the enormous negative values. The feature arises because this valuation method, like the gross premium valuation method, is highly sensitive to the difference between the interest rate underlying the premiums valued and the valuation rate.

5.3.14. If this modification is to be used in practice the relationship between \( i \) and \( g \) must be tightly controlled in the same way as for \( V_3 \). Tables B and C of Appendix V show the effects of defining \( i \) as \( (g - 0.0025) \) and as \( 0.9g \) respectively. The tabulated figures show that in both cases there is an undesirable weakening of reserves at the long terms to run, the first definition of \( i \) giving slightly improved results. It is doubtful whether the modification \( V_4 \) should be allowed, even with such a constraint, it is certainly not suitable for general application.

5.3.15. One interesting feature of the calculations shown in Table A of Appendix V is that, in cases where a negative reserve emerges further increases in \( g \) may reduce the negative reserve. This result arises because as the interest rate increases towards infinity, the reserves must tend to return to zero from their previously negative values.

5.3.16. A study of the figures tabulated in the Appendices is necessarily subjective and their interpretation is inevitably difficult. The figures would, however, appear to suggest that:

1. The net premium method is not as stringent at very high rates of interest as is popularly supposed.
2. None of the suggested modifications to the net premium method would appear to merit general application.
3. The gross premium method of valuation is most unsuitable.

5.4. Valuation Rate of Interest — General Considerations

5.4.1. As pointed out in paragraph 5.2.2, the use of a single valuation rate of interest carries implications both for the assumed return on investments currently held and for the reinvestment rate which will be achieved in respect of future positive cash flow. In
selecting a valuation rate, therefore, the return currently being obtained and resulting from past investment policy, together with expectations regarding the future investment returns to be available (in different investment areas and from future investment policy), must be borne in mind. Furthermore, the extent to which the fund can be considered immunised is an important factor and this will be discussed in paragraph 5.4.5 et seq.

5.4.2. A conservative view would be to fix the maximum interest rate allowed by reference to the income return from admissible assets taken at their market value subject to an increase, in order to take account of the writing-up of fixed-interest securities to their redemption value. A margin should be taken to guard against the possibility of the rate not being maintained during the reinvestment period.

5.4.3. The arguments for this approach are:

1. It takes a realistic view of the state of the fund as at the valuation date.
2. It takes no credit for future growth in dividends or rents, which may not materialise.

5.4.4. There are two grounds on which it might be held that substantially higher rates can be justified. Firstly, it may be argued that investment in equities and property is made only because they are expected in the long run to perform better than gilts and it is unrealistic to insist on only the current income return in conjunction with the market value of the asset. To this it can be replied that it is wrong to anticipate such higher returns in view of the uncertainty of receiving them and that a more desirable approach is to allow them to emerge as future surplus to accord with the investment policy chosen. In particular, in current conditions, the reverse yield gap may be not so much a measure of investors’ expectations of future growth in income as an insurance premium against the risk of hyper-inflation.

5.4.5. Secondly, it may be argued that if careful attention has been paid to immunisation of guaranteed liabilities (including declared bonus) so that the assets consist of (1) gilt-edged investments performing the immunisation function and (2) other assets (which may also include some gilts) providing hoped-for future profits for with-profit policyholders (and shareholders, if appropriate), then it is justifiable to value the immunised assets and liabilities at the current gilt-edged rate and treat the market value of other assets as free reserves.
5.4.6. The second argument is attractive but open to serious objections. To begin with there are a number of unrealistic assumptions underlying immunisation theory—notably that the yield curve is a horizontal straight line which moves only vertically upwards or downwards, and that assets of any desired term may be purchased in any quantity required. The departure of the real situation from this ideal world would make reliance on immunisation dangerous. We have recently seen the yield curve with values of 10% and 17% on the two extremes, which emphasises the unreality of the assumption.

5.4.7. Immunisation is against small interest rate fluctuations only, and we appear now to be living in circumstances where rates fluctuate rapidly and sharply. Maintenance of the immunised condition would involve an extremely active investment policy. This would undoubtedly lead to very real losses as trading would be without regard to market conditions.

5.4.8. The theory ignores the effects of taxation. Due to the tax-free nature of the redemption profit of a gilt-edged security the effective tax rate is not constant but varies as a function of the gross redemption yield (except for irredeemable investments and cash). For the immunisation theory to apply for taxable business, the net rate of interest used for the valuation of liabilities must be based upon net redemption yields.

5.4.9. At sufficiently high rates of interest it is not possible to immunise annual premium business (as is discussed in paragraph 5.3.3).

5.4.10. Due to the obvious difficulty of maintaining a truly immunised position it could be argued that detailed evidence as to the matching of assets and liabilities should be published so that the Actuary can substantiate any assertions he makes regarding immunisation.

5.4.11. The dangers are perhaps best summed up in Redington’s own words: “There should be added a clear warning that the whole examination is theoretical. Not only are there many and serious difficulties in giving practical effect to the theory (amounting in many circumstances to impossibility), but the extent to which it would be wise in practice to adopt the theory is a matter for consideration.”

5.4.12. In conclusion it is felt that unless a margin is taken as referred to in 5.4.2, only in the most exceptional circumstances would it be justifiable in the conservative view for the valuation rate of interest to correspond to the full income return on assets.
5.5. Valuation Rate of Interest—The Department's Proposals

5.5.1. At this point it is appropriate to consider the Department of Trade's proposals with regard to the valuation rate of interest. These are given in Rule 2 of Appendix I.

5.5.2. It should be noted that the proposals permit the use of a higher rate of interest than the maximum considered desirable in the conservative view referred to in paragraph 5.4.2. This arises from the permitted calculation of the maximum rate of interest by reference to actual interest earnings on assets valued at a figure less than market value. This proposal is unsound and its use can disguise an insolvent position as is demonstrated in the following paragraph.

5.5.3. Consider two offices A and B, the business of each consisting of identical single-premium capital redemption policies for 1,000 maturing 20 years from the valuation date. Suppose that the assets of the two offices are identical and that they have a value of 420 determined in accordance with the proposed Rules. Office A has been very conservative in its asset valuation and shows a fund of 235 from which the yield is 8.5%. Ignoring tax and expenses, the proposed Rules would permit the liabilities to be valued at 7.65% giving a figure of 229. Thus a surplus of 6 would be disclosed and an apparent margin retained of no less than 185 between the statutory maximum value of the assets and the balance-sheet value. On the other hand, office B has valued its assets at nearly the maximum allowed, showing a fund of 400. According to the proposed Rules the yield is 5% and the office will be required to value its liabilities at a rate not exceeding 4.2% giving a minimum figure of 439. Thus A appears abundantly solvent and B insolvent although the real financial positions of the two offices are identical.

5.5.4. The suggestion that the maximum rate of interest should be calculated by reference to actual interest earnings is also open to criticism. Admittedly, the Actuary is given certain powers to adjust the calculated rate of yield if it does not give a proper indication of the rate of investment income to be expected in the future. However, in the case of most funds it could be argued that the calculation would not give an appropriate result and thus the proviso virtually nullifies the Rule.

5.5.5. In general, actual interest earnings are not appropriate as they are not necessarily related in any way to the investments held at the valuation date. Furthermore, the flow of investment income
Liabilities of Long-term Insurance Business

can be severely distorted by outside influences such as the postponement of dividend payments by companies that occurred upon the introduction of Advance Corporation Tax.

5.5.6. It is felt that a more realistic indication of future investment income could be obtained by reference to an "in force" figure. The overall rate of yield on the fund would be determined by reference to the yields (to redemption in the case of redeemable securities) of the individual admissible investments, weighted by their market values. The calculations should be on the basis of the expected income for the subsequent year where it is secured by a contractual obligation or in other cases the income receivable by the present or previous owners of that investment during the preceding year.

5.5.7. Having considered the determination of the overall yield of the fund, it is now appropriate to consider the rates of interest to be used for valuation of individual contracts. The proposed rules merely provide that the weighted average of the rates of interest should not exceed a specified maximum. Unfortunately, this is unsound because the weight, being based on the liability at the interest rate used, is not a first-degree function of the rate of interest. The following example demonstrates the dangers.

5.5.8. Office B, referred to in paragraph 5.5.3, worried by its apparent insolvency decides to amend its valuation basis. Half its portfolio consists of old series capital redemption policies for 500 and the other half, new series policies. It proposes to value the old series at 2% giving a reserve of 336 and the new series at 11% giving a reserve of 62. The weighted average rate of interest used is only 3·4%. Thus, although using an apparently "conservative" interest basis, it manages to demonstrate a "surplus" of 2.

5.5.9. Another danger of the principle of the weighted average is that it can be used as a device to circumvent the 6th Principle. Any business subject to guaranteed surrender values could be valued at a sufficiently low rate of interest to ensure that the individual reserves always cover the guarantee. This then enables the remainder of the business to be valued at an artificially high rate of interest.

5.5.10. It is clear that a limit should be placed on the rate of interest used for valuing any individual policy. This could easily be achieved by providing that use of a rate of interest in excess of the overall rate of yield on the fund is only permissible where it is possible to hypothecate assets amongst the various classes of business so that, for each valuation rate of interest used, the yield on the assets
388 Statutory Basis of Valuation of the hypothecated to that business is not less than the valuation rate plus the specified solvency margin.

5.5.11. The final element in the calculation of the gross valuation rate of interest is the solvency margin of 10% of the yield (or 0.8% p.a. gross if greater). This is directly based on the recommendations of the C.E.A. to the E.E.C. Commission. The effects of this margin are shown in Table C of Appendix II. The general pattern of results appears satisfactory. However, the principle of defining a margin of this nature is open to question as there is always a danger that instead of being taken as a minimum it becomes the norm.

5.5.12. So far only the gross rate of interest has been considered. The proposed rules envisage deduction of the “appropriate” rate of tax for business taxed on an (I—E) basis. The term “appropriate” is not defined but it is to be hoped that an “effective” rate of tax may be used that recognises the actual incidence of tax on investment income.

The relationship between gross and net yields for redeemable securities has been referred to in paragraph 5.4.8. Most offices take cognisance of this feature in their investment policy and in many circumstances will sacrifice gross yield in order to secure a higher effective net yield.

5.5.13. Another important tax consideration is the existence of unrelieved management expenses. Quite properly, the proposed Asset Valuation Regulations will not allow such future tax credits to be taken into account as an asset. On the other hand, it would be reasonable to take credit for unrelieved E by discounting at a gross rate of interest in the valuation formula for the period it is anticipated that it would take for the unrelieved E to run off.

5.5.14. In conclusion it is felt that the amount of allowance to be made for tax in arriving at the valuation rate of interest must be at the discretion of the Actuary. On the other hand, it would be appropriate to require the Actuary to justify the use of an effective rate of tax less than the full rate to which the life fund is subject, in circumstances where grossing up by the full rate would result in an interest rate in excess of the permitted maximum.

5.6. Rates of Mortality and Disability

5.6.1. The intention behind the proposed Rule 3 in Appendix I is not clear. The suitability of a particular mortality table for a particular portfolio of business would seem to be a matter that the Actuary who is familiar with the experience of that portfolio is best fitted to decide. It is to be hoped that the Secretary of State will prescribe
Liabilities of Long-term Insurance Business 389

certain tables as standards and if the Actuary wishes to make use of another table a justification of his choice must be included in the Statutory Returns. Any suggestion of prior approval being needed for the use of a non-standard table would seem to lead to unnecessary additional work for an already overburdened department.

5.7. Acquisition Expenses

5.7.1. The proposed maximum Zillmer adjustment of 3% of the sum assured seems quite adequate in normal conditions if no deduction has to be made for tax. It has been argued that a corresponding premium-related measure is required to cope with premium-related commission systems. This would appear unnecessary as the existing formula allows more than adequate scope for a premium-related adjustment in most circumstances.

5.8. Future Expenses

5.8.1. Proposed Rule 6 places the Actuary in a difficult position. Even if hyper-inflation is not expected it is nonetheless “foreseeable”. Furthermore, since the term “rate of expenses” is not defined and, indeed, is hardly capable of definition it is felt that the margin referred to in Rule 6 should be “sufficient in the opinion of the Actuary to take account of increases in the rate of expenses in the future”.

5.9. Guaranteed Discontinuance Values

5.9.1. Rule 7, of course, attempts to give effect to the 6th Principle. At present this is a sensitive issue as the truth of Redington’s warning referred to in paragraph 3.1.5 has recently been fully demonstrated. The proposed Rule does not, however, insist that the value of each contract will not be less than the amount of any surrender value, as it allows the Actuary to make provision “in such other way as is in his opinion appropriate”. It has been suggested that in certain circumstances it would be reasonable to make provision of an amount less than the surrender value.

5.10. Special Types of Contract

5.10.1. There are several types of contract for which the proposed Valuation Rules will not be suitable. The main category is unit-linked business which normally includes neither a bonus loading nor an interest guarantee. Clearly a net premium valuation method is not, in general, suitable for this type of business, but this is provided for in Rule 8.
5.10.2. Another class of business not referred to in the proposed Rules is temporary assurances. A simple net premium valuation of such business does not provide an adequate solvency standard. The considerations are similar to those for some types of general business except that the amount of the claim is defined and, therefore, the claims experience is not affected by inflation. It is felt that for temporary assurances some form of explicit solvency margin is required.

5.10.3. In general, the Authors have concentrated their attention on annual premium whole life and endowment assurances. Time has not permitted a full study of other classes and, in particular, any special problems relating to Group Pensions, Industrial Life Assurance and Permanent Health Assurance have not been considered.

