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EARLY USES OF GRAUNT'S LIFE TABLE

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THE first life table was a brilliant 'guesstimate' by John Graunt (1662) based on mortality data analysed by cause of death but not by age or sex. It was over thirty years before Halley (1693) constructed a table to show "the *odds* that there is, that a Person of that [any] Age does not die in a *Year*". In the meantime several writers tried to use Graunt's figures to make deductions about probabilities of death. This note is a description of these efforts.

Graunt's life table was expressed as follows:

"of 100 ['quick conceptions'] there dies within the first six years	36
The next ten years, or *Decad*	24
The second *Decad*	15
The third *Decad*	9
The fourth	6
The next	4
The next	3
The next	2
The next	1"
["perhaps but one surviveth 76"]	

Summing cumulatively from the end, as Graunt did, we arrive at the abbreviated life table:

x	l_x	Σd_x
0	100	36
6	64	24
16	40	15
26	25	9
36	16	6
46	10	4
56	6	3
66	3	2
76	1	1
80	0	–

As Sutherland (1963) remarks, Graunt "had estimated . . . that of 100 children born alive 36 would die before they were six years old, and it would naturally occur to him to wonder when the remaining 64 would die. He had no information on ages at death from which to estimate this and so . . . he distributed the 64 deaths into a diminishing series. . . ." The important point is that mortality rates distinguished by age were not thought of by Graunt and it seems to have been a pure coincidence that, from the above figures, the 'smooth' ten-year probabilities

$$\begin{aligned} {}_{10}q_x &= \cdot 375 \qquad x = 6, 16, 36 \\ &= \cdot 36 \qquad x = 26 \\ &= \cdot 40 \qquad x = 46 \\ &= \cdot 50 \qquad x = 56 \end{aligned}$$

emerge.

At the time that Graunt wrote, the first small text on probability theory had only recently been published (Huygens, 1657) and there had been no hint that it could be used on other than games of chance. David (1962) describes Huygens's tract, of which a French translation appears in Schevichaven *et al.* (1898, pp. 43–56), and the modern reader is, perhaps, struck by the references to the numbers of 'chances' which each of two or more players has. This is significant because, as David tells us, Huygens received a copy of Graunt's *Observations* in 1662 and seven years later in reply to a letter from his brother wrote: "Ce que je puis conclure de certain par les donnez de la table [of Graunt] c'est que qui gageroit qu'un enfant nouveau né . . . vivra a 16 ans, prendroit le mauvais party et hazarderoit 2* contre 3". (Schevichaven *et al.*, 1898, pp. 57–58). Here is, we believe, the first application of the probability calculus to life contingencies. Notice particularly that Graunt's survivors have become 'chances'.

Huygens's brother showed in a subsequent letter (Schevichaven *et al.*, 1898, pp. 58–59) that Graunt's table could be used to calculate "combien il reste de vie", or the life expectancy, of a person of one of the decennial ages specified by Graunt. Huygens wrote in his rough notes: "Son [i.e. his brother's] calcul est bon pour les rentes viagères."

The next author we believe to have utilized Graunt's table was de Witt (1671). Although the political affiliations of Huygens and de Witt prevented their personal contact (Schevichaven *et al.*, 1901, p. 889) Hudde, 21 times burgomaster of Amsterdam (Schevichaven *et al.*, 1898, p. 71), knew them both well and was in repeated communication with them on matters relating to mortality. We can only assume that he discussed the foregoing Huygens brothers' correspondence with de Witt, for the now famous *Waerdye* of de Witt is best interpreted in its shadow.

This document was inserted in the *Resolutiën* of the States of Holland and West-Friesland for 30 July 1671 (Schevichaven *et al.*, 1898, p. 4) and, with some small changes, was published separately (Rooijen, 1937). It was translated into English by Hendriks (1852, 1853) and part of this translation was reproduced by Walford (1871, Article: Annuities). A translation into French by Chateleux (1937) was presented to the members of the eleventh Congress of Actuaries prior to publication.

The *Waerdye* starts with three Presuppositions. In the second it is assumed that a man is as likely to die in the first as in the second half of a year. Then follow

* By a slip, Huygens wrote 4 here but his rough notes for the letter show he intended 2.

three Propositions the first two of which are essentially the first three of Huygens's (1657) 14 Propositions (David, 1962) expressed without algebraic assistance. The third Proposition repeats some of the assumptions of the Presuppositions and concludes with a Corollary in which a marginal comment refers the reader of material expressed in terms of 'chances' back to the Presuppositions which do not use this terminology. Unless we remember that de Witt is referring to life table decrements as chances we can get into a fine discussion of conditional probabilities depending on survival from age 3 to age x and $x+\frac{1}{2}$ (x an integer).

