GN47: Stochastic Modelling for Life Insurance Reserving and Capital Assessment

Classification
Recommended Practice

MEMBERS ARE REMINDED THAT THEY MUST ALWAYS COMPLY WITH THE PROFESSIONAL CONDUCT STANDARDS (PCS) AND THAT GUIDANCE NOTES IMPOSE ADDITIONAL REQUIREMENTS UNDER SPECIFIC CIRCUMSTANCES

Definitions
Defined terms appear in italics when used in the standard.

Reference	Definition
firm	The insurance company in respect of which stochastic modelling is being used in relation to reserving and capital assessment
FSA	Financial Services Authority
Individual Capital Assessment (“ICA”)	The assessment required by PRU 1.2.26R of the capital which a *firm* needs to hold to meet PRU 1.2.22R (adequate financial resources, including capital resources)
moneyness	the degree to which an option is in or out of the money
WPICC	With-profits Insurance Capital Component

The following terms have the same meaning as in the FSA Handbook of Rules and Guidance:

Long-term insurance business
Principles and Practices of Financial Management

Legislation or Authority
The Financial Services and Markets Act 2000
The FSA Handbook of Rules and Guidance: Integrated Prudential sourcebook (“PRU”)

Application
Life insurance firms using stochastic modelling when reserving for options and guarantees in life insurance policies or assessing the amount of capital required to support *long-term insurance business*.

Author
Life Board

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from which time it ceased to operate (and cannot, with effect from that time, be relied upon) as guidance issued by the Profession. Members needing to comply with professional standards on matters covered by this Guidance Note should in future consult the standards published by the Board for Actuarial Standards.

1. General

1.1 This Guidance Note is drafted in terms which are not addressed to actuaries specifically. Nevertheless, actuaries performing work covered by this Guidance Note are required to apply it according to its classification. However, where a firm requires an actuary to produce work conflicting with this Guidance Note, the actuary may do so provided the work clearly and unambiguously states that the actuary has done so under instructions and that the work does not conform to this Guidance Note.

1.2 If the development of stochastic modelling within a firm is such that one or more material aspects of this Guidance Note are not being complied with, the extent of non-compliance and the alternative adopted should be recorded in the report of the valuation or capital assessment to which it refers. There may be other practices not set out in this note that constitute generally accepted actuarial practice in this area and failure to comply with this note does not necessarily imply failure to follow generally accepted actuarial practice. It is recognised that stochastic modelling is a developing area of practice and firms will need to consider the extent to which plans should be put in place to continue development of stochastic modelling, with particular consideration being given to how all material aspects of this Guidance Note, or justified equivalent alternatives, could be met.

1.3 This guidance note provides guidance on the use of stochastic modelling in the context of PRU 7.3 (Mathematical Reserves), PRU 7.4 (With-profits Insurance Capital Component (WPICC)) and PRU 2.3 (Individual Capital Assessment (ICA)). It also includes some summarised references to, or quotations from, particular provisions of the *FSA* Handbook of Rules and Guidance (the "*FSA* Handbook"), but users should be aware that this is not exhaustive and does not provide a substitute for referring to the *FSA* Handbook.

1.4 It should be read in conjunction with GN44 – 46, which contain guidance on the circumstances in which the use of stochastic modelling is favoured by *FSA* and on some aspects of the way in which modelling should be applied if used.

1.5 Stochastic models are likely to be used in two distinct ways for the purposes covered by this standard. The first is to obtain a 'market-consistent' value of a liability ('market-consistent' is defined in 3.1 below). The second is to establish an amount of assets that will enable the *firm* to meet its liabilities to a desired probability level.

1.6 The types and/or parameterisation of stochastic model which it is appropriate to use may differ according to the purpose of the calculation (e.g. valuation or capital assessment) or the nature of the guarantee (e.g. minimum return from equity portfolio or guaranteed annuity rate).

2. Algorithms for Computing Market-Consistent Values

2.1 Given a market consistent asset model and a liability description, there may be several possible methods for computing the market consistent liability valuation. The possibilities include:

- closed form modelling,
- Monte Carlo simulation,
- other methods, for example:
  - (A) numerical integrations with respect to a density function,
  - (B) finite difference approximations to partial differential equations, for example on a binomial tree or a discrete lattice,
  - (C) exact or approximate application of transform methods, for example the use of complex integration of the relevant Fourier or Laplace transforms.