6. CONCLUSION

"The opinion has been expressed, that the net premium method is one which must inevitably disappear before the advance of true actuarial science. Circumstances, however, seem to favour its continuance."

J. Sorley (1878)17

6.1. The original objective of the Working Party was to seek a modification to the valuation formula that would enable the value of the net liabilities to change with the rate of interest in proportion to corresponding changes in the value of the assets required to immunise the business. The Working Party has failed in this task for the good reason that it is impossible. No one asset or group of assets can immunise a portfolio against large changes in the rate of interest. Unless, therefore, the portfolio of investments is constantly switched, large changes in the rate of interest are bound to result in profits or losses regardless of the liability valuation method adopted.

6.2. The search for a liability valuation basis that on the change in rate of interest leads to proportionate changes in asset and liability values is only capable of satisfaction where the incidence of asset income is exactly equal and opposite to the incidence of liability outgo. This is, of course, the concept of "matching valuation" that was discussed by Benjamin18. However, those that would advocate the use of such a technique as a solvency standard should consider the quotation at the head of Part 5. Since, in practice, this technique has so far only secured limited acceptance it is not realistic to contemplate such a radical departure from normal actuarial practice as a "matching valuation".
6.3. The figures in Appendix II demonstrate that where an office will have difficulty in coping with a large change in interest rates if the Statutory Basis is used, to a considerable extent this is a function of the very real strains to which it will have been exposed. A basis which provides an "acceptable" result can often represent an unacceptable weakening of the valuation assumptions.

6.4. Nevertheless, it is hoped that where an office fails to meet the required standard the authorities will take a sympathetic view of the situation. The most severe financial strain to which a company is subject in inflationary conditions is the runaway escalation of expenses. In order to run its business at an economical level, in broad terms, a company needs to expand its business at the rate of inflation. If an unnecessary standard of adequacy were imposed, resulting in a limitation of expansion, this could in the long term pose a much more serious threat to the company's solvency than the original failure to meet the valuation standard.

6.5. The relative freedom of the United Kingdom market has led to the development of a number of different approaches to the problems of serving the widely differing needs of the public. No one set of Valuation Rules can satisfactorily cover every special case and the Department's proposals must, therefore, be seen as a compromise. Indeed, it could be argued there is a danger that in trying to satisfy all, there may be a failure to satisfy any. Undoubtedly, there will be soundly run companies whose existing practices will not fit easily into the format of the proposed Regulations. As an example, there are a number of offices who for many years have published gross premium valuations in the interests of providing more readily understandable results for their policyholders and the general public. On the other hand, it should be remembered that the proposed Valuation Rules are not intended to provide a basis for the publication of results nor for the distribution of surplus.

6.6. In conclusion it is hoped that the Department will continue to accord special treatment to offices whose particular circumstances will lead to difficulty in complying with the proposed Rules. Ample time should be afforded for completion of the necessary changes in systems and procedures. An unreasonable regard for the letter of the law can only lead to a degree of trouble and expense that will adversely affect the ultimate return to policyholders.
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Liabilities of Long-term Insurance Business


The following papers and publications although not referred to in the text are considered by the Authors to be relevant to the subject matter of the paper. This list is not intended to be exhaustive.

APPENDIX I

PROPOSED RULES FOR VALUING LONG-TERM BUSINESS LIABILITIES

The following Rules were included in Annex B to the Department of Trade's Consultative Note 10 relating to the Insurance Companies Amendment Act 1973 and are published with the Department's permission. The Rules were put forward as a basis for consultation and not as a draft of Regulations.

Rules

(1) The value of a long-term business contract shall be not less than the difference between the present value of the sums payable by the company under the contract according to the contingencies upon which they are payable, including any bonus or addition thereto made before the valuation date, and the present value of the future premiums, if any, calculated in accordance with these rules.

(2) The weighted average of the annual rates of interest used in calculating such present values for all contracts in force on the valuation date shall be lower than the overall rate of yield on the fund by an amount at least one-tenth of that rate of yield and not in any case less than 0.8%, together with such further amount as in the opinion of the actuary is necessary to take account of any foreseeable decrease in the yield on the fund during the term of the contracts in force on the valuation date. For the purpose of this rule:

(a) the weighted average rate of interest shall be obtained by applying as weights to each rate of interest used (grossed up to allow for the appropriate rate of tax when the business is taxed on the basis of investment income less expenses) the total value of the contracts valued in accordance with those rules at that rate of interest, and

(b) the overall rate of yield on the fund shall be obtained by reference to the amounts of interest, dividends, rents and amortisation of redeemable fixed-interest investments credited or debited to the appropriate long-term business revenue account during the year immediately preceding the valuation date and to the balance of the fund shown in that account at the beginning and end of that year. When the value of the assets, including net current assets, of the long-term
Liabilities of Long-term Insurance Business

business at the end of that year calculated in accordance with the rules prescribed is lower than the balance of the fund shown in the account, or where changes in the investments or in the balance in that account during the year were such that the investment income credited does not, in the opinion of the actuary, give a proper indication of the rate of investment income to be expected in future, the overall rate of yield on the fund may be adjusted by him to correspond with those changes.

(3) The rates of mortality and disability to be used for any class of contract shall be rates according to tables recognised as appropriate for the purpose by the Secretary of State.

(4) The premiums to be valued shall, except as provided in subsections (5) and (6), be such premiums as according to the rate of interest and rates of mortality etc. employed in valuing the contract are sufficient to provide for the sums payable by the company under the contract according to the contingencies upon which they are payable, exclusive of any addition thereto for profits, expenses and other charges.

(5) In order to take account of acquisition expenses, it shall be permissible to make an addition to the annual premium to be valued at an amount not greater than the annual equivalent, taken over the whole period of premium payments and calculated according to the rate of interest and rates of mortality etc. employed in valuing the contract, of 3% of the sum assured under the contract.

(6) The difference between the premiums to be valued, including any addition made under subsection (5), and the premiums payable under the contract shall, together with the provision made for expenses under contracts where there are no future premiums, be adequate in relation to the company's current rate of expenses other than acquisition expenses (reduced to allow for the appropriate rate of tax where the business is taxed on the basis of investment income less expenses) and shall contain a margin sufficient in the opinion of the actuary to cover any foreseeable increase in that rate of expenses in future.

(7) The actuary shall make such provision as will in his opinion ensure that the minimum standards prescribed in the above rules may be maintained in future notwithstanding that surrender or paid-up values may be guaranteed under any contract. The
provision shall be made by ensuring that the value placed on each contract is not less than the amount of any surrender value or the present value of any paid-up benefits guaranteed under the contract or in such other way as is in his opinion appropriate. Any negative policy values shall be excluded.

(8) For special types of contract in respect of which any of the above rules is inappropriate, the rules may be modified in a manner appropriate to the special terms of such contracts, provided that the method of valuation adopted by the actuary makes provision which is in his opinion adequate for the sums payable by the company under the contracts, for meeting the company's expenses in connection with the contracts and leaves a margin for contingencies consistent with that provided by the above rules.
### Liabilities of Long-term Insurance Business

#### APPENDIX II

##### TABLE A
Asset values at various net rates of interest

\[
(V = 30a_{\bar{m}} + 1000 \cdot e^n \text{ at rate } g)
\]

<table>
<thead>
<tr>
<th>Term</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
<th>6%</th>
<th>8%</th>
<th>10%</th>
<th>12%</th>
<th>14%</th>
<th>16%</th>
<th>20%</th>
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<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
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<td>874</td>
<td>800</td>
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<td>676</td>
<td>622</td>
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<td>751</td>
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##### TABLE B
Net premium reserves at various net rates of interest

\[
(V_{1}^g = 1000 \cdot V_{x|m}^g / V_{x|3\%}^g)
\]

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<th>3%</th>
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<th>5%</th>
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##### TABLE C
Percentage increases in $V_1$ allowing for interest margin of 0.1g or $\frac{1}{2}$%, whichever is greater

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### APPENDIX III

**Statutory Basis of Valuation of the**

Values of $V_2 = \frac{1000}{t\bar{V}_x^{3\%}} \left[ \frac{t\bar{V}_x^{3\%}}{1 + \frac{g - i}{1 + i} \cdot d_x^{3\%}} \right]$ A49/52 ult.

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Liabilities of Long-term Insurance Business

APPENDIX IV

Values of $V_3 = \frac{1000}{t \cdot 3\%} \left[ \frac{A^g_{x+t:n-t}}{A^0_{x+t:n-t}} \right]$  A49/52 ult.

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<th>14%</th>
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Table B

Interest rate $i = g-0.0025$

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Table C

Interest rate $i = 90\%$ of $g$

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**APPENDIX V**

Values of $V_4 = \frac{1000}{t_{x}^{3\%}} \left[ \frac{A_x^{g}}{x+t:n-i} \cdot p_x^{i} \cdot A_x^{g} \right] \text{A49/52 ult.}$

### TABLE A

**Interest rate $i = 3\%$**

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### TABLE B

**Interest rate $i = g - 0.0025$**

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<tr>
<td>$5V_{30}$</td>
<td>1000</td>
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<td>$35V_{30}$</td>
<td>1000</td>
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<tr>
<td>$45V_{30}$</td>
<td>1000</td>
</tr>
</tbody>
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### TABLE C

**Interest rate $i = 90\%$ of $g$**

<table>
<thead>
<tr>
<th>Policy values</th>
<th>Net rates of interest (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3%</td>
</tr>
<tr>
<td>$5V_{30}:15$</td>
<td>1000</td>
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<tr>
<td>$5V_{30}:30$</td>
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<td>$15V_{30}:30$</td>
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<td>$25V_{30}:30$</td>
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<tr>
<td>$45V_{30}$</td>
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</tbody>
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SYNOPSIS

The Authors are members of a Working Party established by the Councils of the Faculty and Institute to consider the Department of Trade's proposed Rules for valuing the liabilities of long-term business.

The paper opens with a short introduction followed by Part 2 which considers the control systems operating in several overseas territories. Part 3 is largely historical and traces the development of actuarial thought and practice since the war. The negotiations in Europe are also covered. Having set the scene, the Authors move to consideration of the Department of Trade's proposals (the proposed Rules for valuation of liabilities are included as an Appendix to the Paper). After a brief description of the proposed Asset Value Regulations in Part 4 the net premium valuation method is discussed in Part 5 and the problems of large fluctuations in the interest basis are illustrated by examples. Possible modifications to the conventional formula are explored with further examples. The elements of the valuation basis are discussed in turn with particular emphasis on the valuation rate of interest.

The Authors conclude that the problems arising from use of high rates of interest in valuation are not so much a function of the inadequacy of the valuation method as an indication of the real financial strains to which offices are exposed in current conditions. The paper ends with a reminder that with the varied traditions of the British market no one set of Valuation Rules can satisfactorily cover every special case. Any statutory basis therefore must be seen as a compromise, and the Authors touch on special cases where they feel that it would be appropriate for the authorities to exercise their discretionary powers.
Mr. A. N. D. Shaw, introducing the paper, said:—The paper which has been presented for discussion tonight was produced with the greatest of urgency because the Department of Trade is awaiting the views of the actuarial profession on the proposed Regulations for the valuation of liabilities but wishes these Regulations to be issued as soon as possible this year. The Working Party was set up a short four months ago and the time taken from its first meeting till the production of a final draft of the paper was approximately nine weeks. I do not think that Reg Bews or Paul Seymour will object if I mention that it was the perseverance of our Chairman, Richard Wales, that steered the Working Party to the production of a paper in such a short time. The rather limited conclusions we were able to reach in the paper were due essentially to the restrictive time-table to which we had to work.

The Regulations governing the valuation of assets have now been published and it is unfortunate they should impose upon a life office the statutory obligation to value its assets in a manner which is more appropriate to its imminent break-up than to its continuation as a going concern. The standard of adequacy sought by the 1974 Act surely implies taking a view of the assets and liabilities of the company as a continuing entity. Break-up values seem appropriate as far as long-term business is concerned only if it is found that the company is so insolvent that no satisfactory formula for variation of existing contracts can be found, in which case a company must be totally liquidated and the proceeds divided up amongst the policyholders. This last calculation is one of last resort and surely should not have any place in the normal periodic actuarial investigation into the company’s affairs.

Mr. A. D. Shedden, opening the discussion, said:—This evening we are discussing a paper which has been prepared as a commentary on the valuation proposals of the Department of Trade, and these proposals in turn are very largely based on ideas presented in an original paper by R. S. Skerman. I think it is appropriate, therefore, to begin the discussion by considering the so-called “six principles” in the context in which they were first enunciated and note how they relate to the Department of Trade proposals and to the conclusions of the Working Party.

Skerman’s paper propounded five principles for a solvency standard:—

(i) that the liability should be valued by a net premium method or on some other basis producing stronger reserves;
(ii) that an appropriate Zillmerised reserve would be acceptable in order to allow for initial expenses;
(iii) adequate margins over the current rate of expenses should be kept in the valuation of liabilities in order to provide for future renewal expenses;
(iv) that appropriate recognised tables of mortality should be employed;
(v) that the valuation of liabilities should be at rates of interest lower than implicit in the valuation of the assets, with due regard to the incidence of taxation.

In enlarging on these principles, the author propounded what has been taken as a sixth principle—that the net liabilities must in aggregate exceed the surrender values. He also suggested a maximum limit for Zillmerisation, margins of safety in determining the maximum gross rate of interest
Liabilities of Long-term Insurance Business 403

to be used in valuing the liabilities, and the method for determining that rate where more than one gross rate of interest was involved in the valuation.

