The Actuarial Profession
making financial sense of the future

Price Optimisation
Issues & Challenges

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Agenda

- Introduction to Price Optimization
- Lifetime Customer Value (LTCV)
- A Theoretical Comparison
- LTCV vs Short-Term Optimisation
- Conclusions
- Questions

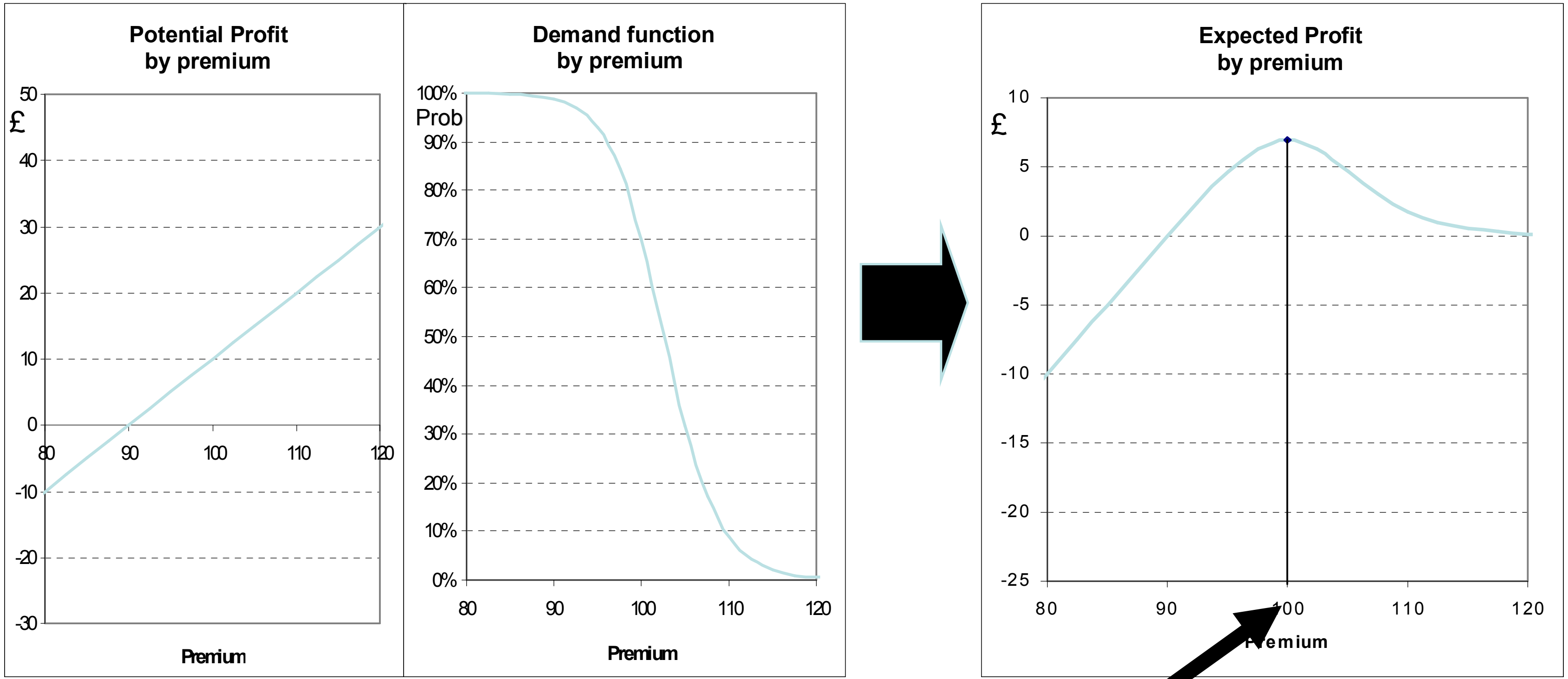
Introduction
Trade off between Profit and Demand

Basic modelling for Price Optimisation allows for the fact that different customers have different losses and react differently to price variations

- Loss model
  - Ultimate loss by customer
- Demand model
  - Probability to buy

Introduction

Trade off between Profit and Demand



Potential profit and demand have opposite effects on the price so that the optimal price is a compromise between the two.

Optimal Price

Introduction
Trade off between Profit and Demand

Optimal pricing is in fact a constrained optimization

We often want to:

- retain at least a fixed share of the market;
- target special segments of customers;
- limit the increase in price in per cent of last year's price;
- assure an adequate level of commissions;
- …

Challenges

How do we define our ideally best share of the market, segment of customers, level of commissions etc. ?

Introduction
Demand Elasticity

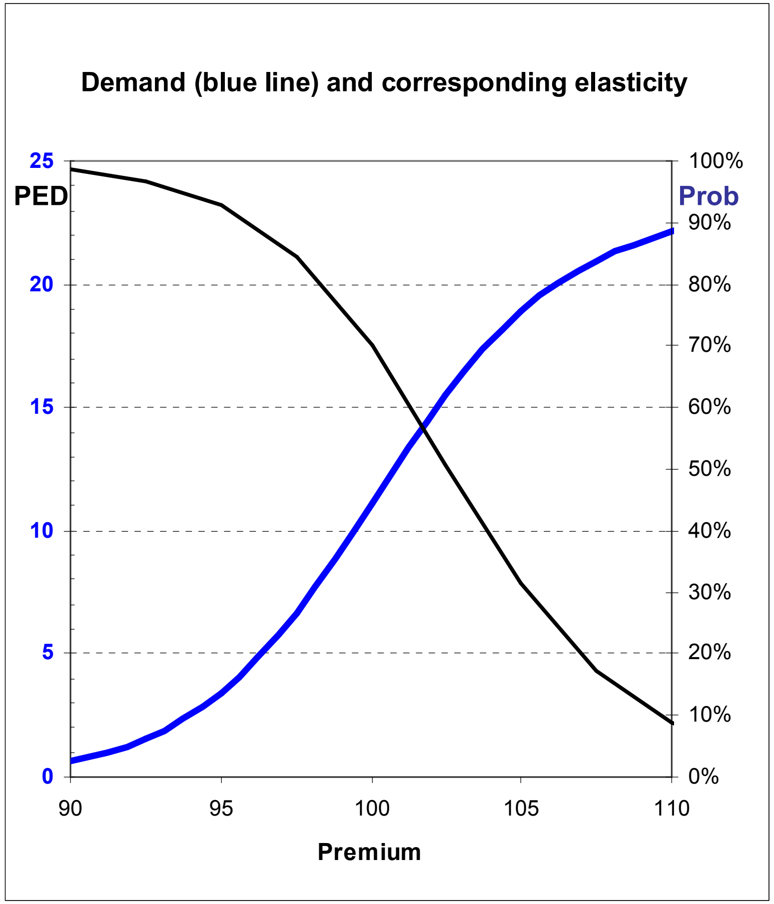
Price Elasticity of the Demand is the sensitivity of the customer demand to price variation.

- Technically, it is the percentage change in demand corresponding to a percentage change in price:

$$E(P) = |\Delta\% D / \Delta\% P| = |\Delta D / \Delta P| * P/D$$

- Elasticity will be implicitly evaluated from our Demand Model.

- **Why is E(P) so important?**



Introduction
Demand Elasticity

- Profit is maximized when

$$E(P) = 1/\ (1 - L/P)$$

- Inelastic demand: E < 1

→ Optimisation will raise prices for such customers

- Elastic demand: E > 1

→ Optimisation will lower prices for such customers (mostly!)

- Many markets are well below 1 (customers rather loyal) . Optimisation will naturally increase prices.

- For other markets (web based, aggregators) demand is very elastic instead!

Introduction
Demand Elasticity

Summary

Elasticity estimation is crucial and extremely difficult because:

- Elasticity varies with
  - customer and policy characteristics,
  - competitors' prices,
  - macroeconomic variables.
- Unstable in time
- Estimating the derivative of the demand is more difficult than estimating the demand function only!

Introduction
Other Challenges

Technical

- Mixed models,
- Splines and GAM,
- Bayesian models.

From Price Optimization to Business Optimization

- **Lifetime value of the customer,**
- **business growth and cross selling,**
- **discount policy,**
- **choice of limit, excess, coverage,**
- **channel**
- **...**

Lifetime Customer Value
What is Customer Value?