2.2 Guidance in the remainder of this note has been prepared in the context of the closed form modelling and Monte Carlo simulation methods above although some is of more general application (e.g. section 9).

2.3 Closed form Modelling

2.3.1 For certain options, it may be possible to use the formula underlying the model selected to derive market-consistent values or percentile values for capital purposes. However, it is necessary to ensure that the formula used reflects both management and policyholder actions, unless these are very limited in possible effect.

2.4 Monte Carlo Simulation

2.4.1 It is equally appropriate to use a risk-neutral probability measure, discounting at risk-free rates, or any other measure (including ‘real world’ measures), discounting using consistent deflators. It is appropriate either to generate independent equally likely simulations or to adopt variance reduction techniques in relation to the model or both.

2.4.2 The sampling errors involved in Monte Carlo simulation should be estimated, either using analytical formulas for standard errors, using increasing numbers of simulations until a number can be identified beyond which additional simulations add little additional accuracy to the valuation or in some other way. A sufficient number of simulations should then be used in the valuation so that the confidence interval is within acceptable levels of materiality. It is not necessary to use the upper bound of the confidence interval as the value.

2.4.3 The pseudo random number generator underlying the model(s) should have been tested to ensure that it produces numbers which display sufficient randomness.

3. ‘Market-Consistency’ in the context of PRU 7.4

3.1 PRU 7.4.169R(1) requires any stochastic approach used for valuing guarantees, options and smoothing when calculating WPICCs to be “market-consistent”. PRU 7.4.170R defines this as “… a model that delivers prices for assets and liabilities that can be directly verified from the market …” and require the model to be calibrated “… to deliver market-consistent prices for those assets that reflect the nature and term of the *with-profits insurance liabilities”*. Additional guidance is given in PRU 7.4.176-180G.

3.2 In the context of with-profits business, assets “reflecting the nature and term” of the liabilities would include those assets the return on which is used to determine policy payouts (i.e. those deemed to constitute the asset shares of the policies being valued). It would also include derivatives, particularly ‘European’ put options on the assets constituting, or reasonably close to those constituting, the asset share if policies contain guaranteed minimum maturity value’ and interest rate swaptions if guaranteed annuity rates are being valued (see also paragraph 5.2 below). The model used should also be capable of reproducing the prices of differently credit-rated stocks (other than those excluded by PRU7.4.87R), if such stocks form a material part of asset shares.

3.3 Except where the contrary is expressly stated in PRU, the basis for the valuation of assets and liabilities is set out in PRU 1.3.5R. In particular, this does not apply to calculation of the mathematical reserves. ‘Market-consistent’ values should be interpreted consistently with this rule. In particular, if the rule requires the use of bid or offer prices rather than mid-market prices, then input parameters or output values should be adjusted to produce the appropriate results, including full allowance for the significant spreads which market-makers may apply to large, infrequently traded over-the-counter instruments. Unless otherwise directed or implied by *FSA* Rules and guidance, it is not necessary to assume that the expected ‘close-out’ cost of an unhedged position after a very short term market shock will be subject to wider than normal price spreads.

3.4 The model used should be one that has been shown to reproduce option prices as at the valuation date sufficiently accurately. Option prices should be reproduced for a range of durations and strikes, and be based on underlying investments, appropriate to the terms and nature of the options or guarantees. It should also be assumed that the issuer is credit risk free (but see 3.5.3 below). Allowance for volatility ‘smile’ (variation of volatility with strike), skew or other aspects of the volatility surface should be made. However, in accordance with 1.2 above, approximate methods may be used. Any such approximations should be expected to be of overall neutral effect and their use should be disclosed in accordance with 1.2 above.

3.4.1 It should not be assumed that calibration to relatively simple traded options necessarily produces a model which reflects the market-consistent prices of more ‘exotic’ (e.g. path-dependent, ‘Bermudan’) options which may more closely represent the options embedded in some types of policy. Consideration should be given to making some checks against any prices which can be obtained for any more appropriate ‘exotic’ options.

