Measuring Investment Risk in Defined Benefit Pension Funds

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An earlier version of this paper, treating in detail investment risk as it applies to Irish pension funds, was prepared to accompany and support the report of the working party of the Society of Actuaries in Ireland, *Report on Investment Risk (2004)*, presented on 24th February. I thank the working party for many useful discussions in honing the ideas presented here. In particular, I single out Tom Murphy, the chairman, who had the difficult task of keeping me to the point, and Pat Ryan who scrutinised and suggested improvements to earlier drafts of the paper. This is a work-in-progress and I welcome comments.

Outline of Paper

Overview

This is not the first paper to treat investment risk in pension schemes. The topic has been contentious and divisive in the actuarial profession in the UK, the US and Canada. The beginnings of the debate in its current form can be traced to the paper of Exley, Mehta, & Smith (1997). In this paper, Exley *et al.* brought to an actuarial audience the approach pioneered by Modigliani & Miller (1958) to the financial structure of companies and adapted to the special case of pension funds by, *inter alia*, Sharpe (1976), Black (1980), and Tepper (1981). Originally the debate centred on how to value pension liabilities and invest pension funds but has since grown to polarise the actuarial community into advocates of financial economics and advocates of the traditional actuarial approach. Writing of US actuaries and their similar approach to valuing and investing pension funds, Bader & Gold (2003) put the importance of the issues in these terms:

> "To protect the pension system and the vitality of our profession, we urge pension actuaries to reexamine and redesign the model. The new model must incorporate the market value paradigm and reporting transparency that is rapidly becoming a worldwide minimum standard in finance."

To simplify the debate to the point of caricature, the 'financial economist' group argue that pension funds should invest primarily in bonds, while the 'traditional actuaries' defend the common high equity exposure. A recent symposium[1] gives an overview of the issues, with Gordon & Jarvis (2003) giving its history and Day (2003) giving a synopsis of the current state of the debate.

The debate has not been confined to a pure professional or academic setting, becoming critically important to the whole pensions industry as the performance of bonds and equities diverged significantly since the start of the millennium. Defined benefit schemes, the backbone of the pensions industry in English-speaking countries, appear to be in crisis and the actuarial profession appears unable to achieve a consensus on the way forward.

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[1] *The Great Controversy: Current Pension Actuarial Practice in Light of Financial Economics Symposium*, Vancouver, British Columbia, Canada, June 24 – 25, 2003, Organised by the The Society of Actuaries (Retirement Systems Practice Area), The Actuarial Foundation and American Academy of Actuaries. Papers available at www.soa.org/sections/pension_financial_econ.html.

This paper goes back to ideas first presented in Arthur & Randall (1989), develops them in a more technical setting, and provides an empirical assessment of the magnitude of risk entailed by different investment strategies. First, we define *investment risk* in the context of actuarial investigations generally. This definition is, we believe, just a formalisation of our intuitive notion of the concept. From the definition, one can quantify the investment risk inherent in any given investment strategy and thereby identify the strategy with lowest investment risk. We further define the notion of *consistency* in the valuation of assets and liabilities (which coincides with the 'no-arbitrage principle' in finance or the 'law of one price' in economics). We show that when assets are valued at market value then the consistency constraint puts a value on some (but generally not all) of the liabilities. In particular, the perfect matching portfolio of assets to meet the liabilities, if it exists, is shown to have zero investment risk irrespective of the investment assumptions underlying the valuation.

We briefly review the different investment objectives of the different parties to the pension scheme. In contrast to the approach generally adopted by financial economists, we take a scheme-centered approach.[2] In particular, our approach finds a role for the trustees, the sponsoring employer and the regulator who, of course, are the parties that typically require the actuarial investigation. We argue that recent changes to the regulatory regime in the UK and Ireland have placed the maintenance of the funding level on discontinuance of at least 100% as a key constraint and investment risk must first be judged against this constraint. These regulations demand assets be valued at market value which, by the consistency requirement, entails that liabilities be valued on market-based methodology. This paper concerns itself primarily with outlining a market-based methodology for the termination liabilities, extending it to the case of liabilities that have no matching asset and, applying the earlier concept of investment risk, measure the investment risk inherent in given investment strategy to meet those liabilities. The method extends easily to a funding plan when assets are valued at

[2] That is, we do not attempt the rather Marxist reduction of the relationships between the various parties to pension schemes into the ultimate struggle of financial interests between shareholder (capitalist) and employee (worker). That reduction, in the manner generally done, leads to the conclusion that bonds are, to a second order, preferable to equities in pension fund investment. That debate, though, is somewhat removed from the topic of this paper, quantifying investment risk.

market value and, for an example, we sketch how this is done under the Defined Accrued Benefit Method (described in McLeish & Steward (1987)) with a one-year control period. Note that investment risk can also be monitored relative to any funding plan and whatever the approach to valuing assets but, of course, it will typically give a different answer as the result is dependent on the funding method and valuation assumptions.

Next comes the main body of the paper. We outline how to identify the asset portfolio that minimizes the investment risk relative to the termination liabilities (the so-called Liability Reference Portfolio or LRP). It is argued that it is been possible to find assets that provide a reasonably close match to pension liabilities, at least for benefits that fall due within the next three decades. The mechanics of finding the LRP are often straightforward, if tedious, for many pension benefits. We extend the approach in the natural way to value the benefits falling due after three decades. We investigate the possible investment strategies to meet the benefits and quantify important aspects of the mismatch distribution (or 'investment variation' as we prefer to call it). In all cases we report that a portfolio of bonds (conventional and index-linked) approximate the LRP.

We illustrate trial-and-error methods that can be used to find the LRP in a single important set of case studies, and quantify the extra risks entailed by other investment strategies. We report the results of these analyses in detail showing that they appear to be reasonably robust across economies over the last 30 years and longer, and reasonably robust when termination liabilities are escalating in line with wages. We get an important insight from this analysis: even long bonds are not long enough to match the liabilities of young scheme members, and investing in such bonds can be as risky as investing in equities. So, in particular, a fully funded relatively immature defined benefit scheme investing in 20-year conventional bonds could quickly develop funding problems on a discontinuance basis. Just as much care must be exercised in matching liabilities by duration as by matching liabilities by asset type.

We point out that finding the LRP for some pension liabilities, especially those defined as the lesser of two amounts either of which can vary in the future, can be non-trivial and might involve dynamic matching strategies. As Palin & Speed (2003) candidly admit:

> "Hedging pension liabilities is difficult, more difficult than we thought when we started to write this paper. We don't claim to have found a full solution to the problem; this paper represents work in progress. However we believe that our method produces reasonable, sensible results for the cases described above, even if we can't explain the answers fully."

Much work remains to be done in determining the LRP for different pension liabilities.

We end the paper by demonstrating that the argument that the risk of equities dissipates with time (so that, at some long-term investment horizon, equities are preferable over other asset classes by any rational investor) is fallacious. This argument, generally known as the 'time diversification of risk', does not hold in that strong a form. True, the expected return from equities might well be higher than other asset classes but, on some measures, so is the risk.

So the most closely matching asset for pension fund liabilities is composed mainly of conventional and index-linked bonds. The LRP is a portfolio that, if history is any guide, has a lower expected long term return than a predominantly equity portfolio.

Our analysis does not allow us to suggest one investment strategy is preferable to another. Investors, including pension funds, are routinely tempted to take risks if the reward (that is, the form of the investment variation distribution) is judged sufficiently tempting. However, pension funds should appreciate the risks involved in alternative strategies and, at a minimum, seek to ensure that the investment portfolio is efficient in the sense that risk cannot be diminished without diminishing reward. To appreciate the risks and ensure that all risks undertaken are reasonably rewarded requires knowledge of the investment variation distribution and, in particular, the Liability Reference Portfolio.

Defining Investment Risk

In this section we attempt to define the concept of investment risk in actuarial investigations. Our intuitive notion that it measures the financial impact when the actual investment experience differs from that expected, holding all other things equal, is given a formal expression. This definition of investment risk is sufficiently general that it can be readily interpreted and applied whether the investigation is into the on-going funding level of the scheme, its solvency position, or the FRS17 balance sheet item. Further, the impact of investment risk on the contribution rate or FRS17 pension cost can, with some elementary calculations, also be assessed. Once investment risk is properly defined, it is straightforward (in theory at least) to measure and attempt to minimise it.

Actuarial investigations into the current financial state and future financing of pension schemes, as envisaged in GN9, typically require the actuary to use his judgement to decide on the most appropriate approach under three broad headings:

1. The valuation methodology, suitable for the purpose of the investigation.
2. Demographic assumptions to estimate the type of benefit and when payable. The quantum of the benefit may depend on the timing of the payment.
3. Financial assumptions, so as to project, in a consistent way, the amount and timing of the liabilities and the asset proceeds.

The financial assumptions generally include inflation in any future year and rates of wage escalation. Further assumptions, compatible with the financial assumptions underlying the projection of the liabilities, are generally required to value the assets. In particular, the valuation rate of interest – the rate of interest used to discount the projected future cashflows to the present time – is generally taken to be an estimate (perhaps erring on the conservative side) of the return over the future that could be expected to be achieved on a portfolio of assets that are broadly suitable to the nature and term of the liabilities.[3]

[3] A concise overview of the traditional actuarial approach to investigating pension funding, especially the consideration affecting the choice of the valuation rate, is given in Paterson (2003).

