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IMPROVING THE PERFORMANCE OF EQUITY PORTFOLIOS

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1. INTRODUCTION AND SUMMARY

THIS paper has the strictly practical objective of devising procedures for managing Equity portfolios to the best advantage.

First, Modern Portfolio Theory, (MPT) which has been developed over the last 35 years with just this objective, is critically examined; from a study of the history of MPT and of its philosophy, principles and practices, the authors conclude that this discipline makes no contribution whatever to improving the performance.

The Index Fund step-up is a sterile concept. Surely a professional investment manager, using modern techniques of data banks, investment analysis, etc. should improve on the Index performance. This objective is in conflict with the Perfect Market Principle. For the less erudite investors, who are, of course, in the majority, the Perfect Market Principle may well apply, at any rate, to the well researched leading shares. However, the Perfect Market Principle should not apply to the erudite, professional investor, well equipped and well advised, who should be able to find many opportunities for situations with undervalued and overvalued shares, both among the leading issues and particularly within the smaller companies that get less research attention.

Having failed to get a solution from MPT, the authors study the possibility of achieving their objective by rigorous investment analysis. They describe in detail an equity market model which appears to offer prospects of estimating the net worth of a share to compare with the current price and to show whether it is cheap or dear. This system is the Equity Market Model designed by Clarkson[1] and already described in *J.I.A.* **110**.

The Clarkson equity model is not put forward as the unique solution to the problem. Similar results can no doubt be achieved by other dedicated analysts, prepared to study individual shares closely, paying attention to the long term trends of growth of earnings, growth of dividends, earnings on capital employed, margins on turnover, etc.

Given a successful technique for identifying cheap or dear shares, the way becomes open to achieve an above-average performance by an active programme of selling overvalued shares and buying undervalued ones. This corresponds to the maintenance of a flowerbed by digging up the weakest specimens and replacing them by more vigorous new plants.

In sections 8 and 9, the authors describe a powerful and accurate method of

monitoring the performance, checking continually on the success or otherwise of the dealing activity. Purchases should outperform the market and sales underperform. If they do not, the manager needs to stiffen up his selection system, or take refuge in an index fund.

2. MODERN PORTFOLIO THEORY

From the references to MPT history in the Appendix, several points emerge:

(1) Under the conditions of the 1950s, with no inflation, with equities yielding more than bonds, and little long term growth of dividends, the original Markovitz scheme could be regarded as acceptable, both in theory and practice.
(2) As inflation develops and dividends begin to rise, the Markovitz theory becomes more difficult to apply as the historical return from equities becomes unrealistic.
(3) The Sharpe version of the MPT theory, is a completely different 'animal' from the Markovitz system. Markovitz was concerned with the market risk of equities relative to bonds. This represented a very heavy risk as an equity portfolio could very well shed, say 80% of its value. Sharpe, dealing only with equities, accepts the market risk and is concerned only with the differential risk between the equities and the Index. For a portfolio, these risks are much reduced by diversification and the likely variation of a normally constituted portfolio of traditional equities might not be much more than plus or minus 10%. Altogether there seems little point in all this fussing about risk, merely to minimize what is a comparatively minor item already.

This point was made very clearly in the discussion on Professor Moore's paper in 1972 (*J.I.A.* **98**). T. Grimes stated as follows:

"The Markovitz 'minimum risk' portfolio was only a portfolio which had a minimum risk if the data was precisely correct, which was very unlikely. Among all the portfolios that a fund might have in practice, there was one that actually had minimum risk, and there was another, probably different, which inaccurate estimates and the Markovitz theory would imply had minimum risk, but all of those practical portfolios had risks so close to the minimum that there was no reason to choose between them. In portfolio selection, common sense in spreading investments was all that was required to achieve the amount of efficiency in avoiding risk."

As regards the return, the historic beta is a crude and unreliable measure. It does not even record the past experience correctly as it includes price changes due to alterations in the equity rating from year to year. Even if the past history could be correctly reported it would in many cases become quite inappropriate due to changing conditions from mergers, takeovers, changes in profit margins, etc., etc. There is in fact a whole class of interesting investments, known as recovery stocks, where the future experience is expected to be quite different from the past performance as shown by the beta.

Whilst certain errors in beta might perhaps be acceptable under the Markovitz programme dealing with large risks and returns, the Sharpe Model is dealing with much smaller items, just the relative returns and relative risks. In these circumstances greater accuracy is required and the errors implicit in the beta system can very well make the whole operation abortive.

As regards the risk factor, the variance measure may be just acceptable in the very short term. For the longer term, it is the growth rate that determines the risk, the variance being of negligible importance. A volatile price structure can even be an advantage, if the share can be bought on cheap terms with consequent lower eventual risk.

The total risk is a combination of the short term dispersion, medium term discrepancies and varying long term trends. Perhaps these three factors may be independent but some co-variance between the medium and long risks cannot be ruled out. Even if these different risks were assessable, their variations would have to be combined with distribution patterns that are impossible to predict. Consequently there does not appear to be any way to determine from past history or other records the financial effect of the risk.

The validity of the Sharpe Model depends completely on the Perfect Market applying 100%. If there is not a Perfect Market, then shares can be assessed as to whether they are cheap or dear. For a cheap share, the risk of underperformance is surely less than for a dear share. Under these conditions the risk element contains a considerable market effect due to short term fluctuations around the norm. A risk factor, just based on historic beta, ignoring this market effect, can be misleading.

Even if the risk and return factors were reliable, there still remain major weaknesses, implicit in the application of the Sharpe Model. One particularly important problem is that of the co-variance between the different share prices. The authors suspect that the facile assumption, so convenient for calculation purposes, that the co-variance can be represented by the difference between the dispersions from the Index is surely likely to be far from the truth. In any event the error element, in a factor taken as the difference between two items, both subject to considerable fluctuations, is surely going to be considerable. If the co-variances are, in fact, suspect, the portfolio risk factor becomes even more dubious.

The mathematical development of the Sharpe theory is based on the assumption of a normal distribution of risk. If the Perfect Market does not apply, and shares can be assessed as cheap or dear, there is surely little doubt that the risk distribution will be distinctly skew.

In going through the detailed mathematics there are several other examples where short cuts have had to be taken, to render the mathematics manageable and where the practical assumptions behind these short cuts could well be unacceptable to an experienced investment practitioner.

In 1948, well before Markovitz, J. B. H. Pegler[3] defined investment policy as "maximizing the expected yield". This yield is net of risk, which is allowed for by the analysts in their estimates and forecasts.

Markovitz rejects this principle, suggesting that choosing investments this way would mean holding only one share, with no diversification. He appeared to be thinking in terms of a portfolio of no more than 10 bonds and shares. (Most of his mathematical treatment is based on analogies with games of chance, with only a limited number of outcomes.) If instead of 10 securities, the investor has available thousands of world-wide equities, the maximum yield range would give ample scope for diversification.

Rejecting the expected yield principle, Markovitz falls back on a system which requires as inputs the gross yield (before risk) and the risk itself as a separate quantifiable item. For a small bond-equity portfolio operating over a one or two year term, this procedure can perhaps be justified. For an all equity portfolio, the return before risk is difficult to assess, and the risk itself as a separate factor is quite impossible to measure.

After much detailed study (see Appendix 1) the authors are forced to the following conclusions:

(1) The Sharpe system uses crude and often misleading measures of both risk and return.
(2) Without the Perfect Market, the mathematical treatment falls down.
(3) Several 'short cuts' in the analysis, particularly the assumptions regarding co-variance are suspect.
(4) The risk factor defies logical assessment.

Altogether the authors are satisfied that this model, using historic betas for return and risk, has no practical application to portfolio management, either to improve the performance or to reduce the risk.

The Rowe Rudd scheme is something of a hybrid combining investment analysis for the return factors with historic betas for the risks. So far we have seen no evidence that this scheme provided any benefits adequate to compensate for the heavy costs.

The Rosenberg version of MPT may perhaps provide some benefit for an 'Index following' Fund to control and reduce risk. There is evidence of this scheme being applied on a considerable scale in the United States of America and to some extent also in the United Kingdom. The authors would like to hear from users of this method as to the extent the benefits, by way of risk reduction, compensate for the considerable costs. In any event the Rosenberg system, dependent as it is on investment analysts for both growth and risk, should not be regarded as MPT, which is supposed to confine its inputs to specific actual items. The Rosenberg version is really a way of applying fundamental analysis to the problem of controlling risk and should be regarded as outside the MPT field.

3. MARKET EFFICIENCY

3.1 *Statistical studies*

MPT supporters attempt to prove that the market is efficient by statistical

studies. If these studies show that out of 100 stocks, 90 appear to be efficient and only 10 'rogues' are inefficient, the market is alleged to be largely efficient and mathematical techniques are then developed to select shares or reduce risk. The experienced investment manager, however, realizes that it is the 10 'rogues' that are likely to cause exceptional profits or losses.

3.2 *Skilled analysts*

The extent to which market efficiency applies must surely depend on the skill and experience of the investor. For a relatively inexperienced analyst and the 100 leading shares that comprise the FTSE Index, the efficient market principle may apply almost 100% in the sense that this analyst cannot do better than random when assessing companies which are so well researched. This might well be the case as regards a large proportion of private investors, institutional managers and stockbrokers' analysts. Even with these prominent shares, the really skilled analyst, using advanced selection techniques, and studying the company particularly closely, should be able to identify certain shares where the current price is either too high or too low in the context of the long term prospects.

Sometimes an experienced analyst will identify a major accounting discrepancy that has been generally overlooked. Some years ago, for instance, Rolls-Royce and Vehicle and General Insurance went bankrupt and the shares became worthless. Well before these disasters both companies had engaged in 'creative accountancy'. With Rolls-Royce it took the form of capitalizing research expenditure, formerly charged against profits. In each case the skilled analysts of a certain firm of stockbrokers immediately appreciated the situation, recognized the danger signal and began a major campaign to get their clients to sell shares. In such a situation the stockbroker concerned does not publicize this information. He works his way round the major clients, selling the shares carefully so as not to upset the market too quickly. It may take perhaps a year for the consequence of this news item to have its full effect.

For smaller companies it is ridiculous to suggest that every possible feature of the accounts, the chairman's statement and the future prospects has been spotted and has been immediately discounted in the price. The specialists who identify a favourable situation will discreetly put their clients into the shares but will not go out of their way to draw attention to the situation. For Stead and Simpson, shoe retailers, the market value of the share capital of this well-managed business was only a fraction of the value of their 200 well-sited freehold shops. This situation persisted for years, until the anomaly was corrected by a period of dramatic outperformance relative to the market.

3.3 *The efficient market hypothesis*

Academic researchers claim that the prices of securities are nearly always 'correct' in relation to published data, and in the MPT literature three levels of efficiency are identified—weak, semi-strong and strong. We regard much of the so-called evidence of efficiency as totally unconvincing. For instance, Cootner[4]

(who was one of the most prominent researchers in the area of weak efficiency) concluded after extensive investigations that future prices were *not* independent of past prices. Cootner in fact believed that 'professionals' could first of all assess the real worth of a share reasonably accurately and then assess when the activities of 'non-professionals' had caused the share price to deviate from its real worth by the maximum likely extent. This led to his 'reflecting barriers' hypothesis, under which share prices tend to move in a channel between upper and lower boundaries. At the semi-strong level, the tests are very indirect and cannot identify whether analysts can compare a share's rating against the fundamental background and successfully assess whether it is dear or cheap. At the strong level, the most widely publicized investigations are those carried on U.S.A. mutual funds by Jensen.[5] We do not believe that Jensen's tests were conducted on proper scientific lines and we are surprised that his conclusions appear to have gone unchallenged.

In Appendix 2 we set out in more detail our criticisms of the Efficient Market Hypothesis literature. After discussing the fundamental approach to share selection, we then set out what we hope will be regarded as convincing evidence that inefficiencies can be detected very easily in the U.K. equity market, even amongst the widely researched shares that comprise the FTSE Index.

4. PERFORMANCE AND RISK

4.1 *Relative strength*

The best way of reducing risk is to improve the performance by successful active management. For this purpose the relative strength (the price movement relative to the index) of all holdings should be monitored weekly, looking for price changes that may indicate anomalies and 'situations'. It is interesting to record that the performance-seeking analyst is now using a form of beta, which has been so discredited in earlier paragraphs. The difference is that the analyst uses the latest value rather than the five year average. Also, instead of making the dangerous assumption that beta is consistent over time, he is looking for changes and is trying to benefit from these movements.

Table 1 sets out the relative strength factors as at 12 January 1987, for a pension fund equity portfolio. Reference to the 12 months' relative strength will show an extraordinary range from a positive figure of 46·7% for Pilkington, to a loss of 36·7% for Avana Group. Almost all of the holdings are in the larger companies, the near blue chips.

4.2 *Cheap or dear shares*

Chart 1 shows the price performance of J. Sainsbury shares relative to the All-Share Index from 1983 to 1987. Although the long term trend is close to the Index, there are successive phases when the price overruns the Index and then falls back below the Index. Over the period the price relative moves between 80 and 108, which is quite a wide range for such a well known and heavily researched

Table 1

Name	Price	% Change in Price Over			% Gain or Loss on Market Over		
		1 mth	3 mths	12 mths	1 mth	3 mths	12 mths
	p						
ASDA-MFI Group	167·00	+10·6	+5·7	+21·0	+3·4	−3·8	−6·5
Avana Group	473·00	+1·9	−8·9	−18·0	−4·7	−17·1	−36·7
Bass	777·00	+5·7	+12·9	+19·9	−1·2	+2·8	−7·4
BAT Industries	492·00	+7·0	+9·3	+58·7	−·0	−·5	+22·6
Beecham Group	454·00	+8·1	+10·7	+36·3	+1·0	+·8	+5·3
Bejam Group	170·00	+4·3	+13·3	+4·9	−2·5	+3·1	−18·9
BOC Group	395·00	+13·8	+30·8	+39·6	+6·4	+19·0	+7·8
British Petroleum	760·00	+11·8	+10·9	+40·0	+4·5	+1·0	+8·1
British Telecom	215·00	+7·5	+13·2	−6·5	+0·5	+3·0	−27·8
BTR	285·00	+2·9	−3·4	+14·6	−3·8	−12·1	−11·5
Bulmer H. P.	145·00	−7·1	−4·6	−5·8	−13·1	−13·2	−27·3
Cable & Wireless	362·00	+13·1	+11·7	+25·3	+5·7	+1·7	−3·2
Eng. China Clays	334·00	+4·0	+10·6	+18·9	−2·7	+·6	−8·2
General Electric	192·00	+11·6	+10·3	+10·3	+4·3	+·4	−14·7
Glaxo Holdings	£11·30	+18·1	+21·2	+47·6	+10·4	+10·3	+14·0
Grand Metropltn.	456·00	−3·2	+0·2	+33·4	−9·5	−8·8	+3·1
Greene, King	275·50	+4·8	+8·9	+45·0	−2·1	−·9	+12·0
Hawker Siddeley	500·00	+14·2	+13·4	+14·4	+6·7	+3·2	−11·6
Hillsdown Hldgs.	236·00	+10·8	+8·5	+67·4	+3·6	−1·3	+29·3
Imp. Chem. Inds.	£11·68	+5·5	+5·3	+55·9	−1·4	−4·2	+20·5
Jarvis Porter	111·00	−1·8	+0·0	N/A	−8·2	−9·0	N/A
Ladbroke Group	398·00	+11·2	+15·0	+21·0	+3·9	+4·7	−6·5
Marks & Spencer	200·00	+11·1	+5·3	+17·6	+3·9	−4·2	−9·1
Pilkington Bros.	623·00	+0·5	+35·4	+89·9	−6·1	+23·2	+46·7
Plessey	190·00	+8·6	+5·6	+10·5	+1·5	−3·9	−14·7
Premier Cons. Oil	43·25	+21·8	+31·1	+58·6	+13·9	+19·3	+22·5
Racal Electronic	186·00	+3·3	+16·3	+5·7	−3·4	+5·8	−18·4
Reckitt & Colman	922·00	+14·0	+18·4	+47·5	+6·5	+7·7	+14·0
Rio Tinto-Zinc	735·00	+10·5	+3·2	+40·8	+3·3	−6·1	+8·8
Sainsbury J.	438·00	+6·1	+7·9	+20·3	−0·9	−1·8	−7·0
Scot & Newcastle	209·00	+1·0	+11·2	+22·9	−5·6	+1·2	−5·0
Shell Transport	£10·31	+8·3	+11·5	+53·2	+1·2	+1·4	+18·4
STC	192·00	+13·6	+21·5	+84·6	+6·2	+10·6	+42·6
Stead & Simpson 'A'	96·00	−2·0	+14·3	+23·1	−8·4	+4·0	−4·9
Storehouse	280·00	+1·8	−8·5	−5·1	−4·8	−16·7	−26·7
Trafalgar House	299·00	+13·7	+8·7	−8·8	+6·3	−1·1	−29·6
Trusthouse Forte	184·00	+4·5	+21·1	+15·0	−2·3	+10·2	−11·2
Utd. Newspapers	427·00	+16·0	+17·0	+50·9	+8·5	+6·5	+16·6
Wellcome	252·00	+17·8	+32·6	N/A	+10·1	+20·7	N/A

share. Ideally the 'genius' investment manager should be able to hold the share over the period when it is overrunning the Index, and then keep out of it during the adverse phase. Obviously the manager, who could anything like achieve this result, would produce an above-average performance. Referring to the relative strength table, the manager, if really inspired should have sold Avana Group at

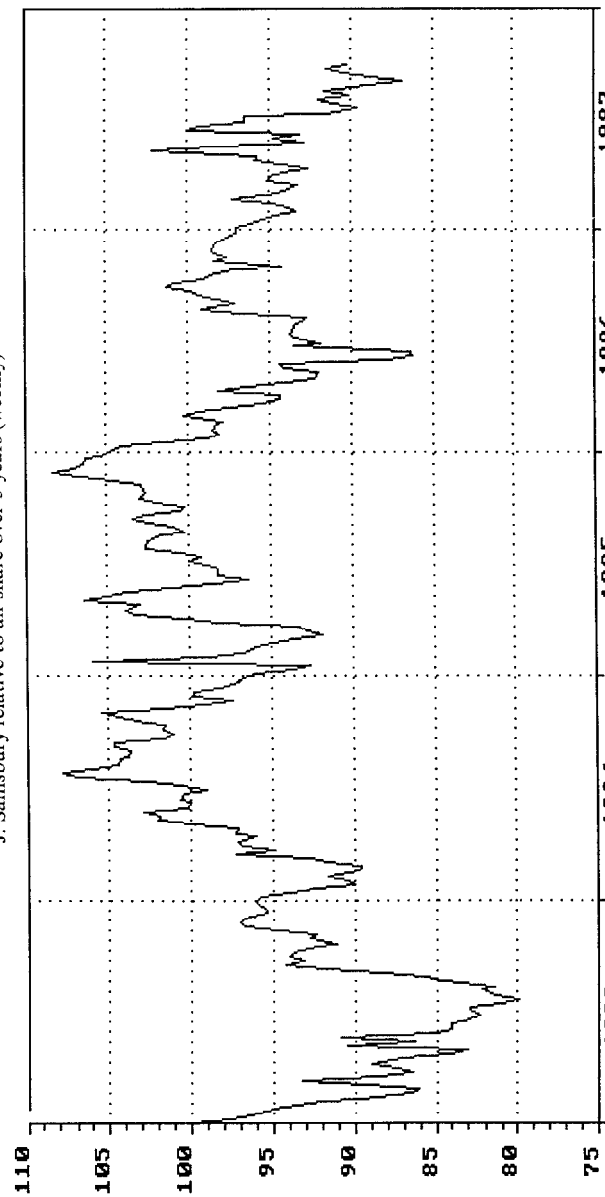

J. Sainsbury relative to all-share over 5 years (weekly)
110
105
100
95
90
85
80
75
1983
1984
1985
1986
1987

Chart 1

the beginning of the year, before it dropped by 36·7% and invested in Pilkington Bros. benefiting from their gain of 46·7%.