3.4.2 Approximations might include using different parameterisations of a constant volatility model to value different model points (in which case the impact of aggregate level management actions needs to be allowed for) or using a single parameterisation calibrated to an appropriately weighted average of volatility with regard to smile, skew and term.

3.4.3 Alternatively, differently parameterised models could be used for different sets of liability model points, and in that case close replication would only be required for options corresponding to the model points to which the parameters are applied. Consistent modelling of management actions between model point sets is necessary.

3.5 In many situations, options may be infrequently traded, or price data may not be available for options of strike, term or credit quality corresponding to the liabilities. In this situation it is acceptable to calibrate a model to the longest available price data, or the closest available *moneyness*, or the nearest available credit quality of issuer. This parameterisation of the model may then be extrapolated to the term, *moneyness* or desired credit quality of the calibration.

3.5.1 Extrapolation should allow for the continuation of any observed trend and, unless there is no trend, unchanging parameters should not be assumed as term, *moneyness* or credit quality become more extreme. On occasions, it may be most appropriate to extrapolate along a curve with a turning point if justified by recent market price observation or underlying economic theory. Any choice between alternative parameters for extrapolation should be justified.

3.5.2 Enquiries should be made as to whether the longest quoted prices have themselves been extrapolated by market -makers rather than based on recent actual trades. If so, the adequacy of the extrapolation relative to the preceding guidance should be considered and adjusted if non-compliant.

3.6 Where the definition of the 'risk-free rate' parameter or 'risk-free curve' for the valuation of the insurance liabilities (see in particular paragraph 5.1.3 of GN45) differs from that implicit in the market price of otherwise relevant options, it may no longer be possible to demonstrate 'market consistency' by direct comparison between the observable market values of particular assets and the values generated for the same options by the liability valuation approach. In such cases a two stage approach to the demonstration of market consistency may be appropriate. In the first stage relatively simple closed form solutions may be parameterised to match the market value of observable options using a consistent discount rate, frequently the swap rate. These closed form solutions and the same parameters should then be reused with the discount rate adjusted to match the selected risk-free rate or curve to establish theoretical market values consistent with the definition of risk-free used in the valuation of the liabilities. These theoretical market values can then be used to validate the 'market consistency' of the liability valuation approach by confirming that the liability approach adequately reproduces those theoretical market values. Alternative approaches such as the calibration of two scenario files (one using a market practice based definition of the discount rate, such as swap rates, and the other maintaining all parameters, but replacing the discount rate with the selected risk-free definition) may be used, but regard should be had as to whether the level of transparency given by such approaches is sufficiently high.

3.7 IPRU(INS) Appendix 9.4A 6(4)(a)(iii) requires the completion of a table of values of specified assets as calculated by the stochastic model used for the purposes of PRU 7.4. This should be done using the same parameterisation of a closed-form model or the same set of simulations for a Monte Carlo approach used to value the liabilities, even though a model calibrated to a risk-free rate as defined in GN45 will not exactly reproduce market-observable prices for the specified assets.

4. Practical use of Market-Consistent Models in the calculation of the WPICC

4.1 Maturity Guarantees

4.1.1 For a policy under which the maturity benefit is the larger of a quantity related in some way to the value of underlying assets and a guaranteed amount (which may increase in future as bonuses are added), ‘assets that reflect the nature and term of the liabilities’ (PRU 7.4.170R) should include appropriate put or call options on the underlying assets.

4.1.2 To the extent that the underlying assets are equities which are invested broadly in line with a recognised index, the model used should normally replicate put option prices on that index at durations and strikes appropriate to the term of the guarantees. The model used should, however, reflect the receipt of dividends as well as capital growth and allow for the impact of tax at an appropriate rate.

4.1.3 To the extent that the assets are fixed interest securities, the approach used will depend upon the degree of matching of liabilities by term. If the fixed interest assets are invested to reproduce cash flows closely matched to those of the liabilities and asset shares are credited with return differentially by term, then little market risk will be present if liability cash flows are as expected. Exact matching cannot be achieved in both the base scenario and the persistency stress scenario and the exposure to market risk in respect of assets matching liabilities subject to persistency stress should be allowed for. For other assets, approximate methods may be appropriate for any residual risk; and allowance should still be made for credit risk.