All of these principles and suggested margins have been incorporated in the proposed Rules and, as far as I can see, the only additions which have been made relate to a specification that the margin for renewal expenses shall cover any foreseeable increase in the current rate of expenses and the provision that the value placed on any contract be not less than the present value of any paid-up benefits guaranteed. In addition, the proposed Rules can be modified for special types of contract.

Turning now to the conclusions of the members of the Working Party, it would seem that, reluctantly, they had not felt able to suggest any practical alternatives to the proposed Rules. They are unhappy about the Rule for determining the weighted average rate of interest, but can suggest no practical alternative except, possibly, to hypothecate assets. They would prefer the maximum rate of yield to be related to the market value of the investments held at the valuation date and would not allow the use of book values of assets in determining this yield; but they do not press this point and, indeed, the proposed Rules require the Actuary to adjust the overall rate of yield on the fund as calculated where, in his opinion, this does not give a proper indication of the rate of investment income to be expected in the future. The Working Party seems to admit that the proposed Rules will have to do and, in effect, asks the Department to exercise a certain amount of clemency in operating the Rules in situations which might cause embarrassment to offices. Some may find these conclusions disappointingly, particularly as it might appear that the proposed Rules will work well in circumstances when they are not really needed, but will tend to break down when extraordinary circumstances prevail. If this is the case, it hardly seems worthwhile to introduce the Rules in the first place.

The Working Party firmly supports the use of the net premium method of valuation, agreeing with Skerman that it has the supreme merit of reserving for future premium loadings and so making automatic provision for future bonus and expenses. One may question whether policyholders' reasonable expectations are necessarily secured by means of a net premium valuation, unless there is some provision for holding stronger reserves for with-profit contracts than for without-profit contracts. On this point the Working Party feels that it is not the function of a control system to seek to make specific provision for future bonuses and considers that the reserving of the premium loadings is sufficient to protect the with-profit policyholder's interests. It would seem to me clear that if an office has got to a position where it cannot reserve on a stronger basis than for solvency, then the interests of with-profit policyholders have been impaired, although this may be considered a risk which they must run if their bonus loadings are to be invested profitably.

While the proposed Rules could thus be criticised in that they might allow an office writing predominantly annual premium with-profit business to meet the solvency standards even though policyholders' expectations have been eroded, the net premium requirement has also been criticised for having an opposite effect on an office writing predominantly annual premium without-profit business, i.e. such an office might be unable to meet the solvency standards even though it may be fundamentally solvent. Such a situation can arise at higher rates of interest because the net premium valued may be considerably smaller than the actual office premium to be received, even after deduction for future expenses. This point was
noted by Skerman, who observed that at very high rates of interest the net premium method of valuation could produce reserves which were too stringent, but his view is challenged by the Working Party in paragraph 5.3.3 of the paper, on the basis of the calculations demonstrated in Appendix II. I do not think this demonstration is particularly fair or realistic, at least for assurance fund liabilities, since the market value of the fixed interest assets would not be determined in the way described and an assurance reserve at a particular net rate of interest would, in practice, be related to an asset valued at the corresponding gross rate. The drop in value of assets relative to liabilities would therefore be more severe than has been illustrated and the rate of interest would have to be very high indeed before the net premium reserves would eventually become lower than the value of the corresponding asset. On the whole, therefore, I would agree with Skerman on this point, rather than with the Working Party.

I do not propose to discuss the various net premium formulae in detail. As I have said, the figures in the Appendices do not seem to me to offer a fair comparison of assets and liabilities but, in addition, the formulae given in paragraphs 5.2.4 and 5.2.5 do not seem to me to be realistic, in that they assume that one might base premiums and reserves on a long-term rate of interest equal to the current return on assets. On the other hand, the formula given in paragraph 5.2.3 has distinct possibilities and I think it may have been rejected too hastily.

The Working Party does not question the original proposition in the Skerman principles that the solvency standard should be based on the current yield of the assets, admittedly less a safety margin. In paragraph 5.5.3, it argues that to base the yield on a value of the fund less than the value of assets could give anomalous results; and in paragraph 5.5.8, it shows how anomalous results can be obtained if the weighted average rate of interest is based on individual rates of interest which differ widely in value. But the tenor of the discussion in Section 5.4 of the paper on the valuation rate of interest is that to fix the maximum rate of interest allowed on the market value of admissible assets is a conservative rule, and that there may be circumstances where one might conceivably be justified in using a higher yield. It seems to me surprising that the Working Party has not questioned this proposition, in the light of the results demonstrated in the Appendices and in view of its own conclusion 6.1 that no one asset or group of assets can immunise a portfolio against large changes in the rate of interest.

The original solvency standard principles were apparently made in the context of an office whose liabilities and assets were matched. The author recognised that it was not easy to involve the relative lengths of the term of the assets and liabilities in a solvency standard, but considered that it was unnecessary in practice to bring this into account in view of the numerous margins inherent in applying the principles. These would ensure that there was ample early warning to correct the situation if interest rates were to change. However, I doubt if this view can be maintained with interest rates at their present level and in view of the very large changes we have seen over the past few years. The higher the rate of interest the more difficult it becomes to immunise the liabilities, and at the current levels it is, in fact, only possible to immunise immediate annuities or deferred annuities with two or three years to go until vesting. To immunise anything else one would have to postulate an asset with increasing interest payments and it would be unacceptable to treat an equity as fitting this description for the purposes of a solvency valuation. The remarks in paragraph 5.4.4 would seem to support this latter point.
Liabilities of Long-term Insurance Business 405

It is possible to view a valuation of actuarial liabilities as a reassessment of the premium rates on existing contracts in the light of the existing assets and current projections for future investment of premiums and interest. It may be that for certain types of single premium liability, suitably matched by appropriate assets, the reinvestment assumptions are of minor importance, but for long-term annual premium business, particularly if the average duration in force is short, it would seem to me that the rate of interest assumed for future investment ought to be consistent with that inherent in current premium rates for the same class of business. The greater the difference between the current yield obtainable on new investments and the rate of interest that one might safely assume would be averaged over the term of an annual premium contract, the more important it is in my view to make a distinction between them. It would seem to me that the theoretically correct method to use in valuing such annual premium business is a modification of the method given in paragraph 5.2.3, such that the rate $g$ would be assumed to be earned for the outstanding term of the assets (these might be hypothecated assets) and the rate $i$ would be something akin to the long-term rate of interest used for new without-profit annual premiums. In order to avoid unnecessary stringency the net premiums should not be at rate $i$ but should represent (for without-profit policies) the actual gross premium less the appropriate loading for future expenses and contingencies. The dangers of capitalising future profits are not so great with such a modification of the normal gross premium reserve formula, and as interest rates rise there is a further compensating feature in that any provision for future increases in expenses should become relatively larger. Such a provision has an appreciable effect on the reserves for very long-term contracts at early durations and can easily result in reserves that exceed net premium reserves.

It follows from these remarks that I would advocate the use of an appropriate reinvestment interest rate which might possibly have to be re-specified from time to time, and I would further advocate that the use of a rate of interest based on the current yield on the assets be limited so as to take account of the outstanding term of the assets; at the same time I would permit a gross premium valuation of without-profit business. In my judgment it remains to be seen whether the use of net premium reserves, as presently proposed, would produce approximately the same effect.

I am in sympathy with the remarks of the Working Party given in conclusion 6.3. One of the effects of having a statutory valuation basis ought to be that offices will tend to invest in assets suitable to the liabilities which they have assumed and, possibly, vice versa. The proposed Rules make no mention of the assets and it would seem to me to be perfectly possible for an office to appear solvent under these Rules, or indeed under the modifications that I have suggested, and yet be inherently insolvent because its assets are not suited to its liabilities. The experience of the past two or three years must surely bear this out.

It would be undesirable for the Government to regulate as to what are considered to be suitable assets. Inevitably this would lead to constraints which in most cases would be unnecessary and inhibiting. It is up to the Actuary to perform his valuation in the light of the assets covering the liabilities, and in completing the present abstract of valuation under the Insurance Act he is required to say to what extent he has done so. This must surely involve considering the effect on his valuation surplus of a change in interest rates. Need I add that the concomitant of considering assets in relation to a valuation of liabilities is considering investment
policy in relation to a premium basis? I suspect that our techniques for doing this kind of exercise are not sufficiently developed and I would suggest that as a profession we must investigate such matters more deeply.

I would not like to end my discussion without congratulating the Working Party on its paper which, as has been said, represents the fruit of an enormous amount of work done in a very short space of time. In my opinion, more work requires to be done before we can be satisfied that the proposed Rules are suitable, or can specify alternative ones, and I am sure that the discussion this evening will point the way to further investigations. In the meantime, we are surely indebted to the Working Party for setting out the problems clearly for us and demonstrating the many pitfalls that beset the unwary in coping with valuations at very high rates of interest.

Mr. K. M. McKelvey:—We are tonight discussing a new departure in U.K. life assurance. Section 34 of the 1974 Act empowers the Minister to call for an actuarial investigation at any time while section 37 gives him power, also at any time, to act not only "to protect policyholders or potential policyholders against the risk that the company may be unable to meet its liabilities", but also against the risk that a life company may be unable "to fulfil the reasonable expectations of policyholders". I presume that this last refers to bonuses on with profit business, though it could (with my wholehearted support for the life office) also refer to surrender values now offered by some well-known offices represented tonight. In addition to the power to call for an investigation the Minister has the power (and clearly plans to use it) to prescribe (i) what are admissible assets, (ii) how these are to be valued and (iii) how liabilities are to be valued.

His object in using these powers will presumably be to ascertain whether the company is solvent to the extent of being able to meet its contractual liabilities and also to the extent of being able to "fulfil the reasonable expectations" of with-profit policyholders. For this purpose a net premium valuation of liabilities is proposed with the admissible assets taken at market value. Such a valuation of the assets and liabilities of a life office could be described as "(i) valuing a premium the office will not receive and (ii) using (at least currently) a rate of interest at which it is least likely to invest on average in future, while (iii) taking assets at a value at which the office has neither need nor intention to sell them".

You will perhaps have inferred that I am not a devotee of a net premium valuation basis for the assessment of the solvency of a life office, even though I have read and re-read Mr. Skerman's paper. I have been a devotee for a very long time of joining the Common Market and I think it is slightly unfortunate that I get the impression somewhere about a third of the way through this paper this evening that our four authors (who have done such a good job in such a short time) confess that it does not really matter what any of them or us think, we will be committed to a net premium liability valuation basis for the purposes of proving solvency to the authorities because we are one of "the Nine", and any other basis would cause repercussions outside the U.K.

I would like to enlarge on the changing conditions that Mr. Shedden mentioned. For the whole of the 19th century, life office investments were virtually confined to fixed interest stocks and mortgages. The purchasing power of the currency was, relatively, very stable. What had cost 1d. in 1840, posting a letter, cost 1½d. when I was a schoolboy (and that was 95 years later even though that is difficult to believe). In the following 30 years (and I am talking in old pennies, of course) the cost of posting a letter
will have gone up from 1½d. to 1s 5d. Between 1840 and 1967 the rate of interest seldom varied, if at all, outside the range of 2½ and 6%.

Right from the early part of the 19th century until 1960 the object, nearly always, of a life office valuation was “the orderly release of surplus”—but seldom the establishment of solvency.

With that sort of background, net premium valuation of liabilities with assets at “book or market whichever is the less” probably served well enough, and towards the end of that long period—getting on towards the end of it anyway—Raynes wrote for the Institute a daring paper on the possibility of investing life office funds in equities.

Then came a third period in the U.K. Firstly, it involved the foundation of a relatively large number of life companies for whom solvency was not a theoretical concept; secondly, it involved a currency being progressively debilitated by erosion of purchasing power and, partly in consequence, an increasing tendency for life companies to invest in ordinary shares and property. We are now in a fourth period; it is like the third one except that the rate of interest available on matched gilt-edged investments moves violently and the currency is melting at a faster rate than ever. Clearly (to me anyway), this of all these four periods is the one where the traditional net premium valuation of liabilities is the least appropriate. Yet we are on the verge of being committed to use it as proof, with the onus of proof upon the members of this profession, that a life company is solvent.

May I give you a few brief specific reasons why I dissent from the approach proposed in the draft Regulations? I may add that as a consulting actuary I am not concerned, as are most of your members here tonight, with the problems of a 100-year-old mutual life office.

(1) To take assets at market value (and here I am repeating in different words something Mr. Shedden touched on) and to derive the valuation rate for liabilities therefrom seems to me no more defensible than to base office premiums for long-term policies upon the rate of interest ruling at the date of commencement of the policy. The rate on 20 year Government stocks (F.T.-Actuaries Index) was 11¼% per annum on the last day of 1973 and 16¼% per annum on the last day of December 1974. Why should that fact have exactly the same effect upon the average rate of interest one is to assume to apply over the remaining lifetime of the business on the books?

(2) Even more difficult is the fact that, if one uses a net premium valuation of liabilities, the chance vagaries of market movements of asset values affect not only the valuation rate of interest but also the net premium. One is then confronted by two alternative dilemmas when, as now, interest rates are very high. Either one does a true net premium valuation—and may show the office insolvent (because the future premium taken credit for is only about two-thirds of the office premium less bonus loadings) or one modifies the net premium so as to be based on a lower interest rate—maybe something like the one underlying the office premiums; one then throws up very large negative values. If one adds all these back, insolvency may be the result and I believe a spurious insolvency; if one adds less than all of them back, one has the problem of assessing “less than”.