If a procedure could be developed to produce for ordinary shares explicit values of 'current worth' for comparison with current prices, the shares would be classified as to whether they are cheap or dear. The cheap shares can then be expected to outperform the Index and the dear shares to underperform. Putting it another way, the cheap shares represent the lowest risk and the dear shares offer the greatest prospect of underperformance.

5. FUNDAMENTAL ANALYSIS

In this section we develop various principles of fundamental analysis into a formalized framework to produce explicit values of 'current worth' and to obtain expressions which describe the variation of prices. For convenience, this development is carried out along the lines set out in three earlier technical papers by the present authors. We summarize below the gilts model developed by Clarkson,[6] the investment trust selection system used by Plymen & Prevett,[7] and the equity model developed by Clarkson.[1]

5.1 *The gilts model*

The gilts paper develops a price model for gilt-edged stocks which assumes only that the structure of prices is such that no 'obvious anomalies' exist. This assumption is developed into the following more precise statement:

For stocks of the same term to maturity, no switches exist which result in:

(i) a higher amount at maturity for the same, or higher, annual income; or
(ii) a higher amount of annual income for the same, or higher, amount at maturity.

Taking the price of a stock to be a smooth function $P\ (n, g)$ of its term to maturity, n, and coupon, g, this gives:

$$\frac{1}{\partial i}\frac{1}{P} < 0$$

$$\frac{\partial^2}{\partial i^2}\frac{1}{P} \leqslant 0$$

where $i = \dfrac{g}{P}$.

The most general solution of these partial differential inequalities is:

$$\frac{1}{P} = 1 + h(n) + f(n) \int_0^{\frac{g}{P} - i_0} \lambda\,(n, t)\, dt \qquad (1)$$

where (a) i_0 is a positive constant,
(b) $h(n)$ is a continuously differentiable function of n,
(c) $f(n)$ is a positive continuously differentiable function of n,
(d) $\lambda(n, t)$ is a positive smooth function of n and t with:

$$\text{(i)}\ \lambda\,(n, 0) = 1$$

$$\text{(ii)}\ \frac{\partial}{\partial t}\lambda\,(n, t) \leqslant 0.$$

Considerable care has to be taken in the selection of these auxiliary functions. Too loose a fit, which would result from having too high a number of variable parameters, would mean that the model had poor predictive properties. Conversely, a very poor fit would mean that important features of the price structure could not be represented by the model. The basic form of the model for long-dated stocks uses the following auxiliary functions:

$$h(n) = a_0 + \frac{a_1}{n} + \frac{a_2}{n^2}$$

$$f(n) = b_0 n + b_1 n^2$$

$$\lambda(n, t) = \exp\{-(c_0 + c_1 n)t\}$$

$$i_0 = \text{yield on irredeemables}$$

The next stage is to define a criterion of best fit. Since it is proportionate rather than absolute deviations of the actual price P from the expected price P that is important, this criterion is to minimize $\Sigma\, e_r^2$, where e_r is the proportionate price error $Pr/\hat{P}r - 1$.

Although equation (1) is transcendental in nature, iterative procedures which converge rapidly to a unique solution have been developed. This gives the set of expected prices P. In assessing whether a stock is dear or cheap, the absolute value of the proportionate price error is not a reliable guide, since this price error can be significantly different from zero for a long period. Instead, Mean Absolute Deviation techniques are used, with a parameter of ·027 for the daily geometric moving average and a value of 1·6 times the Mean Absolute Deviation for the distance between the moving average and the upper and lower control limits. Chart 2 shows the final chart for Treasury $9\frac{1}{2}\%$ 1999. Experience over many years has shown that this type of control theory chart is an exceptionally powerful method of identifying short term anomalies in the prices of gilt-edged stocks.

It will be noted that this type of approach is identical in nature to the 'reflecting barriers' approach suggested by Cootner in section 3 (above). Cootner, however, could only surmise that these reflecting barriers existed; the gilts model allows them to be calculated directly (as the upper and lower control limits) from the actual prices of stocks.

It is obvious that the nearer the proportionate price error is (to a control limit), the more likely it is that the next price movement will take the price error back

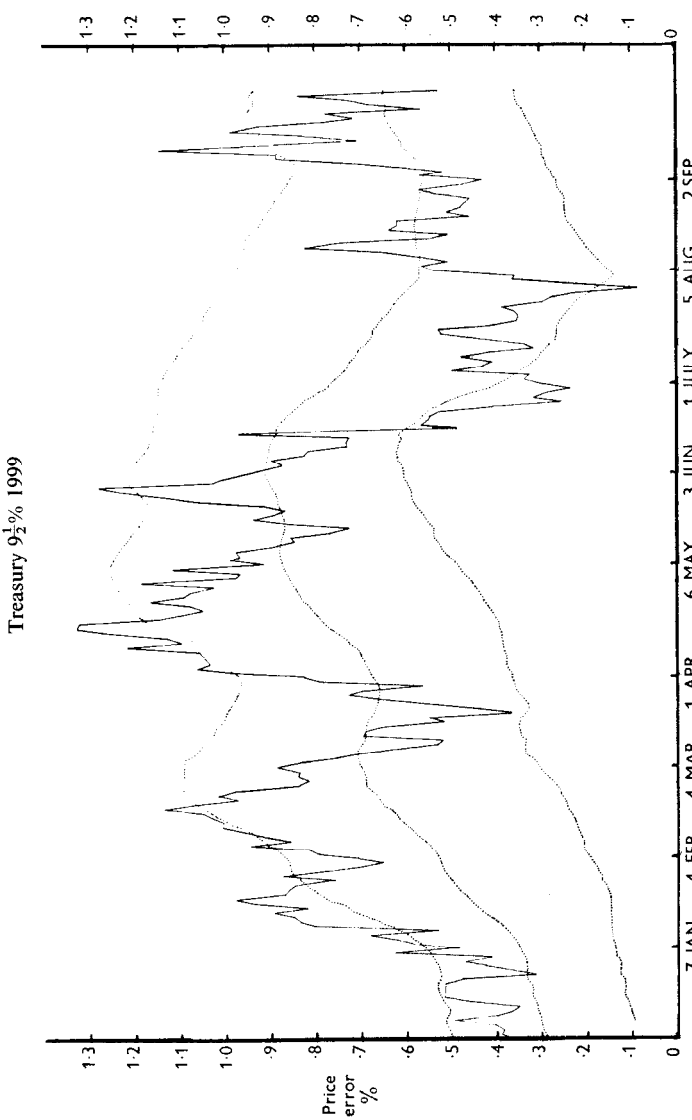

Treasury 9½% 1999
Price error %
1·3
1·2
1·1
1·0
·9
·8
·7
·6
·5
·4
·3
·2
·1
0
7 JAN
4 FEB
4 MAR
1 APR
6 MAY
3 JUN
1 JULY
5 AUG
2 SEP

Chart 2

towards its moving average. A reasonable description of the price formation process is thus:

$$e_{t+1} = e_t - \theta\,(e_t - \bar{e}_t) + x_{t+1},$$

where $\bar{e}_t$ the moving average value of e_t,
θ is a constant, with $0 < \theta < 1$,
and x_t is a random error term with expected value zero.

The day to day change in the price error (and hence in the actual price of the stock relative to a market index) is thus the sum of two components, a random component and a systematic component in the direction of the moving average. On the basis that we want the systematic component to 'explain' as much of the price variation as possible, an obvious estimation procedure for θ is to find those values which minimize Frequency $\{|x_t| < \varepsilon\}$ for various small value of ε. Tests have shown that, for a typical long-dated stock, the value of θ is approximately ·1.

5.2 *Investment trusts*

Plymen & Prevett describe how Mean Absolute Deviation techniques can be used to assess the cheapness or dearness of investment trust shares. In this case the statistic that is analysed is the percentage discount of the share price to the estimated net asset value. Estimated asset values are calculated by adjusting the latest official asset value per share (normally published monthly by the company) to allow for the subsequent movements in the relevant market indices. The Mean Absolute Deviation calculations are carried out with a value of 1·6 times the Mean Absolute Deviation for the distance between the moving average and the upper and lower central limits. A typical Mean Absolute Deviation chart is reproduced as Chart 3. The general pattern is very similar to that for gilts in Chart 2, except that the control limits are very much further apart. There are numerous reasons why this can be expected to be the case:

(i) there are estimation errors in the asset value per share used in the calculations;
(ii) dealing expenses are much higher than for gilts;
(iii) marketability is much poorer than for gilts;
(iv) occasionally there is a material change in the nature of the trust which makes the previous discounts irrelevant (e.g. a change in the investment policy, unitisation proposals, or a take-over bid for the company).

For all these reasons, a relatively large movement in the discount is necessary before there is the prospect of a sufficiently large switching profit to offset these various uncertainties.

As with gilts, the Mean Absolute Deviation technique as applied to investment trusts is identical in nature to the 'reflecting barriers' approach suggested by Cootner. In this case, the 'current worth' is simply the latest published asset value per share updated in line with the subsequent movements in the relative market indices; no estimates involving any significant degree of judgement are required.

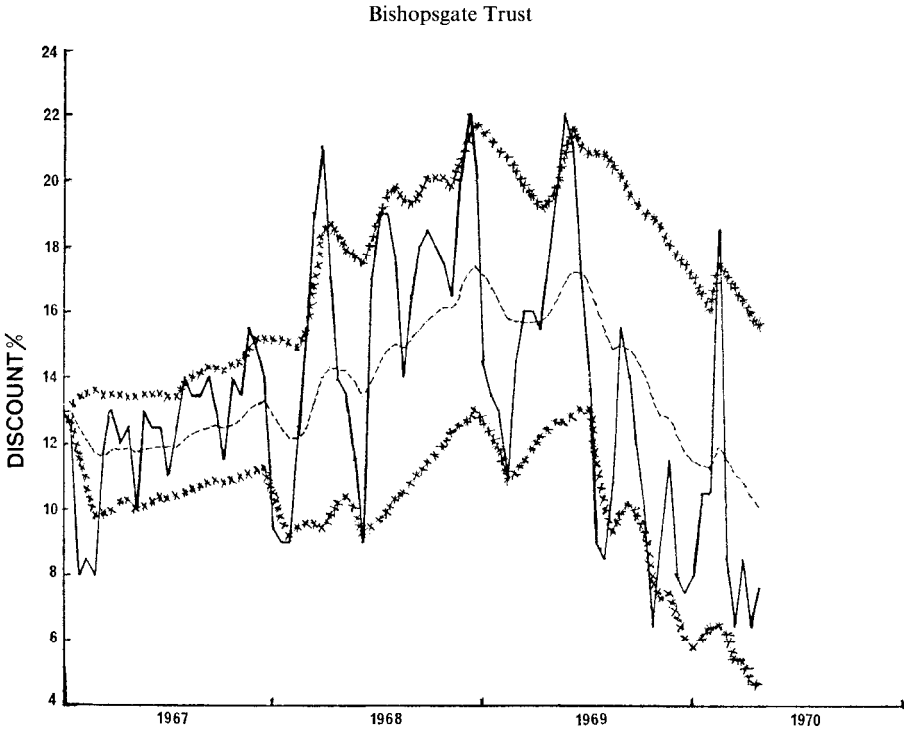


Chart 3

5.3 *The equity model*

When we proceed from investment trust shares, which represent a special class of equity shares, to equity shares in general, the difficulties involved in determining a satisfactory estimate of 'current worth' increase by several orders of magnitude. However, the Clarkson equity paper shows how the general principles adopted in the gilts model can be extended step by step to the equity market.

The starting point of the gilts model is the assumption that stocks vary in terms of only two attributes, term to maturity n and coupon g, and that the preferences of different investors together with their switching activity result in the prices of all stocks in the market being very close approximations to the prices determined by a certain function $P\ (n\ ,g)$. In the equity model, the starting point is the assumption that shares vary in terms of only three attributes, the historic (i.e. last declared) earnings per share E, the historic dividend per share D, and a ranking measure G of the future rate of growth of earnings per share. Obtaining the price

model is therefore equivalent to identifying a function $P(E, D, G)$ which represents actual prices as accurately as possible.

$$\text{Clearly } P(kE, kD, G) = k\,P(E, D, G),$$

$$\text{and hence } P(E, D, G) = E \cdot P\left(1, \frac{D}{E}, G\right).$$

We can therefore express the price function as $E \cdot P(R, G)$, where R is the dividend pay-out ratio D/E, and hence we need only consider this function of two independent variables.

Clearly, $\partial/\partial G\ P(R, G) > 0$, since investors will pay more per unit of current earnings if the future growth rate is higher. Also $\partial/\partial G\ P(R, G) > 0$, since investors will pay more per unit of current earnings if the dividend is higher and the expected future growth rate is the same. These two partial differential inequalities are the counterparts of those used in the gilts model in that they determine in very general terms the properties of the price model. The next stage is to define $P(R, G)$ in terms of auxiliary functions. The formulation chosen is:

$$P(R, G) = r + (1-r)e^{-d(\frac{1}{R}-1)}(1+g)^G,$$

where r and g are fitted parameters.

To simplify the practical operation of the model, the growth rate attribute G is deliberately chosen to be a ranking measure (which can remain constant for a significant time) rather than an absolute rate of growth which might have to be revised every time the aggregate rate of earnings growth for the whole market was changed. However, where a significant proportion of a company's profits are derived from overseas activities or exports, changes in the rate of U.K. economic growth relative to that of other major economies and changes in the exchange rate could change its growth ranking relative to a company with minimal overseas exposure. The growth function is therefore generalized to:

$$(1+g)^G(1+a \cdot A),$$

where A is a measure of the overseas exposure of a particular company and a is a fitted parameter. The model is now:

$$P = E\left[r + (1-r)e^{-d(\frac{1}{R}-1)}\right](1+g)^G(1+a \cdot A).$$

(We ignore for simplicity various minor adjustments for gearing, special factors, etc., which are described in the equity paper.)

As in the case of the gilts model, the process of statistical fit involves minimizing the sum of the squares of the proportionate price errors. Again the weekly price error is analysed in term of a Mean Absolute Deviation control chart using the same parameter values (·13 and 1·6) as for investment trusts. Chart 4 shows the final chart for Whitbread. The control limits for equity shares are in general much wider than for investment trusts. This is obviously what

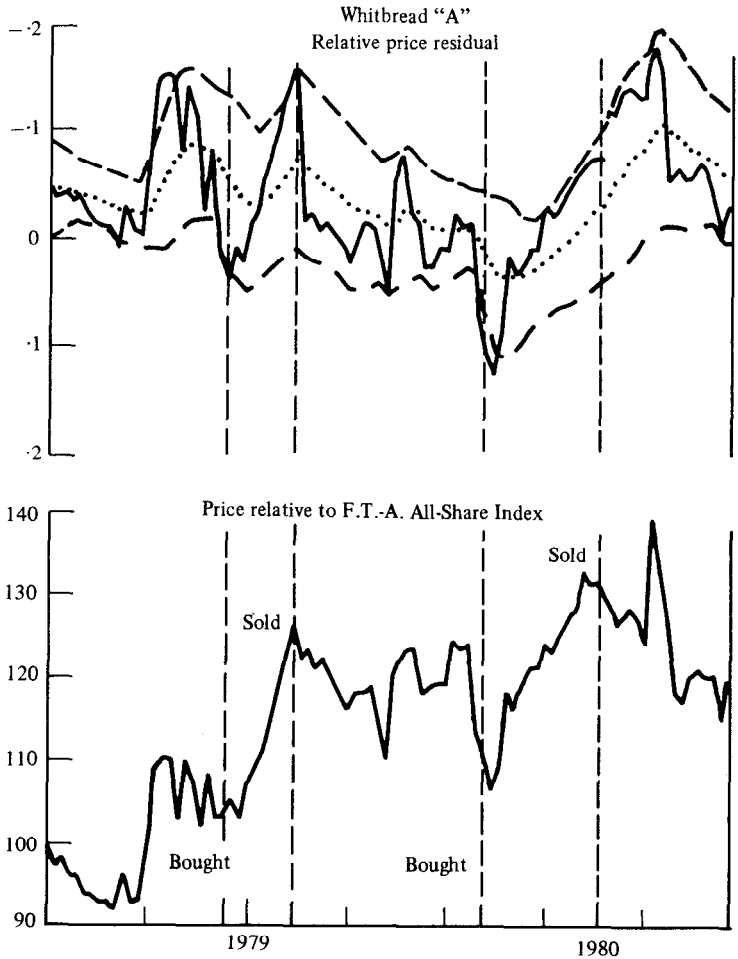


Chart 4

would be expected, since the estimation errors, the differences of opinion between different investors, and the differing investment time scales of different investors are likely to generate much more 'random noise' for a single equity share than for an investment trust share. Appendix 3 contains the section from the equity paper which describes in detail how the control chart is used in conjunction with fundamental analysis to identify buying and selling opportunities.