Once the three components of the valuation (1,2 and 3 above) are specified then the investigation allows the actuary to report not only on the current state of the finances but generally also to outline, on the valuation assumptions, the future course of the pension scheme's finances. This point is key as it allows future valuations to assess the divergence of the actual experience from that expected (which might then help suggest timely corrective actions).[4]

To readily see this, let us assume that the valuation methodology is the form of a discounted cashflow approach. When t = 0, we call it the present time, and t > 0 a future time. Let A_t be the forecast cashflow from the assets at time t, L_t be the forecast liability cashflow at time t and *i* be the valuation rate of interest. Now, the reported valuation result at time 0, expressing the surplus (excess of the present value of the assets over the present value of the liabilities), denoted $X_{0,}$ can be written as:

$$\begin{aligned} X_0 &= \sum_{t=0}^{\infty}(A_t - L_t)(1+i)^{-t} \\ &= \sum_{t=0}^{p}(A_t - L_t)(1+i)^{-t} + \sum_{t=p}^{\infty}(A_t - L_t)(1+i)^{-t} \\ &= \sum_{t=0}^{p}(A_t - L_t)(1+i)^{-t} + (1+i)^{-p}\sum_{t=p}^{\infty}(A_t - L_t)(1+i)^{-t+p} \qquad \text{(I)} \\ &= \sum_{t=0}^{p}(A_t - L_t)(1+i)^{-t} + (1+i)^{-p}\sum_{s=0}^{\infty}(A_t - L_t)(1+i)^{-s} \\ &= \sum_{t=0}^{p}(A_t - L_t)(1+i)^{-t} + X_p^0(1+i)^{-p} \end{aligned}$$

where

p is the time that the next actuarial valuation falls due

If the experience of the scheme in the inter-valuation period is exactly in line with that assumed at time 0, and the method and assumptions in the valuation undertaken at time *p* are also the same, then the valuation result at time *p* will be $X_0(1+i)^p$.[5]

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[4] The so-call 'actuarial control cycle'.

[5] This can readily be seen, as the cashflow surplus in the inter-valuation period will be invested (or disinvested) at the valuation rate of interest, accumulating at time *p* to $(1+i)^p\sum_{t=0}^{p}(A_t - L_t)(1+i)^{-t} = \sum_{t=0}^{p}(A_t - L_t)(1+i)^{p-t}$ and this amount is to be added to discounted value of all the yet unrealised asset and liability cashflows at time p, namely X_p^0. The total value at time p is then

Consider X_0. We shall assume that this is a number.[6] So, under this simplifying assumption, X_0 is a constant, representing the surplus (if positive) or deficit (if negative) of assets relative to liabilities at the present time identified by the specified (deterministic) valuation methodology.

It is generally possible to form a reasonable apportionment of the difference of the valuation result at the next valuation date from that expected from the valuation at time 0 (i.e., $X_0(1+i)^p$) into that due to either

- (i) the actual demographic or financial assumptions in the inter-valuation period differing from that assumed, or,
- (ii) that due to a changed valuation method or basis applied at time *p*.

In particular, it is possible to form a reasonable assessment of the financial impact of the actual investment experience relative to that expected, other things being held the same.

Let X_{0-p}^i denote the result of the valuation at time 0, under the same methodology and assumptions as underlying the valuation result, X_0, at time 0 but reflecting the actual investment experience of the scheme in the inter-valuation period. Then $X_{0-p}^i - X_0$ measures the financial impact at time 0 of how the actual investment experience up to time *p* differed from that assumed in the original valuation at time 0. Obviously, if it turns out that the actual investment experience bears out the assumed experience in the inter-valuation period then $X_{0-p}^i = X_0$, so $X_{0-p}^i - X_0$ takes the value zero. We shall call $X_{0-p}^i - X_0$ the 'investment variation' up to time *p*.

$\sum_{t=0}^{p}(A_t - L_t)(1+i)^{p-t} + X_p^0$, which is just the right-hand side of equation (I) above multiplied by $(1+i)^p$, whence the result.

[6] If this is allowed be a non-constant random variable then we call the valuation methodology used stochastic otherwise the valuation approach is said to be deterministic. Note that a stochastic valuation is representing some part of the assets and/or liabilities as a non-trivial random variable at time 0. We shall discuss only deterministic valuation methods in the sequel to simplify the analysis but, as should be clear by the end, the results carry through (with relatively straightforward extensions) when applied to stochastic valuation approaches.

Investment variation, so defined, is a non-trivial concept. It measures the financial impact at time 0 created when the actual investment experience up to time *p* differs from the investment assumptions underlying the valuation at time 0.

We also need to formalise what we mean by saying that assets and liabilities are valued on a consistent basis. By a 'consistent valuation method' we mean that the present value of a cashflow of a given amount at time t is the same up to a change in sign, whether the cashflow is positive (an asset) or negative (a liability).

We list some of the consequences of investment variation so defined:

(1) Investment variation affects both sides of the balance sheet in general – the present value of both assets and liabilities - as some assumptions used to value the assets can also affect the size of the liabilities.

(2) If we have *perfect matching* of assets to liabilities,[7] then any consistent valuation method will always report the investment variation to be zero.

> The present value of the assets at time 0 (i.e., $\sum_{t\geq 0} A_t(1+i)^{-t}$) might vary with the investment assumptions but $\sum_{t\geq 0}(A_t - L_t)(1+i)^{-t} = \sum_{t\geq 0} 0.(1+i)^{-t} = 0$ and hence $\sum_{t\geq 0} L_t(1+i)^{-t}$ must vary in direction proportion to $\sum_{t\geq 0} A_t(1+i)^{-t}$. Hence, in aggregate, a gain (loss) on the assets relative to that expected is exactly offset by an increase (decrease, respectively) in the value of the liabilities relative to that expected. In short, perfect matching of asset and liability cashflows has zero investment variation, irrespective of the experienced or the assumed investment conditions.

(3) Investment variation can be altered by changing the composition of the asset portfolio, as this changes the nature and timing of the future asset cashflows. In particular, the closer the asset proceeds are to the liability outgo in timing, amount, and nature[8] then the closer to zero the investment variation.

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[7] In the technical sense that $A_t = L_t$, for all t, independent of any investment assumptions.

[8] By matching by nature, we mean that unforeseen economic influences ("shocks") have a similar effect on the magnitude of both assets and liabilities.

(4) Other than the uniformly zero variation in (2) above, the magnitude of the investment variation depends on the valuation approach, the demographic aasumptions, and the other non-investment financial assumptions as all these enter into its measurement of both X_0 and X^i_{0-p}.

(5) Of course, the actual valuation at time *p* of the scheme will generally be different from X^i_{0-p} suitably inflated as

(i) the experience of the scheme in the inter-valuation period is unlikely to be in line with the non-investment assumptions, leading to gains or losses at the next valuation on the non-investment financial assumptions and the demographic assumptions.

(ii) The valuation methodology or the underlying assumptions at time *p* could change in the light of experience or other factors consequent on the passage of time so that, at time *p*, the valuation approach or basis is changed.

Suppose, we make the following three assumptions:

(1) Assets are to be valued at market value

(2) There exist assets that perfectly match the liabilities (in the technical sense earlier).

(3) We demand our valuation methodology be consistent (in the technical sense earlier).

Then, it follows that the investment variation is the present value of the increase in the surplus, discounted at the rate of interest equal to the return on the market value of the matching asset. That is, the experienced inter-valuation rate of interest is the return on the market value of the matching asset. To see this, we can rewrite X_0 as:

$$\begin{aligned}
X_0 &= \sum_{t=0}^{\infty}(A_t - L_t)(1+i)^{-t} \\
&= \sum_{t=0}^{\infty} A_t(1+i)^{-t} - \sum_{t=0}^{\infty} L_t(1+i)^{-t} \\
&= \sum_{t=0}^{\infty} A_t(1+i)^{-t} - \sum_{t=0}^{p} L_t(1+i)^{-t} - \sum_{t=p}^{\infty} L_t(1+i)^{-t}
\end{aligned}$$

This latter expression can now be used to calculate X^i_{0-p} with knowledge of what the actual investment experience will be over the inter-valuation period. The first term of

X^{i}_{0-p} in the above decomposition is simply the market value of the assets at time *p* [by assumption (1)], discounted at the valuation rate of interest experienced over the period (which has to be determined). At time *p*, the present value of the future liabilities (the last term in the above equation) is equal to the market value of the matching asset at that time [by consistency, assumption (3)]. Now, we need to find the experienced valuation rate in the inter-valuation period. Note that if there was a matching asset at time 0 and the scheme held the matching asset then the investment variation would be 0 (irrespective of what happened in the inter-valuation period). Hence the experienced valuation rate in the inter-valuation period can now be seen as the market return on the matching asset over the inter-valuation period. The upshot is that the investment variation is the present value of the extent to which the increase in the value of the assets exceeded the increase in the liabilities over the inter-valuation period, discounted at the rate of return on the matching asset over the period.[9] We see immediately from this that investment variation is positive only if the increase in the market value of the actual assets exceeds the increase in the market value of the matching asset.[10]

Investment variation at time 0 can be viewed as a stochastic process, $X^{i}_{0-p} - X_0$, indexed in *p*, as the term X^{i}_{0-p} cannot in general be evaluated at time 0 from the then available information. Viewed in this way, the investment variation at time 0, up to time *p*, is random variable. This is the more practically useful concept - the ex ante distribution at the present time. However, in order to define it, it was necessary to first define how to calculate the ex post investment variation. In short, investment variation up to time *p* can only be measured at time *p*, before that it may be modelled as a random variable with an associated distribution.

So $X^{i}_{0-p} - X_0$, when viewed at time 0 is a random variable, so it has an associated distribution. Insofar as pension schemes have been unwilling or unable to perfectly

[9] This expresses, in more technical terms, the 'Main Guiding Principle' of Arthur & Randall (1989) that states "that if there is a rectifiable mismatch, a relative change in market values of the matched and mismatched assets should be reflected in the valuation result" (Section 5.1).

[10] Or, as expressed in Arthur & Randall (1989), "the Main Guiding Principle merely reaffirms an earlier fundamental principle, namely that if you are mismatched and you get your forecasts wrong then you have to pay the penalty" (Section 2.5).

match the asset and liability cashflows, in the technical sense earlier, then investment variation will have a non-constant distribution. The mean of this distribution captures the bias in the original investment assumptions – a positive mean implies that the original investment assumptions were conservative (as, on average, the experienced conditions turn out better than originally forecast). Some prefer to give a single number to capture the notion of riskiness in a distribution, often using some parameter that measures the spread of the distribution, such as its standard deviation, its semi-variance, or its inter-quartile spread. Often this summary measure is called 'investment risk'.[11] Alternatively, one can apply some other measures such as report the value below which there is a specified low probability of the outcome falling (the so-called 'Value-at-Risk'), which combine the mean and the standard deviation in a particular way.[12] The key point to be made is that the distribution of $X^i_{0-p} - X_0$ is a more foundational concept and maintains more information than any summary spread statistic. We do not enter on the wider discussion of the most appropriate measure to apply to the investment variation distribution to capture our intuitive notion of risk but adopt the generally accepted measure of standard deviation. So we identify investment risk as the standard deviation of the investment variation distribution (to a first order approximation).