(3) To a smaller extent the same problems would exist in reverse if the rate of interest currently ruling were 2% per annum. One’s true net premium would for many policies exceed the office premium (less bonus loadings where appropriate). It would presumably have to be restricted. Reserves would be increased artificially (rather as they are when one eliminates negative values in conditions of high ruling interest rates) and solvency could be spuriously understated or unstated.
(4) In theory, I am well aware, an office with exactly adequate and perfectly matched investments is solvent whatever the valuation rate of interest, provided the same rate is applied to assets and liabilities. But this would not be shown to be so, in current conditions with assets at market value and liabilities valued at corresponding interest rates, unless one were to use a gross premium method or something closely resembling it and refrain from eliminating all negative values.

I prefer, as they say, "not to start from here".

I would first like to define what I mean by "the surplus". It is "those assets in the long-term fund without which the office could still meet its contractual liabilities" and, if you wish, "fulfil reasonable expectations, etc.". The value of that surplus (i.e. of those assets) is quite another thing. It depends upon the purpose of valuation which could be (1) for purchase of the life company, in which case the value is the market value, (2) for declaration of a cash bonus, in which case the value is again market value, (3) for declaration of reversionary bonus, in which case the value is the one placed on the surplus assets if one values them at the valuation rate of interest, or (4) for solvency, in which case the value is relatively unimportant provided only that it is positive.

We know the business on the books and we know the investment portfolio. We also know some other things, namely:

(i) that there is no risk that the policies will all be surrendered tomorrow (though the risk of this will be maximised if a section 29 notice is served by the Minister) and

(ii) that all the investments will not be sold tomorrow. Indeed, if reasonably well matched, they will be held to fulfilment.

We do not know what will happen to expenses in future and we do not know the average rate of interest which will rule in markets over the remaining lifetime of the business on the books on the valuation date.

We have some general agreement, I believe, that one should use the same rate of interest to discount projected liabilities as is used to discount future asset proceeds. Mr. Gunlake's 1960 Presidential Address to the Institute indicated quite rightly that commutation functions and discounting were an actuarial accident of the pre-computer era. What the actuary is really trying to do is to project the fund and ascertain whether it looks like going at any future time "into the red", including, if you like, some future rate of bonus among the items of future benefit outgo.

If one puts together all these thoughts, one values for solvency on the following lines (I assume no significant guaranteed options). One assumes a valuation rate of interest. It must be an assumption because no one can know the average rate of interest destined to rule over the next 30 years or so. One does not vary this valuation rate of interest often, any more than one varies "annual premium" premium rates often. As a suggestion, one assumes between 6 and 7% per annum gross. Provided that one's assets are reasonably matched as to term and provided that one does not use a net premium method (with intractable problems of spurious insolvency or negative values or both), the absolute level assumed will not make an enormous difference.

The direction of research which I wish to suggest for this subject would proceed along these lines:

(1) Having valued liabilities on a gross premium basis at a "neutral" rate of interest one turns to the assets.
(2) One splits the liabilities (valued on a gross premium basis) into broad categories such as:

(a) short-term assurances;
(b) medium-term assurances;
(c) long-term assurances;
(d) immediate annuities;
(e) taxable deferred annuities;
(f) tax-free deferred annuities.

Where these relate to participating classes the liability includes explicit allowance for future bonuses.

(3) One splits the assets into corresponding groups:

(a) short fixed-interest stocks;
(b) medium fixed-interest stocks;
(c) long fixed-interest stocks;
(d) equities and property.

(4) One values, for example, asset group (a) (short fixed-interest stocks) on a basis consistent with that used for liabilities under short-term assurances in order to ascertain what proportion of those assets is needed to cover the accrued reserves for that type of liability.

(5) During this process one will be testing each asset group for its matching as to date with the liabilities group with which one is dealing. If there is no match, one's asset valuation must make stringent assumptions about realisation or reinvestment terms, as appropriate.

(6) In such a process, how does one deal with the equity type of asset? Should one assume any increase in income and capital value? On two conditions, I think that the answer is "yes". The conditions are, first, that one takes some account in allowing for future expenses (on a "closed fund" basis) for future inflation and, second, that one has made adequate allowance for future bonuses in valuing liabilities.

(7) Having worked one's way right through the list either (a) one finds that one has run out of assets before covering all liabilities so that the office is insolvent and needs a cash infusion equal to the market cost of buying the missing assets or (b) one finds that one has covered all the liabilities without using up all the assets. The unused assets are the surplus. I have already discussed what will be "the value" of that surplus.

This is a contribution to a discussion, not a paper. You will not therefore expect me to have tied up all the loose ends. But, until I am convinced that a net premium valuation with assets at market value can tell the authorities as much about the solvency or insolvency of a life office as the realistic calculations which I have tried to outline, I remain most unhappy about the way we are apparently to be led.

I would like to make two comments to close with. The first one is (again Mr. Shedden has touched on this)—What happens to investment policy under the proposed regulations? There is a danger of the mathematical tail of the statutory valuation basis wagging the dog of real financial health of the life office. Suppose the managers have £100,000 to invest, and suppose that a third of it represents bonus loadings. If they invest it all in a matched gilt they secure a redemption yield of 16% per annum, but if they invest one third in equities or property and their Actuary must assume no future increases in dividends or rents, they secure a lower overall yield for the Actuary's valuation of liabilities (due next day). With an old office this likely does no more than hold down bonuses a year or two. With a young one it may make all the difference between statutory solvency and statutory insolvency.
If I may close with a more general question I would ask what an actuary is to do if he conscientiously believes that a valuation based upon statutory regulations does not present a correct picture of his company's condition? If he is a company employee, his position is very difficult indeed. If he is a consulting actuary, he may be wisest to decline to carry out those parts of the actuarial function governed by statute.

Mr. J. M. G. Smart:—I very much regret that I haven't had time to put all the thought I would like into this fascinating subject of valuation, but nevertheless I would like, supporting Mr. McKelvey and perhaps taking up some of his challenge as to what might alternatively be done, to eject the whole question of solvency from the territory of valuation as we usually understand the word, and see if I can spot where it would then find a home, because it seems to me that we are not clear what we are trying to do. In other words, what question are we trying to answer? What do valuation, solvency, surplus, reserves and so on mean, and given anything you like about them, what can you tell about the company under scrutiny? Because this is what we are usually trying to do, surely—tell whether this company is a sound recipient for our money. This may or may not be the same as asking whether the company is solvent, there being a big difference in our attitude depending on whether our interest is as a with-profit or non-profit policyholder or perhaps as an annuitant.

Take valuation. If it means discounting to the present day various quantities with their associated probabilities, why do we think this is the only or even the best way? We traditionally value assets and liabilities separately. Why? Paragraph 1.1 of the paper indicates that this will be perpetuated. Why? Is it not the difference between the assets and the liabilities in time and amount that is important? The valuation we do at present is, I think, something we do to please accountants. Similarly with valuation of assets, today's price is ephemeral and useful only as a guide to the income to be expected. What really does change, as I think Mr. McKelvey asked, if you discover that today's price turns out to be something different? You weren't going to sell that asset anyway.

Solvency. This doesn't mean that the value of the assets is greater than or equal to the value of the liabilities. It means the ability to meet your liabilities. Though you are accustomed to saying that, for example, your liabilities are £200m and your assets are £220m or whatever, your liabilities are most certainly not £200m. Your liability is to pay out such and such amounts in event of this or that at sundry dates in the future. Are you sure your £220m, available now (perhaps) if you sold everything, can do that? The answer is almost certainly no, because valuation, and solvency, are matters not of fact but of probability; and therefore to prevent all insolvencies is not possible, nor, I think, desirable.

"Surplus" is also not meaningful, since we have shown in the past that we ignore it when the amount doesn't suit our requirements. It is another accounting concept, as is "reserves".

I mention all this because when reading the paper I had to check back to see if perhaps it was written 50 years ago by a Part II student named Procrustes. This chap has obviously just learned the delightful trick of combining everything in commutation function form, and in the euphoria surrounding his new skill he is torturing everything to fit it, whether relevant or not. The only reason I can think of for valuing to present values is that taxation of the funds is based on this, which seems a severe case of the tail wagging the dog.
To show how futile it all is, I should like to take a very simple example. We have a certain liability outgo of 120 in 2 years' time. Ignore premiums, expenses and everything. Our two alternative certain asset incomes are $A_1$ of 110 in 1 year's time, or $A_2$ of 121 in 2 years' time. Now the question is, are we solvent in either case? The point is that with $A_2$ there is no need to assume anything, no need to value anything. We are solvent. Indeed, "valuing" can easily cloud the issue. For example, taking the $\frac{3}{4}$% margin suggested in the paper, say valuing at 10½% for the asset and 10% for the liability, the values out on the asset and liability are $99\cdot1$ and $99\cdot2$ respectively, and you would say we were insolvent. Joke! Now look at the other asset, $A_1$. On the same valuation basis, its value comes out at $99\cdot5$ compared with $99\cdot2$ of liability so you would say we were solvent. But are we? The discounting process has disguised what we are actually assuming. The 10½% rate is no doubt based on last year's income divided by today's price (although what relevance that has is a bit obscure), but what we really need to assume is that the 110 we shall receive in 1 year's time can be increased with certainty to 120 in 2 years' time. Unfortunately the method doesn't tell us that, or estimate its probability; it merely presents a delusion of fact. Thus that simple example shows that traditional valuation methods are not the proper answer.

Even in the paper we have criticism after criticism of the net, and for that matter gross, premium methods, but there is still no concerted effort to find a better one. They are shown to give unreasonable results; we are trying to mix past and future by presenting solvency as an accounting exercise. The main reason for the net premium method, apart from "it's aye been done", seems to me its automatic exclusion of negative values, but the assumptions involved in discounting (on top of other necessary assumptions) separate any conclusions far too far from the premises which give rise to them. The argument that in good times the net premium method releases surplus smoothly and usefully doesn't hold water—so does any other method so long as it is unchanged. Meantime, there are no useful by-products in the way of hints to the Investment Manager, Agency Manager, Actuary and so on.

So, having discovered that Procrustes, and no doubt his dog, are barking up the wrong tree, let us stop torturing the old methods and start afresh, retaining all relevant dimensions to the problem, including time.

Conventional valuation compares mean expected values of assets and liabilities, but we should surely be looking for something more like $(\text{mean} + 2\sigma)$ of the difference. Now this is not easy to do, although it is easy for mortality on the liability side, but it points a moral. If we are short of assets, we can reduce the value of $(\text{mean} + 2\sigma)$, though not of the mean itself, by reducing $\sigma$ by improving the matching position. In the ultimate, if the emerging income exactly matches in amount and time the emerging outgo, $\sigma$ is reduced to zero, near enough, and our asset requirements (in terms of present values) are at a minimum.

It is suggested in the paper, 5.1.3, that it isn't the function of control to provide for future bonuses, but surely it is if bonus expectations are being used in new business quotations, to prevent fraud.

Anyway, it is all a devious way of avoiding reality. Let us sweep it away and do an actuarial job, a real actuarial job—not a Part II job and not a Part IV job either, it being somewhat tarred with the same brush, but a Part IVB type of job. Our business lies in the future and we must seek our answers there too. So make such assumptions as are necessary to find out the amounts and dates of expected income and outgo, and compare them.

Forget meantime about how the fund could be immunised; take it as it
comes. Allow for all items specifically—expenses, bonuses, exercise of options of all sorts against us, and so on.

What assumptions do we have to make? The answer is, very few. Mortality, obviously, and surrender rates, which are easy. Expense units should also be easy. The troublesome one is the rate of inflation each year over the next 100 years or so; this sounds impossible, and no doubt is, but the point is that we have to do it anyway (implicitly or explicitly) in any valuation, so let's face up to it. Probably we require a formula deriving the expected undated gilt yield from this inflation rate. We might need a recognition of the variation of surrender rates with inflation, and relationships of the surrender value and premium bases with these other items, but they are all fairly automatic. For equity shares we could reasonably assume a dividend increasing at a rate say half the shortfall of the current yield from the current gilt yield—we could do this even for a solvency valuation as this must be a better estimate of income than that provided by the current yield unadjusted. Now that doesn't add up to many assumptions. They are very difficult but they are realistic—this is what actually happens.

All this gives us figures of income and outgo each year and we should be interested only in the difference each year between income and outgo; immediately the whole assessment problem is reduced. By using the matching valuation technique of Benjamin and others (so cavalierly discarded in 6.2; incidentally the quotation at the head of section 6 I find rather amusing—we wonder why we have a fuddy-duddy image!) with twin rates of interest, say the calculated gilt-edged long term rate plus or minus 2%, or some such margin, we can find whether we have surplus values or not. All the problems about choosing rates of interest, Zillmerising, escalating expenses and so on melt away and come out in the wash. Problems like that in the second half of 5.3.4 vanish because the "profit", if there is such, remains unascertained; our expected income and outgo are unchanged, and it leaves them unchanged. The method also avoids the problem in 6.3 mentioned by Mr. Shedden.

We can then value the surplus back to the present day and ratio it to the value of all liabilities on the same basis—the basis doesn't matter very much—to give a "degree of solvency" or insolvency. In drawing conclusions from this we must remember we need an estate of the office to provide for the continuance of the office as a competitive entity and for the provision of a few $s of this and that; perhaps such margins should all be included as liabilities. Full prospective bonus would be allowed for and full premiums taken credit for. Anyway, asset A emerges triumphant in all circumstances and the other one has appropriate doubts cast on it, depending on how many $s we take into account, which is all as it should be.