5.4 *Random noise*

The parameters used in the Mean Absolute Deviation control charts are

identical for all three classes of security, and the general patterns of each control chart are very similar. The only significant difference is the vertical scale; the control limits are very close for gilts, much wider for investment trust shares, and very wide for equity shares. This suggests that the price formation process is similar in all three cases, in that investors attempt to assess the 'correct' price by reference to the fundamental attributes, with the magnitude of the 'random noise' being a reflection of the scale of the estimation errors involved in quantifying the fundamental attributes.

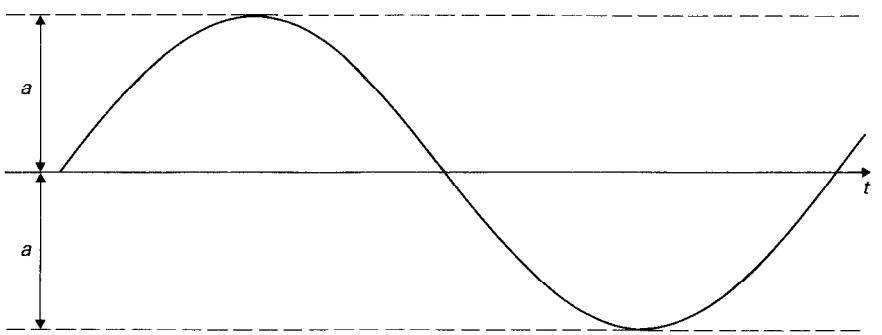
6. THE PRICE FORMATION PROCESS

The equity model incorporates three quite distinct components of price movement, and the relative magnitude of each is considered below.

6.1 *Short term*

This is the 'random noise' element, and results from short term factors and 'sentiment' causing the share price to deviate from an 'intrinsic worth' value that can be justified on the basis of fundamental attributes. In a typical case, the upper and lower Mean Absolute Deviation control limits are around 15% apart.

Suppose that the variation of the price error around the moving average follows a sine wave pattern as below.



Then the Mean Absolute Deviation calculated over a complete cycle

$$\int_0^{2\pi} |a \sin\theta t|\, dt \div \frac{2\pi}{\theta}$$

$$= \frac{2\theta}{\pi} \int_0^{\frac{\pi}{2}} a \sin \theta t \, dt$$

$$= \frac{2a}{\pi}.$$

The distance between the moving average and each control limit is a, which is k times the Mean Absolute Deviation where $k = \pi/2 \doteqdot 1{\cdot}57$. This is very close to the value of 1·6 which is found from practical experience to be the most suitable value for gilts, investment trusts and equities. We therefore conclude that, in any theoretical work, the frequency distribution of the future value of the proportionate price error can be represented as above by a sine wave.

For practical purposes, it is the combination of the position of the price error relative to the control limits together with an appraisal of all the relevant fundamental considerations which leads to a judgement of whether a share is 'dear' or 'cheap' on a short to medium term view.

6.2 *Medium term*

The model contains three fitted parameters which correspond to investors' preferences in the aggregate for:

(i) earnings growth
(ii) dividend pay-out ratio
(iii) overseas exposure.

If the price model incorporates variable parameters for the attributes $A, B, C \ldots$, then a suitable measure of the relevant importance of attribute A is obtained by observing changes over time in the price sensitivity $S(A)$ defined by:

$$S(A) = \frac{P(Auq, Bm, Cm, \ldots) - P(Alq, Bm, Cm, \ldots)}{P(Am, Bm, Cm, \ldots)},$$

where Auq, Am and Alq are the upper quartile, median and lower quartile values at A respectively.

On the basis of this measure, it is found that the earnings growth rate and overseas exposure factors are normally much more important than the dividend pay-out ratio factor. This is discussed in much more detail in the equity paper, but it may be useful to comment further on the behaviour of the earnings growth rate factor.

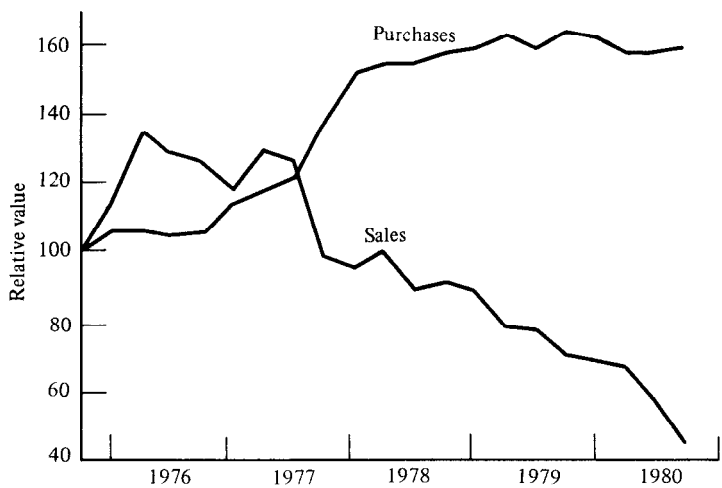
An increase in the earnings growth rate factor is equivalent to the PE ratios of high growth companies increasing proportionately by more than the PE ratios of low growth companies. This is an effect which might be taken to be equivalent to an increase in general confidence in line with a rising market trend. However, in May, June and July 1979 the earnings growth sensitivity increased at a time when the market was in a strong downward trend. This seemed inconsistent with past experience and general expectations, and the situation was investigated closely at the time. The conclusion was that, in a period when wage costs were rising rapidly and interest rates were very high, the profits and cash flow of low growth companies had come under severe pressure in the new economic environment whereas the profits of high growth companies were only marginally affected.

If the U.K. equity market entered a 'bear phase' for any significant period of time it is highly likely that the PE ratios of high growth shares would, in

proportionate terms, fall very much more than the PE ratios of low growth shares. Since the general market trend has been upwards since January 1975, the behaviour of PE ratios in recent years is not likely to be a reliable guide to what would happen in a bear market.

6.3 *Long term*

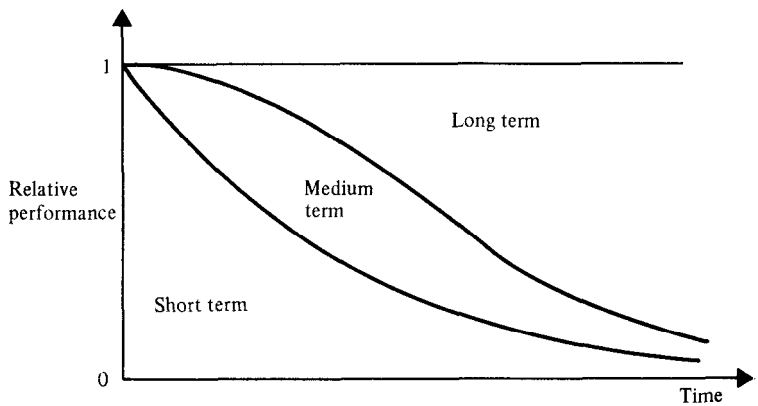
The long term component of price movement corresponds to the change over time of attribute values such as earnings and dividends per share. Since the short term and medium term components are in essence fluctuations about some central value, it would be reasonable to expect this long term component to be the dominant factor for price movements over periods of, say, two years or longer. The equity paper describes an example which shows how powerful this effect can be. When the model was being set up and growth rankings were being calculated for the first time, it appeared that many capital-intensive companies were likely to show very much poorer growth than previously as the result of the likelihood that high rates of inflation would persist for several years. This, however, was not discounted in the ratings of these companies. Accordingly, holdings in many of these companies were sold during August and September 1975 and reinvested in companies expected to show higher earnings growth. The performances of the sales and repurchases are shown below.



The sales outperformed the market very strongly until the second quarter of 1976, which suggested that the majority of investors disagreed with the rationale for the switch. Thereafter their relative performance declined marginally for a year and then began an exceptionally steep decline which continued for more than three years. The purchases, on the other hand, having moved only slightly

ahead of the market over the first year, outperformed significantly for the next two years and then levelled off. After five years, the value of the purchases was three and a half times the value of the sales.

It is apparent from the above discussion that the relative importance of the various components varies with the period of time over which price changes are being analysed. Over a very short period the 'random noise' is dominant; over a period of two years or more the price change due to changes in earnings and dividends is by far the most important. This relative importance is represented in diagrammatic form below.



For convenience these three components of price movement have been discussed in the context of the equity model. However, it is apparent that nearly all of the research work of stockbroking firms and others is along parallel lines. Share recommendations are often described as being short term (i.e. is the share price materially different from its expected value as a resut of short term factors) or long term (is the current rating in line with its long term growth prospects, bearing in mind the quality of its products and its management, etc.). Also, market strategists attempt to assess the relative attractiveness of various classes of shares (e.g. high earnings growth, high overseas exposure, etc.) in the light of the expected stockmarket and economic background.

6.4 *Testing the model*

Some will argue that this resolution of price movements into various components is irrelevant, since there is no evidence that any methods of fundamental analysis can achieve consistently better than random results. We disagree. To test the Efficient Market Hypothesis using a much simplified version of the equity model, we asked three leading firms of London stockbrokers to rank the long term earnings growth of the FTSE Index constituents on a scale of 1 (very low) to 10

(very high), with 10 shares in each growth category. The average of their growth ratings was taken as the value of G in the simplified equity model:

$$\hat{P} = E\,(1 + g)^{G-5\cdot5}$$

and one particular firm's estimates of earnings per share were used for the only other attribute, E.

Applying this model as at 24 June 1987 to the non-financial FTSE companies researched by all three brokers gave 14 'buys' and 14 'sells'. The average performance of each group relative to the FT-Actuaries All-Share Index at fortnightly intervals is set out below:

Duration

	2 weeks %	4 weeks %	6 weeks %	8 weeks %	10 weeks %	12 weeks %
Buys	−·7	−·3	−·8	−·6	+ 1·6	−·2
Sells	−2·5	−3·3	−3·9	−3·6	−4·3	−4·1
Δ	+ 1·8	+ 3·0	+ 3·1	+ 3·0	+ 5·9	+ 3·9

Random sampling investigations on the relative performances of all the 71 stocks at duration 12 weeks suggest that there is less than a 3% probability that a differential performance of 3·9% could have arisen by chance. Fuller details of this elementary test of efficiency are given in Appendix 4. We regard these results as convincing evidence that the Efficient Market Hypothesis is indefensible so far as the U.K. equity market is concerned, and we challenge any proponents of market efficiency to conduct their own investigations along similar lines.

7. RISK AND RETURN

In many areas of applied mathematics, practical methods for the solution of day to day problems tend to concentrate on expected values of the underlying probability distributions and virtually ignore other possible outcomes. In actuarial work, much of the subject of life contingencies is concerned with expected values of future mortality and interest rates. Where the underlying probability distributions are subsequently well defined (this is normally the case for q_x), confidence limits, variances and other similar measures can be calculated.

In investment analysis, where the underlying probability distributions are very difficult to identify, the final assessments generally relate to a central or most likely outcome, ignoring the effect of risk.

Although risk is generally seen to be equivalent to the likelihood of a significantly poorer than average return, it is very difficult to arrive at an acceptable theoretical definition of investment risk. Nearly all the work that has been carried out on risk has been within the confines of MPT, where it is axiomatic that risk is the variability of the return as measured by its variance or standard deviation.

We regard this MPT definition of risk as totally unsatisfactory. Suppose, for example, that we have two shares A and B, where the returns to a particular future date depend on certain scenarios $X_1, X_2, \ldots X_N$. For each scenario X_i, the return on share A (which is always positive) is twice the return of share B. Since the return on share A is always greater than the return on share B, any reasonable investor will regard share A as 'less risky' than share B regardless of the respective variances of return.

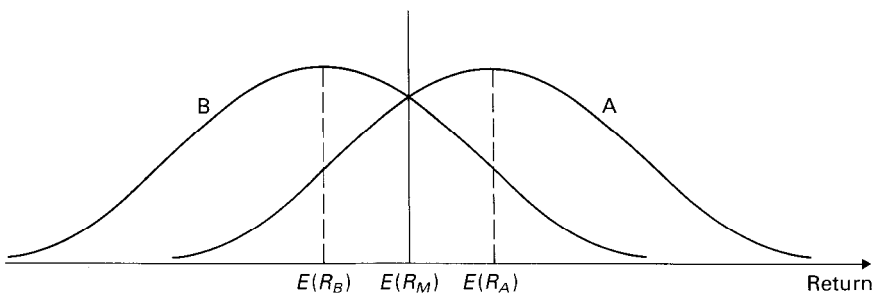
This example suggests the following axiom:

If share A has a higher expected return than share B over a particular time scale, the risk on share A is lower than the risk on share B if the probability density function for all values of return below the expected return of B is smaller for A than for B.

This in turn suggests the following integral measures:

$$\text{Risk on share } A = \int_{-\infty}^{E(R_B)} [E(R_A) - r]^2 \, P_{E(R_A)-r} \, dr.$$

$$\text{Risk on share } B = \int_{-\infty}^{E(R_B)} [E(R_B) - r]^2 \, P_{E(R_B)-r} \, dr.$$



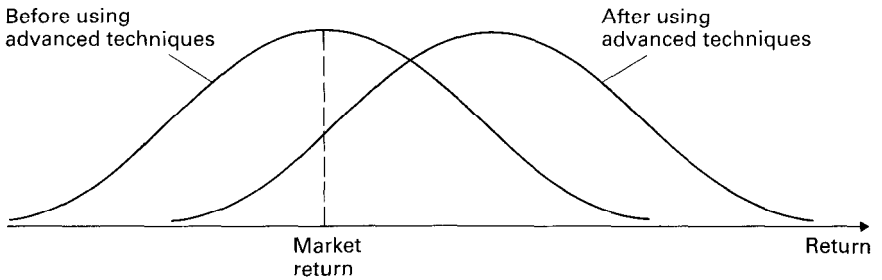
If the expected market average return $E(R_M)$ is known, an 'absolute' measure of risk can be obtained by integrating from $-\infty$ to $E(R_M)$.

This theoretical definition of risk may appear to be a useful step forward, but the various probability density functions are so complex, and depend on so many fundamental variables, that further analysis along these lines is pointless. In section 6 we have shown how the return on a share involves three components, each of which must be analysed separately.

It might be argued that historical data alone can be used to estimate the dispersion of various distributions, but we also reject this approach as unsatisfactory.

We then come to the main conclusion of the paper, namely that if we look after the expected return the risk will look after itself. In other words, if advanced

analytic techniques can assess the likely returns on shares more accurately than most market participants, then the probability distribution of the return on a portfolio 'moves to the right' and improves the expected return while simultaneously reducing the risk.



A fuller discussion of the complexity of the risk factor is given in Appendix 5.

8. IMPROVING THE PERFORMANCE

From section 7 above, it is clear that the best way to reduce the risk and simultaneously improve the performance is to set up an active programme of selling overvalued shares and buying undervalued ones.

Shares with a poor relative strength over the last three to six months are picked out for close analysis and investigation. The manager refers to all relevant data, such as price charts, P/E relatives, the cheapness and dearness from a market model etc., to decide whether the recent adverse performance is temporary or is likely to persist. Obviously if the company seems to be encountering a long term adverse trend the shares should be sold. Conversely, of course, if the adverse trend is due to some temporary setback, a case may be made for buying some more. In principle the objective is to get out of shares that are likely to underperform and into those likely to 'beat the Index'.

9. MONITORING THE PERFORMANCE

In practice many professionally managed funds reveal a high degree of activity. If combined with a poor performance the conclusion must be that the costs of this intensive activity are greater than any benefit to the performance. The situation clearly calls for an investigation, on the following lines.

The profit from the original portfolio as it stands, say at the beginning of the year, is calculated, month by month. The profit from every purchase is analysed to see whether the purchases have been cheap and whether they have performed better than the Index. Finally the sales are examined in the same way to see whether they were dear at the time of sale and whether they underperformed the market afterwards. These studies are done by indexation, i.e. for every purchase

Table 2. *Performance from 31 March 1983 to 23 August 1984*

Group	Market Value at 23 August 1984	Units	Indexed Value	Result
'Old'	2635072	5218·682	2662154	(27082)
Purchases	1100612	2089·843	1066070	34542
Trades Completed		(11·922)	(6081)	6081
	3735684	7296·603	3722143	
Deduct Sales	(1121085)	(2204·813)	(1124719)	3634
Current	2614599	5091·790	2597423	17175

If portfolio at 31 March 1983 had been left unchanged	
Value at 23 August 1984 would have been	£27,082 below 'Indexed Value'
Current Portfolio worth	17,175 above 'Indexed Value'
Gain from management (after all expenses)	£44,257
As % of 'old'	1·68%

or sale the amount involved is divided by the index of the day to give the number of units. Subsequently the number of units is multiplied by the Index to indicate the performance of an 'Index Fund'. With this set up it is possible to monitor continually the results of the whole portfolio, of the original portfolio at the beginning of the year and of the subsequent purchases and sales. An example of this operation is given in Table 2, dealing with the results of the Insurance Orphans Fund from March 1983 to August 1984.

Over the 18 month period the old fund, as it stood in March 1983, underperformed the Index by about 1%. This suggests that the distribution of the shares at that time, largely the responsibility of the previous manager, left something to be desired from the point of view of performance. The purchases outperformed the Index by some 3% and this result was further improved by a small profit on a couple of in and out trades. The underperformance of the sales was of a marginal nature, around $\frac{1}{4}$%.

In this case the switching activity distinctly improved the performance, suggesting that the manager had sufficient selection skill to ensure that the Perfect Market did not apply to his operations!

If however the activity had not made a profit, and the portfolio performance deteriorated, the conclusion would have been that the manager was not sufficiently skilled. The Perfect Market would appear to have been appropriate in this case. The Manager should improve his selection skills, or else take refuge in an index type fund.