- **Customer value measures the value that customers, or particular customer segments, have added to the business and could add in the future.**
- **Measurement, whether implicit or explicit, of customer value, whether by the company, its management or the broker, forms part of most decision making processes within insurance. This process is dynamic and always changing.**
- **Measuring customer value and implementing a strategy based on customer value can allow companies to optimise the profit generated by policyholders.**
- **The table summarises the different components that can be included for the measurement of customer value. The list is not exhaustive.**

The present value of future profits and/or historic profits	Gross premium Net premium	The number and amounts of claims past and projected
Retention rate (absolute or relative to the market)	Expenses	Marketing spend
Cover type/extent of policy	Volatility of cashflows	Number of products held
Cross-sell	True risk premium	Competitiveness
Distribution channel	Broker	Reinsurance requirements
Cost of capital	Investment Income	Underwriting cycle

Lifetime Customer Value

A Simple Model

Example

- Time T = 1 (next renewal)

GWP at last renewal x inflation factor

\- GRP at last renewal x inflation factor

\- Expenses (per policy x inflation factor)

\+ Investment Income

\- Cost of Capital

\- Net Reinsurance cost

- Time T = 2

GWP at last renewal x inflation factor^2

\- GRP at last renewal x inflation factor^2

\- Expenses (per policy x inflation factor^2)

\+ Investment Income

\- Cost of Capital

\- Net Reinsurance cost

x 1 - Lapse Rate*

Aggregation

The sum of these values over 5 years time will then be discounted back to the next renewal date allowing for survival (MTC and lapses) between and allowing for the time value of money.

*** Lapse Rate**

Varies by Product / Tenure / Age / Distribution Channel / Location etc.

Lifetime Customer Value

Benefits

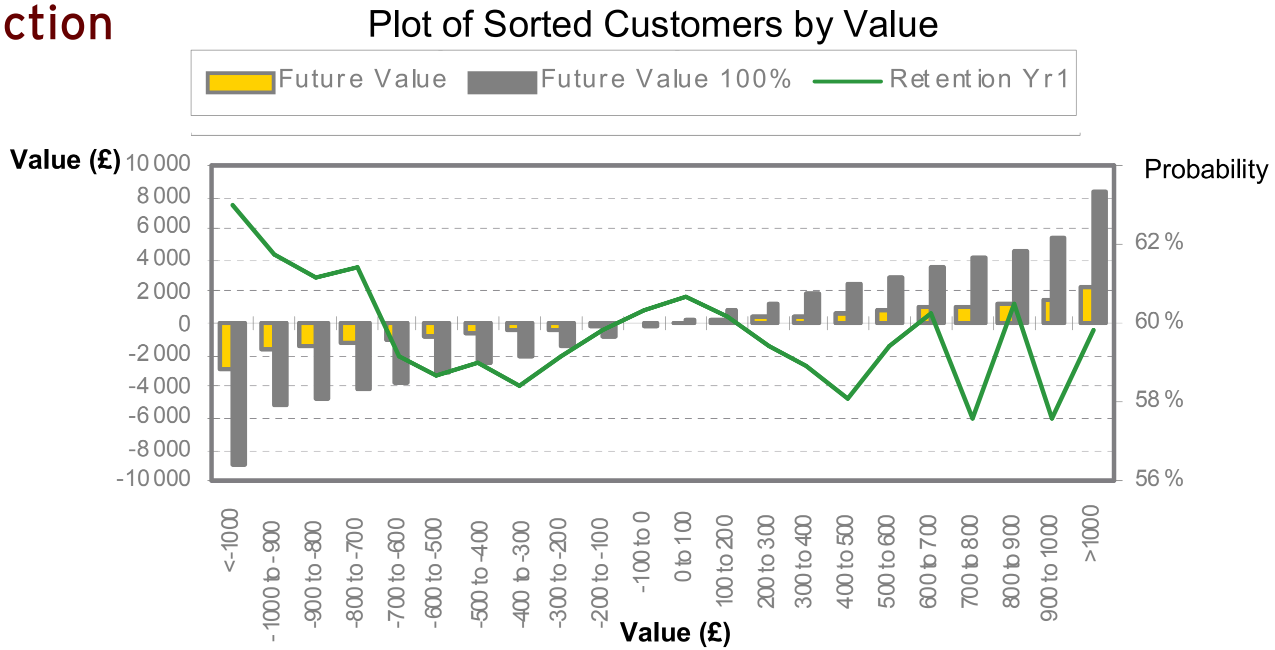
Customer value impacts marketing, underwriting, pricing and functions. Main benefits are:

- Consistency
  - Helps businesses develop consistent customer value models.
  - Supports definition of a consistent customer data system specification.
  - Customer value is the main driver for consistent tactical decisions.
- Efficiency
  - Focus on value creating customers.
  - Enables models and development effort to be shared between the business.
  - Improved management information.
- Insight
  - Provides tools for evaluating strategic options and segment value.
  - Provides real value outputs to support delivery of differentiated customer treatments.
  - Supports segment prioritisations.

Lifetime Customer Value

Examples

Adverse Selection



Graph shows the customer value (future value, yellow bar), the customer value assuming 100% retention (future value 100%, grey bar) and the first year retention probability (green line -values measured on left axis).

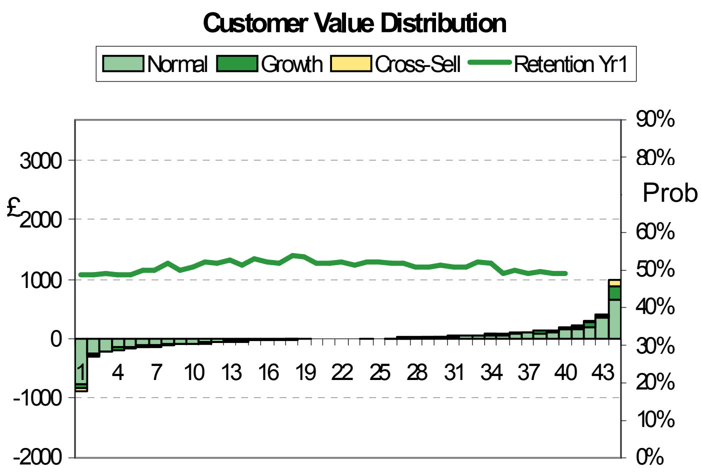
- Note the inverse relationship between retention and customer value, showing some extent of adverse selection.

Lifetime Customer Value
Examples

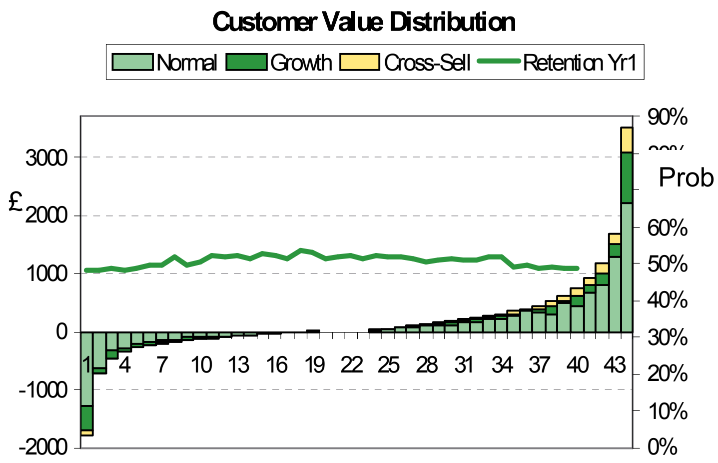
Growth and Cross Sell

The plots show the customer value distributions for two different products offered by the same insurer.

Product 1



Product 2

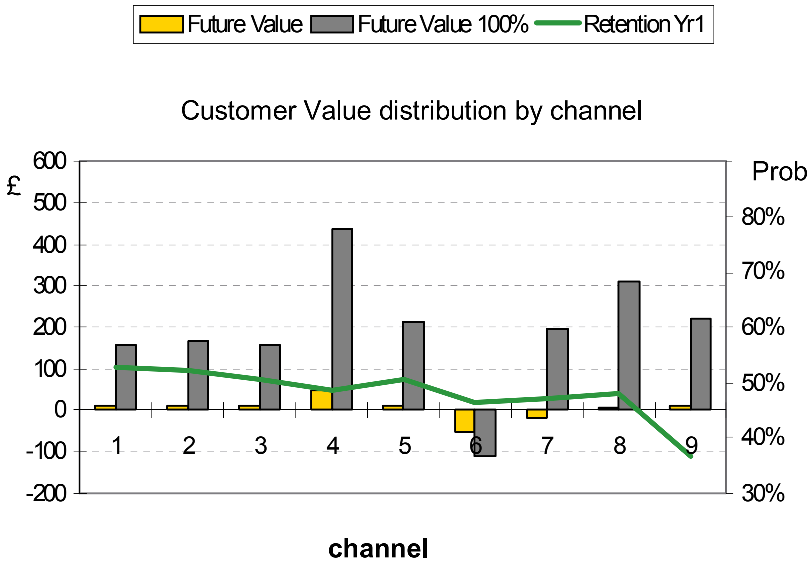


It is possible to break down customer value by the different components modelled. Here we look at how much cross-selling and internal product growth contribute to the customer value.