4.1.4 However, if the fixed interest assets are pooled (i.e. an identical return is attributed to all policies independent of term), then stochastic modelling would be appropriate, using a model which is capable of reproducing swaption prices for a range of exercise dates corresponding to the range of policy guarantee dates and of tenors (lengths of underlying swaps) corresponding to the outstanding terms of the assets intended to be held at the guarantee date. Allowance should also be made for credit risk.

4.1.5 Where no established derivative market in properties exists, it is impossible to calibrate market-consistently, and the use of historical parameters for calibration may be a suitable alternative. A key parameter is volatility. The volatility observed from the progression over time of surveyors' valuations, which are typically used in property market indices, contains significant elements of smoothing relative to similar, although sparser, observations from realised sale prices. Volatility parameters derived from the historical movement of indices based on valuations should be adjusted to remove the effects of such implicit smoothing.

4.1.6 For any asset class, calibration should be adjusted to allow for increased volatility where there is bias relative to the index used (e.g. a territorial bias relative to an international index or a sector bias relative to a national index) or where there are individual large holdings.

4.1.7 If option prices are not available on a particular index at certain durations but are available on an index which may share some similar characteristics, an appropriately adopted parameterisation to that second index may be used.

4.1.8 In practice, models are likely to be used which simultaneously allow equity, fixed interest and property returns to vary stochastically. It is necessary in such circumstances to make assumptions about covariances between the different elements (this may include covariances between the equity markets and interest rates of different countries). It is unlikely that many current covariances can be deduced from quoted financial instruments; accordingly, where this is the case, appropriate historic averages should be taken into consideration, along with any relevant theoretical implications of the underlying economic model being used.

4.1.9 Policies such as with-profits bonds may provide policyholders with the option to encash policies at certain dates (e.g. 10^{th} anniversary) for a greater value than would apply at other times (e.g. without the application of a market value reduction). If there is one such date, it should be assumed that a high proportion of policies are encashed on that date if the option is more than trivially in the money at that date. If there is more than one such date, a proportion of policies should be assumed to be encashed at each, taking into account relevant past experience, if any, and the value of the option at each date. If there is an expectation that encashment rates might change in the future (e.g. because of increased communications to policyholders about the value of such options), appropriate allowance should be made. The effect of a valuable option on encashment rates on non-option dates close to the option dates should also be taken into account. See also paragraph 7.1.2.

4.1.10 Where regular amounts are being withdrawn from policies on favourable terms (e.g. without the application of a market value reduction), this should also be modelled unless it can be shown that to do so would not result in a materially different value of the options associated with the policy.

4.1.11 If it would make a material difference to the reserve calculated, models should allow for a proportion of policyholders dying in accordance with recent own or industry experience, allowing for the continuation of any trends. If enhanced, or reduced, benefits are payable on death, this should be reflected in the model.

4.2 Guaranteed Annuity Rates (GARs)

4.2.1 A portfolio of GARs have some similarities in form from an economic perspective with a portfolio of swaptions with a range of exercise dates, tenors and strike rates and with quantum equal to the value of the cash fund of the underlying policy on vesting. However, in most circumstances, the quantum depends upon persistency, take-up rate of pension at vesting, the then market values of the assets constituting the asset shares of the policies and the expected future progress of mortality rates.

4.2.2 It is therefore appropriate to calibrate stochastic models to interest rate swaptions. Account should be taken of the different profiles of the cash flows from a portfolio of life annuities and a portfolio of swaps.

4.2.3 The model used should be calibrated to reproduce swaption prices as closely as possible across as much as possible of the range of swaptions which replicate the liability portfolio. In particular, the greatest accuracy should be achieved at the exercise dates and tenors and strike rates which represent the majority of the liabilities by value, subject to the availability of reliable derivative prices. If reliable prices are not available for a material part of the liabilities (e.g. because the strike rates required are significantly different from those currently available), then adequacy of the model should be tested relative to the available prices and theoretical justification documented of the adequacy for the prices actually required.