Now even that is not the end. The real question we are aiming to answer is a bit obscure, but must be something like—What is your degree of solvency now and what will it be in a year's time, assuming that everything (mortality and so on) transpires as you have assumed and that all financial options available against you have been exercised against you? For Department of Trade control, they could pose the same questions, but no doubt without having to allow for any future bonus unless bonus is being used in projections. Incidentally, it is a pleasant thought that perhaps the Department of Trade could be made to lay down the inflation rates to be incorporated in the method. Having thus acquired a distant early warning system, the Department of Trade could then concentrate their control on checking the premium rates, quality of management, and prospects generally of doubtful companies. They could also ask such companies to report their degrees
Liabilities of Long-term Insurance Business 413

of solvency more frequently. A good going concern would probably not boast about its degree of solvency in case, due to various randomnesses, it was reduced next year. It could, however, say it was "top grade", or whatever the Department of Trade allowed.

Mr. L. W. G. Tutt:—Along with others I would like to add my tribute to this evening's paper. It clearly reflects a considerable amount of work on the part of the authors and, personally, I found the arithmetical results in Appendices II to V particularly interesting. The figures therein indicate the results of formulae which start with a strict net premium valuation formula and are then modified, from an interest aspect, through various stages to a point where the net premium reserve is calculated at the rate of interest representing the current return on assets but with the net premium calculated at a rate of interest representative of an expected return on future investments. Figures relative to the last formula appear to suggest to the authors, according to them in paragraph 5.3.16, that the gross premium method of valuation is most unsuitable.

In such regard, it is, of course, well known that when office premiums are utilised which involve bases less favourable than considered appropriate at a later valuation date, such an approach can capitalise future surplus arising from the difference in the bases. This is broadly what is illustrated in Appendix V. However, the fact remains that it is the office premium that is received and it may seem to some artificial to assume anything else. Having said this, clearly an office cannot validly take credit for the value of the whole office premiums without debiting at least all the loadings in them, bonus loadings, expense loadings, contingency loadings and so on, and indeed possibly debiting an increased loading if, for example, expense experience has worsened relative to expected. A similar point appears to be made by the authors in paragraph 5.3.9 relative to their first modification of the net premium method.

In fact, does one of the main emphases of the results in Appendix V seem to be not that the gross premium method is necessarily most unsuitable, but that under any valuation method $g - i$ should in effect be constrained? In practice, amongst the traditionalists, both the advocates of the net premium method and the bonus reserve method think so, but a point is that whereas under the strict net premium method $g - i$ is taken as 0, under the bonus reserve method $g - i$ can, in effect, be adjusted appropriately to meet individual circumstances which can, in effect, result in $g$ being less than $i$, which is conceivably desirable in some circumstances.

Incidentally, I regard the term bonus reserve method as an unfortunate misnomer because the bonus reserve is merely one of the incidental by-products of the basic approach.

What is termed the bonus reserve method possesses, most importantly, the attribute of being explicit by the setting up of specific appropriate separate items of reserve within the aggregate, and it is flexible. As is indicated in the paper, similar numerical results under the net premium method can be achieved by a variety of modifications to it, but some of them like the method itself may be merely implicit. Also, the overall yield reduced by 10% as mentioned in Appendix I, can vary from time to time, and an alteration in the valuation interest basis to reflect this could result under the net premium method in a degree of immediate capitalisation.

Nevertheless, it is the net premium method which has been favoured in this evening's paper, in connection with some proposed regulations. It is sometimes thought that some regulations may lead to more regulations, and the expression is used in paragraph 2.1.2 of the paper of "bowing to the
Statutory Basis of Valuation of the inevitable ". Consequently, there may conceivably be a feeling in some quarters that what has been suggested may merely be regarded as another step along a path of imposed rigidity, the next relatively short step along that path possibly involving a requirement in connection with a now agreed net premium method, of a specified inflexible rate of interest, and so align with some places elsewhere.

Miss M. C. Allanach:—I would first like to thank you for inviting me to this evening’s meeting and also for allowing me, as another visitor, to take part in your discussions on this highly topical subject. I would say, at the outset, that I am in substantial agreement with the authors’ support for the net premium method as a basis for the proposed Rules. This is perhaps not surprising, considering the “stable” from which I come! I would, however, like to add my own reasons for this view. (Perhaps these are essentially practical reasons although I am not quite sure, in the context of the earlier speaker who criticised them, which “Part” of the Examinations this puts me back to, probably a very early one!) It does seem to me that, for the particular purpose of Regulations for an adequacy standard, the net premium method, with some of the modifications that have been suggested (together with those variations permitted by the Regulations) has two great practical advantages. One is that the bases are readily definable and can be used relatively universally by all offices. I, myself, think it is very difficult actually to define bases for a bonus reserve valuation (or gross premium valuation) in such a way that it can be readily applied and defined for Act/Regulation purposes for any office. Taking the expenses aspect as an example, and ignoring the question of bonus loadings, it only needs a 1% variation in the expense loading assumed to vary the allowance for expenses by some 10% or 15% or more. There is also the need, from office to office, if they do a gross premium valuation, to make variations in the formulae for the frequency of payment. I do realise, of course, that Regulation (6) proposes that the Actuary should take a view as regards current and expected future expense rates and the supporters of the bonus reserve method will say that in fact this Regulation requires a similar assessment, when doing a net premium valuation, to the one that would have to be made when doing a bonus reserve valuation. But, against the background of requiring a definable basis, it seems far better to define the actual bases, in a net premium method, for interest, mortality and so on, and then to compare the adequacy of the resulting margins with expense ratios as determined by the office, rather than to fix the expense margin for valuation purposes in the first place.

I do appreciate, of course, that in the present circumstances of very high rates of interest, this method does lead to tremendous problems because of the fall in the net premium as interest rates rise and I think there could be a case for trying to provide some sort of “floor” to the premium used. I don’t, however, altogether go along with the authors’ views when they refer to the possibility of using an adjusted rate of interest: I wonder whether perhaps the better approach might not be to link the basis for determining the “floor” with the expense margin? I think, however, that it is also worth noting that, as interest rates go up to the level that they have recently reached, this is also indicative of a high rate of inflation and it is at just such a time that the margin between the office and net premium should, in fact, be increased. So, I do feel that the net premium method itself, although it does produce such very stringent answers at these levels, does in fact vary with interest rate changes in a direction which might well be considered appropriate to meet increasing expenses.
This leads me on to the other important aspect of the proposed Rules and to another equally important criterion, namely that they should have some automatic form of adjustment or interaction as conditions change. This in fact they do because, as interest rates rise and asset values fall, a higher yield on the fund will emerge. Other people have criticised the fact that using this rate in a net premium method does not produce an exactly matching change in liabilities, which of course I accept, but at least the method proposed in the Regulations does provide for using a higher valuation rate of interest automatically as asset values fall.

Now, if one considers what other alternatives might be incorporated in Regulations (that is assuming that some sort of statutory basis is inevitable) it does seem to me that the alternative would almost certainly be that fixed bases would be incorporated. No doubt these bases would, eventually, be varied from time to time, as conditions changed, but whenever one has a fixed basis or a fixed rate of interest, say, written into Regulations, it seems to me that it would be very much more difficult to vary it and there would probably be a much greater time lag in varying it as conditions changed. I therefore feel that even with all the admitted imperfections of the proposed Regulations, (and the first speaker did touch on a number of these points in this connection which were made in Skerman's original paper) it does appear that the net premium method, as qualified in the proposed Regulations, does meet two important criteria, namely that the bases can be readily defined and that it has got a fair degree of automatic flexibility. I would therefore personally go along with the proposed Regulations as an adequacy standard for good conduct of the business.

Mr. A. P. Limb:—I would like to comment on the proposed Rule No. 2 in particular, the proposal that varying rates of interest may be selected by the Actuary to value various classes of business in force, provided that the overall interest income required in the ensuing year under the valuation bases selected by the Actuary, to apply to the classes of business with which he is concerned, shall be covered with a suitable margin by the interest, dividends and rents expected to be receivable. One might, I suppose, uphold the simple view that the assets of the office are held en bloc to meet its liabilities, that no particular group of assets is legally hypothecated to one particular class of liability, and so one should value all liabilities at the same gross rate and allow properly for tax. The protagonist of this viewpoint would say that he can only assume that the current mix of investments will be repeated in future by the office, so that must determine the unique valuation rate to be used. This point of view is, however, somewhat unrealistic. An office markedly changing the mix of its liabilities would presumably change its investment policy also, and thus alter the current return on its investment portfolio. And so investments are, to some extent, tailored to the mix of liabilities and so hypothecated notionally if not legally. At the other end of the scale is the approach outlined in paragraph 5.4.5 of the paper and thereafter commented upon in the following paragraphs in terms of which I personally strongly approve. The approach apparently sanctioned in proposed Rule (2) lies, I suppose, between the two extremes mentioned above. Plainly, using this approach, a variety of quite different figures for the same collection of liabilities can be produced with a given portfolio of assets. The authors comment in paragraphs 5.5.7–5.5.10 on the problem. I am concerned to emphasise that not only is the proviso suggested by the authors in 5.5.10 a very sensible one, as it seems to me, but there are also corollaries to be borne in mind. The assets thus hypothecated should be sensible, bearing in mind...
the nature of the liabilities, and should be of a suitable date distribution. All the limitations of the immunisation theory are, of course, relevant. Investment changes must be made bearing in mind the notional allotment of assets underlying the spread of valuation bases used; the extent to which attention would have to be paid to this would of course depend on the overall investment reserve which the office possesses. I should end this section of my remarks by admitting that I find it difficult to see how to suggest an improvement on proposed Rule No. 2, but as it stands, it leaves, I believe, a good deal unsaid.

On a second topic I would also like, if I may, to comment on remarks which have been made by previous speakers, and have indeed been made in the profession for many years, on the conflict, if it might so be described, between the net premium approach and the gross premium or bonus reserve approach. I believe that this conflict is to some extent an illusory one. What we are concerned with, as a previous speaker has said, is to see that in the event we can meet the liabilities as they fall due, and no doubt pay due regard to the reasonable expectations of policyholders. No matter which valuation approach is used, the same underlying long-term problem is the one with which we are seeking to concern ourselves, and we are seeking to assess an office’s position in regard to its real liabilities. One can appreciate the advantage of the net premium valuation as a basis for Regulations, since it has certain elements of built in conservatism and since it is much easier to define. One could sympathise with any regulatory authority called upon to scrutinise and comment on explicit assumptions made by large numbers of offices regarding future inflation rates as they might affect expenses, for example. None the less, this kind of problem, the effect of inflation of expense rates and so on, is the real problem with which we are faced. The fact that it isn’t mentioned explicitly in the net premium valuation hides conveniently the fact that implicitly it has been taken account of by the automatic operation of the net premium valuation system. As the previous speaker pointed out, in times of high interest rates the margin between the gross premium and the net premium becomes larger, and this is precisely the kind of situation in which one would expect a continuance of a high level of inflation and so the situation in which one would need a larger margin between the gross premium and the premium valued, if no specific liability in regard to future expenses is to be included in the valuation. The question which has to be answered, of course, is whether this implicit allowance for future expenses and an implicit allowance for a future expense loading are marching in step in the Actuary’s view as they should in changing conditions. The gross premium valuation seeks to make explicit assumptions about such matters, the net premium valuation makes these assumptions implicitly, but they are made none-the-less. For my own part, in assessing the position of an office, I would like to have available both net premium valuations and gross premium valuations so that I could assess the extent to which implicit allowance for future problems has been made in the net premium valuation, and try to consider whether or not such implicit allowance is reasonable, overly conservative or perhaps rash. At the end of the day, however, whether a net premium or a gross premium valuation is used, it seems to me that the interpretation of the valuation rests as it always has done on the Actuary’s professional, but admittedly subjective, judgment.

Mr. J. Plymen:—I feel that the discussion is developing into the age-old argument between the net premium and the bonus reserve and other methods of valuation. What we really should be talking about is setting
Liabilities of Long-term Insurance Business

up the statutory valuation basis. This is an extremely difficult task. In every other country where there is a statutory valuation basis, it succeeds in distorting the management of the insurance business, introducing all sorts of confusions and stultifications. It can handicap the investment policy and slow down development, as has been mentioned in the paper. Surely, what we are trying to do is to set down a statutory basis, which is a minimum solvency valuation. We are not talking about the valuation that is going to be used by the office for determining what its bonuses are going to be. Obviously, the minimum basis will have to be worked out, but the real valuation used for distribution of bonuses will be a different one. Under present extremely tricky conditions, determining bonuses is obviously far more difficult than it has ever been in the past, and for this purpose one wants the most realistic valuation, possibly one of those "cash-flow" valuations, so difficult to set down in print and to submit to the authorities; however, for publication in the company’s accounts, surely a realistic bonus reserve valuation should be used. Probably, the Actuary will make a statement to the effect that this is stronger than the official statutory minimum. If we have to make both the statutory minimum valuation as well as the bonus reserve valuation (for policy purposes), I hope we don’t then get a third valuation in the form of a net premium valuation cooked up to produce the same answer as the bonus reserve basis. I have heard sinister suggestions to this effect. The third valuation would be like a fan dancer—it would conceal more than it reveals. It would be quite unhelpful to an external analyst. I hope that in presenting this material adequate information will be given. If we have a published bonus reserve valuation, give us all the details of it, preferably clearly published in a separate Actuary’s Report, so that those external commentators who do try and understand these things will be able to make use of the material.