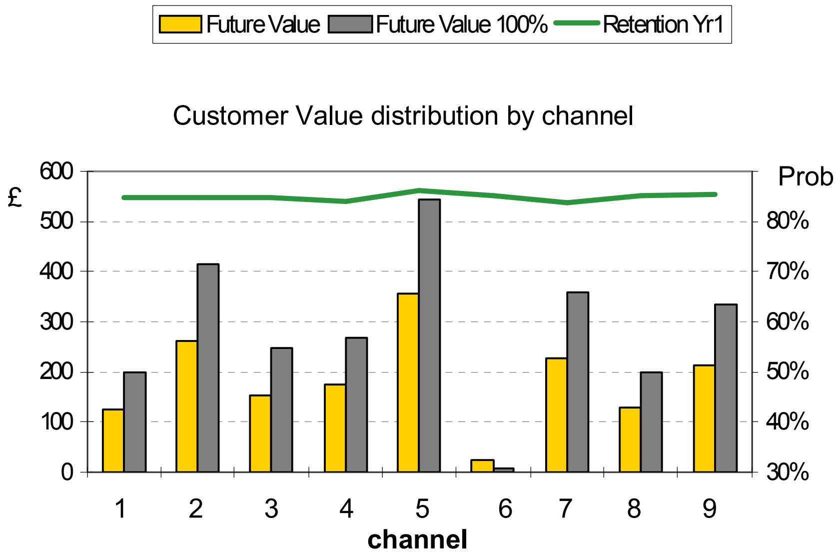
Lifetime Customer Value
Examples

Channel

Product 1



Product 2



Customer value can also be split by channel or intermediary. This can tell us what segments contribute more to the business than others.

A Theoretical Comparison
A Demand Model

- **What if there is no-data to fit our models ?**
  - New company start-up
  - New market: new region, new segment of customers etc.

- **How can we price our products?**
  - Expert judgment only!

- **It turns out that expert judgment –and some additional assumptions- implicitly determines a demand model !**

A Theoretical Comparison
A Demand Model

- A theoretical result
  - Assume that for a given segment of customers, an expert gives us the expected loss, the optimal (1yr) price and the corresponding propensity to buy.
  - Assume that the demand follows a logistic model and the best price is obtained by ordinary PO.

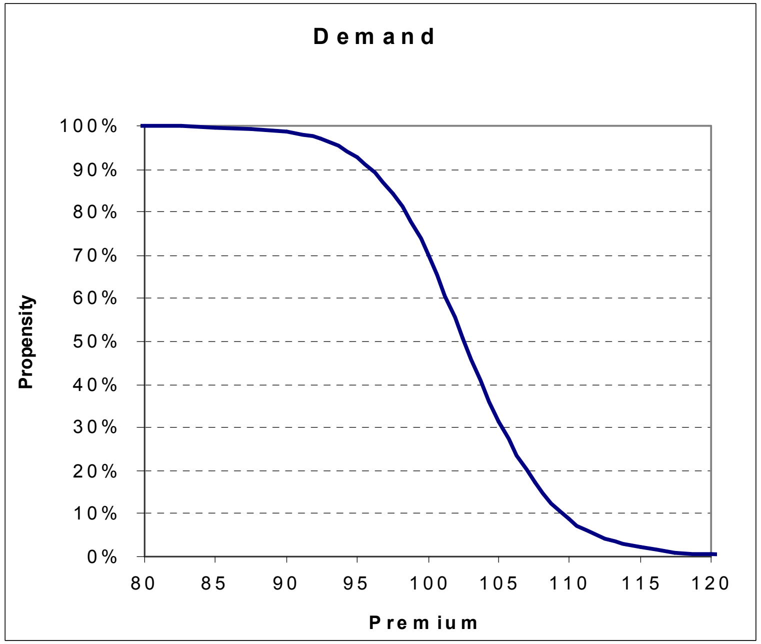
  - Then, there exists only one **regular** demand function that corresponds to the given benchmarks.

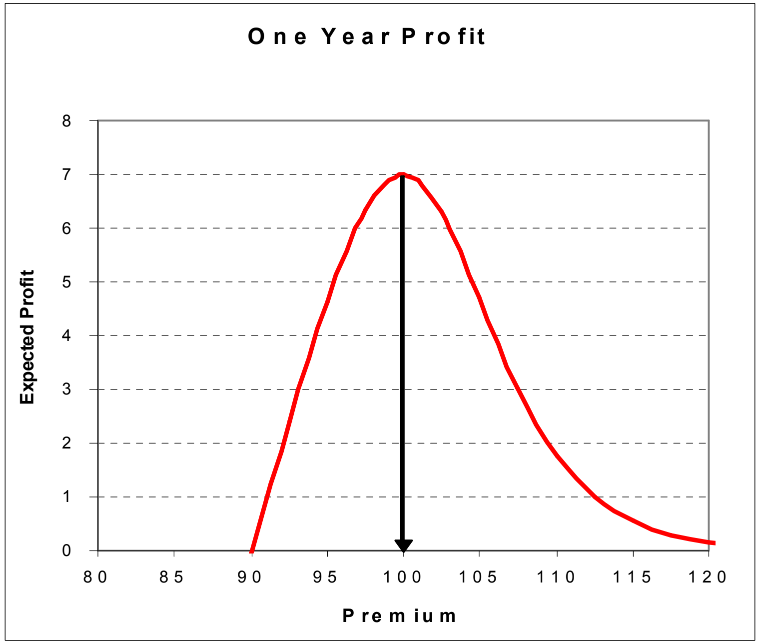
- This means that given these 3 numbers, we can draw the entire demand function!

A Theoretical Comparison
A Demand Model

- For example:

Benchmarks		
Optimized Premium	Propensity	Loss Ratio
100	70%	90%





A Theoretical Comparison
LTCV simplified

- Multi-year optimisation: mimics executives’ business planning by optimising a moving window of N years into the foreseeable future
- This results in a set of optimal prices for the next N years that consider the long term implications of current pricing on future earnings
- The net present value (NPV) of earnings is used as a profit indicator

	Year 0	Year 1	Year 2	…	Year N
Life time	Origination	Renewal 1	Renewal 2	…	Renewal N

LTCV (NPV) = $D_0 \cdot M_0 + D_0 \cdot D_1 \cdot M_1/(1+i) + D_0 \cdot D_1 \cdot D_2 \cdot M_2/(1+i)^2 \ldots$

D_n = Demand in year n, M_n = Margin in year n, i = Interest rate

A Theoretical Comparison
LTCV simplified

		Year 0	Year 1	Year 2	…	Year N
Life time		Origination	Renewal 1	Renewal 2	…	Renewal N
LTCV	=	$D_0 \cdot M_0$	$+ \; D_0 \cdot D_1 \cdot M_1/(1+i)$	$+ \; D_0 \cdot D_1 \cdot D_2 \cdot M_2/(1+i)^2$	…	
LTCV*	=	$D_0 \cdot M_0$	$+ \; D_0 \cdot D_0 \cdot M_0$	$+ \; D_0 \cdot D_0 \cdot D_0 \cdot M_0$	…	

Assumptions

- Demand model remains the same.
- Price inflation and claim inflation balance out with Profit inflation.

A Theoretical Comparison
Long term PO vs short term PO

- We have a model for a single segment of customers that is easy to calculate and therefore handy for comparisons.

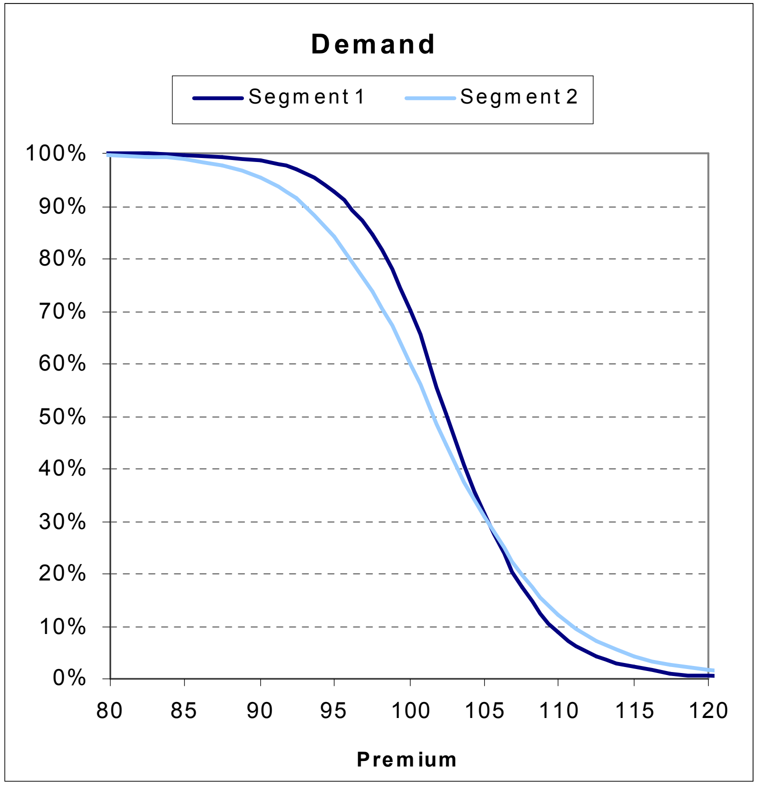
- Lets compare the previous example with another segment having the same optimal price, the same LR but a different demand curve.