4.2.4 It is normally necessary to model both the maturity benefit and the GAR simultaneously using appropriate correlations, although it may be possible to model each separately and combine the results using appropriate analytical techniques.

4.3 Risk Capital Margin

4.3.1 In the calculation of the *risk capital margin, it* is necessary to revalue liabilities in scenarios of changed prices.

4.3.2 PRU 7.4.50G states, amongst other things, that a *firm* using a stochastic approach should "… keep recalibration in the post-stress scenarios to the minimum required to reflect any change in the underlying risk-free yields". In particular, for the purposes of determining the RCM, it is appropriate to ignore the likelihood of increased volatility at times of large market movements.

4.3.3 It is also necessary to assume that any hedging assets (e.g. equity put options or interest rate swaptions) are revalued in line with the changed prices, ensuring that allowances for issuer credit risk are preserved (or, in the case of credit stress, appropriately adjusted).

4.4 Reserves for Smoothing etc

4.4.1 A reserve (or possibly an asset) in respect of smoothing is an element of the realistic balance sheet. This should normally be calculated stochastically.

4.4.2 *FSA* encourages in PRU 7.4.178G an holistic approach to stochastic modelling. Other items on the realistic balance sheet which may be calculated stochastically include inflation in expenses (because expenses may impact upon guarantee or option costs), profits or losses from early terminations, regular or terminal charges against or credits to asset share, misselling compensation and investment expenses.

4.4.3 The value of future profits from non-profits business should be calculated on a stochastic basis, in respect of market risk, if there is material exposure to unhedged guarantees or options inherent in the projected profit stream.

5. Use of Stochastic Models in Individual Capital Assessment

5.1 This section does not apply to the use of stochastic models for the calculation of market consistent values of liabilities for the purposes of PRU2.3.14G. See paragraph 4.11 of GN46.

5.2 Choice of Model

5.2.1 It is necessary to ensure that the probability distribution used can properly reproduce the more extreme historically observed behaviour of the variable being modelled. If a lognormal or other simple model cannot do this adequately, either a model exhibiting more appropriate skewness and kurtosis should be used or the simpler model should be used with adjusted parameters to provide sufficient outcomes in the relevant tail.

5.2.2 It should be recognised that there will be limited historical observations of the more extreme tail outcomes, even for the most common economic variables. A considerable degree of uncertainty will therefore exist in the behaviour of the tails of distributions. Extreme value theory may be of use in supporting a particular approach for some types of risk.

5.2.3 The probability level to be used should be selected (unless, of course, the model is being used to solve for the probability level supported by the capital available). A number of factors need to be considered in making such a determination, including:

- the financial strength which the *firm* wishes to demonstrate (i.e. its risk appetite);
- the period over which the assessment of the *firm's* ability to meet its liabilities is being made;
- published or private guidance from the *FSA*, including PRU 2.3.14G, where emphasis is given the one-year 99.5% probability level as the likely *FSA* intervention point for the issuance of individual capital guidance.

Further guidance is given in GN46.

5.2.4 The method used to combine distributions of different variables to arrive at a combined model to enable the determination of the amount of capital which satisfies the level derived in 5.2.3 is of key importance. For example, there may be a stronger observed or anticipated relationship between variables in more extreme stress scenarios. If this is the case, consideration should be given as to whether simple combination approaches (i.e. involving a fixed correlation) or assumptions of independence are adequate. Further guidance is given in GN46.

5.3 Parameterisation

5.3.1 To obtain 'real world' outcomes, it is generally appropriate to calibrate models with reference to actual historic parameters. Maximum Likelihood Estimation ("MLE") may be appropriate in some circumstances. However, particularly where fit to the tail of a distribution is more important than the overall fit, alternative techniques such as quantile matching may be more appropriate.

5.3.2 The length of the historic data may be limited by availability (e.g. the UK property market). In which case, all available data should be used. In other cases, data may be available going back much longer (e.g. UK gilt yields). An assessment should be made of the data available and the effect that different lengths of observation period would have. The selected parameter should also be consistent with the *firm*'s underlying future economic expectations.