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APPENDIX 1: MODERN PORTFOLIO THEORY

1. MARKOVITZ

During the 1950s, Markovitz[8],[9] published his theses on the subject of portfolio selection and the degree of risk. At that time American investment portfolios were largely bonds with only a small proportion of common stocks. The shadow of the 1929 crash still hung over the equity market. Investors had the choice of a wide range of low risk investments, particularly municipal bonds, often tax exempt. Markovitz' objective was to assist with the determination of the best bond-equity mix. He was not usually concerned with a large number of separate investments. A considerable proportion of the portfolio might be in near riskless securities, the equities being represented as a group, by what he described as 'a risky portfolio'. For each security he calculated the average performance, year on year, over say ten years. The standard deviation of the performance was calculated and the square of this represents the variance which is used as the direct risk criterion. Inflation had at that time scarcely been heard of and equities were regarded as carrying little growth prospects. The high yield of the equities compensated for the risk of dividend cuts. Under these rather static conditions, the average performance of the different stocks and the variance of this performance can be regarded as giving a reasonably fair indication of the performance and the risk factors. When averaging the variance for a portfolio, it was necessary to include the co-variance factors between each pair of stocks. This was not a very formidable task, as many of the investments were near riskless, contributing little to the variance.

Bearing in mind that between 1929 and 1933 the Dow Jones Index dropped by 80%, the equity risk was considerable indeed. In fact in 1929 for many investors, operating on margin or investing in geared up investment trusts, their savings were wiped out. Even though equities yielded very much more than bonds and riskless securities, it was necessary to diversify the portfolio away from equities into lower yielding bonds so as to protect the investors' savings in the event of another 1929 style crash.

During the 1960's, inflation appeared on the scene with consequent growth of share prices and dividends. Equity yields declined relative to bond yields. With equity dividends more likely to go up than down, the risk distribution for equities became skew. This meant that the variance of the equity price became a less satisfactory measure of the risk.

2. SHARPE

As time went by, the fear of a 1929 disaster receded and the cult of the equity developed in America as it did in this country. Under these circumstances the original purpose of the Markovitz principle, the choice of the bond-equity mix

became irrelevant. Clearly the Markovitz principles had to be developed so as to cope with the problem of selecting an all equity investment programme. This meant coping with a much larger number of securities. At this stage the problem of the co-variances became a major obstacle (for only ten securities the number of co-variances is 55). At this stage Professor Sharpe[10] suggested that the variance of each constituent should be calculated against a standard portfolio i.e. an Index. The co-variance of individual securities, with other equities, was restricted to the difference between their dispersion from the Index. Without the Sharpe adjustment, the Markovitz technique, was quite impracticable.

The change-over to the 'Sharpe' system, in effect, takes the market fluctuation out of the picture (i.e. the investor has to stand the risk of the equity market as a whole). The remaining risk, that of underperforming the Index, is small compared with the large risk difference between equities and bonds. An equity portfolio, taken as a whole, could well drop in value by 60–80% under extreme conditions. Within the equity portfolio, some shares will underperform and some outperform. The average dispersion of the result from the Index will be zero. If the portfolio has, say, 50 shares with broadly level weightings, 100% loss of one constituent will only reduce performance overall by 2%. A 100% loss does not normally come without warning. The professional manager should, in most cases, be able to spot the deterioration in the company's status and to sell the shares before the loss becomes important.

For an all equity portfolio, really a completely new form of portfolio theory is needed. The risk and return factors now need to be studied to see whether they are appropriate under the somewhat different conditions.

The one year return for an equity includes an element representing the growth of dividends and earnings, combined with a rating element representing the effect of a change in the rating of the shares, relative to the market. For a well managed company, with profits and dividends growing consistently at an above average rate, it is almost certain that the status, reflected by the price/dividend ratio, or price/earnings ratio, will gradually improve as the company's quality becomes appreciated. In such a case, the return figures from the price statistics will include this element of improving status, which is not necessarily going to apply in future.

Under 'Markovitz' conditions the variance can be regarded as a fair measure of the risk. Once inflation, with corresponding dividend and earnings growth, takes over, the risk of loss long term declines and the variance is increasingly irrelevant as a risk criterion. If as seems logical, for a long term investor, the risk is regarded as the dispersion from the market return, this means in effect the dispersion of the long term growth factor from the norm. Under these conditions, it is difficult to see what purpose is served by looking at the variance factor for risk measurement.

Clearly, with both growth and risk measurements increasingly suspect as inflation rises, the Sharpe form of portfolio theory loses all credibility for portfolio selection and risk measurement.

3. ROSENBERG

Around the mid 1970s, the theoretical and practical weakness of the Sharpe version of MPT began to be appreciated. Since then, major refinements in measurement process have been developed by Barr Rosenberg, Professor of Administration at the University of California at Berkeley, (see 'Modern Portfolio Theory; The Principles of Investment Management' by Rudd and Clasing[11] 1982).

Rosenberg no longer adheres 100% to the perfect market theme as shown by the following quotes:

"Because the market is reasonably efficient, it will not be possible to easily identify securities which are greatly undervalued, all that can be hoped for is to locate a great many slightly undervalued securities."

"If the market is reasonably efficient, then traditional analysis i.e. organizing the research department by industry, performing the long range projections and so on is likely to be rewarding only to the extent that it is better than anybody else's."

Clearly Rosenberg accepts the fact that the Market is far from perfect as regards the really skilled analyst. Even the moderate analyst is able to identify slightly undervalued situations. Recognizing therefore the analyst's contribution, he replaces the return function in his analysis by estimates of the long term return produced by various methods. One analytical method that he mentions is the dividend discount model. Clearly he regards the alpha and beta functions, heretofore used for the return factors, as useless.

Secondly, Rosenberg takes the view that the historical betas are of very little use as shown by the following quote.

"Unfortunately there is no reason to believe that the average value of beta in the past is going to be the best prediction of beta in the future. If beta is to be used to help form judgements on future security returns, then a predictive beta and not a historical beta is required."

In practice Rosenberg uses a most complicated and time consuming process for calculating what he calls a 'good' beta. He calculates the beta by using some 46 fundamental descriptors, based on special features of the company concerned and the industry concerned. Finally he combines the information obtained from these predictive betas with that derived from the market related information. Again he is resorting to the investment analysts to improve his risk functions.

The original concept of the MPT theory was that the inputs should all be derived from hard market information without using anything of a subjective nature. Rosenberg seems to be abandoning this theme and is now prepared to use input material derived to a certain extent from subjective analysis.

Thirdly, Rosenberg changes the concept of risk and the method of assessing it. He splits up the total 'variance' risk into two parts: (1) market related risk, and (2) specific non-market related risk.

The first item in market related risk represents the extent to which the price movement is correlated with the market; in effect it represents the difference

between the stock beta and 1·0. This risk cannot be reduced by diversification. The specific non-market related risk is quite different. It is special to each security, reflecting the extent to which the security price movement is affected by specific factors special to the company. This last type of risk represents perhaps 30% of the variance risk for an individual stock. For the portfolio, however, the risk is reduced directly by diversification and if the number of securities is more than, say 50, this factor nearly disappears, coming down to say 1% of the total portfolio risk.

It is probable that Rosenberg has changed the risk definition from the original basis, which reflected the risk of capital loss to a basis which reflects the prospect of the performance departing from that of the Index, either positively or negatively.

The major application of the Rosenberg technique is to compare the portfolio with an Index and to perhaps make changes in it so that its performance may in future be closer to the Index. Certainly changes can be made to reduce the risk but they will surely also reduce the return. For an individual security within the portfolio, it is possible to calculate the contribution to the overall portfolio risk and return that is due to the security. If the security has a high risk and at the same time a high return, this system will reveal the size of the possible benefits and possible losses that may arise from the stock.

There is no doubt that some of the basic weaknesses of the Sharpe system have been reduced by the improved inputs used by Rosenberg. However, the basic system still suffers from the two basic problems:

(1) no contribution is made to improve performance, the whole emphasis is on reducing the risk;
(2) the risk cannot be quantified in any way; the only way of using the risk factor is to take the ratio of the return to the risk.

It is strange that so much trouble seems to be taken to reduce the risk, which for a decent sized portfolio more or less disappears with diversification.

4. ROWE RUDD

In 1978 Rowe Rudd, Stockbrokers, attempted to apply a form of the Rosenberg technique (but using historical betas only) to the U.K. market. For 1,500 shares, beta was calculated in the usual way over a five year period. Clients submitting a portfolio for vetting, were asked to rank the shares according to the expected relative growth rate on a 1–10 scale. When particulars of the portfolio had been fed into the computer, it was possible to operate a number of routines, designed to improve the performance by re-shuffling the size of the holdings, but keeping the shares the same. One of these routines aimed to minimize the risk, whilst another offered to maximize the ratio of return, divided by risk. Various constraints could be applied as regards the maximum and minimum percentage holding in each share and the extent of the turnover. This operation is described in detail in 'Modern Portfolio Theory and Financial Institutions' by Desmond Corner and David G. Mayes.[12]

This project does not attempt to eliminate weak holdings or introduce attractive new ones—it merely shuffles the weightings, regarding the existing holdings as sacrosanct. If implemented, every one of the holdings appears to change, benefiting the stockbroker but not necessarily the client. This project is a prime example of a most elaborate and expensive mathematical, computerized edifice being erected on distinctly shaky foundations, bearing in mind that the quality of the growth inputs must be questionable and the values of the Historic betas are equally suspect.

The authors have no knowledge how much business was done under this plan, but it did not appear to have any long term success with the institutions as Rowe Rudd & Co. ceased trading as stockbrokers in 1981.

APPENDIX 2: MARKET EFFICIENCY

Many MPT applications, including the Capital Asset Pricing Model and the analysis of risk, are based on the assumption that capital markets are highly efficient, in the sense that markets interpret and process information in such an efficient manner that the prices of securities are nearly always 'correct' in relation to published data.

1. TESTS OF EFFICIENCY

This efficiency concept is referred to as the Efficient Market Hypothesis, and in the MPT literature three levels of efficiency are identified:

Weak efficiency

The market is efficient in the weak sense if future share price movements are independent of previous movements. In such a market, a study of past prices cannot help predict future prices.

Semi-strong efficiency

The market is efficient in the semi-strong sense if share prices respond instantaneously and in an unbiased manner to new information. In such a market, it is futile for investors to attempt to identify undervalued or overvalued shares on the basis of published information.

Strong efficiency

The market is efficient in the strong sense if share prices fully reflect not only published information but also relevant information including data not yet publicly available.

In many of the more recently published books on the Efficient Market Hypothesis, the empirical evidence—although referred to only in summary terms—is presented in such a manner that any reader who is not an investment professional will generally be led to believe that there is a voluminous and coherent body of evidence to show that the Efficient Market Hypothesis has been subjected to, and passed, various stringent tests. Keane,[13] for example, refers to:

(i) the random walk studies of random or patternless price behaviour which led to the concept of 'weak' efficiency;
(ii) various tests of the speed and efficiency of the market's reaction to new information;
(iii) various studies which failed to find any professionally managed funds that performed better than the average on a consistent basis;
(iv) the conclusion, in 1978, of Professor Michael Jensen, a prominent market economist:

> "I believe there is no other proposition in economics which has more solid empirical evidence supporting it than the Efficient Market Hypothesis."

However, a detailed study of all the original research papers reveals that neither the coherence of the results nor the stringency of the tests are as convincing as implied by the summary papers of later writers.

2. WEAK LEVEL TESTS

Consider first of all the evidence of efficiency at the weak level. When early researchers studied commodity prices, market indices and individual share prices and failed to identify any profitable trading strategies, it became generally accepted that future share price movements were independent of past prices. In the late 1950's the 'Brownian Motion' model was considered to be the most satisfactory representation of share price behaviour.

However, there were two key sets of results which suggested that this model was inappropriate. Firstly, Mandelbrot[14] showed that distributions of price changes were more peaked and had longer tails than would be expected for the normal distributions implied by the simple 'Brownian Motion' model. He suggested that a new class of distributions known as stable-Paretian should be used instead. Secondly, Cootner[4] concluded that future prices were *not* independent of past prices. He put forward the hypothesis that 'professionals' could identify the 'current worth' of a share with reasonable accuracy and that sometimes as a result of the activities of 'non-professionals' the actual share price would depart sufficiently from the current worth for the professionals to be able to profit from the share price being more likely to move towards its current worth in future than to move further away from it. If changes in 'current worth' were random in nature, then the resulting price changes would also be random, but Cootner suggested that the professionals' estimates of 'current worth' would change gradually over time rather than in discrete jumps. He therefore concluded that price movements would tend to consist of relatively long segments each of which was constrained by 'reflecting barriers'. Throughout one of these segments, share prices would not be random but would behave as if there were reflecting barriers which forced them back towards their average value. Cootner tested this hypothesis by calculating transition matrices of price changes. His results confirmed that prices tended to move back towards a central value significantly more frequently than would be expected on the pure 'random walk' model, but his method was insufficiently precise to formulate trading strategies by which to profit from this departure from randomness.

He also realized that his investigations were not conclusive as the tests did not permit a definitive test of statistical significance. Steiger[15] then took over where Cootner left off and tested Cootner's hypothesis in a more rigorous manner. He concluded that the existence of the type of non-randomness postulated by Cootner was indeed confirmed by the results. It is therefore very surprising, to say the least, that this work of Cootner and Steiger is never referred to by proponents of the Efficient Market Hypothesis.

3. SEMI-STRONG LEVEL TESTS

At the semi-strong level, where efficiency would imply that all fundamental analysis based on publicly available data is futile, the tests are very indirect in nature. Rather than addressing the question of whether the application of specific criteria for investment appraisal can produce consistently superior results, the tests concentrate on whether prices react quickly, and in an unbiased manner, to certain easily quantifiable types of new information such as announcements of dividends, earnings and stock splits. Tests such as these cannot identify whether analysts can compare a share's rating against the fundamental background and assess whether it is dear or cheap.

But perhaps the most obvious criticism of the semi-strong tests in the MPT literature is that no attempt is made to examine share selection models which have been put forward by practitioners as systematic ways of achieving superior performance. The equity selection model described by Weaver & Hall[16] is a good example of a formalized set of investment appraisal rules which could be regarded as convincing evidence that the market is inefficient at the semi-strong level. With all models of this type, the rankings that are obtained are highly dependent on the input data, many of which are estimates of future earnings or of earnings growth rates. Accordingly, proponents of MPT might suggest that the input data are too subjective in nature for any superior performance to be acceptable as evidence of market inefficiency. This view is misguided for two reasons. Firstly, some of the success of the method could be because it applies certain criteria consistently across all the shares analysed in a manner that is impossible with a single human brain. If this is the case, above-average performance will still result if the input data represented only the 'market consensus' values rather than values which contained some superior insight into the future. The existence of this 'added value' from consistency could be tested by running the model using input data from practitioners other than those directly involved with the development of the model. No such tests appear in the MPT literature. Secondly, practical experience of models of the Weaver & Hall type shows what factors are most important in explaining share prices movement; by concentrating analytic resources on improving the estimates of future values of these factors, enhanced performance should result. Again this point has been ignored by proponents of the Efficient Market Hypothesis.

4. STRONG LEVEL TESTS

At the strong level, the evidence of efficiency is even more suspect. It is argued that if inefficiencies exist, then at least some market professionals should be able to exploit them and achieve above-average performance for the funds they manage. The tests carried out purport to show that there is no evidence that any expertly managed funds can perform consistently better than average. The conclusions of Jensen regarding U.S.A. mutual funds are frequently quoted in

this connection, but there are four quite separate aspects of his investigations which we regard as unsound:

1. Long term returns are adjusted for risk as measured by the short term variability of price. We regard this measure of risk as totally unsatisfactory.
2. It is assumed that the Capital Asset Pricing Model provides a realistic description of equity market behaviour.
3. There is a very serious lack of homogeneity in the sample as regards both the observation periods (which vary from 10 to 20 years) and also the investment profiles of the funds (which vary from bond funds to high technology funds).
4. Jensen bases many of his conclusions on a comparison of actual and expected frequency distributions of Student 't' values despite being aware that the usual normality assumptions may not be valid.

More generally, it is difficult to believe that the behaviour of U.S. mutual funds many years ago is in any way relevant to the efficiency of the U.K. market at present. Much more direct tests can be devised using statistics from the U.K. unit trust industry. For example, Table 3 shows that there were eight unit trusts with 90% or more in U.K. equities which showed significant and consistent outperformance in each of the years 1984, 1985 and 1986. If the U.K. market were indeed efficient at the strong level it is highly improbable that such results could have arisen by pure chance.

Table 3. *Unit trusts. 90% or more invested in U.K. Equities 1 January 1984 to 1 January 1987.*

Percentage gain—year by year

Trust	1984	1985	1986
A	40·8	30·8	41·1
	(10·6)	(12·0)	(15·2)
B	42·7	43·9	33·0
	(12·5)	(25·1)	(7·1)
C	36·4	31·2	43·7
	(6·2)	(12·4)	(17·8)
D	38·6	29·4	34·8
	(8·4)	(10·6)	(8·9)
E	38·8	27·2	35·9
	(8·6)	(8·4)	(10·0)
F	44·4	27·9	39·9
	(14·2)	(9·1)	(14·0)
G	39·0	38·1	29·9
	(8·8)	(19·3)	(4·0)
H	37·7	28·1	35·7
	(7·5)	(9·3)	(9·8)
FTA-All Share, income reinvested	30·2	18·8	25·9

The figures in parentheses indicate the arithmetical percentage improvement on the yearly index performance.

APPENDIX 3: ANALYSIS OF THE RELATIVE PRICE RESIDUAL

Chart 2 shows the graphs for Whitbread 'A' of the relative price residual (together with moving average and control limits) and the price relative to the F.T.-Actuaries All-Share Index for the period January 1979 to September 1980. The vertical scale of the relative price residual is reversed, so that penetration of the upper control limit indicates that the share may be dear and penetration of the lower limit indicates that the share may be cheap.

Towards the end of March 1979 the share price rose strongly against the market on the news that the company had been granted a 1p a pint interim rise on beer prices by the Price Commission and was to seek a further increase of 2p a pint, and the relative price residual rose above the upper control limit. However, in view of the likelihood of a Conservative victory in the imminent general election, this penetration of the control limit was not interpreted as a 'sell' signal.

When the promised abolition of the Price Commission was taken into account in profit projections after the general election, the relative price residual returned to the middle of the range. Following revisions to earnings and dividend projections after the 1978–79 preliminary results were announced in May 1979, the relative price residual penetrated the lower control limit. Since the company had indicated its confidence in the future by announcing with the results a £230 million investment programme, this penetration of the control limit was interpreted as a 'buy' signal and holdings were increased early in June at the point of the graphs shown by the vertical broken line.