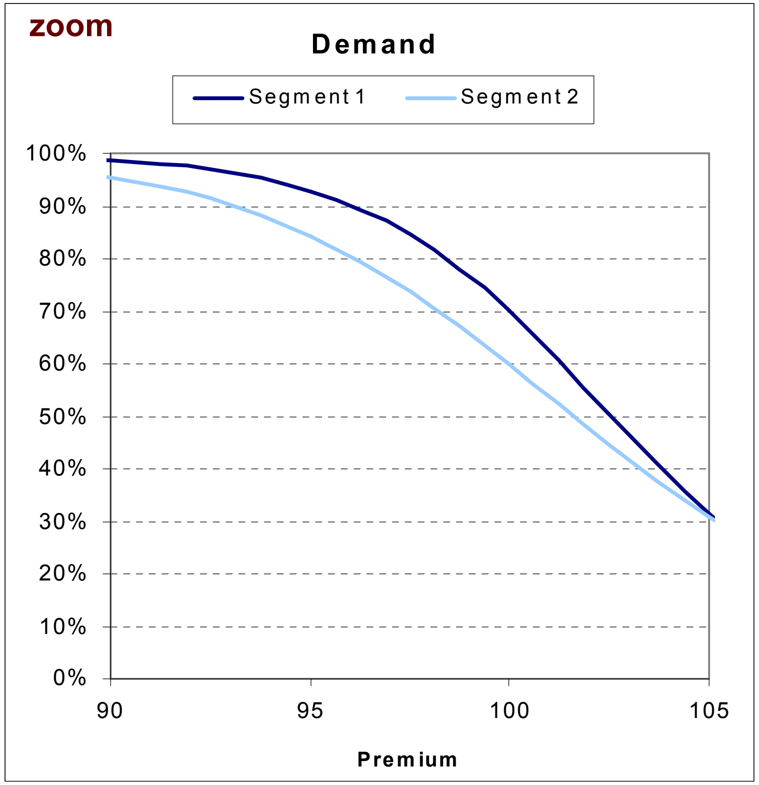
- Example 1:

Benchmarks		
Optimized Premium	Propensity	Loss Ratio
100	70%	90%
100	60%	90%

A Theoretical Comparison
Long term PO vs short term PO

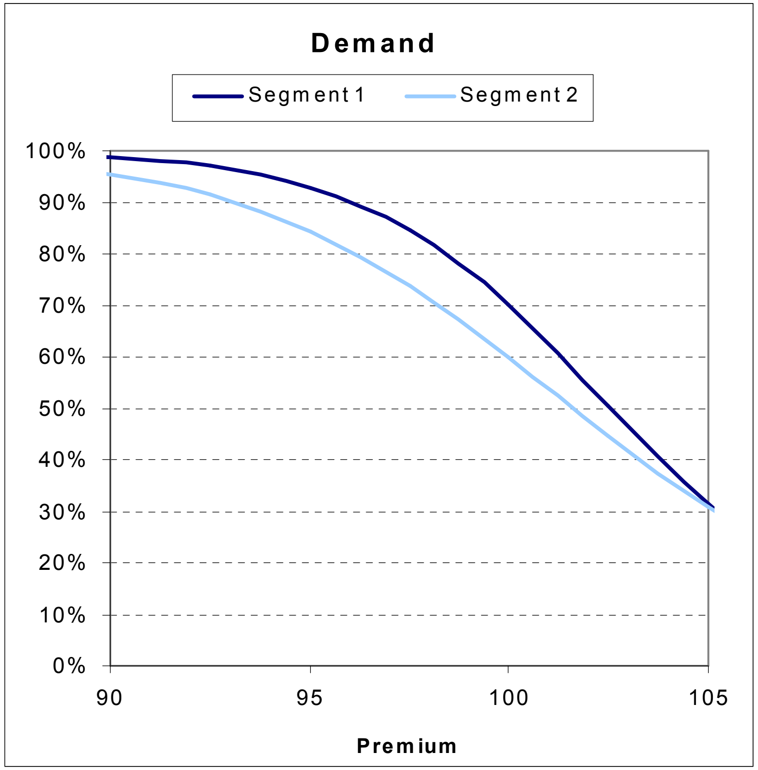
- (… Ex.1) Comparing two segments of customers having identical optimal price and expected loss, but different demand.

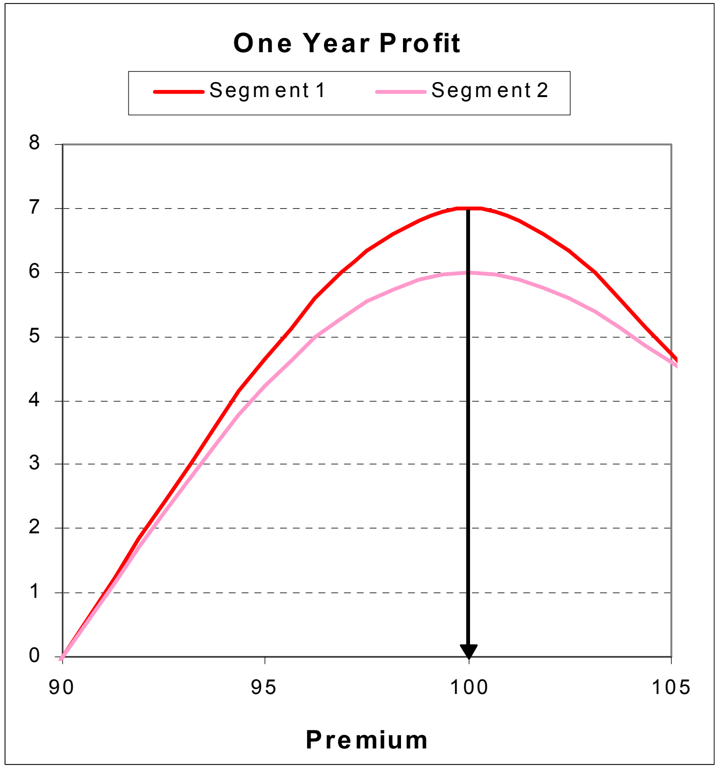




A Theoretical Comparison
Long term PO vs short term PO

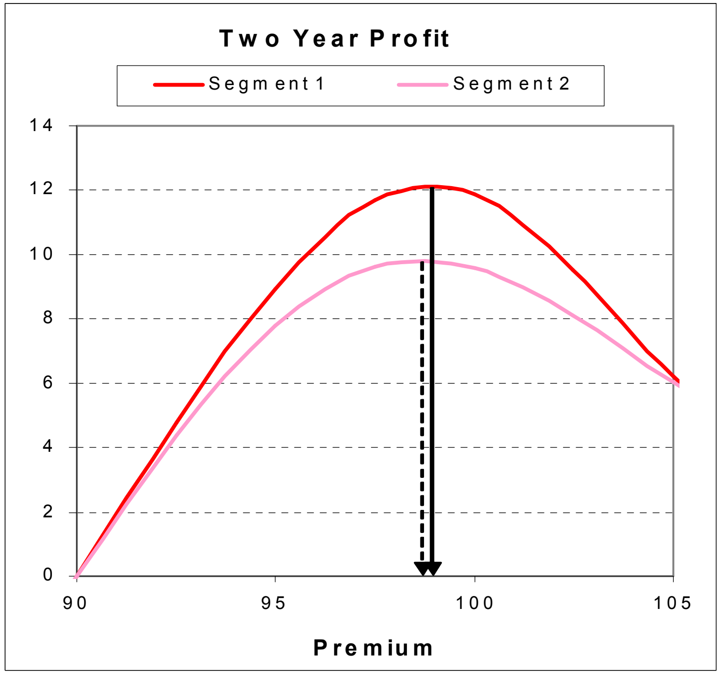
- (… Ex.1) Note: consumer 2 is more elastic (locally)

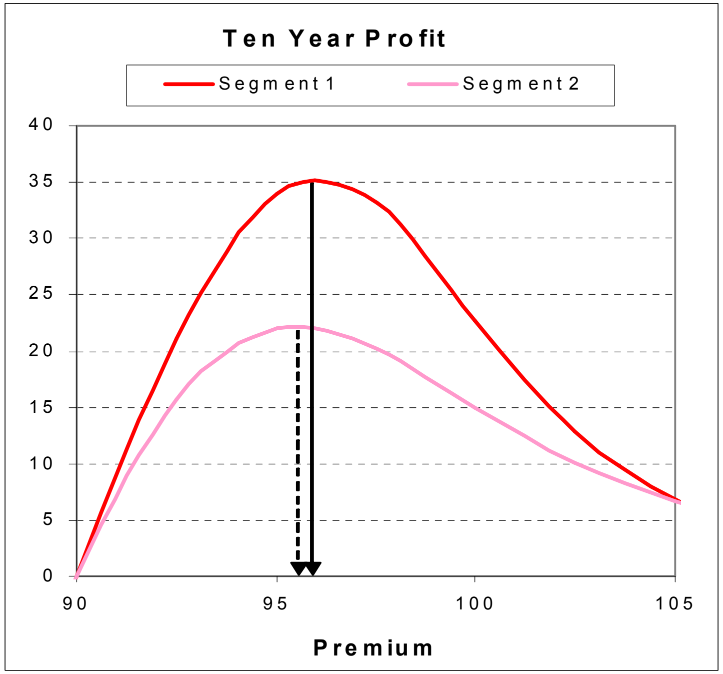




A Theoretical Comparison
Long term PO vs short term PO

- (… Ex.1) Lifetime Optimal Price
  - decreases when LR < 100% (obvious)
  - decreases more for more (locally) elastic consumers
  - … but the difference is small in this example.