In the corollary following the Third Proposition appear the successive values of

$$10^7(v^{1/2}+v^{2/2}+v^{3/2}+\ \ldots\ +v^{n/2}),\ n\ =\ 1,2,3,\ldots 200$$

at 4% effective, certified as correct by two government accountants. Ignoring the power of 10 this would be written $2a^{(2)}_{\overline{n/2}|}$

The calculation of a life annuity value to a child aged 3 is made by considering the value of an annuity to each of the deaths in d_x and summing for all x. It is effected by writing down 10^7 times the numerical value of each term within the Θ's of the following expression:

$$\sum_{0}^{99} 2a^{(2)}_{\overline{n/2}|}+\frac{2}{3}\sum_{100}^{119} 2a^{(2)}_{\overline{n/2}|}+\frac{1}{2}\sum_{120}^{139} 2a^{(2)}_{\overline{n/2}|}+\frac{1}{3}\sum_{140}^{153} 2a^{(2)}_{\overline{n/2}|}\text{ with } a^{(2)}_{\overline{0}|}\ =\ 0.$$

taking the sums, multiplying by the fractions, performing the additions, dividing by 128 (namely, $100\times 1+20\times\frac{2}{3}+20\times\frac{1}{2}+14\times\frac{1}{3}$) and, finally, dividing by 20 to obtain $10^6a^{(2)}_3=16001607$, a million times a unit half-yearly life annuity.

If we multiply the foregoing expression by 6 to dispose of fractions, and insert the denominator we get

$$\alpha^{(2)}_3\ =\ \left[\ 6\sum_{0}^{99}+4\sum_{100}^{119}+3\sum_{120}^{139}+2\sum_{140}^{153}\right]2a^{(2)}_{\overline{n/2}|}/(2\times 768)$$

and we see the four adult 'chances', Σd_x, in Graunt's life table. However, while Graunt's chances applied to the 10 years following 36, 46, 56 and 66, respectively, de Witt's apply to the 50 years following age 3, the 10 years following ages 53 and 63 and the seven years following age 73. Can we explain these discrepancies?

Although only nine years had elapsed since the publication of Graunt's (1662) *Observations* the Supplement (or Appendix) to de Witt's (1671) *Waerdye* states (Hendriks, 1853) that he "had very carefully extracted from the registers of your Lordships *some thousands of cases* [italics inserted by Hendriks] of persons upon whose lives annuities have been purchased, with the memoranda up to the last due-dates to which the life annuities have been paid to each". Now if there was anything wrong with Graunt's guesses it was between ages 6 and 46 (Westergaard, 1901, p. 32) and it would be here that de Witt might seek to correct him from his own observations, particularly since Graunt's death 'chances' were excessive. Unfortunately, we do not have de Witt's annuitant observations but the very first extant mortality data, relating to 1495 nominees of subscribers to annuities issued by the Dutch United Provinces government in 1586–90, were sent by Hudde to de Witt in a letter dated 31 July 1671, one day later than the date of the *Waerdye*. Judging by the summary life table prepared by Schevichaven *et al.* (1901) de Witt's own data would have shown relatively constant mortality 'chances' (d_x values) between ages 3 and 53 with no particular upward or downward trend. What more natural than for de Witt to extend Graunt's 6 'chances' backwards year by year from age 36 to age 3. As for the choice of age 3 at which to start the annuity de Witt's Second Presupposition fixes this as the age at which a man has achieved his 'full vigour' so that Graunt's decennial starts are shifted by three years. Finally, we observe that de Witt's last seven years carry him to age 80 which is the age that Graunt chose to terminate his table. While these are no more than speculations we find it difficult otherwise to explain the sequence 6, 4, 3, 2 occurring at the ends of both Graunt's and de Witt's tables.

The *Waerdye* concludes with a declaration by J. Hudde that he has "attentively read and examined" the foregoing and that he was "of opinion that the method employed . . . is perfectly discovered, and that the conclusion made therefrom . . . depends upon solid and incontestable mathematical foundations . . ." (Hendriks, 1852).

Now although the calculations represent an impeccable, if prolix, method of obtaining $a^{(2)}$ based on certain assumptions about the values of d_x in a life table, de Witt made what appears to be a slip of the pen when he described these assumptions verbally in his Third Presupposition. He writes (Hendriks, 1852):

it is not likely, with respect to another man of like constitution or state of body, that the latter should die in less than a year or half-year of the said vigorous time of his life; whilst this likelihood or chance of dying in a given year or half-year of the ten first following years, namely from 53 to 63 years of his age, taken inclusively, does not exceed more than in the proportion of 3 to 2 the likelihood or chance of dying in a given year or half-year during the aforesaid vigorous period of life. . . .

I then presuppose that the greatest likelihood of dying in a given year or half-year of the second series of the ten following years (that is, from 63 years to 73, taken one with the other, rather than in a given year or half-year of the period of the vigour of life), cannot be estimated at more than double, or as 2 is to 1; and as the triple, or as 3 is to 1, during the seven following years, that is, from 73 years to 80.

These multiples appear to be the inverses of the fractions used by de Witt in the calculations described above. For those who see de Witt's "likelihood or chance of dying" (Hendriks's translation of "apparentie of dat hazardt van sterven") as a veritable rate of mortality the problem of interpretation is aggravated. Rooijen (1937) gives references to the authors who have participated in a free-for-all including Orchard (1852) who seems to have been the first to notice the contradiction and condemned de Witt's work out of hand. We believe that de Witt would naturally have made the calculations first, possibly discussing them with

Hudde, and then wrote out his description of them for his political peers. Between the calculations and the Report occurred the infelicitous explanation based, possibly, on a lapse of memory.

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