5.3.3 Similarly, correlations between variables should be calculated over longer and shorter periods and the results compared. To the extent that there have been material changes in level between different time periods, correlations should be selected consistent with the *firm's* underlying future economic expectations.

5.3.4 The actuarial profession has derived the following table of maximum acceptable values of £1 invested free of tax in equities (UK and overseas, in sterling terms, combined), over various periods, with gross dividends reinvested, divided by the value of £1 invested free of tax in a risk-free asset over the same period ('excess returns') at various percentiles of the excess return. Models used should not produce equity excess returns higher than those contained in the table at the probability level selected in accordance with paragraph 5.2.3 above (or at that closest to it or those on either side of it if the level selected is not in the table) at 'relevant duration(s)'. The model should also not produce excess returns greater than the values in the table in more than 50% of the 'relevant entries'. The table does not necessarily apply to less well diversified portfolios (or portfolios with a material exposure to developing economies), where more extreme tail values may be appropriate.

Excess Total Equity Returns				
	0.5th	***2.5th***	***5th***	***10th***
1 yrs	**0.57**	**0.69**	**0.75**	**0.82**
5 yrs		**0.50**	**0.59**	**0.70**
10 yrs			**0.53**	**0.68**
25 yrs				**0.78**

The equity return percentiles have been calculated using data from a survey of sixteen markets by Dimson, Marsh and Stauton. All time periods 1900-2003 have been used. Results for periods longer than 1 year have been calculating using a bootstrap approach.

5.3.5 For the purposes of 5.3.4, the ‘relevant duration(s)’ is or are:

- for a projection for a fixed number of years, that number of years
- for a projection until all but an immaterial liability remains, the periods until the most material numbers of relevant options or guarantees are expected to be exercised or to expire.
- for an instantaneous stress, one year.

5.3.6 For the purposes of 5.3.4, the ‘relevant entries’ are those for the 0.5^{th} and higher percentiles for durations less than or equal to the ‘relevant duration(s)’.

5.3.7 For the avoidance of doubt, the requirements of 5.3.4 must be met for the equity return unaggregated with other risks, notwithstanding the fact that the effective percentile equity return after aggregation with other risks will be lower.

5.3.8 The table in 5.3.4 will be revised periodically. However, as it is not intended to reflect recent implied market expectations of the relevant probabilities at the date of coming into force of the Guidance Note, revisions should not be expected to reflect changes in such market expectations. Nevertheless, the actuarial profession accepts that, in the circumstances where an adjustment to the calculation of the *resilience capital requirement* or the *risk capital margin* applies reflecting the *equity market adjustment ratio* relevant to those calculations, the model requirements under paragraph 5.3.4 may tend to remove the ‘time to respond’ given by the *equity market adjustment ratio.* In such circumstances FSA rules for those two capital items effectively reduce the fall to be tested. When such reductions apply, it is reasonable to permit the use of a less onerous test than the table in 5.3.4. The excess return produced by the model may, in such circumstances, be up to the figure in the table, divided by a factor (105%- *equity market adjustment ratio)* subject to a minimum divisor of 90% and a maximum divisor of 100%.

5.3.9 There is no explicit restriction on the choice of equity models; in particular, mean-reverting models may be used and no maximum equity risk premium is prescribed. However, all models should normally satisfy the requirements of paragraph 5.3.4 above, subject to the exception in paragraph 5.3.8.

5.3.10 In the context of an ICA, it is necessary to use models for all asset classes which reflect real-world parameters yet which are still arbitrage free. Models which may introduce a minor theoretical arbitrage opportunities as a result of interpolation between points derived from an arbitrage-free model are nevertheless permitted, provided that no aspect of the modelling takes advantage of these opportunities.

5.3.11 Most published interest rate models are designed to calculate market-consistent financial instrument prices under risk neutral assumptions. Care should be taken to ensure that they are used in an appropriate way in a ‘real-world’ projection.

5.4 **Credit Risk**

5.4.1 For fixed-interest stocks (other than stocks issued or guaranteed by EU governments or by the US Treasury) then it is likely to be necessary to model variation in prices and default rates/recoveries. Where possible, models should be calibrated to historic spread variation, rerating, default and recovery experiences. The additional risks associated with any lack of diversity should also be modelled.