During the next 8 weeks the share price rose by 25% relative to the market and early in August the relative price residual penetrated the upper control limit. At this point, when it looked as though the shares had become overvalued against the market, the position was reviewed carefully to determine whether in fact holdings in the company should be reduced. The fairly optimistic current year earnings and dividend prospects had been reflected in the input data, the benefits to accrue from the Chiswell Street property development had been taken into account by means of a special factor of 1·08, and the price-earnings relative was standing at 151 as against highs for 1976, 1977 and 1978 of 135, 127 and 130 respectively. It was judged that the shares had in fact risen to an unsustainably high rating, and accordingly holdings were reduced at the point shown by the second vertical broken line.

Despite the announcement of highly satisfactory interim results in November 1979 and the upgrading of estimates of the value to Whitbread of the Chiswell Street development, the share price declined sharply against the market in January 1980, causing the relative price residual to penetrate the lower control limit. The background news was again reviewed in detail. Although it appeared that the weakness in the share price had been caused by fears of consumer resistance to increased beer prices, the earnings and dividend estimates

incorporated in the model reflected some deterioration in the trading background and it was judged that the price weakness had been overdone. Holdings were therefore increased at the point shown by the third vertical broken line.

In the following 3 months the shares rose by 25% relative to the market, taking the relative price residual very close to the upper control limit. When the trading background was once again reviewed ahead of the 1979–80 preliminary results in May, it appeared that the general squeeze on consumer expenditure and the very sharp increases in beer prices might soon have a serious effect on sales volume. Since the 1979–80 results tended to confirm this view, holdings were reduced immediately after the announcement of these results at the point shown by the fourth vertical broken line.

Apart from a short-lived rally towards the end of June, the share price steadily underperformed the market after these results. At the annual general meeting in July, the chairman warned shareholders that "If the group suffered a similar fall away in business as had been seen in June and July, some elements of the group's investment plan would have to be postponed".

This example highlights the sensitivity of share prices, even in well researched sectors like breweries, to short-term changes in market sentiment. During the relatively short period under review, two buying opportunities were identified and in both cases selling opportunities after 25% outperformance were identified a few months later. On all four occasions, holdings of Whitbread 'A' shares were increased or decreased accordingly.

APPENDIX 4: EFFICIENCY TEST ON THE CONSTITUENTS OF THE FTSE 100 INDEX

When investment practitioners attempt to produce evidence to show that the U.K. equity market is inefficient, proponents of the Efficient Market Hypothesis invariably reject this evidence on the grounds that it does not meet academic standards of rigour. The grounds most often quoted for this rejection are:

(i) the qualitative nature of input data (i.e. the tests cannot be duplicated by other researchers)
(ii) the qualitative manner in which 'buy' and 'sell' recommendations are obtained (i.e. there is no set of algorithms by which the cheapness or dearness of shares can be assessed)
(iii) bias in assessment classes (e.g. 'buys' which comprise mainly high yielding shares could outperform merely because high yielding shares in general have performed well over the period; the same could apply to high growth shares as against low growth shares in a rising market)
(iv) lack of statistical significance of any differential performance (i.e. apparently good performance might not be inconsistent with random sampling from a universe of shares showing widely differing individual performances).

To avoid these potential criticisms, a test involving the constituents of the FTSE 100 Index was carried out on the following lines:

1. Three major London stockbroking firms were asked to rank the expected future long term earnings growth rates of these companies on a scale of 1 (very poor) to 10 (very good), with 10 companies in each category.
2. Earnings per share estimates for the latest reported financial year (E_0), for the current financial year (E_1), and for the subsequent financial year (E_2) were obtained from one of these stockbroking firms.
3. Since the accounting policies and methods of calculating earnings per share varied widely amongst financial companies (particularly with banks as a result of differing treatment of Third World loan provisions), all financial companies were excluded on account of this lack of homogeneity.
4. Non-financial companies which were not researched by all three stockbroking firms were excluded, leaving 71 companies in the investigation.
5. For these 71 companies, the estimated long term earnings growth rate G was taken as the average of the three ranking values in (1) above, and the earnings per share base E was taken as

$$E = \cdot 5\left(1 - \frac{t}{12}\right) E_0 + \cdot 5E_1 + \cdot 5\frac{t}{12} E_2,$$

where $(3+t)$ was the nearest number of months from the latest financial

year end. (It is assumed that preliminary results are announced 3 months after the financial year end.)

6. Because of the great importance to the U.K. equity market of the General Election outcome on 11 June 1987, the investigation was not carried out until after that date. Earnings estimates and share prices as at 24 June 1987 were used.
7. In terms of a much simplified version of the equity model, the expected price $\hat{P}$ is given by:

$$\hat{P} = E \cdot (1 + g)^{G - 5 \cdot 5}$$

where g is a fitted parameter.

8. For simplicity, a graphical rather than a full least squares fit was used. On a scatter diagram of $P \div E$ versus G (where P was the actual price), smooth upper and lower control limit curves were drawn so that 14 shares, evenly spread over the range of G, were above the upper control limit and 14 shares, again evenly spread over the range of G, were below the lower control limit.

The average performances (relative to the FT-Actuaries All-Share Index) of the 'buys' (the 14 shares below the lower control limit) and of the 'sells' (the 14 shares above the upper control limit) were calculated at fortnightly intervals thereafter. These performances, together with the change in the All-Share Index, were as follows:

Duration

	2 weeks %	4 weeks %	6 weeks %	8 weeks %	10 weeks %	12 weeks %
Buys	−·7	−·3	−·8	−·6	+1·6	−·2
Sells	−2·5	−3·3	−3·9	−3·6	−4·3	−4·1
Δ	+1·8	+3·0	+3·1	+3·0	+5·9	+3·9
FT-A All-Share	+4·0	+4·0	+2·7	−2·2	+0·1	+1·7

Since the general market behaviour over the period was highly erratic (rising to a peak in July, falling in very nervous trading for much of August, then stabilizing and moving ahead again) the consistent outperformance of the 'buys' over the 'sells' cannot possibly be attributed to any general market trend.

It is of course always possible that apparently good performance is not inconsistent, at some predetermined confidence level, with random selection from a sample of shares showing highly diverse individual returns. The full list of

71 relative (percentage) performance outcomes at duration 12 weeks is set out below:

24·9	20·2	14·2	14·0	13·9	13·5	10·7	10·0	9·5	9·4	9·1	6·9
6·4	5·7	5·6	5·1	5·0	5·0	3·6	3·5	3·1	2·9	1·5	1·4
1·2	·6	·1	−·5	−·6	−·7	−1·2	−1·7	−1·7	−2·4	−3·2	−3·3
−3·6	−3·8	−4·0	−4·6	−4·6	−4·9	−5·7	−5·7	−5·8	−6·4	−6·7	−6·9
−7·0	−7·5	−7·8	−8·1	−8·5	−8·7	−9·1	−9·3	−10·4	−11·0	−11·2	−11·9
−12·2	−12·3	−12·5	−12·5	−12·6	−12·8	−15·0	−15·2	−16·0	−19·7	−22·0	

Random sampling investigations suggest that the probability that a differential performance of 3·9% could have arisen purely by chance is less than ·03.

APPENDIX 5: RISK AND RETURN

1. MODELLING THE RETURN

From the indexes it is possible to make up a Model of return and risk.

All-share index

Date	Price	Yield %	Dividend
23.6.86	801·5	3·91	31·3
23.6.87	1136·2	3·08	35·0
Dividend	31·3		+11·6
	1167·5		
	+45·64		

Over the year from 23 June 1986 to 23 June 1987, the Index moved from 801·5 to 1136·2. This made for a return from capital and dividends over this exceptional year of 45·6%. This return is made up of three components, first a component of 3·9% being the dividend yield at 23 June 1986. Secondly there is the growth component, the gain in price due to the increase of dividend with the market returns assumed unaltered. This amounts to 11·6%. Finally the balance of the return 30·1% can be regarded as that fraction of the return that is caused directly by movement in the share market rating.

Traditionally, with a moderate degree of inflation, 5% or so, as at the present time, profits, dividends and share prices tend to rise more or less in line with the inflation (refer to Plymen's contribution to TFA in February 1987).[17] As long as inflation at this level continues we would expect equity prices, based on the index, to have a long term upward trend. Over 1986/87 the return on the Index, 45·6%, is made up of 15·5%, approximately one-third, from the underlying growth of dividends. The remaining 30·1% is due to the re-rating of the market consequent on a dramatic fall from 3·91% to 3·08% in the dividend yield over the year. For individual securities, the growth element will be greater or less than that from the Index. The balance of the return is, of course, geared to the market and moves up and down in line with the progress of the Index price.

$$\text{Return} = \alpha G + \beta V$$

G = Index growth rate V = Index variability

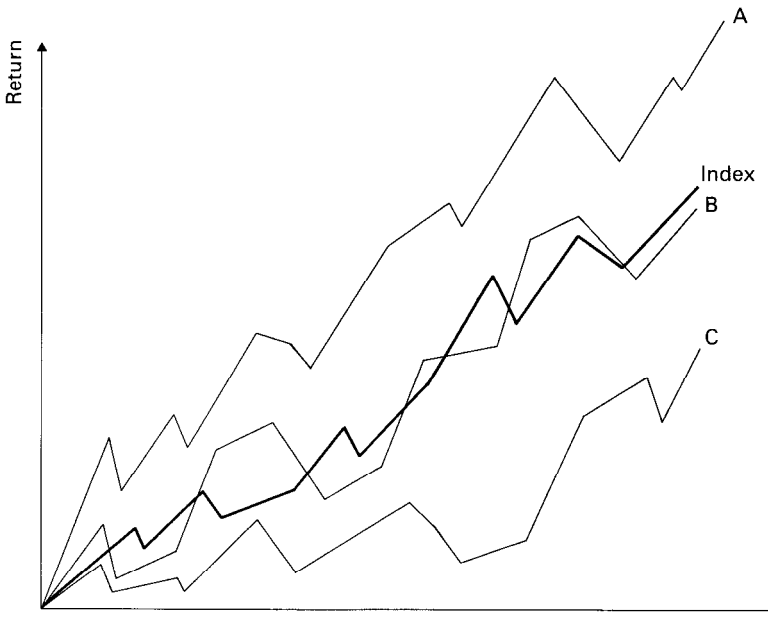
G = growth factor β = market factor

For an individual share, the relative rating may change, making for sharp price movements. Clearly the risk factor includes an element dependent on the short term price fluctuations. A 'cheap' share has better performance prospects with less risk than a dear share.

For a highly geared individual security, an above average performance, is

expected under favourable conditions. Hence the effect of the high gearing comes out in the alpha or growth factor. Consequently the beta factor reflects the variance of the security from special factors other than gearing and dividend growth.

The growth factor on the other hand reveals some clue to the risk element. Presumably the company with above average growth rate of dividends, profits, etc., is likely to have a below average rate of capital loss. Conversely, if the growth rate is below average, the contingency of loss and underperformance becomes significant (see Chart below).



Share A is clearly cheap, long term, as it outperforms the Index steadily. Conversely, Share C must be dear long term, as shown by its underperformance.

Share B follows the Index closely over the years, with the usual fluctuations as the price under and over runs the Market.

When setting up a portfolio, clearly it is best to buy shares of class A if enough can be found that are really cheap long term. Failing these, the next objective should be shares of class B, preferably bought when they are cheap short term. Category C shares should not normally play any part in the portfolio.

2. ANALYSING THE PROFIT PICTURE

Suppose that an analyst is assessing the likely relative performance over the

next twelve months of a small pharmaceutical company, most of whose profits come from one highly successful drug. The analyst's assessment might be:

"Buy. The share price is likely to outperform the market by 10% over the next twelve months as the result of continuing very strong growth in earnings per share."

While the recommendation appears to imply that only one outcome (10% relative outperformance) is possible, the analyst will readily admit that various other outcomes could occur. An obvious possibility is that earnings per share growth—on the assumption of no significant change in the demand for the company's products—is either somewhat above or somewhat below the analyst's expectation. If this growth is 10% higher, the relative outperformance might be 20%. If it is 10% lower, which could lead to a lower rating for the share, the share price might underperform by 10% over the period. The analyst, when asked to assign probabilities to these three outcomes (growth as expected, 10% higher and 10% lower) might quote ·5, ·25 and ·25 respectively. The expected value of the outperformance would then be 8%. This is lower than his central expectation, since the magnitude of the additional return implied by higher than expected growth is lower than the magnitude of the reduction in return implied by lower than expected growth.

If questioned further about other possible outcomes and their associated probabilities, the analyst might enumerate the following:

Scenario	Relative performance	Probability
The company is taken over	+100	·05
Bid speculation arises but no bid appears	+20	·1
Intense competition with other drugs arises	−30	·15
New drugs make its products obsolete	−70	·1

To accommodate these further possible outcomes, the probabilities of earnings growth being as expected, 10% higher and 10% lower are reduced to ·3, ·15 and ·15 respectively. The expected outcome, as calculated below, is now a relative performance in line with the market, which is 10% below the analyst's central expectation.

Scenario	Relative performance %	Probability	Contribution to expected return
Growth as expected	+10	·3	3
Growth 10% higher	+20	·15	3
Growth 10% lower	−10	·15	−1·5
Company taken over	+100	·05	5
Bid speculation	+20	·1	2
Intense competition	−30	·15	−4·5
Products obsolete	−70	·1	−7
Expected return			0

Three immediate disadvantages of this type of analysis are obvious. Firstly, the list of outcomes considered is far from exhaustive. Secondly, the probabilities assigned to each outcome are subject to a very wide margin of estimation error. If, for example, the probabilities of the more extreme outcomes are changed from ·05, ·1, ·15 and ·1 to ·1, ·2, ·1 and ·05 respectively, the expected return becomes an outperformance of 12%. Thirdly, the performance is related to the short term price movement; no allowance is made for the change in the rating, which can have a dramatic effect on the price, the return and the risk.

3. LONG TERM TRENDS

In the long term, there is a risk that a share underperforms significantly if its actual growth in earnings falls short of the expected growth. Such a shortfall can be resolved into two essentially independent components:

(i) estimation errors on correct economic assumptions;
(ii) failure to allow for changes in economic conditions.

Where a company has achieved a high historic rate of growth it is particularly essential to watch out for this growth rate slowing down for either 'disaster' reasons (e.g. Guinness, Thalidomide/Distillers) or mere competitive pressures.

The equity paper contains a classic case of (ii); a change in economic conditions (i.e. high interest rates, high inflation rates) was identified in 1975 as being likely to cause very material underperformance of engineering shares. Engineering shares became 'more risky' and were sold; the performance figures in the paper speak for themselves in this instance.

4. THE "DISASTER" RISK

Dramatic underperformance can be caused by a variety of circumstances such as insolvency, fraud and serious financial or marketing misjudgements.

Most of these 'disaster' risks are difficult to assess and will not be revealed by any study of past share prices as recent share price movements are unlikely to have included periods when any of these near disasters have actually occurred. In addition, the risk of a large loss is not compensated by any significant prospect of a super profit. Consequently the normal distribution is unlikely to apply to disaster risks!

5. THE NATURE OF RISK

So far we have studied the performance trends and the variability of these and their affect on the risk, without considering the shape of the risk distribution. For many of the risks described above, the distribution is clearly skew rather than normal. For most companies there may be a significant risk of a disaster, but little corresponding prospect of a balancing bonanza.

When attempting to quantify the 'risk', allowance must surely be made for the positive factors as well as the negative ones. In fact in 50% of all cases, the outcome is likely to be better than the market return rather than worse, i.e. the risk item is positive.

For practical purposes we suggest that the analyst should concentrate, not on trying to measure the risk, which is so difficult, but in assessing the value of the 'dispersion', i.e. the occurrence of an outcome significantly *different* from the market return.

The dispersion factor clearly has a short term element affected by day to day price fluctuations, combined with a longer term element dependent on the progress of future profit margins. This latter item is clearly a matter for the analyst's judgement. For a successful company, with a good market share and a high profit margin, the prospect of shrinking margins could well be greater than the possibility of further improvement. Conversely, a relatively unsuccessful company, operating to a minimum margin, may well become a recovery situation, with expectations of better margins and with a dispersion factor positive.

6. THE GUN SIGHT ANALOGY

The study of risk has many analogies with the problems of aerial gunnery and the design of gun sights, with which Plymen was concerned when serving as an R.A.F. scientist at Fighter Command during the war.

Consider first the problem of the short term price fluctuation. The analogy here is with a group of riflemen shooting at a target 200 yds away. After the shooting their target cards are collected for examination. Their hits will be scattered within a 'circle of confusion' of varying size, according to their skill. On this particular occasion, one of the riflemen was an elderly farm worker with poor eyesight, who never hit the target at all. For the others, their results varied according to their experience, their eyesight and the accuracy of their ancient World War I rifles. The analogy with investors trying to assess the correct price of a share is strong. Some investors may be likened to the short sighted farm worker, with their idea of the proper price scattered at random over a wide area. Other investors will have varying degrees of skill and access to different levels of statistical information. The really good analyst may be likened to the sniper with a telescopic gunsight, who can pin-point the price with some degree of accuracy. Presumably the perfect market and random walk exponents assume that all investors are like the farm worker with poor eyesight. If this were the case then future price movements might indeed be random. However, if there are some better marksmen and closer estimators of the proper price, their buying or selling will surely move the price towards the 'norm' so that future price movements will not be random.

Next consider the medium term price movements. Here the best analogy is with air to air combats. Again the short term price movements can be compared

with the skill of the attacking fighter pilot and the erudition of his equipment. Medium term price movements arise from varying assumptions about the short term progress of profits and dividends. Such fluctuations can be compared with the medium term 'jigging about' of the attacked plane, taking evasive action. As regards the longer term price movements, this can be compared with the direction and speed of the relative movement of the target. (The target could have been at any angle at between 0 and 90° and travelling at various speeds from 300–600 mph.) Shooting in these circumstances is similar to duck shooting in that the marksman has somehow or other to estimate the rate of change of angular movement and apply an 'aiming off' adjustment. However, for the last year of the war, the fighters were equipped with gyroscopic gunsights, which included a crude form of mechanical computer. The pilot had to track the enemy aircraft in his gunsight for 5 to 10 seconds. The computer measured the rate of angular movement and calculated the amount of aim-off so as to hit the target. The technique of the gyroscopic gunsight is analogous to the systems used by skilled analysts, studying closely all the long term trends of the profits, dividends, earnings on capital employed, etc., to assess the proper price.