Mr. J. K. Scholey:—This paper has been produced at short notice and, in some aspects, I feel that a good deal longer period of consideration would have been beneficial.

The first point I would like to stress is that valuation regulations are no substitute for professional integrity in safeguarding a life office. Many of the points about valuation rates of interest and the rates of interest used for different sections of the portfolio can be dealt with by the exercise of professional judgment rather than by minute regulations. I have three main points of criticism. The first is the uncritical acceptance of the net premium method and the values it provides. It seems to me little short of astonishing that a life office can be regarded as insolvent for the sole reason that the rate of interest secured on new investments is higher than had been expected. How can this be right? The answer is that of course it can’t. The authors try to sweep away the problem first by saying there isn’t a problem, and secondly by saying that even though there is, no other method is any better. I take the authors up on their statement in paragraph 5.3.6 that for a mixed portfolio of policies of short, medium and long outstanding terms the strains and releases introduced by the use of the net premium method may well roughly cancel out. Taking, as I did, a mixed business of endowments, ranging from 15 years to 30 years original term, and a portfolio of investments related to the maturity dates, I found that the value of the net premium reserve dropped, if you are going from a 3% to a 6% basis, by 18% but the value of the investments fell by 24%. On a business originally valued at £100m there was a shortfall of £6m.

The authors put forward three other methods. Formula (4) I regard as out, and hardly warranted consideration. Formula (3) is dismissed with the
Statutory Basis of Valuation of the

phrase "it will be seen immediately that in all cases the reserves appear far too weak". This is quite unproven. I assume it is based on the fact that for endowment assurances the ratios of $V_3$ to $V_1$ are lower than the asset ratios of matched investments, but if higher interest rates are a benefit to an office, ought not the valuation basis to show this, which it does in the use of $V_3$. In any case, I think that the formula given for $V_3$ is not correct. The authors have put up an Aunt Sally and shot it down, which I think is not justified in the context. The denominator should be on the rate of interest more or less implicit in the premium rates, although this isn't essential, whereas the authors take it as the expected return on future investments. Much more important, they have used in the numerator the rate of interest equal to that currently being earned, whereas more correctly they should have brought in an investment rate which is lower than $g$ taking account of future investment prospects. $V_3$ is not too bad a basis, and I think that in the proposals Mr. Shedden made regarding modification of $V_3$ he is getting to the modified value of $V_3$ that I am suggesting. As printed, of course, $V_3$ suffers from the criticism that it brings in a net premium which is not related to the premium being secured.

Now, I am not concerned whether a basis is weak or strong in relation to any other basis. I am concerned that it should be a truthful basis, and I think one of the further qualifications for any valuation basis is that it shouldn't mislead. If there is strength, it should say there is strength; if there is weakness, it should say there is weakness. So I think there are two characteristics which such a basis should have. Firstly, that the assets which are held can be readily and properly compared with the value of the part of the sum assured and declared bonuses which premiums received to date can be assumed to have provided. That is, on the basis set out in the premiums or on such more stringent bases as circumstances dictate, for example, if interest rates are falling. Secondly, that there is a proper margin in either future premiums or in interest earnings on present investments, or both, to provide profits on the basis currently being declared.

This leads me on to a consideration of pensions business. This hasn’t been mentioned so far. I wonder if it is because net premium valuations have no relevance and could only be misleading. In pensions business you don’t know the future sum assured or the future premiums. All you know is the position to date and what has been promised so far. The question is—do the assets match those? So we look at the accrued position, and I claim that it is also the only rational way to look at life insurance, when you are considering a minimum position—I’m not talking about the methods we usually use. Turning to the authors’ conclusions then in 5.3.16, I find their assertion that the net premium method is not as stringent at very high rates of interest as popularly supposed, to be unproven. I also find their assertion that modifications to the net premium methods don’t merit general application is insufficiently examined. The authors state in 6.1 that it is impossible to frame a valuation method under which liabilities change correspondingly in value to a change in immunised assets. It may be so if exact correspondence is the test, but I think it is possible to frame a method which gives reasonable correspondence and which is not misleading. $V_3$, or $V_3$ corrected in the way proposed, I think, would be reasonable.

Now, I think the subject tonight is too important and too urgent for trying to find smooth ways of putting forward strongly divergent opinions, and I am conscious, Mr. President, that I have been blunt at times. I hope the authors, for whom I have the very greatest respect, will accept my thanks for the illuminating and informative way they have placed the subject before us tonight.
Mr. G. G. Torkington:—My remarks this evening are mainly directed against criticism of the net premium method of valuation, as detailed in Section 5 and exemplified in the Appendices, and I am grateful to the authors for setting these out so extensively. To my mind the great virtue of the net premium method is that it does not capitalise profits from future office premiums and avoids negative values, which are undesirable in themselves and a nuisance to the Actuary who is trying to compare net liabilities on different bases. The inability of the conventional net premium method to match the revaluation of liabilities with the revaluation of assets has become painfully obvious under recent financial conditions. The conventional method implicitly assumes that the new valuation rate of interest and the relative net premiums apply both before and after the valuation date; this is not in accordance with the facts, from which I conclude that the failure of the net premium method on a change of interest does not lie in its underlying philosophy but in the application thereto. Proposed Rule (4) is not sufficiently precise on this point; if, as I think, its "premium" means "premium from inception of the policy" this leads to the conventional net premium valuation. Without some such additional specification, e.g. allowance for assets held or previous net liability, Rule (4) would merely be an inverted definition of Rule (1), and the amount of the net premium could be chosen at will. I have attempted to find a method of overcoming these difficulties including application of some of the formulae set out by the authors, eventually deciding that the answer must be in the form of an adjustment to the old valuation net premium so as to permit future valuations on a prospective basis, using the new net premium (until it is changed), and enable the expense margin under Rule (6) to be directly ascertainable without the further calculation of the equivalent net premiums which would be required if, for example, Formulae (2) or (3) were adopted. By equating the revised value of a matching redeemable asset to the net liability on the revised basis, but with an adjusted net premium, it is possible to derive a formula for the adjusted net premium. I was not greatly surprised to find that this formula is identical to a formula which appeared in Appendix I to my Faculty Paper of February 1974 (T.F.A., 34 p. 49) if the office functions are replaced by the corresponding valuation net functions. This particular formula was designed to calculate the revised office premium on change of class of assurance or alteration in term, and it now appears that it can also be used to calculate the revised valuation net premium on a change of interest basis. It is also in a form which throws some light on the Appendix II phenomenon mentioned by the authors in paragraph 5.3.3. It would, at this stage, be useful to look at Appendix II and note my formula for reference; you will be glad to hear that it can be defined adequately in basic English without any actuarial symbols. It is that

"Alteration in Policy net premium" = "Sum at risk" times "Difference in net premium rates for a new policy".

For those who wish to have the formula, it is

\[ \Delta P = (S - V_0) \left( P_x^f \cdot e^{-\delta t} - P_x^o \cdot e^{-\delta t} \right). \]

At short durations the adjusted net premium will automatically be close to the revised conventional net premium and as the "sum at risk" reduces to zero near maturity the adjusted premium from that stage will be close to the original net premium; this is as it should be. As a result the conventional reserve is normally higher than the adjusted reserve. However, for long-term assurances at short durations the factor "premium rate difference" may increase faster than the factor "sum at risk" reduces, particularly...
at high rates of interest; in this case the adjusted net premium may be
less than the conventional net premium and the conventional reserves under
Table B of Appendix II for $SV_{30}$ will be too low—they are actually below
the irredeemable index under Table A where the interest rate is 8% or more.

Calculations of adjusted reserves for an interest rate of 12% produced
figures very close to those in Table A, after allowance for mortality. At
interest rate 20%, for $SV_{30}$, Table B gives an index of 76 against the
Corresponding Table A index of 159 and the minimum figure of 150 for an
irredeemable asset. Table A assumes dividends yearly in arrear, but Table
B assumes premiums yearly in advance and at this very high rate of interest
the effect of these different assumptions becomes significant; I therefore
based my calculations on continuous functions—the indices of 76 and 159
now both come out as 171 as against 163 for irredeemable assets and all
three indices have fallen properly into place.

These calculations can be computerised provided each policy's valuation
net premium is put on the computer record. The premium adjustment
formula was deliberately designed to be flexible; it now seems to be able to
cope with policy alterations or changes in valuation (or surrender value)
interest rates or changes in mortality, or even all three at the one time. It is
also in such a form that different interest rates could be allocated to different
unexpired terms, if any actuary required this nicety.

The question arises—Is the expense of the preceding adjusted net premium
valuation justifiable? I think it is, since at least one source of error would
be removed from a problem which is difficult enough for the Actuary; also,
even if the errors demonstrated in Appendix II look small, they
relate to
the reserve and would constitute a much larger percentage of the surplus.

As regards with-profit policies, a net premium valuation at a rate of
interest below the non-profit valuation rate can satisfy the Department of
Trade's overall requirements but I think we still have to do some type of
bonus reserve valuation to ensure as far as possible that the current bonus
rate can be met at future declarations. In my opinion the bonus reserve
method has been subject to criticism for the wrong reason, as it has for too
long been shackled to office premiums; the logical method would seem to be
to use the net premium method with the same interest rate as for non-profit
policies, but with a conservative bonus rate injected into the valuation net
premiums and the assurance valuation factors. Adjustments to this special
with-profit net premium on a change of interest and/or bonus allowance
could be calculated from the premium adjustment formula with the aid
of a computer; in the pre-computer era the amount of calculation would
have been prohibitive.

One argument for using office premiums is that altered policies do not
require the complicated calculations which have to be made under the
conventional net premium method; the disadvantage of the latter is removed
under the adjusted net premium method as under this the altered and
unaltered policies are treated alike. In the case of paid-up assurances
where there is a rise in interest rates application of the premium adjustment
formula will produce negative adjusted net premiums and provision might
have to be made for this feature, which is essentially correct.

May I conclude by saying that I have dealt with only one of the problems
stated in this paper; as regards the other (perhaps more difficult) problems,
such as selection of the proper rates of interest after allowance for taxation,
I am in general agreement with the authors' comments and conclusions.

Mr. J. Young, closing the discussion, said:—Much of the discussion this
evening has been concerned with alternative methods of valuation, some-
times of valuation for solvency and sometimes of valuation for surplus; many suggestions have been made and I am sure, Mr. President, that you will not expect me, in closing the discussion, to adjudicate on these methods and to produce the answer that actuaries have been seeking for about 200 years! Nor, I think, can we expect Mr. Wales, who is speaking after me, to do this; after all he and his working party were given terms of reference which were limited to that, as Mr. Plymen reminded us, of the statutory valuation basis and I shall try to limit my remarks to that basis.

It is interesting to note from the terms of reference that the question is not whether there should be a statutory basis for valuation of liabilities, for that seems to be a fait accompli. That is very remarkable indeed; after all actuaries of my generation, and of at least four generations previously, have been brought up in the profound belief in the concept of freedom in judging the proper valuation basis with due disclosure of the facts; and certainly in the hundred years following the enactment of 1870 that system has worked in the sense that the stability of life assurance companies was never in doubt, epitomised very well by the physicist, Sir Arthur Edington, in the 1930's who, seeking in every day life some ideal of certainty, cited a well managed life office as a model.

One of the interesting facts mentioned by the working party is that just 25 years ago only a handful of British offices had been established for less than 50 years, and over a half for more than 100 years. During the earlier lifetime of these offices problems and uncertainties confronted their actuaries. These were perhaps different in character and extent from those facing the profession now, but they were real enough to cause the actuaries to satisfy themselves on what they considered was the first priority, namely solvency in the conditions then ruling and anticipated. Their solution was to build up reserves in the asset valuations, in the liability valuations and in contingency funds. Of course, that was achieved frequently at the expense of previous generations of policyholders, since the transfers from surplus which built up these margins took the form of a levy rather than a loan. Perhaps our predecessors were over-cautious, and while more recent generations of actuaries have certainly been concerned with solvency first, they have also paid increasing attention to equity and to the proper emergence of surplus. I think the words of Redington in his much quoted paper are very pertinent. He said "it is perhaps desirable to admit to ourselves that if there is a sufficiently large break in the rate of interest there is no single valuation basis which will adequately answer the two basic questions: 'is the company solvent?' and 'is surplus emerging satisfactorily?' In such circumstances it becomes necessary to divorce the two questions and to concentrate, in the public presentation of results, on an adequate answer to the solvency question, dealing separately with the question of emergence of surplus by internal investigation."

Also remarkable is the way the proposed statutory basis has confronted the profession. I won't say suddenly, because five of the six principles were established by Skerman in 1966 and, as mentioned by one speaker, his first principle did include the elements of the sixth principle which deals with guaranteed surrender values. But there has been in the last eight years very little investigation or discussion by the profession on the feasibility of the method then proposed; that is until now, and now only because to quote paragraph 1.3, "the period of rapid and quite unprecedented change through which we are passing has led to the need to reappraise these principles". Incidentally, I find this comment disturbing because principles, once established, should be capable of operating in all circumstances, existing and foreseeable. Why then are they being questioned now?
The reason is given in paragraphs 5.1.5 and 5.1.6, that in today's conditions of very high interest rates which have caused a diminution in asset values, the use of a corresponding rate of interest in a net premium valuation of the liabilities is likely to prove too stringent, because the liability valuation does not necessarily give rise to a proportionate decrease in the calculated liability. As Mr. Shedden noted, Skerman commented on just that point in his paper, and indeed Redington also in his paper of 1952. It is interesting that the example Redington gave there of a "so large" swing in interest rates was from $2\frac{1}{2}\%$ net to $3\frac{1}{2}\%$ net. What therefore was perhaps unexpected is the extent to which interest rates have increased, and especially the suddenness of the increase in the recent past. Much has been said this evening on the subject of the interest rate and the yield on the assets, particularly by Messrs. Shedden, McKelvey, Limb, Scholey and Torkington. It is clear that this is a source of some anxiety and perhaps should be the subject of further research.

