Projection of mortality rates as illustrated by Weibull distribution
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Building mortality models

Possible forms of mortality models:
- Projection of past trends without adjustment
- Projection of past trends with adjustment
  - Adjust for changes in risk factors
  - Application to alternative populations
- Projections from medical cohort data by disease condition, allowing for effects of treatment and disease interaction

Development of actuarial mortality models

- Gompertz - $Bc^x$
- Makeham – $A + Bc^x$
- Kannisto – $Bc^x / (1 + Bc^x)$
  - Other variants by Beard, Perks
- Each model approximates actual mortality experience either in specified calendar year or over lifetime of individual
Weibull distribution

- Published in 1939 by Waloddi Weibull
- Probability density function
  \[ f(x; \lambda, k) = \begin{cases} \frac{k}{\lambda} \left( \frac{x}{\lambda} \right)^{k-1} e^{-\left( \frac{x}{\lambda} \right)^k} & x \geq 0, \\ 0 & \text{otherwise} \end{cases} \]
- Cumulative distribution function
  \[ F(x; \lambda, k) = 1 - e^{-\left( \frac{x}{\lambda} \right)^k} \]
- \( \lambda \) is scale parameter, and \( k \) is shape parameter
- 3 parameter version available

Uses of Weibull distribution

- Weibull distribution normally used as a reliability distribution to model:
  - Material strength
  - Times-to-failure of electronic and mechanical components
- More recent application to model various phases of mortality experience

Value of “slope” or “shape” parameter
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- Effect of value of slope parameter:
  - <1 – Infant mortality or burn-in
  - 1 – Random failures
  - 1-4 – Wear out failure
  - 3.5 – Similar to Normal distribution
  - >4 – Old age or rapid wear out

Weibull distribution in mortality models

  - Comparison of Weibull to Kannisto
- Application of mortality models to Japan – SOA’s Living to 100 (2005) – Ozeki
  - Use of Mixed Weibull distributions at different ages
  - Use in construction of Japanese life tables

Weibull distribution in mortality models

- Complementarity between survival and mortality - Weon
  - Weon model derived from Weibull distribution using age-dependent shape parameter
- Analysis of trends in the age-specific shape of mortality curves for populations in the US and Japan – SOA’s Living to 100 (2005) – Dugan et al
- Analysis of trends in mortality near or during retirement for four European countries – ICA (2006) – Humble et al
Source of mortality experience and choice of measure of mortality

- Population mortality experience by individual age and for both sexes taken from Human Mortality Database over the period 1960 to 1999 for:
  - England & Wales
  - USA
  - Japan
  - France
  - Italy
- Probability of death from starting age – chose age 50 - for each subsequent age from vertical life table

Method for fitting Weibull mortality models

- Method of percentiles
  - Fit Weibull cumulative probability distribution to actual experience at selected percentiles
  - Chose 50th and 95th percentiles to highlight old age mortality
- Comparison of parameter values from Weibull mortality models in different calendar years

E&W males – appropriateness of fit

![Graph showing fit of Weibull distribution to probability of death at specified age - E&W males in calendar year 1999]
E&W females – appropriateness of fit

Specimen parameter values for Weibull mortality models in different calendar years

Investigation period affecting comparisons between actual and projected mortality
Investigation period affecting comparisons between actual and projected mortality

Comparisons of male life expectancy from Weibull mortality model

Comparisons of female life expectancy from Weibull mortality model
England & Wales Male population mortality improvements

- 4.0%-4.5%
- 3.5%-4.0%
- 3.0%-3.5%
- 2.5%-3.0%
- 2.0%-2.5%
- 1.5%-2.0%
- 1.0%-1.5%
- 0.5%-1.0%
- 0.0%-0.5%

England & Wales Female population mortality improvements

- 3.5%-4.0%
- 3.0%-3.5%
- 2.5%-3.0%
- 2.0%-2.5%
- 1.5%-2.0%
- 1.0%-1.5%
- 0.5%-1.0%
- 0.0%-0.5%

USA Male population mortality improvements

- 3.0%-3.5%
- 2.5%-3.0%
- 2.0%-2.5%
- 1.5%-2.0%
- 1.0%-1.5%
- 0.5%-1.0%
- 0.0%-0.5%
USA females - mortality improvements from Weibull mortality model

Further areas for consideration over application of Weibull mortality model

- Differing views over scope for further improvements in life expectancy
- Application to insured population experience
- Quantifiable differences between future expectation and past trends for prevalence of known risk factors e.g. smoking
- Adjustment where appropriate for publicly available disease specific models, with recent attention on cardiovascular mortality

Broken limits to life expectancy