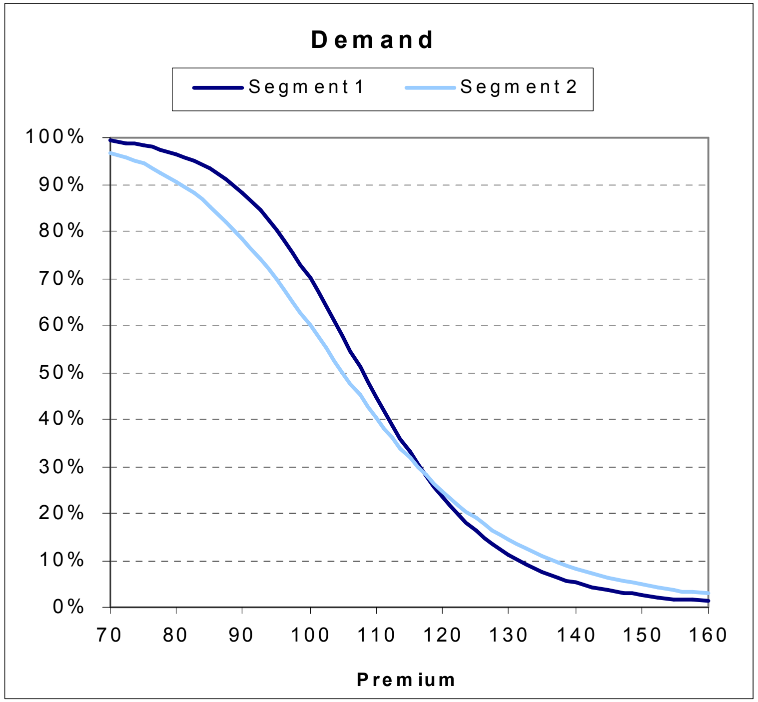


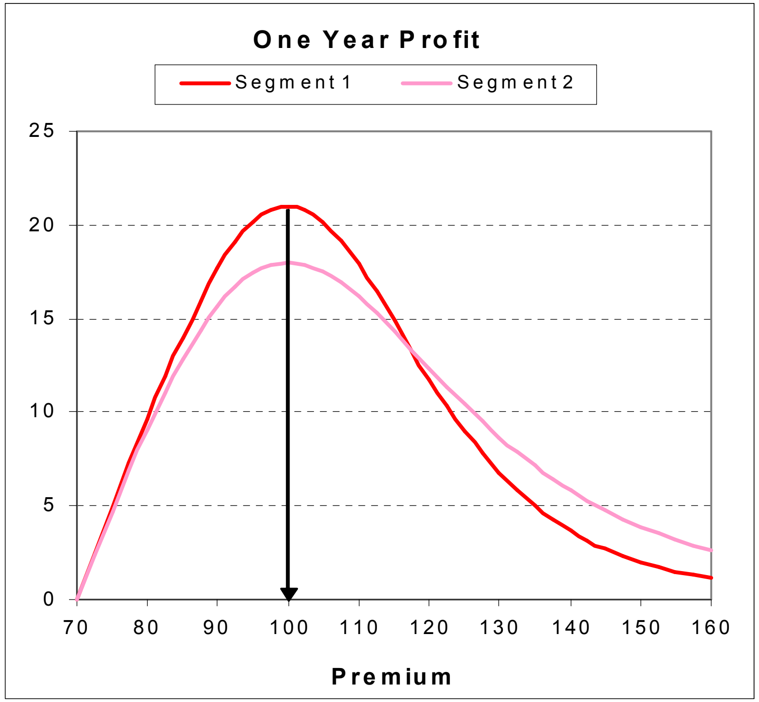


A Theoretical Comparison
Long term PO vs short term PO

- Example 2. Lower LR

Benchmarks		
Optimized Premium	Propensity	Loss Ratio
100	70%	70%
100	60%	70%





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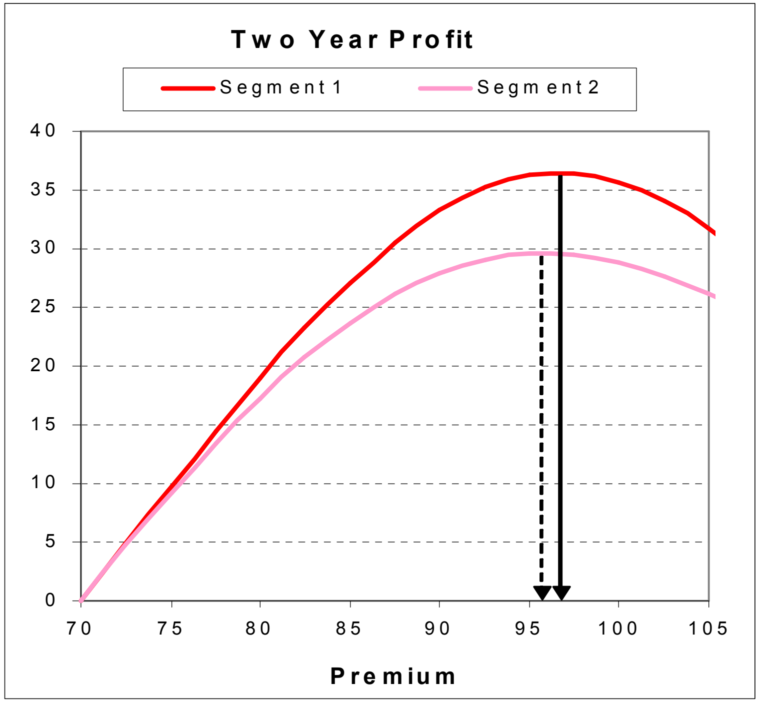
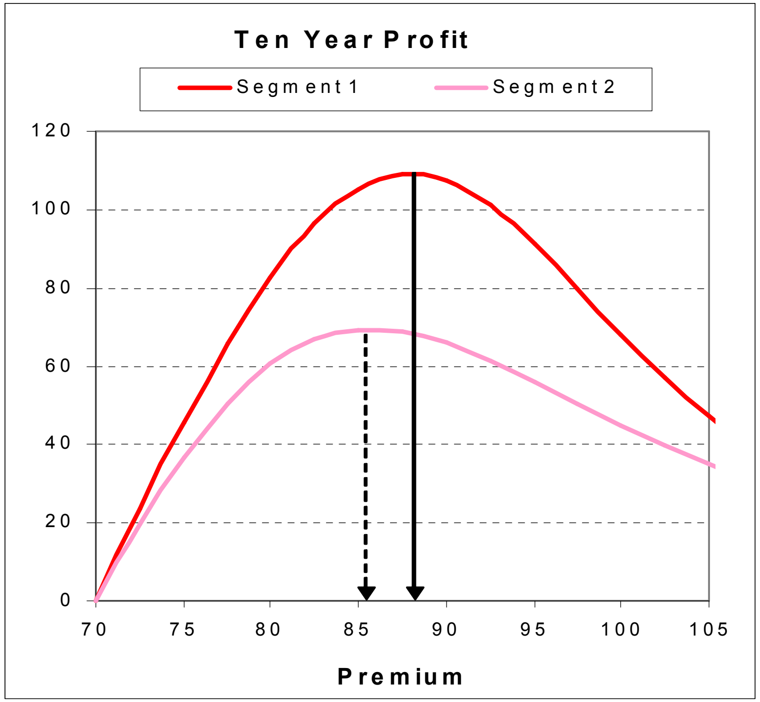
A Theoretical Comparison
Long term PO vs short term PO

- (… Ex.2) Bigger profit margins
  - => lifetime discounts are more relevant.
  - => segments more differentiated.





