TFA 39 (1984-1986) 419-420

CORRIGENDUM

"Some Applications of the Poisson Distribution in Mortality Studies," by W. F. Scott, M.A., Ph.D., *T.F.A.* 38 (1982), 255-263.

The following statement and proof of theorem 1.1 should be substituted for those given in the above paper.

Theorem 1.1

Let n be a positive integer, let $h = \mathrm{T}/n$ and let the age-range $[x, x+\mathrm{T}]$ be divided into n sub-intervals each of length h. Let a life age x be followed until age $x+\mathrm{T}$ with "replacement" at the end of each of these intervals; that is, for each $j = 0, 1, 2, \ldots, n-1$, a life aged $x+jh$ is followed for a time-period of length h, and if he exits (by any mode of decrement) a new life will replace him at age $x+(j+1)h$. Let $\theta^\alpha(n)$ denote the number of exits by mode α between ages x and $x+\mathrm{T}$.

The limiting distribution of $\theta^\alpha(n)$ as $n\to\infty$ is the Poisson with parameter $\int_x^{x+\mathrm{T}} \mu_y^\alpha dy$.

Proof. By [4; p.206], the characteristic function of $\theta^\alpha(n)$ is

$$\phi_n(t) = \prod_{j=0}^{n-1} \{1+(e^{it}-1)_h(aq)_{x+jh}^\alpha\}$$

It is therefore sufficient to show that, for all real t,

$$\lim_{n\to\infty} \phi_n(t) = e^{\lambda(e^{it}-1)}$$

where $\lambda = \int_x^{x+\mathrm{T}} \mu_y^\alpha dy$ (cf. [4; p.96, p.204].)

Let t be chosen; by lemma 1.2, there is n_0 such that if $n>n_0$, ${}_h(aq)_{x+jh}^\alpha \leqslant \frac{1}{4}$ for $j = 0, 1, 2, \ldots, n-1$.

Let $n>n_0$; we have $|(e^{it}-1)_h(aq)_{x+jh}^\alpha| \leqslant \frac{1}{2}$ for $j = 0, 1, 2, \ldots, n-1$,

and

$$\phi_n(t) = \prod_{j=0}^{n-1} \exp\{\log[1+(e^{it}-1)_h(aq)_{x+jh}^\alpha]\}$$

$$= \exp\left\{\sum_{j=0}^{n-1} \log\,[1+(e^{it}-1)_h(aq)_{x+jh}^\alpha]\right\}.$$

It is therefore sufficient to show that, as $n\to\infty$,

$$\sum_{j=0}^{n-1} \log\,[1+(e^{it}-1)_h(aq)_{x+jh}^\alpha] \to \left[\int_x^{x+\mathrm{T}} \mu_y^\alpha dy\right](e^{it}-1).$$

To do this, we note that for all complex numbers z such that

$$|z| \leqslant \tfrac{1}{2}, \ |\log(1+z) - z| \leqslant |z|^2.$$

Hence

$$\left| \sum_{j=0}^{n-1} \log[1 + (e^{it} - 1)_h(aq)^{\alpha}_{x+jh}] - \left[\sum_{j=0}^{n-1} {}_h(aq)^{\alpha}_{x+jh} \right] (e^{it} - 1) \right|$$

$$\leqslant |e^{it} - 1|^2 \cdot \sum_{j=0}^{n-1} [{}_h(aq)^{\alpha}_{x+jh}]^2$$

$$\leqslant 4\mathrm{M}^2\mathrm{T}h, \text{ using lemma 1.2,}$$

$$\rightarrow 0 \text{ as } n \rightarrow \infty.$$

Also,

$$\int_x^{x+\mathrm{T}} \mu^{\alpha}_y dy = \lim_{n\rightarrow\infty} \sum_{j=0}^{n-1} \mu^{\alpha}_{x+jh} \cdot h$$

$$= \lim_{n\rightarrow\infty} \sum_{j=0}^{n-1} {}_h(aq)^{\alpha}_{x+jh}, \text{ by lemma 1.1.}$$

This completes the proof theorem 1.1.