Appendix I draws attention to a major limitation of our definition of investment variation (and the associated investment risk) for pension funds. Practitioners must overcome the limitation in each case, the manner being dependent on unique characteristics of the scheme, sponsoring employer, and the business of the sponsoring employer.

[11] The investment variation distribution in those cases where the proceeds from the assets perfectly match the liability cashflows is uniquely characterised as being a constant (i.e., a degenerate distribution), so it will give a value of zero for any metric attempting to measure the spread of a distribution.

[12] Of particular importance in the probability distribution is its extreme left tail behaviour, which gives the probability of a reduction to the surplus of any given large amount. Such an event might cause sudden and severe financial strain to the cost-bearer, perhaps threatening the continuance of the scheme. Appropriate measures for such extreme risks include, for symmetric distributions, the kurtosis or higher even moments if they exist.

Discussion on the Different Investment Risks for Irish Pension Schemes

The primary objective of the pension fund is to pay all the promised benefits as they fall due. There are many ways to achieve this end, so invariably the primary objective is supplemented with some secondary aim. Members and other beneficiaries of the scheme with a fixed contribution rate might, for instance, phrase the unique objective of the pension fund as to pay all the promised benefits as they fall due with this objective to be achieved with the utmost certainty – that is, they might stress the financial security of the scheme. The sponsoring employer (if financing the balance of the cost) might adopt the objective of meeting the promised benefits at the lowest regular financing cost. The trustees of the scheme could perhaps adopt the twin aims of wanting the utmost security of payment of the promised benefits and wanting the scheme to continue indefinitely (hence maximising the payout to current and future beneficiaries). This trustees' objective entails that they must pay attention to the desire of the employer in providing the benefits at the lowest cost, as they do not want to frustrate the employer's objective to such a degree that it provokes the employer to reduce future benefits or terminate the scheme altogether. A formal manner of modelling this trustees' objective is to model the employer's future contribution rate as an asset of the scheme and the trustees' wish to maximise the value of this asset, subject to adequate security of benefits accrued to date.

The main point is that the beneficiaries, the sponsoring employer, and the trustees can all be envisaged as in agreement on the primary objective but at odds on secondary aims. This somewhat stylised manner of viewing pension schemes shows that the trustees could be pictured as arbiters of the inherent conflict of secondary aims between the cost-bearer and the beneficiary (as to reduce the pace of financing inevitably reduces the security of the promised payments). The fundamental cause of the tension arises from the design of such schemes, where it could be argued, before the change in regulation in the UK last year that made a deficit on discontinuance of the scheme a debt to the employer, that the employer enjoyed the investment variation if positive but, if negative, retains the option of winding-up the scheme.[13]

[13] The opposite view of the tension in Exley *et al.* (1997), which sees the member as gaining on positive investment variation with the chance that the benefits will be improved but never suffering on a negative investment variation. No doubt, the reality is more complex than either of our summaries

Aside from the beneficiaries, the trustees, and the employer, there is a fourth party to pension provision: the regulator.[14] The environment in which pension funds operate has been transformed over the last few decades. Materially regulations in Ireland and the UK demand that a member is entitled to a certain minimum benefit on wind up or if a member leaves prior to retirement age (that might well be greater than that promised under the original benefit rules of the scheme). Further, regulations have also been made introducing the requirement that pension schemes must have an actuary undertake a periodic review to ensure that the assets of the scheme taken at market value exceed the liabilities on termination (as defined in the regulations). In Ireland, if this investigation reveals a shortfall then it must be disclosed to interested parties and a short-term funding plan agreed with the newly established regulator, the Pensions Board, to make good the deficit. Typically, the liabilities on termination are defined to be the benefits payable under the rules (as amended in the case of the early leaver above) with in-service members assumed to be early leavers at the valuation date.

The regulations have fundamentally altered pension fund financing. Pension schemes now have at least two investment targets. One target is to provide, as before, from the existing assets and the members' and sponsoring employer's future contribution rates, the benefits promised under the rules when they fall due (with, as noted earlier, the different interested parties augmenting this primary objective with different sub-objectives). The other target is to ensure that, at any valuation date, the value of the assets exceed the value of the liabilities on the termination of the scheme. Now, prior to these regulatory changes, the value of the termination liabilities was in general a small percentage of the value of the assets of the typical scheme, so that the second target could be ignored in practice when financing the scheme and setting the investment strategy. However, now the termination liabilities of defined benefit pension funds form a high percentage of the value of the assets so that attention must now be given to this latter target. This creates an investment dilemma because the

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with, perhaps, the value of an investment variation to each party being dependent on the relative negotiating strength of each, which varies in time.

[14] See, for instance, Whelan (2003), for a fuller discussion of the role of the regulator and the likely long-term effect of the current regulatory regime on Irish defined benefit pension funds. Some of the ideas presented below are a development of arguments presented in that paper.

investment strategy that produces an acceptably low probability of showing a deficit on future termination valuations may not coincide with the investment strategy that produces the employer's desired low regular financing cost[15] and hence might be detrimental to the trustees' secondary aim in prolonging the scheme.

The question naturally arises as to how one prioritises the twin investment objectives identified above. In Ireland, the more stringent requirement is that imposed by the regulator, as if the scheme can always demonstrate it is at least 100% funded on a discontinuance basis then, trivially, it can always pay the promised benefits. So the primary objective of the interested parties is satisfied on the regulatory basis. Also, the secondary objective of the beneficiaries is also satisfied if the scheme is always 100% funded on the regulatory basis. Further, if at any time the scheme falls below the 100% funding level on discontinuance, then the future funding plan is no longer at the sole discretion of the employer in Ireland but the plan must be approved by the Pensions Board. So for the employer to maintain autonomy in the financing arrangements, it is necessary to maintain at least a funding rate of 100% on discontinuance. Accordingly, for Irish pension funds, the two investment objectives can be prioritised with the funding level of discontinuance being the constraint (i.e., must be met) and minimisation of the financing cost a target subject to this constraint. Consequently, the focus of investment variation (and therefore the associated investment risk) will in the first instance relative to the termination liabilities and the funding level on discontinuance.

[15] From the formal definition of investment variation defined in the last section, it is clear that investment variation depends on the liabilities. As the liabilities change from a valuation assuming the scheme to be on-going and a valuation on assumed termination, clearly the investment variation distribution is altered. In particular, the zero risk investment strategy if it exists, will not in general be the same for a valuation assuming determination and a valuation with the on-going assumption.

Identifying the Liability Reference Portfolio

In this section we show how to apply the concept of investment variation in some case studies. When necessary we use historic data from the UK, Irish and US capital markets to assess the investment variation associated with different investment strategies. When no matching asset exists we give several descriptors of the investment variation distribution from the historic data – including the key measures of its geometric mean and its standard deviation. These latter two summary measures give an illustration of the relative rewards of the different strategies and, to a first approximation, the risks associated with the strategies. We call the asset portfolio that minimises the variance of the investment variation distribution the Liability Reference Portfolio (LRP).

Theoretical Considerations

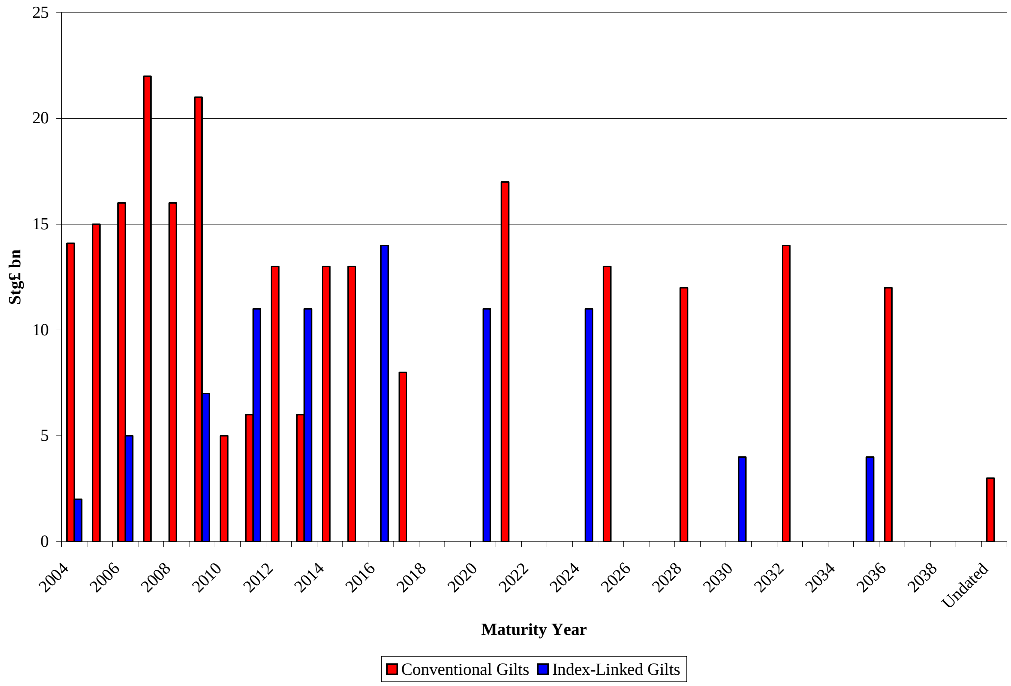
For concreteness, let us consider a person aged 55 who is a member of a scheme that promises, based on his past service of 20 years, a pension from age 65 of one-third his salary at the time of retirement, the pension subject to a fixed rate of increase while in payment. Let us further assume that the man will die on his 85th birthday.

Now, we shall assume, for illustration that the value of his preserved benefit (that is, the benefit on termination) is to be revalued by the lesser of inflation or 4% in any year, up to vesting at age 65. Given that the average revaluation rate can be reliably estimated (a point treated later), then the liability is a series of (estimated) nominal amounts falling in a regular pattern, beginning in 10 years' time and ending in 30 years' time.