7. SHORT, MEDIUM, AND LONG RISKS

The error factor involves three elements, short, medium and long term (see paragraph 6 in the main text). The total error is, of course, a combination of the three. The question arises as to whether these three movements show any correlation. On the analogy of the gun firing, the correlation between the short and medium term movement appears unlikely. However, there could be some correlation between the medium term movement of dividends and the long term trend of these factors. For all these factors, the risk depends very much on the phase of the error cycle. If say the short term price movement is shown to be moving up and down between two trend lines, the risk of a downward movement is surely greatest when the price is towards the top of the range. Clearly the risk distribution will be skew and the usual assumptions of a normal distribution, permitting mathematical treatment, will be quite inappropriate.

8. CONCLUSIONS REGARDING RISK

From sections 5 to 7 above, it is clear that the shape of the risk factor and of its distribution is incredibly complex, with three largely independent components, each likely to have a skew distribution. With this elaborate structure, there is no way of assessing the risk from any available statistics, and mathematical analysis is completely impossible. In these circumstances, it is difficult to see how the MPT system, which purports to measure and control risk, can have any possible validity.

ABSTRACT OF THE DISCUSSION

Mr J. Plymen (introducing the paper): To find under-valued shares that will really outperform the market and improve the performance of a portfolio requires the study of a great mass of regular statistics. These studies involve some measure of success and in general one can break down the operations into various components which can be monitored all the time and one can try and improve the techniques, the methods and the success of these various systems. Operational research I maintain is just a scientific way of putting together statistical information; it needs good statistics analysed by professionals who really know what is going on in the underlying operations.

The methods we describe in §§8 and 9 are systems for monitoring portfolio performance, improving the performance by a sort of replanting system and so on. This is just an application of the O/R principle. The unitization means that we have got the continual comparison of the performance with the index, and the market model gives great assistance for deriving long-term situations. The study of the 'buys' and 'sells' is performed over one-and-a-half to two years, the purpose of this technique being to improve the long-term performance, and not chasing short-term performance. On the other hand, what I call the alpha/beta system, appears to use suspect statistics, analysed by mathematicians who seem largely interested in the short- rather than the long-term because this system is based on the one-year return. I do not criticize the more modern developments, as envisaged by Rosenberg and Barra, which use investment analysis thoroughly and rigorously. However, it is difficult to comment on these systems because the data is not readily accessible.

Index funds are very popular in America and are beginning to be developed here, but I see no reason to concentrate on the mathematics and the procedures for an Index fund. If one wants to perform more or less in line with the Index then one should set up a portfolio that is the same 'shape' as the Index, ensuring that it has the proper proportion of the major shares like BP, Shell, Telecom and so on, and that the industrial spread is somewhat on the lines of the Index. Then use the procedures we outline in §§8 and 9 for maintaining and improving the performance. Experience shows that this system which involves continual monitoring and watching the performance, picking out the shares that are underperforming and going into ones that are doing a bit better, tracks the Index extremely well. The difference between our system and the Index funds is that our system (with a bit of luck) will overperform the Index and the Index funds are designed not to do that.

The comments about the Efficient Market are based on the United Kingdom and American markets.

Mr Clarkson has updated certain of the statistics in our paper to show a chart of the performance of his cheap and dear shares.

Mr R. S. Clarkson, F.F.A.: The figures below update the performance of the buys and sells in $6.4.

	Duration (weeks)							
	14 %	16 %	18 %	20 %	22 %	24 %	26 %	
Buys	−·1	+·5	−1·9	−2·4	−2·1	−·8	+·9	
Sells	−3·6	−5·2	−4·3	−1·3	−·3	−1·2	−3·8	
Δ	+3·5	+5·7	+2·4	−1·1	−1·8	+·4	+·7	
F.T.-A. All-Share	+5·5	+4·7	−26·0	−28·6	−27·1	−28·3	−21·8	
	28 %	30 %	32 %	34 %	36 %	38 %	40 %	42 %
Buys	+·3	+·2	−·3	−·6	+·1	−·2	−·8	+·2
Sells	−4·3	−4·1	−4·7	−6·1	−4·9	−5·4	−5·6	−3·9
Δ	+4·6	+4·3	+4·4	+5·5	+5·0	+5·2	+4·8	+4·1
F.T.-A. All share	−20·7	−21·7	−20·9	−21·7	−19·2	−18·2	−21·1	−19·0

The figures in the paper cover the period to the middle of September. The market collapsed in October and interestingly enough for four weeks thereafter the performance of the sales overtook the performance of the buys. By December the out-performance of the buys re-established itself; it rose to 4% and then remained between 4% and 5% for most of the subsequent period. Having set up our test of buys and sells ahead of what was to prove a very traumatic period, we were very relieved that the performance figures remained satisfactory.

Mr J. Plymen: On page 663, Table 3 we show the performance of eight unit trusts. I have now reviewed the figures to 1 January 1988.

For the year 1986 the average performance of the index was plus 25·9% and the average over-performance of the eight trusts was 10%. For the year 1987 the over-performance of these selected trusts has gone up. The index figure for the year is plus 6·9% (that is capital plus net income gain). For these trusts, the figure for the yearly gain of the trust varies from 13·2% to 35·8%. It is quite fantastic that one of these has gained 35·8% over the year, a year with a fall in market values in October. For 1987, if the 6·9% is excluded from the trust performances and the resultant figures averaged, the result is 15%. These trusts continued their over-performance under the extraordinary conditions of 1987.

These trusts have been invested with a certain emphasis perhaps on smaller companies and on recovery stocks, also 5 out of the 8 are specific High Income Trusts; (a High Income stock means investing in stocks with a recovery potential). I must admit that some of the over-performance is due to the fact that the type of shares they invest in on the whole are likely to out-perform the index to a small extent. But they are not small companies, the trusts are of significant sizes, the smallest one is now worth about £15 to £17 million, and the largest one is worth more than £200 million. As they only have about 50 holdings, their average holdings are large so they cannot be holdings in small companies because they would not be able to deal in them. It seems to me that these results prove that it is possible to find professional investors who continually do better than the market. It would be interesting to know how it is that these people are so successful. I rather wonder whether it is not that they are using some of the systems that we recommend in §§ 8 to 9!

Mr A. J. Banks (opening the discussion): The authors seem to have chosen an enticing title perhaps to gain the interest of the investment professional. In the investment world actuaries have achieved a prominent position in the analysis and management of fixed interest portfolios. However, in the area of equity investment there are numerous participants all with their own views on how to 'beat the market' and so this paper is bound to be contentious.

The authors do us a service by highlighting strengths and weaknesses of some investment models and, more importantly, their underlying assumptions.

Unfortunately the subject of Modern Portfolio Theory (MPT) is full of jargon, which could alienate half the potential readership. The authors have provided a brief history of the subject on page 632 and in Appendix 1 which helps to guide the reader through the subject without the full mathematical development. However their definition of MPT is very narrow, covering what could be described as the original theory. The authors state that using historic betas for risk and return has no practical application to portfolio management, either to improve the return or to reduce the risk. This is difficult to refute but it perhaps neglects some of the continued development of the subject. MPT has been extended so that betas are adjusted for fundamental data such as gearing of a company—these have become known as bionic betas. The bounds of MPT have grown over time to include performance measurement, index funds, divident discount models and options and possibly even gilt models. MPT is more often associated with risk control (or diversification). The authors, whose main concern is return, ignore risk, saying that there is little point in fussing about a minor item. It seems to me that this places them and the business schools (who give greater weight to this factor) at opposite ends of a spectrum. To give some examples. The definition of risk is of major significance to pension funds and to insurance companies for a variety of reasons—e.g. the risk of having to increase contribution rates for a pension fund when the company cannot perhaps afford it and the risk of a general insurance company being regarded as insolvent.

MPT risk (which uses historic variance of return as a measure) is unlikely to equate to the above risks. However the author concedes that this type of measure of risk may be of use in the short term.

In practical terms it is used extensively in very short-term investments—in options, futures and in program trading, particularly in the United States of America. These contracts are very short term, ranging from 1 day to 9 months and so the measure of risk is appropriate. The authors highlight the important problem in MPT of co-variance between different share prices. An example would be gold shares which tend to move together and not in line with the equity market as a whole. But the original MPT theory has been adapted to cope with those practical realities.

The efficiency of Equity Markets is another topic covered by the authors. They regard the test by Jensen as unscientific. This echoes reality. There must be few people who believe in the so called 'strong theory' of efficient markets; that prices reflect not only all known information which is published but also information which is not yet published. If prices did reflect all information, I think there could be no such crime as insider trading, as every bit of information would be in the price. So I side with the authors on this point, the equity market is not a perfect market in the same sense as the gilt market. Trading costs between buying and selling shares can be significantly higher than for gilts, perhaps it could be 1% to 2% even for the largest companies. Just after the crash in October the bid/offer spread on RTZ was 10% which is a major company in the F.T.S.E. Index and even if there were inefficiencies in the market at that time with undervalued shares, you may have found that the costs of buying and selling may have swamped those inefficiencies. So it is not only important to find the inefficiencies but actually make sure that you can make money out of them and this may not be possible if costs really are that high.

Turning onto another subject, the authors mentioned the stockbroking firm of Rowe Rudd which tried to market a service in the U.K. based on an improved version of MPT. A conference was held in 1980 at which over 60 institutions did attend and apparently three of them did subscribe to the service so it was not an entire failure. After some research I discovered that one of the founders had gone to the U.S.A. and this time set up a similar business very successfully. So the personalities involved have moved on and the service in the U.K. ceased in the early 1980s.

Turning to the U.S.A. this is really where the real appetite of MPT exists with billions of dollars using the services of consultants all with their own improved version of MPT and as Mr Plymen said they are not really going to tell you about it until you actually subscribe to the service. In the U.K. the London Business School and DATASTREAM have been operating services for some years. In fact I also discovered that a firm in the late 1960s, W. I. Carr, tried to market a service in the U.K. again based on similar lines. Looking through some press cuttings on the subject I found that a Lex column in the *Financial Times* in 1978 was commenting both on betas and on Rowe Rudd and it had a statement that "fund managers outside the U.S.A. would be able to stick to more familiar market indicators like sun spots or hemlines if they choose but the new service should be a value to those large diversified funds which because of their size are obliged to remain pretty well fully invested for most of the time". This seems to be borne out in practice as large index funds have appeared in the U.K. and some of those are using MPT while others rely on more basic analysis, as Mr Plymen said, by analysing portfolios by sector.

Section 4 moves on to the return aspect rather than the risk aspect. The table shows a wide dispersion of equity returns which may be of interest to those unfamiliar with the moves in share prices which can occur over quite a short period. In the authors' example spectacular profits can be made by successful dealing, this time not using MPT but relying on their 'genius' fund manager. At the beginning of the period the 'genius' fund manager had sold all his shares in Avana and switched it into Pilkingtons which was the best performing share and did remarkably well out of it. Well I hope he reversed the switch because Pilkingtons are the same price today as they were in 1986 when that example ended. But in fact Avana which is a small food company has risen about 86% because it has been taken over by RHM. I would add that not only must you be able to identify cheap and dear shares but you must watch them constantly..

Models are used extensively in the gilt market and the Clarkson model mentioned in this paper serves to demonstrate why actuaries are so prominent in the field of fixed interest investment. The authors' equity models in the paper attempt to solve an even harder practical problem of which shares to buy and sell and it would be interesting to know how many institutions actually use equity models. Clearly if an institution is using fundamental analysis it would be sensible to have some form of model even if it was just a management tool which relied on the regular appraisal of the investments.

The interesting part to me was on page 635 and in the Appendix where the efficient market hypothesis is tested. In the example, three stockbrokers each ranked 71 shares in terms of their long-term earnings growth. With the information and the application of the model 14 companies became buys and 14 companies became sells. They were then charted over fortnightly periods. Is this an apparent contradiction looking at long-term earnings over two-week periods?

As we saw from the results the buys out-perform slightly and the sells under-perform, but what I am interested in is what the brokers were actually recommending at the time. They are commercial organizations; they don't just predict or rank earning forecasts; they would have given a recommendation either to buy the shares, sell the shares or hold onto them. So did the model actually produce worse results or better results than simply taking the average of the stockbrokers' recommendation? Perhaps if two of the brokers said buy and one of them said sell, you simply would have taken that as a buy. It is not to say that it was not a very useful experiment, it does highlight how you should actually follow through a model to test your theories.

Section 7 returns to the subject of risk and starts to develop an interesting theory with the two dispersions of returns. The authors did not pursue it but perhaps it is a subject for further study.

Section 8 is short being the authors stating the obvious—you simply buy the cheap shares and you sell the dear ones.

Section 9 gives another example of the inefficiencies of markets. Although this may not be accepted as convincing proof, it does demonstrate quite a neat way of measuring investment performance. Effectively it turns the investments into a unit trust. Tax and dividends have been excluded in all the examples, but clearly these are practical items which all of us must take into account. It may have been the case that the shares outperformed by 1% but you missed out by a 2% loss in income.

This paper has done an excellent job in questioning the theories and the assumptions used in investment models based on quantitative information. However, I suspect that the firms which offer advice or those which use these models will continue to extend their influence as they have done in the U.S.A. There is a vast amount of data now available and there are computers to analyse it. It may be the case that legislators actually turn to this form of analysis to provide some objective measure of diversification of investment portfolios. The narrowly defined MPT in this paper may not be of practical use but the theoretical idea that it introduces a trade-off between risk and return is appealing.

Mr D. Damant (a visitor): MPT is a very vague term and in line with a comment already made it has been too narrowly defined in this paper. It is in fact merely a systematic and scientific approach to the market. This approach depends on the efficiency of markets. Only if markets are reasonably efficient is it worthwhile scrutinizing the data with care. By adopting a systematic approach you can capture the added value. As this process continues, we shall see more and more similarities between traditional investment methods and MPT.

The method proposed by the authors of this paper could be considered as an approach to a MPT. Therefore the sentence in which they conclude that this discipline makes no contribution whatever to improving performance, is false unless you define MPT in a rather unuseful way.

The authors attack the efficient market theory but their model is perfectly in line with it. Their comments on beta are reasonable and it seems to me something which needs further work.

The authors could have made a better attempt at attacking the efficient market theory. In fact, the strong form is altogether better supported than they suppose in their Appendix on the strong level tests. The recent statistics on pension fund performance look extremely powerful. I have done the unit trust performance analysis in very much more detail and think that some quite interesting results come out. These do not negate the efficient market theory, but confirm the fact that there are inefficiencies in a generally efficient market.

Finally although the performance of portfolios is important, I believe that the nature of the capital markets in this country is more important. If our markets are reasonably efficient in the way discussed it means that capital is being allocated efficiently. This is not often perceived to be the case by many companies, society or many politicians. As a result of the reactions against the efficient market theory as in this paper and as a result of the crash of 1987 there is a tendency to seize on any argument which can try to persuade us that the stock market is swayed by fashions, that it takes short-term views and

that there are bubbles. There are some positive lines of thought in this paper and I am extremely sorry that it has been put in a framework which attacks the efficiency of the market, which is something of tremendous political and social importance.

Mr A. T. Adams: My general comments are as follows:

1. *Risk.* I agree with the criticisms of the general use of variance of short-term monetary returns for risk. Whilst such an approach is extremely convenient, it is normally unrealistic in practice, certainly for most institutional investors. Academic researchers are aware of these criticisms but usually choose to ignore them. The situation is not helped by the fact that performance measurement in the real world is usually carried out over short periods, even for long term investors such as pension funds.
2. *Efficient market hypothesis.* Stock market efficiency is not a black and white issue. We are concerned with how close the markets in shares are to efficiency. A lot of what is said on market efficiency in the paper is simply part of that debate.

   It is true that claims made in the past by many academics concerning equity market efficiency were not justified by the results of tests which had been carried out. However, over the past five years or so the academic literature has been full of discussion of market inefficiencies as the previous speaker mentioned. They have been listening more and more to the views of practitioners and those 'hard line' efficient market theorists still in existence on 19 October last year have been forced to change their views.
3. The most recent paper (rather than book) written by an academic researcher which is referred to in the paper is dated 1968 but a lot of good research has been carried out in the last twenty years.

There are a number of specific points I wish to mention:

1. I do not agree with the authors' view on what seems to constitute a perfect market. The strict assumptions which define a perfect market are not necessary for an efficient market. Nobody has ever suggested that the U.K. equity market is a perfect market. In particular 'less erudite' investors do not exist in a perfect market.
2. Given that the paper is concerned with Improving the Performance of Equity Portfolios, I cannot see the relevance of § 5.1 which is concerned with Clarkson's gilt model.
3. Section 3 of Appendix 1 concerning Professor Rosenberg's work states "one analytical method that he mentions is the dividend discount model. Clearly he regards the alpha and beta functions heretofore used for return factors as useless." I do not agree with the logic of that argument.
4. In Appendix 2, the statement "it is highly improbable that such results could have arisen by pure chance" in referring to the eight unit trusts in Table 3 should be tested using a scientific statistical approach.
5. The graphical fit in Appendix 4, used "for simplicity" should be constructed using the least-squares fit.
6. I am not happy with the analogy in Appendix 5 since it appears to show a lack of understanding of the ideas behind efficient markets. The greater the number of competing skilled analysts in the market, the more efficient the market. And in an efficient market, new information which reaches the market at random is immediately impounded in share prices and hence share prices follow a random walk.

Professor A. D. Wilkie: I am sure that others will deal very ably, as some already have, with the authors' discussion of some of the ideas of modern financial economics to which they themselves have made some substantial contributions in their way. I will only say that it is not a useful treatise which explains MPT and its advantages and disadvantages, and I would not recommend it for general reading.

From Mr Plymen's introductory remarks I gather that the authors' criticisms are more of those who have attempted to simplify sound theoretical models out of all recognition in order to purvey their own particular brand of investment advice.

I should like to concentrate on the positive part of the paper from § 5 onwards. In each of three

cases: gilts, investment trusts and ordinary shares, the authors postulate a similar model. The supposed true price of the securities is determined externally in some way and the movement of the deviation of the actual price is then studied.

In each case the authors suggest using the Mean Absolute Deviation (or MAD) statistics. Perhaps I can mention that, although the MAD was a widely used statistic during the nineteenth century, in various papers in the 1890s Karl Pearson introduced the concept of standard deviation which has generally overtaken the use of MAD in statistical work since then. But perhaps the authors prefer not to be associated with twentieth century statistical ideas any more than twentieth century financial economics.

