First a bit on CAMRADATA

CAMRADATA:
600 Managers
4,023 products
85 Consultants
Over 300 Pension Scheme Trustees
Over 5,000 online reports run since June 2009
Manager League Tables
Fund Flows Reports
Preliminaries

• Starting points

Transformations

Clockwise rotation as a matrix

\[
\begin{bmatrix}
  x' \\
y'
\end{bmatrix} =
\begin{bmatrix}
  \cos(\theta) & \sin(\theta) \\
  -\sin(\theta) & \cos(\theta)
\end{bmatrix}
\begin{bmatrix}
  x \\
y
\end{bmatrix}
\]

Scaling as a matrix

\[
\begin{bmatrix}
  x' \\
y'
\end{bmatrix} =
\begin{bmatrix}
  A & 0 \\
  0 & B
\end{bmatrix}
\begin{bmatrix}
  x \\
y
\end{bmatrix}
\]

Clockwise rotation and scale as a matrix

\[
\begin{bmatrix}
  x' \\
y'
\end{bmatrix} =
\begin{bmatrix}
  A & 0 \\
  0 & B
\end{bmatrix}
\begin{bmatrix}
  \cos(\theta) & \sin(\theta) \\
  -\sin(\theta) & \cos(\theta)
\end{bmatrix}
\begin{bmatrix}
  x \\
y
\end{bmatrix}
\]
Actuarial Maths

Uncorrelated random number to correlated numbers

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \sqrt{\text{covariance matrix}} \times \begin{bmatrix} x \\ y \end{bmatrix}
\]

Transforming uncorrelated random number to correlated numbers

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix} L_1 & 0 \\ L_2 & L_3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}
\]

Our maths is in fact a transformation

Transforming uncorrelated random number to correlated numbers

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix} \sqrt{\text{EigenValue}_1} & 0 \\ 0 & \sqrt{\text{EigenValue}_2} \end{bmatrix} \text{inv}[\text{EigenVectors}] \begin{bmatrix} x \\ y \end{bmatrix}
\]

Covariance eigen vectors rotate and covariance eigen values scale
So covariance is a geometric transformation!
Simple example

0.5 in Y

Distance = Risk Between X and Y in uncorrelated space

\[ Distance = \sqrt{(0.5^2 + 0.5^2)} = 0.7071 \]

0.5 in X

Correl = 1.0, Length = 0.000
Correl = 0.5, Length = 0.500
Correl = 0.0, Length = 0.707
Correl = -0.5, Length = 0.666
Correl = -1.0, Length = 1.000

What is the influence of Pythagorean cults?

"Establish the triangle and the problem is two-thirds solved": Pythagoras
Can we think in triangles?

Normal view
Notion of distance

(Average (square(Relative Returns Data - Mean))

Link to dimensionality

How many assets do I really hold?

How many dimensions are there?

$$[V, D] = \text{eig}(\text{corrcoef}(\text{mxRet}))$$

$$n\text{Dimensions} = \text{sum}(\text{min}(D(:, 1)))$$
Example Uncorrelated Assets?

\[ [V,D] = \text{eig}([\text{eye}(3)]) \]

\[ \text{sum}(-\text{min}(D(:,1))) = 3 \]

\[ [V,D] = \text{eig}([\text{ones}(3)]) \]

\[ \text{sum}(-\text{min}(D(:,1))) = 1 \]

Equation for correlation

\[
\begin{align*}
\text{cov}(X, Y) &= \frac{1}{n} \sum (x_i - \bar{x})(y_i - \bar{y}) \\
\rho_{(x,y)} &= \frac{\sigma_{(x-y)}^2 - \sigma_x^2 - \sigma_y^2}{-2\sigma_x\sigma_y}
\end{align*}
\]
Cosine Rule

\[ \cos(\theta) = \frac{(C - B)^2 - (C - A)^2 - (B - A)^2}{-2(C - A)(B - A)} \]

\[ \rho_{(C-A,B-A)} = \frac{\sigma^2_{(C-B)} - \sigma^2_{(C-A)} - \sigma^2_{(B-A)}}{-2\sigma_{(C-A)}\sigma_{(B-A)}} \]

270° in a triangle?

\[ \sum (\theta_i) = 180^\circ \]

\[ \theta_{(x,y)} = \cos^{-1}(\rho_{(x,y)}) \]

Uncorrelated assets have these angles:

<table>
<thead>
<tr>
<th></th>
<th>Asset 1</th>
<th>Asset 2</th>
<th>Asset 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets 1</td>
<td>90</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>Assets 2</td>
<td></td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>Assets 3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \sum (\theta_i) = 270^\circ \]
Not a flat triangle, a sphere

Uncorrelated
A bit of correlation

More....
A lot of correlation

0 degrees, dimensions = 1

Real world visualisation I

Actual Number of Assets, ndimensions = 2.4
95 degrees FTALLSh(R)
95 degrees MUHTUK(R)
57 degrees BMUL15V
Real world visualisation II

Risk is radius

Only positive?
Radius = risk
But this is an average triangle

\[ \frac{1}{\text{sum(logic criteria)}} \sum (x_{\text{logic criteria}} - \bar{x})(y_{\text{logic criteria}} - \bar{y}) \]

We can disaggregate triangle

Splitting out covariance:
Equity Market Up (Green), N dima = 2.4
Equity Market Down (Red), N dima = 2.6
Quartiles of equity movements

- Equity Market Q1 (Green), N dims = 2.3
- Equity Market Q2 (Green), N dims = 2.3
- Equity Market Q3 (Red), N dims = 2.7
- Equity Market Q4 (Red), N dims = 2.4

Extremes of equity movement

- Equity Market Top 5 profile, N dims = 2.2
- Equity Market Bottom 5 profile, N dims = 2.1
Joint top bottom quartiles

Jointly up/down Equity, High Yield, Government Fixed

Economic cycle?

Cyclical indicators? Normalised indicators

US FEDERAL FUNDS RATE (AVG)
US UNEMPLOYMENT RATE (SAJ)

Jan58 Jan63 Jan68 Jan73 Jan78 Jan83 Jan88 Jan93 Jan98 Jan03 Jan08
Establish the economic wave

Smoothed federal funds rate-unemployment

Plot as an economic cycle

"The motion of God is circular": Pythagoras
Can we disaggregate covariance by point on clock

Financial Space Time

Economic cycle ... Data is 97 to present

.....moving round the circle

Green = equity up; Red = equity down
97 to June 2011 using this measure

This is a visualisation of financial space-time

- Round Balls = less correlation
- Rugby Balls = more correlation
- Big Balls = more risk