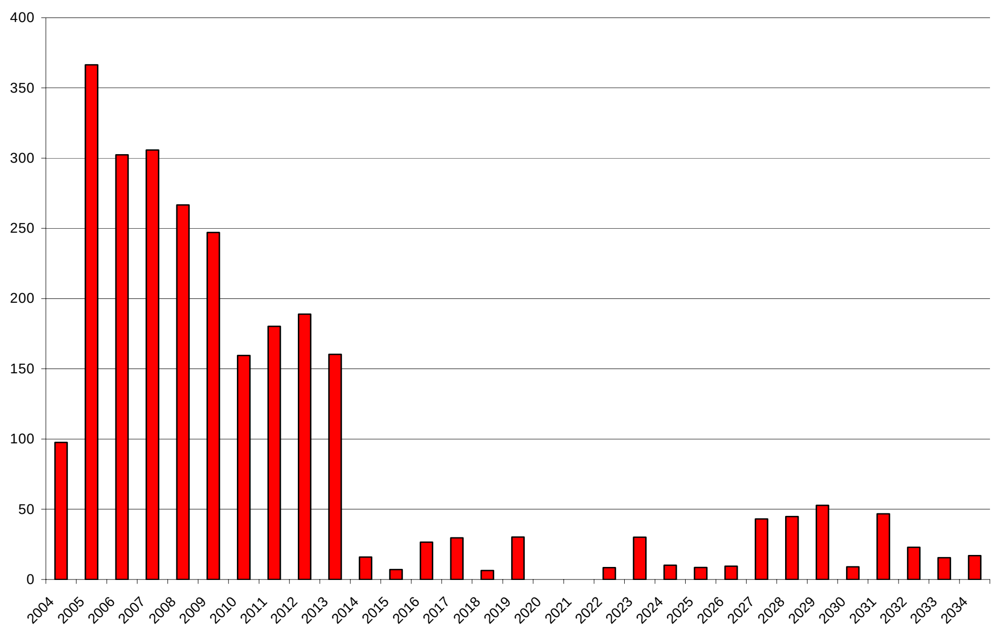
From our definition of investment variation and investment risk earlier, it is clear that to minimise the investment risk then we want to find an asset portfolio that provides an income that most closely matches this liability stream. Whether these liabilities are nominal or real in sterling, euro, dollar, there is arguably a sufficiently deep market in conventional and index-linked bonds so that a near perfect matching portfolio can be constructed.

The maturity profile of the sterling-denominated and euro-denominated sovereign debt markets is shown below.

Graph 1a: Outstanding Nominal Amount of Sterling-denominated conventional and index-linked (inflation-adjusted) Government bonds, by Calendar Year of Maturity, Stg£ Billions (as at end March 2004)



Graph 1b: Outstanding Nominal Amount of Euro-denominated Government bonds over 1 year, by Calendar Year of Maturity, € Billions (as at September 2003)



The graph thus indicates that a pattern of fixed amounts falling due anywhere within the next three decades can adequately be matched by euro-denominated bonds, especially now that that many such bond issues are strippable.[16] Similar remarks hold for sterling and dollar bond markets. It follows that we can identify a bond portfolio matching the revalued preserved payments due to the 55 person, subject only to the extent that we can reliably estimate the rate of revaluation prior to vesting. This points to a predominantly bond-based portfolio to match the termination liabilities.

However, the benefits due to the same 55 year-old person, assuming the scheme continues and he stays in service to normal retirement age is a pension of half his salary at the time of retirement, the pension subject to a fixed rate of increase while in payment. In order to estimate the payments falling due after 10 years' time now requires us to estimate the person's wage increases over the next decade. This problem can be decomposed into estimating the general rate of inflation over the next decade and the real rate of wage increase. The latter might be estimated to a reasonable accuracy leaving us to allow for the rate of inflation over the next decade. The development of the index-linked bond markets over the last couple of decades allow for a portfolio to be constructed that match a pattern of such real payments in the UK, Eurobloc and US economies up to, coincidentally, 30 years into the future.

The above considerations have allowed us to identify in general terms the most closely matching portfolio to the stylised pension liabilities in our example. The portfolio mix depends on whether the termination liability or on-going benefits are targeted, but it still comprises only conventional bonds and eurozone index-linked securities. In particular, a role for equities has not been identified in the most closely matching portfolio as the proceeds from equities are not known in advance. If we generalise our example to consider pension payments linked to inflation, or persons aged over 55, then clearly similar arguments above apply and we again identify portfolios consisting of just bonds (conventional and index-linked) to be the closest matching portfolio to the liabilities.

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[16] Stripping means trading each coupon or principal payment of the bond as a separate asset – each a bullet bond. The sovereign euro bonds are generally strippable, with France issuing such bonds since 1991, Germany since 1997, followed by many others (including Ireland) in more recent years.

So the liability reference portfolio for members over the age of 55 with benefits subject to fixed or inflation-linked escalation can be estimated by the methods above.[17]

For persons younger than 55, there is no sovereign guaranteed security matching payments falling due after about three decades in the major economies, whether nominal or real. However the market allows us to provide a nominal amount or inflation-linked amount in three decades' time and this can be used as a stepping-stone to provide payments falling due after the three decades. Applying this logic entails that solving for the most closely matching asset for nominal or index-linked liabilities after 30 years is perhaps best done by extrapolating the yield curve beyond the present cut-off and price on the basis that longer dated securities at the extrapolated yield exist.

The investment strategy to allow for these very distant payments would be to invest the estimated amount in the longest dated bonds. Of course, extrapolation of the yield curve introduces another risk, the risk related to the extent of the extrapolation. However, if the weight of the liabilities of a scheme falls due within the next three decades[18] then this extrapolation technique will produce an acceptable error as a proportion of the total liability. A key question is how much investment variation is increased with the extrapolation technique and the associated investment strategy proposed above. When the liabilities are linked to inflation then we cannot, unfortunately, reliably back-test how well the extrapolation method proposed above would have worked as index-linked stocks have only been in issue since 1981 in UK, since 1997 in US, since 1998 in France. We can, though, assess the extrapolation technique when the liabilities are fixed in monetary amount. To make this assessment, we derived the empirical investment variation associated with different investment strategies over the last century in the following case studies.

---

[17] We would not like to give the impression that is always straightforward. It can be non-trivial to estimate the liability reference portfolio of some pension payments, particularly those expressed as the lower of two amounts. Much work remains to be done in this area.

[18] This is often the case with defined benefit schemes as the liability increases, other things being equal, with the greater the age of the member, the longer the past service and the higher the salary. However, the extent to which it holds true is dependent on the maturity of the scheme.

Case Studies: Evaluating the extrapolation technique to members under 55 years of age

Case Study 1: A 40 year old is promised a non-escalating pension from age 65 of a fraction of his salary at the time of retirement. Let us further assume that the person will die on his 85th birthday. Assume that the termination liability is the pension amount based on his current salary to be revalued by the lesser of inflation or 4% in any year, up to vesting at age 65. From the above discussion, we shall take the valuation rate of interest equal to the gross redemption yield on the 30 year bullet bond, and the rate of escalation of the benefit pre-retirement is assumed to be 2½% (this is not a material assumption, as discussed later). Finally, we assume at time 0 that the valuation result is that the value of the assets, assessed at market value, is identical to the (discounted) value of the liabilities. We wish to calculate the investment variation when the investment strategy is to invest totally in either (a) the UK equity market, (b) a conventional 20 year bond, (c) a bullet (or stripped) bond with a single payment in 30 years, or (d) short-term cash. The period between valuations is taken to be a calendar year (i.e., $p = 1$). Returns and yields for the UK market were sourced as follows: 20 year gilt yields and returns and also cash returns were sourced from Baclays Capital Equity Gilt Study 2003 for the period after 1962, prior to that yields at the year end and interest rates during the year were sourced from Mitchell (1988) and the return on a notional 20 year bond and cash calculated as outlined in Whelan (2004). The annual UK equity market returns were sourced from Dimson *et al.* (2004).

The liabilities are the termination liabilities under the Pensions Acts, assuming the scheme was wound up at time 0. The rate of escalation in the year following the valuation was assumed to be 2½% (although, as noted later, this assumption is not material to the result). From before, we know that the investment variation is the present value of the extent to which the increase in the value of the assets exceed the increase in the liabilities over the year, the rate of discount (or inter-valuation rate of interest) being the rate at which the liabilities increased over the year. In the example, the inter-valuation rate of return, i_v, is given by:

$$i_v = \frac{\frac{(1.025)^{65-41}}{(1+i_1)^{65-41}}(1.025)(Pen)\bar{a}_{\overline{20|}}^{@i_1}}{\frac{(1.025)^{65-40}}{(1+i_0)^{65-40}}(Pen)\bar{a}_{\overline{20|}}^{@i_0}} - 1$$

where,

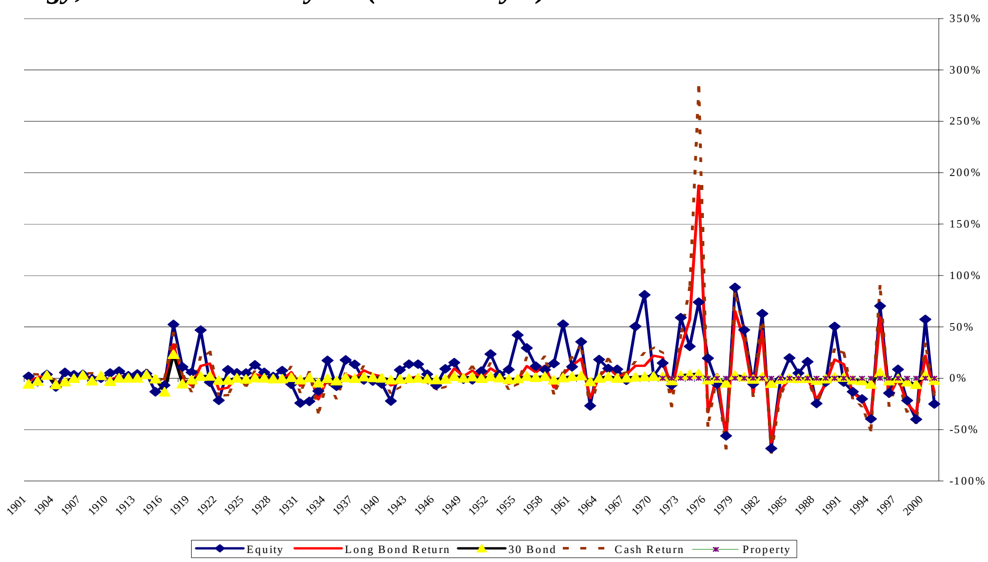
i_j is the valuation rate of interest at time j (that is, the gross redemption yield on the 30 year bullet bond)

Pen is the pension on termination at time 0, payable from age 65.