A Theoretical Comparison

Long term PO vs short term PO

- Summary on four examples

increase in profit

increase in demand

1

Benchmarks		
Optimized Premium	Propensity	Loss Ratio
100	70%	90%
100	60%	90%

2

Benchmarks		
Optimized Premium	Propensity	Loss Ratio
100	70%	70%
100	60%	70%

3

Benchmarks		
Optimized Premium	Propensity	Loss Ratio
100	90%	90%
100	80%	90%

4

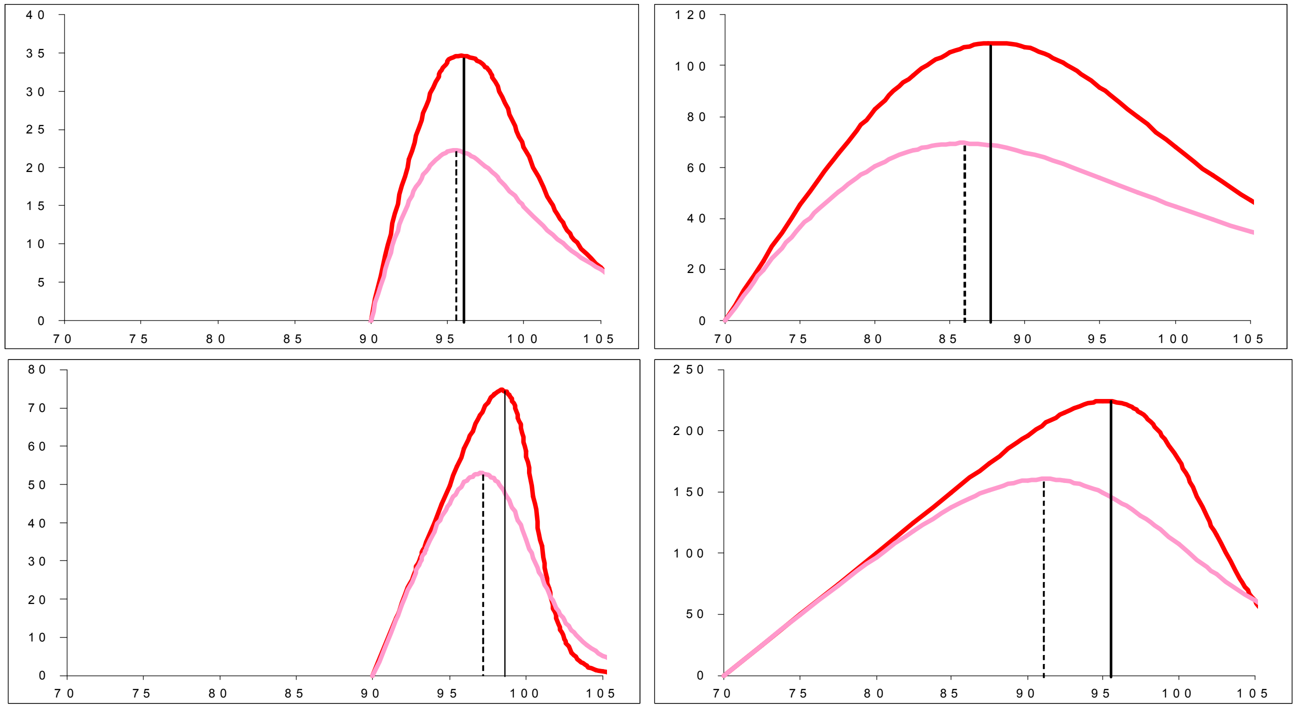
Benchmarks		
Optimized Premium	Propensity	Loss Ratio
100	90%	70%
100	80%	70%

A Theoretical Comparison
Long term PO vs short term PO

- more profit → more discount & more segmentation
- more demand → less discount & more segmentation

increase in profit

increase in demand



A Theoretical Comparison
Summary

- A theoretical model for demand has been used to compare optimal prices on 1yr vs lifetime horizons.
- Lifetime pricing recommends for relatively small discounts in highly competitive and high risk markets, with very little differences between customers.
- For increasing demand, discounts are even smaller but customers are more differentiated.
- Finally, on more profitable segments both discounts amount and customers differentiation increase.

A Theoretical Comparison
Summary

- **Summary**
  - Results resemble the activity that underwriters do in practical deals.
    - Assessing customer propensity
    - Assessing expected loss
    - Fixing a "standard" price on a 1yr profit basis
    - Assessing lifetime customer value
    - Determining the proposition
  - Lifetime pricing may be useful to define standard guidelines for discount policies on a quantitative basis.

Practical Optimisation
Main Elements

	Year 0	Year 1	Year 2	…	Year N
Life time (policy)	Origination	Renewal 1	Renewal 2	…	Renewal N
Profit (NPV) =	$D_0 \cdot M_0$ +	$D_0 \cdot D_1 \cdot M_1/(1+i)$ +	$D_0 \cdot D_1 \cdot D_2 \cdot M_2/(1+i)^2$		…
Local constraints:	Range(P_0)	Range(P_1)	Range(P_2)		
Global constraints:	$\frac{\sum_{p=1}^{\#Offers} D_0(P_0)}{\#Offers}$	$\frac{\sum_{p=1}^{\#Policies_0} D_0 \cdot D_1(P_1)}{\#Policies_0}$	(e.g. Renewal Rates)		
Trends:	Claims$_0$	Claims$_1$	Claims$_2$		…
Trends:	Market$_0$	Market$_1$	Market$_2$ (e.g. Competitor rates)		…
Trends:	Demand$_0$	Demand$_1$	Demand$_2$ (change in elasticity!)		…

Huge extra complexity that can come in:
- Prices might follow a certain rating structure instead of being individual for each client!

P_n = Premium in year n, D_n = Demand in year n, M_n = Margin in year n, i = Interest rate

Practical Optimisation
Main Elements

- Main elements of <u>Long-Term</u> optimisation:

	Year 0	Year 1	Year 2	…	Year N
Life time (policy)	Origination	Renewal 1	Renewal 2	…	Renewal N
Profit (NPV) =	$D_0 \cdot M_0$ +	$D_0 \cdot D_1 \cdot M_1/(1+i)$ +	$D_0 \cdot D_1 \cdot D_2 \cdot M_2/(1+i)^2$		…
Local constraints:	$\text{Range}(P_0)$	$\text{Range}(P_1)$	$\text{Range}(P_2)$		…
Global constraints:	Less relevant the <u>longer</u> the horizon N				…
Trends:	Claims_0	Claims_1	Claims_2		
Trends:	Market_0	Market_1	Market_2 (e.g. Competitor rates)		…
Trends:	Demand_0	Demand_1	Demand_2 (change in elasticity!)		…

Trends are very important, but difficult to set right!

Practical Optimisation
Main Elements

- Main elements of <u>Short-Term</u> optimisation:

	Year 0	Year 1	Year 2	…	Year N
Life time (policy)	Origination	Renewal 1	Renewal 2	…	Renewal N
Profit (NPV) =	$D_0 \cdot M_0$ +	$D_0 \cdot D_1 \cdot M_1/(1+i)$ +	$D_0 \cdot D_1 \cdot D_2 \cdot M_2/(1+i)^2$		…
Local constraints:	Range(P_0)	Range(P_1)	Range(P_2)		
Global constraints:	$\frac{\sum_{p=1}^{\#Offers} D_0(P_0)}{\#Offers}$	$\frac{\sum_{p=1}^{\#Policies_0} D_0 \cdot D_1(P_1)}{\#Policies_0}$	(e.g. Renewal Rates)		…
Trends:	$Claims_0$	$Claims_1$	$Claims_2$		…
Trends :	**Less relevant the <u>shorter</u> the horizon N**		(e.g. Competitor rates)		…
Trends:	$Demand_0$	$Demand_1$	$Demand_2$ (change in elasticity!)		…

Global constraints very important to reflect future value of customers

Practical Optimisation
LTCV vs Short Term

LTCV Optimisation	Short-Term Optimisation

• **Pros**
• Natural, easy to explain
• Theoretically best approach
• Long term business projections as a by product
• Fair!?
• **Cons**
• How to set trends?
  • Individual: claims, additional policies, cover changes, personal changes
  • Global: market, finance, demand
• Still only first year prices are applicable
• More risky to base prices on uncertain future
• Heavy computations
	• **Pros**
	• Results based on most recent information of book (reliability)
	• Flexibility in the future (market adjustments, changes in demand, policy development, price politics)
	• Less computation, results come faster
	• Can run purely deterministically
	• **Cons**
	• “Exit” constraints have to be imposed to value future profits (e.g. volume of book)
	• How to set constraints best?
	• General criticism of missing LTCV view

Practical Optimisation
LTCV vs Short Term

- LTCV optimisation is only “hypothetical” pricing of the future
- It cannot be applied to a real book (“Uncertainty of Claim”) after the first year!
- In fact we know of no P&C insurer that determines prices over a Life-Time horizon using LTCV optimisation!
- “Short-Term” Optimisation with “exit” constraints is the “de facto” standard of the P&C market
- Is LTCV optimisation of no real use?

