Outline

- Cash flow structure, optionality and pay-off
- Overview of the OTC market in 2008
- Valuation methodologies – models of the yield curve
  - Black’s formula
  - Implied volatilities
- Data sources – the swap curve
- Hedging interest sensitive liabilities with swap and swaptions
  - Rho, rhoga, vega, volga, rhova
- Exotic swaps
- GAO hedges: impact of mortality
- Liability valuation basis impact: marked to market calibration of yield curve and volatility. Evidence from WP insurers
Relevance to Life Insurance Liabilities

- Insurers are structurally long in interest rates on marked to market liabilities
  - Guaranteed Annuity Options: Mortality linked receiver swaption with an asset linked nominal
  - Annuities: Mortality linked bond portfolios
- Liability matching strategies utilise opposite exposures
  - Gilts
  - Corporate Bonds
  - Receiver Swaps
  - Receiver Swaptions
- The relevance of swaps and swaptions is therefore two fold
  - As hedging instruments
  - As reference instruments for marked to market calibration

Receiver Swap Cash Flows

10 year interest swap arranged in '07: Purples receive fixed pay floating, Reds receive floating pay fixed.

Fixed>Floating
Reds pay Purples (Fixed-Floating)

Floating>Fixed
Purples pay Reds (Floating-Fixed)

Receiver Swaption Cash Flows

5 into 5 interest rate swaption purchased by Purples. Receive fixed pay floating. Purples exercised in 2011.
Receiver/Payer Relationships

- Payer Swap + Receiver Swap = 0
- Receiver Swaption - Payer Swaption = Deferred Receiver Swap
- ATM: Receiver Swaption = Payer Swaption
- ATM Strike = Forward Swap Rate

Value Caps

<table>
<thead>
<tr>
<th></th>
<th>Max</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Receiver Swap</td>
<td># Payments x Strike x Nominal</td>
<td>Nominal</td>
</tr>
<tr>
<td>Receiver Swaption</td>
<td># Payments x Strike x Nominal</td>
<td>0</td>
</tr>
</tbody>
</table>

Swaption Pay-off at Exercise
**Overview of the OTC Swap and Swaption Market**

- **USA**
  - Swaps: $200,000, Swaptions: $50,000

- **Germany**
  - Swaps: $250,000, Swaptions: $100,000

- **France**
  - Swaps: $220,000, Swaptions: $70,000

- **Japan**
  - Swaps: $180,000, Swaptions: $60,000

- **UK**
  - Swaps: $150,000, Swaptions: $40,000

- **Switzerland**
  - Swaps: $120,000, Swaptions: $30,000

**Turnover Change 2007/2004**

- **Swaptions**
  - USA: +50.0%, Germany: +100.0%, France: +200.0%, Japan: +250.0%, UK: +300.0%, Switzerland: +350.0%

- **Swaps**
  - USA: +50.0%, Germany: +100.0%, France: +200.0%, Japan: +250.0%, UK: +300.0%, Switzerland: +350.0%
Overview of the OTC Swap and Swaption Market by Country

International Interest Rate Derivatives OTC Market

- USA
- Germany
- France
- Japan
- UK
- Switzerland

BIS Triennial Survey 07

Overview of the OTC Swap and Swaption Market by Country

Local GBP Swaps Volume

Dealers, 41,299
Other Financial Institutions, 18,212
Non Financial Companies, 1,188

Dealers
Other Financial Institutions
Non Financial Companies

Valuation Methods

- Swap
  - Closed Form
    - Fully Fitted Initial Yield Curve
- Swaption
  - Closed Form/Monte Carlo/Finite Differences
    - Fully Fitted Initial Yield Curve
    - Stochastic Yield Curve Model
    - Market Consistent Calibration
Swap Valuation

\[ V_{\text{swap}} = N \left( g \left( \sum_{j=1}^{T} ZCB \left( s_j \right) \right) + ZCB \left( S \right) - 1 \right) \]

\[ V_{\text{swap}} = 0 \]

\[ G_{\text{Swap}} = (1 - ZCB \left( S \right)) \left( \sum_{j=1}^{T} ZCB \left( s_j \right) \right) \]

---

Exotic Swap Valuation Example

- Receiver Swap with Asset Linked Nominal
- Designed to achieve a holistic hedge of a GAO liability

\[ V_{\text{swap}} = \frac{Ne^{-\sigma^2 T}}{ZCB(T)} \sum_{j=1}^{T} ZCB(0,j) \left( k - g'(j,T+1) \right) \exp(\sigma_j^2 + \rho_{jk}) \]

---

Receiver Swaption Valuation in Closed Form

- Short Rate Model
- Bond Pricing Equation
- Closed Form Solution
- Available Models
- Closed Form Solution for a Swaption
- Analytic Calibration to Yield Curves and Swaption Volatilities
- Realistic Yield Curve Dynamics
Receiver Swaption Valuation: Monte Carlo

\[ \text{Fix}(s, s, t) = k \sum_{j=1}^{\infty} P_{s+t} \]
\[ \text{Float}(s, s, t) = 1 - P_{s+t} \]
\[ \text{Swaption}(s, s, t) = \max(\text{Fix}(s, s, t) - \text{Float}(s, s, t), 0) \]
\[ \text{Swaption}(0, s, t) = \frac{1}{n} \sum_{j=1}^{n} \left( \text{Swaption}(s, s, t) \frac{D_j(s)}{D(0)} \right) \]

Receiver Swaption Valuation: Black's Formula

\[ V_{\text{swaption}} = \sum_{j=1}^{\infty} \text{CB} \left( s, \frac{g + N(-d_1)}{\sigma \sqrt{T}}, \frac{g + N(-d_2)}{\sigma \sqrt{T}} \right) - f \ast N(-d_2) \]
\[ d_1 = \frac{\ln(f / g) + \sigma^2 T / 2}{\sigma \sqrt{T}} \]