The inter-valuation rate of interest can be seen as the hurdle rate of return that assets must exceed to show a positive investment variation over the year.

Using the historic statistics of the UK capital markets, we investigated over each calendar year in the 20th century the investment variation for the 40 year old, assuming the assets to be invested in different asset classes. We assumed that the yield on the 30 year bullet bond was the same as the yield on the long bond. The result shows the investment variation in each calendar year for each investment strategy, standardized by dividing the investment variation by the value of the liability at time 0.

Graph 2: Standardised Investment Variation for 40 Year Old for each Investment Strategy, in each calendar year (Case Study 1)



The graph tends to be dominated by the large positive investment variation posted by many mismatching investment strategies over the 1970s and early 1980s (the first and second oil shocks which markedly raised inflation which, in turn, caused bond yields to rise). Apparent from the graph, though, is that the spread of the empirical distribution appears non-stationary – that is, the spread evolves in time.[19] The implication for those attempting to forecast the distribution of the investment variation for each asset class is that it is difficult and past experience is only a loose guide to the future experience.

Table 1 sets out summary statistics to describe the key features of the empirical investment variation, with figures calculated for the whole 20th century, the second half of the 20th century, and those reflecting the experience since 1970.

Table 1: 40 Year Old: Summary Statistics of the Empirical Investment Variation Distribution, over 20th Century, the second half of the 20th century, and 1970-2000

	Based on an Investment Strategy of 100% in…			
	Equity	Long Bond	30 Year Bullet Bond	Cash
20th Century				
Mean	8.0%	2.9%	0.0%	4.9%
Median	5.0%	0.9%	0.2%	1.5%
Geometric Mean	4.6%	0.2%	-0.1%	-0.1%
Stan. Dev.	26.7%	26.5%	3.5%	38.2%
Skew	0.6	3.4	2.3	4.0
Excess Kurtosis	1.5	23.6	20.0	28.0
Since 1950				
Mean	13.0%	5.5%	0.1%	9.4%
Median	8.8%	1.9%	0.3%	2.0%
Geometric Mean	7.2%	0.3%	0.1%	0.1%
Stan. Dev.	34.4%	36.5%	2.5%	52.3%
Skew	0.1	2.5	-0.4	2.9
Excess Kurtosis	-0.1	11.7	0.3	14.4
Since 1970				
Mean	8.9%	5.5%	-0.3%	10.2%
Median	-1.0%	-2.5%	-0.2%	-1.6%
Geometric Mean	1.2%	-2.5%	-0.3%	-4.1%
Stan. Dev.	39.6%	46.5%	2.9%	66.4%
Skew	0.2	2.0	0.0	2.4
Excess Kurtosis	-0.6	6.9	-0.3	8.8

[19] This is not surprising as there is considerable evidence that returns from capital markets form a non-stationary time series (e.g., Loretan & Phillips (1994)), even in the weak sense.

We can make the following remarks:

(1) The 30-year bullet bond is the closest match to the liability (of those tested) as the investment variation has lowest standard deviation for this asset type. Hence the 30-year bullet bond is close to the Liability Reference Portfolio. Equities and long bonds have roughly equal risk, while cash is higher again.

(2) Note in particular that a 20 year conventional bond (which, of course, has a weighted average duration much lower than 20 years) is a term mismatch from the 30 year bullet bond (which has a weighted duration of 30 years), and on the historic simulation above, this term mismatch has introduced almost as much risk as equity investment.[20]

(3) While the figures change whether one looks at the last 30, 50 years or the last 100 years, the relative ordering of the different asset classes in terms of risk (or reward) are largely unaltered. The estimated investment risk is very high and dependent on the sample period for equities, bonds, and cash. This points to the need for considerable judgement in estimating the future risk of the different classes – history is only a partial guide.

(4) One of the assumptions in calculating the figures in the table above was that inflation subject to a cap of 4% over the year following the valuation was 2½%. The upper limit of possible outcomes is 4% that, if applied, would deduct about 1½% from the mean, median, and geometric mean figures above and leave all the other figures unaffected. This shows that the results of our analysis are not particularly sensitive to estimating this figure, once deflation of any severity is considered unlikely.

(5) The skew of the investment variation has been non-negative, which ensured that the mean exceeds the median. The geometric mean of the data, which corresponds to the annualised rate over the period, is arguably the more relevant average for actuarial investigations. The above table shows us that, historically, investing in the most closely matching asset of those studied (the 30 Year Bullet Bond) has involved a sacrifice to return only in the case of equity investment.

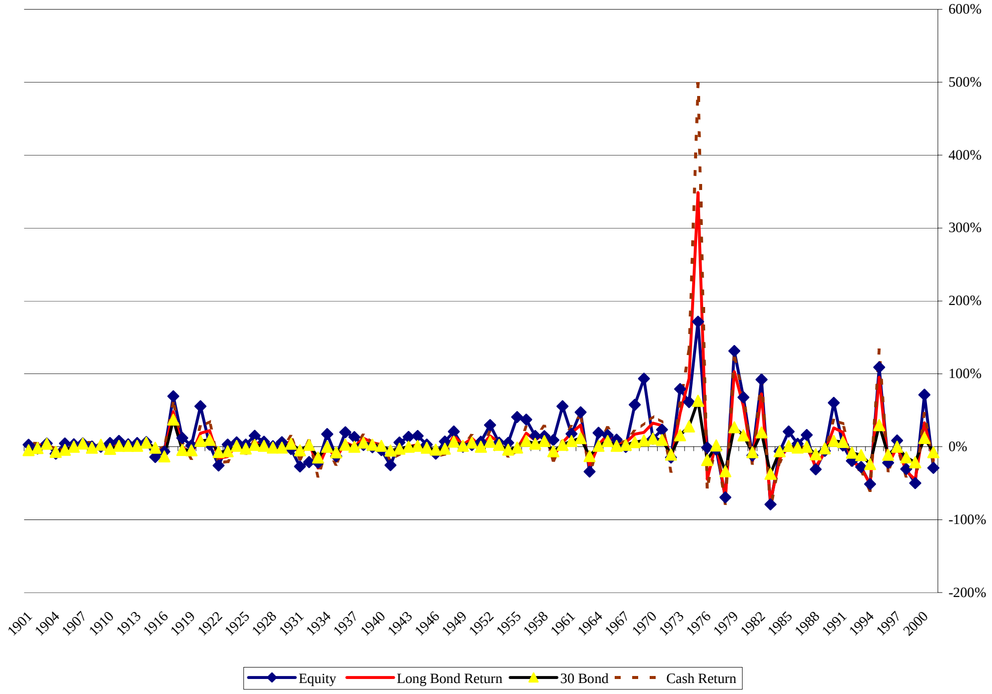
[20] Note that the returns from the long bond and the 30-year bullet bond are highly correlated but the variability of the former is much lower than the latter, which leads the mismatch.

(6) Note that there is no simple relationship between risk and return, as cash and the long bond exhibit higher risk and lower return than the matching asset. This immediately entails that, in actuarial applications, there is not necessarily a compensation for assuming extra risk. Accordingly, actuarial advice can add real value by identifying the idiosyncratic risk of the scheme (that is the deviation with respect to the liability reference portfolio) and exploiting the uniqueness of this risk measure to increase return without increasing risk.

Modelling 30 Year Old Member

We can apply the very same model above but now to a 30 year old. The results are as follows, in graphical and tabular form.

Graph 3: Investment Variation for 30 Year Old for each Investment Strategy, in each calendar year (Case Study 1)



Note the 30 year bullet bond – the longest on the market – is not long enough to match the liability so we witness investment variation arising from the term mismatch. The fluctuations in investment variation for the 30 year bullet bond tend, as is apparent from the table overleaf, to be lower than that of the other asset classes.

Table 2: 30 Year Old: Summary Statistics of the Empirical Investment Variation Distribution, over 20th Century, second half of the 20th century, and 1970-2000 (inclusive), Case 1

	Based on an Investment Strategy of 100% in…			
	Equity	Long Bond	30 Year Bullet Bond	Cash
20th Century				
Mean	10.8%	6.2%	0.9%	9.3%
Median	4.7%	1.5%	1.1%	2.5%
Geometric Mean	4.8%	0.4%	0.1%	0.1%
Stan. Dev.	37.3%	44.0%	12.6%	60.4%
Skew	1.4	4.9	1.1	5.6
Excess Kurtosis	4.1	36.8	6.8	43.2
Since 1950				
Mean	18.1%	11.5%	1.6%	17.5%
Median	9.8%	2.9%	1.3%	3.8%
Geometric Mean	7.5%	0.5%	0.3%	0.3%
Stan. Dev.	49.0%	60.8%	16.2%	83.2%
Skew	0.8	3.5	0.7	4.0
Excess Kurtosis	1.2	18.7	3.6	22.3
Since 1970				
Mean	15.0%	13.6%	0.4%	21.4%
Median	-1.1%	-3.4%	-1.1%	-2.5%
Geometric Mean	0.0%	-3.6%	-1.5%	-5.2%
Stan. Dev.	59.4%	77.6%	20.3%	106.0%
Skew	0.8	2.8	0.8	3.2
Excess Kurtosis	0.4	11.2	1.9	13.6

We note in the historic study above that equities appear better than 20 year conventional bonds as the risk is lower but the reward is higher. As one would have expected from the earlier discussion, the risk of all asset types studied in meeting the termination liability is increased when compared with that of the 40 year old.