The authors then postulate that a "reasonable description of the price formation process" is given by the formula for e_{t+1} towards the end of § 5.1. A little algebra demonstrates that this is a formula that in Box and Jenkins' terminology is described as an ARIMA (1, 1, 1) model. If indeed the price residuals for each of the three types of security do follow this sort of time series model, this fact would be interesting, but the authors do not seem to analyse their residual in these terms. Nor do they quote the values so that others can analyse them using modern time series techniques.

In the case of the first two classes of security the authors seem only interested in the deviation from the true price and not in how that true price in itself changes. The yield curve for interest rates is hidden in the auxiliary function $h(n)$, but how that is structured and how it changes do not seem to be the authors' concern. Equally they do not seem to be concerned with how the net asset value of investment trusts is formulated or how it changes, but only in how the discount relative to that asset value moves.

When we turn to the equity model we do find a fuller discussion of the price formation process. The authors then state a version of Ross' Arbitrage Pricing Model, roughly what they describe in Appendix 1.3 under the name of Rosenberg. Instead of using a large number of different factors, the authors restrict themselves to three: earnings, pay-out ratio and earnings growth rate. They then simplify this to state that the Price is equal to the Earnings times the P/E ratio and the P/E ratio is expressed as a function of pay-out ratio and growth rate.

My own stochastic asset model explicitly formulates Price as Dividend times P/D ratio or Dividend divided by Divident Yield, a very similar formulation. The authors' more elaborate formulation is a useful step in the same direction.

In § 6 the authors introduce the potentially useful idea that, as they put it: "the frequency distribution of the future value of the proportionate price error can be represented as above by a sine wave". After clarifying in my own mind the obscure terminology in which this is expressed, I came to the conclusion that we were discussing a probability distribution defined over the range $-a$ to $+a$, where the distribution function is proportionate to arc sine (x/a), and the density function is proportionate to $1/\sqrt{1-(x/a)^2}$. This is a symetrical U-shaped density function with the density tending to infinity at the limits $+a$ and $-a$. I suppose it may have its uses but the authors do not explain whether the residuals in their three security models have in fact been found to follow this sort of distribution. If they have, it would be a remarkable finding worth publicising more clearly.

In § 7 the authors begin to discuss a way in which probability distributions can be ranked. Their example, where security A gives double the return of security B in all possible circumstances is persuasive, but it suggests to me the concept of 'stochastic dominance' rather than the so called axiom—surely rather a definition—that they give.

Distribution A stochastically dominates distribution B if the distribution function for A is lower than (or on occasion equal to) the distribution function for B at every point. Using this definition the distribution of returns for share A would stochastically dominate that for share B and indeed any reasonable investor would prefer the outcome of share A.

Unfortunately, for many pairs of distributions neither clearly dominates the other, so stochastic dominance is rather seldom available to help our choice. It is indeed defined for two normal distributions which have the same variance, and different means. But if the variances are different, neither distribution dominates the other, however far apart the means are.

I had thought that the authors had perhaps found the key in the integral measures they then quote, representing what we might call the second downside moments, but I find the notation of these formulae somewhat confusing, I suppose that p represents density functions but I should have

thought that the subscripts should simply be r, rather than what is shown. The risk on share B is defined wholly in terms of its own distribution, and if the distribution is symmetric it is just the variance. The risk on share A, however, introduces integration only up to the mean of B. I am not sure whether the function to be integrated should be the square of the deviation from the mean of A or from the mean of B. But in either case it surely must be described as the risk on share A, relative to share B.

Given either of these definitions, we could say that share A dominates share B if the risk on share A relative to share B is less than the risk on share B relative to itself. Unfortunately this does not prevent share B in certain circumstances also dominating share A if the risk on share B relative to share A is less than the risk on share A relative to itself.

Unless the authors can show that their definition of risk implies a strict order relationship between distributions then I think we must fall back to the original difficulties that Markovitz first discussed, that there are at least two aspects of a probability distribution, the location and the dispersion, however you define them, that may be of relevance to an investor. There may be other aspects of the distribution, represented perhaps by the third or higher moments that an investor may wish to take into account. If all we know about the distributions are the measures of locations and dispersion then all we can reasonably say is that all rational investors will prefer a higher location or a lower dispersion all other factors being equal.

The authors state their main conclusion "that if we look after the expected return the risk will look after itself". This is a quite reasonable approach for a particular investor, whether an individual or an institution. It is equivalent, in the jargon of financial economists, to the investor being risk-neutral and it helps to identify the portfolio that he would choose on his efficient frontier. But it is not reasonable to suggest that all other investors are in fact risk neutral nor that they should be.

I am grateful to the authors for having challenged some of the assumptions of Modern Financial Economists, whether expressed in the rather narrow form they assume for MPT or in its wider context.

Mr M. McIvor: The authors' paper concentrates on that subject which is often the principal determinate of product selection by clients. Performance and how it can be improved are key issues.

The concluding sentence of the introduction and of the paper itself are key. The manager should improve his selection skills or else take refuge in an index fund. I would suggest that for most money managers this is no longer a choice as he is either providing an investment management product offering performance in line with an index or he is offering selection skills of one form or another.

The paper is orientated towards investment management in a single equity market and yet that is no longer the usual investment constraint. October 1987 showed once again that the adage "if America sneezes the world catches a cold" has some credence. Many experienced money managers would expect to achieve higher returns by selecting between markets than by expecting to contribute higher added value from selection within a single market. The advent of financial futures for equities, bonds and currency markets, and competitive U.K. style program trading make this management style so much more practical than even a few years ago.

Before moving to stock selection, I suggest that the modelling approach presents as many challenges at the inter-market level as it does intra market. I strongly support all forms of analysis, fundamental, technical or MPT risk analysis techniques. All are means of representing information or experience. The key is the choice of tool for the job.

With the benefit of that master-tool 'Hindsight' we often see that major market moves are not dependent on analysis of historic information or news items. Frequently it is a change in political will or priority, taxation or regulation which will be the spur. Did the lenders of billions of U.S. dollars to South American States seriously judge the political framework? The nature of this spur applies equally at the company level. Most companies have experienced major management changes in the last five years. Takeover activity has been rife and company sales of major divisions are commonplace, as corporate goals are re-focussed. The product life-cycle has shortened. Is it reasonable to expect the analytic tools portrayed in this paper to give substantial help? A review of the top ten Irish companies over the last ten years makes very interesting analysis for those who have the time.

In addressing the question of performance, on page 5, the example of Rolls Royce is given and the "prospect of a year for the consequence of news to have its effect". How many money managers will be willing to subscribe to accepting a drag on performance for 6 or 9 months in the expectation of a gain a year on? I suggest there are far fewer today than there were five or ten years ago. Investment performance is scrutinized quarterly or even more frequently now; timing is of the essence and I suggest that the Quickstep beat of 'Slow-Slow-Quick-Quick-Slow' with share price action in the 'Quick-Quick' intervals is the challenge that the market offers most often.

Although Appendix 5 includes a scenario model which requires significant effort to construct and to manage, it does give a hint of the range of prospects which the manager is addressing. Practitioners should take heart from John Galsworthy who said "If you do not think about the future you cannot have one".

There are many interesting aspects of this paper and the authors are encouraging the development of frameworks within which managers may practise and reminding us that existing approaches need regular re-appraisal.

Mr P. G. Scott: This paper reminds us that there are no black boxes in investment. There are no black boxes that you can just use without the input of other information and come out with strategies that would beat the market or beat competitors.

The paper concentrates on 'risk' as used by MPT. It is easy to get too tied up in a sterile argument about whether this is the appropriate definition of risk. In MPT the definition used is an analysis of variance. I believe that asset allocation is one area where analysis of variance can help. There are others. For example the pension fund trustees have got a considerable dilemma at the present time as to what is the appropriate equity exposure to maintain within their pension funds. An analysis of variance can help in that regard. It can also help in the communication of portfolio structures to clients.

In § 7 of the paper, we are reminded that the authors feel that the definition of risk is totally unsatisfactory. They go on to suggest an alternative which is "the risk on share A is lower than the risk on share B if the probability density function of all functions for all values of return below the return B is smaller for A than for B". I suggest if I tried to explain that to my client then I might not get too favourable a reaction. What I can do if I use an analysis of variance is to help explain what has happened to their portfolios and what may happen in future scenarios.

I believe that analysis of variance can help in the management control of portfolio managers in a delegated environment. It is extremely difficult to analyse the level of diversification being held within say, a Japanese portfolio. It is quite easy within the U.K. when one knows most of the share components but when two thirds of the names are quite unknown to you it is quite difficult. Analysis of variance can help in that regard as well. I feel that the tone of the paper is wrong to the extent that it attacks risk analysis, or analysis of variance. It is quite correct to say that analysis of variance will not help very much in equity selection. What it will do is help in other aspects of portfolio management and that seems to be the missing feature from the paper.

Mr E. N. Lambert, F.F.A.: Is the authors' method not self defeating, in that, like indexation, the more wide spread their method is used, the less it would satisfactorily perform its stated objective to improve absolute or relative performance?

Mr J. Heron (a visitor): I am totally in agreement with the authors' attack on the efficient market hypothesis, but believe we should recognize the market as offering a genuine information or learning process (as Professor Hayek describes markets). The starting point for practical purposes of analysing an equity involves the twofold opposing processes of first trying to understand the market's view of the stock, that is from its current return, from the price charts, press comments, personal contacts and so forth; and secondly, testing that market view initially with what Appendix 1, § 3 of the paper calls models or what one might possibly call basic professional statistical disciplines on the Weaver Hall lines.

I would suggest that EMH is misguided because it assumes a logical mechanistic process by which what it calls facts are translated by the market into the presumed correct share price. I would suggest

that EMH would be more acceptable if its proponents could turn their binoculars round, and propose that the market offers the best view we have of the stock for the time being until we demonstrate that view to be wrong. I would add that this process demands more than just a formalized set of consistent appraisal rules, because each company and its situation are unique in some appreciable degree. For example, the success of the investment program described in § 6.3 did depend upon what in Appendix 2, § 3 is called "a superior insight into the future"; and the examples in § 3.2 upon a superior analysis of accounts (as with the Rolls Royce and Vehicle and General cases.)

Mr M. D. Kemp: I think the key point to the whole discussion is the validity of the efficient market hypothesis. As one of the speakers said, it is just a hypothesis and therefore a starting point for working with. I think the efficient market hypothesis is generally accepted as not wholly accurate but still reasonably true. If, however, it is approximately true, then markets may be expected to behave approximately in line with the way they would if the efficient market hypothesis were to be wholly accurate. What this reveals to me is that although not impossible it is very difficult for investment managers to achieve the sort of additional returns that would be available if markets were not approximately efficient.

Mr A. Grant: A followed dimension from the authors' paper is that if you have their sort of approach or one of the many other alternatives it helps a great deal in organizational matters. If you have a disciplined system then you can use that as a building block. One should have a system which allows one to work within the U.K. equity market, in the U.K. Gilts market and overseas markets. This is not meant to be the overall result. That some proposition applies is appropriate, but there are organizational problems of clients choosing fund managers and then knowing just what it is they are mixing.

There are also problems within large management groups in that the chairman of an investment management organization may have some difficulty in controlling his fund managers, all of whom want to do entirely different things. So one has a need for some discipline of form and also the monitoring of systems. Those described by the authors are useful in keeping track of internal management styles. One, which Mr Plymen and I had experienced many years ago, was the extremely simple proposition that we chose some stocks on a regular basis and reported every three months to a particular client. Over a period of a number of years that client wanted to know what happened to the recommendations and the interesting factor was to trace the persistence or lack of persistence of the recommendations. Broadly what we found was that the buyers beat themselves over a 6 months and 9 months and later period: that the degree of our performance faded after about 9 months when I was doing it, so at least it showed that I knew my time horizons. That kind of knowledge is useful, it is exactly the same as Mr Plymen has described in his following through purchases and sales, and I was glad to see that in the chart following through.

There are difficulties in describing what MPT is, but I think that it is a question of knowing which tool to use at which time. It is very much a question of having a disciplined approach, if only to discipline your colleagues, but being able to switch flexibly within that discipline.

Mr A. J. Wise (closing the discussion): The authors have reminded us in their paper that in investment matters, as in so many aspects of the world, nothing is as simple as mathematical theories might encourage us to believe. MPT is a good example. It is a theory of investment which clearly does not give a complete representation of reality. The authors refer in particular to that version of MPT which relies on the hypothesis of an efficient market. They point to inefficiencies in the equities market which an expert investor can exploit to his advantage. These conclusions seem to be well accepted to judge from tonight's discussion. They are perhaps almost self-evident.

In practice investment managers do not appear to rely for their stock selection on the basic version of MPT to which the authors refer. It would seem that inefficiencies in the market are widely recognized by practitioners and that various different methods are used to obtain added value from stock selection. The authors properly pointed out that the equity selection method described in their paper is not the unique solution to the problem. The authors suggest that variants like the Rosenberg system should be regarded as outside the MPT field. Mr Banks and Mr Damant thought that the

authors are being too narrow here. This is just a matter of semantics, of saying exactly what we mean by Modern Portfolio Theory. Thirty-six years after the original Markovitz paper, perhaps the time has come to drop the word 'modern', and to accept that Portfolio Theory now reaches further horizons than those of Markovitz and Sharpe.

Mr Plymen spoke, in introducing the paper, about improving upon the performance of an index fund. Mr Lambert implied that if more and more people were to follow a particular method of striving to beat the index, such as the authors' method, then the market inefficiences would presumably reduce to the point where that very method, net of costs, ceased to add any value compared with an index fund.

This raises the broader investment issues to which Mr McIvor referred. There are questions of investing in index funds, in property, overseas etc. In order to deal with these issues satisfactorily, one needs a still broader definition of risk which copes with events like the occasional equity crash. Mr Scott pointed to the application of the analysis of variance in this area. The authors give a definition of risk in § 7 which Professor Wilkie related to a principle known as stochastic dominance. A broader and more satisfactory risk definition would, in my opinion, necessarily take due account of the nature of the fund's liabilities.

The way ahead would be found by using the best modern ideas for stock and sector selection along with the best modern ideas in portfolio theory.

The President (Mr M. H. Field, C.B.E.): Investment performance is certainly a subject of interest and concern to all actuaries as indicated by the discussion. Actuaries are accustomed to and indeed are trained to develop practical conclusions from intractable and incomplete data. Further, actuaries are, or should be, accustomed to recognizing situations where the data is insufficiently reliable to justify the application of a particular analysis or where the technique has been so simplified in the interest of facilitating its operation, as to render the conclusions invalid, and perhaps both sides could be judged not entirely guilt-free in that connexion. However, it is wholly appropriate that the discussion should have been in this hall.

The authors are critical of the Efficient Market Hypothesis (EMH). However a test of any attribute based on particular data is only valid, if it is valid at all, so long as those who produce the data are not influenced by the testing to be based on it. The intelligence test seems to have some value until candidates are trained for it; closer to home, the assessment of the strength of a Life Office from the Statutory Returns and the use of surrender value levels to assess the impact of initial expenses are reasonable tests until the Offices manipulate, as they can, that data to produce a more attractive impression; in the context of the paper, a company can order its affairs and their presentation so as to affect the market view of its operations-induced random noise. And the authors refer to the skilled analyst doing better for his clients by not publishing his findings.

However, central to their argument is the restatement of the investment objective—to achieve an above-average performance. This is an objective that most investors would find palatable. But, as an aside, the analogy of the well tended flower bed troubled me. Quite often in my own garden I have noticed a particular plant making a spectacular showing in one season only to find that it was actually in its death throes—I trust this is merely the common problem of following an analogy too far.

Finally, I do formally express my own and the meeting's appreciation of the authors for their paper tonight.

Mr R. S. Clarkson (replying): We had clearly anticipated that we would attract quite a bit of criticism from the hardline supporters of MPT and market efficiency; it is a great surprise to us that everyone is now concentrating on showing how inefficient the market is. Only Professor Wilkie and Mr Wise commented on what we regard as possibly the key implication of our paper for the future of risk theory, namely our suggested new definition of risk. Professor Wilkie went into some detail about our counter-example lacking generality on account of stochastic dominance and Mr Wise said it might be an interesting approach, but neither followed it up.

Mr Banks said there was a contradiction between our using long-term growth rates and looking at the investment performance of our buys and sells in the short term. We disagree. We used information from stockbrokers which was more or less the current concensus in the market; we were then showing

in a deterministic way how you could apply a test of efficiency to this information and designed that test so that others could replicate it.

How did we adjust for dividends? In all of the examples there was no significant difference in the dividend yield so any adjustment for dividend would not invalidate our results.

I was very surprised at Mr Damant's worry that we are giving people ammunition to attack market efficiency. The market efficiency hypothesis is a hypothesis on which useful theories might be constructed if it in anyway reflects what happens in the real world. We show that there is considerable evidence against the market being efficient.

Mr Adams said that the gilt model was irrelevant given the title of the paper. Again we would disagree; the gilts model was mentioned as a relatively straightforward logical construction which we could then build upon using very similar principles to derive the equity model.

Professor Wilkie said that our time series might be ARIMA (1, 1, 1). The reason that sounded so familiar was that I explained at the Faculty some years ago that I had done a lot of work using Box Jenkins time series analysis on the residuals for the gilts model and had come to the conclusion that these might be either ARIMA (4, 0, 0), (4, 1, 0), (0, 1, 2) or (1, 1, 1); Professor Wilkie mentioned the last of these.

There has been considerable discussion since the October 'Crash' about market efficiency and whether there might be positive as well as negative feedback, and accordingly I have a suggestion for Professor Wilkie for his stochastic model. The equation that I mentioned at the Faculty discussion and again in this paper, states that the daily price error is the previous error plus a term which takes it back towards its moving average plus a random term. What I suggest might be a better model in practice is one which includes a positive feedback item. If you look in the paper at the control chart for the investment trust example, once the price residual comes off one control limit, it does not come back only to the average but tends to shoot straight through and go to the other control limit much more often than would be expected by chance. Building a normal variable where the mean is a trend value equal to a short term moving average $Tr(e)$ of the rate of change of the price residual gives;

$$e_{t+1} = e_t - \theta(e_t - \bar{e}_t) + N(Tr(e_t), \sigma).$$

I have a suspicion that this would be a much better form of model on which to base future stochastic work.