5.4.2 Credit risk should be modelled for all fixed interest stocks, not just those backing with-profits liabilities.

5.4.3 Both market and credit risk should be incorporated in the same model if non-governmental stocks for a material part of the assets of the *firm*. This will allow for appropriate correlations to be incorporated between adverse credit and equity scenarios.

5.5 **Volatility Risk**

5.5.1 Assets and liabilities subject to valuation by stochastic means will change in value as option-implied volatility changes. Allowance for this risk should be made in ICAs (unless hedged). *Firms* should assess an appropriate amount of capital to hold against this risk in the ICA (which might, for example, be derived by experimentation from different stochastic runs with different volatility parameters or through the use of a model that simulates variations in implied, rather than observed, volatility).

5.6 **Inflation Risk**

5.6.1 It will generally be appropriate to model inflation stochastically, especially where significant exposure exists to administration expenses recovered from charges which are not price index linked (e.g. as a percentage of funds under management). The relevant income from charges should be modelled using consistent stochastic assumptions.

5.6.2 Inflation risk should also be modelled stochastically if a material quantity of price index linked policy liabilities exist and adequate close matching assets are not held.

6. Stochastic Modelling for Mathematical Reserves

6.1 For with-profits business with options (especially GARs) for which a stochastic model is to be used for calculating the mathematical reserves for the options, the guidance above for obtaining a market-consistent valuation should be followed.

6.2 Where stochastic techniques are to be used for material numbers of unit-linked policies with significant maturity guarantees, it should not be assumed that the simple model previously recommended by the Maturity Guarantees Working Party is appropriate. Instead, the guidance applicable to the market value of with-profits guarantees in this GN should be followed, adapted as necessary and with the addition of an appropriate prudence margin.

6.3 Similarly, adapted guidance from this GN should be used where stochastic models are used to calculate the mathematical reserves for GARs attaching to unit-linked policies and non-profit endowments.

7. Matters relevant to all uses of stochastic modelling under PRU

7.1 Management and Policyholder Actions

7.1.1 In most cases, it will be necessary when projecting liabilities and assets stochastically to assume that *firms* will react to future adverse or favourable investment or other scenarios by making changes to its practices for factors such as asset mixes (see paragraph 7.2 below), bonus rates, surrender values, charges, etc. in accordance with the principles set out in their *Principles and Practices of Financial Management* ("PPFMs") It should not be assumed that it will be possible to make changes to those principles. A time interval should be allowed for practical and regulatory constraints on the timing of changes. Allowance must be made for the cost of any such changes.

7.1.2 It is also necessary to assume that some policyholder behaviours will change, especially in more extreme scenarios. In particular, lapse rates and option take-up rates may change as options and guarantees become more or less valuable. See also paragraph 8.2 below.

7.1.3 Further guidance on management and policyholder actions is given in GN45 and GN46.

7.2 Hedging

7.2.1 *Firms* may have either purchased hedging assets or have adopted a dynamic hedging strategy.

7.2.2 For ICAs, unless the hedging can be demonstrated to be robust across a wide range of scenarios, apart from immaterial differences, hedging assets should be stochastically projected consistently with the liabilities. In particular, the modelling should highlight any material imperfections of hedging (e.g. if the hedge relates to a different interest rate to that determining the liabilities, is subject to exchange rate risk or is for a different duration to the liabilities). This necessitates the ability to model the mismatched risks, including the correlations between them.

7.2.3 When calculating the price of hedging assets in stressed scenarios, care should be taken to ensure consistency between the assumptions underlying the asset pricing model and the liability model, including consistency of shape of the long end of the yield curve.

7.2.4 Dynamic hedging strategies (e.g. delta hedging of guarantee liabilities) should be taken into account in stochastic models if they form part of the ongoing management of the *firm* and are permitted under the PPFM, except where *FSA* rules require the assumption of an instantaneous price change.