In the meantime, the Working Party has cast doubts on some of the features of the proposed basis, which, by and large, have not really been contradicted in the discussion. The Working Party concludes that it is impossible to modify the net premium valuation formula in such a way that would enable the valuation of the liabilities to vary with the interest rate corresponding to variation in the value of the assets; and I confess even to a feeling of relief at this conclusion because for the purposes of this exercise, like Miss Allanach, Mr. Limb and Mr. Torkington, I do prefer a net premium method for the purpose in question; and like Mr. Plymen, I prefer a net premium method that looks like one. I doubt, in any case, whether any major modifications specific to the United Kingdom would be generally acceptable.

I am prepared to go along with the comment of the Working Party in paragraph 6.3, that where an office will have difficulty in coping with a large change in interest rates under the statutory basis, to a considerable extent that is a function of the very real strains to which it is exposed. It should be noted that this conclusion has been derived from an examination of a non-profit situation and if we consider such an office it will necessarily be one owned by shareholders. For such an office with a portfolio of wholly or even predominantly guaranteed long-term liabilities there must be no doubt about its ability to meet its obligations to its policyholders. Old established offices of this type probably will have, in addition to their original shareholders' capital, free reserves—perhaps even substantial ones—and if such is the case these reserves will have been built up over the past by ploughing back as fresh capital monies which belonged to the shareholders. So much the better, and probably these offices will be able to satisfy the statutory valuation basis, stringent though that may be, and at the same time the income from the free reserves passes to the shareholders, either as dividends or used in the formation of additional reserves.

The corollary to this is for an office old or new, which has little or no free reserves to put itself in that position of satisfying the statutory valuation basis by obtaining the necessary finance from its shareholders or, if necessary, from new shareholders. This may seem harsh, but I believe that such an office is better to be over, rather than under, capitalised to ensure the security of the long-term policyholders beyond any doubt in what in these days is a very changeable, and at high interest rates somewhat unpredictable, economic climate. After all, the shareholders hope to derive a profit from the office's operations, and this will happen provided the terms on which the business has been obtained are sound. This situation is enhanced if the office also issues with-profits contracts where the shareholders' risk is smaller.
The circumstances are different for a mutual office where there are no shareholders to provide any necessary additional finance. However, the principle remains the same, and again should be directed first towards solvency. But a mutual office which has to set up additional reserves required by the regulations may find that the emergence of surplus as suggested by internal investigations—possibly perfectly sound bonus reserve valuations—is affected and is therefore in a difficult position, for it would be faced with a conflict on the matter of equity and its with-profit policyholders’ reasonable expectations. Again this may not be a problem for the old established office, but it is possible to visualise an office perhaps of more recent origin which has so far eschewed building up what it considered to be over-stringent reserves at the expense of the equitable treatment of its present policyholders. If this office’s distribution method is based on the uniform reversionary bonus system, it has probably published net premium valuations designed to control the emergence of surplus in the light of its policyholders’ reasonable expectations. This, in effect, will have meant using a smaller valuation rate of interest for its with-profit classes, in order to defer the emergence of the surplus to meet the rising cost of bonus in future. Such an office may well find difficulties in releasing surplus to pay bonuses which its internal investigations suggest it could safely declare and maintain. This office might be tempted, in accordance with proposed Rule (2), to value its with-profit business in the same way as its non-profit classes at the rate of interest yielded by the assets less the margin; but if current high rates of interest and depreciation of assets persist it may sooner or later be faced with the fact that the continued need to satisfy the statutory basis will prevent sufficient surplus being released to maintain the bonus previously estimated. In other words, such an office would be faced with cash-flow problems in the future and it is here that I think the bonus reserve method can give a truer picture than the net premium method, especially if valuations are made at several rates of interest. Yet from the overall point of view of security, the mutual office is no different from its shareholder-owned counterpart because difficulty in meeting the statutory valuation basis may with the mutual office also reflect real strains. Logically it should be required to conform, and in that case it is really being compelled to do what many offices have voluntarily done in the past when they withheld surplus from policyholders to create substantial free reserves.

I am not suggesting that this should be done by expropriating permanently monies that rightly belong to policyholders and are needed to satisfy their reasonable expectations. It is not possible to dictate a universal solution which will apply in changing conditions for the varying status and characteristics of offices. What may have to be investigated is the bonus system itself, so that any drafts on surplus could be repaid later. In equity, these drafts should be treated on the lines of interest-free loans repayable when the policy leaves the fund as a claim. In other words, the Actuaries of these offices may have to modify the bonus systems to allow for deferment of bonus, possibly introducing some form of terminal bonus. It may well be that in such cases full compliance with and adjustment to the statutory conditions would need time, and I echo the sentiments of the Working Party that for this and for the other good reasons mentioned in the paper the Department of Trade should allow ample time for this process to be carried out in an orderly way.

May I conclude, Mr. President, by adding my congratulations to the members of the Working Party for the magnificent job they have done in such a short time at a difficult time of the year.
Mr. F. R. Wales, replying to the discussion, said:—There is an awful lot to reply to. First of all, I would like to say that Alastair Shaw made some very kind remarks at the beginning. I would like to emphasise that this paper was very much a team effort and without everybody in the team really contributing in the way they did, it would never have been possible for us to produce the paper on time.

I am very glad that Mr. Young drew attention to our terms of reference. A number of comments have been made this evening which implied that we accepted the net premium method too readily. Our terms of reference, though, were very specifically related to the net premium method and to have departed from it we would have had to have held extremely strong views against it, which we don’t. Mr. Young and Mr. Plymen also drew attention to a very important point and that was the object of the Valuation Regulations. The Regulations aren’t intended to produce a valuation basis which is suitable for publication, nor are they intended to produce a basis which is suitable for bonus distribution. They are merely meant to provide a standard of good conduct against which companies can be measured by the supervisory authority. That is to say, that if the Actuary carried out his valuation, he is expected to certify that the liabilities he produces are not less than those that would have applied had he calculated them on the statutory basis.

Another thing that worries me is that many comments that are made about the problems we are facing today with asset depreciation, seem to imply that the people who make them are trying to find a valuation basis that will suit the assets they have got, and they are not trying to see whether the assets they have got are the right ones in the first place. Therefore I was very glad of Mr. Plymen’s warning that we shouldn’t be trying to “cook” the basis.

I would now like to move on to Section 5. First of all I would like to turn to the opener’s remarks. He referred to the comments about the Actuary’s discretion with regard to the rate of interest to be assumed, and he seemed to think that we meant that the Actuary could take a rate of interest higher than that arrived at by reference to the assets held at the valuation date. That wasn’t in our minds at all. We felt that if the rate of interest earned on the fund at its market value at the valuation date was higher than the Actuary felt could be maintained, then he was under obligation to value at a lower rate. In fact, as was mentioned by Mr. Scholey, this statutory basis is not meant to be a substitute for the Actuary’s judgment. Again, the opener did refer to the statutory basis as being a solvency standard. This is not so. It is a standard of good conduct. It is something more than a measure of mere solvency. I also found that the opener had misunderstood a point which is absolutely fundamental, and this is the question of the use of net rates of interest. He mentions that the market when fixing the price of redeemable securities considers gross rates. I consider that the method by which the market arrives at its values is totally irrelevant to the holder of a security. The holder of a security must measure what the value of that security is to him and the only possible way of doing this is by consideration of net income. After all, this is the criterion that any investor employs in deciding whether to purchase one particular stock instead of another. If anybody has any doubts on this particular question and still feels we were wrong to use net rates of interest, I would be very happy to enter into further discussions with him on the point. I might add that this was a point that did trouble us in the Working Party, since initially we had conflicting views, but we were completely agreed on the point in the end.
The opener also referred to 5.3.3. He implied that we had rejected this approach. Now, this is not true. We liked this approach because it helped to show what one was doing rather more clearly than just assuming a level rate of interest, but we didn’t feel that the results which flowed from it were conclusive evidence that it ought to be incorporated in the Valuation Rules.

He also said that he would like to see this approach brought in with a specified re-investment rate in the Regulations. I think this has certain dangers, because if you examine the figures in the Appendix, you will find that the lower the re-investment rate specified, the lower the resulting reserves, which I am sure would place the Government Actuary in a quandary.

He also made a comment about our statement that fixing the valuation rate of interest by reference to the yield on the fund was taking a conservative view. We did say this, but we said “by reference to”, we didn’t say that you take the actual yield on the fund, and we specifically pointed out that you must make suitable allowance for reinvestment.

Mr. McKelvey made many points that indicated his abhorrence of the net premium method. He also protested particularly about the linking of the valuation rate to current yields. One of the points he made was that this method was most unkind to a young office. Actually, it isn’t necessarily, if you look at our figures. For large changes in the rates of interest, since it is impossible to immunise a young office, you will find that increases in the rate of interest will in fact act to the office’s benefit, which is common sense, because in a young office the investment of future premiums is rather more important than the terms you’ve secured for the few premiums that have already been paid. In particular, I couldn’t accept Mr. McKelvey’s comments about the effects on investment policy. I feel that if the effects of the investment of one year’s bonus loading are going to be so significant in assessing the company’s solvency, then it certainly can’t afford the risk of investment in equities.

Mr. Scholey gave us a dramatic example of the dangers of the net premium valuation but, unfortunately, he didn’t tell us how he had worked out his sums, so I would prefer not to accept his comments until I have further details. He did plead, however, for some allowance in $V_2$ for a lower future investment rate but, in fact, that is formula $V_2$. The point here is that the formulae we have set out do follow, I hope, a logical progression, starting from the position where you assume a level rate of interest and then moving through the concept of a high initial investment rate and a lower re-investment rate to progressively weaker bases.

Mr. Smart talked a lot about the exams which showed he had considerable familiarity with them. Do I dare ask in what capacity he gained this familiarity? In particular, he has queried the whole concept of valuation as we know it. I don’t think he has been very fair to us, because in paragraph 6.2, we made the very point that he made, that the only real way of gaining a true insight into an office’s solvency is to compare the incidence of asset income and liability outgo year by year. This is a matter which has already been discussed in a paper to the Institute to which we referred. Don’t be put off by the title of the paper—nor by the very complex formulae which appear in it. I think Sidney Benjamin’s work on the subject is extremely interesting and well worth reading. The practical problem is that nobody has followed up his paper and, as we say, the techniques only secured limited acceptance. It just isn’t realistic to contemplate trying to incorporate such a concept in statutory rules at the present time. Nor is it realistic to consider putting off the drafting of the statutory rules until the subject has been adequately explored. I think we have got to face practical realities. The Insurance Companies Act 1974 provides for
Statutory Basis of Valuation of the statutory regulations and there is strong political pressure for their intro-
duction. I think that we would be putting our heads in the sand if we thought we could just talk away for the next ten years to find the perfect solution.

Mr. Smart, though, seemed to imply that matching is fairly easy. I would ask him one question—How would he and his technique deal with flexible endowment assurances? Also he did make the point that he was prepared to accept insolvencies from time to time. I am not. I speak personally here and I don’t know if my fellow authors will agree—I hope they would. We just have to look at the political consequences of the recent cases to realise what a dangerous thing it is if a life assurance company goes insolvent. However small the company, its effects can be quite disastrous on the industry as a whole.

Next, I would just like to make one or two points about the gross premium valuation. Cox and Storr-Best made a very important point. Actuaries have been shocking prophets. If you use a gross premium method, the whole thing hinges upon the very subjective prophecies you make about the future. It is totally sensitive to your future interest assumption. Personally I prefer to look upon the gross against net premium valuation argument as being the prospective approach against the retrospective approach because in reality a net premium method is a retrospective one. It does not assign a specific value to the future but merely tests whether future premiums are adequate by comparing them with the net premiums on the valuation basis. I think there is a lot of intuitive appeal to a method which tries to get the right answer on an accrued basis and very largely leaves the future to look after itself, having tested that the office is at least charging adequate premiums.

Again, it was said that it was anomalous that an increase in the rate of interest would mean a stricter standard of solvency due to this idea that the net premium reserve gets stronger as the interest rate rises. I would make the point, “what about the mismatching situation? ” If an office is mismatched, particularly one, say, with a lot of short-term single premium business, a rise in the rate of interest and a consequent fall in asset values means real insolvency if the office has invested too long.

Finally, one quick comment about Mr. Torkington’s comments about the idea of allowing for bonus loading in the net premium valuation. This in fact was one of the main theses in the paper by Mr. Carroll to the Australian Institute that is included in the list of references. He discussed precisely this idea and I can recommend this paper.

The President (Mr. J. G. Wallace):—Thank you very much, Mr. Wales, for the really excellent way in which you summed up the discussion tonight.

I think, Ladies and Gentlemen, you will agree that we have had a first-class discussion from Members and Visitors on a subject which is of great interest to all actuaries and life offices. The present financial circumstances in the U.K. and the possible entry into the European Economic Community have made the production of such a paper as we have had tonight most timely, and I think this situation has been recognised by the presence of a record audience for the Faculty.

