PRACTICAL EXPERIENCES IN STOCHASTIC RESERVING

27 JULY 2007, LONDON

Stephan Westphal
Towers Perrin Tillinghast
Structure

Stochastic Reserving: The starting point (Introduction)

Stochastic Reserving: Opinions, Shortcomings and Solutions

Stochastic Reserving: How to get the most out of it
Assumptions used to make insurance decisions need to consider more than just one estimate.

Variations in the past have to be observed and considered.

Change-over to stochastic reserving methods.
Stochastic Reserving: The starting point

- Loss development is a stochastic cash flow of payments
- The observed paid losses are but one specific realisation of the underlying stochastic process
- The task of projecting reserves is to derive an assessment of the future development of this cash flow, given the data observed so far

- Deterministic methods provide a point estimate of claim liabilities
  - “Best estimate” is usually chosen judgementally
  - Applying multiple deterministic methods can give a “range of estimates”

- Stochastic reserving starts with solving questions like:
  - How far can the future loss development deviate from my expectation?
  - And how likely is that?
Structure

Stochastic Reserving: The starting point (Introduction)

Stochastic Reserving: Opinions, Shortcomings and Solutions

Stochastic Reserving: How to get the most out of it
Some Assertions:

- Stochastic reserving does not add (much) value!
  - “We already use (deterministic) best practice reserving techniques.”
  - “Stochastic methods are complicated to understand and to use. I do not have the time to read all those papers.”
  - “Ranges can easier be estimated by stressing factor selections.”

- Stochastic methods (often) produce implausible results!
  - “My data is not suitable for stochastic methods.”
  - “The indicated range is too big / too narrow.”
  - “Even the projected mean is considerably different from my best estimate.”

- So why bother with stochastic reserving methods?!
Excursion: GIRO 2005 Reserve Uncertainty Survey

“What did we learn?”
- Using methods/models is easy
- Understanding methods/models is more difficult
- Understanding method/model output can be very difficult […]
- No “correct” method/model apparent
- Wide range of results from different methods/models
- Range still wide even when same method/model used
- Data issues will distort results
- […]
Take a step back: The deterministic approach

Indicated Unpaid Claim Liabilities as at 31 December 2006

The General Deterministic Approach

- Usually starts with a chainladder projection of paid (and incurred) data
- (Almost) never ends there…
- Excellent knowledge of the underlying business is essential
- Applying actuarial expertise and judgement when selecting development patterns and projection methods is crucial for deriving sensible results
- Point Estimate usually picked judgementally
- Possible estimation of a range based on a) results of different projection methods or b) varying assumptions (e.g. loss ratios, tail)

Advantages

- Easy to understand and apply
- Established

Disadvantages

- Does not include confidence intervals
- No quantitative answer
- Simplistic and highly judgemental
The Stochastic Approach:
Reserves are really distributions

General Approach - Stochastic
- Estimate probability distribution
- Based on statistical methods
- Approach varies and depends on the actual data

Advantages
- Produces estimates of confidence intervals
- More complete description of loss generating process
- Feeds other analyses (ERM, DFA, QIS)
- Can approximately separate parameter and process risk

Disadvantages
- Involves relatively complex statistical analyses
- There are many different approaches, but the variety is not yet developed well enough as to have a general agreement among actuaries on which approach is best practice
- Some exposures not amenable to this approach (e.g. APH)
Conclusion

- Stochastic reserving techniques are an emerging technology
- There are some challenges
- Big opportunities for more refined information about claim liabilities make the extra effort worthwhile
- Naïve application of stochastic models seldom provides good results (as with deterministic models)
- Testing the model assumptions against the data is essential!
Structure

Stochastic Reserving: The starting point (Introduction)

Stochastic Reserving: Opinions, Shortcomings and Solutions

Stochastic Reserving: How to get the most out of it
Testing Model Assumptions

- The most common stochastic methods are based on the basic chainladder assumptions:
  - Mack’s model
  - Over-dispersed Poisson (→ GLM, Bootstrapping)

- These models have only limited allowance for actuarial judgement

- Hence it is not possible to judgementally overrule parameter calculations as is done in the deterministic Loss Development Method

- **Testing and verifying the underlying model assumptions is crucial!**
The Reserving Problem

Loss triangle data

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Best Model?
Ideally, minimize parameter and model uncertainty

Additional Information
- Trends
- Exposure data
- Pricing information

Process Uncertainty
Model Uncertainty
Parameter Uncertainty
### Some Sample data (German Motor Liability)

<table>
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<tr>
<th>AY/DY</th>
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Cumulative Payments, Figures in mn EUR

**Plotted Link Ratios**  
Can we identify any trends in this chart?
A first glance at the data: Check for Linearity in the Functional Form of Development Year Trends

- **Graphical judgment:**
  Plot cumulative (or incremental) claims for year \( d+1 \) versus cumulative claims for year \( d \) for all accident years.