- Answer: It is of use!
  - Predominantly to calculate some “LTCV”
  - Also maybe there are some key results that can be used for improving Short-Term Optimisation!

Practical Optimisation
Examples: One customer only

- Comparing LTCV & One-Year optimisation for only one customer (or segment):

	Year 1	Year 2	Year 3	…	Year N
LTCV optimisation	Renewal 1	Renewal 2	Renewal 3	…	Renewal N

Profit (NPV) = $D_1 \cdot M_1 + D_1 \cdot D_2 \cdot M_2/(1+i) + D_1 \cdot D_2 \cdot D_3 \cdot M_3/(1+i)^2$ …

No global constraint

	Year 1
1Y optimisation	Renewal 1

Profit (NPV) = $D_1 \cdot M_1$

Global constraint: $D_1(P_1)=RR_1(LTCV)$ **Renewal rate of year 1 taken from LTCV optimisation!**

Practical Optimisation
Examples: One customer only

- Let us make the following mild assumption:
  - Demand curve is monotonic!

- Then the optimised price P_1 of the first year is identical for both optimisations (LTCV and 1Y with Volume exit constraint= RR_1(LTCV))!

- Proof: There is only one price P_1 that satisfies $D(P_1)=RR_1(LTCV)$

- Conclusion(*): It is sufficient to know the volume of an LTCV optimisation after the 1st year to get the same pricing results with just 1Y optimisation, if the book of business is relatively homogenous (behaving like one customer)!

Some more little assumptions: Each customer is projected identically into the future (like with deterministic projections)

Practical Optimisation
A Real Book

General Settings & Assumptions (Optimisation)

Pricing example: Real Motor Renewal Book (Europe)

- **Settings and Assumptions of Price Optimisation – General**
  - **Type of optimisation:** Prices have been optimised individually
  - **Interest rate:** High & low interest rate (0% & 10%) on future profits assumed
  - **Local constraints:** Premiums can vary from one to the next year (relative to previous premium) by
    - No claims in previous year: Max[-20%,-150] to Min[+16%,+60]
    - With claims in previous year: 0% to Max[+80%,+480]
  - **Future trends:** Age, tenure + year of projection
    claims, risk, demand model & market situation remain unchanged
  - **Projection:** Deterministic (functional development)

LTCV

- Optimisation horizon: 1Y-5Y & 8Y
- No "exit" or inter-year constraints

Short-Term (1Y)

- Optimisation horizon: 1Y
- "Exit" constraint: Renewal Rate (1Y) of LTCV optimisation

Practical Optimisation
A Real Book

Results with <u>0%</u> interest rate: Optimal <u>volumes</u> <u>higher</u> than for 10% interest rate

In general the optimal volume is higher with

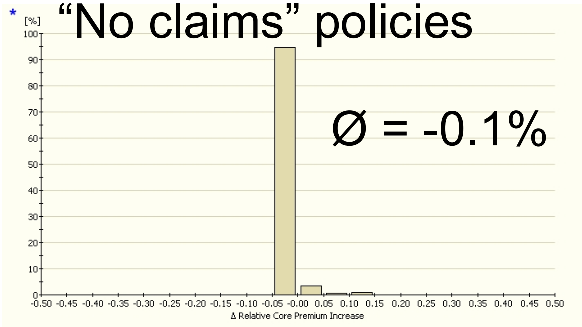
- Increasing future profits (improving claims ratios)
- Decreasing interest rates
- Increasing horizon
- Higher elasticity

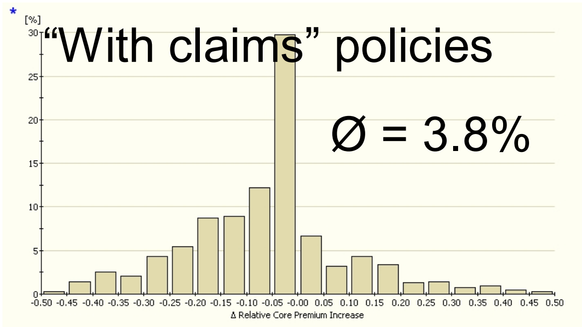
Efficient Frontier

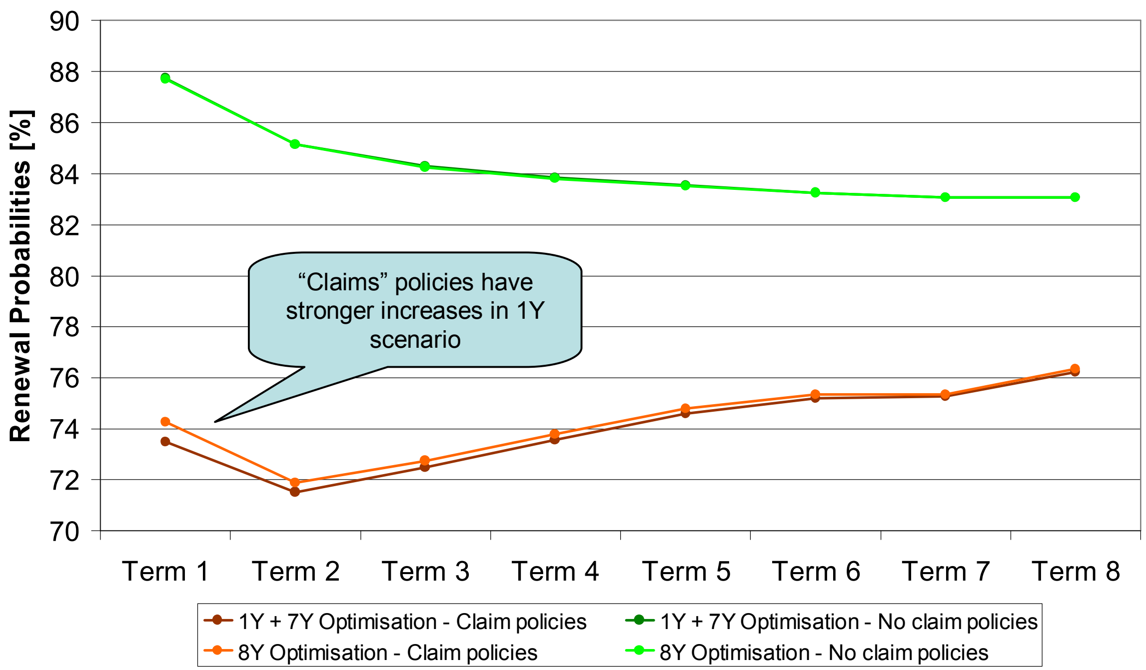
Practical Optimisation
A Real Book

- What are the differences after the first year (price differences, mix of business & profit)?

Results: Difference between 1Y with optimal volume and 8Y Optimisation







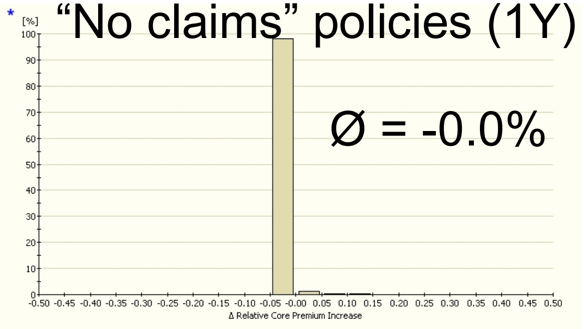
Differences on the graphs shown as: [1Y with constraint] – [8Y]