I am sure that the discussion will be closely studied by both the Department of Trade and the Government Actuary. This meeting and the meeting at the Institute next week provide a forum at which the actuarial profession and others are able to express their views. We are therefore particularly grateful to the authors for preparing a paper which has enabled these discussions to take place and for producing it so speedily.
I would like you all, therefore, to join with me in according them a very hearty vote of thanks indeed.

Mr. J. M. G. Smart wrote:—In view of the late hour I did not (in spite of strong temptation) wish to interrupt Mr. Wales’ closing remarks in order to answer the question he put to me about how to deal with “flexible” endowment assurances. I should like to reply now. Briefly the answer is “Anything he can do I can do better”, i.e. if he can cope with any type of policy on a net premium method I can cope at least as well, and almost certainly better, on an emerging sum method. In the particular case of flexible endowments the solution is, as usual, extremely simple; I stick to facts and, using the same multiple decrement approach as for all policies, have no difficulty at all in assessing the expected income and outgo each year—naturally I choose the rates of surrender (or “maturity”) on as conservative a basis as seems prudent. There is of course no problem in calculating the amount to be paid out in any year of exit.

While I am at it I might give another simple example which shows how irrelevant a net premium method is. If our sole liability is a non-profit endowment assurance of £4,000 maturing in 15 years’ time, with no guaranteed options and subject to an annual premium of £100 payable throughout, that should be all we need to know (and is all I need to know) because that, and that precisely, defines our liability. But the net premium man needs to know when the policy started and, if it has arrived at its present state by, say, alteration—from a whole-life policy, he not only needs to know this but much other detail so that he can tie himself in suitable knots. But all this extra information has no relevance; the liability is the same whatever the history, since our contract is prospective, always prospective.

Some speakers seemed to suggest that while the emerging sum method was suitable for internal use in connection with bonus declaration, it was not suitable for a statutory method. But this is blindly accepting what has “aye been done” and ignoring the purpose of a statutory method, which is presumably to enable the D.o.T. (and other skilled investigators) to check the state of solvency of the company. Now checking the ability of the company to stand a bonus rate of nil is merely a special case of checking its ability to stand a bonus rate of x%, and any method satisfactory for the latter should also satisfy the former. Introducing the artificiality of net premiums is bad enough for the bonus declarations of a continuing office; when we are approaching the insolvency stage it could lead to wildly erroneous conclusions. It is done because it is easy, not because it is right.

Mr. J. K. Scholey wrote:—During the discussion I was asked where I got the figures from which I quoted in my remarks. The ratio of reserves was obtained from the tabulated values for the A 49/52 Table; basically I used the ratios set out in Table B of Appendix II, but had to get the absolute values to deal with the mix of business I took. The ratios of assets I took from Table A of the Appendix. The full calculations are set out below.
Comparison of changes in asset values and liabilities under a net premium valuation method, consequent on a change in interest rates from 3% to 6%

[No account is taken of any additional margins, e.g. 10% of 3% is less than 10% of 6% (the required margin under the proposed regulations) and it is assumed that assets mature when the policies mature, that they yield 3% net and that their value at 3% equals the amount of the 3% reserve.]

<table>
<thead>
<tr>
<th>Term of policy</th>
<th>Years in force</th>
<th>Assumed sum assured</th>
<th>Liabilities on A49/52</th>
<th>Assets on 6% basis</th>
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<tr>
<td></td>
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<td>3% Reserve</td>
<td>6% Reserve</td>
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<td>Overall ratios</td>
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<td>0.82</td>
<td>0.76</td>
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</table>

On the $V_2$ basis the liabilities percentage would be approximately 79%, and on $V_3$ about 73%, taking $i$ at 3% and $g$ at 6%.

Mr. C. Hymans wrote:—In the first place I would like to say that I was very intrigued with the elegant mathematics used by the authors of the paper. I thought they gave a very neat solution to the mathematical problem. However, I felt that this was really rather theoretical because what the authors were trying to do was to bend the net premium valuation to produce something which is not really a net premium valuation, on the grounds that a gross premium valuation, or alternatively a bonus reserve valuation, is not acceptable to the Government Actuary's Department. I am very puzzled about this, because it seems to me that the only realistic way of assessing the financial strength of an insurance company is to do a valuation on something that is on the prudent side of a realistic (i.e. solvency) valuation basis. We already have in the draft valuation basis a rule for the maximum rate of interest that can be used for valuation, and, as far as future expenses go, it seems to me that it is right that the Actuary should be sufficiently competent and professional to assess how much of the premium is going to be required. This seems to me a very much more simple and common sense view. I do not know what is
the official objection to other than a net premium valuation, but, since
the Government Actuary has said that he is available to discuss with the
Actuary any problems he may have, it seems to me that if the Actuary
was in any doubt that a basis on the above lines which he proposed to use
was in any way less adequate than it ought to be, then surely he could
discuss the problem with the Government Actuary.

If the valuation is to prevent the position arising where the company is
unable to meet its obligations to policyholders, there are other considerations
which could be much more important than the valuation method, of which
the most important is likely to be the suitability of the investments.
The only other point I would like to make is that it does seem to me
to offend common sense that the assets should be valued on a solvency
basis while the liabilities should be valued on the basis of a going concern.
Surely the only reasonable way to do a valuation is to ensure that the assets
and liabilities are valued on consistent bases.

Mr. C. O. Beard wrote:—The authors have rejected the bonus reserve
valuation method as a possibility. I agree that the bonus reserve method
can be a very dangerous weapon, but if used intelligently it becomes a
valuable tool.

The authors have generally referred to an immunisation model. For
purposes of illustration I would like to consider an office in a position where
its existing investments are precisely matched to its existing liabilities, with
maturing investments plus interest income equalling maturity and death
claims less premiums each year, tax and expenses being ignored. Again to
simplify the model I make the assumption that the redemption yield on the
various investments will always be independent of the term of the invest-
ment whatever the redemption yield may be in the future.

With this model we know that if the rate of interest changes, the change
in value of the assets will be exactly equal to the change in the discounted
value of the liabilities. This statement does of course presuppose a realistic,
i.e., gross premium, valuation of the liabilities.

It is instructive to consider what lies behind this model. With a very
large increase in the rate of interest the value of the assets may reduce by
say 50%. The gross premium liability in respect of policies near maturity
will reduce by a much smaller proportion, but that will be counterbalanced
by a very large proportionate swing in the liability in respect of renewable
policies having a long outstanding term. Indeed, large negative values can
arise on renewable policies with long outstanding terms, as mentioned by
the authors in 5.3.13.

Thus the matching theory depends on negative values being allowed to
arise. Incidentally, it will be noted that an emerging cost valuation could
hide negative values. Actuarial tradition decrees that negative values
should be excluded because they represent the anticipation of future profits,
and yet by definition the office I have considered shows no profit or loss
from the change in interest rates. Surely, therefore, this cannot be the correct
reason for excluding negative values.

The solvency of the office I have described is not, on the face of it,
affected by a change in rates of interest, yet in times of high interest rates
a solvency valuation is likely to lead to negative values, and if they are
excluded the office will appear insolvent.

What is the correct approach to negative values? If a policy has a
negative liability and negative values are not excluded, the valuation
reserve is inadequate to cover the amount payable on the policy if it is lapsed
or surrendered on the valuation date. Similarly, if that policy lapses or is
surrendered at a future date, the amount available under the valuation basis at that future date may be inadequate.

This leads to the suggestion that if substantial negative values arise under a particular bonus reserve valuation basis, wastage rates by duration in force should be included in the valuation formula. Whilst I am assuming that surrender value bases have not been guaranteed, it is necessary to include a scale of surrender values in the valuation formula to associate with the wastage rates. Whether the scale is the office's current scale or a lower scale which the office would be prepared to change to is a matter for consideration.

It will be noted that, rather than carrying out such a highly complicated valuation, it should be possible to include an additional reserve for policies which now, or in the future, would have reserves on the valuation basis less than the amount the office would wish to pay on discontinuance. The additional reserve would take into account the deficit from time to time and the probabilities of discontinuance.

This additional reserve would cause insolvency of the office I have used as a model since by definition it has no free reserves. However, that in turn leads to the necessity to reconsider the present matching theory.

It seems to me that the matching theory should be amended to allow for future discontinuances, the probabilities thereof and the amounts to be paid on discontinuance. If it were so amended, offices' theoretical mean investment terms would be significantly shorter than under existing theory, perhaps so much so that where immunisation is possible under existing theory, matching may be possible under the amended theory. In the model office I have used the assets would then depreciate less on a given increase in the rate of interest, and the value of the assets would equal the value of the liabilities whatever the rate of interest, provided a bonus reserve valuation basis allowing for future wastage is used.

I have not previously mentioned any guaranteed surrender and fully paid values which may have been granted but due allowance would, of course, have to be made for them in the valuation.

I have attacked the problem of a mere demonstration of solvency from first principles using a bonus reserve valuation method. From the model I have given, I feel that rules which would stand the test of time can be developed for a reasonable standard of adequacy using a bonus reserve valuation method.

Dr. W. F. Scott wrote:—Mr. Smart has mentioned that the valuation of liabilities should take account of random variations in future mortality. I would like to point out that these fluctuations may readily be dealt with by the formulae of Pollard and Pollard (J.I.A., 95, p. 79), who give examples in life assurance and pensions business using A1949/52 ultimate mortality with 2½% and 5% interest. The formulae given in that paper could, with various extensions and a suitable allowance for duplicates, be adopted in practice; the value of the liabilities would be taken as not exceeding

\[ V_1 = V_0 + k\sigma, \]

where \( V_0 \) is the mean, or conventional, reserve (whether calculated by the net premium or another method), \( \sigma \) is the standard deviation of the reserve, and \( k \) is a number to be fixed by the supervisory authorities. Other things being equal, the percentage addition to \( V_0 \) varies inversely with the square root of the size of the office, and is therefore greater for smaller than for larger offices, and if formula \( V_1 \) were adopted the statutory or recommended basis need be less stringent than for formula \( V_0 \) alone. It is not, however,
claimed that these variations in mortality rates are the most important
source of fluctuation in valuations, merely that they may be dealt with.

The authors subsequently wrote:—We were gratified by the very full
discussion of the paper and feel that there is little we can usefully add.
However, Mr. Scholey’s written submission of figures in support of the
assertions he made in the course of the discussion requires further comment.

We accept that Mr. Scholey’s figures are correct but do not accept that
the example he has chosen is particularly appropriate to the circumstances
under consideration. Firstly, since the paper was written in the context
of the interest rates prevailing towards the end of 1974, our concern was
not with movements from 3 to 6% net. The increase in market rates of
interest to the 10% gross level occurred some time ago and had been covered
without the need for most offices to weaken their reserves at all. Our
conclusion that the strains and releases would roughly cancel out was,
therefore, reached in the context of a much larger increase to 10% net or
more. Our feeling was that the main concern of the profession was with the
effect of the net premium method in the context of these extreme changes.

Admittedly, if a 10% valuation rate is applied to Mr. Scholey’s portfolio
an even greater strain occurs. This is because his model portfolio is for a
relatively short average outstanding term and would not appear to cor-
respond to the normal mix of business for an established life assurer in the
U.K. market. We suspect that an office with such a mature portfolio would
have had a disappointing new business performance over recent years and,
indeed, would probably be experiencing considerable problems with in-
creased expenses. It could, therefore, be argued that the strain arising from
the net premium method is an indication of the real strains the office is
suffering in inflationary conditions. However, it is accepted that this is,
nevertheless, “rough justice”, as in no way can it be argued that there is a
direct relationship between the valuation strain and the excess expenses.

To illustrate the effects of variations in the mean term of the liability
portfolio, we have carried out some sample calculations. The results are
given in the following table, which shows reserves as a percentage of asset
values for three different models at different estimated net rates of interest:

<table>
<thead>
<tr>
<th>Model</th>
<th>Mean outstanding term</th>
<th>Net Rates of Interest (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3%</td>
<td>6%</td>
</tr>
<tr>
<td>Scholey</td>
<td>15 years</td>
<td>100</td>
</tr>
<tr>
<td>Authors 'A'</td>
<td>25 years</td>
<td>100</td>
</tr>
<tr>
<td>Authors 'B'</td>
<td>30 years</td>
<td>100</td>
</tr>
</tbody>
</table>

The first model is Mr. Scholey’s sample portfolio and the figures bear out the
comments made in paragraph 5.3.4. Our ‘A’ was based on an office,
subject to the following assumptions, when it reaches the stationary
condition:

- **New Business mix** = Term (years) = 15 20 25 30 Whole Life
  - New API% = 5 10 15 40 30
- **Sum Assured** = API x term
- **Lapses/deaths** = 5% per annum
- **Inflation** = 5% per annum
- **Real growth in new API** = 5% per annum
- **Decrement in S.A. by duration in force** = 15% per annum (i.e. halved every 5 years)

The long-term model ‘B’ is based on the same set of assumptions but
restricting the duration in force to five years, i.e. assuming the office is new.
As can be seen from the table, in example ‘A’ the strains and releases cancel out in a move from 6 to 10% net which, as indicated above, was the type of movement and mix of business with which we were concerned. The long-term example shows a considerable relief on the move from 6 to 10%, almost cancelling out the strain that arises in the 3 to 6% range.

In conclusion, we feel that in the paper it was made clear that our view was the net premium valuation method achieved “rough justice”, and there would certainly be cases where it did not provide a suitable standard of adequacy. In our view the example submitted by Mr. Scholey was one of the exceptions and we would not consider it to be a typical portfolio of business.