7.2.5 Imperfections exist in dynamic hedging (e.g. 'gap' risk - it may not be possible to transact the necessary sales or purchases at the assumed price in the required quantity; infrequently rebalanced delta hedging does not provide an exact match) particularly for rapid, large changes. If 'gap' protection has been purchased, then, subject to an allowance for counterparty risk, 'gap' risk may be ignored. Otherwise, models should make allowance for these imperfections, which should take into account any readily available public or internal knowledge of market capacities and spreads in recent times of rapid price change.

7.2.6 Any strategy which involves the purchase of hedging assets in the future, either to replace existing assets on expiry or if certain scenarios arise, needs to adequately allow for the purchase prices of the hedging assets at those times or in those scenarios. This applies particularly to rolling 'gap' protection and to strategies which involve the purchase of derivatives in the event of specific market scenarios arising. Prices should be consistent with implied future market conditions at the purchase date (e.g. taking account of the forward yield curve). For PRU 7.4 purposes, it is not necessary to assume that volatilities or skews change in adverse scenarios. However, for ICA purposes, it is necessary to make allowance for these risks.

7.3 Frequency of Iteration

7.3.1 For *firms* with in-force business spread out over many years and little short-term path-dependency, annual model iterations may be sufficient for long-term projections. However, for shorter-term projections and where hedging strategies or other short-term path dependencies may otherwise fail to be modelled adequately, shorter periods are likely to be more appropriate. Awareness should be demonstrated of the effect of shorter periods and approximate adjustments, with disclosure under 1.2 above, made if longer periods continue to be used despite evidence that shorter periods would lead to a materially different result.

7.3.2 If there are significant amounts of guarantees or options which are close to the money at the valuation date and which have exercise dates within the next year or two, it will be appropriate to value these options and the capital required to support them using more frequent iterations to ensure that adequate value is placed upon them.

7.4 Tax

7.4.1 In all uses of stochastic models, it is necessary to ensure that appropriate allowance is made for tax. The actuarial profession has yet to develop guidance in this area. However, particular attention should be paid to ensuring that the tax treatment of more extreme scenarios is appropriate (e.g. the ability to relieve losses, the ability to index capital gains, the acceleration of the realisation of capital gains, the overall tax basis of the *firm*, the actual 'BLAGAB' ratio as opposed to that based on the ratio of realistic basis liabilities).

8. Insurance Risks

8.1 Mortality and Morbidity Risk

8.1.1 It is generally not necessary to value stochastically the variation in the number of deaths from year to year under insurances or annuities, due to the effect of the law of large numbers. However, if small portfolios of large risks after reinsurance or retrocession are held, the extreme outcomes of which are material in the context of the *firm* overall, it may be appropriate to do so. Similarly, if non-proportionate reinsurance is accepted, stochastic modelling may be necessary to establish appropriate market-consistent reserves and capital for ICAs.

8.1.2 In the ICA, it may be appropriate to model uncertainties in the future trend of annuitant longevity for different cohorts using a stochastic model. This may provide a more appropriate capital assessment, especially for GARs, than simply combining a conservative deterministic assumption with a stochastic financial model alone. It may also be appropriate to model mortality change trends for large portfolios of fixed-rate term assurance.

8.1.3 The extent to which any stochastic model allows for ‘large-scale’ events such as a major epidemic or a major medical advance in the prevention of ageing should be considered and, if necessary, additional provisions made in accordance with GN44-46 as appropriate.

8.1.4 If stochastic modelling is used for critical illness or income protection morbidity, which may be appropriate for fixed rate policies, then the model should allow not just for randomness in the number of claims or recoveries but also for the risk of adverse trends, particularly in critical illness diagnosis. Consideration should be given to the possibility of positive correlation between claims and adverse economic scenarios.

8.2 Persistency and Option Take-up Risk

8.2.1 It is possible to make assumptions about stochastic persistency or take-up rates of options, etc. However, it is unlikely that much, if any, relevant past experience is available from which to calibrate any model. It is therefore not a requirement to model such factors stochastically for any purpose of PRU. However, if such modelling is done, it should be consistent with the static or dynamic deterministic assumptions which would otherwise be used.

8.2.2 Persistency and option take-up rates should relate dynamically to relevant factors in stochastically generated scenarios. Further guidance on policyholder actions of this nature is given in GN45 and GN46.