The case studies to date have treated the termination liabilities on the assumption that the scheme is terminated at the valuation date. This then allowed us to consider the distribution of the investment variation over a year if different investment strategies were used. However, if the scheme remains open, then under the assumptions in our case study, the liability will increase by

(a) the excess of the increase in salary over the increase in pension in deferment,

(b) the increase in pensionable service,

(c) other factors capturing how the unfolding experience differs from the other financial and demographic assumptions used to estimate the liabilities (e.g., new entrants).

In practice, of course, almost all schemes will continue so, arguably, the investment strategy that is best adopted is not the one that best matches the termination liabilities at one instant in the past but the one that best matches the increase in the termination liabilities assuming the scheme is not wound up.

We investigated each of the investment strategies previously under this new scenario. In order to do so we needed to make the following assumptions

(ii) the wage increase in any calendar year is 2% above inflation for that year. Thus the rate of increase of the termination liabilities assuming the scheme is not terminated is (1 + wage increase) /(1 plus the lower of 4% or the rate of inflation over the year) times the rate of increase of the termination liabilities assuming it is terminated, all other things being equal.

(iii) The increasing pensionable service can be accurately allowed for in advance as it deterministic. This creates a factor (greater than unity) that multiplies the liability factor on scheme termination. We ignore this factor as it varies from scheme to scheme and can be estimated in advance.

(iv) The experience of the scheme is in line with that assumed in calculating the termination liabilities in all other matters (e.g., no new entrants).

Note the similarity between the approach above and the on-going funding plan known as the ‘defined accrued benefit method’ described and discussed in McLeish & Steward (1987).

We can redo the previous analysis with these new assumptions, which we term Case Study 2.

Graph 4: Investment Variation for 40 Year Old for each Investment Strategy, in each calendar year (Case Study 2)

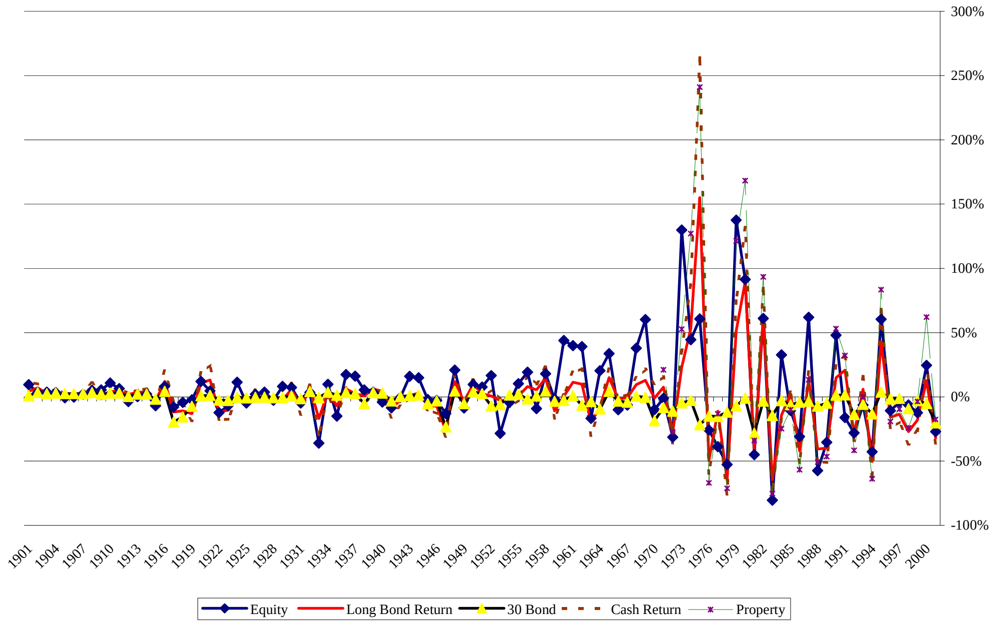


Table 3: 40 Year Old: Statistics of the Empirical Investment Variation Distribution, over 20th Century and since 1970 (inclusive), Case Study 2

	Based on an Investment Strategy of 100% in…			
	Equity	Long Bond	30 Year Bullet Bond	Cash
20th Century				
Mean	4.3%	-0.7%	-3.7%	1.3%
Median	4.3%	-0.8%	-2.6%	-0.5%
Geometric Mean	0.8%	-3.8%	-4.0%	-4.3%
Stan. Dev.	26.1%	25.7%	7.4%	37.0%
Skew	0.4	2.7	-0.9	3.7
Excess Kurtosis	1.5	19.3	5.2	25.8
Since 1950				
Mean	13.0%	5.5%	0.1%	9.4%
Median	8.8%	1.9%	0.3%	2.0%
Geometric Mean	7.2%	0.3%	0.1%	0.1%
Stan. Dev.	34.4%	36.5%	2.5%	52.3%
Skew	0.1	2.5	-0.4	2.9
Excess Kurtosis	-0.1	11.7	0.3	14.4
Since 1970				
Mean	1.8%	-1.5%	-7.4%	3.2%
Median	-5.7%	-10.3%	-5.7%	-10.4%
Geometric Mean	-6.1%	-10.2%	-7.5%	-12.5%
Stan. Dev.	38.4%	44.9%	5.6%	64.7%
Skew	0.3	1.8	-1.1	2.3
Excess Kurtosis	-0.4	6.0	2.4	8.1

The 30 year bullet bond is still found, of the assert strategies assessed above, to be closest to the Liability Reference Portfolio, and the ranking of the other asset classes in terms of risk remains the same as the first case study (in fact the figures for risk are of the same order of magnitude as those earlier). The means and other measures of the central location of the distribution of the standardised investment variation are altered significantly (as could be expected) but, again, the relative ranking is very similar to the result of Case Study 1.

Perhaps the surprise in the results presented above is that equities do not fare better in the risk comparisons, as equities, if a good inflation hedge, could have been expected to match liabilities increasing in line with wage inflation (which, is closely related to inflation). The hypothesis that there is a positive relationship between inflation and nominal return on stocks (so that they both move up and down together) is generally known as the Fisher Hypothesis, after the mathematical economist, Irving Fisher. While there is mixed evidence that the Fisher hypothesis holds for Irish and UK equities, it has been shown not to hold in equities markets generally. In particular, equities have not demonstrated themselves an inflation hedge in the US and the major euro equity markets, although there is some evidence to support a weak link in the UK econony.[21] In short, no consistent positive relationship is evident between equity returns and inflation in most economies.

We also did the same analysis for the 30 year old and present the results in the following table.

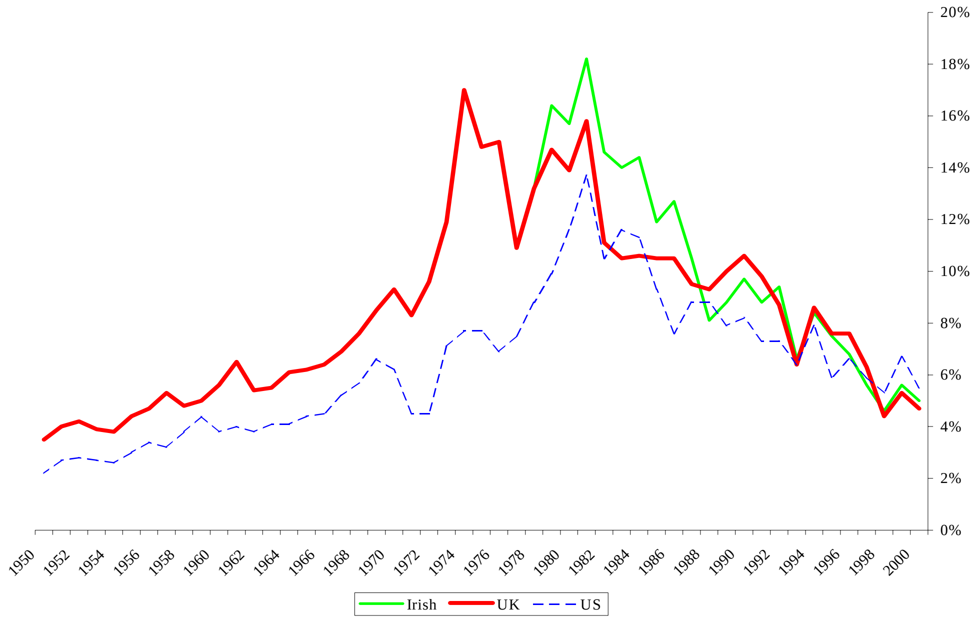
[21] For international evidence across 26 equity markets capturing more than 60% of the capitalisation of all equities in the world over the period 1947-1979, see Gultekin (1983*)*. For Ireland, see the working paper by Ryan (2002).