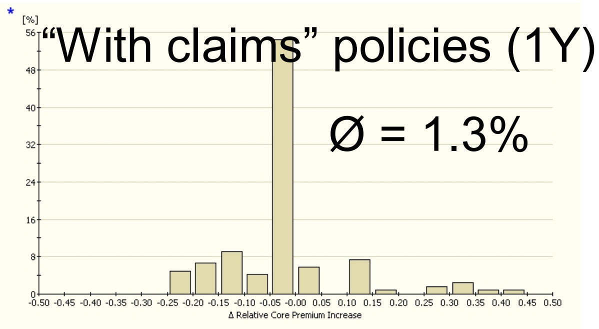
Practical Optimisation

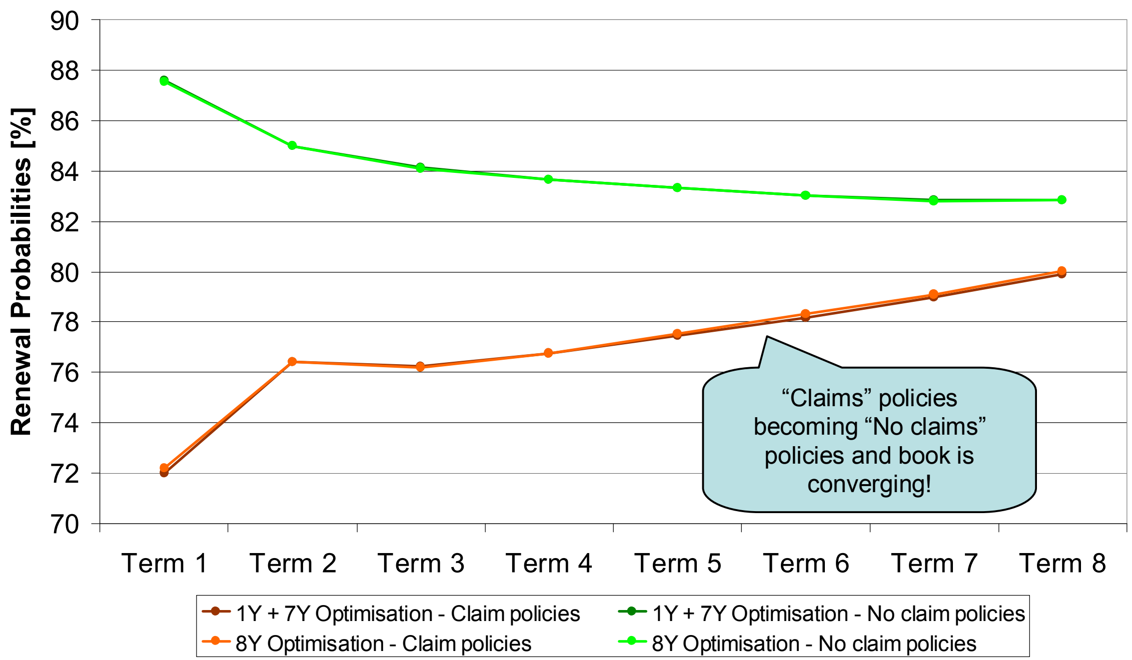
A Real Book

- Results here: After 1Y all policies are assumed to have no claims!
  - Book more homogeneous and thus results of 1Y & 8Y closer together

Results: Difference between 1Y with optimal volume and 8Y Optimisation





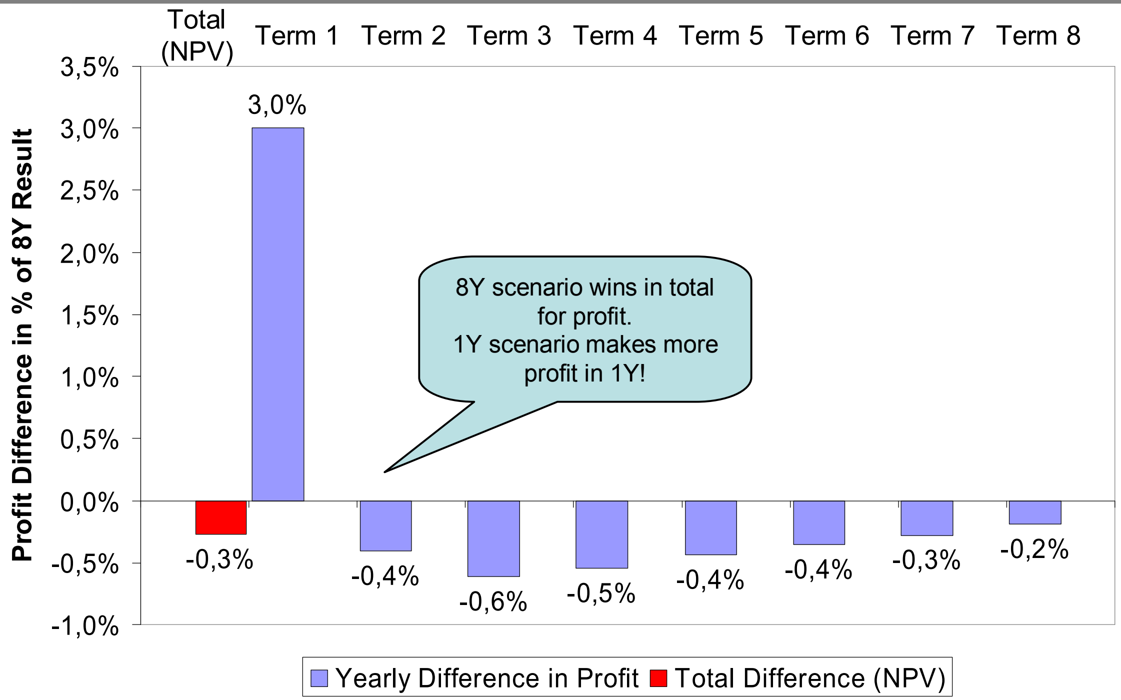


Differences on the graphs shown as: [1Y with constraint] – [8Y]

Practical Optimisation
A Real Book

- What are the differences after the first year (price differences, mix of business & profit)?

Results: Difference between 1Y with optimal volume and 8Y Optimisation

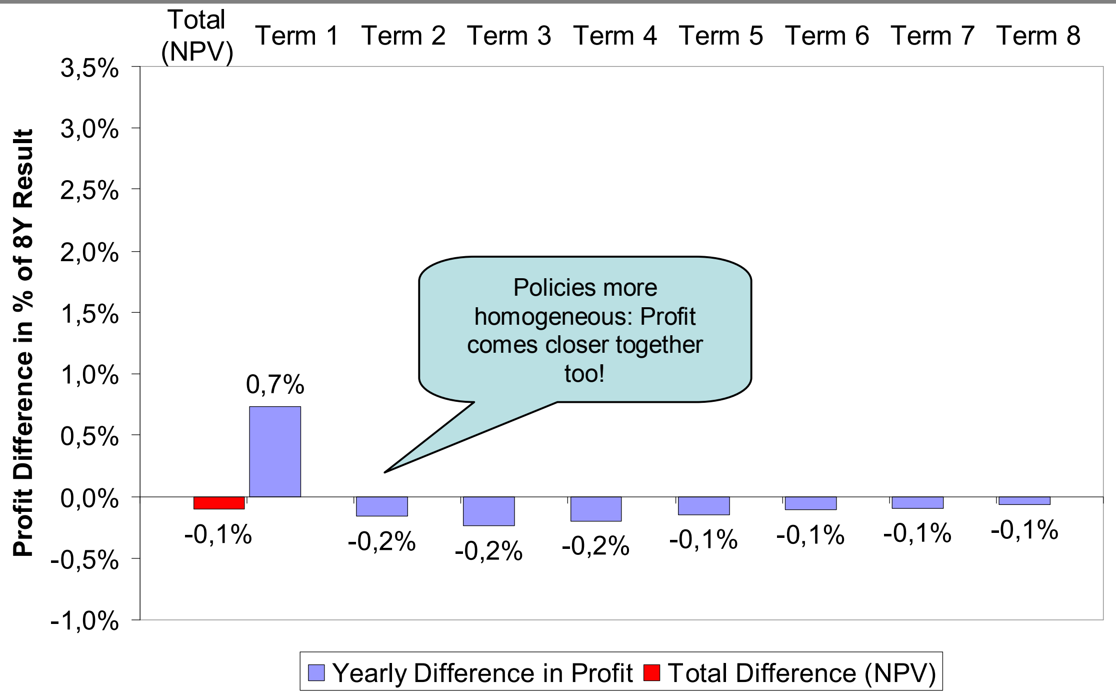


Differences on the graphs shown as: [1Y with constraint] – [8Y]

Practical Optimisation
A Real Book

- Results here: After 1Y all policies are assumed to have no claims!
  - Book more homogeneous and thus results of 1Y & 8Y closer together

Results: Difference between 1Y with optimal volume and 8Y Optimisation



Differences on the graphs shown as: [1Y with constraint] – [8Y]

Practical Optimisation
A Practical Approach to LTCV

Suggestion for LTCV optimisation from the previous conclusions:

- **Recipe**
  - Take a small sample of the book
  - Calculate LTCV optimisation on sample (simple trends, assumptions)
  - Take from it the first year Renewal Rate RR(1)
  - Plug this rate into 1Y optimisation as a global “exit” constraint
  - Calculate the total book with this 1Y optimisation

Conclusions

- LTCV & 1Y optimisation with “exit” constraint are not that different, if
  - The exit constraint is the first year renewal rate of LTCV
  - The book is sufficiently homogeneous
- Volume of book is more robust than a more uncertain individual LTCV value
- Clever optimisation is the goal, not optimisation for the sake of optimisation!
- What we have not covered here:
  - How reliable is a long term projection in real life?
  - Is the additional effort of setting LTCV up worth the trouble?
  - Prices, politics and market change year by year (even week by week)!
  - Weather forecasts are now reliable for 3 days!

Questions