Few speakers apart from Professor Wilkie and Mr Wise appreciated the significance of our suggested alternative definition of risk. In the paper we are saying that if we have satisfactory axioms which sum up how the real world works, and if we also have a suitable method for forward projection we may then get useful results in important application areas such as funding rates for pension schemes and the solvency of life offices. We maintain that there are so many short cuts in the Markovitz approach that it is no more than a special case of risk analysis and is not really a sufficiently broad framework on which to develop further work. If you take how Markovitz approached the problem for stocks of the same expected return, he was using the variance as his measure of risk to compare them. If you take our definition, it is the downside moment of the deviation, which gives not the variance but the semi-variance. If you go back to the early literature of MPT, rather than the Markovitz papers themselves, it is quite clear that Markovitz wanted to use semi-variance for comparing shares of the same expected return, which would have been the same definition as ourselves. If the distribution is symmetric you get the same answer; if, however, the distribution is not symmetric then you get misleading answers. But Markovitz could not use semi-variance in his model as the computational difficulties were too severe. Instead, he had to use the whole variance of the share and bring in the quite artificial assumption that there is a utility function which describes the trade-off between additional risk and additional return.

I think Professor Wilkie has underestimated the power of our counter-example, where any rational investor can choose between two shares without knowing any further information. One has twice the expected return of the other, and in every case it gives the higher return. However, its variance is four times that of the other. Yet the whole of MPT is founded on the variance being the *be all* and the *end all*.

Mr Wise said how important it is to continue portfolio selection type work in actuarial areas of application, particularly pension scheme funding and life office solvency. Mr Plymen and I maintain

that MPT is not appropriate. Our counter-example falsifies the assumption that investors choose between portfolios on the basis of risk as defined by variance of return. A much more general framework is needed for risk before any further work can be done.

Professor Wilkie has produced a very elaborate and worthwhile stochastic model for actuarial use. It is certainly the best framework that has been put forward to date, but in many ways I do not think it is a robust enough model as it tends to be too stable in the long term.

Hence my suggestion to Professor Wilkie and others. Perhaps by building in, as I have suggested, a positive feedback element, we could replicate speculative bubbles and other real life factors. Investment practitioners might then accept that these models give satisfactory answers to the practical problems.

WRITTEN CONTRIBUTIONS

Mr S. J. Green: The trouble with MPT is that it has never been properly defined. When I first heard of it nearly twenty years ago it was a synonym for the Efficient Market Hypothesis, which is now generally accepted as being unsound in all but perhaps its weakest, and consequently, most useless form. If there were any lingering doubts then this paper must have given them the *coup de grâce*, and for this reason alone Clarkson and Plymen have rendered a great service to the investment community.

There is one section of the paper where I would cross swords with the authors. I do not think that they have drawn enough on the work of Professor Barr Rosenberg, who it would now seem has similar views to those of the authors.

With the aid of the substantially more extensive investment data available in the U.S.A. and the better real-time computing power which as been developed in the last ten years, Rosenberg has travelled the full circle from the Efficient Market Hypothesis which he experimented with until 1977, to finding anomalous pockets of inefficiency over the next few years, and finally correlating these into a "pervasive web of related anomalies" more recently. From conversations with one of his colleagues, together with certain presentation documents, I understand that studies of the U.S.A., U.K., Japanese and Canadian markets have revealed factors which consistently bias the returns which could normally have been expected from an efficient market. Some of those factors are more important than others and not all of them have the same relative weight in all of the four markets studied.

Rosenberg's associates now make use of these factors in devising model portfolios to out-perform the market indices rather than to match them. They fully accept that different managers may give different, including zero, weight to some of these factors but could still expect to out-perform the market.

It would seem, therefore, that as a result of his extensive research Rosenberg has returned to fundamental analysis, through using vastly superior data and in a much more scientific manner. Since his techniques are now not significantly different in style, if perhaps in content, to those of the authors, or even those of Weaver and Hall of 1967, I am surprised that the authors still consider him to be an exponent of MPT, whatever that may mean. On the contrary I regard Rosenberg's work as providing strong support for Clarkson and Plymen.

This paper should be required reading for any serious student of investment and those academics whose positions are not too rigid for them to question the myths which they have been perpetuating for years.

Professor P. Praetz:

1. I can see no evidence for this paper's conclusion that MPT does not improve performance.

   Index funds are better than portfolios of only a few undiversified securities as it is more risky. It is also better as the alternative suggested would have more search costs, transaction costs and risk due to the much more active portfolio strategy being adopted which investors may not prefer due to the chance of a large loss.

2. Given the massive computational burden of Markowitz in inverting a variance-covariance matrix 25 years ago, the Sharpe approach was a very impressive simplification with a single measure for the risk of a share. Better betas will come from discounted prices (exponential smoothed) or the Kalman filter is a useful model for changing risk and avoids the former methods problems.

   Total risk is normally diversifiable risk and non-diversifiable risk. While cheap and dear shares may exist, methods of identifying and exploiting them can be dangerous. Most shares have strongly variable risks. A covariance matrix can be added to the Sharpe model. Normality or a quadratic utility function of the Sharpe model is also needed.

   Markowitz, about 40 years ago, was severely constrained by the need to use quadratic programming. Sharpe used a simplified approach to avoid the above problem. He used standard statistical measures of risk and return which I do not feel are crude and often misleading. The perfect market is a necessary simplifying assumption, as is the covariance one. The risk factor does not defy logical assessment to me as it is the regression coefficient of a share's return on that of the market.
3. The small firm effect and other effects such as day of the week and January are discussed in Keim.[1] He finds many very small effects which while inconsistent with the CAPM represent very small deviations from it. It is doubtful if they can lead to extra profits. Non-liquid markets and often extra large costs of buying and selling make use of small stocks unprofitable. Their actual risk is often much higher with problems in measuring beta, which are often much less of a problem for major stocks.

   Work form efficiency tests share returns by statistical tests such as autocorrelation coefficients and runs tests. These are in Praetz[2] who supports efficiency. Non-statistical tests often used are charts and filters. Levy[3] discusses the absence of profits in charting and filters, Praetz[4] shows the bias in filters which make them very unprofitable.

   Semi-strong form efficiency studies share returns and other information—e.g. annual reports or the money supply.

   Strong form efficiency studies mutual fund managers, investment analysts, financial journalists and corporate executives, who all should possibly have some superior performance.
4. Relative strength reduces profits by active trading. Cheap or dear shares are often a high risk strategy which can lead to excessive trading, which would make it consequently less profitable.
5. The gilts model assumes no obvious anomalies exist. This is a similar assumption to that of the CAPM and Sharpe's model.
6. This section has short, medium and long term's artificial assumption on variable behaviour. The brokers failure may say little on efficiency.
7. Risk and return ignores the sampling behaviour of both quantities.
8. Given the very high uncertainty in determining cheap or dear shares and their probably changing their categories, an index fund or a buy and hold strategy may do better after accounting for the extra risk and costs of trading.
9. Monitor the performance: Table 2 over 1983–1984 shows that the perfect market was not operating. Well next time it probably will be especially if it includes the 1987 crash in the share market.

Comments on Appendix 2: section 1 (Tests on Efficiency).

Jensen[5] says that no other proposition in economics has more empirical support than the efficient market hypothesis. This is the introduction to the symposium on some anomalous evidence regarding market efficiency. It has 8 papers, most of which show large irregularities in relationships between securities yields and yield-surrogates, systematic abnormal returns after quarterly earnings announcements, information content on discounts and premiums on closed-end fund shares, empirical tests of boundary conditions for CBOE options, the information content of option prices and a test of market efficiency, the market valuation of cask dividends, split information, stock returns and market efficiency, 1 and 2.

The Mandelbrot paper is about distributions of share returns. It should not be in a section on weak tests. His stable paretian with infinite variance has been discredited badly by Praetz.[6]

Praetz[2], for a U.K. actuarial audience, has argued for the random walk. Praetz[7] gives very low correlation coefficient for 4 U.K. share indices and 2 bond series. The overall support for random behaviour is very strong. Benjamin[8] in the maturity guarantees working party shows some low correlation in the de Zoete index, which would have been lower except for its construction.

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Mr J. G. Spain: The authors are to be commended for their bravery in presenting their paper to a sceptical audience, and for providing us with a welcome opportunity to reconsider an important subject.

It seems to me that there are two key words in the paper, these being 'risk' and 'performance'. I assume that everyone will accept that performance has to be 'monitored'. Further, many would like to see risk factored into this exercise, taking account of the nature of the liabilities.

This is the concept upon which I wish to concentrate, bearing in mind the authors' words at the bottom of page 651 of their paper, namely that ". . . risk will look after itself". In my view, this is not a sensible conclusion, as it totally ignores the variation in risk-return profiles which should be acceptable to different types of investor.

As pointed out by the authors, risk has not been at all well-defined. Whatever it is, it cannot be avoidable, for the simple reason that no one knows what is going to happen.

There are certain types of risk, which can readily be identified. For example, we have *default risk*, which would arise where anticipated repayments of capital (or income) could not be made, following a loss by the borrower. Then, we have *dividend risk*, where it is recognized that a dividend may be reduced below expectations (or even passed entirely).

Inflation risk exists in that the total return may not match a selected inflation target, as with trustees of 'final pay' pension plans.

There is *markets risk*, which is the risk that a sale may be forced when prices are lower than expected.

One must bear in mind *currency risk*, which reflects the fact that losses can be made as a result of exchange rate fluctuations. There is also the associated *other markets risk*, reflecting factors such as less well regulated markets, or being subject to different legislation (say forced nationalization, or taxes which cannot be reclaimed).

The final type of risk I wish to distinguish is what I call *satisfaction risk*, where an investor believes that all is well with the portfolio, when the published return may have been calculated taking no account of the nature (term and type) of the liabilities against which the assets are being accumulated. This could also be described as *over-optimism risk*.

To adapt the authors' R.A.F. analogies, consider the navigator's job, which is to tell the pilot where he has been, is now, and is going. To achieve this implies continual monitoring, testing 'actual' against 'expected', a concept with which we are familiar.

It seems to me that, by concentrating upon volatile uncertain market values, performance monitoring services manage to attempt perhaps the first two parts of the navigator's job, but definitely not the third. If the monitoring exercise were modified so as to give an 'expected', we would then see that *satisfaction risk*, can be taken as being broadly measured by the difference between 'actual' and 'expected'.

Using this terminology, the 'actual' value would be the market-value denominated return, commonly calculated. However, the 'expected' value would be based upon the progress of 'discounted values' over the period, *using consistent discounting assumptions at all stages.*

Even actuaries will immediately say that this cannot work, and that the introduction of 'discounting assumptions' inevitably brings with it the problem of subjectivity, implied by the choice of assumptions.

In fact, having tested the concept, I have found that the approach I suggest (written up elsewhere) is far from subjective or biased, and does appear to be robust. I hope to have the opportunity of backing up my claim with hard evidence.

The joint authors subsequently wrote: Several speakers made the point that they regarded our definition of Modern Portfolio Theory as being too narrow and suggested that it should include such developments as the Weaver & Hall model and the gilts and equity models described in the paper. However, we deliberately limited our definition to those methods which have as their main cornerstones the efficiency of markets and a Markovitz-type definition of risk involving the variance of the expected return. Our definition of MPT corresponds exactly to the most important concepts of the subject as described in the preface to the book of readings by Lorie & Brealey:

> "The essays that we selected were distinguished for their lucid explanation of three pivotal ideas. These ideas are: (1) capital markets in the U.S.A. are highly efficient—meaning that current prices reflect in an unbiased way what is knowable about the companies whose securities are traded; (2) portfolio management is a different subject from security analysis (security analysis is designed to indicate the likely or possible returns from investing in particular securities, whereas portfolio management has to do with the selection and surveillance of a bundle of securities that match the aspirations, fortitude and tax status of the beneficiary); and (3) the relative prices of securities are determined by the expected return to the investor and also by the uncertainty about the return. Modern theory has taught us something about the way in which the market determines the premium for enduring uncertainty or risk."

Mr Adams drew attention to the fact that the latest MPT paper in our references was dated 1968. Again this was deliberate; we wished to focus attention on the important early papers in the area rather than to review the (often highly technical) papers of more recent date, largely from the Rosenberg school which 'blend in' a considerable amount of fundamental research.

Risk and Return

Professor Wilkie suggests that for gilts and investment trusts we are only interested in the deviation from the true price and not in how that true price in itself changes. The gilts paper does in fact contain a very full description of how price changes can be resolved into five quite separate components, of which three relate to changes in the general structure of the market. In the present paper the gilts model is introduced primarily as a starting point from which the more complicated equity model can be derived. Similar comments apply to the investment trust model: the original paper discusses factors such as gearing and the 5-year management rating as well as the discount on net asset value.

Mr Scott confuses the use of variance of return as a measure of dispersion (and hence of risk) with analysis of variance which is a statistical tool that shows which factors are the most important in a least-squares regression model. In § 6.2 we describe measures of price sensitivity which provide similar information to that obtained from a full analysis of variance, but variance in this context has nothing to do with the use of variance of return as a measure of risk.

A very interesting philosophic point was raised by Mr Lambert, namely that our selection methods are likely to be self-defeating. This, we believe, is highly unlikely for two reasons. Firstly, different investors, operating essentially the same model, may use different values for the various fundamental

inputs and hence identify different groups of cheap and dear shares. Secondly, we do not anticipate that the use of formalized investment models will ever dominate the equity market. Even for gilts, where the subjective element is far less important, institutional investors who operate their own model on a daily basis are in the minority.

We are grateful to Mr Grant for highlighting one of the great advantages of a formalized system of assessment and performance monitoring, namely that it introduces a very necessary degree of discipline into the whole investment management process. We also agree that an element of flexibility is required when interpreting the results obtained from a formalized investment system.

Mr S. J. Green suggests that we should have given more attention to the work of Rosenberg. Paragraph 3 of the Appendix covered the 'Efficient Market' portion of the Rosenberg program—that part which we regarded as coming under the orbit of MPT. In 1982 Rosenberg, after abandoning the concept of the Efficient Market, transformed his system from an Index Fund risk reducer to a powerful method of improving performance. The procedure is to calculate the 'expected return'. relative to the index return, from historical data, using not only price records but allowing also for special features of the company concerned (gearing, earnings growth, dividend yield, market capitalization etc.). This analysis is done in enormous detail using the 46 descriptors mentioned in Paragraph 3 of the Appendix. Where the expected return is higher than the corresponding index figure, the share is classified as cheap. The cheapness of some of the shares may be ascribed to a particular descriptor, such as the small company effect or the benefit of higher yield etc. Pinpointing the cheap or dear groups of share is of course most useful from the point of view of policy and tactics.

In practice our market model uses much the same procedures, obtaining the expected return from a combination of earnings, dividends and the long-term growth rate of earnings and dividends. The difference is that our long-term growth rate is not derived solely from rigorous analysis of all the past data but includes a contribution based on investment analysis of the latest trends and prospects of the share concerned. It is likely that our market model, bu making more use of forward-looking analystic material, gives a more precise measurement of the cheapness or dearness of individual shares than does Rosenberg.

Several speakers suggested that we had virtually ignored risk since it was a relatively minor item, and indeed Professor Wilkie interpreted our main conclusion (namely that if we look after the expected return the risk will look after itself) as implying that we assume all investors are risk-neutral. These are quite erroneous interpretations of our approach to risk. In § 7 we suggest that the MPT definition of risk is totally unsatisfactory and should be replaced by a quite different conceptual basis which defines the risk of one share relative to another in terms of relative likelihoods of significantly poorer than average performances. We develop one particular axiom from this conceptual framework and suggest how the approach can be developed further in terms of integral measures. On this basis, as explained towards the end of § 7, an improvement in the expected return in which the entire probability distribution 'moves to the right' also reduces the risk.

The Efficient Market

Mr Damant and other speakers seem to be claiming on the one hand that the market is efficient and on the other that pockets of inefficiencies can be easily identified. Presumably Mr Damant wishes to argue (1) that the market is near enough efficient for fundamental analysis to be self-defeating and (ii) that he is nevertheless able to offer a 'tilt' fund which will perform better than a fully indexed fund by exploiting pockets of inefficiency (the most powerful of which is normally claimed to be the 'small company effect'). Several other speakers also referred to the hypothesis as being a simplified statement of the real world and then said that the existence of anomalies does not necessarily mean that the hypothesis should be rejected.

Further powerful evidence against the Efficient Market Theory has been provided by Professor Rosenberg. S. J. Green points out that Rosenberg, who accepted the efficient market until 1977, has since then travelled full circle and turned the whole emphasis of his advanced analytical and statistical techniques to finding cheap or dear shares and then using the 'tilt fund' procedure to improve the performance. By statistical analysis of historic data, he is able to establish certain factors which make for a good performance. The individual holdings are then altered so as to increase the emphasis on these favourable factors, i.e. tilting the holdings in the right direction. Rosenberg is in effect endorsing

our theme, namely that dedicated and determined analysts can identify significant inefficiencies and can exploit them profitably.

In a publication dated July 1987 from Barra International some figures are given for the success of these tilt portfolios in the U.S.A. and Japanese markets. Over the period July 1980 to June 1985, a 'Factor Tilts Portfolio' gave an annualized return of 20·01% compared with 16·34% for the Standard & Poor's 500 Index; for the Japanese market the tilt portfolio returned 18·55% p.a. compared with 14·88% p.a. for the Tokyo Stock Exchange First Section Index.

Our analysis of the prices of the eight unit trusts from 1984 to 1987 shows a quite spectacular over-performance relative to the indices. As most of these trusts appear to give emphasis to investment in high yielding equities, this study can perhaps be regarded as another successful application of the Tilt Fund technique, and another proof of market inefficiencies. This example and our efficiency test in Appendix 4 seem to us to be conclusive evidence that prices do not follow a random walk of the type described by Mr Adams.

Finally, Mr Wise pointed out that in other scientific disciplines such as physics there comes a point at which so many anomalies have accumulated that the hypothesis should be abandoned and a search made for a more satisfactory successor. We believe that the time has come for academics and practitioners alike to accept:

(1) that the Efficient Market Hypothesis is not a reasonable statement of reality:
(2) that the price formation process within a capital market involves cyclical short-term price changes whose future values can to a degree be predicted using elementary control theory techniques; and
(3) that research workers, instead of arguing about obsolete theory, should follow the example of Rosenberg and ourselves and concentrate their attention on developing the powerful new methods now available for finding and exploiting market inefficiencies.