Table 4: 30 Year Old: Summary Statistics of the Empirical Investment Variation Distribution, over 20th Century, second half of 20th century and from 1970-2000 (inclusive), Case Study 2

	Based on an Investment Strategy of 100% in…			
	Equity	Long Bond	30 Year Bullet Bond	Cash
20th Century				
Mean	7.1%	2.6%	-2.8%	5.7%
Median	3.9%	0.1%	-1.0%	0.3%
Geometric Mean	0.8%	-3.8%	-3.7%	-4.5%
Stan. Dev.	36.3%	42.8%	13.0%	58.9%
Skew	1.1	4.6	-0.4	5.4
Excess Kurtosis	3.4	34.8	3.3	42.4
Since 1950				
Mean	12.5%	5.9%	-4.0%	11.9%
Median	6.7%	-1.0%	-2.7%	0.5%
Geometric Mean	1.0%	-6.1%	-5.4%	-7.2%
Stan. Dev.	48.1%	59.3%	15.8%	81.6%
Skew	0.7	3.4	-0.1	3.9
Excess Kurtosis	0.8	17.7	2.2	21.6
Since 1970				
Mean	7.9%	6.5%	-6.6%	14.3%
Median	-10.9%	-8.5%	-8.3%	-10.5%
Geometric Mean	-7.9%	-12.1%	-8.7%	-15.0%
Stan. Dev.	57.9%	75.8%	19.5%	104.1%
Skew	0.8	2.7	0.3	3.2
Excess Kurtosis	0.2	10.7	0.9	13.2

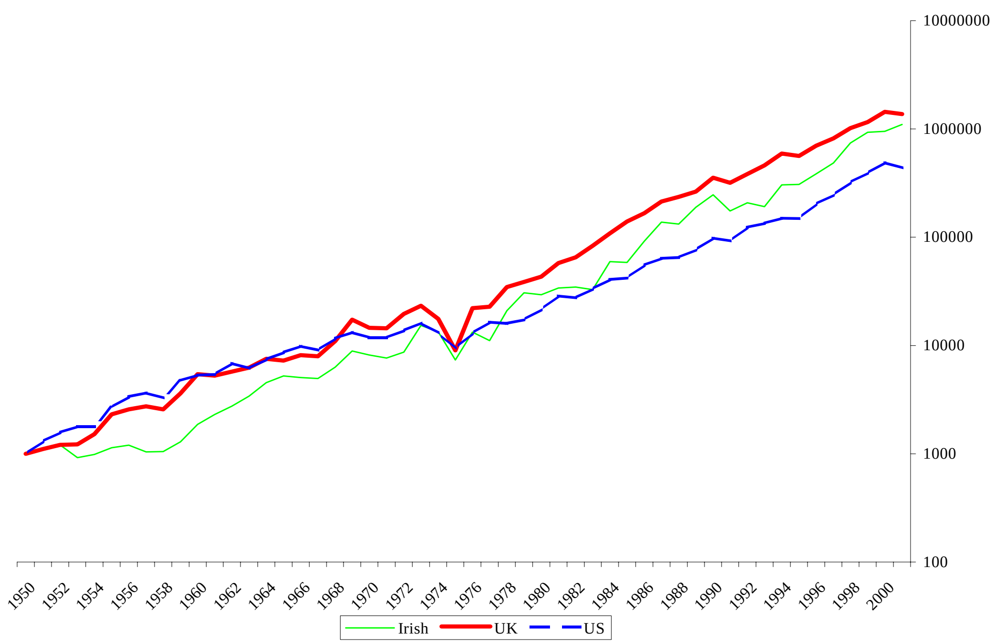
The above case studies used only historic UK figures. We did the analysis again using historic figures from the US and Irish capital markets to ensure our findings are robust across markets. The sources of data for US markets was Barclays Capital Equity Gilt Study 2003 while that for Ireland is based on the dataset fully described in Whelan (2004). First, we graph the evolution of the gross redemption yield on 20 year sovereign bonds in each economy from 1970 to 2000.

Graph 5: Long Bond Gross Redemption Yield, US, UK and Ireland, Year Ends, 1950-2000 (inclusive).



The performance of the equity markets over the same period is graphed below.

Graph 6: Equity Market Total Return Indices, US, UK, and Ireland, Year Ends, 1950-2000 (Log Scale)



Finally, under Case Study 1 above we redid the analysis for the 40 year old but this time assuming the experience of the US and Irish markets from 1950 to 2000. To aid comparison, we show the figures from the UK experience alongside.

Table 5: 40 Year Old: Summary Statistics of the Empirical Investment Variation Distribution, 1950-2000 (inclusive), US, UK and Irish Experiences, Case Study 1

	Based on Investment Strategy of 100%…			
	Equity	Long Bond	30 Year Bullet Bond	Cash
US Market				
Mean	13.6%	4.2%	0.3%	6.2%
Median	11.7%	1.7%	0.4%	2.8%
Geometric Mean	8.3%	1.5%	0.2%	1.1%
Stan. Dev.	34.1%	24.7%	2.5%	33.7%
Skew	0.2	1.2	0.1	0.9
Excess Kurtosis	-0.5	4.4	0.9	2.3
UK Market (from Table 1 earlier)				
Mean	13.0%	5.5%	0.1%	9.4%
Median	8.8%	1.9%	0.3%	2.0%
Geometric Mean	7.2%	0.3%	0.1%	0.1%
Stan. Dev.	34.4%	36.5%	2.5%	52.3%
Skew	0.1	2.5	-0.4	2.9
Excess Kurtosis	-0.1	11.7	0.3	14.4
Irish Market				
Mean	14.6%	6.1%	0.1%	11.2%
Median	0.6%	4.0%	0.6%	5.1%
Geometric Mean	6.7%	0.4%	0.1%	0.3%
Stan. Dev.	44.1%	38.6%	2.4%	57.0%
Skew	1.0	2.0	-0.5	2.6
Excess Kurtosis	1.3	7.7	-0.1	10.8

The tables above bear out the lessons from UK capital markets.

Time Diversification of Risk Argument

The previous analysis has compared the actual investment experience with that expected over periods of one year and, for that analysis, reported descriptive statistics for the empirical distribution of the investment variation. A natural question is whether the implications of our empirical investigation would significantly alter if the time period over which the distribution of the empirical variation was assessed increased from one to three or more years. In particular, some have advanced the argument that equity investment is preferable in the long-term but not necessarily the short-term, so if our review period was *p* years, where *p* is a 'large' number, then the equity investment strategy would have better risk and reward characteristics.

The problem in testing this hypothesis empirically is that we have a limited history of capitalism so that as *p* increases the number of non-overlapping intervals quickly decreases. We have only 10 distinct non-overlapping decades in the 20th century, which would give just 10 data-points in the empirical distribution.[22] However, we can resolve the problem with a simple model of the investment variation distribution. We treat one model below but note that the insight it gives applies to a very wide category of models.

The empirical distribution given in the tables earlier was the standardised investment variation over one year, or equivalently, the distribution of the percentage change in the funding level (on termination). Let Y be a random variable with this distribution. Then the funding level at time 1 (F_1), given it was 100% funded at time 0 is

$$F_1 = 100(1+Y)$$

A simple model for the funding level at time p (F_p) is

$$F_p = 100(1+Y_1)(1+Y_2)...(1+Y_p)$$

where each Y_i is independent of the others and has same distribution as Y. Now,

$$\ln F_p = \ln 100 + \ln(1+Y_1) + ... + \ln(1+Y_p)$$

[22] Worse, we know that the underlying distribution has changed over this time period (that is, is non-stationary over such a long period) thus making such a sample of questionable value to forecast the future.

Let us further assume $\ln(1+Y)$ is normally distributed with mean μ and variance σ^2. Then $\ln F_p$ is normally distributed and F_p is lognormally distributed. Then, from the well-known parameterisation of the lognormal, we can write

$$E[Y] = e^{\mu+\frac{1}{2}\sigma^2} - 1 \tag{I}$$

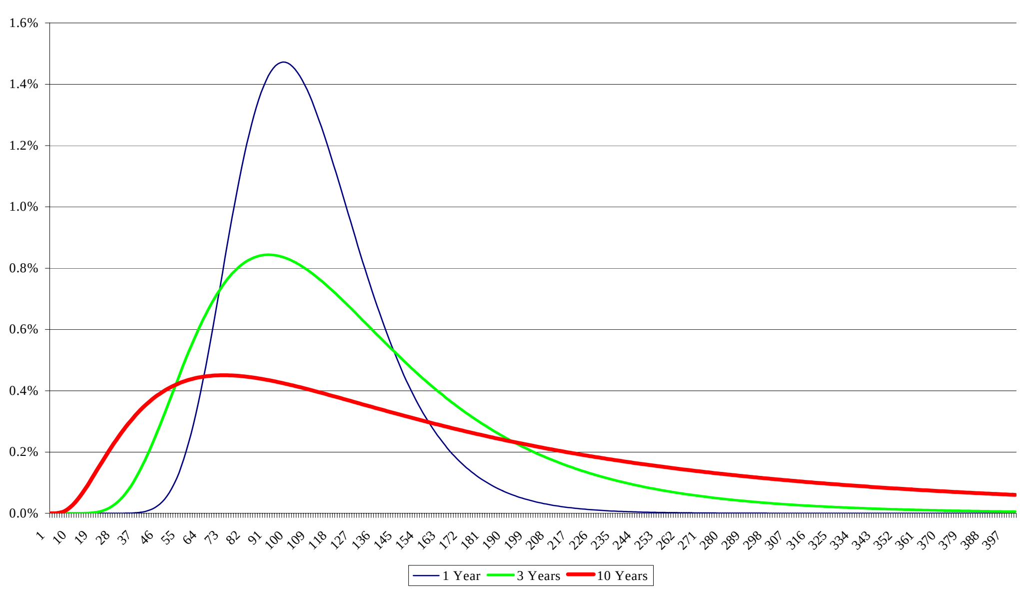
and

$$Var[Y] = e^{2\mu+2\sigma^2} - e^{2\mu+\sigma^2} \tag{II}$$

We have already estimated $E[Y]$ and $Var[Y]$ and so can solve the above equations for μ and σ^2. In particular, we can solve for the distribution of F_p.

