GLOSSARY OF TERMS – UTILITY THEORY

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Opportunity set :- the opportunities or options open to the investor.

Indifference curves :- indicate the investors preferences on a relative scale. The curves connect points of indifference (equal happiness) where the investor is indifferent (equally happy) as between outcomes.

Utility function/Preference function :- a function which assigns a numerical value to the various possible outcomes which arise as a result of the various investor options. Normally the outcome will be measured in terms of resulting wealth and the utility function will be a function of wealth, say U(W).

Risk averse, risk neutral, risk seeking :- in terms of the utility function U(W) means $U''(W) \;<=>\; 0$.

Decreasing (constant, increasing) absolute risk aversion :- investor decreases (keeps constant, increases) the absolute amount invested in risky assets as his wealth increases (stays constant, decreases). Absolute risk aversion is measured by $A(W) = -\dfrac{U''(W)}{U'(W)}$ and decreasing absolute risk aversion has $A'(W) < 0$ etc.

Pratt-Arrow measure of relative risk aversion :- another name for relative risk aversion.

Decreasing (constant, increasing) relative risk aversion :- investor decreases (keeps constant, increases) the relative amount invested in risky assets as his wealth increases (stays constant, decreases). Relative risk aversion is measured by $R(W) = -\dfrac{W.U''(W)}{U'(W)}$ and decreasing relative risk aversion has $R'(W) < 0$ etc.

> *Example ; (i) the quadratic utility function $U(W) = W\text{-}bW^2$ exhibits increasing absolute risk aversion and increasing relative risk aversion (ii) the log utility function $U(W) = \ln W$ exhibits decreasing absolute risk aversion and constant relative risk aversion.*

First order stochastic dominance of portfolio A over portfolio B :- the investor prefers more to less ($U'(x) > 0$) and the cumulative distribution function of A lies on or below that of B and sometimes is below B i.e. $F_A(x) \leq F_B(x)$ and $F_A(x) < F_B(x)$ for at least one x. The expected utility of A is then greater than that of B.

Second order stochastic dominance of portfolio A over portfolio B :- the investor prefers more to less and the investor is risk averse and the cumulative of the cumulative distribution function of

A lies on or below that of B and sometimes is below B i.e. $\int_a^x F_A(s)ds \le \int_a^x F_B(s)ds$ with strict inequality holding for some value of x. The expected utility of A is then greater than that of B.

Third order stochastic dominance of portfolio A over portfolio B :- the investor prefers more to less ($U'(x) > 0$) and the investor is risk averse ($U''(x) < 0$), the third derivative of the investor's utility function is positive ($U'''(x) > 0$ which ensures A'(W) > 0 i.e. decreasing absolute risk aversion), the mean return of A is greater than that of B (i.e. $\int_a^b F_A(s)ds > \int_a^b F_B(s)ds$) and the cumulative of the cumulative of the cumulative distribution function of A lies on or below that of B and sometimes is below B (i.e. $\int_a^x \int_a^t (F_A(s) - F_B(s))dt.ds \le 0$) with strict inequality for some value of x. The expected utility of A is then greater than that of B.

Risk measure :- a functional (i.e. a mapping from the random variable X to the real numbers R) $\rho(X) : X \rightarrow [0,\infty)$

Value at Risk (VaR) :-a **risk measure** namely the barrier that the losses would breach a certain proportion of the relevant time-scale. If the timescale is a day, then $V_{2.5\%}$ is the barrier that the losses would exceed in 2.5 days out of 100.

Conditional Tail Expectation (CTE):- a **risk measure** namely the expected loss, conditional on the barrier being breached. In the above example it would be the average loss in the 2.5days that the barrier is breached.

Distorted risk measure (DRM) :- a **risk measure** computed from the c.d.f. of the loss.
If the c.d.f. is *F(x)* then the distorted risk measure relative to the function g (normally convex) is $\int_{u=0}^{u=\infty} g\{F(u)\}du$. It will be noted that when f is linear then $\int_{u=0}^{u=\infty} F(u)du$ is equal to the expected value of the loss.

Coherent risk measures :- risk measures which satisfy the coherency axioms :-

1. Bounded above by maximum loss i.e. $\rho(X) <= \max(X)$,
2. Bounded below by mean loss $\rho(X) >= E(X)$,
3. Scalar additive and multiplicative $\rho(aX+b) = a\rho(X) + b$,
4. Sub-additive $\rho(X+Y) <= \rho(X) + \rho(Y)$.

VaR satisfies all axioms except 4 but CTE and DRM satisfy all 4 axioms.