- Log-Normal swap rate
- Forward risk-neutral
- Universal quoting convention

Simple Forward Risk-Neutral Model for a GAO

\[ \begin{bmatrix} \ln S_4 \n S_4 \end{bmatrix} \begin{bmatrix} \sigma_1 \n \sigma_2 \n \rho \sigma_1 \sigma_2 \n \sigma_1 \sigma_2 \end{bmatrix} = \begin{bmatrix} \mu_1 \n \mu_2 \n \rho \sigma_1 \sigma_2 \end{bmatrix} \]

\[ \text{cov}(S_4, S_4) = \sigma_1^2 \sigma_2^2 \rho \sigma_1 \sigma_2 \]
Receiver Swaption Valuation: Implied Volatilities

- Inverting closed-form solution for log-Normal swap rate
- Infer corresponding implied volatility
- Important benchmark for cross product/quote comparison
- Important for extended model validation

Models and Prices

- Market Model
- Swap Curve
- Black's Volatility
- Swaption Price
- Swap Price
- Possible Insurer's Model
- Gilt Curve
- Black's Volatility
- Swaption Price

Swap over Gilts Spread

- No counter party risk at inception for ATM swaps
- Max exposure = current swap value
- Demand/supply type arguments for gilts
Data Sources: Swap Curves

- Pit falls
- Interpolation
- Spot rates vs swap rates
- Market compounding convention
- Day counts

Yield Curves

EURO Yield Curve FR 31 December 2007

Methodology: http://www-cfr.lbs.cam.ac.uk/archive/PRESENTATIONS/seminars/lent20/ASMITHｙｉｅｌｄ.pdf

Data Sources: Implied Swaption Volatilities

- Pit falls
- Log-Normal vs Normal vola
Interest Rate Sensitive Exposures

<table>
<thead>
<tr>
<th></th>
<th>Long Position</th>
<th>Short Position</th>
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<tbody>
<tr>
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<td>●</td>
<td>●</td>
</tr>
<tr>
<td>Payer Swap</td>
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<td>●</td>
</tr>
<tr>
<td>Bonds/Annuities</td>
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</tr>
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<td>●</td>
<td>●</td>
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</tbody>
</table>

Net value moves in the same direction as interest rates
Losses when interest rates go up

Net value moves in the opposite direction to interest rates
Losses when interest rates go down

Interest Rate Volatility Sensitive Exposures

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</thead>
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<td>●</td>
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Net value moves in the same direction to volatility
Losses when volatility goes down

Net value moves in the opposite direction to volatility
Losses when volatility goes up

Exposure Hedging

<table>
<thead>
<tr>
<th></th>
<th>Interest Rate Sensitivity</th>
<th>Interest Rate Volatility Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liabilities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annuities</td>
<td>●</td>
<td>X</td>
</tr>
<tr>
<td>GAOs</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td>Assets</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonds (Long)</td>
<td>●</td>
<td>X</td>
</tr>
<tr>
<td>Receiver swap (Long)</td>
<td>●</td>
<td>X</td>
</tr>
<tr>
<td>Receiver option (Long)</td>
<td>●</td>
<td>X</td>
</tr>
</tbody>
</table>

Losses when the driver goes up
Losses when the driver goes down
Hedged Position Construction

- Net Asset Value is a function of interest rate level and interest rate volatility: $N_{AV}(r, \sigma)$

\[
\Delta N_{AV} = \frac{\partial}{\partial r} N_{AV}(r, \sigma) + \frac{\partial}{\partial \sigma} N_{AV}(r, \sigma)
\]

- Net Asset Value will be unresponsive to changes in interest rate and volatility if the asset portfolio hedges liabilities. "Greeks" based hedge.

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Volatility Sensitivity: Why Second Order Terms Matter

Interest Rate Vol Shift

Sensitivity to Vol Increases as Interest Rates Increase

Interest Rate Shift

NAV Model: A Simple Longevity Book

GAO

Annuities

% of Base Case

Parallel IYC Shift (bp)

NAV Model: Addition of a Downside Swap Hedge

100% Bonds + ATM Receiver Swap (0 Cost)

Parallel IYC Shift (bp)
Minimal Solvency II Hedge

- NAV m. Euro 100% in Bond 1
- Minimum Hedge Composition:
  - Bond 1
  - Bond 2
  - Swaption

Swaption dominates the simple strategy everywhere.

Minimal Solvency II Hedge

- Stable under Volatility Stress

Minimal Solvency II Hedge: Drastic Longevity Shock Impact

- Opposite longevity exposure missing

- Insolvency

- Longevity Shock

- Minimal rate parallel shift
### Marked to Market Valuation: Significance of Assumptions

<table>
<thead>
<tr>
<th></th>
<th>Yield Curve</th>
<th>Interest Rate Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annuities</td>
<td>![Red Circle]</td>
<td>![Gray Circle]</td>
</tr>
<tr>
<td>GAO</td>
<td>![Red Circle]</td>
<td>![Yellow Circle]</td>
</tr>
<tr>
<td>Bonds</td>
<td>![Red Circle]</td>
<td>![Gray Circle]</td>
</tr>
<tr>
<td>Swaps</td>
<td>![Red Circle]</td>
<td>![Gray Circle]</td>
</tr>
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<td>![Red Circle]</td>
<td>![Yellow Circle]</td>
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**Conclusions**

- Non-dealer financial companies are important players in the UK swap market
- Insurers’ interest rate sensitive exposures can be profoundly manipulated with OTC long term interest rate derivatives
- Hedging schemes must be carefully evaluated
- Yield curve is the single most significant valuation input