In this example (for \( d=1 \)), the linearity assumption seems to fit well!
The estimated intercept is not significantly different from zero. This is a necessary requirement for the Chain-Ladder models.

- **Statistical measure:**
  \( R^2 \) value = Fraction of the variance that can be explained by the linear relationship.
  In the above example: \( R^2 = 79\% \).
Other common tests use a (model-specific!) definition of residuals

\[ R_{s,d+1} = \left( C_{s,d+1} - f_{d+1} C_{s,d} \right) / \left( \alpha_{s,d+1} \sqrt{C_{s,d}} \right) \]  "Standardised Residuals"

Remarks:

- Also called “Pearson Residual”.
- This definition of residual is used in all models that are based on chainladder assumptions, in particular:
  - Mack’s Chain Ladder
  - Over-dispersed Poisson, etc → GLM
  - ODP-Bootstrapping (without volatility coefficient)
Testing Model Assumptions: Independence of Residuals

- The standardised residuals should be independent.

- A necessary condition is that the residuals for two consecutive development years are independent or uncorrelated.

- In this particular example:
  - Graphically, a slight positive correlation may be identified (significant?).
  - R²-Value: very low (~ 20%).

Remark:
Note that independence of the residuals is also a necessary condition so that bootstrap for Chain-Ladder may be applicable.
Testing Model Assumptions: Distribution of Residuals

- Consider the cumulative distribution function for the residuals:
  - *What is the underlying distribution?*
  - *Outliers?*

**In our example:**
- Plot shows comparison of
  - Empirical distribution (diamond)
  - Fit with Normal distribution (─)

- The “periodic” deviations from the Normal curve are likely to be caused by calendar year trends.
- No outliers

Average = -0.05
Median = -0.22
Stdev = 0.91
In our example, all stochastic methods provide reasonable results.

<table>
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<tr>
<th>Year</th>
<th>Mack B.E. Reserve</th>
<th>S.E.</th>
<th>GLM B.E. Reserve</th>
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- **GLM**: Over-Dispersed Poisson (ODP) parameterization used.
- **Bayesian**: Based on ODP.
  - Prior information: Gamma distributed ultimate loss ratio with
    - Average = 75%; Stdev = 8%

Note: All calculations without tail extrapolation.
But even in our data we could have observed some trends in the residuals:

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*Note: The table shows the trends in accident year and calendar year for the years 1996 to 2004.*

*Figure: The Actuarial Profession, making financial sense of the future*
What to do if things are not that easy –
How to deal with significant trends in the data?

- Option 1: Apply judgement (e.g. Practical method)
- Option 2: De-trend data before model input:
  - Introduce trends such that residuals appear independent of accident year, calendar year or development year.
  - Trends introduced should be explainable.
  - Estimate chainladder factors and volatility parameters (scale-independent!) based on de-trended data.
  - It may be difficult to re-introduce the trends on results!
- Option 3: Incorporate trends into the model:
  - Use GLM’s since they offer a much richer parameter space.
  - This is not complicated!
Conclusion

- The available stochastic methods are based on different approaches and can lead to different results.

- A solid understanding of the models is essential.

- As models are more complex, they require the actuary to be more rigorous in assessing how well a model fits the given data.

- The actuary is responsible for selecting the appropriate approach, depending on the specific situation.

- Not only mathematical and statistical arguments count. Experience, expert knowledge and additional, non-actuarial knowledge is also necessary.
Thank you!

Neue Weyerstr. 6
50676 Köln
Phone: +49 221 921234-0
Fax: +49 221 921234-42

71 High Holborn
London WC1V 6TH
Phone: +44-20-7170-2000
Fax: +44-20-7170-2222

Email: stephan.westphal@towersperrin.com
www.towersperrin.com/tillinghast