The distribution of the standardised investment variation over one year has been approximated by our empirical study earlier and we might assume, for concreteness, that it has a mean of 8% and a standard deviation of 30%. Solving (I) and (II) above gives μ=0.04 and σ=0.27. The density function of the funding level at time *p*, where *p=1*, *p=3* and *p=10*, is graphed below:

Graph 7: Probability Density Function of Funding Level, when Viewed at end of 1,3 and 10 years, assuming Log-Normal Distribution (see above)



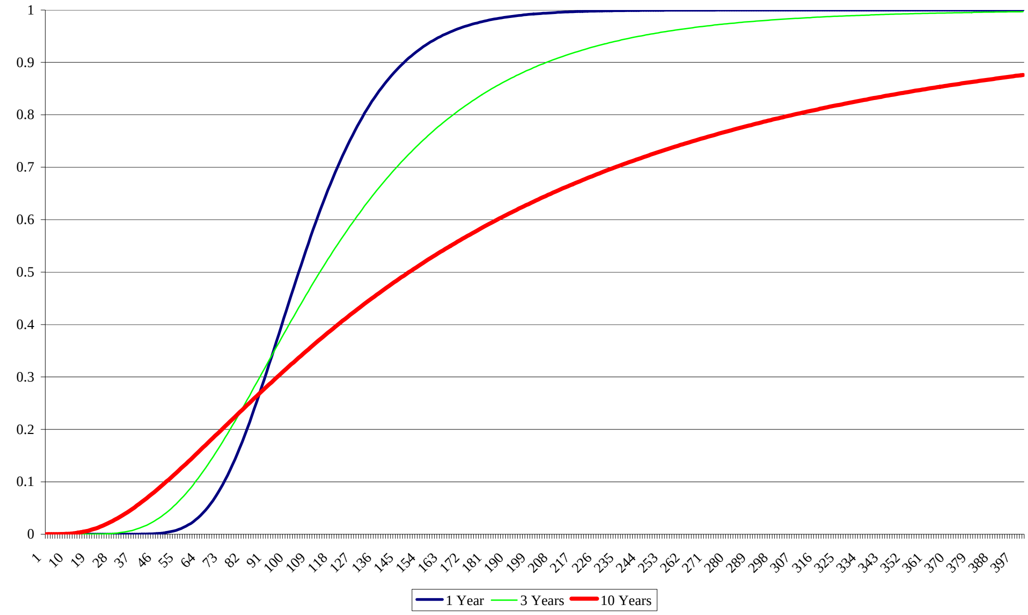
We note that the distribution of possible outcomes is wider when the review term increases ('the expanding funnel of doubt') and, in particular, that the probability of a very low funding level is higher the greater the period between reviews. From the above graph of the funding levels, a rational investor need not necessarily favour the outcome when p=10 (or, more generally, when p is large) over the outcome when p=1. When p=10, the expected value is increased but so too is the probability of an extremely poor outcome. A particularly risk averse investor could conceivably prefer the outcome when p=1 over when p=10.

We can investigate the above remarks in a more formal setting. Given two distributions, the condition that

$$F_1(x) \leq F_2(x), \forall x$$

is described as the first order stochastic dominance (FSD) of $F_1(x)$ over $F_2(x)$, where the $F_i(x)$ are distribution functions. A return distribution that first order dominates another is preferred by any wealth maximiser regardless of their utility function. The distribution functions of the funding levels for each forecast period are graphed below.

Graph 8: Cumulative Distribution Function of Funding Level, when Viewed at end of 1,3 and 10 years, assuming Log-Normal Distribution



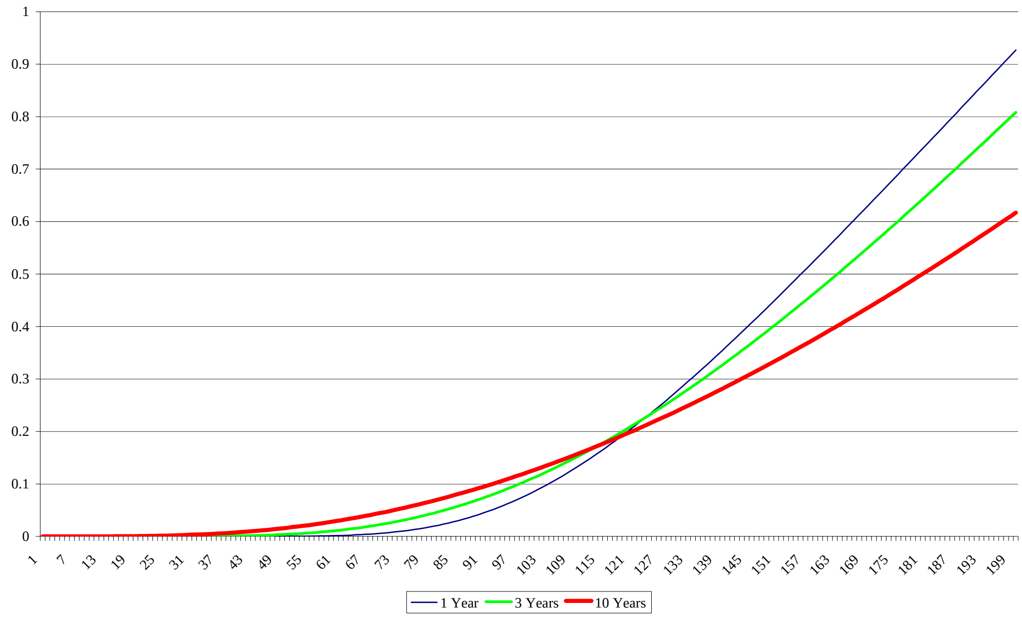
So, clearly, no distribution for any p stochastically dominates any of the others.

A less stringent condition in comparing two distributions is second order stochastic dominance (SSD), with $F_1(x)$ said to dominate $F_3(x)$ by SSD if and only if

$$\int_{-\infty}^{x} F_1(y)dy \leq \int_{-\infty}^{x} F_3(y)dy, \forall x$$

It can be shown that investors who are both nonsatiated and risk averse can be shown to prefer the payoff of $F_1(x)$ over $F_3(x)$.[23] Again, under our model earlier, we can show that no distribution for any p stochastically dominates to second order any of the others. Graph 10, capturing the area under the distribution functions up to the 400% funding level, demonstrates this.

Graph 9: Area under Cumulative Distribution Function of Funding Level ($\int_{-\infty}^{x} F_p(y)dy$), when p= 1,3 and 10 years, assuming Log-Normal Distribution



[23] See, for instance, Eichberger & Harper (1997).

Conclusion

The above argument, though different in detail, leads to a conclusion very similar to that advocated in Speed, Bowie *et al.* (2003) – that the most closely matching asset for pension fund liabilities is composed mainly of conventional and index-linked bonds.[24] It also makes clear that there is generally no simple matching asset for pension fund liabilities and some judgement must be used in identifying the closest matching portfolio. We note, in particular, that the above argument leads to a portfolio that, if history is any guide, has a lower expected long term return than a predominantly equity portfolio.

Note, in particular, that our foregoing analysis does not allow us to suggest one investment strategy is preferable to another. Investors, including pension funds, are routinely tempted to take risks if the reward (that is, the form of the investment variation distribution) is judged sufficiently tempting. However, pension funds should appreciate the risks involved in alternative strategies and, at a minimum seek to ensure that the investment portfolio is efficient in the sense that risk cannot be diminished without diminishing reward. To appreciate the risks and ensure that all risks undertaken are reasonably rewarded requires knowledge of the investment variation distribution and, in particular, the Liability Reference Portfolio.

[24] Our different arguments hopefully overcome the objections in Hill (2003) to the conclusions in Speed, Bowie *et al*. (2003).

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Appendix I: Limitations of Proposed Definition of Investment Variation (and the associated Investment Risk)

The above definition of investment variation in actuarial valuations (and the associated investment risk) has some limitations. Many such limitations arise from the fact that we have ignored the important relationship between the finances of the sponsoring employer and the scheme finances. A full treatment of the problem would model, not just the distribution of the difference between the value of the assets and that of the liabilities at any point in time, but also the coincidence of risk between a shortfall being revealed at any future date and the ability (and, if possible to model, the willingness) of the sponsoring employer to fund the shortfall at that time. We do not treat this fuller formulation as it demands knowledge of the dynamics of the sponsor's business (and so is specific to the sponsor) and how this is related to the dynamics behind the valuation of assets and liabilities.

We can, however, make some general points on this limitation. First, as a hypothetical case, consider a pension fund with a high exposure to the business of the sponsoring employer. Such an investment strategy increases significantly the twin risk of a shortfall in the value of the assets over the liabilities just when the sponsoring employer is unable to make up the shortfall.[25] In fact, in this case, members might lose their jobs and part of their pension entitlements if the employer fails.[26] Now, in a less extreme case, the performance of the equity-based portfolio could be correlated to some degree with the fortunes of the sponsoring employer. Consider, for instance, the difficulties faced by a small company in the high-technology sector, sponsoring a pension fund over the couple of years since March 2000. Here, we have the same or at least similar underlying factors creating financial stress in the pension fund and also causing strains to the profitability of the sponsoring employer. This is an instance of a significant fall in the value of the portfolio occurring at an inopportune time for the employer – once again (but now with less certainty) adversely affecting the security of the members' pension entitlements just when those pension assets could be called

[25] For this reason, the Pensions Acts impose limits on the level of 'self-investment' (as this practice is called) allowed by approved pension schemes.
[26] The Pensions Act 1990 outlines a priority ranking of members' pension entitlements on the termination of the scheme. The Act sets out that members in receipt of a pension have first call on the assets, followed by members in active service. Accordingly, any shortfall in the value of assets to liabilities will be felt more acutely by active members.

upon.[27] The extent to which these points are material to any particular scheme and sponsoring employer depend, *inter alia*, on the relative surplus of the value of scheme assets over the value of its liabilities (as, other things being equal, the greater the relative surplus the less likely a deficit will be revealed) and the volatility of the employer's profits (as, other things being equal, the less volatile the employer's profits the less likely they will experience poor profits when the scheme is in deficit). (Arguably, a bond-based portfolio of suitable maturity profile ensures that the twin risks of a deficit revealed in the pension funds and, at the same time, the employer is particularly financially constrained are largely independent or perhaps even negatively correlated with one another.)

A case can perhaps be made that Irish pension funds to date have not fully exploited asset types or investment strategies that are uncorrelated or negatively correlated with the financial health of the sponsoring employer. Whelan (2002) treats the case of the National Pensions Reserve Fund, outlining an argument that the Fund should underweight its exposure to indigenous Irish industries and those sectors of the world equity market in which the Irish economy has already a high exposure (such as the pharmaceutical and technology sectors). In particular, pension funds could widen the search of asset types from the traditional categories to include others such as actively managed currency funds or hedge funds (see, for instance, Caslin (2002)) which have little correlation with either the other mismatched assets of pension funds or the financial strength of the employer and might reduce the standard deviation or thin the tail of the investment variation distribution.

The general point made in this subsection is that the very same portfolio could have quite different risk characteristics depending on the nature of the business of the sponsoring employer – account should properly be taken of the relationship between the value of the portfolio and the financial strength of the sponsoring employer.

[27] Indeed, with the new disclosures demanded of companies under the accounting standard FRS 17, a deficit revealed in the pension fund could precipitate a financial crisis for the employer (say, by reducing their credit rating) and, if the deficit was caused by a sudden collapse of equity values, this is likely just at a time when equity capital is expensive and difficult to raise.